# PDF dependence of benchmark cross sections in p-p collisions at the 7 TeV LHC

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CERN PH-TH

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# Fixed-order collinear factorisation at hadron colliders

- The "standard" pQCD framework: holds up to formally power-suppressed ("higher-twist") terms $\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/Q^2)$.
- Expand $\hat{\sigma}_{ab}$, $P_{aa'}$ and $\beta$ as perturbative series in $\alpha_S$ ($\mu_R = \mu_F = Q$).

$$\sigma_{AB} = \sum_{a,b=q,g} \left[\hat{\sigma}^{\mathrm{LO}}_{ab} + \alpha_S(Q^2)\hat{\sigma}^{\mathrm{NLO}}_{ab} + \ldots\right] \otimes f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2)$$

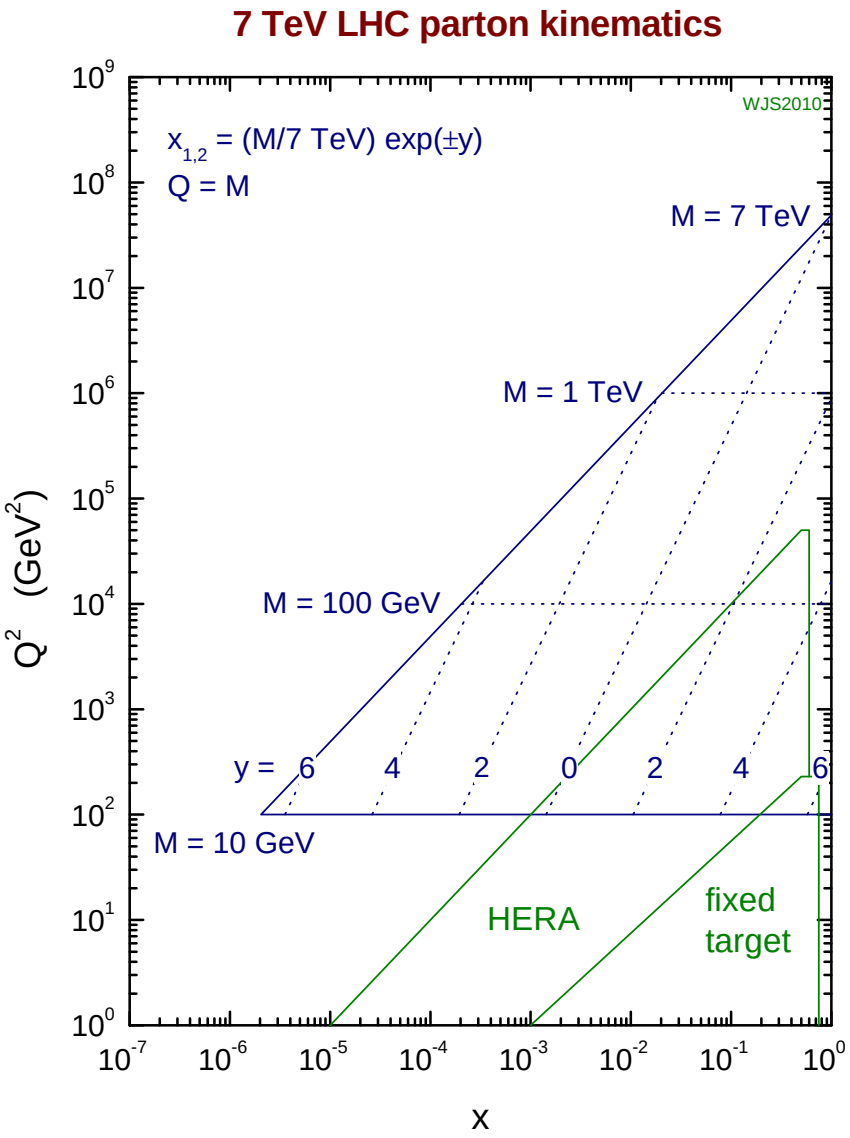
PDF evolution:
$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} \left[P^{\mathrm{LO}}_{aa'} + \alpha_S P^{\mathrm{NLO}}_{aa'} + \ldots\right] \otimes f_{a'/A}$$

$\alpha_S$ evolution:
$$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\mathrm{LO}}\alpha_S^2 - \beta^{\mathrm{NLO}}\alpha_S^3 - \ldots$$

- Need to extract input values $f_{a/A}(x, Q_0^2)$ and $\alpha_S(M_Z^2)$ from data.
- Structure functions in deep-inelastic scattering (DIS):

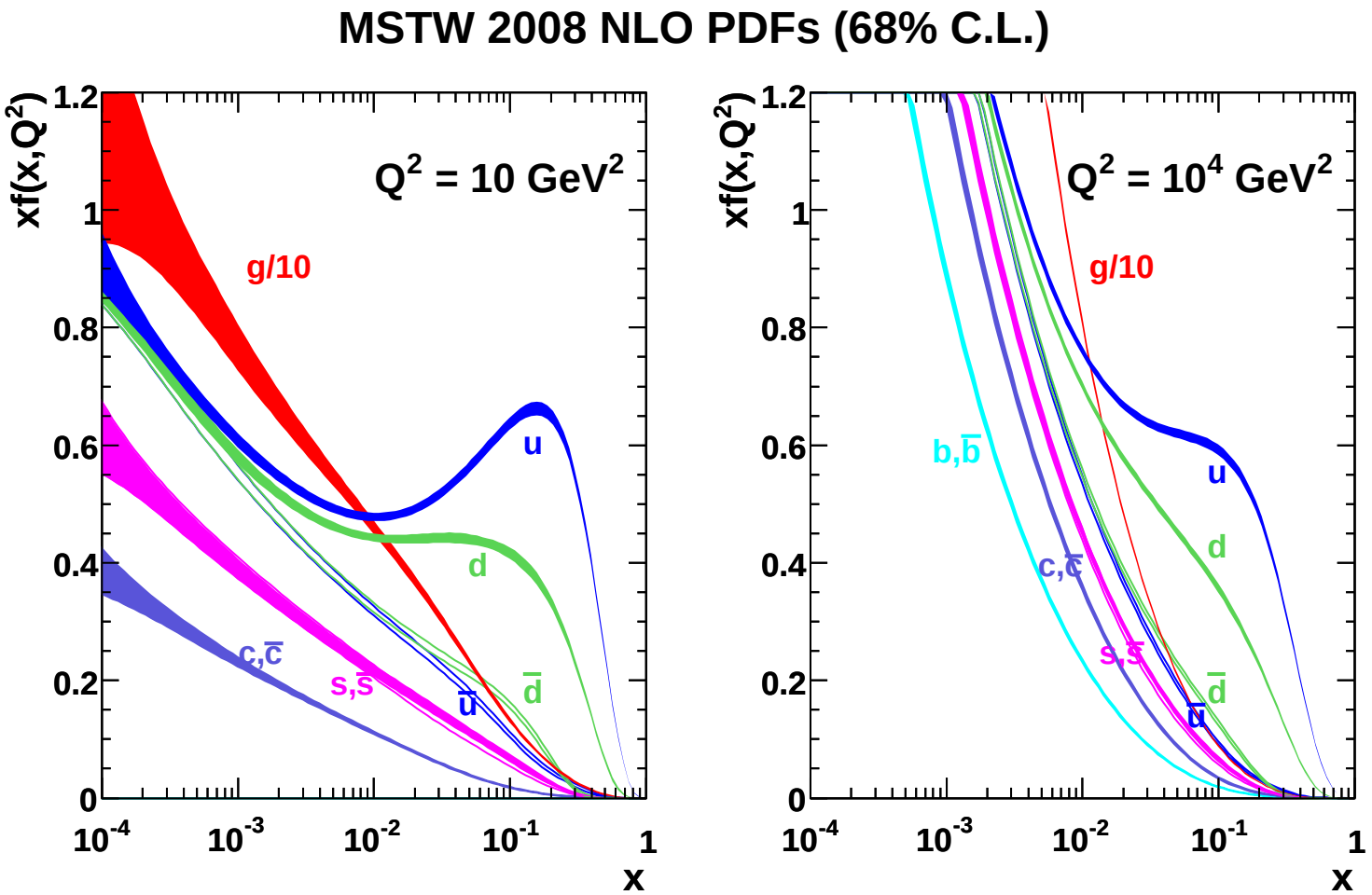
$$F_i\left(x_{\mathrm{Bj}}, Q^2\right) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}, \quad C_{i,a} = C^{\mathrm{LO}}_{i,a} + \alpha_S C^{\mathrm{NLO}}_{i,a} + \ldots$$

# From HERA *et al.* to the LHC




- PDFs are **universal**.
- Fit existing data from HERA and fixed-target experiments, together with Tevatron data.
- HERA $ep$ (H1, ZEUS).
- Fixed-target experiments: $\ell p$, $\ell d$ (BCDMS, NMC, E665, SLAC), $\nu N$ (CCFR, NuTeV, CHORUS), $pp$, $pd$ (E866/NuSea).
- Tevatron $p\bar{p}$ (CDF, DØ).
- DGLAP evolution gives PDFs at higher $Q^2$ for LHC.

# Example of PDFs obtained from global analysis




- **Error bands** shown are obtained from propagation of experimental uncertainties on the fitted data points.

# MSTW 2008 PDFs [http://projects.hepforge.org/mstwpdf/]

A. D. **M**artin, W. J. **S**tirling, R. S. **T**horne, G. **W**att

- "*Parton distributions for the LHC*"
  [Eur. Phys. J. C **63** (2009) 189, arXiv:0901.0002]
- "*Uncertainties on $\alpha_S$ in global PDF analyses and implications for predicted hadronic cross sections*"
  [Eur. Phys. J. C **64** (2009) 653, arXiv:0905.3531]
- "*Heavy-quark mass dependence in global PDF analyses and 3- and 4-flavour parton distributions*"
  [Eur. Phys. J. C **70** (2010) 51, arXiv:1007.2624]
- "*The effects of combined HERA and recent Tevatron $W \to \ell\nu$ charge asymmetry data on the MSTW PDFs*"
  [DIS 2010 proceedings, arXiv:1006.2753]

Other PDF fitting groups: CTEQ/CT ($\to$ M. Guzzi), NNPDF, HERAPDF ($\to$ A. Cooper-Sarkar), ABKM, GJR.

