HIGGSPLOSION

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- VVK & Spannowsky 1704.03447
- VVK 1705.04365

SM: Unitarity, Hierarchy and HIGGSPLOSION

- Before the discovery of the Higgs boson, massive Yang-Mills theory violated unitarity — problem with high-energy growth of 2 -> 2 processes

- Discovery of the (elementary) Higgs made the SM theory self-consistent

- But a new unitarity problem — caused by the elementary Higgs bosons — appears to occur for processes with large final state multiplicities n >> 1

- Plus: Higgs brings in the Hierarchy problem: radiative corrections push the Higgs mass to the new physics (high) scale:

$$m_h^2 \simeq m_0^2 + \delta m^2_{\rm new}$$

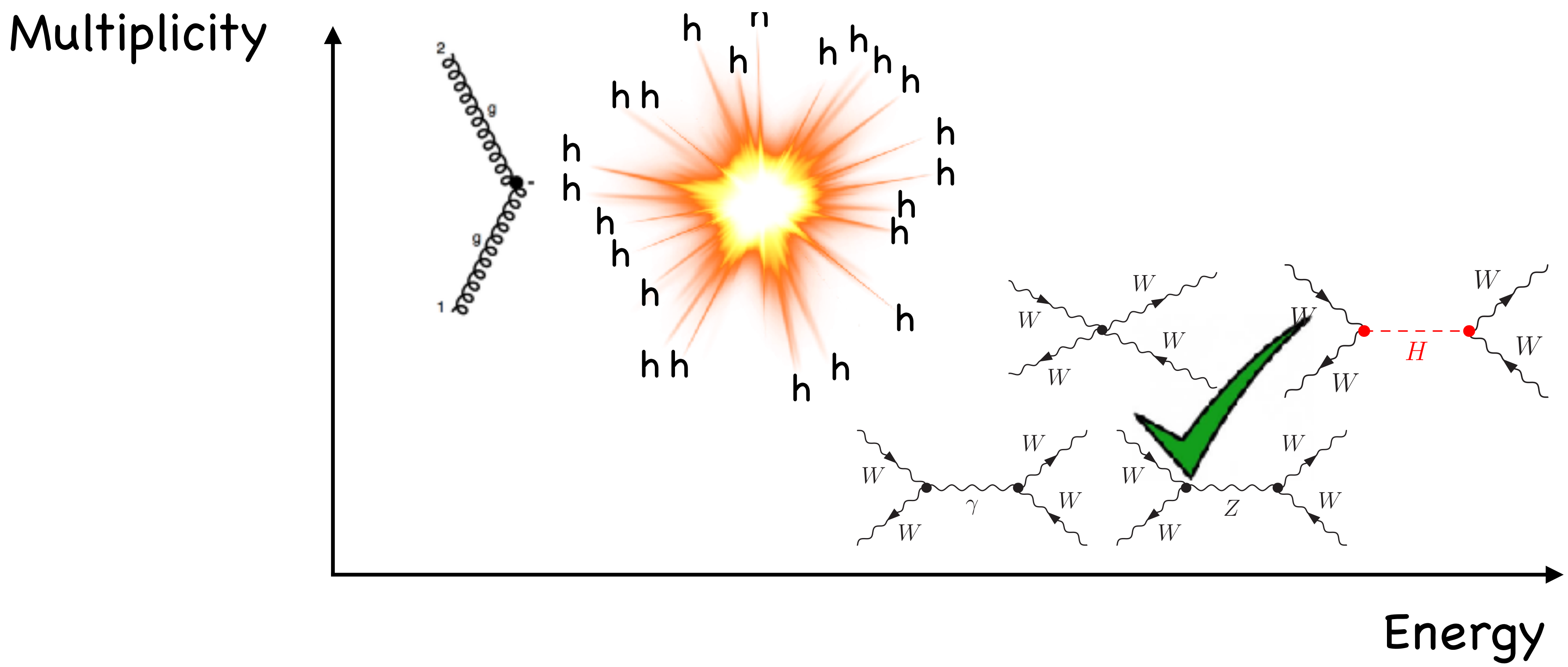
- In this talk: consider n~100s of Higgs bosons produced in the final state n x lambda >> 1. Investigate scattering processes at ~ 100 TeV energies

- HIGGSPLOSION offers a solution to both: it restores the unitarity of high-multiplicity processes and dynamically cuts off the values of the loop momenta contributing to the radiative corrections to the Higgs mass.

SM: Unitarity, Hierarchy and HIGGSPLOSION

After the discovery of the Higgs boson - complete Standard Model

Situation at tree-level



Perturbative unitarity violated at very high multiplicities → model inconsistent (at high multiplicities)

\+ Hierarchy problem (Loop level)

$$m_h^2 \simeq m_0^2 + \delta m_{\text{new}}^2$$

HIGGSPLOSION and HIGGSPERSION

- At high energies (100 TeV range), production of multiple Higgs and vector bosons becomes kinematically possible

- HIGGSPLOSION: Cross-sections computed in weakly-coupled perturbation theory become unsuppressed above certain critical values of n and E

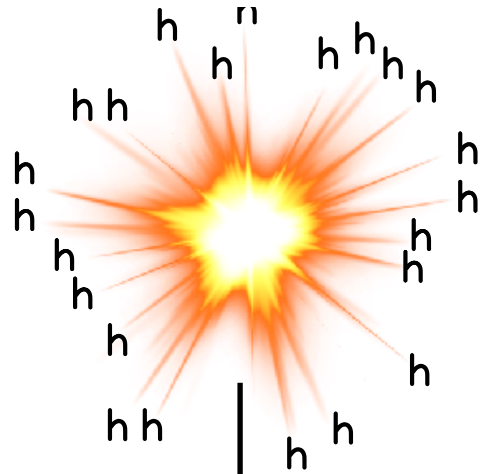
- This also applies to partial decay widths of highly-energetic states

- But there are no violations of perturbative unitarity due to the related HIGGSPERSION mechanism [exponential growth is tamed above Ec]

- [Similar considerations should also apply to high-multiplicity longitudinal W and Z production]

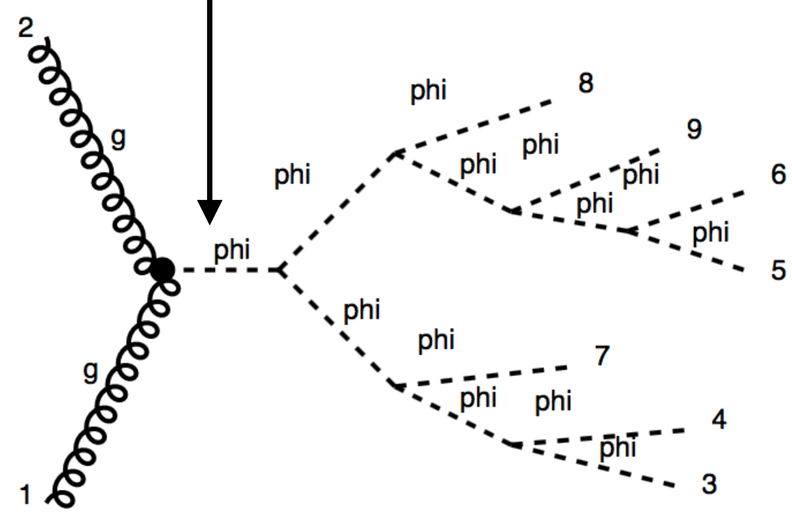
HIGGSPLOSION and HIGGSPERSION

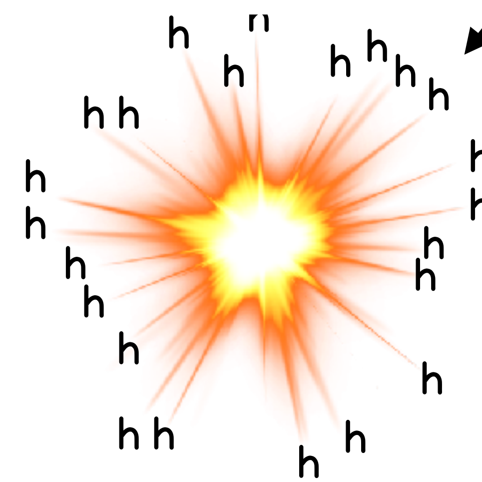
Include self-energy loops



$$\Gamma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon)\ , \qquad \text{and} \qquad \sigma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon)$$

Tree-level LO sketch:

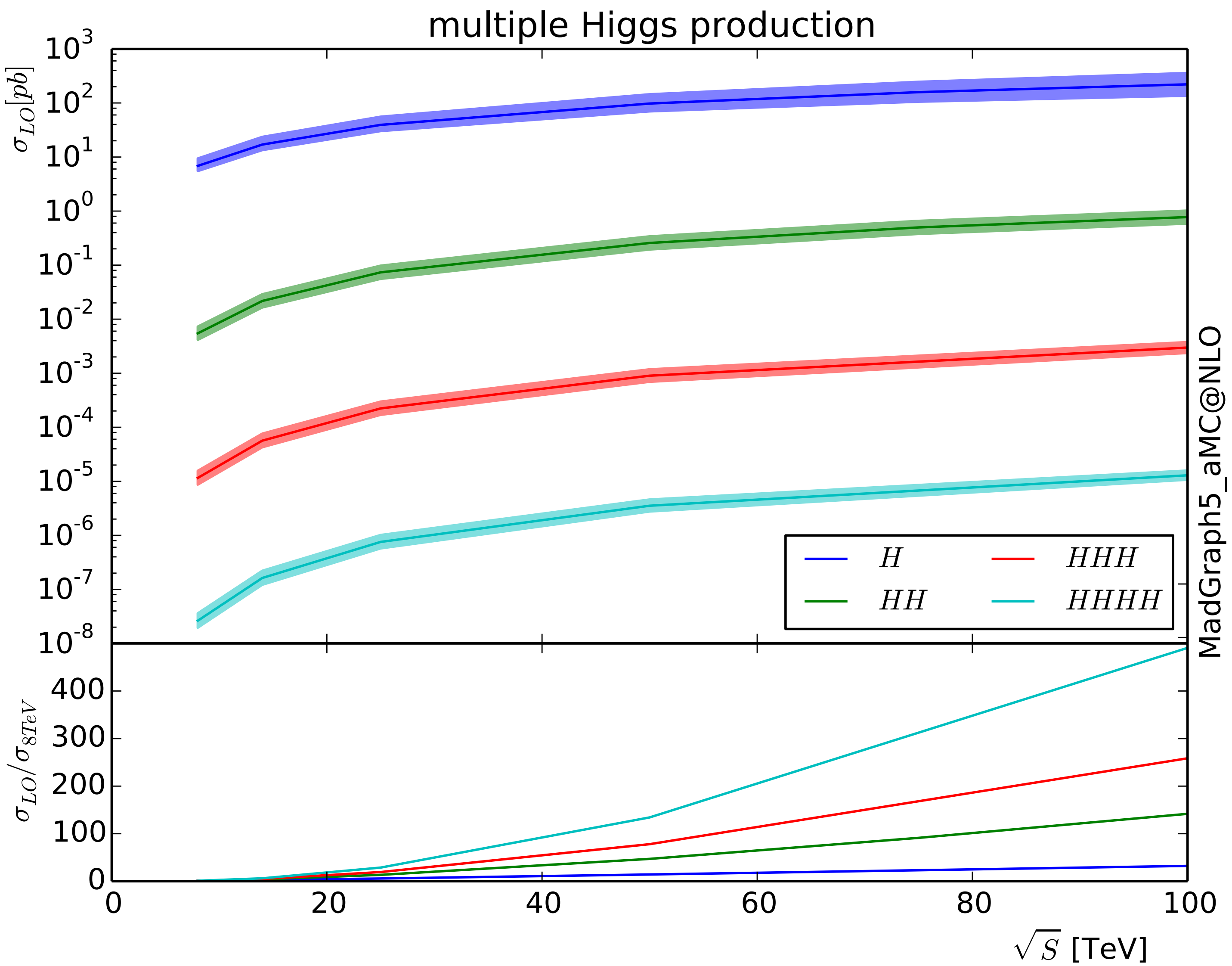




$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - M_h^2 + i\, M_h\, \Gamma(p^2)} \times \mathcal{M}_{h^*\to n\times h}$$

