0.1 Access to the ZEUS 2002 PDFs

The ZEUS PDFs are sets of PDFs with errors extracted from a global NLO-QCD fit in the DGLAP formalism to ZEUS data and fixed target data, as described in hep-ex/0208023 (DESY-02-105).
The DGLAP equations were solved at NLO in the $\overline{\mathrm{MS}}$ scheme with the renormalisation and factorisation scales chosen to be Q^2.
The ZEUS-S fit was done using the Thorne-Roberts heavy quark scheme- (Thorne Roberts Variable Flavour Number Scheme -TRVFN) but PDFs are also available using a Zero Mass Variable Flavour Number Scheme (ZMVFN) and a Fixed Flavour Number Scheme (FFN) with 4-flavours. For each of these schemes the PDFs and their uncorrelated (statistical plus uncorrelated systematic) error bands, correlated systematic error bands, and total error bands are available.
These parton distributions are publically available in various formats:
1)a central PDF set plus eigenvector PDF parameter sets from which the errors may be calculated easily.
2)a central PDF parameter set plus covariance matrices
3)as x, Q^2 grids of PDF values for for the parton momentum distributions: gluon, sea, xu_v, xd_v, $x\bar{u}$, $x\bar{d}$, $x\bar{s}$, $x\bar{c}$, $x\bar{b}$. Grids of various structure functions and reduced cross-sections are also available.

0.1.1 Eigenvector PDF sets

Eigenvector PDF sets represent the most efficient and compact way of storing the information on the PDF errors. The technique was first suggested by CTEQ (hep-ph/0101032) and CTEQ6 errors are given in this form (hep-ph/0201195). MRST have recently issued their PDF parameters in this form (hep-ph/0211080).
The errors on the parton distribution parameters are encapsulated in the error matrix (covariance matrix) of the fit. This may be diagonalised and its eigenvalues represent the squared errors on the combinations of parameters which are the eigenvectors. The results of the fit can be summarised as one central PDF set (S) and $2 \times N_{pdf}$ eigenvector PDF sets for the errors, where N_{pdf} is the number of free PDF parameters. For the ZEUS-S fit $N_{pdf} = 11$. These eigenvector PDF sets represent excursions up and down along each of the N_{pdf} eigenvector directions by an amount equal to the error on the corresponding combination of the original PDF parameters, i.e the square-root of corresponding eigenvalue. These eigenvector PDF sets are labelled S_i^+ and S_i^- for each of the $i = 1, N_{pdf}$ eigenvalues.
The error on a quantity $F(S)$, which is a function of the PDF parameters, (for example, such a quantity could be a PDF distribution, a structure function or a cross-section) is then calculated from

$$\Delta F^2 = \Sigma_i \left(\frac{F(S_i^+) - F(S_i^-)}{2} \right)^2$$

i.e. the value of F is calculated from the parameter sets S_i^+ and S_i^-, exactly as it is for the central set, and then the difference between its value for these two sets gives the error on F due to eigenvector i. These are then simply added in quadrature.
These eigenvector PDF sets are supplied in data files as follows
For the FFN heavy quark scheme

- *parcen_ff.dat* is the 11 free PDF parameters at their central values
- *instat_ff.dat* is a file with 22 sets of such 11 parameters in the order S_i^+, then S_i^- for the $i = 1, 11$ eigenvector directions; for uncorrelated (statistical plus uncorrelated systematic) errors
- *insys_ff.dat* is a file with 22 sets of such 11 parameters in the order S_i^+, then S_i^- for the $i = 1, 11$ eigenvector directions; for correlated systematic errors
- *intot_ff.dat* is a file with 22 sets of such 11 parameters in the order S_i^+, then S_i^- for the $i = 1, 11$ eigenvector directions; for total errors

For the ZMVFN scheme the files carry the same names with *_ff* repleaced by *_zm* and for the TRVFN scheme they carry the same names with *_ff* replaced by *_rt*.

