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Higgsplosion

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IPPP Durham

- VVK & Spannowsky 1704.03447, 1707.01531
- VVK, Reiness, Spannowsky, Waite 1709.08655
- & with Scholtz & Spannowsky — to appear
- VVK 1705.04365

- Before the Higgs discovery, massive Yang-Mills theory violated perturbative unitarity — problem with high-energy growth of 2 -> 2 processes

- Discovery of the (elementary) Higgs made the SM theory self-consistent

- The Higgs brings in the Hierarchy problem: radiative corrections push the Higgs mass to the new physics (high) scale: $m_h^2 \simeq m_0^2 + \delta m_{\rm new}^2$

- In this talk: consider n~100s of Higgs bosons produced in the final state n lambda >> 1. Investigate scattering processes at ~ 100 TeV energies.

- HIGGSPLOSION: n-particle rates computed in a weakly-coupled theory can become unsuppressed above critical values of n and E. Perturbative and non-perturbative semi-classical calculations. n! ~ exponential growth with n or E. (Scale n~E/m).

- A new unitarity problem — caused by the elementary Higgs bosons — appears to occur (?) for processes with large final state multiplicities n >> 1

- HIGGSPLOSION offers a solution to both problems: it restores the unitarity of high-multiplicity processes and dynamically cuts off the values of the loop momenta contributing to the radiative corrections to the Higgs mass.

Organisation of the talk:

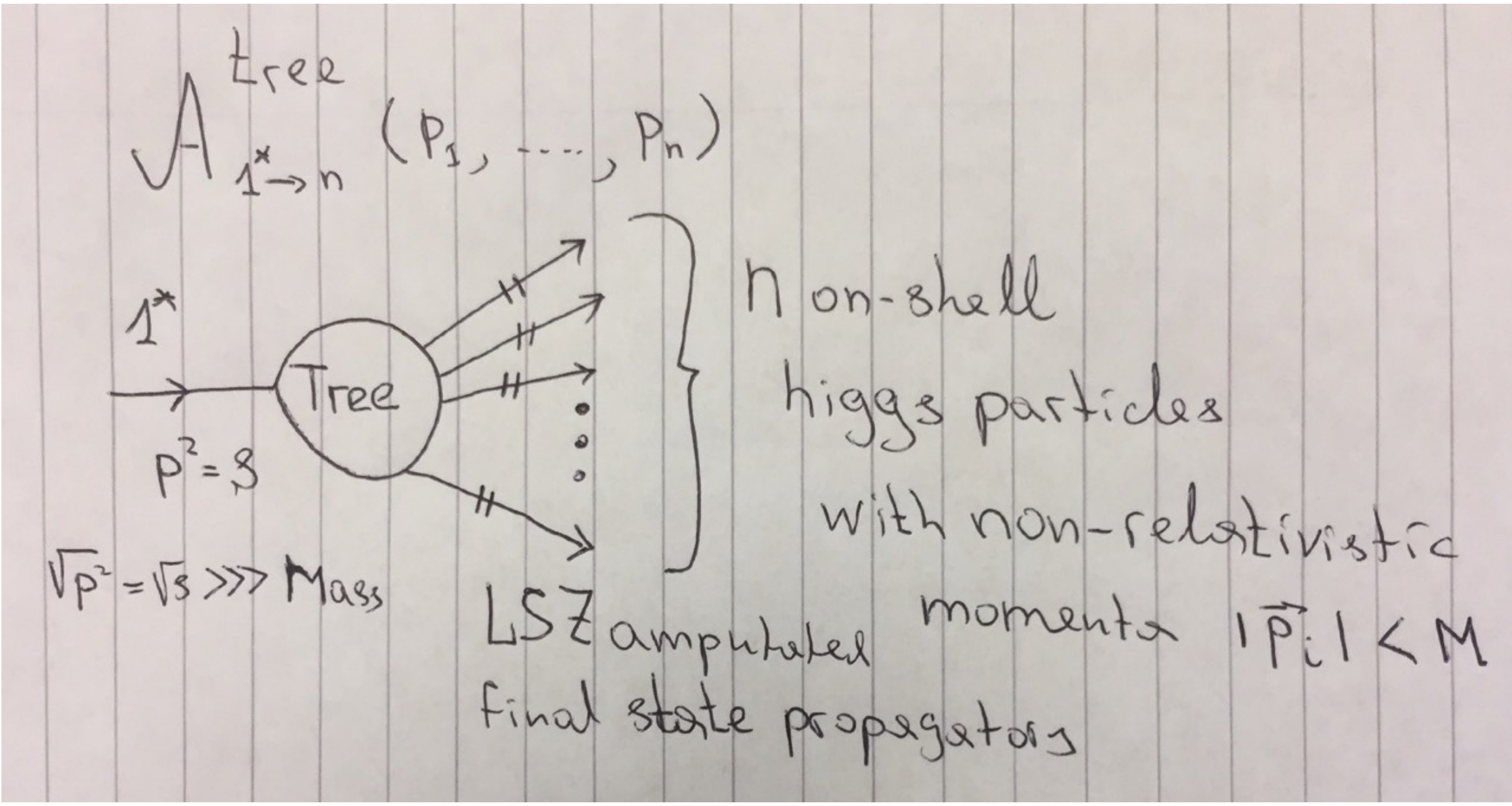
1. main part

- Main idea and simple expressions for n-point amplitudes and rates
- Interpretation of tree-level results and inclusion of quantum effects: loops and semiclassical methods. Higgsplosion and Higgspersion.
- Some phenomenology / inc. future colliders => Summary

2. in more detail (depending on time)

- Full propagator & Higgsplosion/Higgspersion
- All tree-level amplitudes from classical solutions
- The semi-classical approach for computing quantum effects
- Effects of Higgsplosion on Precision Observables

Compute 1 -> n amplitudes @LO with non-relativistic final-state momenta:



see classic 1992-1994 papers:
Brown; Voloshin;
Argyres, Kleiss, Papodopoulos
Libanov, Rubakov, Son, Troitski

more recently: VVK 1411.2925

$$\mathcal{L} \;=\; \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2$$

prototype of the SM Higgs
in the unitary gauge

Tree-level $1^* \to n$ amplitudes in the limit $\varepsilon \to 0$ for any n are given by

$$\mathcal{A}_n(p_1, \ldots p_n) \;=\; \boxed{n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}}} \left(1 - \frac{7}{6}n\varepsilon - \frac{1}{6}\frac{n}{n-1}\varepsilon + \mathcal{O}(\varepsilon^2)\right)$$

factorial growth

amplitude on the n-particle threshold

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \;=\; \frac{1}{n}\,\frac{1}{2M_h^2}\sum_{i=1}^{n} \vec{p}_i^{\,2}$$

kinetic energy per particle per mass

In the large-n-non-relativistic limit the result is

$$\boxed{\mathcal{A}_n(p_1, \ldots p_n) \;=\; n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \exp\left[-\frac{7}{6}n\varepsilon\right], \quad n \to \infty,\; \varepsilon \to 0,\; n\varepsilon = \text{fixed}}$$

Can now integrate over the n-particle phase-space

The cross-section and/or the n-particle partial decay Γ_n

$$\Gamma_n(s) \; = \; \int d\Phi_n \, \frac{1}{n!} \, \left|\mathcal{A}_{h^* \to n\times h}\right|^2$$

The n-particle Lorentz-invariant phase space volume element

$$\int d\Phi_n \; = \; (2\pi)^4 \delta^{(4)}(P_{\rm in} - \sum_{j=1}^{n} p_j) \, \prod_{j=1}^{n} \int \frac{d^3 p_j}{(2\pi)^3 \, 2p_j^0} \, ,$$

in the large-n non-relativistic limit with $n\varepsilon_h$ fixed becomes,

$$\Phi_n \; \simeq \; \frac{1}{\sqrt{n}} \left(\frac{M_h^2}{2}\right)^n \, \exp\left[\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi} + 1\right) \, + \, \frac{n\varepsilon_h}{4} \, + \, \mathcal{O}(n\varepsilon_h^2)\right]$$

We find:

$$\boxed{\Gamma_n^{\rm tree}(s) \; \sim \; \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) \, + \, \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) \, - \, \frac{25}{12}\, n\varepsilon \, + \, \mathcal{O}(n\varepsilon^2)\right]}$$

Son 1994;
Libanov, Rubakov, Troitskii 1997; more recently: VVK 1411.2925

- The n! growth of perturbative amplitudes is not entirely surprising: the number of contributing Feynman diagrams is known to grow factorially with n. [In scalar QFT there are no partial cancellations between individual diagrams (unlike QCD).]

- Important to distinguish between the two types of large-n corrections:

(a) present case where the *leading-order* tree-level contribution to the 1*->n Amplitude grows factorially with the particle multiplicity n of the final state.

(b) *higher-order* perturbative corrections to some leading-order quantities

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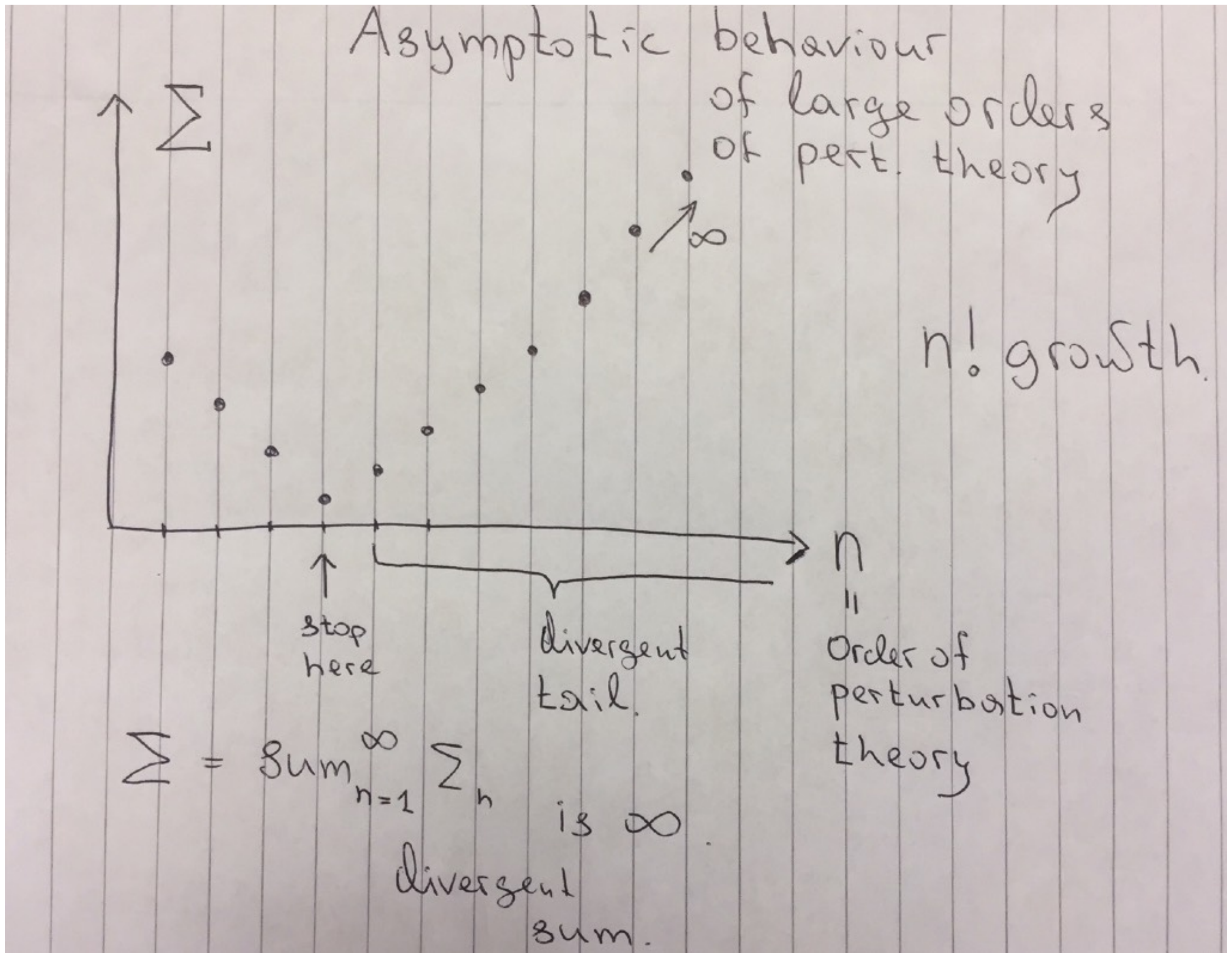
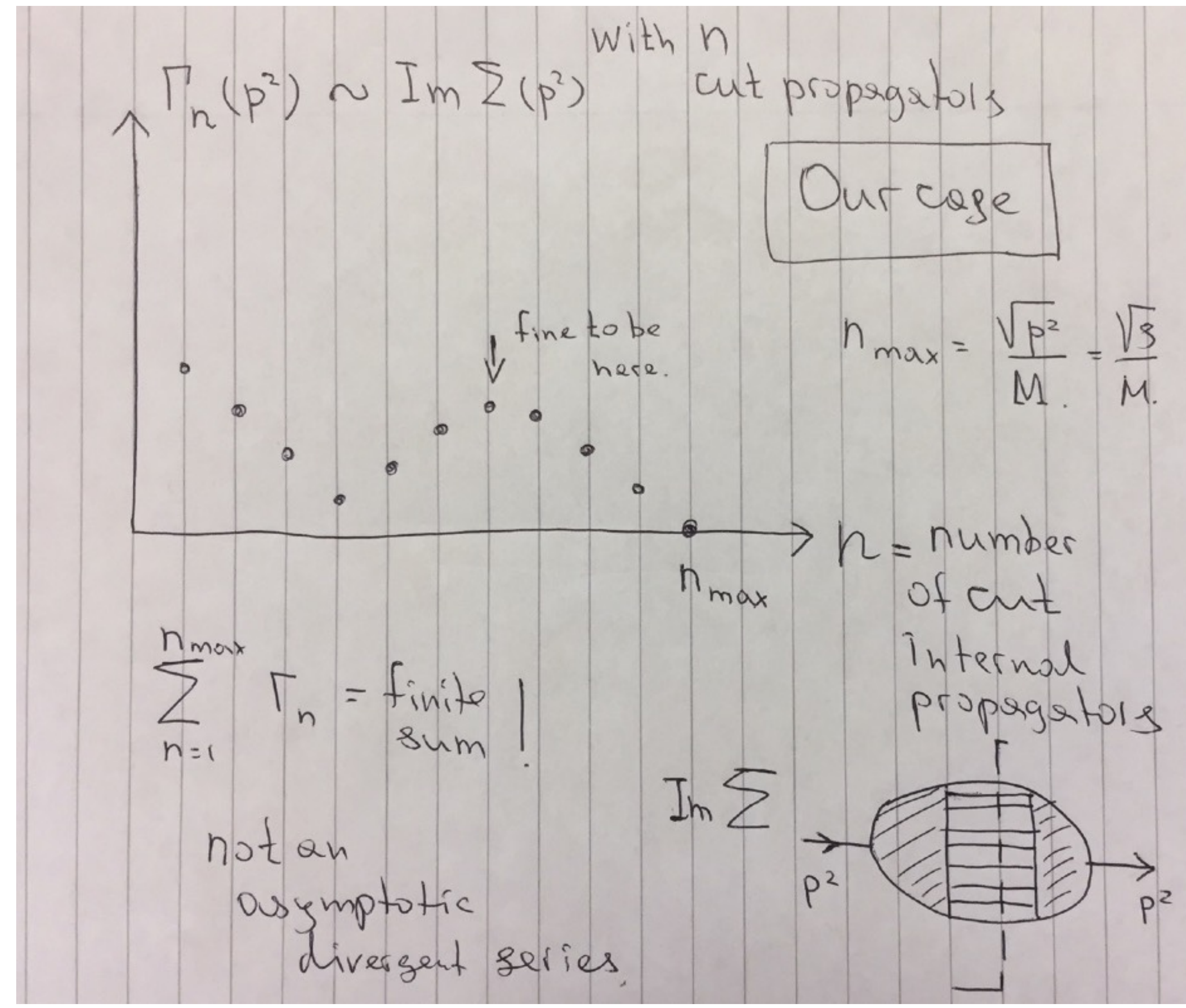
- These amplitudes were first studied in the 90s in scalar QFTs