$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 m_t^2 \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{s^2 + M_h^4 \mathcal{R}^2} \times (2\lambda)^{n-1}\, \mathcal{R}_n$$

Warm-up: 1,2,3,4 Higgs bosons production
8 TeV < E < 100 TeV



Up to 4 Higgses with
MSTW2008 pdf set &
MadGraph5_aMC@NLO
for different energies

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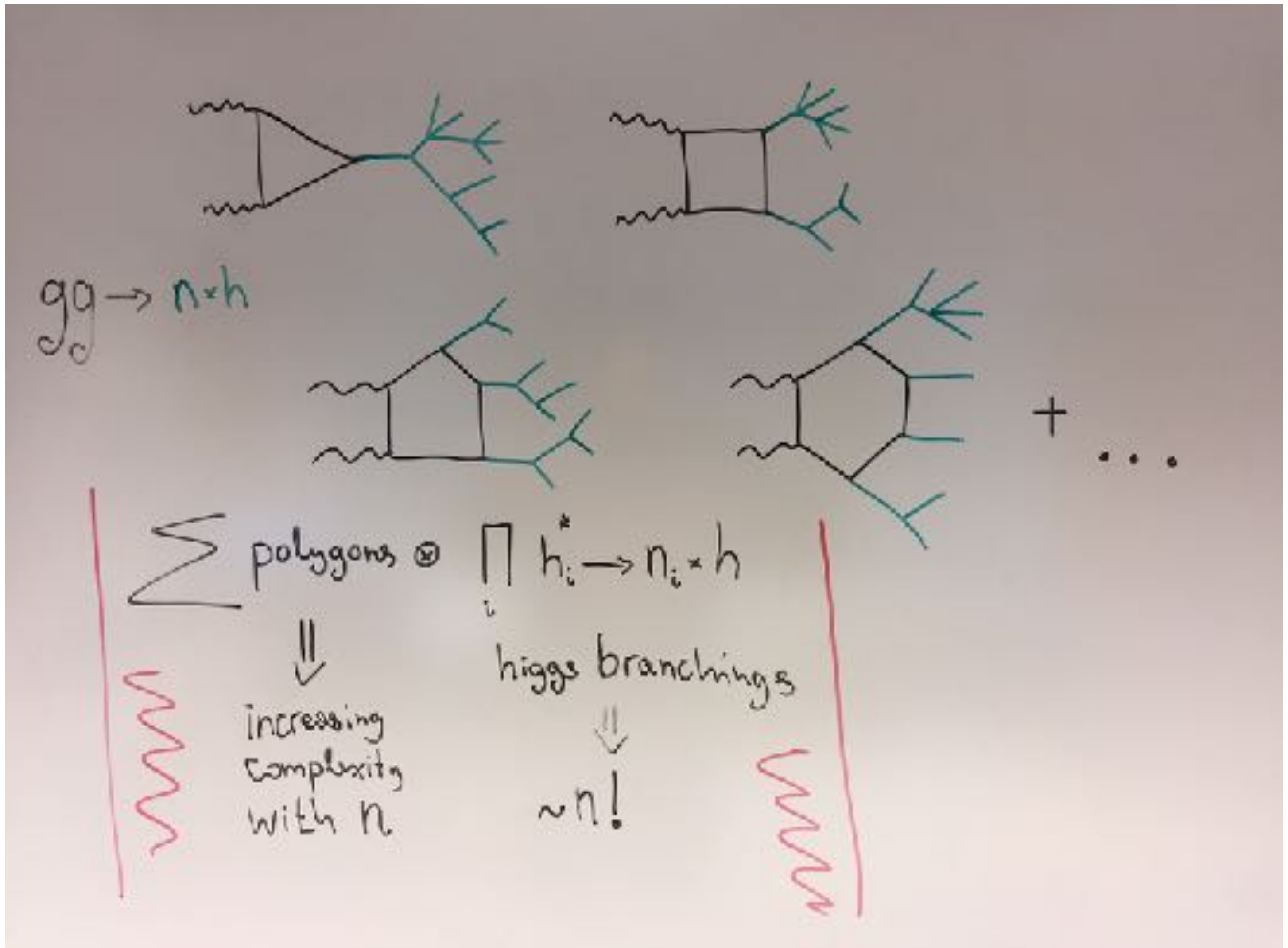
- What if ~100 Higgs bosons are produced in the final state at 100 TeV ?

Gluon fusion multi-Higgs production at large n



Two immediate problems to address:

1-loop polygons with up to n-2 edges increasing technical complexity

1-> n x h tree-level (& loop-corrected) Higgs branchings grow as n!

Will address this first

$$\mathcal{A}_{gg\to n\times h} = \sum_{\text{polygons}} \mathcal{A}^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\ldots+n_k=n} \prod_{i=1}^{k} \mathcal{A}_{h_i^*\to n_i\times h}$$

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Tree-level n-point Amplitudes on mass threshold

The amplitude $\mathcal{A}_{1\to n}$ for the field ϕ to create n particles in the ϕ^4 theory,

$$\mathcal{L}_\rho(\phi) \;=\; \frac{1}{2}\,(\partial\phi)^2 - \frac{1}{2}M^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \rho\,\phi\,,$$

is derived by applying the LSZ reduction technique:

$$\langle n|\phi(x)|0\rangle \;=\; \lim_{\rho\to 0}\left[\prod_{j=1}^{n}\lim_{p_j^2\to M^2}\int d^4x_j e^{ip_j\cdot x_j}(M^2-p_j^2)\frac{\delta}{\delta\rho(x_j)}\right]\langle 0_{\rm out}|\phi(x)|0_{\rm in}\rangle_\rho\,.$$

Tree-level approximation is obtained via $\langle 0_{\rm out}|\phi(x)|0_{\rm in}\rangle_\rho \longrightarrow \phi_{\rm cl}(x)$ where $\phi_{\rm cl}(x)$ is a solution to the classical field equation.

On **mass threshold** limit all outgoing particles are produced at rest, $\vec{p}_j = 0$ and we set all $p_j^\mu = (\omega, \vec{0})$ and $\rho(x) = \rho(t) = \rho_0(\omega)\, e^{i\omega t}$. Hence,

$$(M^2-p_j^2)\frac{\delta}{\delta\rho(x_j)} \longrightarrow (M^2-\omega^2)\frac{\delta}{\delta\rho(t_j)} = \frac{\delta}{\delta z(t_j)}\,,$$

$$z(t) \;:=\; \frac{\rho_0(\omega)\, e^{i\omega t}}{M^2-\omega^2-i\epsilon} \;:=\; z_0\; e^{i\omega t}\,, \quad z_0 = \text{finite const}$$

Tree-level amplitudes in phi^4 on mass threshold

Brown 9209203

The generating function of tree amplitudes on multiparticle thresholds is a classical solution. It solves an ordinary differential equation with no source term,

$$d_t^2\phi + M^2\phi + \lambda\phi^3 \;=\; 0\,.$$

The solution contains only positive frequency harmonics, i.e. the Taylor expansion in $z(t)$,

$$\phi_{\rm cl}(t) \;=\; z(t) \;+\; \sum_{n=2}^{\infty} d_n\, z(t)^n\,, \qquad z := \; z_0\, e^{iMt}$$

Coefficients d_n determine the actual amplitudes by differentiation w.r.t. z,

$$\mathcal{A}_{1\to n} \;=\; \left(\frac{\partial}{\partial z}\right)^n \phi_{\rm cl}\Big|_{z=0} \;=\; n!\, d_n \qquad \textbf{Factorial growth!!}$$

$$\phi_{\rm cl}(t) \;=\; \frac{z(t)}{1-\frac{\lambda}{8M^2}\, z(t)^2} \qquad \mathcal{A}_{1\to n} \;=\; n!\, \left(\frac{\lambda}{8M^2}\right)^{\frac{n-1}{2}}$$

Tree-level 1*->n Amplitudes on multi-particle mass thresholds determined by classical solutions that are uniform in space: h(x,t)=h(t)

Lagrangian for the scalar field:

$$\mathcal{L}(h) = \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 ,$$

prototype of the Higgs in the unitary gauge

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h = -\lambda h^3 + \lambda v^2 h ,$$

has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$h_{\rm cl}(t) = v\, \frac{1 + \frac{z(t)}{2v}}{1 - \frac{z(t)}{2v}} , \quad \text{where} \quad z(t) = z_0\, e^{iM_h t} = z_0\, e^{i\sqrt{2\lambda}\, v\, t}$$

$$h_{\rm cl}(t) = 2v \sum_{n=0}^{\infty} \left(\frac{z(t)}{2v}\right)^n d_n = v + 2v \sum_{n=1}^{\infty} \left(\frac{z(t)}{2v}\right)^n ,$$

i.e. with $d_0 = 1/2$ and all $d_{n\geq 1} = 1$.

$$\mathcal{A}_{1\to n} = \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} = \boxed{n!\,(2v)^{1-n}}$$

Factorial growth

L. Brown 9209203

Factorial growth of large-n scalar amplitudes on mass thresholds: E=nm

- The n! growth of perturbative amplitudes is not entirely surprising: it reflects the large-n behaviour of perturbation theory:

- [Use of classical solutions is equivalent to summing over tree-level Feynman diagrams; the number of contributing Feynman diagrams is known to grow factorially with n]

- Important to distinguish between the two types of large-n corrections:

(a) *higher-order* perturbative corrections to some leading-order quantities

(b) our case where the *leading-order* tree-level contribution to the 1*->n Amplitude grows factorially with the particle multiplicity n of the final state.

- The n! growth of n-point perturbative Amplitudes persists also above the threshold => can integrate over n-particle phase space to obtain cross-sections

- This was studied in the 90s in scalar QFTs (Voloshin; Son; Libanov, Rubakov, Troitski; …)

- But now realised that the characteristic energy scale for EW applications starts in the 50-100 TeV range. FCC would provide an exciting challenge to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.

- [Critical energy scale above which the production may be unsuppressed is ~50-100 TeV]

Similar story also holds in the Gauge-Higgs theory for tree-level amplitudes on multi-particle mass thresholds

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These equations are solved by iterations (numerically) with Mathematica. The double Taylor expansion of the generating functions takes the form:

$$h_{\rm cl}(z,w^a) \;=\; 2v \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} d(n,2k) \left(\frac{z}{2v}\right)^n \left(\frac{w^a w^a}{(2v)^2}\right)^k ,$$

$$A^a_{L\,\rm cl}(z,w^a) \;=\; w^a \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a(n,2k) \left(\frac{z}{2v}\right)^n \left(\frac{w^a w^a}{(2v)^2}\right)^k ,$$

where $d(n,2k)$ and $a(n,2k)$ are determined from the iterative solution of EOM.