In order to use these PDF sets it is necessary to know how the parton distributions were parameterised and used for the ZEUS fits. The parton distribution functions (PDFs) are parameterised at Q_0^2 by the form

$$xf(x) = p_1 x^{p_2}(1-x)^{p_3}(1+p_5 x)$$

The NLO DGLAP equations are used to evolve the parton distributions to all values of Q^2. The input scale is $Q_0^2 = 7$ GeV^2, but backward evolution can be performed to fit lower-Q^2 data. The PDFs give a good description of data down to $Q^2 \sim 1\text{GeV}^2$, but note that the low-x gluon becomes negative for $Q^2 < 1.8\text{GeV}^2$.
The parton momentum distributions parameterised are: u valence, $xu_v(x)$; d valence, $xd_v(x)$; total sea, $xS(x)$; gluon, $xg(x)$; and the difference between the d and u contributions to the sea, $x\Delta = x(\bar{d} - \bar{u})$. The total sea at Q_0^2 is made from the flavours up, $xu_{\text{sea}}(x)$, down, $xd_{\text{sea}}(x)$, strange, $xs_{\text{sea}}(x)$ and charm, $xc_{\text{sea}}(x)$, as follows

$$xs_{\text{sea}}(x) = 0.2xS(x)$$

$$xu_{\text{sea}}(x) = 0.4xS(x) - 0.5xc_{\text{sea}}(x) - x\Delta(x)$$

$$xd_{\text{sea}}(x) = 0.4xS(x) - 0.5xc_{\text{sea}}(x) + x\Delta(x)$$

where the symbols u_{sea}, d_{sea}, s_{sea}, c_{sea} include both quark and antiquark contributions to the sea for each flavour. The suppression of the strange sea to 20% of the total sea is consistent with neutrino-induced dimuon data from CCFR. The charmed sea is treated according to the chosen heavy quark scheme.
The following parameters were fixed:

PDF	p_1	p_2	p_3	p_5
xu_v	(1.69)	0.5	$4.00 \pm 0.01 \pm 0.08$	$5.04 \pm 0.09 \pm 0.64$
xd_v	(0.96)	0.5	$5.33 \pm 0.09 \pm 0.48$	$6.2 \pm 0.4 \pm 2.3$
xS	$0.603 \pm 0.007 \pm 0.048$	$-0.235 \pm 0.002 \pm 0.012$	$8.9 \pm 0.2 \pm 1.2$	$6.8 \pm 0.4 \pm 2.0$
xg	(1.77)	$-0.20 \pm 0.01 \pm 0.04$	$6.2 \pm 0.2 \pm 1.2$	0
$x\Delta$	$0.27 \pm 0.01 \pm 0.06$	0.5	(10.9)	0

Table 1: Table of PDF parameters at Q_0^2, as determined from the ZEUS-S fit. The first uncertainty given derives from statistical and other uncorrelated sources and the second uncertainty is the additional contribution from correlated systematic uncertainties. The numbers in parentheses were derived from the fitted parameters through the number and momentum sum-rules

- p_1 for xu_v and xd_v were fixed through the number sum-rules and p_1 for xg was fixed through the momentum sum-rule;
- $p_2 = 0.5$ was fixed for both valence distributions, since there is little information on the low-x valence shapes. Allowing this parameter to vary produces values which are consistent with 0.5.
- the only free parameter for the $x\Delta$ distribution is its normalisation, p_1, because there is insufficient information on its shape when using only DIS data Thus, $p_2(\Delta) = 0.5$, $p_3(\Delta) = p_3(\mathrm{Sea}) + 2$ were fixed and $p_5(\Delta) = 0$; the normalisation $p_1(\Delta)$ was found to be compatible with the measured value of the Gottfried sum-rule.
- for the gluon distribution, p_5 was set to zero, since this parameter was found to be highly correlated to the p_3 parameter of the gluon. Allowing this parameter to vary in the fit produced values which are consistent with zero.