  - But now it is realised that the characteristic energy scale for EW applications starts in the 50-100 TeV range. FCC would provide an exciting challenge to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.

Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma_n(s)

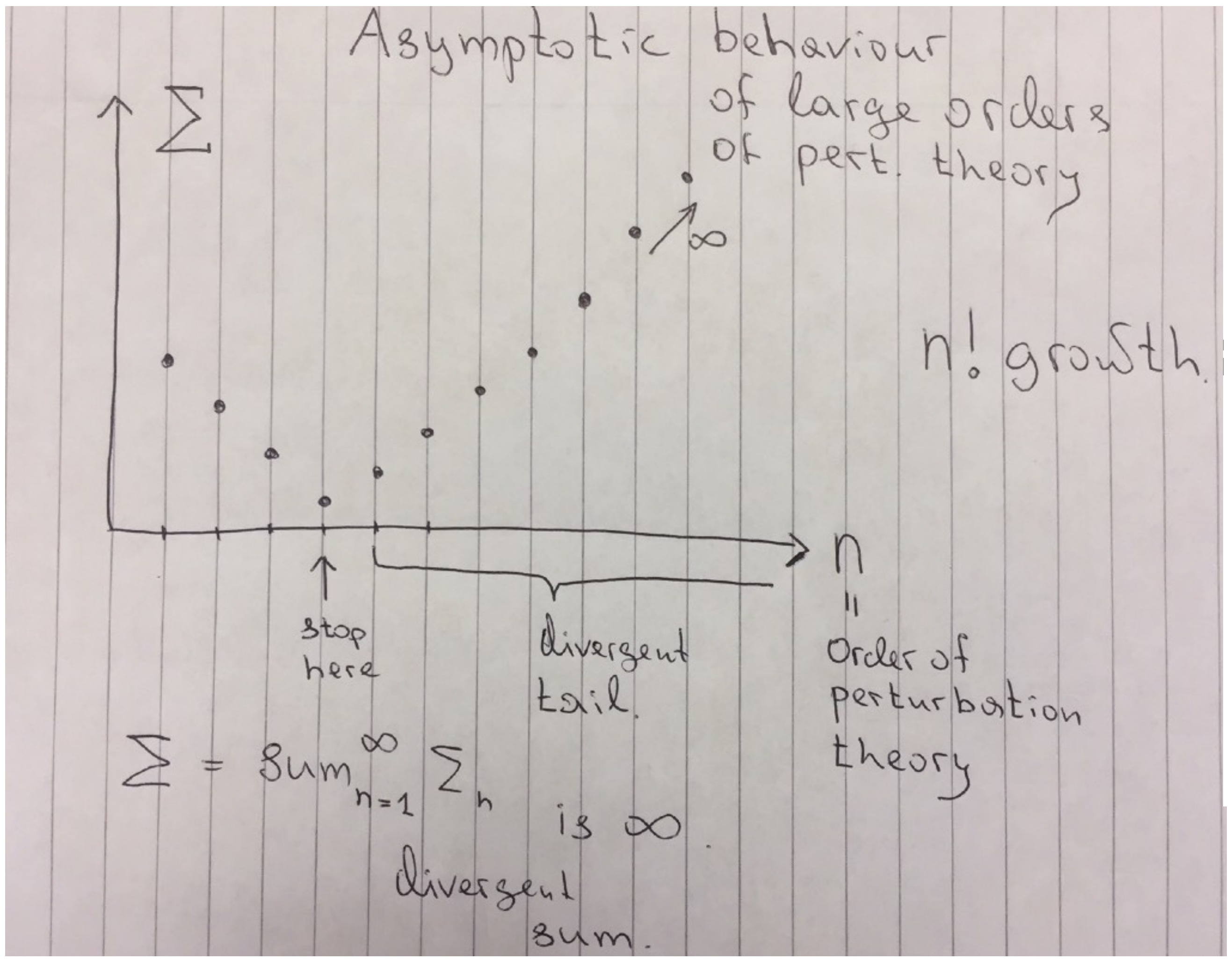


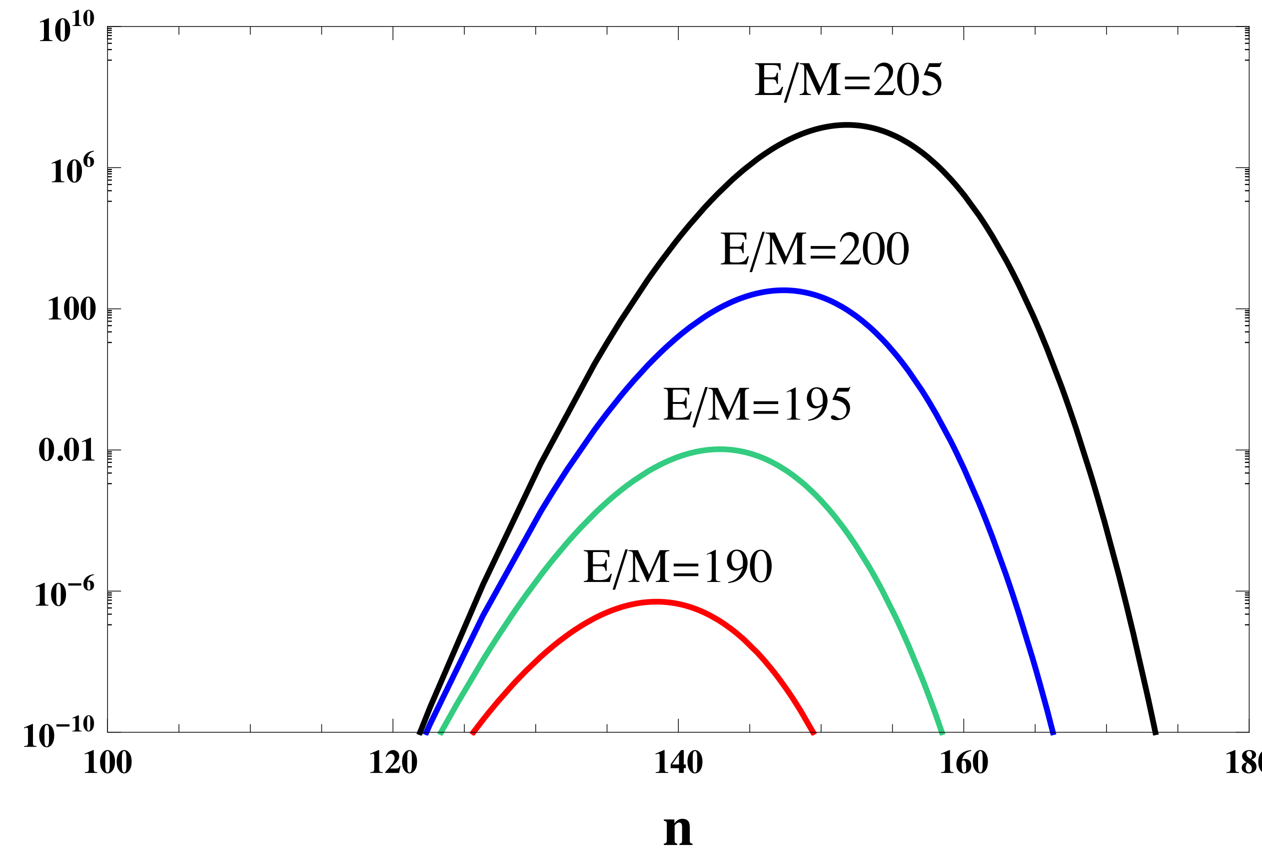


Not the same types of beasts

It is the decay width Gamma_n(s) which is the central object of interest and the driving force of Higgsplosion.

Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma_n(s)





For Gamma_n(E^2) we'll find this

Not the same types of beasts

Perturbative as well as semi-classical calculations result in the exponential form for the n-particle width Gamma ~ exp[F_holy_grail]

- Libanov, Rubakov, Son, Troitsky; Son: 1994-1995

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

bare cross-section [ignoring the width effect for now]

$$\boxed{\Gamma_n(s) \propto \exp\left[n\left(\log\frac{\lambda n}{4}-1\right)+\frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\,n\,\varepsilon\right]}$$

More generally, in the large-n limit with $\lambda n =$ fixed and $\varepsilon =$ fixed, one expects

$$\boxed{\Gamma_n(s) \propto \exp\left[\frac{1}{\lambda}\,F_{\rm h.g.}(\lambda n,\varepsilon)\right]}$$

[e.g. Libanov, Rubakov, Troitsky review 1997]

where the *holy grail* function $F_{\rm h.g.}$ is of the form,

$$\boxed{\frac{1}{\lambda}\,F_{\rm h.g.}(\lambda n,\varepsilon) \;=\; \frac{\lambda n}{\lambda}\,(f_0(\lambda n)+f(\varepsilon))}$$

In our higgs model, i.e. the scalar theory with SSB,

$$\boxed{\begin{array}{lcll} f_0(\lambda n) & = & \log\dfrac{\lambda n}{4}-1 & \text{at tree level} \\ f(\varepsilon) & \to & \dfrac{3}{2}\left(\log\dfrac{\varepsilon}{3\pi}+1\right)-\dfrac{25}{12}\,\varepsilon & \text{for } \varepsilon \ll 1 \end{array}}$$

Can also include *loop corrections* to amplitudes on thresholds:

The 1-loop corrected threshold amplitude for the pure n Higgs production:

$$\boxed{\phi^4 \text{ with SSB}: \quad \mathcal{A}_{1\to n}^{\text{tree+1loop}} = n!\,(2v)^{1-n}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)}$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$\boxed{\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\text{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]}$$