By repeatedly differentiating these with respect to z and w^a for the Higgs to n Higgses and m longitudinal Z bosons threshold amplitude we get,

$$\mathcal{A}(h \to n\times h + m\times Z_L) = \; (2v)^{1-n-m}\, n!\, m!\, d(n,m)\,,$$

and for the longitudinal Z decaying into n Higgses and $m+1$ vector bosons,

$$\mathcal{A}(Z_L \to n\times h + (m+1)\times Z_L) = \; \frac{1}{(2v)^{n+m}}\, n!\, (m+1)!\, a(n,m)\,.$$

Factorial growth reemains (in n and in m) !

Tree-level Amplitudes *above mass thresholds* are determined by recursive solutions to classical equations — now include the kinematic dependence

$$-\,(\partial^\mu\partial_\mu + M_h^2)\,\varphi \;=\; 3\lambda v\,\varphi^2 \;+\; \lambda\,\varphi^3$$

This classical equation for $\varphi(x) = h(x) - v$ determines directly the structure of the recursion relation for tree-level scattering amplitudes:

$$(P_{\rm in}^2 - M_h^2)\,\mathcal{A}_n(p_1\ldots p_n) \;=\; 3\lambda v \sum_{n_1,n_2}^{n} \delta^n_{n_1+n_2} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)},\ldots,p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\ldots p_{n_2}^{(2)})$$

$$+\lambda \sum_{n_1,n_2,n_3}^{n} \delta^n_{n_1+n_2+n_3} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)}\ldots p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\ldots p_{n_2}^{(2)})\,\mathcal{A}_{n_3}(p_1^{(3)}\ldots p_{n_2}^{(3)})$$

Away from the multi-particle threshold, the external particles 3-momenta $\vec{p}_i$ are non-vanishing. In the non-relativistic limit, the leading momentum-dependent contribution to the amplitudes is proportional to $E_n^{\rm kin}$ (Galilean Symmetry),

$$\mathcal{A}_n(p_1\ldots p_n) \;=\; \mathcal{A}_n + \mathcal{M}_n\,E_n^{\rm kin} \;:=\; \mathcal{A}_n + \mathcal{M}_n\,n\,\varepsilon\,,$$

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \;=\; \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n}\vec{p}_i^{\,2}\,.$$

In the non-relativistic limit we have $\varepsilon \ll 1$.

Above the n-particle thresholds:
solution of the recursion relations

$$\varepsilon = \frac{1}{n\, M_h}\, E_n^{\rm kin} \;=\; \frac{1}{n}\, \frac{1}{2M_h^2}\, \sum_{i=1}^{n} \vec{p}_i^{\;2}$$

$$\mathcal{A}_n(p_1 \ldots p_n) \;=\; n!\,(2v)^{1-n}\left(1 - \frac{7}{6}\, n\, \varepsilon \,-\, \frac{1}{6}\, \frac{n}{n-1}\, \varepsilon + \mathcal{O}(\varepsilon^2)\right).$$

An important observation is that by exponentiating the order-$n\varepsilon$ contribution, one obtains the expression for the amplitude which solves the original recursion relation to all orders in $(n\varepsilon)^m$ in the large-n non-relativistic limit,

$$\mathcal{A}_n(p_1 \ldots p_n) \;=\; n!\,(2v)^{1-n}\, \exp\left[-\frac{7}{6}\, n\, \varepsilon\right], \quad n \to \infty\,, \quad \varepsilon \to 0\,, \quad n\varepsilon = \text{fixed}\,.$$

Simple corrections of order ε, with coefficients that are not-enhanced by n are expected, but the expression is correct to all orders $n\varepsilon$ in the double scaling large-n limit. The exponential factor can be absorbed into the z variable so that

$$\varphi(z) = \sum_{n=1}^{\infty} d_n\, \left(z\, e^{-\frac{7}{6}\,\varepsilon}\right)^n\,,$$

- VVK 1411.2925

remains a solution to the classical equation and the original recursion relations.

Can now integrate over the phase-space

Phase-space integration

n Higgs bosons & m vector bosons,
take m=0 below:

$$\sigma_{n,m} \;=\; \int d\Phi_{n,m}\, \frac{1}{n!\, m!}\; \left|\mathcal{A}_{h^*\to n\times h+m\times Z_L}\right|^2 \;,$$

The n-particle Lorentz-invariant phase space volume element

$$\int d\Phi_n \;=\; (2\pi)^4\delta^{(4)}(P_{\rm in}-\sum_{j=1}^{n}p_j)\,\prod_{j=1}^{n}\int \frac{d^3p_j}{(2\pi)^3\,2p_j^0}\,,$$

in the large-n non-relativistic limit with $n\varepsilon_h$ fixed becomes,

$$\Phi_n \;\simeq\; \frac{1}{\sqrt{n}}\left(\frac{M_h^2}{2}\right)^n \exp\left[\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi}+1\right) \;+\; \frac{n\varepsilon_h}{4} \;+\; \mathcal{O}(n\varepsilon_h^2)\right]\;.$$

Repeating the same steps now including vector boson emissions,

$$\sigma_{n,m} \;\sim\; \exp\Bigg[2\log d(n,m) \;+\; n\left(\log\frac{\lambda n}{4}-1\right) \;+\; m\left(\log\left(\frac{g^2 m}{32}\right)-1\right)$$
$$+\;\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi}+1\right) \;+\; \frac{3m}{2}\left(\log\frac{\varepsilon_V}{3\pi}+1\right) \;-\; \frac{25}{12}\,n\varepsilon_h \;-\; 3.15\,m\varepsilon_V \;+\; \mathcal{O}(n\varepsilon_h^2+m\varepsilon_V^2)\Bigg]$$

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In general: Methods based on classical solutions result in the exponential form for the n-particle cross-section: exp[F_holy_grail]

- Libanov, Rubakov, Son, Troitsky; M Voloshin; …

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

bare cross-section [ignoring the width effect for now]

$$\boxed{\sigma_n \propto \exp\left[n\left(\log\frac{\lambda n}{4}-1\right)+\frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}n\,\varepsilon\right]}$$

More generally, in the large-n limit with $\lambda n =$ fixed and $\varepsilon =$ fixed, one expects

$$\boxed{\sigma_n \propto \exp\left[\frac{1}{\lambda}F_{\rm h.g.}(\lambda n,\varepsilon)\right]}$$

[e.g. Libanov, Rubakov, Troitsky review 1997]

where the *holy grail* function $F_{\rm h.g.}$ is of the form,

known function at tree level

$$\boxed{\frac{1}{\lambda}F_{\rm h.g.}(\lambda n,\varepsilon) = \frac{\lambda n}{\lambda}\left(f_0(\lambda n)+f(\varepsilon)\right)}$$

known at eps<<1

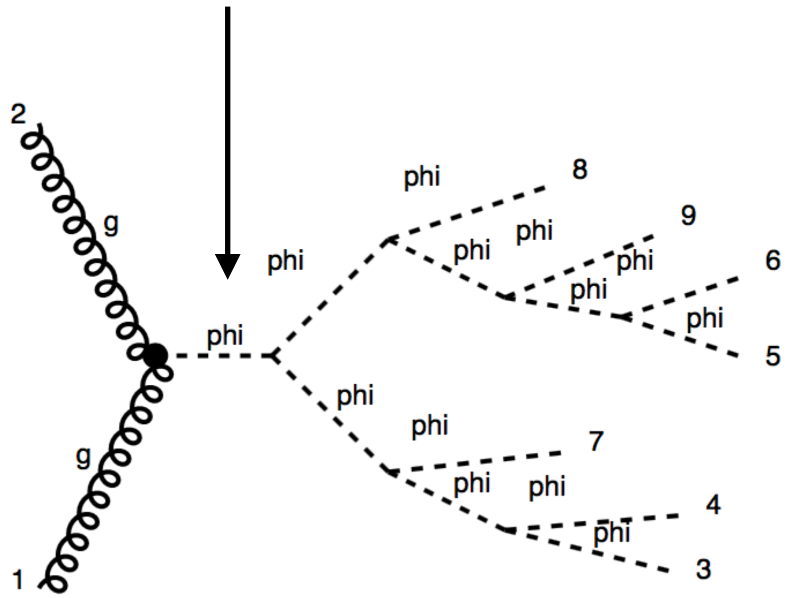
In our higgs model, i.e. the scalar theory with SSB,

$$\boxed{\begin{array}{rclr} f_0(\lambda n) & = & \log\frac{\lambda n}{4}-1 & \text{at tree level} \\ f(\varepsilon) & \to & \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\varepsilon & \text{for } \varepsilon\ll 1 \end{array}}$$

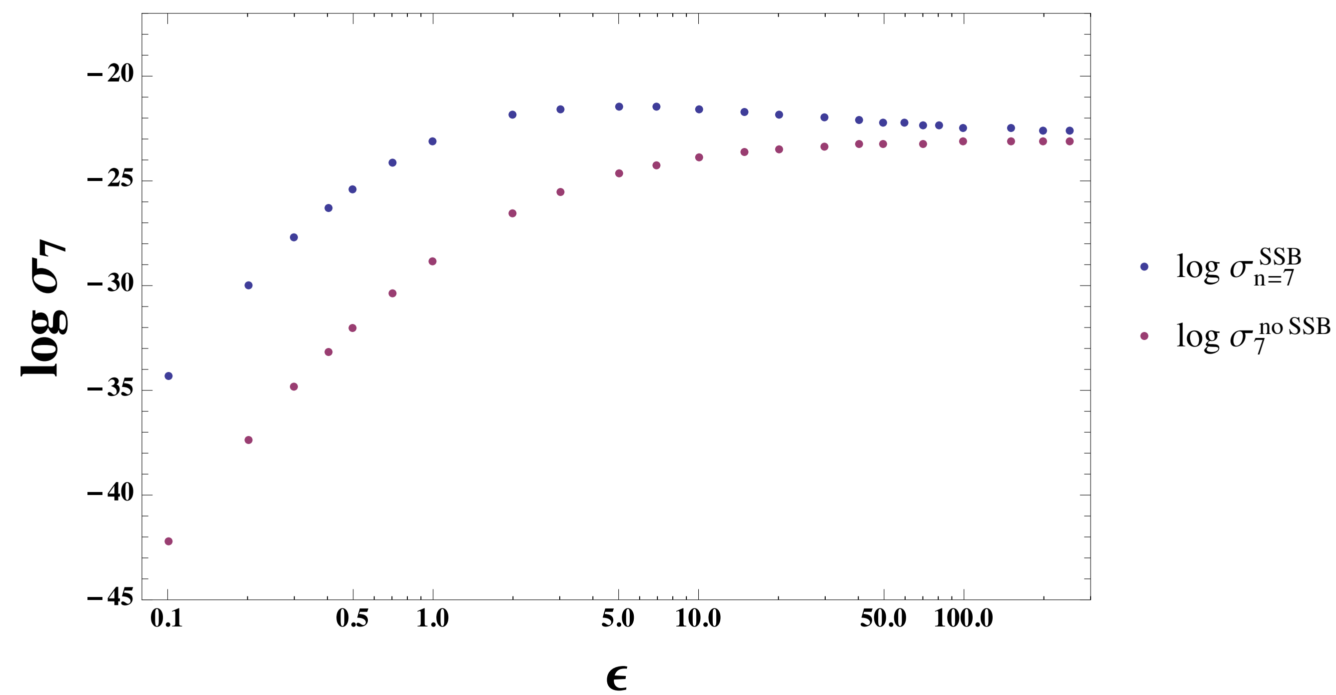
Next step:
compute $f(\varepsilon)$ for any epsilon