**Rest of this talk: update to PDF4LHC benchmark exercise.**

# Background and motivation for benchmark exercise

- January 2010: LHC Higgs Working Group formed and requested recommendation from **PDF4LHC Working Group** on PDFs and $\alpha_S$ values (and their uncertainties).
- Use most recent public NLO PDFs from all fitting groups to calculate LHC benchmark processes: $W^{\pm}$, $Z^0$, $t\bar{t}$, $gg \to H$.
  **Aims:**
  1. Establish degree of compatibility and identify outliers.
  2. Compare cross sections at **same** $\alpha_S$ values.
  3. To what extent are differences in predictions due to different $\alpha_S$ values used by each group, rather than differences in PDFs?
- Results initially presented in talk by G.W. at PDF4LHC meeting at CERN on 26th March 2010 and formed basis for subsequent PDF4LHC *Interim Report* [arXiv:1101.0536] and PDF4LHC *Interim Recommendations* [arXiv:1101.0538] used in the *Handbook of LHC Higgs Cross Sections* [arXiv:1101.0593].

# PDF4LHC WG Interim Recommendations [arXiv:1101.0538]

**Steering Committee:** Botje, Butterworth, Cooper-Sarkar, De Roeck, Feltesse, Forte, Glazov, Huston, McNulty, Sjöstrand, Thorne.

## NLO prescription (68% C.L. uncertainties)

- "For the calculation of uncertainties at the LHC, use the envelope provided by the central values and PDF+$\alpha_S$ errors from the **MSTW08**, CTEQ6.6 and NNPDF2.0 PDFs."
- "As a central value, use the midpoint of this envelope."

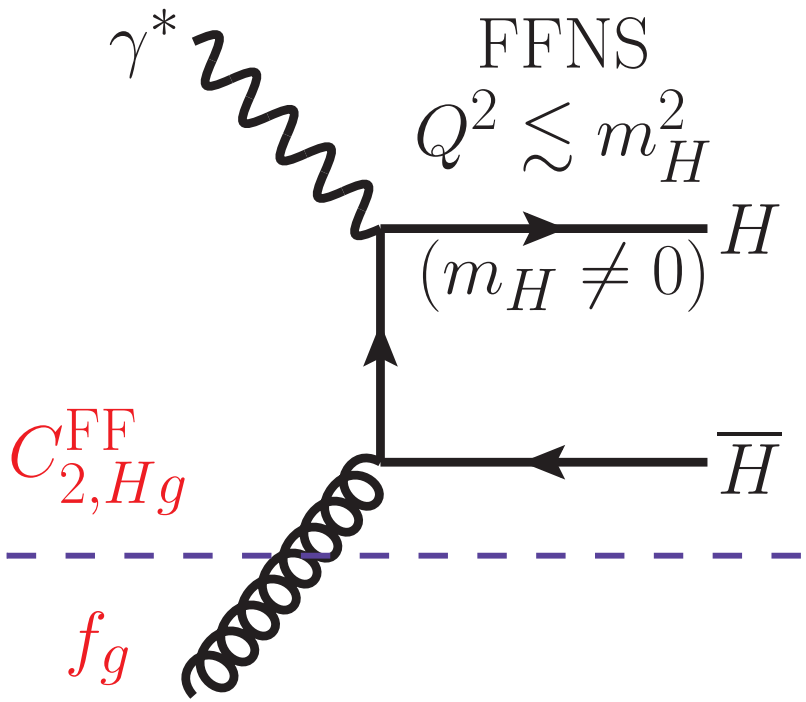
## NNLO prescription (68% C.L. uncertainties)

- "As a central value, use the **MSTW08** prediction."
- Rescale the **MSTW08** NNLO uncertainty by "the factor obtained by dividing the full uncertainty obtained from the envelope of **MSTW**, CTEQ and NNPDF results at NLO by the **MSTW** uncertainty at NLO" ($\sim 2$ for $gg \to H$ at LHC).

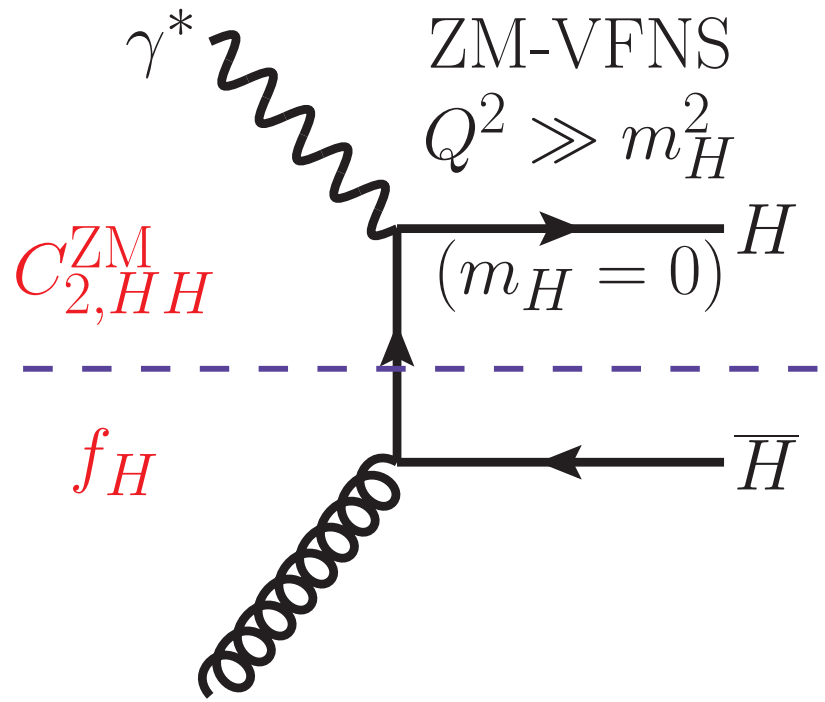
# Motivation for updating PDF4LHC benchmark exercise

- Original exercise (at NLO) completed by G.W. in March 2010.
- **New NLO PDFs** have appeared in the last year:
  - CTEQ6.6 $\rightarrow$ CT10 [arXiv:1007.2241]
  - NNPDF2.0 (ZM) $\rightarrow$ NNPDF2.1 (GM) [arXiv:1101.1300]
  - HERAPDF1.0 $\rightarrow$ HERAPDF1.5 [H1prelim-10-142, ZEUS-prel-10-018]
- **Comparisons at NNLO**, where $W$, $Z$, $gg \rightarrow H$ known exactly. Public code HATHOR [arXiv:1007.1327] for approx. NNLO $t\bar{t}$.
- First LHC data on $W$, $Z$, $t\bar{t}$ (not yet with full 2010 statistics).
- Lepton charge asymmetry from $W \rightarrow \ell\nu$ decays (ATLAS, LHCb).
- Constraints from Tevatron jet data on high-$x$ gluon (and $\alpha_S$).
- Many results shown for the first time: paper in preparation.

# Heavy quark contribution to DIS structure function $F_2$




Fixed flavour number scheme




Zero-mass variable flavour number scheme

- **G**eneral-**m**ass **v**ariable **f**lavour **n**umber **s**cheme (**GM-VFNS**) interpolates between two well-defined regions: **FFNS** for $Q^2 \leq m_H^2$, **ZM-VFNS** for $Q^2 \gg m_H^2$.

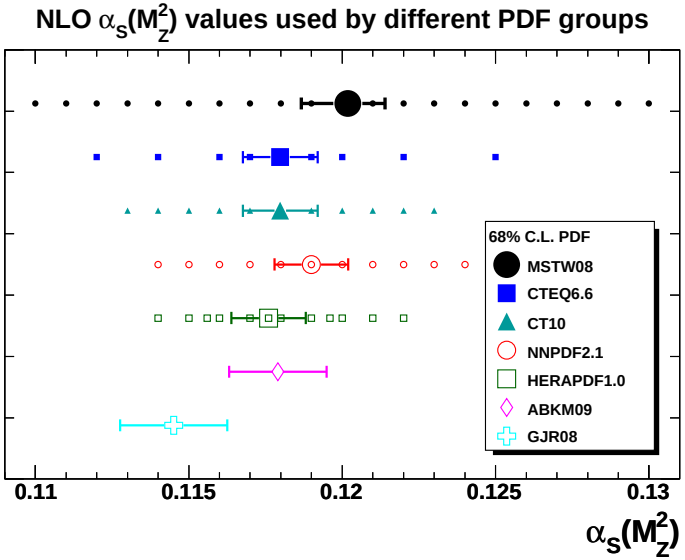
# Current status of proton PDFs from different groups