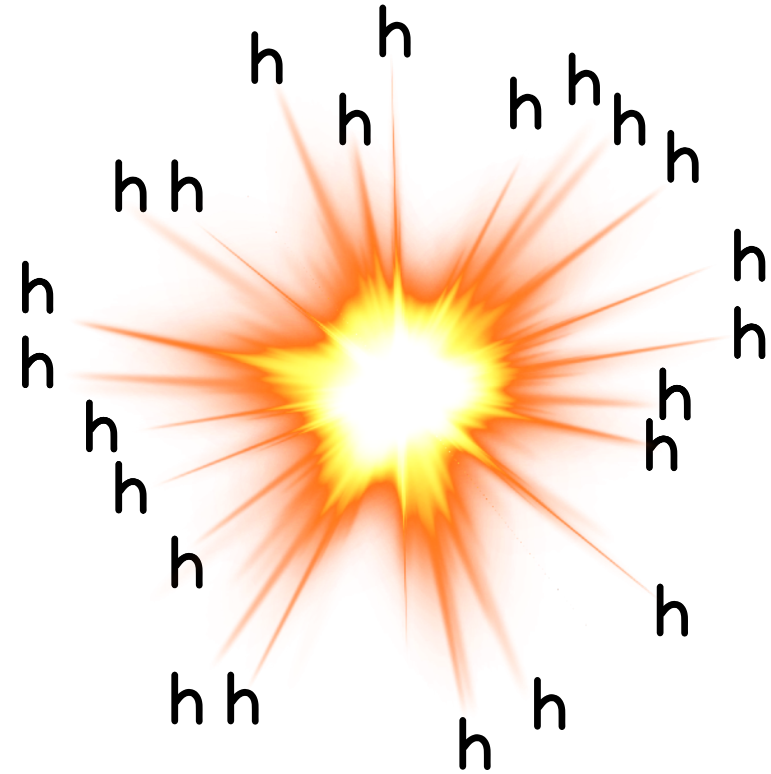
in the limit $\lambda \to 0$, $n \to \infty$ with λn fixed. Here B is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*): $B = +\lambda n \frac{\sqrt{3}}{4\pi}$

$$\boxed{\begin{aligned} f_0(\lambda n) &= \log\frac{\lambda n}{4} - 1 + \lambda n\frac{\sqrt{3}}{4\pi} + \mathcal{O}(\lambda n)^2 \\ f(\varepsilon) &\to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\varepsilon \qquad \text{for } \varepsilon \ll 1 \end{aligned}}$$

Explosive growth of 1->n process

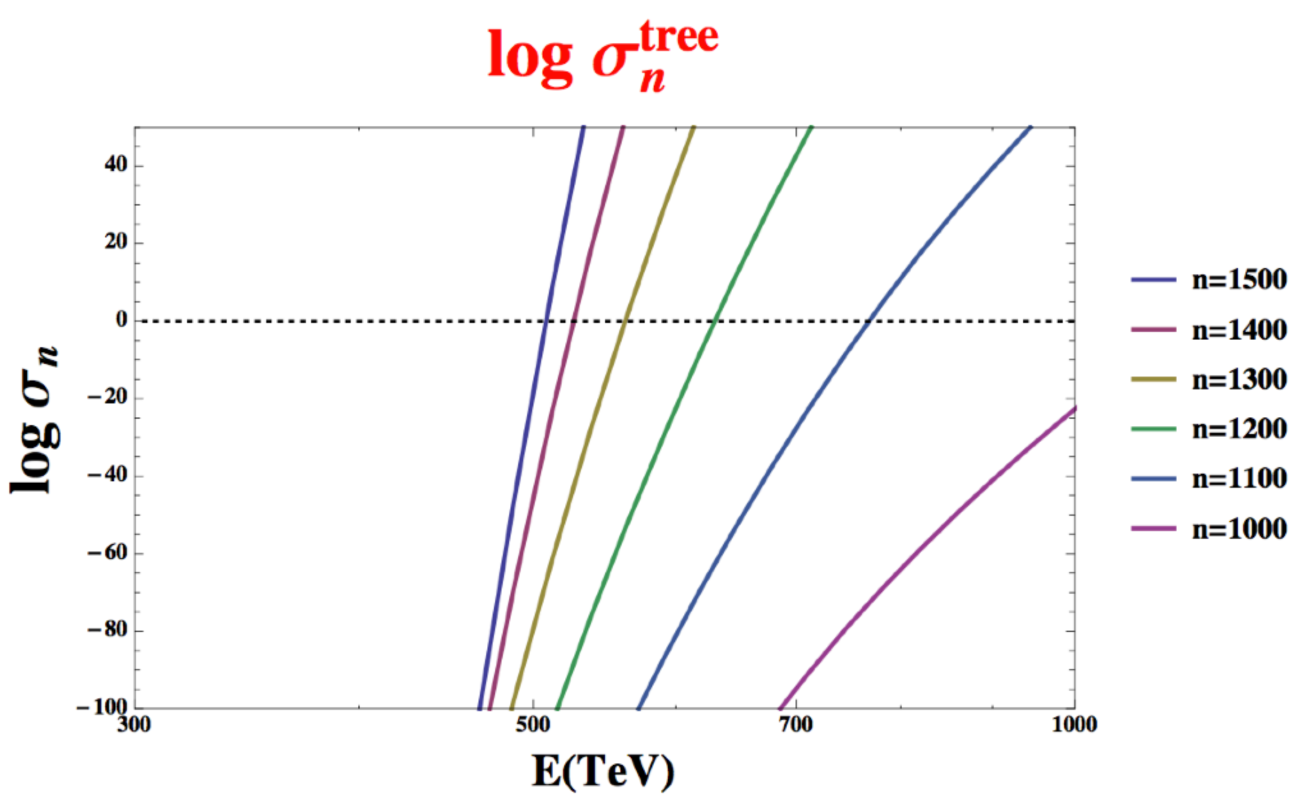
$$\mathcal{R}_n(s) \;:=\; \frac{1}{2M_h^2}\int d\Pi_n |\mathcal{M}(1\to n)|^2$$

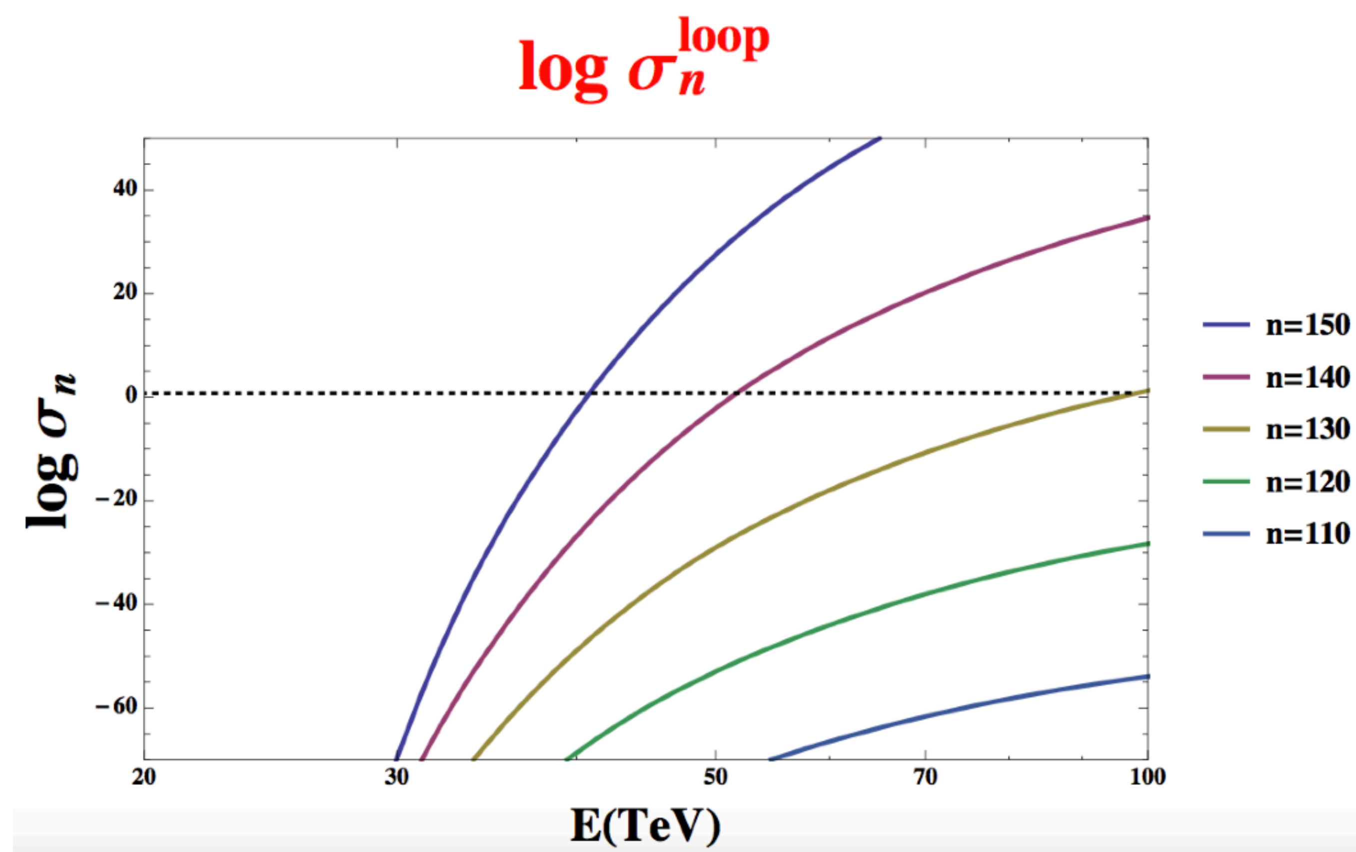
$$\mathcal{R}(\lambda; n, \varepsilon) \;=\; \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) + \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}n\varepsilon\right]$$



Extreme energy dependence for $1^* \to$ n cross section

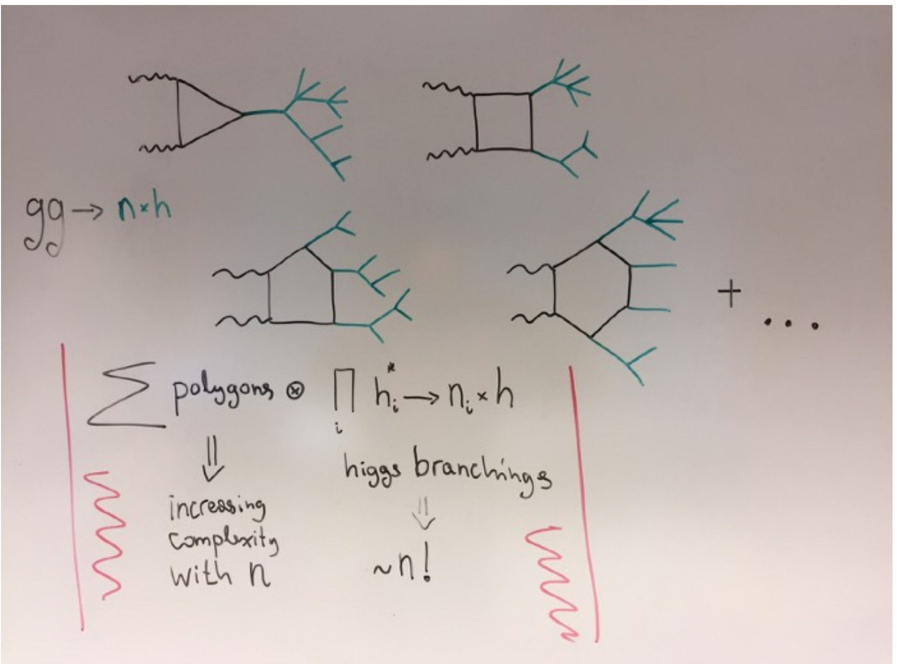
including 1-loop result reduces 'ignition' scale