1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)

ignoring the width in the propagator for now



$\log \sigma_7^{\text{tree}}$ SSB & no SSB



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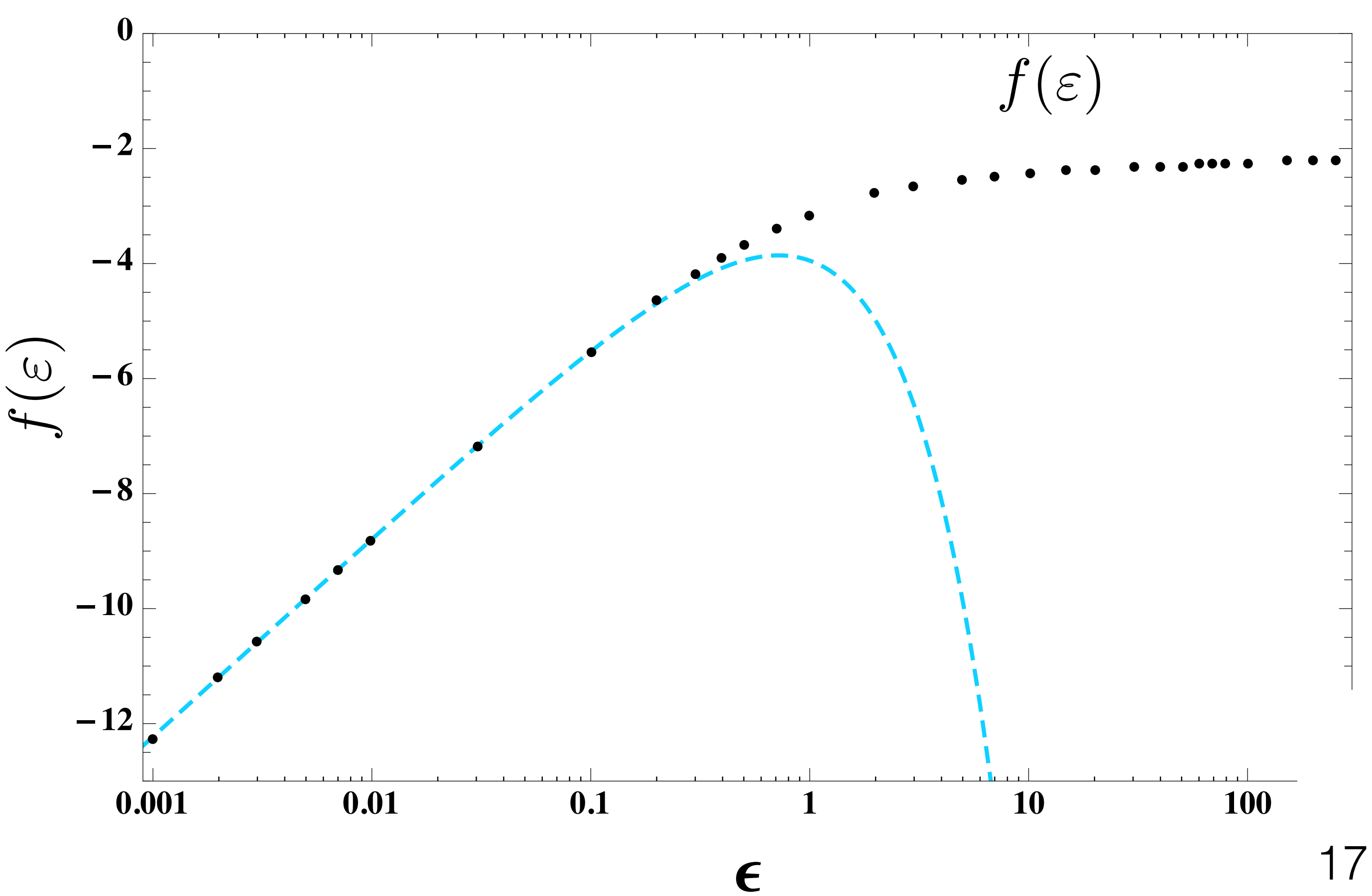

2. Assume large n, subtract $f_0(\lambda n)$ and extract $f(\varepsilon)$ using

$$\frac{1}{n}\log\sigma_n = \frac{1}{\lambda n}F_{\text{h.g.}}(\lambda n, \varepsilon) = f_0(\lambda n) + f(\varepsilon)$$

$$f_0(\lambda n) = \log\frac{\lambda n}{16} - 1$$

$f(\varepsilon)$ asymptotes to a const at large eps (highly relativistic final state)

Can also include *loop corrections* to amplitudes on thresholds:

The 1-loop corrected threshold amplitude for the pure n Higgs production:

$$\boxed{\phi^4 \text{ with SSB}: \quad \mathcal{A}_{1\to n}^{\text{tree+1loop}} = n!\,(2v)^{1-n}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)}$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$\boxed{\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\text{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]}$$

in the limit $\lambda \to 0$, $n \to \infty$ with λn^2 fixed. Here B is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*): $B = +\lambda n\,\frac{\sqrt{3}}{4\pi}$

$$\boxed{\begin{aligned} f_0(\lambda n) &= \log\frac{\lambda n}{4} - 1 + \lambda n\,\frac{\sqrt{3}}{4\pi} \\ f(\varepsilon) &\to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\,\varepsilon \qquad \text{for } \varepsilon \ll 1 \end{aligned}}$$

In Summary: first purely at tree level

1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)
2. Scale to large n using the known n-dependence in the holy grail without including the leading-loop factor in the exponent

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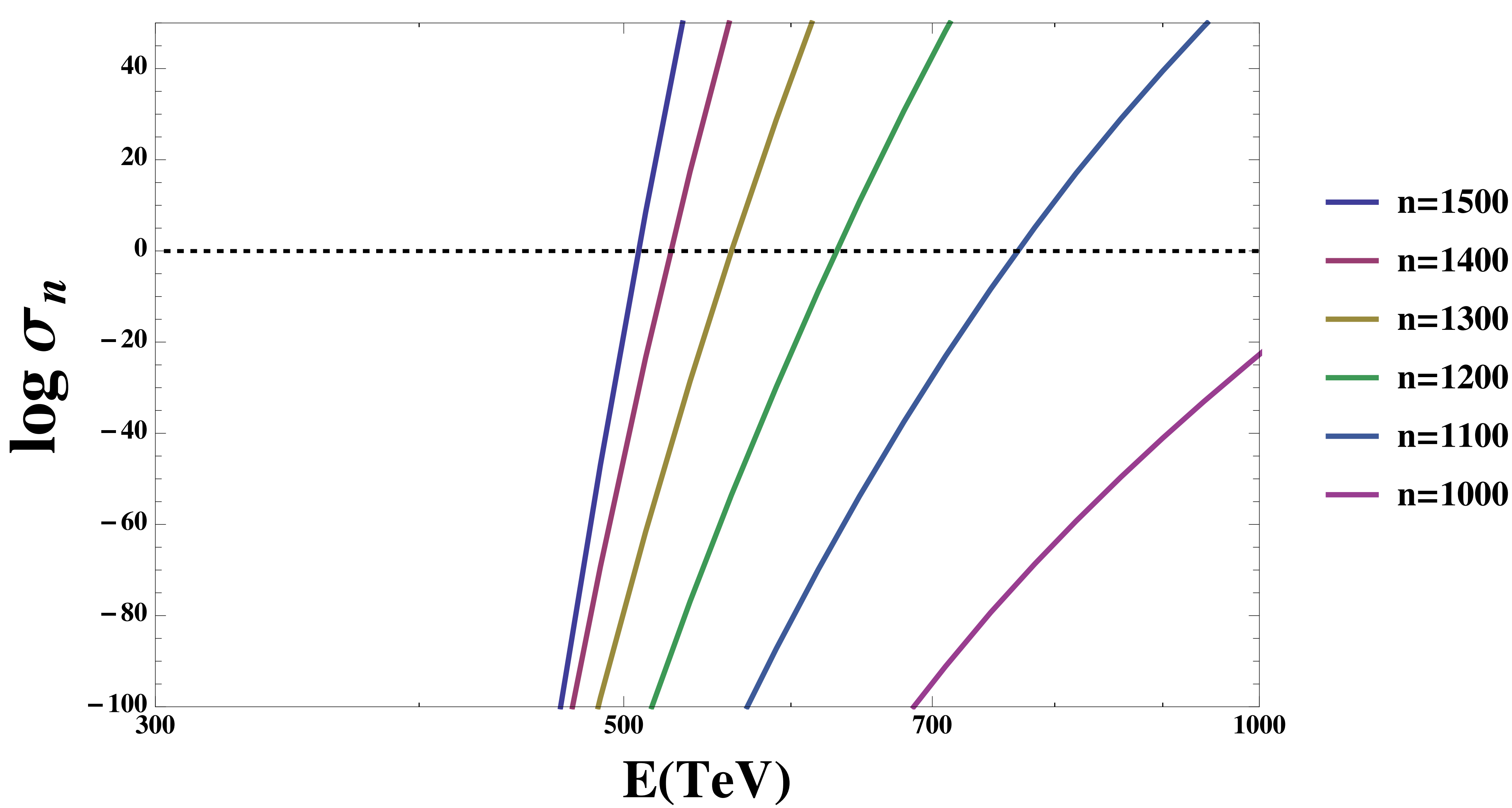
bare cross-section
[ignoring the width
effect for now]

$\log \sigma_n^{\rm tree}$



In Summary [taking into account leading order loop effects]:

1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)
2. Scale to large n using the known n-dependence in the holy grail including the leading-loop factor to the exponent $+\lambda n \frac{\sqrt{3}}{4\pi}$