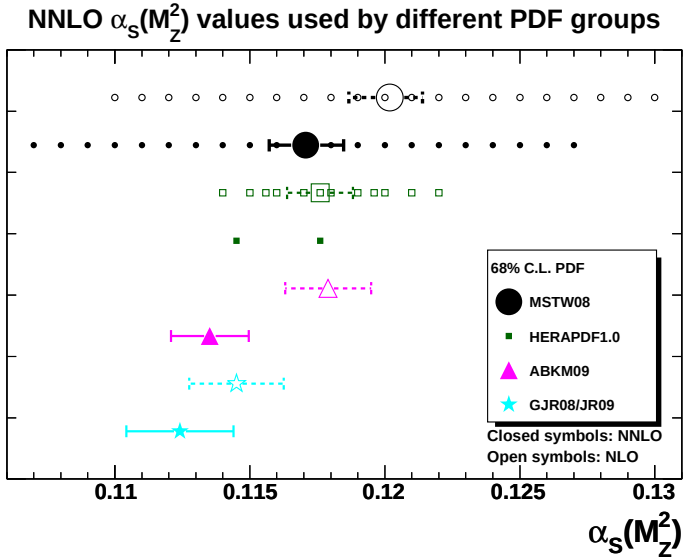
- Highlight major differences in data and theory between groups:

| | **MSTW08** | **CTEQ6.6/CT10** | **NNPDF2.1** | HERAPDF1.0 | ABKM09 | GJR08/JR09 |
|---|---|---|---|---|---|---|
| HERA DIS | ✔ | ✔ | ✔ | ✔ | ✔ | ✔ |
| Fixed-target DIS | ✔ | ✔ | ✔ | ✗ | ✔ | ✔ |
| Fixed-target DY | ✔ | ✔ | ✔ | ✗ | ✔ | ✔ |
| Tevatron $W,Z$ | ✔ | ✔ | ✔ | ✗ | ✗ | ✗ |
| Tevatron jets | ✔ | ✔ | ✔ | ✗ | ✗ | ✔ |
| GM-VFNS | ✔ | ✔ | ✔ | ✔ | ✗ | ✗ |
| NNLO | ✔ | ✗ | ✗ | ✔ | ✔ | ✔ |

- *Three* groups with **NLO** global fits, but only *one* at **NNLO**.
- CTEQ6.6 only uses Tevatron Run I data, not Run II.
- Only CT10, NNPDF2.1 and HERAPDF1.0 use *combined* HERA.
- NNPDF2.0 (ZM-VFNS) $\longrightarrow$ NNPDF2.1 (GM-VFNS), now allowing meaningful comparison to other NLO global fits.

# Values of $\alpha_S(M_Z^2)$ used by different fitting groups



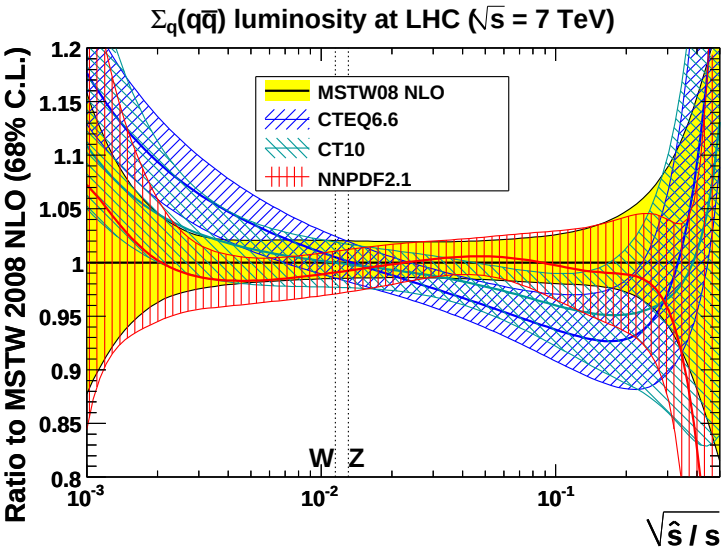

- $\alpha_S(M_Z^2)$ for MSTW08, ABKM09 and GJR08/JR09 fitted.
- $\alpha_S(M_Z^2)$ for other groups applied as an external constraint.
- Smaller symbols indicate alternative $\alpha_S(M_Z^2)$ values provided.
- Fitted NLO $\alpha_S(M_Z^2)$ always larger than NNLO $\alpha_S(M_Z^2)$: attempt to mimic missing higher-order corrections.

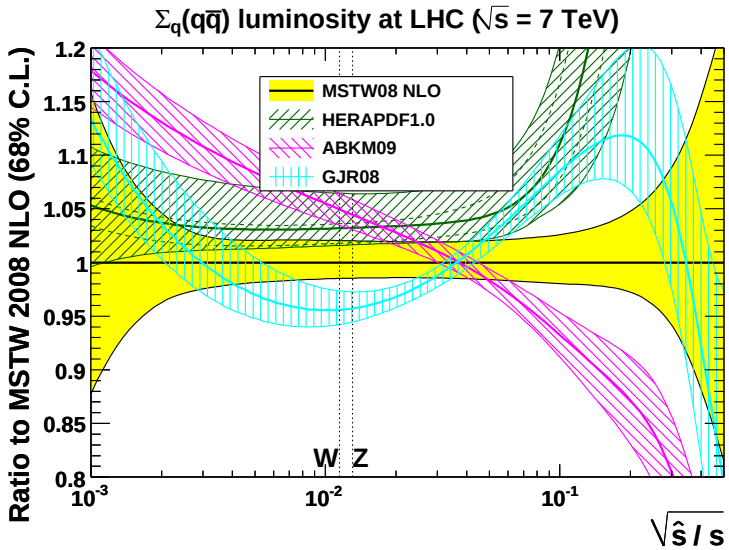
# Settings for NLO and NNLO benchmark cross sections

- Aim to isolate PDF (and $\alpha_S$) dependence.
  $\Rightarrow$ Use **same** code for all PDF sets with common settings.
- PDF+$\alpha_S$ uncertainties (at 68% C.L.) computed using recommended prescription of each fitting group.
- No attempt made to evaluate other theoretical uncertainties.
  Single scale choice, $\mu_R = \mu_F = M_W, M_Z, M_H, m_t$.
- Treatment of heavy quarks in 5-flavour ZM-VFNS.
- Only present *total* cross sections, not differential distributions.
- On-shell $W$ and $Z$ production times leptonic branching ratios.
- $t\bar{t}$ production (without decay) and $m_t = 171.3$ GeV.
  HATHOR [arXiv:1007.1327] for approximate NNLO calculation.
- $gg \rightarrow H$ (no decay) via $t$-quark loop ($m_t$ dependence at LO).
  NNLO corrections from GGH@NNLO [R. Harlander].

# Ratio of quark–antiquark luminosity functions

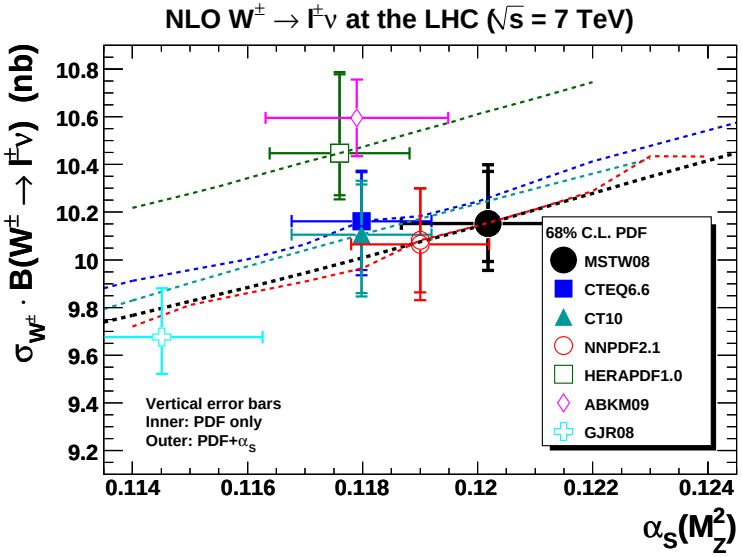
$$\frac{\partial \mathcal{L}_{\Sigma_q(q\bar{q})}}{\partial \hat{s}} = \frac{1}{s}\int_\tau^1 \frac{\mathrm{d}x}{x} \sum_{q=d,u,s,c,b} \left[ f_q(x,\hat{s}) f_{\bar{q}}(\tau/x,\hat{s}) + (q \leftrightarrow \bar{q}) \right], \quad \tau \equiv \frac{\hat{s}}{s}$$

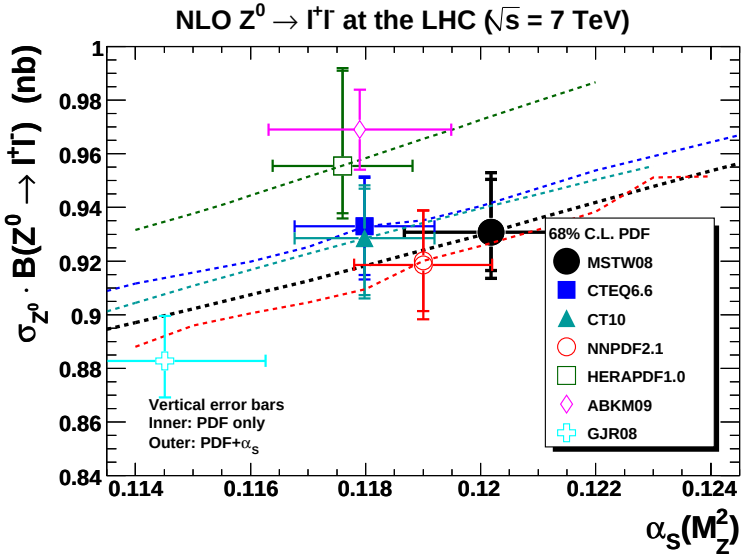


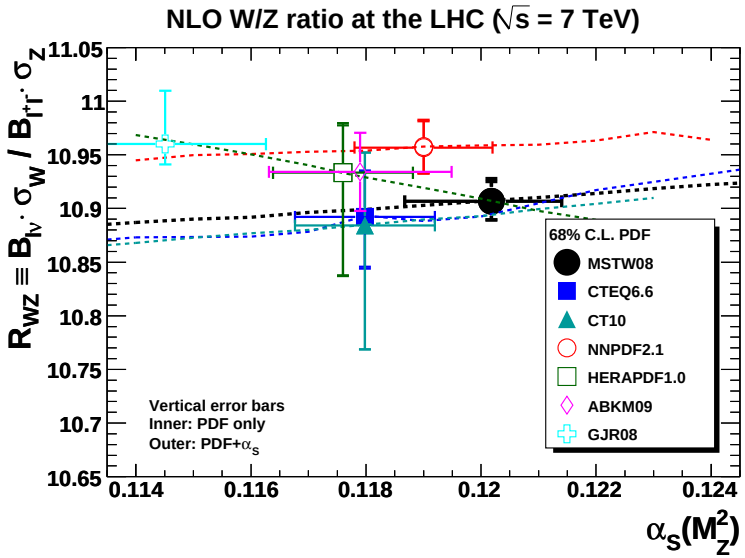

- Relevant values of $\sqrt{\hat{s}} = M_{W,Z}$ are indicated: good agreement for global fits (left), but more variation for other sets (right).