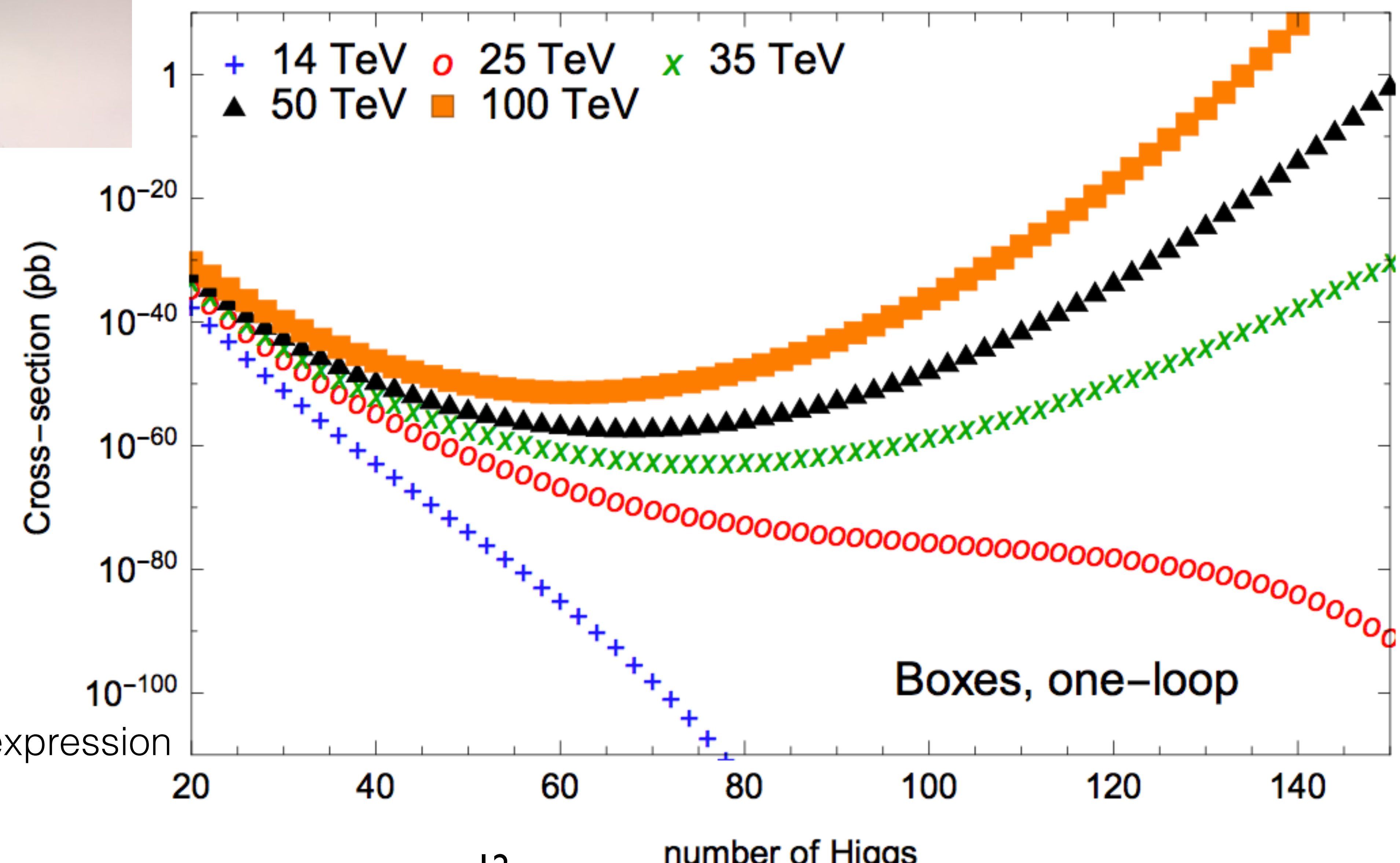


Really need to switch to the regime of lambda n >>1

Was argued that these results can be used to assess what collider energy needed to test where perturbation theory becomes strong [in gluon fusion]



Degrande, VVK, Mattelaer '16



using the 1-loop improved expression

Semi-classical approach for computing the rate R(1->n,E)

- DT Son1995

Multi-particle decay rates Γ_n can also be computed using an alternative semi-classical method. This is an intrinsically non-perturbative approach, with no reference in its outset made to perturbation theory.

The path integral is computed in the steepest descent method, controlled by two large parameters, $1/\lambda \to \infty$ and $n \to \infty$.

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.$$

The semi-classical computation in the regime where,

$$\lambda n = \text{fixed} \ll 1\,, \quad \varepsilon = \text{fixed} \ll 1\,,$$

reproduces the tree-level perturbative results for non-relativistic final states.

Remarkably, this semi-classical calculation also reproduces the leading-order quantum corrections arising from resumming one-loop effects.

Semi-classical approach for computing the rate R(1->n,E)

$$\Gamma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon)$$

The semiclassical approach is equally applicable and more relevant to the realisation of the non-perturbative Higgsplosion case where,

$$\lambda n = \text{fixed} \gg 1\,, \quad \varepsilon = \text{fixed} \ll 1\,.$$

This calculation was carried out for the spontaneously broken theory with the result given by,

- VVK 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) \,=\, \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2}\log\frac{\varepsilon}{3\pi} - \frac{25}{12}\varepsilon\right)\right],$$

Higher order corrections are suppressed by $\mathcal{O}(1/\sqrt{\lambda n})$ and powers of ε.

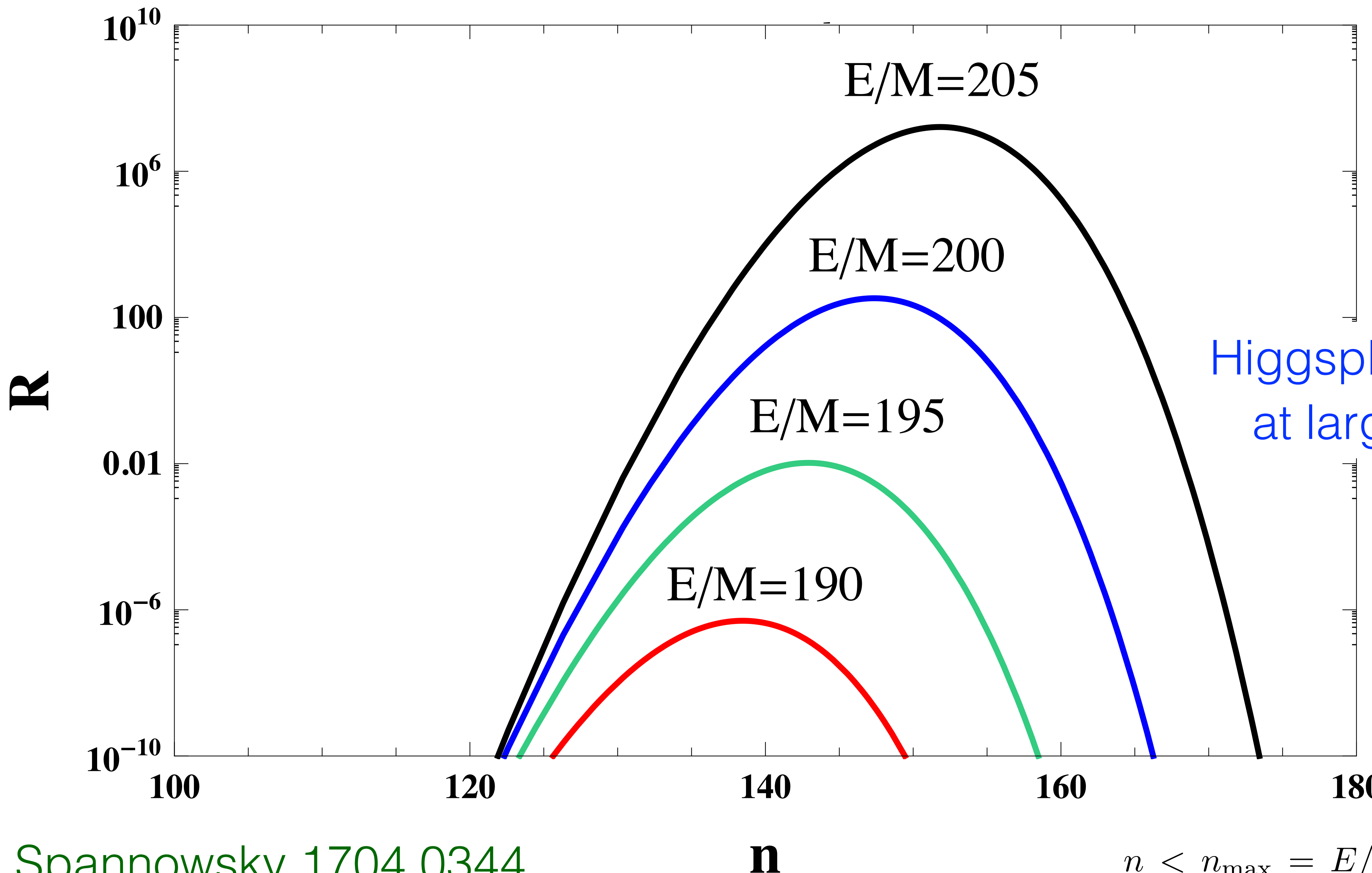
Thus we have computed the rate R in the large lambda n limit:

using the semi-classical approach and the thin-wall approximation

VVK 1705.04365

$$\mathcal{R} = \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + \boxed{3.02\sqrt{\frac{\lambda n}{4\pi}}} - 1 + \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) - \frac{25}{12}\varepsilon\right)\right]$$

$\lambda n \gg 1 \quad$ small ε



Higgsplosion realised at large lambda n

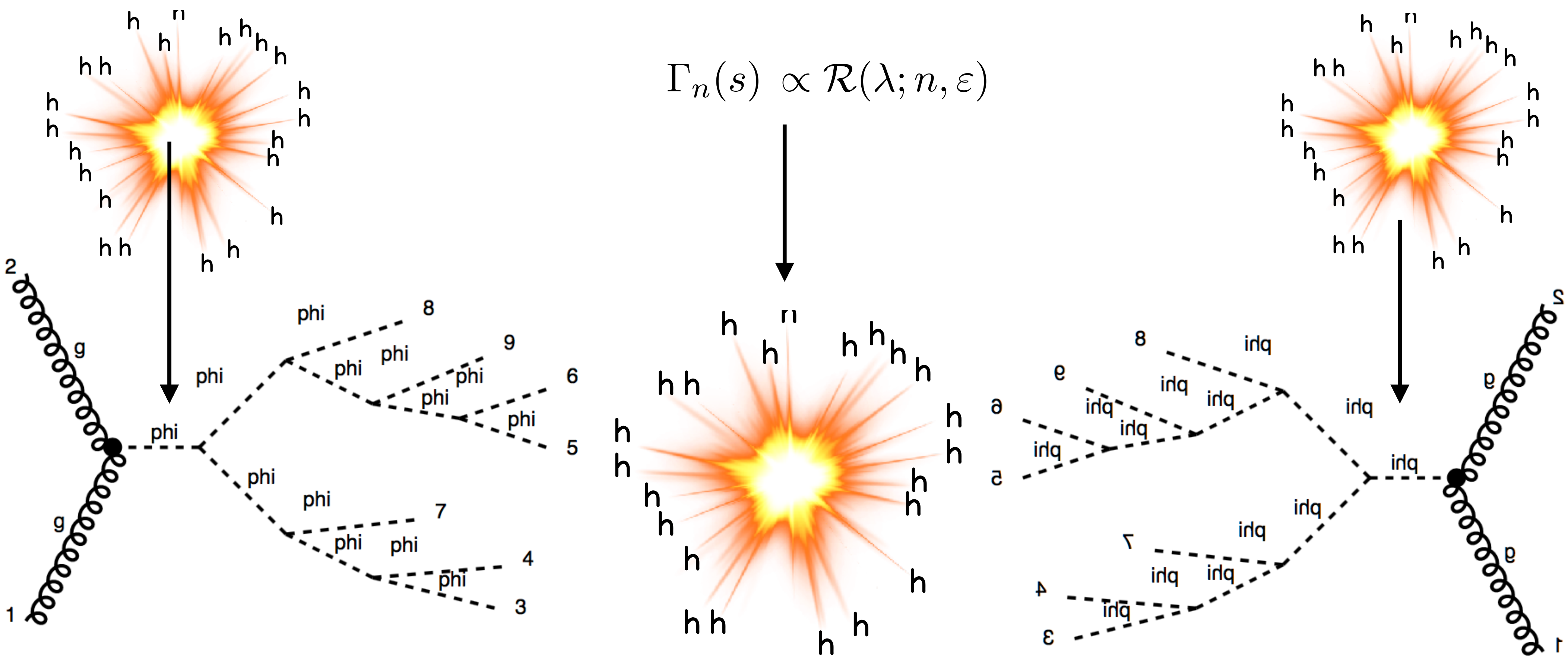
VVK & Spannowsky 1704.0344

$n < n_{\max} = E/M_h$

HIGGSPLOSION and HIGGSPERSION

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - m_h^2 - Re\tilde{\Sigma}(p^2) + i m_h \Gamma(p^2)} \times \mathcal{M}_{h^* \to n\times h}$$

Include self-energy



$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 \frac{m_t^2}{m_h} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{(s - Re\Sigma(s))^2 + m_h^2 \Gamma^2(s)} \times \Gamma_n(s)$$

VVK & Spannowsky 1704.0344

- Unitarity restored!

Summary of the main idea

The Dyson propagator (continued to Euclidean space) is,

$$\Delta_R(x_1, x_2) \;=\; \langle 0|\phi(x_1)\phi(x_2)|0\rangle \;=\; \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{p^2 + m^2 + \Sigma_R(p^2)}\, e^{ip_0\Delta\tau \,+\, i\vec{p}\Delta\vec{x}}\,.$$

When the theory enters the Higgsplosion regime, the self-energy undergoes a sharp exponential growth,

$$\Sigma_R(p^2) \;\sim\; \begin{cases} 0 & : \;\text{for } p^2 < E_*^2 \\ \infty & : \;\text{for } p^2 \geq E_*^2 \end{cases}$$

The loop momentum integral becomes cut off by Σ outside the ball of radius E_*

$$\Delta_R(x_1, x_2) \;=\; \int_{p^2 \leq E_*^2} \frac{d^4p}{(2\pi)^4}\, \frac{1}{p^2 + m^2}\, e^{ip_0\Delta\tau \,+\, i\vec{p}\Delta\vec{x}}$$

$$\sim \begin{cases} 1/|\Delta x|^2 & : \;\text{for } 1/E_* \ll |\Delta x| \ll 1/m \\ E_*^2 & : \;\text{for } |\Delta x| \lesssim 1/E_* \end{cases} \,.$$

Summary of the main idea

A conventional wisdom: in the description of nature based on a local QFT, one should always be able to probe shorter and shorter distances with higher and higher energies.

Higgsplosion is a dynamical mechanism, or a new phase of the theory, which presents an obstacle to this principle at energies above E_*.

E_* is the new dynamical scale of the theory, where multi-particle decay rates become unsuppressed.