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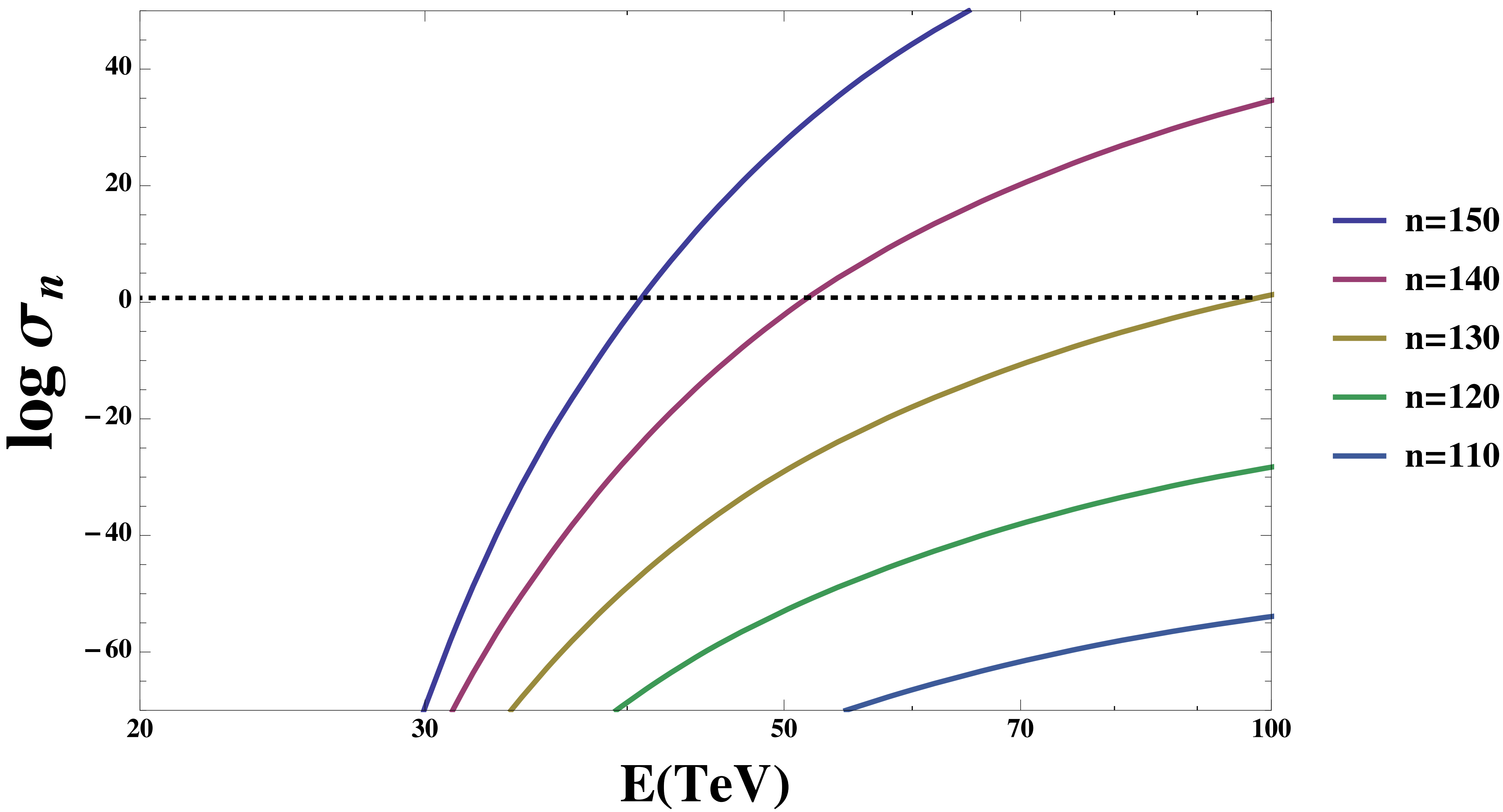
bare cross-section [ignoring the width effect for now]

$$\log \sigma_n^{\text{loop}}$$



Very recently: computed the rate R in the large lambda n limit:

using the semi-classical approach of Son and the thin-wall approximation

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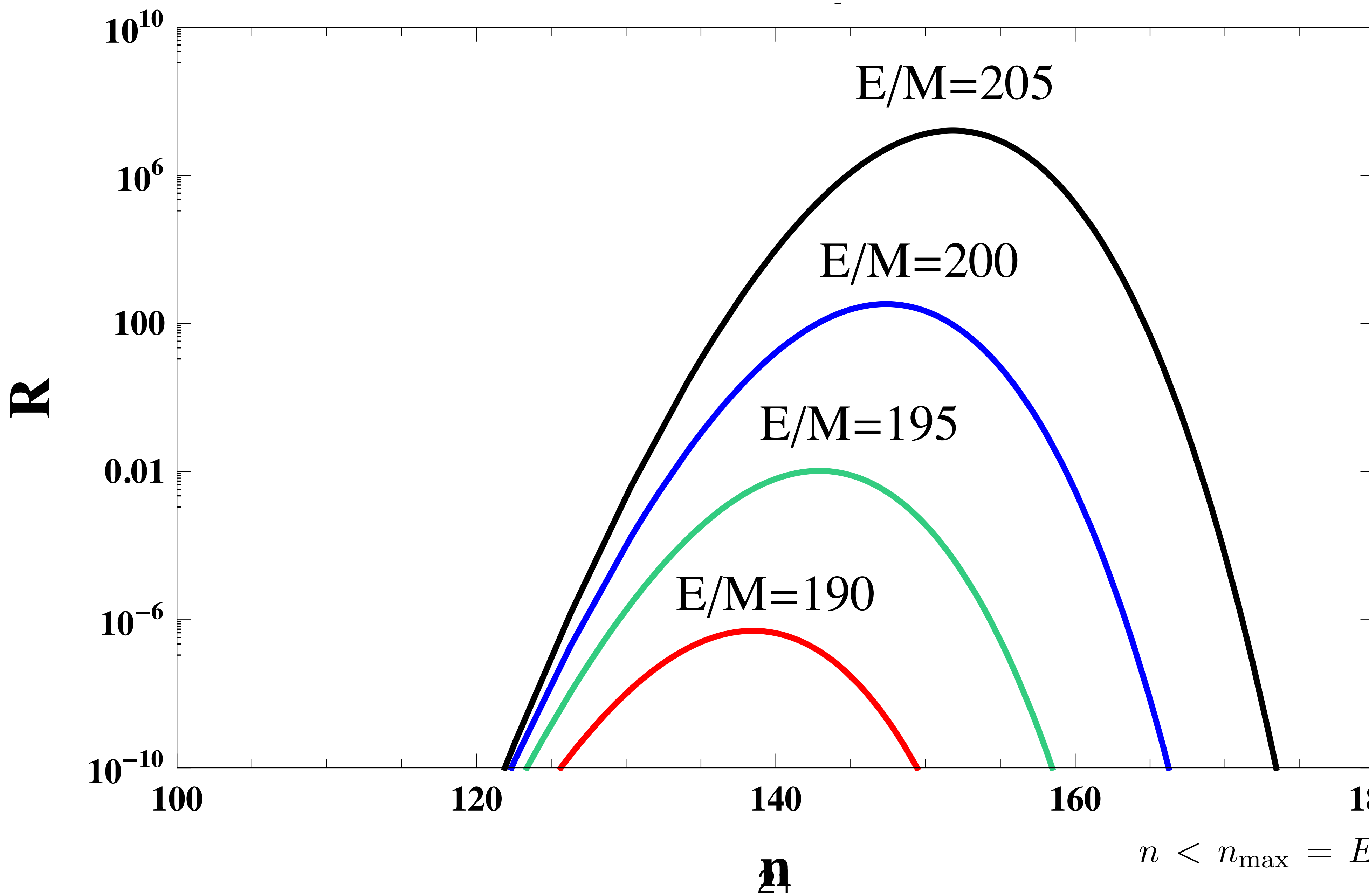
$$\mathcal{R} = \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + \boxed{3.02\sqrt{\frac{\lambda n}{4\pi}}} - 1 + \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) - \frac{25}{12}\,\varepsilon\right)\right]$$

$\lambda n \gg 1 \quad \text{small } \varepsilon$



$n < n_{\max} = E/M_h$

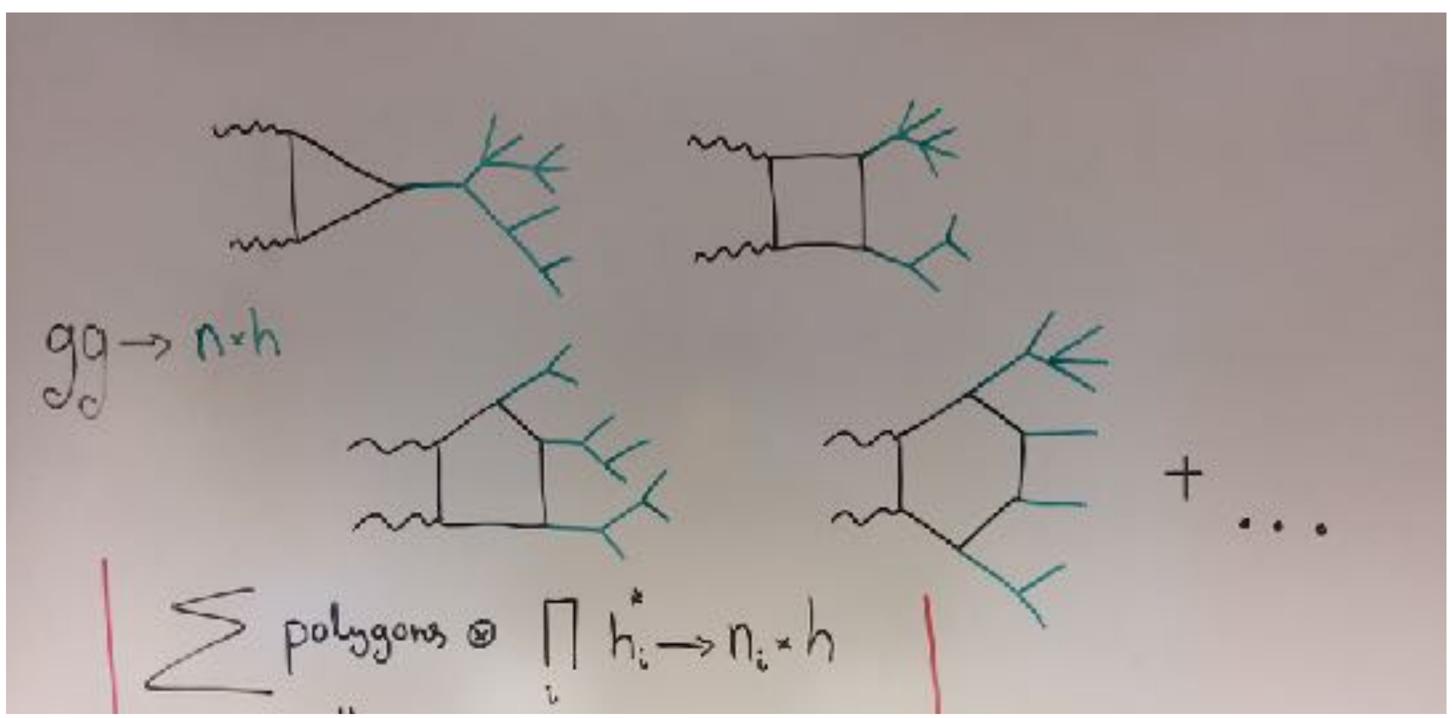
Now: full Gluon fusion process including polygons

$$\mathcal{A}_{gg\to n\times h} = \sum_{\text{polygons}} \mathcal{A}^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\ldots+n_k=n} \prod_{i=1}^{k} \mathcal{A}_{h_i^*\to n_i\times h}$$

1-loop polygons
Compute numerically
in the high-energy limit where E>> all kin scales