# NLO $W^{\pm}$ and $Z^0$ total cross sections versus $\alpha_S(M_Z^2)$



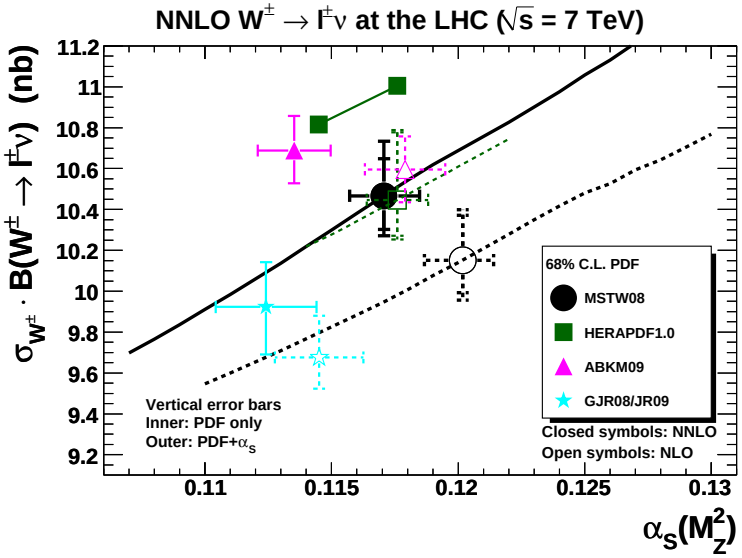



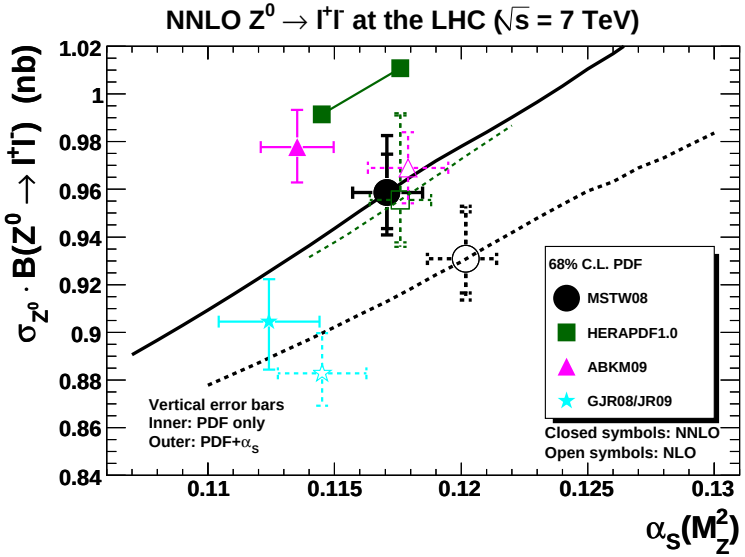

- Global fits in good agreement for $\sigma_{W^{\pm}}$ and $\sigma_{Z^0}$ (left plots).
- Small uncertainties in predictions for $W/Z$ ratio:



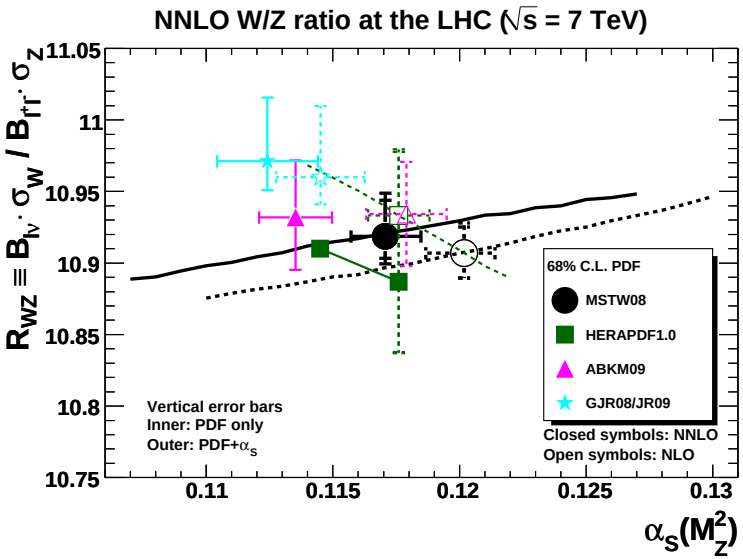

- Agrees with existing data from ATLAS [arXiv:1010.2130] and CMS [arXiv:1012.2466].

# NNLO $W^{\pm}$ and $Z^0$ total cross sections versus $\alpha_S(M_Z^2)$

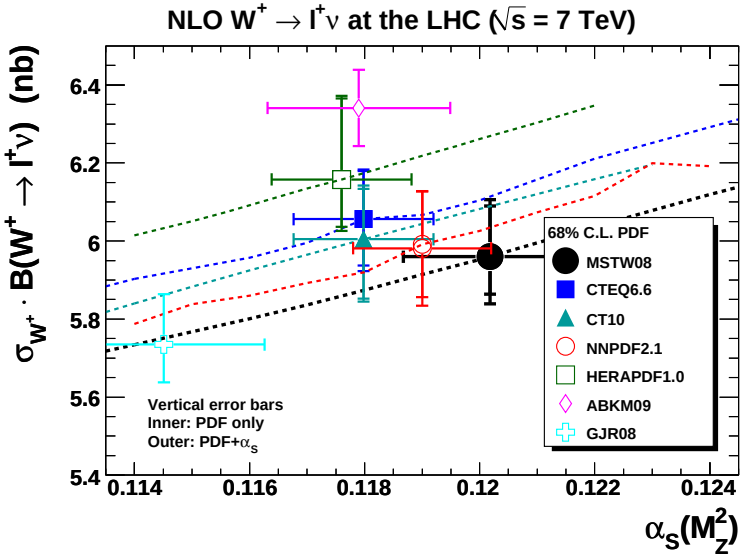


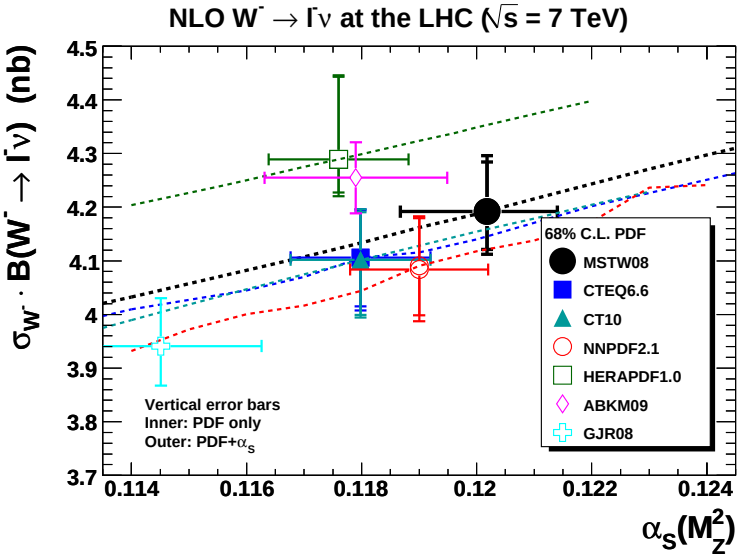




- NNLO corrections reduced by taking different $\alpha_S(M_Z^2)$ values at different orders.
- $W/Z$ ratio insensitive to NNLO corrections (and $\alpha_S$):