Schematically, $E_* = C\,\frac{m}{\lambda}$, where C is a model-dependent constant of $\mathcal{O}(100)$. This expression holds in the weak-coupling limit $\lambda \to 0$.

Higgsplosion

At energy scales above E_* the dynamics of the system is changed:

1. Distance scales below $|x| \lesssim 1/E_*$ cannot be resolved in interactions;
2. UV divergences are regulated;
3. The theory becomes asymptotically safe;
4. And the Hierarchy problem of the Standard Model is therefore absent.

Consider the scaling behaviour of the propagator of a massive scalar particle

$$\Delta(x) \;:=\; \langle 0|T(\phi(x)\,\phi(0))|0\rangle \;\sim\; \begin{cases} m^2\, e^{-m|x|} & : \;\text{for } |x| \gg 1/m \\ 1/|x|^2 & : \;\text{for } 1/E_* \ll |x| \ll 1/m \\ E_*^2 & : \;\text{for } |x| \lesssim 1/E_* \end{cases} ,$$

where for $|x| \lesssim 1/E_*$ one enters the Higgsplosion regime.

This is a non-perturbative criterium. Can in principle be computed on a lattice.

Higgsplosion

Loop integrals are effectively cut off at E_* by the exploding width $\Gamma(p^2)$ of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \lll E_*^2$.

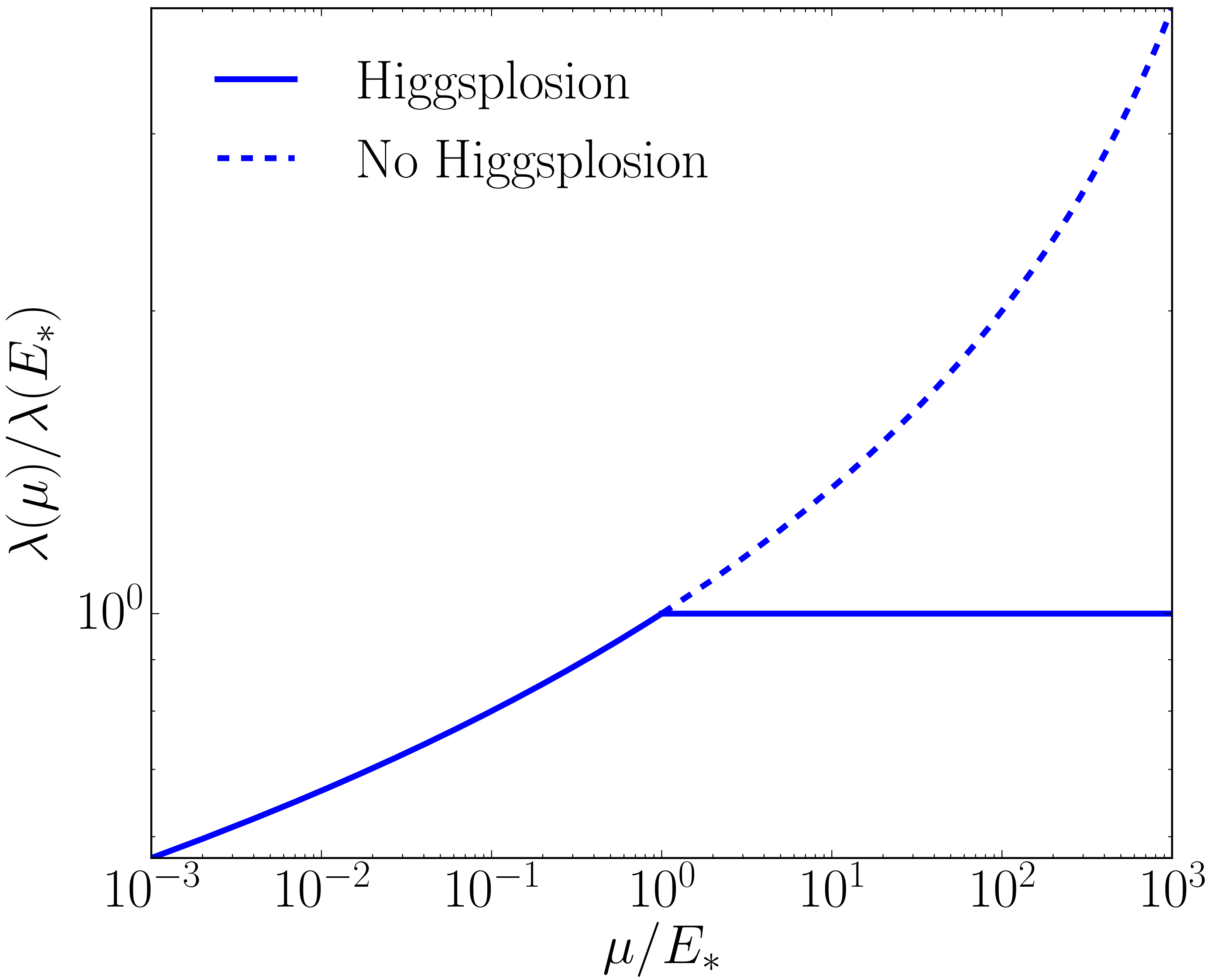
The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the n soft particle quanta of the same field ϕ.

VVK & Michael Spannowsky 1704.03447, 1707.01531

Asymptotic Safety

For all parameters of the theory (running coupling constants, masses, etc):

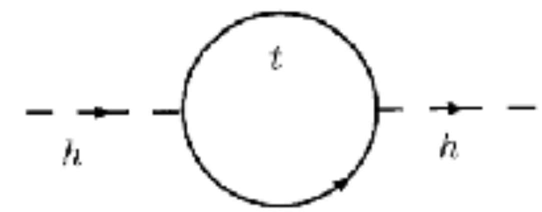


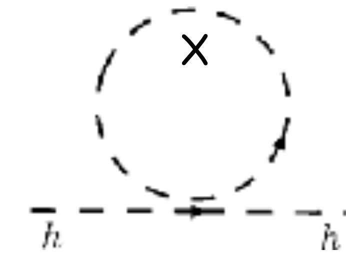
UV fixed point

Higgsploding the Hierarchy problem

X=heavy state

$$\mathcal{L}_X = \frac{1}{2}\partial^\mu X\,\partial_\mu X - \frac{1}{2}M_X^2\,X^2 - \frac{\lambda_P}{4}\,X^2h^2 - \mu X h^2$$





$$\Delta M_h^2 \sim \lambda_P \int \frac{d^4p}{16\pi^4}\,\frac{1}{p^2 + M_X^2 + \Sigma_X(p^2)} \propto \lambda_P\,\frac{E_\star^2}{M_X^2}\,E_\star^2 \quad \ll \lambda_P M_X^2\,.$$

Due to Higgsplosion the multi-particle contribution to the width of X explode at $p^2 = s_\star$ where $\sqrt{s_\star} \simeq \mathcal{O}(25)\mathrm{TeV}$

It provides a sharp UV cut-off in the integral, possibly at $s_\star \ll M_X^2$

Hence, the contribution to the Higgs mass amounts to

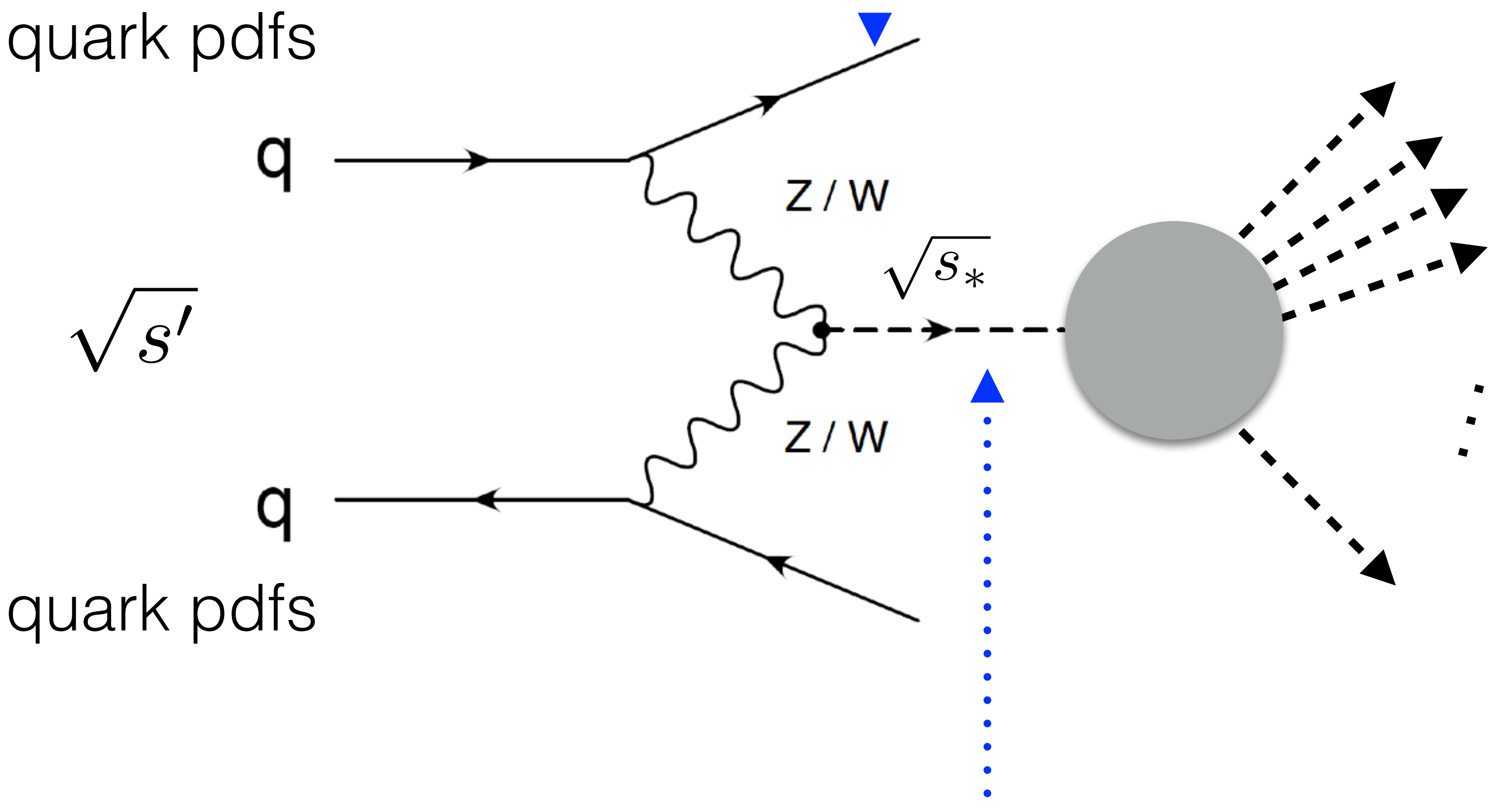
$$\text{For}\quad \Gamma(s_\star) \simeq M_X \quad \text{at} \quad s_\star \ll M_X^2 \quad \Longrightarrow \quad \Delta M_h^2 \propto \lambda_P\,\frac{s_\star}{M_X^2}\,s_\star \ll \lambda_P\,M_X^2$$

and thus mends the Hierarchy problem by $\left(\frac{\sqrt{s_\star}}{M_X}\right)^4 \simeq \left(\frac{25\,\mathrm{TeV}}{M_X}\right)^4$

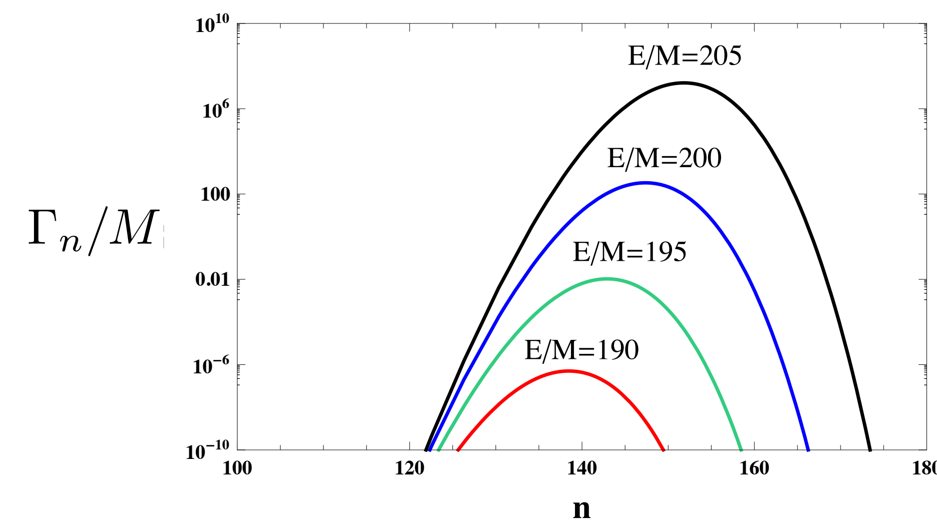
Prospects of *direct* observation of Higgslposion