MadGraph5_aMC@NLO

1*->n multi-Higgs processes.
Already computed



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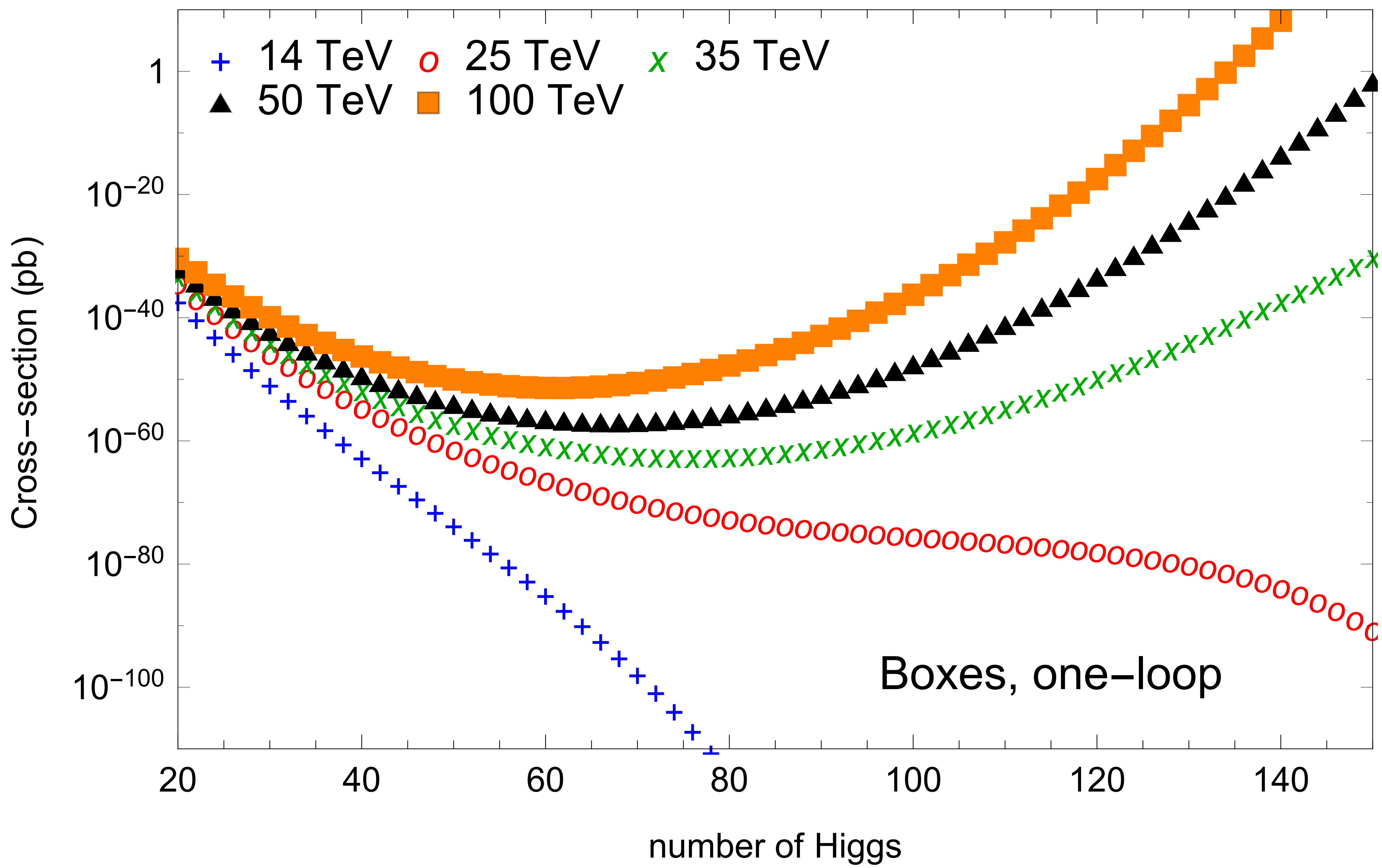
Finally: combine with the multi-Higgs branchings & convolute with gluon PDFs

Results: 20 to 150 Higgs bosons @ different collider energies



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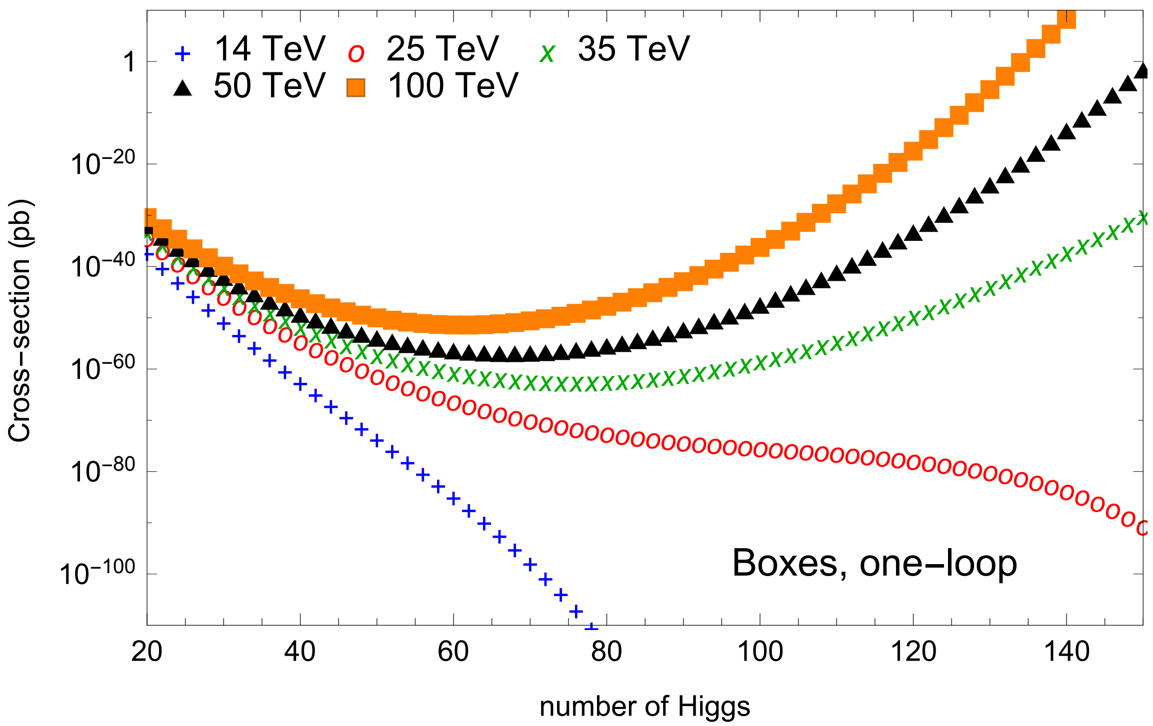
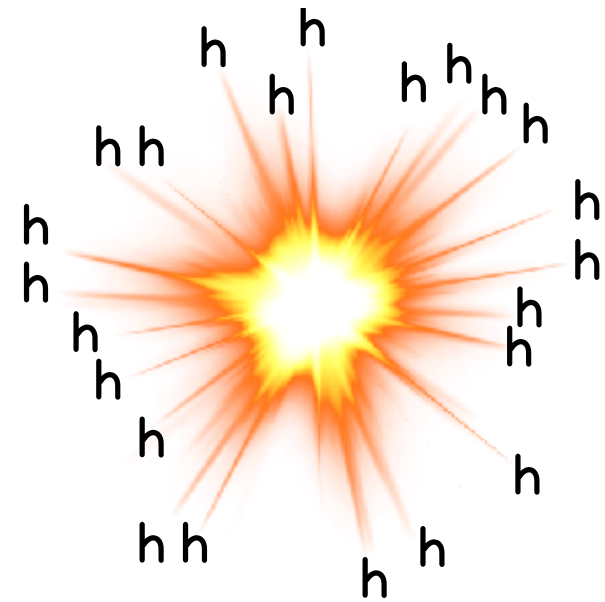
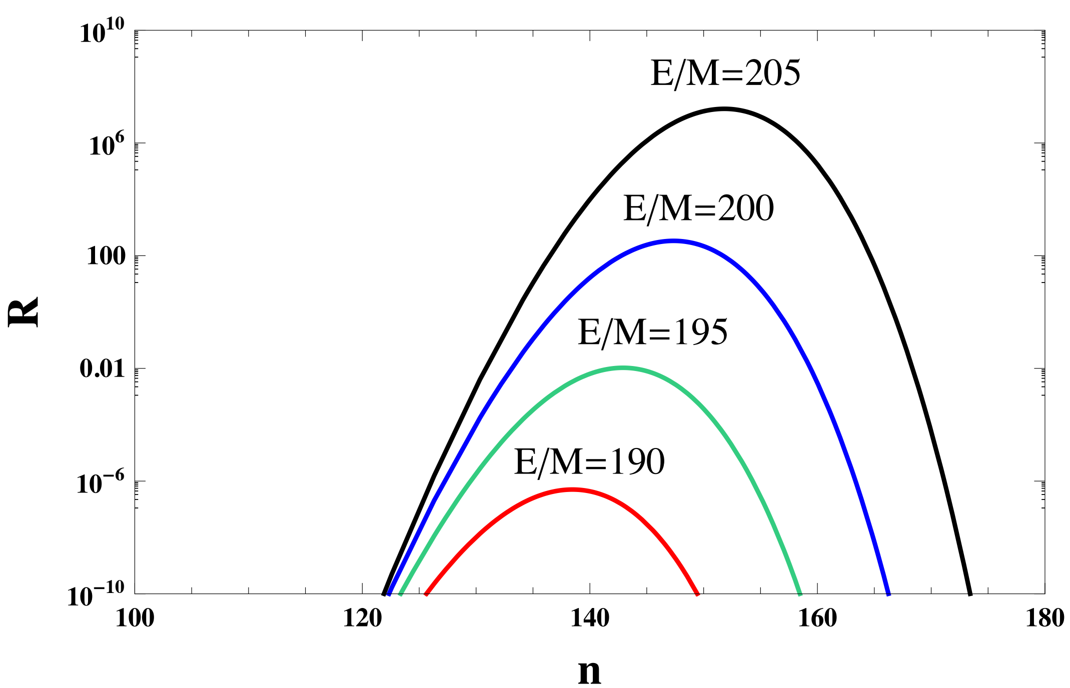
1. Can reach observable level at ~50-100TeV