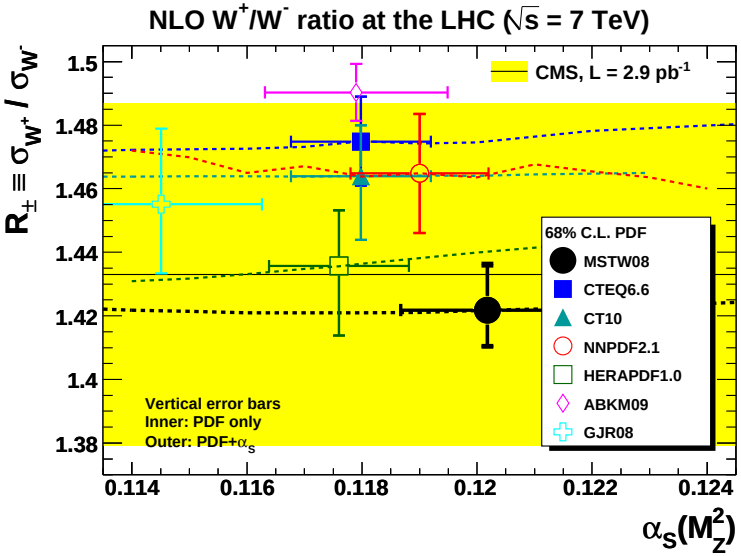


# NLO $W^+$ and $W^-$ total cross sections versus $\alpha_S(M_Z^2)$



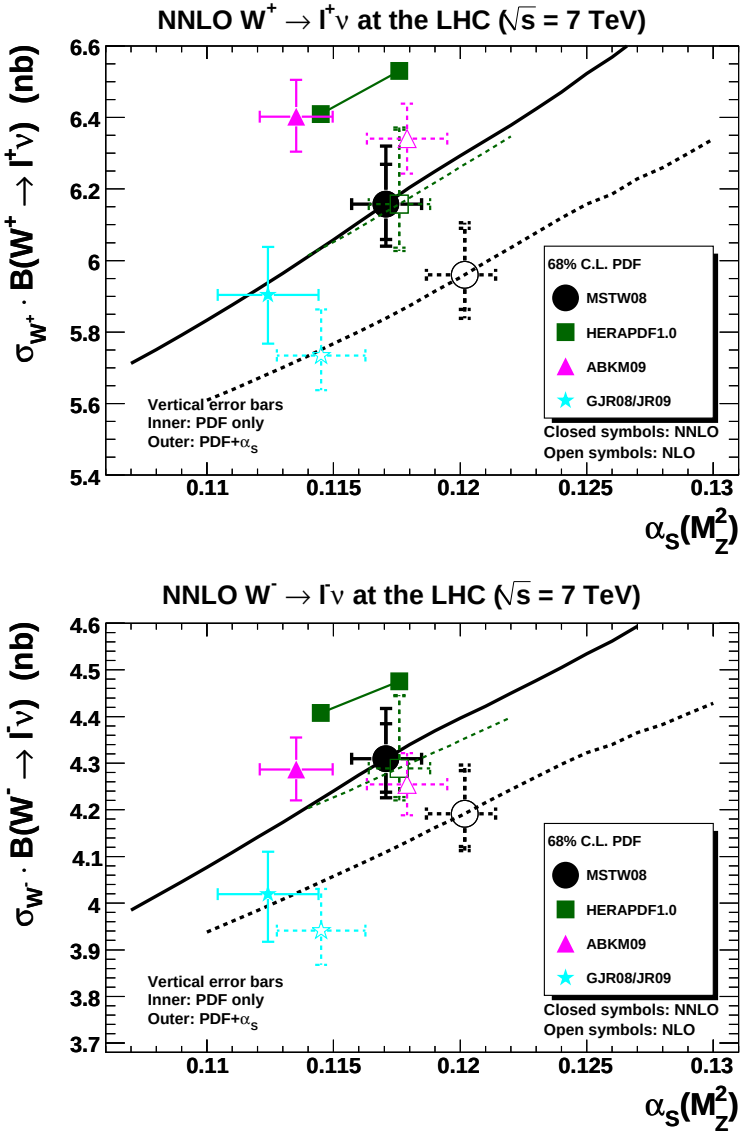




- Slightly more spread in separate $\sigma_{W+}$ and $\sigma_{W-}$.
- Reflected in $W^+/W^-$ ratio (sensitive to $u/d$ ratio):

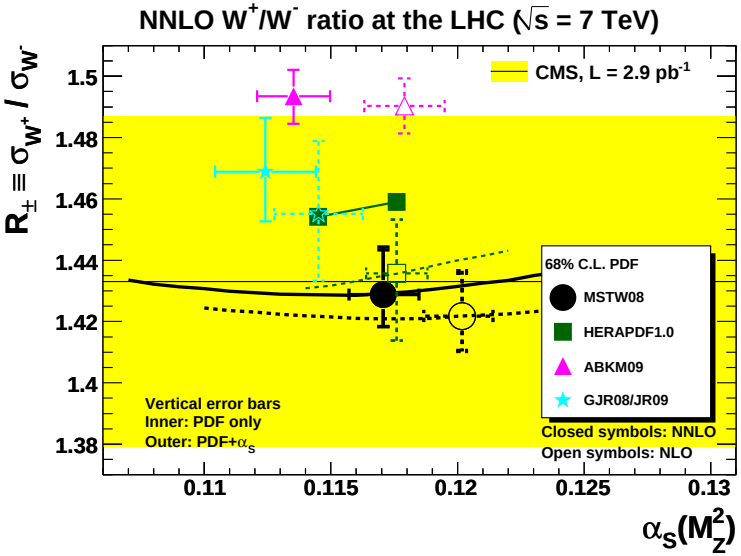



- Yellow band on plot is CMS measurement [arXiv:1012.2466].

# NNLO $W^+$ and $W^-$ total cross sections versus $\alpha_S(M_Z^2)$

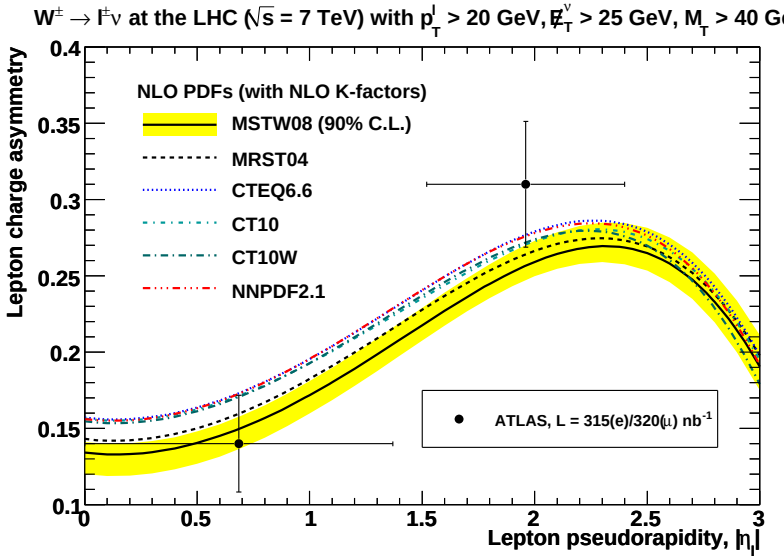


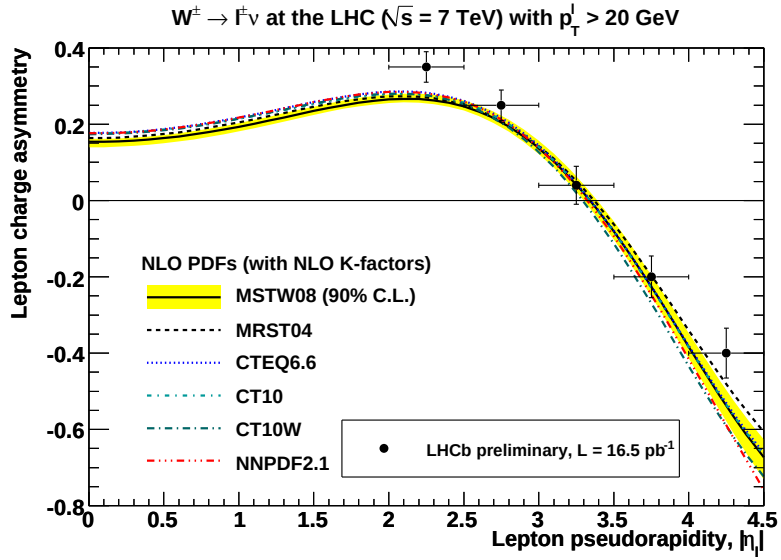

- $W^+/W^-$ ratio insensitive to NNLO corrections (and $\alpha_S$):

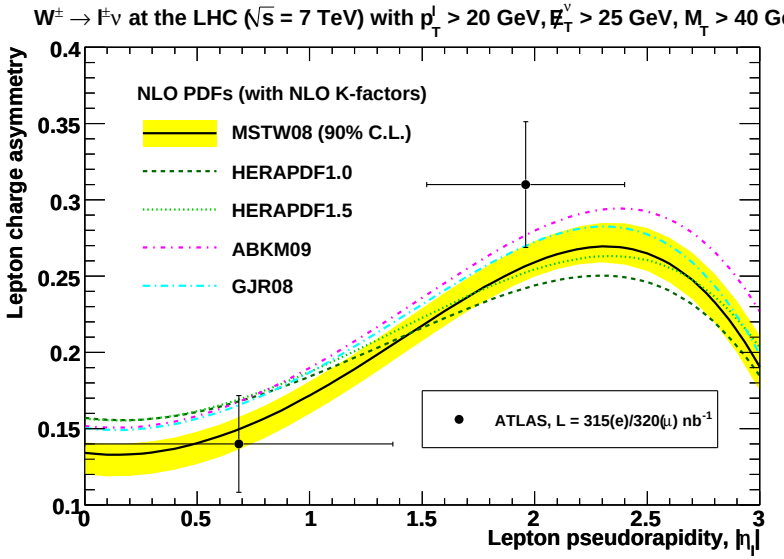


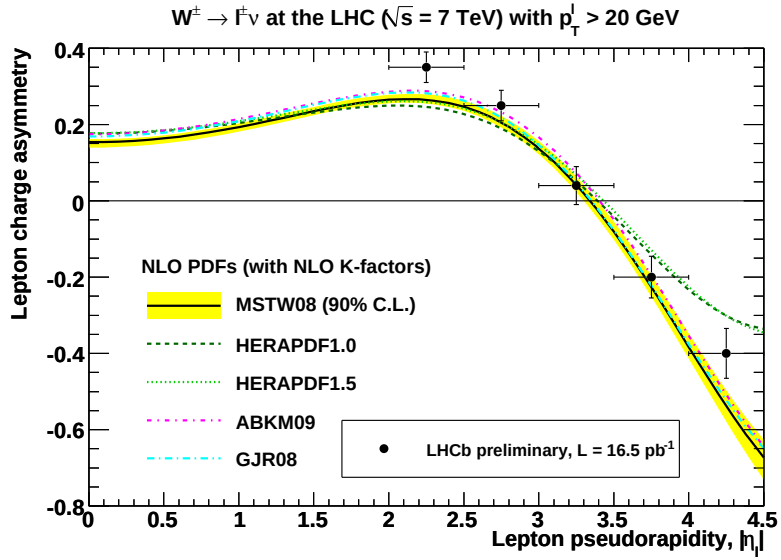

- Yellow band on plot is CMS measurement [arXiv:1012.2466].