Vector boson fusion at high-energy pp colliders (FCC)

energy excess over $\sqrt{s_*}$ carried away by jets



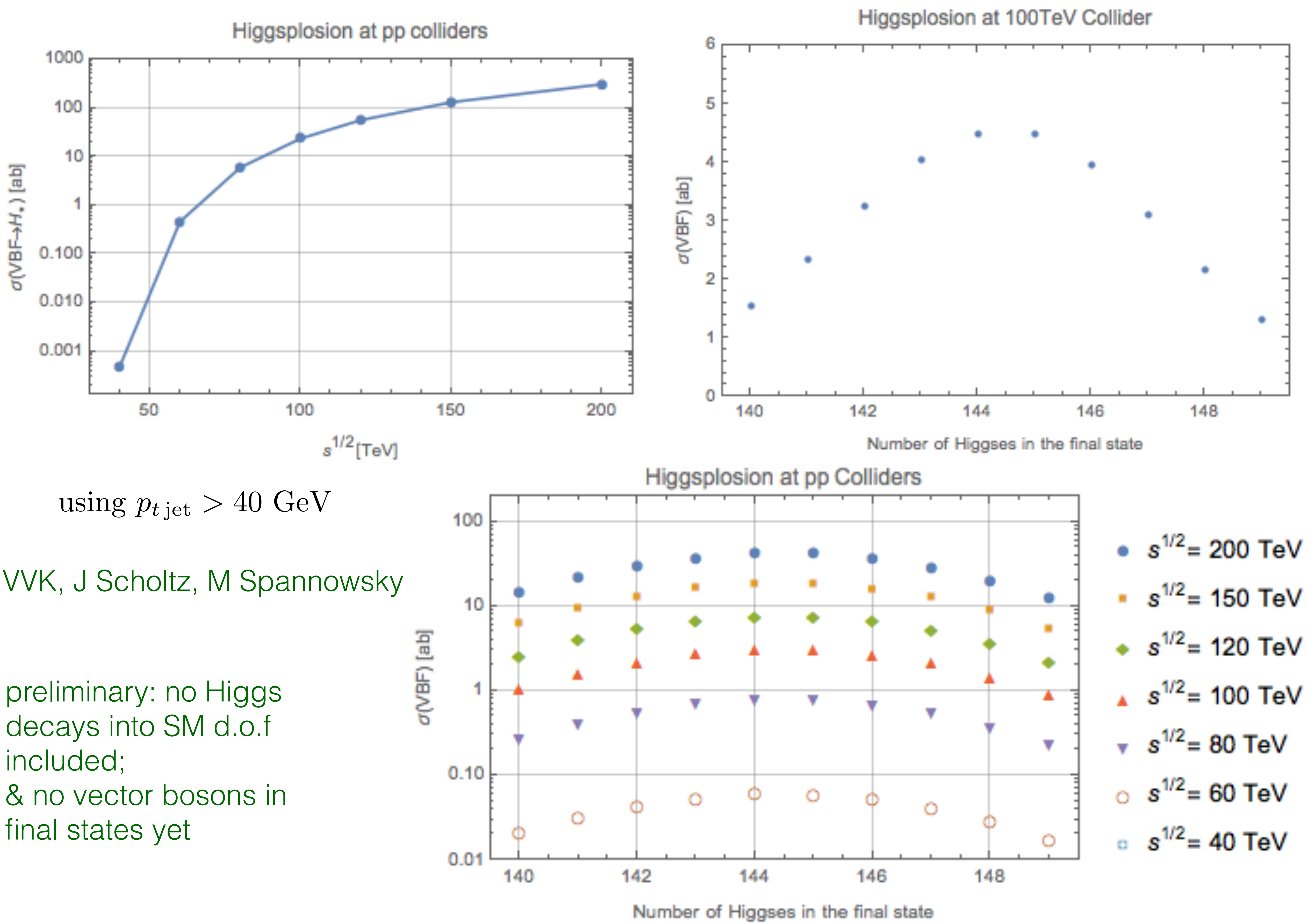
n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$



$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$

Propagator with Higgspersion at $\sqrt{s_*}$

Vector boson fusion at high-energy pp colliders (FCC)



using $p_{t\,\mathrm{jet}} > 40$ GeV

VVK, J Scholtz, M Spannowsky

preliminary: no Higgs decays into SM d.o.f included;
& no vector bosons in final states yet

Summary

- The Higgsplosion / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplosion energy).
- UV-finiteness => all coupling constants slopes become flat above the Higgsplosion scale => automatic asymptotic safety
- [Below the Higgsplosion scale there is the usual logarithmic running]
- 1. Asymptotic Safety
- 2. No Landau poles for the U(1) and the Yukawa couplings
- 3. The Higgs self-coupling does not turn negative => stable EW vacuum
- No new physics degrees of freedom required — very minimal solution

Now in more detail

- Full propagator & Higgsplosion/Higgspersion
- All tree-level amplitudes from classical solutions
- The semi-classical approach for computing quantum effects to Gamma_n at large lambda n
- Effects of Higgsplosion on Precision Observables

a) The Propagator and Higgspersion basics

In a generic QFT model with a massive scalar consider:

1. The Feynman propagator of ϕ is the 2-point function,

$$\Delta(p) \,=\, \int d^4x\, e^{ip\cdot x} \langle 0| T\left(\phi(x)\,\phi(0)\right)|0\rangle \,=\, \frac{i}{p^2 - m_0^2 - \Sigma(p^2) + i\epsilon}\,,$$

2. The self-energy $\Sigma(p^2)$ is the the sum of all 2-point (1PI) diagrams,

$$-i\,\Sigma(p^2) \,=\, \sum -(1\text{PI})-\ .$$

   In perturbation theory,

$$\frac{i}{p^2 - m_0^2 - \Sigma(p^2)} \,=\, \frac{i}{p^2 - m_0^2} + \frac{i}{p^2 - m_0^2}\sum_{n=1}^{\infty}\left(-i\Sigma(p^2)\,\frac{i}{p^2 - m_0^2}\right)^n\,.$$

   But the expression for the full quantum propagator on the left is valid no-perturbatively.

3. The physical (or pole) mass m is defined as the pole of the quantum propagator,

$$m^2 - m_0^2 - \Sigma(m^2) \,=\, 0\,, \quad \text{or} \quad m^2 \,=\, m_0^2 + \Sigma(m^2)\,.$$

a) The Propagator and Higgspersion basics

4. The field renormalisation Z_ϕ is determined from the slope of $\Sigma(p^2)$ at m^2,

$$Z_\phi = \left(1 - \frac{d\Sigma}{dp^2}\bigg|_{p^2=m^2}\right)^{-1} .$$

Using the definition of the pole mass and the renormalisation constant,

$$\Delta(p) = \frac{iZ_\phi}{p^2 - m^2 - Z_\phi[\Sigma(p^2) - \Sigma(m^2) - \Sigma'(m^2)(p^2 - m^2)]} .$$

5. The renormalised quantities $\Delta_R(p)$ and $\Sigma_R(p^2)$ are,

$$\boxed{\begin{aligned} \Delta_R(p) &= Z_\phi^{(-1)}\,\Delta(p)\,, \\ \Sigma_R(p) &= Z_\phi\,\left(\Sigma(p^2) - \Sigma(m^2) - \Sigma'(m^2)(p^2 - m^2)\right) . \end{aligned}}$$

Hence, the result for the renormalised propagator in terms of all finite quantities is,

$$\boxed{\Delta_R(p) = \frac{i}{p^2 - m^2 - \Sigma_R(p^2) + i\epsilon} .}$$

a) The Propagator and Higgspersion basics

6. The optical theorem provides the physical interpretation of the $\mathrm{Im}\,\Sigma$,

$$\mathrm{Im}\,\Sigma_R(p^2) \;=\; -\,m\,\Gamma(p^2)\,,$$

with the decay width being determined by the partial widths of n-particle decays at energies $s \geq (nm)^2$,

$$\Gamma(s) \;=\; \sum_{n=2}^{\infty} \Gamma_n(s)\;, \qquad \Gamma_n(s) \;=\; \frac{1}{2m}\int \frac{d\Phi_n}{n!}\,|\mathcal{M}(1\to n)|^2\,.$$

7. The origin of Higgsplosion is that $\Gamma_n(s)$ grows factorially with n in the large-n limit, $\frac{1}{n!}|\mathcal{M}_n|^2 \sim n!\lambda^n \sim e^{n\log(\lambda n)}$. When n scales linearly with the available energy, $n \sim \sqrt{s}/m$, this translates into the exponential dependence of the decay rate $\Gamma(s)$ on $\sqrt{s}$.

8. Hence in a Higgsploding theory, the propagator,

$$\boxed{\Delta_R(p) \;=\; \frac{i}{p^2 - m^2 \;-\; \mathrm{Re}\,\Sigma_R(p^2) \;+\; im\Gamma(p^2) \;+ i\epsilon}\,,}$$

is effectively cut off at $p^2 \geq E_*^2$ by the exploding width $\Gamma_n(p^2)$.

a) The Propagator and Higgspersion basics

6. The optical theorem provides the physical interpretation of the $\mathrm{Im}\,\Sigma$,

$$\mathrm{Im}\,\Sigma_R(p^2) \;=\; -\,m\,\Gamma(p^2)\,,$$

with the decay width being determined by the partial widths of n-particle decays at energies $s \geq (nm)^2$,

The driving force of Higgsplosion is the partial width, i.e. the 1-> n particle decay width Gamma_n(p^2) at large p^2. It gives rise to the Imaginary part of the self-energy.

We don't know much about the Real part from first principles, it could be small or large, but even if large, it cannot cancel the Imaginary part contribution. [Note that one cannot use the usual dispersion relation to relate Real and Imaginary parts as cannot close contour at infinity]

8. Hence in a Higgsploding theory, the propagator,

$$\boxed{\Delta_R(p) \;=\; \frac{i}{p^2 - m^2 \;-\; \mathrm{Re}\,\Sigma_R(p^2) \;+\; \boxed{im\Gamma(p^2)} + i\epsilon}\,,}$$

is effectively cut off at $p^2 \geq E_*^2$ by the exploding width $\Gamma_n(p^2)$.

b) Tree-level n-point Amplitudes on mass threshold

The amplitude $\mathcal{A}_{1\to n}$ for the field ϕ to create n particles in the ϕ^4 theory,

$$\mathcal{L}_\rho(\phi) = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}M^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \rho\,\phi\,,$$

is derived by applying the LSZ reduction technique:

$$\langle n|\phi(x)|0\rangle = \lim_{\rho\to 0}\left[\prod_{j=1}^{n}\lim_{p_j^2\to M^2}\int d^4x_j e^{ip_j\cdot x_j}(M^2-p_j^2)\frac{\delta}{\delta\rho(x_j)}\right]\langle 0_{\rm out}|\phi(x)|0_{\rm in}\rangle_\rho\,.$$

Tree-level approximation is obtained via $\langle 0_{\rm out}|\phi(x)|0_{\rm in}\rangle_\rho \longrightarrow \phi_{\rm cl}(x)$ where $\phi_{\rm cl}(x)$ is a solution to the classical field equation.

On **mass threshold** limit all outgoing particles are produced at rest, $\vec{p}_j = 0$ and we set all $p_j^\mu = (\omega,\vec{0})$ and $\rho(x) = \rho(t) = \rho_0(\omega)\,e^{i\omega t}$. Hence,

$$(M^2-p_j^2)\frac{\delta}{\delta\rho(x_j)} \longrightarrow (M^2-\omega^2)\frac{\delta}{\delta\rho(t_j)} = \frac{\delta}{\delta z(t_j)}\,,$$

$$z(t) := \frac{\rho_0(\omega)\,e^{i\omega t}}{M^2-\omega^2-i\epsilon} := z_0\;e^{i\omega t}\,,\quad z_0 = \text{finite const}$$

b) Tree-level amplitudes in phi^4 on mass threshold

Brown 9209203

The generating function of tree amplitudes on multiparticle thresholds is a classical solution. It solves an ordinary differential equation with no source term,

$$d_t^2\phi + M^2\phi + \lambda\phi^3 \;=\; 0\,.$$

The solution contains only positive frequency harmonics, i.e. the Taylor expansion in $z(t)$,

$$\phi_{\rm cl}(t) \;=\; z(t) \;+\; \sum_{n=2}^{\infty} d_n\, z(t)^n\,, \qquad\quad z \;:=\; z_0\, e^{iMt}$$

Coefficients d_n determine the actual amplitudes by differentiation w.r.t. z,

$$\mathcal{A}_{1\to n} \;=\; \left(\frac{\partial}{\partial z}\right)^n \phi_{\rm cl}\Big|_{z=0} \;=\; n!\, d_n \qquad \textbf{Factorial growth!!}$$

$$\phi_{\rm cl}(t) \;=\; \frac{z(t)}{1-\frac{\lambda}{8M^2}\,z(t)^2} \qquad\quad \mathcal{A}_{1\to n} \;=\; n!\,\left(\frac{\lambda}{8M^2}\right)^{\frac{n-1}{2}}$$

b) Tree-level amplitudes for a scalar theory with SSB

Lagrangian for the scalar field:

$$\mathcal{L}(h) = \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 ,$$

prototype of the Higgs in the unitary gauge

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h = -\lambda h^3 + \lambda v^2 h ,$$

has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$h_{\rm cl}(t) = v\,\frac{1+\frac{z(t)}{2v}}{1-\frac{z(t)}{2v}} , \quad \text{where} \quad z(t) = z_0\, e^{iM_h t} = z_0\, e^{i\sqrt{2\lambda}\, v\, t}$$

$$h_{\rm cl}(t) = 2v \sum_{n=0}^{\infty} \left(\frac{z(t)}{2v}\right)^n d_n = v + 2v \sum_{n=1}^{\infty} \left(\frac{z(t)}{2v}\right)^n ,$$

i.e. with $d_0 = 1/2$ and all $d_{n\geq 1} = 1$.

$$\mathcal{A}_{1\to n} = \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} = \boxed{n!\,(2v)^{1-n}}$$

Factorial growth

L. Brown 9209203

Factorial growth of large-n scalar amplitudes on mass thresholds: E=nm

Similar story also holds in the Gauge-Higgs theory for tree-level amplitudes on multi-particle mass thresholds

VVK 1404.4876

These equations are solved by iterations (numerically) with Mathematica. The double Taylor expansion of the generating functions takes the form:

$$h_{\rm cl}(z, w^a) \;=\; 2v \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} d(n,2k) \left(\frac{z}{2v}\right)^n \left(\frac{w^a w^a}{(2v)^2}\right)^k ,$$

$$A^a_{L\,\rm cl}(z, w^a) \;=\; w^a \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a(n,2k) \left(\frac{z}{2v}\right)^n \left(\frac{w^a w^a}{(2v)^2}\right)^k ,$$

where $d(n,2k)$ and $a(n,2k)$ are determined from the iterative solution of EOM.