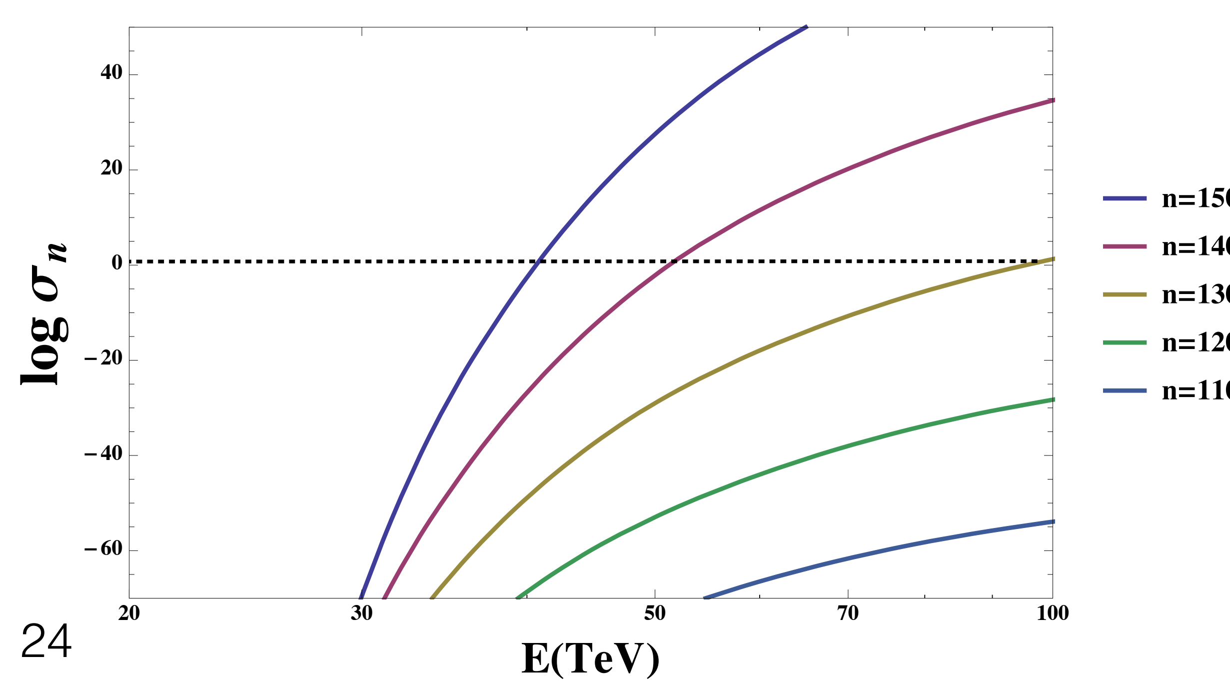
Multi-Higgs processes



2. Explosive growth at even higher energies ?



$\log \sigma_n^{\text{loop}}$



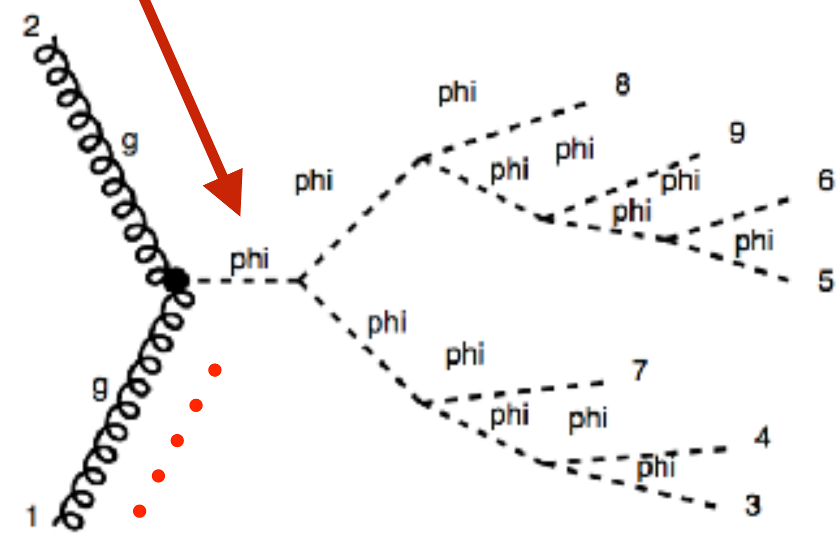
2. Explosive growth at even higher energies — No

Include the width effect in the cross-section

previous calculations neglected 'width'/self-energy contribution to scalar propagator

Analytic expression for process

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - M_h^2 + i\,M_h\,\Gamma(p^2)} \times \mathcal{M}_{h^*\to n\times h}$$



results in limit $s \gg M_h^2,\, m_t^2$ in

$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 m_t^2 \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{s^2 + M_h^4 \mathcal{R}^2} \times (2\lambda)^{n-1}\,\mathcal{R}_n$$

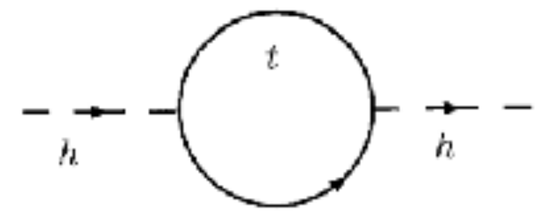
For large R cross section small $\sigma_{gg\to n\times h} \sim \begin{cases} \mathcal{R} & : \ \text{for } \mathcal{R} \lesssim 1 \\ 1/\mathcal{R} \to 0 & : \ \text{for } \mathcal{R} \gg 1 \text{ at } s \to \infty \end{cases}$

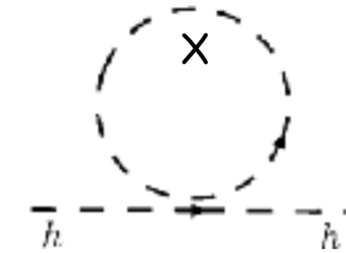
-> no violation of perturbative unitarity for large multiplicities

Higgsploding the Hierarchy problem

X=heavy state

$$\mathcal{L}_X = \frac{1}{2}\partial^\mu X\,\partial_\mu X - \frac{1}{2}M_X^2\,X^2 - \frac{\lambda_P}{4}\,X^2h^2 - \mu X h^2$$





$$\Delta M_h^2 \sim \lambda_P \int \frac{d^4p}{16\pi^4}\,\frac{1}{M_X^2 - p^2 + i\,\mathrm{Im}\,\Sigma_X(p^2)}$$

$$= \lambda_P \int \frac{d^4p}{16\pi^4}\left(\frac{M_X^2 - p^2}{(M_X^2 - p^2)^2 + (\mathrm{Im}\,\Sigma_X(p^2))^2} - \frac{i\,\mathrm{Im}\,\Sigma_X(p^2)}{(M_X^2 - p^2)^2 + (\mathrm{Im}\,\Sigma_X(p^2))^2}\right)$$

Due to Higgsplosion the multi-particle contribution to the width of X explode at $p^2 = s_\star$ where $\sqrt{s_\star} \simeq \mathcal{O}(25)\mathrm{TeV}$

→ It provides a sharp UV cut-off in the integral, possibly at $s_\star \ll M_X^2$

Hence, the contribution to the Higgs mass amounts to

$$\text{For}\quad \Gamma(s_\star) \simeq M_X \quad\text{at}\quad s_\star \ll M_X^2 \quad\Longrightarrow\quad \Delta M_h^2 \propto \lambda_P\,\frac{s_\star}{M_X^2}\,s_\star \ll \lambda_P\,M_X^2$$

and thus mends the Hierarchy problem by $\left(\frac{\sqrt{s_\star}}{M_X}\right)^4 \simeq \left(\frac{25\,\mathrm{TeV}}{M_X}\right)^4$

Summary 1/2

- The SM Higgs boson introduces the Hierarchy problem
- and the perturbative unitarity problem for high-n production
- The SM heals itself and resolves both these problems if the Higgsplosion mechanism is operative

$$\text{For} \quad \Gamma(s_\star) \simeq M_X \quad \text{at} \quad s_\star \ll M_X^2 \quad \Longrightarrow \quad \Delta M_h^2 \propto \lambda_P \, \frac{s_\star}{M_X^2} \, s_\star \ll \lambda_P \, M_X^2$$

$$\sigma_{gg\to n\times h} \sim \begin{cases} \mathcal{R} & : \text{ for } \mathcal{R} \lesssim 1 \\ 1/\mathcal{R} \to 0 & : \text{ for } \mathcal{R} \gg 1 \text{ at } s \to \infty \end{cases}$$

- No new physics degrees of freedom required — very minimal solution
- Predictions / falsifiability of Higgsplosion at 100 TeV (FCC). Applications to Cosmology, reheating,
- Leptogenesis, Flavour structure, Axions, (no) GUTs, etc

Summary 2/2

- The Higgsplosion / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplosion energy).
- UV-finiteness => all coupling constants slopes become flat above the Higgsplosion scale => automatic asymptotic safety
- [Below the Higgsplosion scale there is the usual logarithmic running]
- 1. Asymptotic Safety
- 2. No Landau poles for the U(1) and the Yukawa couplings
- 3. The Higgs self-coupling does not turn negative => stable EW vacuum
- No new physics degrees of freedom required — very minimal solution

Backup slides

Full Propagator with running width:

$$\Delta_\phi(p) \;=\; \int d^4x\, e^{ip\cdot x} \langle 0|T\left(\phi(x)\,\phi(0)\right)|0\rangle \;=\; \frac{i}{p^2 - m_0^2 - \Sigma(p^2) + i\epsilon}$$

$$m^2 - m_0^2 - \Sigma(m^2) \;=\; 0\;, \quad \text{or} \quad m^2 \;=\; m_0^2 + \mathrm{Re}\,\Sigma(m^2)\,.$$

$$\Delta_\phi(p) \;=\; \frac{i}{p^2 - m^2 - [\Sigma(p^2) - \Sigma(m^2)]} \;=\; \frac{i}{p^2 - m^2}\left(\frac{1}{1 - \frac{d\Sigma}{dp^2}\big|_{p^2=m^2} + \mathcal{O}(p^2 - m^2)}\right)\,,$$

$$Z_\phi \;=\; \left(1 - \frac{d\Sigma}{dp^2}\bigg|_{p^2=m^2}\right)^{-1}$$

$$\Delta_\phi(p) \;\simeq\; \frac{iZ_\phi}{p^2 - m^2 - iZ_\phi\,\mathrm{Im}\,\Sigma(p^2)} \;=\; \frac{iZ_\phi}{p^2 - m^2 + i\,m\,\Gamma(p^2)}\,,$$

$$\Delta_R(p) \;=\; \frac{i}{p^2 - m^2 - i\,\mathrm{Im}\,\Sigma_R(p^2)} \;=\; \frac{i}{p^2 - m^2 + i\,m\,\Gamma(p^2)}\,.$$

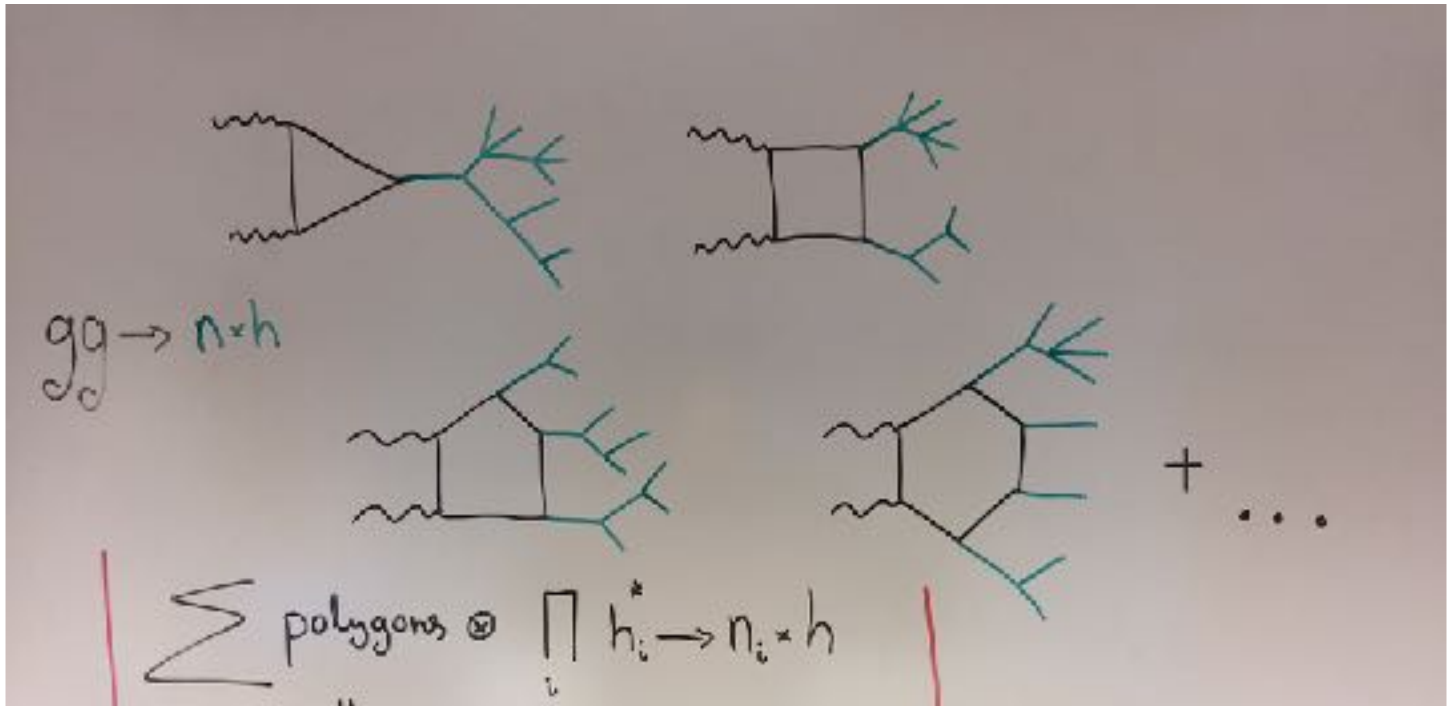
Gluon fusion process including polygons

$$\mathcal{A}_{gg\to n\times h} = \sum_{\text{polygons}} \mathcal{A}^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\ldots+n_k=n} \prod_{i=1}^{k} \mathcal{A}_{h_i^*\to n_i\times h}$$