# $W^{\pm} \to \ell^{\pm}\nu$ charge asymmetry from ATLAS and LHCb

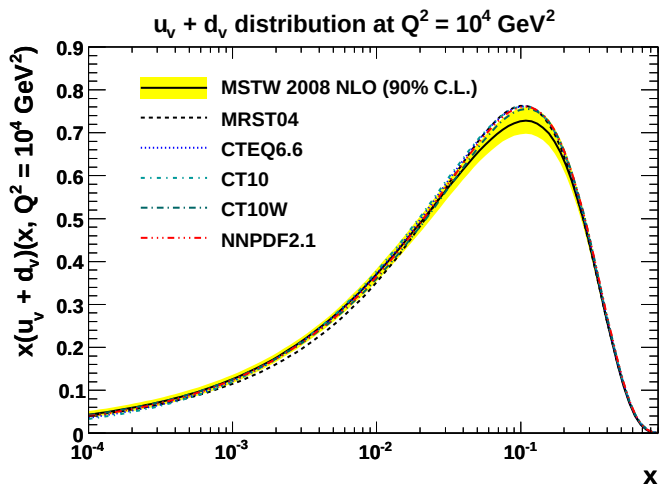


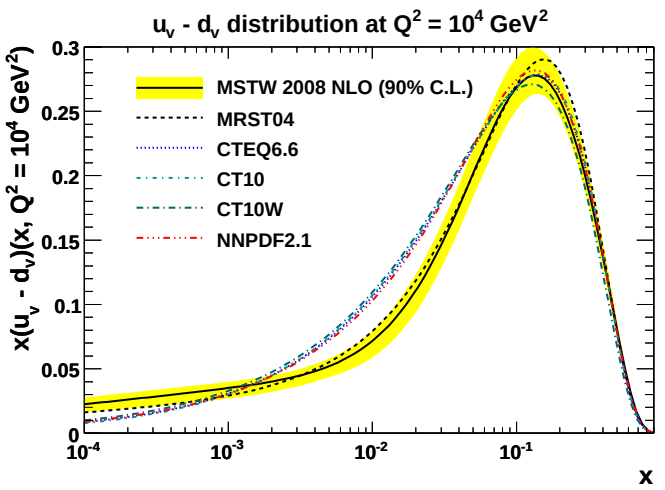



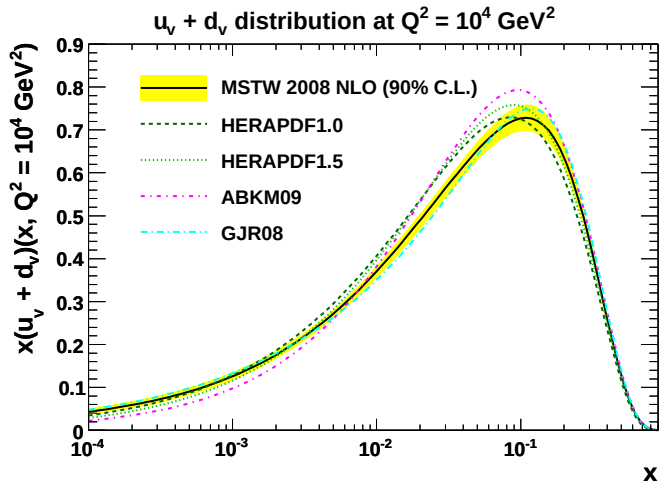



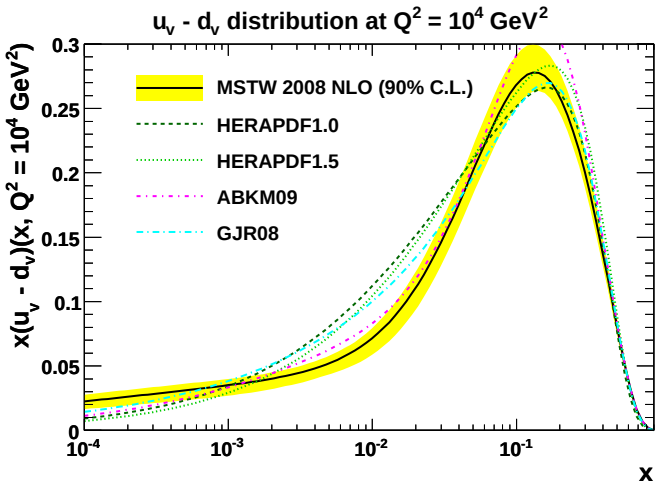



# Comparison of $u_v \pm d_v$ for different NLO PDFs



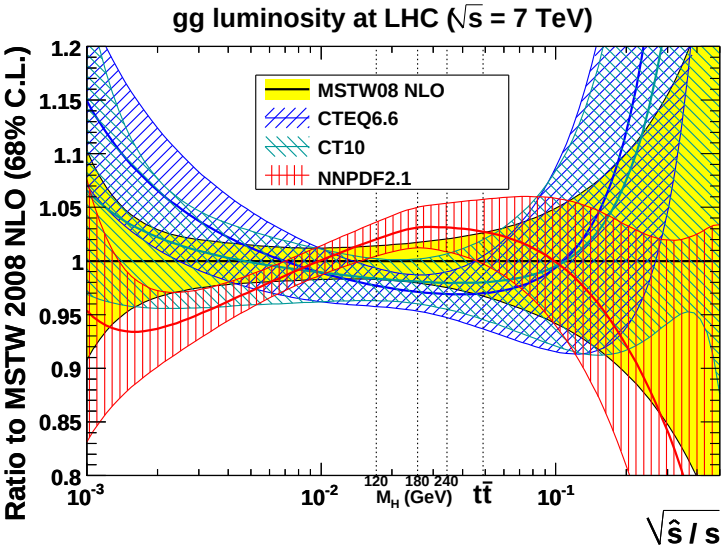



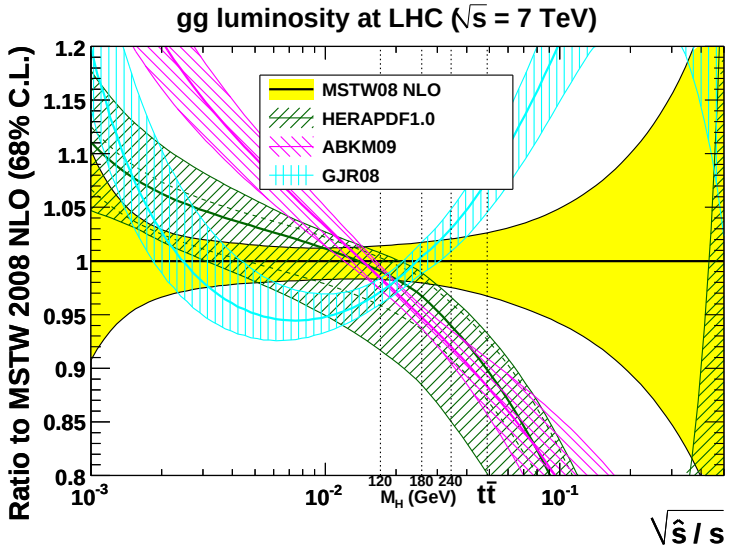




- MSTW08 has input $xu_v \propto x^{0.29 \pm 0.02}$ and $xd_v \propto x^{0.97 \pm 0.11}$.
  Many other groups **assume** equal powers $\Rightarrow$ parameterisation bias.

# Ratio of gluon–gluon luminosity functions

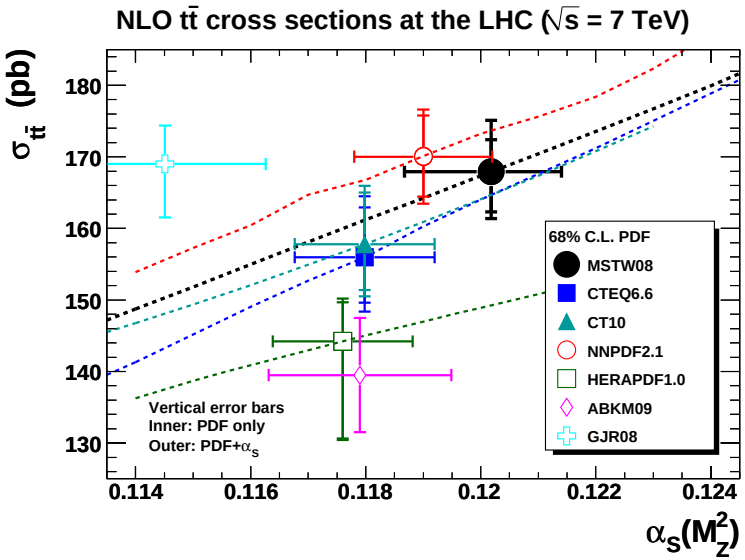
$$\frac{\partial \mathcal{L}_{gg}}{\partial \hat{s}} = \frac{1}{s}\int_{\tau}^{1} \frac{\mathrm{d}x}{x} f_g(x,\hat{s}) f_g(\tau/x,\hat{s}), \quad \tau \equiv \frac{\hat{s}}{s}$$

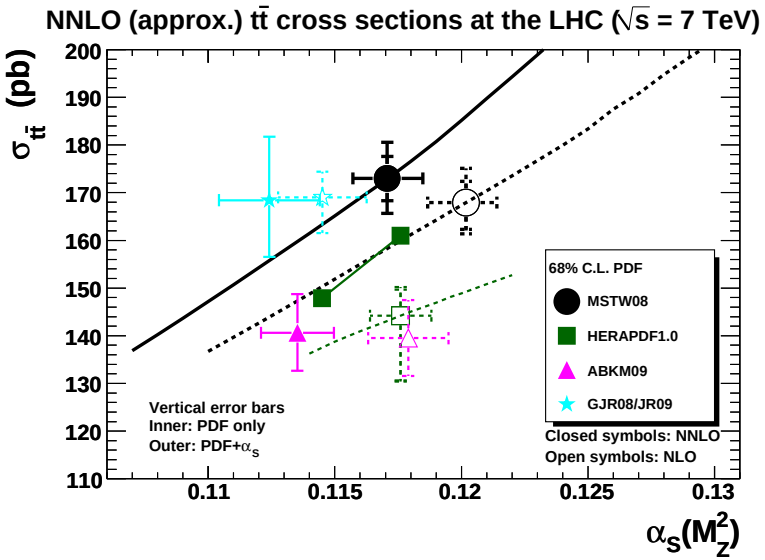



- Relevant values of $\sqrt{\hat{s}} = M_H, 2m_t$ are indicated: reasonable agreement for global fits (left), but more variation for other PDF sets (right), particularly at large $\sqrt{\hat{s}/s}$.