By repeatedly differentiating these with respect to z and w^a for the Higgs to n Higgses and m longitudinal Z bosons threshold amplitude we get,

$$\mathcal{A}(h \to n\times h + m\times Z_L) = \,(2v)^{1-n-m}\, n!\, m!\, d(n,m)\,,$$

and for the longitudinal Z decaying into n Higgses and $m+1$ vector bosons,

$$\mathcal{A}(Z_L \to n\times h + (m+1)\times Z_L) = \,\frac{1}{(2v)^{n+m}}\, n!\,(m+1)!\, a(n,m)\,.$$

Factorial growth reemains (in n and in m) !

Tree-level Amplitudes *above mass thresholds* are determined by recursive solutions to classical equations — now include the kinematic dependence

$$-\left(\partial^{\mu}\partial_{\mu}+M_h^2\right)\varphi \,=\, 3\lambda v\,\varphi^2 \,+\, \lambda\,\varphi^3$$

This classical equation for $\varphi(x) = h(x) - v$ determines directly the structure of the recursion relation for tree-level scattering amplitudes:

$$(P_{\rm in}^2 - M_h^2)\,\mathcal{A}_n(p_1\ldots p_n) \,=\, 3\lambda v \sum_{n_1,n_2}^{n} \delta^n_{n_1+n_2} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)},\ldots,p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\ldots p_{n_2}^{(2)})$$

$$+\lambda \sum_{n_1,n_2,n_3}^{n} \delta^n_{n_1+n_2+n_3} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)}\ldots p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\ldots p_{n_2}^{(2)})\,\mathcal{A}_{n_3}(p_1^{(3)}\ldots p_{n_2}^{(3)})$$

Away from the multi-particle threshold, the external particles 3-momenta $\vec{p}_i$ are non-vanishing. In the non-relativistic limit, the leading momentum-dependent contribution to the amplitudes is proportional to $E_n^{\rm kin}$ (Galilean Symmetry),

$$\mathcal{A}_n(p_1\ldots p_n) \,=\, \mathcal{A}_n + \mathcal{M}_n\,E_n^{\rm kin} \,:=\, \mathcal{A}_n + \mathcal{M}_n\,n\,\varepsilon\,,$$

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \,=\, \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n}\vec{p}_i^{\;2}\,.$$

In the non-relativistic limit we have $\varepsilon \ll 1$.

Above the n-particle thresholds: solution of the recursion relations

$$\varepsilon = \frac{1}{n\, M_h}\, E_n^{\rm kin} = \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n} \vec{p}_i^{\;2}$$

$$\mathcal{A}_n(p_1 \ldots p_n) \;=\; n!\,(2v)^{1-n}\left(1 - \frac{7}{6}\,n\,\varepsilon \,-\, \frac{1}{6}\,\frac{n}{n-1}\,\varepsilon + \mathcal{O}(\varepsilon^2)\right).$$

An important observation is that by exponentiating the order-$n\varepsilon$ contribution, one obtains the expression for the amplitude which solves the original recursion relation to all orders in $(n\varepsilon)^m$ in the large-n non-relativistic limit,

$$\mathcal{A}_n(p_1 \ldots p_n) \;=\; n!\,(2v)^{1-n}\,\exp\left[-\frac{7}{6}\,n\,\varepsilon\right], \quad n \to \infty\,, \quad \varepsilon \to 0\,, \quad n\varepsilon = {\rm fixed}\,.$$

Simple corrections of order ε, with coefficients that are not-enhanced by n are expected, but the expression is correct to all orders $n\varepsilon$ in the double scaling large-n limit. The exponential factor can be absorbed into the z variable so that

$$\varphi(z) = \sum_{n=1}^{\infty} d_n\, \left(z\, e^{-\frac{7}{6}\,\varepsilon}\right)^n\,,$$

- VVK 1411.2925

remains a solution to the classical equation and the original recursion relations.

Can now integrate over the phase-space

c) The main idea of the semi-classical set-up:

- DT Son1995

$\mathcal{R}_n(E)$ is the probability rate for a local operator $\mathcal{O}(0)$ to create n particles of total energy E from the vacuum,

$$\mathcal{R}_n(E) \;=\; \int \frac{1}{n!} d\Phi_n \, \langle 0| \, \mathcal{O}^\dagger \, S^\dagger \, P_E |n\rangle\langle n| \, P_E \, S \, \mathcal{O} \, |0\rangle$$

P_E is the projection operator on states with fixed energy E.

$$\mathcal{O} \;=\; e^{j\, h(0)}\,,$$

and the limit $j \to 0$ is taken in the computation of the probability rates,

$$\mathcal{R}_n(E) \;=\; \lim_{j\to 0} \int \frac{1}{n!} d\Phi_n \, \langle 0| \, {e^{j\,h(0)}}^\dagger \, S^\dagger \, P_E |n\rangle\langle n| \, P_E \, S \, e^{j\,h(0)} \, |0\rangle\,.$$

Note: non-dynamical (non-propagating) initial state $\mathcal{O}|0\rangle$.
The semi-classical (steepest descent) limit:

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.$$

Evaluate the path integral in this double-scaling limit.
n enters via the coherent state formalism.

c) Semi-classical approach for computing the rate R(1->n,E)

- DT Son1995

1. Solve the classical equation without the source-term,

$$\frac{\delta S}{\delta h(x)} \;=\; 0$$

   by finding a complex-valued solution $h(x)$ with a point-like singularity at the origin $x^\mu = 0$ and regular everywhere else in Minkowski space.

2. Impose the initial and final-time boundary conditions,

$$\lim_{t\to -\infty} h(x) \;=\; v\,+\int \frac{d^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\;a_{\mathbf{k}}\,e^{ik_\mu x^\mu}$$

$$\lim_{t\to +\infty} h(x) \;=\; v\,+\int \frac{d^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\left(b_{\mathbf{k}}\,e^{\omega_{\mathbf{k}}T-\theta}\,e^{-ik_\mu x^\mu}\;+\;b^*_{\mathbf{k}}\,e^{ik_\mu x^\mu}\right)$$

c) Semi-classical approach for computing the rate R(1->n,E)

- DT Son1995

3. Compute the energy and the particle number using the $t \to +\infty$ asymptotics of $h(x)$,

$$E = \int d^3k \, \omega_{\mathbf{k}} \, b^*_{\mathbf{k}} \, b_{\mathbf{k}} \, e^{\omega_{\mathbf{k}} T - \theta}, \qquad n = \int d^3k \, b^*_{\mathbf{k}} \, b_{\mathbf{k}} \, e^{\omega_{\mathbf{k}} T - \theta}.$$

At $t \to -\infty$ the energy and the particle number are vanishing. The energy is conserved by regular solutions and changes discontinuously from 0 to E at the singularity at $t = 0$.

4. Eliminate the T and θ parameters in favour of E and n using the expressions above. Finally, compute the function $W(E, n)$

$$W(E,n) = ET - n\theta - 2\mathrm{Im}S[h]$$

and thus determine the semiclassical rate $\mathcal{R}_n(E) = \exp\left[W(E,n)\right]$

c) Semi-classical approach for computing the rate R(1->n,E)

- DT Son1995

In practice: Match the two branches of the solution $h_1(\tau,\vec{x})$ and $h_2(t,\vec{x})$ on a complexified time surface $\tau = \tau_0(\vec{x})$.

$h_1(\tau,\vec{x})$ and $h_2(t,\vec{x})$ are finite regular solutions with boundary conditions

$$\begin{aligned}\lim_{\tau\to+\infty} h_1(\tau,\vec{x}) - v &= 0\\ \lim_{t\to+\infty} h_2(t,\vec{x}) - v &= \int \frac{d^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\left(b_{\mathbf{k}}\, e^{\omega_{\mathbf{k}}T-\theta}\, e^{-ik_\mu x^\mu} + b^*_{\mathbf{k}}\, e^{ik_\mu x^\mu}\right).\end{aligned}$$

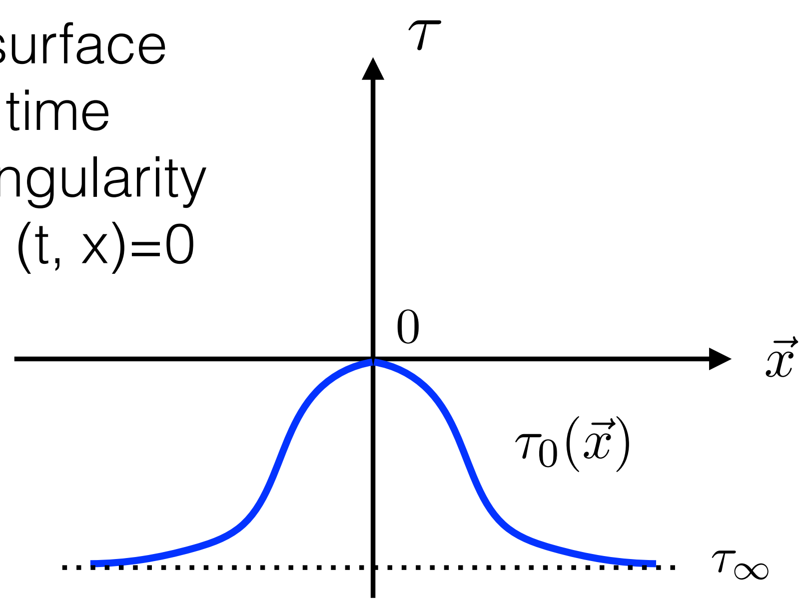
The Euclidean action of the complete solution $h(x)$ along our complex-time contour is obtained by extremizing the integral

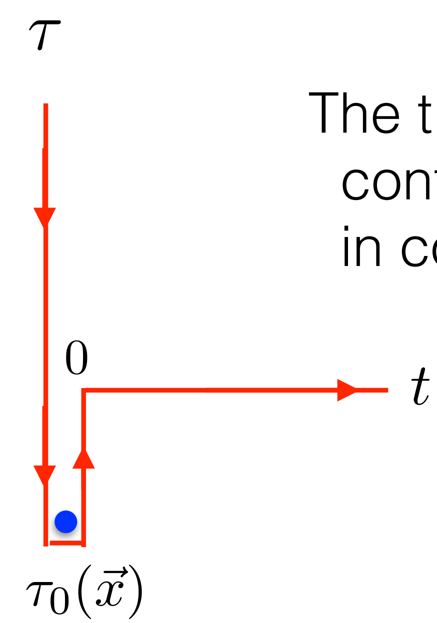
$$S_{\mathrm{Eucl}}[\tau_0(\vec{x})] = \int d^3x \left[-\int_{+\infty}^{\tau_0(\vec{x})} d\tau\, \mathcal{L}_{\mathrm{Eucl}}(h_1) - \int_{\tau_0(\vec{x})}^{0} d\tau\, \mathcal{L}_{\mathrm{Eucl}}(h_2) - i\int_0^\infty dt\, \mathcal{L}(h_2)\right]$$

over all surfaces $\tau = \tau_0(\vec{x})$ (containing the origin).