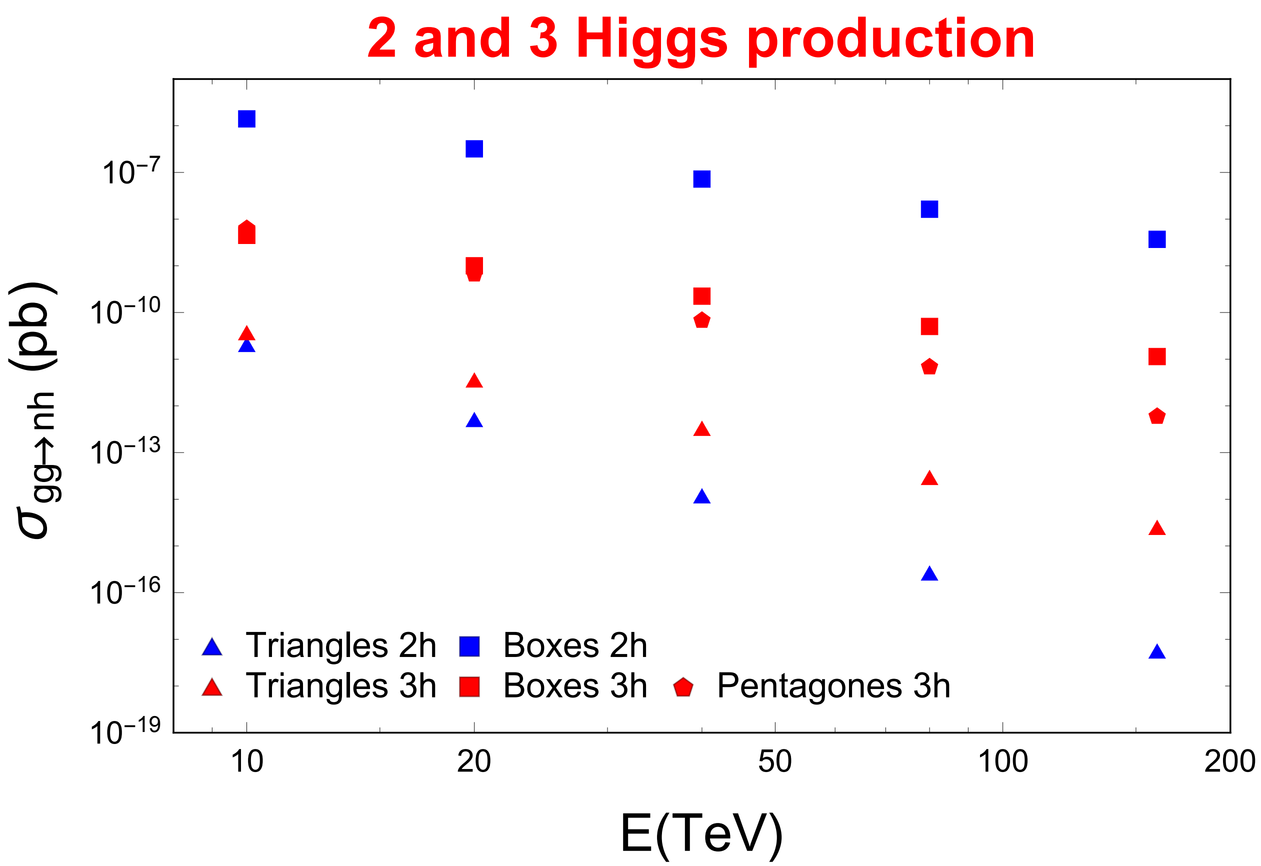
1-loop polygons
Compute numerically
in the high-energy limit where E>> all kin scales

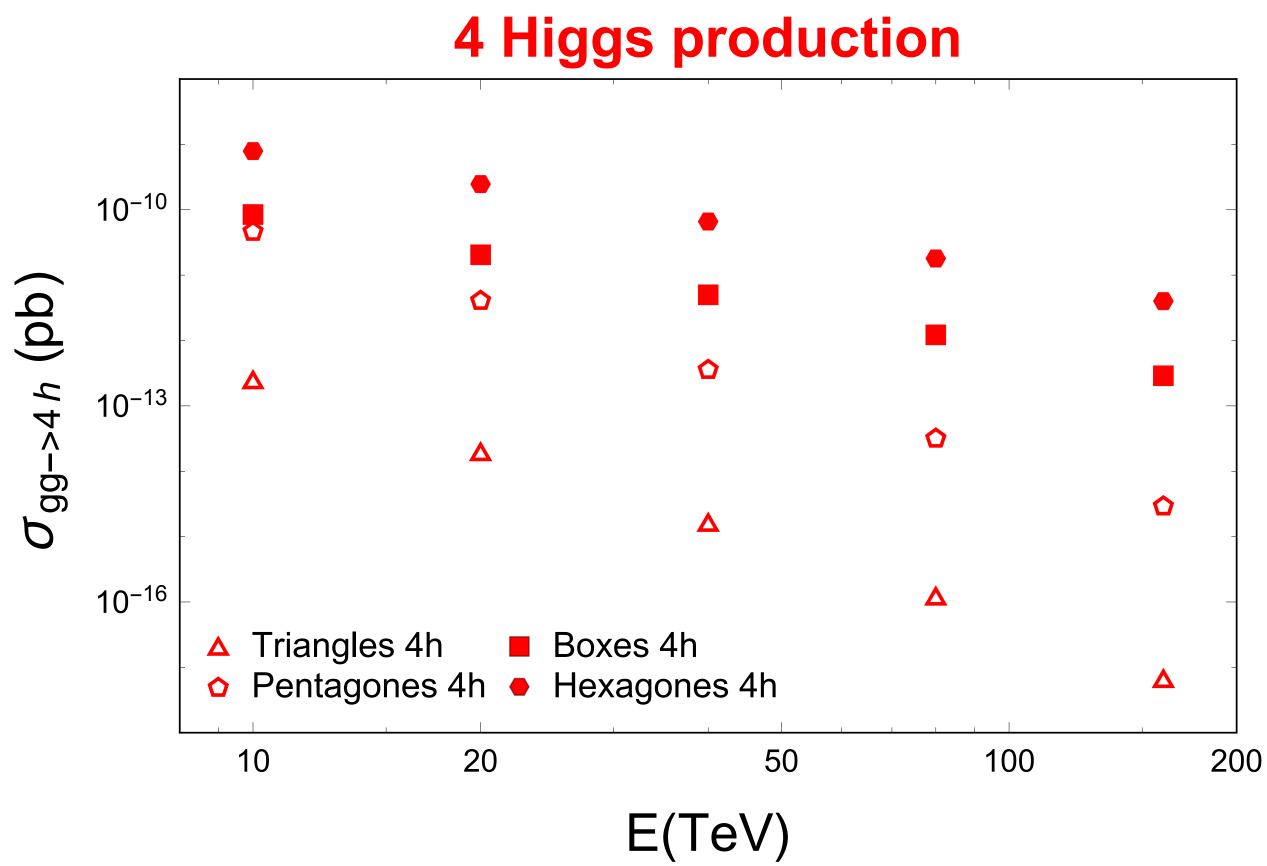
MadGraph5_aMC@NLO

1*->n multi-Higgs processes.
Already computed



Polygon contributions:





$s \gg m_t\,,\, M_h$ limit

	$\sigma_{gg\to hh}$	$\sigma_{gg\to hhh}$	$\sigma_{gg\to hhhh}$
Triangles	$y_t^2 \frac{m_t^2 M_h^2}{s^3} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^4}{v^4}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^6}{v^6}$
Boxes	$y_t^4 \frac{1}{s}$	$y_t^4 \frac{1}{s} \frac{M_h^2}{v^2}$	$y_t^4 \frac{1}{s} \frac{M_h^4}{v^4}$
Pentagons	–	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right)$	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$
Hexagons	–	–	$y_t^8 \frac{1}{s}$

- Compare to the high-energy scaling behaviour resulting from the effective vertices:

	$\sigma^{\rm eft}_{gg\to hh}$	$\sigma^{\rm eft}_{gg\to hhh}$	$\sigma^{\rm eft}_{gg\to hhhh}$
$\alpha_s {\rm tr}(G_{\mu\nu}G^{\mu\nu})\, h^1$	$\frac{M_h^2}{v^2} s^0$	$\frac{M_h^4}{v^4} s^0$	$\frac{M_h^6}{v^6} s^0$
$\alpha_s {\rm tr}(G_{\mu\nu}G^{\mu\nu})\, h^2$	s	$\frac{M_h^2}{v^2} s$	$\frac{M_h^4}{v^4} s$
$\alpha_s {\rm tr}(G_{\mu\nu}G^{\mu\nu})\, h^3$	–	s^2	$\frac{M_h^2}{v^2} s^2$
$\alpha_s {\rm tr}(G_{\mu\nu}G^{\mu\nu})\, h^4$	–	–	s^3

- The pattern established for polygons with 2+k edges:

$$(2+k){-}\text{polygons}: \qquad \sigma_{gg\to n\times h} \propto \frac{1}{s}\, y_t^{2k} \left(\frac{M_h}{v}\right)^{2(n-k)} \times \begin{cases} 1 & : \; k = \text{even} \\ \frac{m_t^2}{s} \log^4\left(\frac{m_t}{\sqrt{s}}\right) & : \; k = \text{odd}\,. \end{cases}$$

- allows to associate the full 1-loop result from rank-(2+k) polygons in the high E limit to the effective vertex - now including the form-factors - via:

$$\mathcal{V}_k = C_k \, \frac{\alpha_s(\sqrt{s})}{\pi} \, {\rm tr}(G_{\mu\nu}G^{\mu\nu}) \left(\frac{y_t\, h}{\sqrt{s}}\right)^k \times \begin{cases} 1 & : \; k = \text{even} \geq 2 \\ \frac{m_t}{\sqrt{s}} \log^2\left(\frac{m_t}{\sqrt{s}}\right) & : \; k = \text{odd} \geq 3\,. \end{cases}$$

- For h substitute the classical solution generating functional to represent subsequent Higgs branchings. Ck constants are known (computed). [More detail in the paper.]