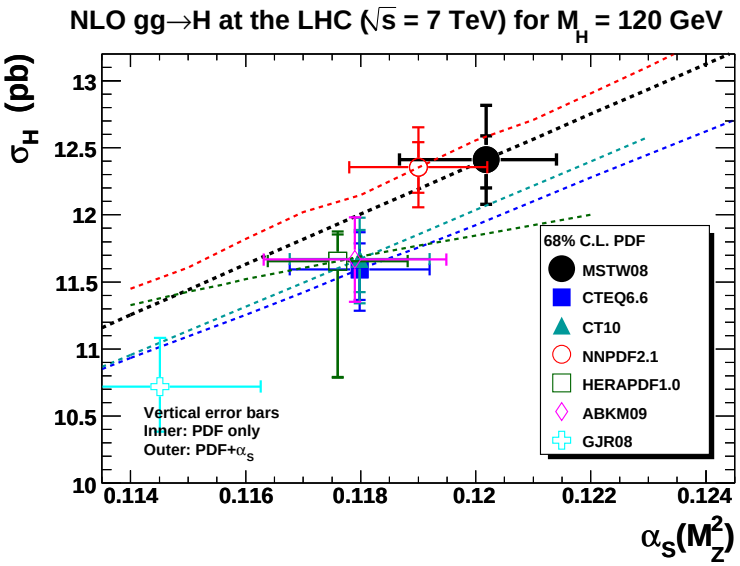
# $t\bar{t}$ total cross section versus $\alpha_S(M_Z^2)$

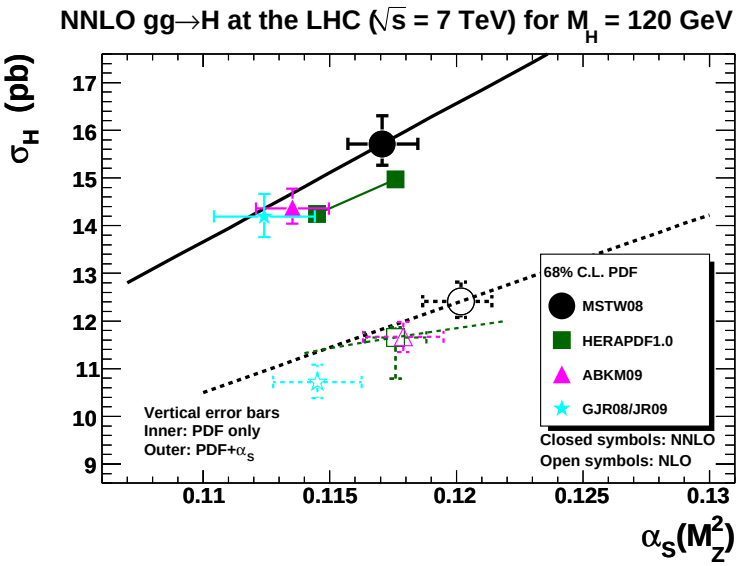


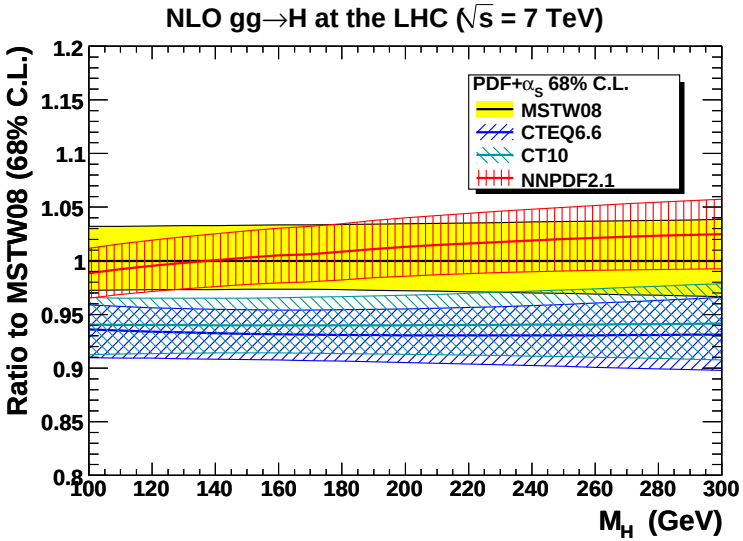



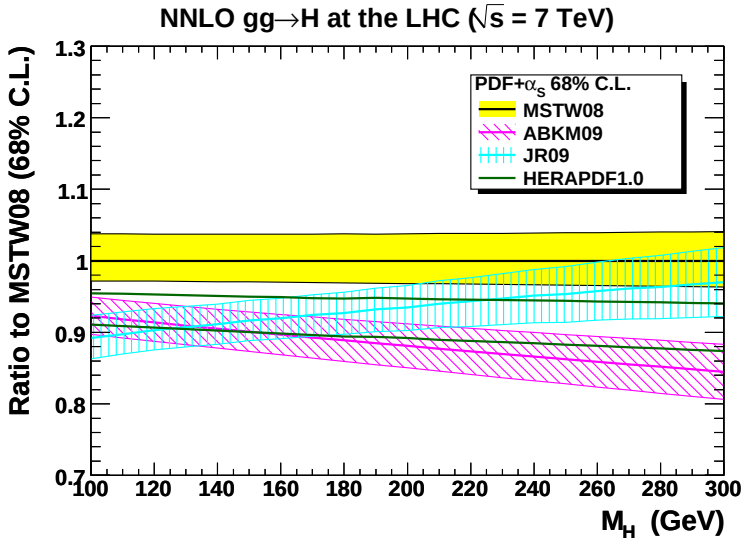

- More than 80% of NLO $t\bar{t}$ cross section from $gg$ channel at the 7 TeV LHC (almost 90% at the 14 TeV LHC).
- Strong dependence on gluon distribution and $\alpha_S$.
- Use default settings for approximate NNLO calculation with HATHOR public code [Aliev *et al.*, arXiv:1007.1327].
- Approximate NNLO corrections seem small taking different $\alpha_S(M_Z^2)$ at different orders, but many "NNLO" choices.

# $gg \to H$ total cross section versus $\alpha_S(M_Z^2)$ and $M_H$



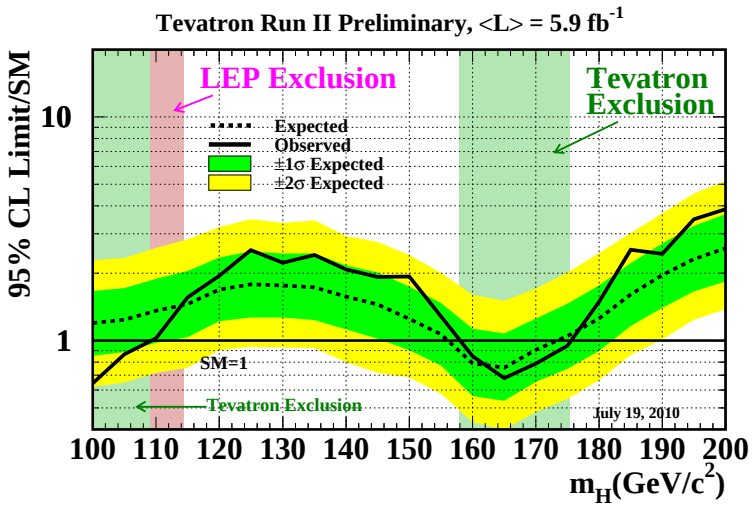



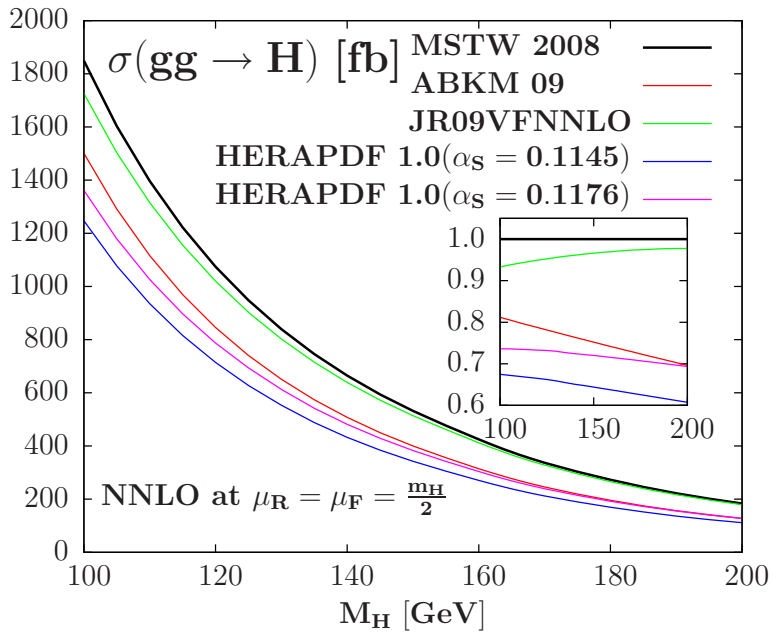



# "An Attack To The Tevatron Higgs Bounds!"

(Slide title from blog post by T. Dorigo.)