There are thus 11 free parameters in the ZEUS-S fit. The value of $\alpha_s(M_Z) = 0.118$ is fixed and $\alpha_s(Q^2)$ is calculated to 2-loop accuracy.
The evolution was performed using the program QCDNUM and the evolution equations were written in terms of quark-flavour singlet and non-singlet distributions (made from the sea and valence quark distributions) and the gluon momentum distribution.
The parton distributions must be convoluted with coefficient functions in order to calculate structure functions and cross-sections. The coefficient functions are specific to the heavy-quark formalism used. The routines of QCDNUM can be used to perform this convolution for the FFN and ZMVFN schemes. For the TRVFN scheme additional routines are needed (*rt.f* and *jacksmith.f*).
Table 1 gives the central values of the 11 free fit parameters of the TRVFN fit, and their errors. The values of the fixed parameters and the PDF parameters which are functions of the fitted parameters are also given.

The user routine *qcd_results.f* illustrates how the PDF sets are used. This routine constructs the PDFs from the PDF parameters and evolves them in Q^2. It also performs the convolutions necessary to calculate many functions of the PDF parameters which are of interest like structure functions, and reduced cross-sections. These are output onto x, Q^2 grids of central value and up and down errors.
This routine needs QCDNUM and the Thorne Roberts routines (*rt.f* and *jacksmith.f*), as well as various cernlib/mathlib functions.
The following linux link command puts these together
$f77 - g - o\ qcdres\ qcd\_results.f\ \ rt.f - L. - lqcdnum1612 - L/cern/pro/lib - lmathlib - lpacklib - lkernlib - lgrafX11 - L/usr/X11R6/lib - lX11$

0.1.2 Covariance matrices

For some users it may be more convenient to use a single central PDF set and the covariance matrices of the fit. In this case the error on a quantity $F(S)$ is calculated from

$$\Delta F^2 = \Sigma_{ij} \frac{\delta F}{\delta p_i} V_{ij} \frac{\delta F}{\delta p_j}$$

where V_{ij} is the covariance matrix. Such covariance matrices are available for the three different heavy quark schemes
For the FFN heavy quark scheme

- *parcen_ff.dat* is the 11 free PDF parameters at their central values
- *statcov_ff.dat* is the 11*11 covariance matrix for uncorrelated (statistical plus uncorrelated systematic) errors
- *syscov_ff.dat* is the 11*11 covariance matrix for correlated systematic errors
- *totcov_ff.dat* is the 11*11 covariance matrix for total errors

For the ZMVFN scheme the files carry the same names with *_ff* replaced by *_zm* and for the TRVFN scheme they carry the same names with *_ff* replaced by *_rt*
In order to use these PDF sets and covariance matrices it is necessary to know how the parton distributions were parameterised and used for the ZEUS fits. This is explained above in Sec. 0.1.1

0.1.3 PDF grids

For some purposes it may be easier to use x, Q^2 grids of PDF values or structure functions, reduced cross-sections etc.
Such grids are available for the FFN, ZMVFN and TRVFN heavy quark schemes For each of these schemes grids are only available for total errors. (The separate contributions can be generated from the eigenvector PDf sets using *qcd_results.f*
For each function F of the PDF parameters, the information for each point of the x, Q^2 grid is supplied as:F, $F + \Delta F$ and $F - \Delta F$.

The grids are 161*161 points in $Q2, x$ constructed with the $Q2, x$ values given by

```
DO I=0,160
IF(I.LE.72)THEN
Q2=10**(5D0/120D0*I)-7D-1
ELSE
Q2=10**(2D0/88D0*(I-72)+3D0)
ENDIF
DO J=0,160
IF(J.LE.80)THEN
X=10**(6D0/120D0*J-6D0)
ELSE
X=10**(2D0/80D0*(J-80)-201D-2)
ENDIF
```

such that the grid ranges from 0.3 to 10^5 in Q^2 and 10^{-6} to 0.98 in x, with logarithmic spacing which is designed to become finer at high x and high Q^2.
The PDFs available are: gluon, sea, xu_v, xd_v, $x\bar{u}$, $x\bar{d}$, $x\bar{s}$, $x\bar{c}$, $x\bar{b}$.
For the FFN heavy quark scheme;