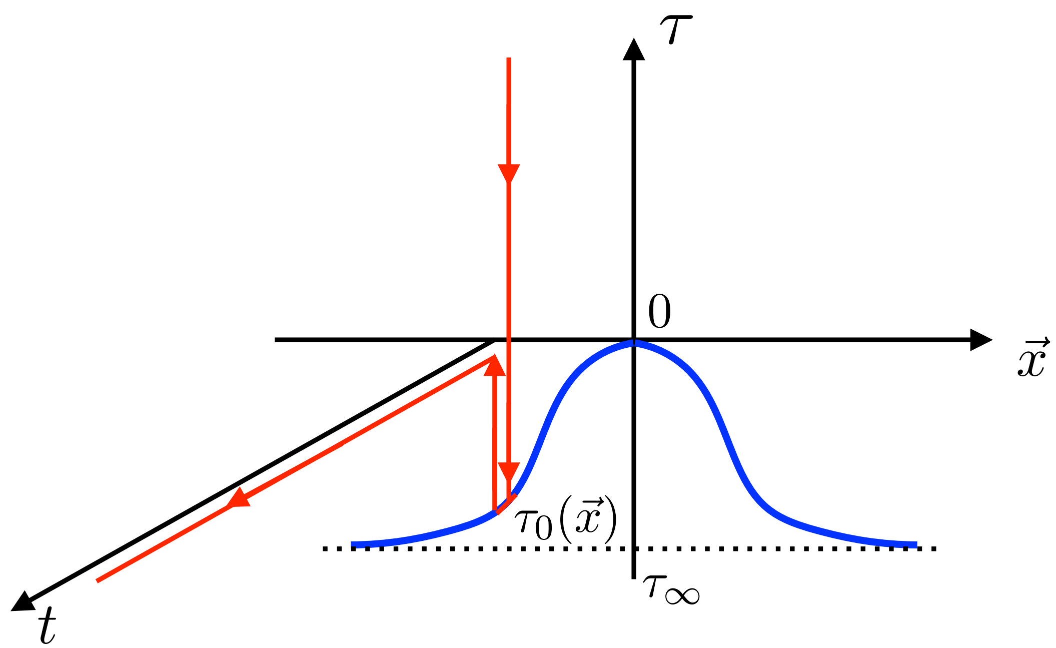
c) Semi-classical approach for computing the rate R(1->n,E)

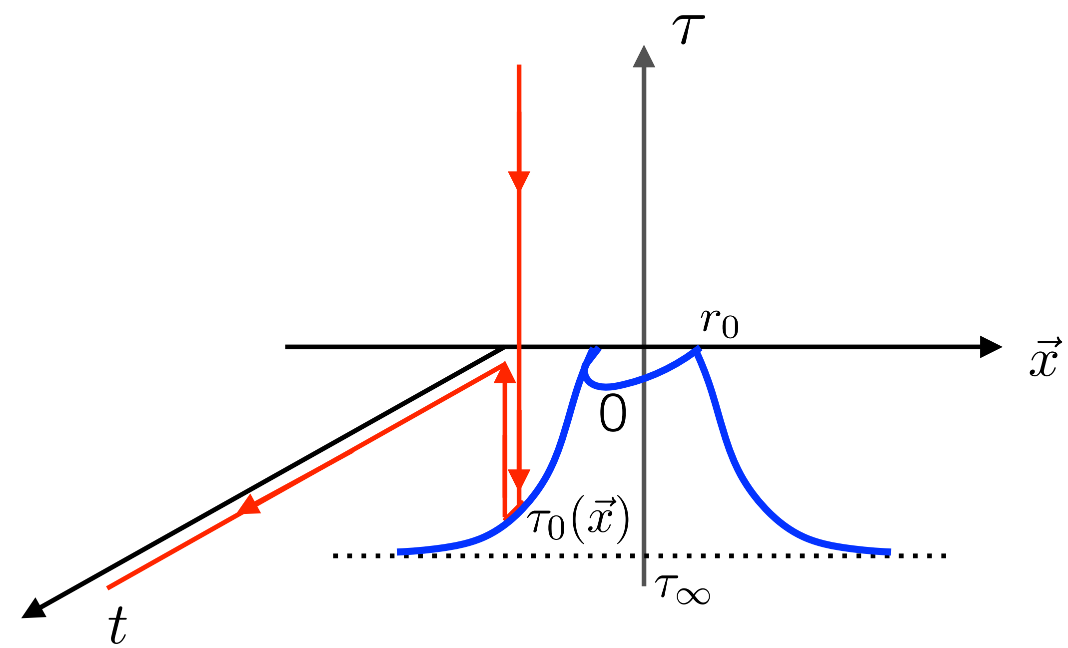
The matching surface
in Imaginary time
containing the singularity
of h at the origin (t, x)=0





The time-evolution
contour of h(t,x)
in complex time

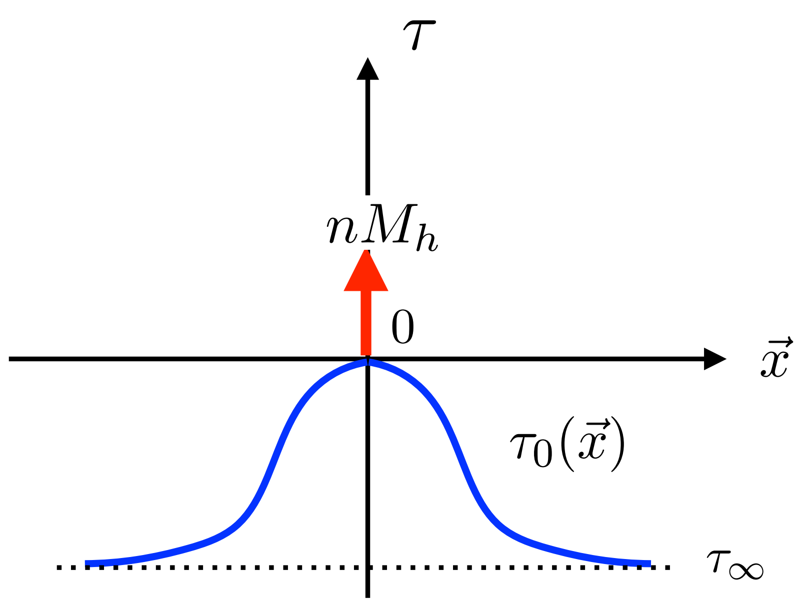




The matching surface
as the domain wall

…the domain wall of thin-wall bubbles
of radius r> r_0 (critical radius)

c) Semi-classical approach for computing the rate R(1->n,E)



$$
\begin{aligned}
W(E,n;\lambda)^{\text{tree}} &= \frac{\lambda n}{\lambda}\left(f_0(\lambda n) + f(\varepsilon)\right) \\
f_0(\lambda n) &= \log\left(\frac{\lambda n}{4}\right) - 1 \\
f(\varepsilon)|_{\varepsilon\to 0} &\to f(\varepsilon)_{\text{asympt}} = \frac{3}{2}\left(\log\left(\frac{\varepsilon}{3\pi}\right) + 1\right)
\end{aligned}
$$

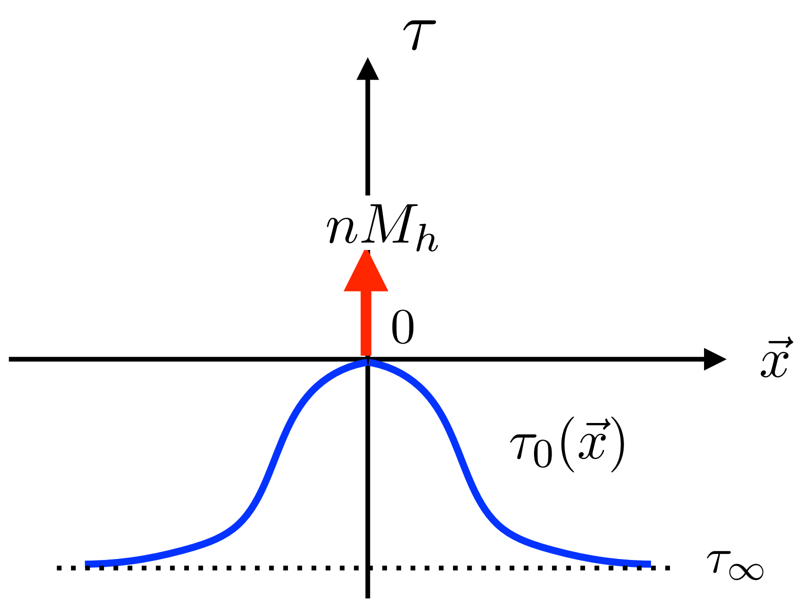
$$
h(\tau,\vec{x}) = v\left(\frac{1 + e^{-M_h(\tau-\tau_\infty)}}{1 - e^{-M_h(\tau-\tau_\infty)}}\right) + \delta h(\tau,\vec{x})
$$

$$
W(E,n;\lambda) = W(E,n;\lambda)^{\text{tree}} - 2nM_h\tau_\infty - 2(S_{\text{Eucl}}[\tau_0(x)] - S_{\text{Eucl}}[0])
$$

The quantum correction to the tree-level result W^{tree} is

$$
\boxed{
\begin{aligned}
\frac{1}{2\lambda}\,g(\lambda n) &= -nM_h\tau_\infty - \text{Re}(S_{\text{Eucl}}[\tau_0(x)] - S_{\text{Eucl}}[0]) \\
&= nM_h|\tau_\infty| - \text{Re}(S_{\text{Eucl}}[\tau_0(x)] - S_{\text{Eucl}}[0])
\end{aligned}
}
$$

c) Semi-classical approach for computing the rate R(1->n,E)



$$W(E,n;\lambda)^{\mathrm{tree}} = \frac{\lambda n}{\lambda}\left(f_0(\lambda n) + f(\varepsilon)\right)$$

$$f_0(\lambda n) = \log\left(\frac{\lambda n}{4}\right) - 1$$

$$f(\varepsilon)|_{\varepsilon\to 0} \to f(\varepsilon)_{\mathrm{asympt}} = \frac{3}{2}\left(\log\left(\frac{\varepsilon}{3\pi}\right) + 1\right)$$

$$h(\tau,\vec{x}) = v\left(\frac{1 + e^{-M_h(\tau-\tau_\infty)}}{1 - e^{-M_h(\tau-\tau_\infty)}}\right) + \delta h(\tau,\vec{x})$$

$$W(E,n;\lambda) = W(E,n;\lambda)^{\mathrm{tree}} - 2nM_h\tau_\infty - 2(S_{\mathrm{Eucl}}[\tau_0(x)] - S_{\mathrm{Eucl}}[0])$$

Using the thin-wall bubble solution in the $\lambda n \gg 1$ limit we get

$$\frac{1}{\lambda}g(\lambda n) := \Delta W(E,n;\lambda) = \frac{1}{\lambda}(\lambda n)^{3/2}\frac{2}{\sqrt{3}}\frac{\Gamma(5/4)}{\Gamma(3/4)} \simeq 0.854\, n\sqrt{\lambda n}$$

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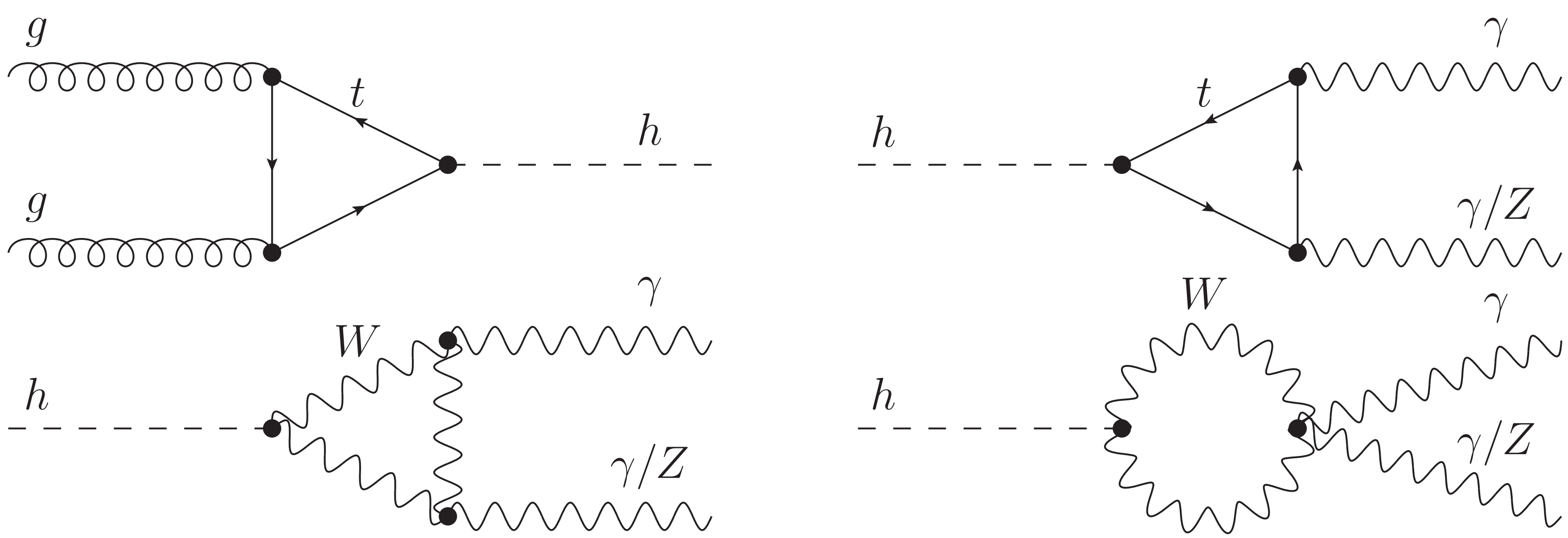
d) Effects of Higgsplosion on Precision Observables

- VVK, J Reiness, M Spannowsky, P Waite 1709.08655

Here focus on a class of observables which have no tree-level contributions



At LHC energies effects of Higgsplosion are small (next slide).

However O(1) effects can be achieved for these loop-induced processes if the interactions are probed close to ~ 2E*.

d) Effects of Higgsplosion on Precision Observables

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