- Tevatron exclusion of SM Higgs boson at 95% C.L. for $158 < M_H < 175$ GeV [arXiv:1007.4587]. Theory prediction used MSTW 2008 NNLO PDFs.




- Baglio, Djouadi, Ferrag, Godbole [arXiv:1101.1832]: other PDFs can lower Higgs cross section by up to 40%, requiring more than twice as much Tevatron data to recover the same sensitivity.

# Tevatron jet data as a constraint on the high-$x$ gluon

- $gg \to H$ at Tevatron sensitive to high-$x$ gluon distribution.
- Main data constraint from Tevatron data on inclusive jets: only included in MSTW NNLO fits, **not** by other groups.
- **Problem:** NNLO $\hat{\sigma}$ unknown, approximate with NLO $\hat{\sigma}$ and 2-loop threshold corrections [Kidonakis and Owens, hep-ph/0007268].
- Take different scale choices $\mu_R = \mu_F = \mu = \{p_T/2, p_T, 2p_T\}$ as some indication of the theoretical uncertainty.
- Important to account for *correlated* systematic uncertainties of experimental data points [CTEQ6, hep-ph/0201195]:

$$\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left( \frac{\hat{D}_i - T_i}{\sigma_i^{\text{uncorr.}}} \right)^2 + \sum_{k=1}^{N_{\text{corr.}}} r_k^2, \quad \text{where } \hat{D}_i \equiv D_i - \sum_{k=1}^{N_{\text{corr.}}} r_k \, \sigma_{k,i}^{\text{corr.}}.$$

- Restrict shift in luminosity uncertainty, $r_k \in [-1, +1]$.

# Description of CDF II inclusive jet ($k_T$) data [hep-ex/0701051]

- Values of $\chi^2/N_{\text{pts.}}$ with (without) accounting for correlations:

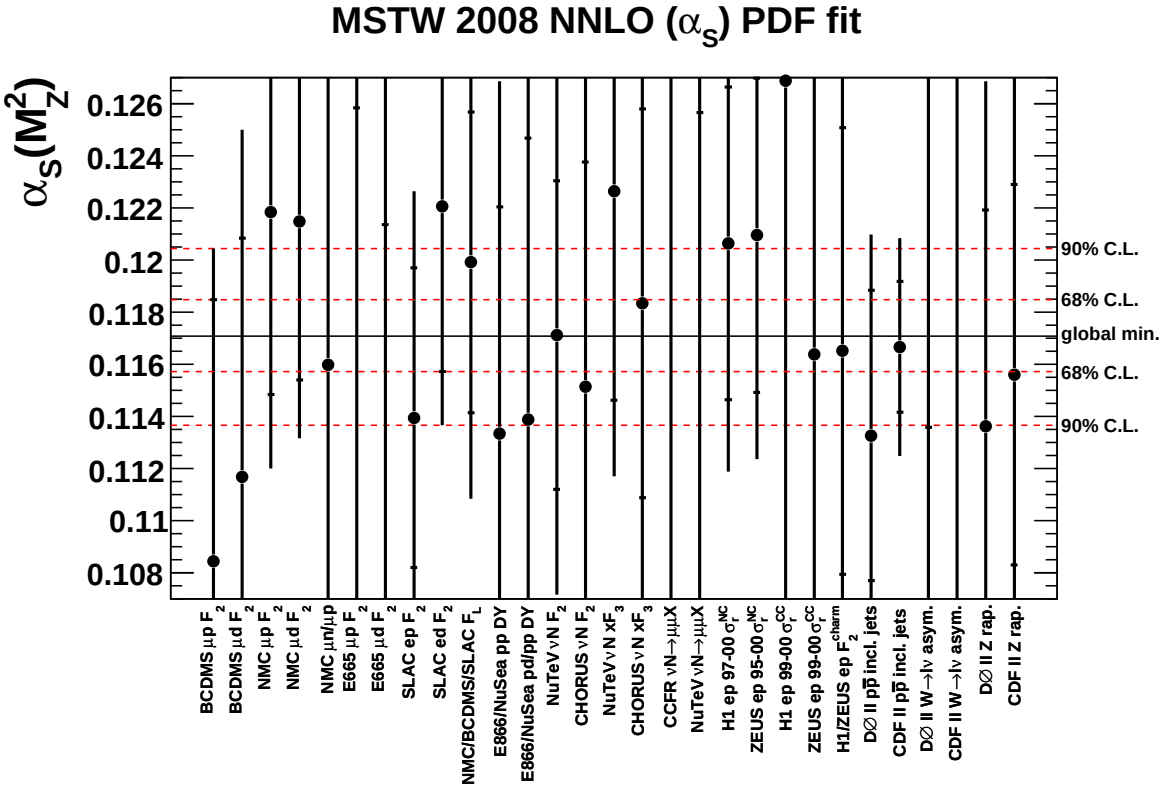
| NLO PDF | $\mu = p_T/2$ | $\mu = p_T$ | $\mu = 2p_T$ |
|---|---|---|---|
| MSTW08 | 0.75 (0.30) | 0.68 (0.28) | 0.91 (0.84) |
| CTEQ6.6 | 1.25 (0.14) | 1.66 (0.20) | 2.38 (0.84) |
| CT10 | 1.03 (0.13) | 1.20 (0.19) | 1.81 (0.84) |
| NNPDF2.1 | 0.74 (0.29) | 0.82 (0.25) | 1.23 (0.69) |
| HERAPDF1.0 ($\alpha_S = 0.1176$) | 2.43 (0.39) | 3.26 (0.66) | 4.03 (1.67) |
| ABKM09 | 1.62 (0.52) | 2.21 (0.85) | 3.26 (2.10) |
| GJR08 | 1.36 (0.23) | 0.94 (0.13) | 0.79 (0.36) |

| NNLO PDF | $\mu = p_T/2$ | $\mu = p_T$ | $\mu = 2p_T$ |
|---|---|---|---|
| MSTW08 | 1.39 (0.42) | 0.69 (0.44) | 0.97 (0.48) |
| HERAPDF1.0 ($\alpha_S = 0.1145$) | 2.64 (0.36) | 2.15 (0.36) | 2.20 (0.46) |
| HERAPDF1.0 ($\alpha_S = 0.1176$) | 2.24 (0.35) | 1.17 (0.32) | 1.23 (0.31) |
| ABKM09 | 2.55 (0.82) | 2.76 (0.89) | 3.41 (1.17) |
| JR09 | 0.75 (0.37) | 1.26 (0.41) | 2.21 (0.49) |

- Similar trends for CDF II (cone) data and DØ II (cone) data.

# Common lore that DIS-only fits prefer low $\alpha_S$. Is it true?

**ABKM09:** $\alpha_S(M_Z^2) = 0.1135 \pm 0.0014$, cf. **MSTW08:** $0.1171 \pm 0.0014$.




- Not all DIS data sets prefer low $\alpha_S(M_Z^2)$ values.
- True only for BCDMS, and for E665 and SLAC *ep* data.
- NMC, SLAC *ed* and HERA data prefer high $\alpha_S(M_Z^2)$ values.

# What is $\alpha_S$ from only DIS in the MSTW08 NNLO fit?

[Studies prompted by question from G. Altarelli, December 2010]

- Known that $\alpha_S$ is **anticorrelated** with low-$x$ gluon through scaling violations of HERA data: $\partial F_2/\partial \ln(Q^2) \sim \alpha_S\, g$. Then $\alpha_S$ is **correlated** with high-$x$ gluon through *momentum sum rule*.
- DIS-only fit gives $\alpha_S(M_Z^2) = 0.1104$ (BCDMS-dominated), but input $xg < 0$ for $x > 0.4$ due to lack of data constraint. $\Rightarrow F_2^{\rm charm} < 0$ and $\chi^2/N_{\rm pts.} \sim 10$ for Tevatron jets.
- DIS-only fit without BCDMS gives $\alpha_S(M_Z^2) = 0.1193$. Global fit without BCDMS gives $\alpha_S(M_Z^2) = 0.1181$. DIS-only fit fixing high-$x$ gluon parameters similar to global fit.
- Force input $xg > 0$: $\alpha_S(M_Z^2) = 0.1157$, but $\Delta\chi^2_{\rm global} = 80$.
- **Conclusion:** Tevatron jet data vital to pin down high-$x$ gluon, giving smaller low-$x$ gluon and therefore larger $\alpha_S$ in the global fit compared to a DIS-only fit, at the expense of some deterioration in the fit quality of the BCDMS data.

# Summary

- Now reasonably good agreement between NLO *global* fits from **MSTW08**, CT10 and NNPDF2.1, all using GM-VFNS.
- More variation with other PDF sets, particularly at NNLO where **MSTW08** is the only *global* PDF fit available.
- $W^+/W^-$ ratio and $W^\pm \to \ell^\pm \nu$ charge asymmetry close to disfavouring some PDF sets. Results with full 2010 statistics will be interesting: expose parameterisation bias in $u_v - d_v$?
- $t\bar{t}$ and $gg \to H$ sensitive to $gg$ luminosity and $\alpha_S(M_Z^2)$, but can use Tevatron jet data to *discriminate*. Fits not including Tevatron jet data should be used with caution (if at all).
- Tevatron Higgs exclusion analysis using **MSTW08** is sound.