- *upval_ff_tot.dat* is the grid for the total errors on the xu_v distribution
- *dnval_ff_tot.dat* is the grid for the total errors on the xd_v distribution
- *gluon_ff_tot.dat* is the grid for the total errors on the gluon distribution
- *seaqk_ff_tot.dat* is the grid for the total errors on the sea distribution
- *ubar_ff_tot.dat* is the grid for the total errors on the $x\bar{u}$ distribution
- *dbar_ff_tot.dat* is the grid for the total errors on the $x\bar{d}$ distribution
- *sbar_ff_tot.dat* is the grid for the total errors on the $x\bar{s}$ distribution

Note that there are no $x\bar{c}$ or $x\bar{b}$ distributions in this scheme. For the ZMVFN scheme the grids carry the same names with *_ff* replaced by *_zm*, and for the TRVFN scheme they carry the same names with *_ff* replaced by *_rt*. These two schemes also have grids: *cbar_zm_tot.dat*, *bbar_zm_tot.dat*, *cbar_tr_tot.dat*, *bbar_tr_tot.dat*, for the $x\bar{c}$ and $x\bar{b}$ distributions
The structure functions available currently are those for γ exchange in the $e^{\pm}$ NC processes: F_2^{γ}, F_L^{γ}, F_2^{charm} and those for γ and Z_0 exchange in the $e^{\pm}$ NC processes: F_2, F_L and xF_3. For the FFN heavy quark scheme;

- *f2_ff_tot.dat* is the grid for $F_2^{\gamma}(epNC)$
- *fl_ff_tot.dat* is the grid for $F_L^{gamma}(epNC)$
- *f2c_ff_tot.dat* is the grid for $F_2^{charm}(epNC)$

- $f2nc\_ff\_tot.dat$ is the grid for $F_2(epNC)$
- $flnc\_ff\_tot.dat$ is the grid for $F_L(epNC)$
- $xf3nc\_ff\_tot.dat$ is the grid for $xF_3(epNC)$

For the ZMVFN scheme the grids carry the same names with $\_ff$ replaced by $\_zm$, and for the TRVFN scheme they carry the same names with $\_ff$ replaced by $\_rt$. (For TRVFN the Charm structure function needs an update)
The reduced cross-sections available are: $\tilde{\sigma}(e^+pCC)$, $\tilde{\sigma}(e^-pCC)$, $\tilde{\sigma}(e^+pNC)$, $\tilde{\sigma}(e^-pNC)$, for HERA running with lepton beam energy 27.5GeV, proton beam energy 920GeV.
For the FFN scheme;

- $epnc\_ff\_tot.dat$ is the grid for the reduced cross-section $\tilde{\sigma}(e^+pNC)$
- $emnc\_ff\_tot.dat$ is the grid for the reduced cross-section $\tilde{\sigma}(e^-pNC)$

For this scheme the CC cross-sections should not be used since the W exchange was not implemented fully in QCDNUM. For the ZMVFN scheme the grids carry the same names with $\_ff$ replaced by $\_zm$, and for the TRVFN scheme they carry the same names with $\_ff$ replaced by $\_rt$. These schemes also have the grids: *epcc_zm_tot.dat*, *emcc_zm_tot.dat* and *epcc_tr_tot.dat*, *emcc_tr_tot.dat* for the reduced cross-sections $\tilde{\sigma}(e^+pCC)$, $\tilde{\sigma}(e^-pCC)$.
An example of the use of such grids is given in the PAW kumacs *qerr_x.kumac*, *emccerr.kumac*, which need the routine $f2qcdfine.f$.
Ken's routines provide a more sophisticated interface routine?