



# Saturation and Diffraction at the LHC and the EIC

by ECT\*, Trento/I 27 Jun-1 July 2022

HUNTING FOR QCD PHENOMENA IN THE FORWARD

**Work in Progress  
(Testing the waters)**



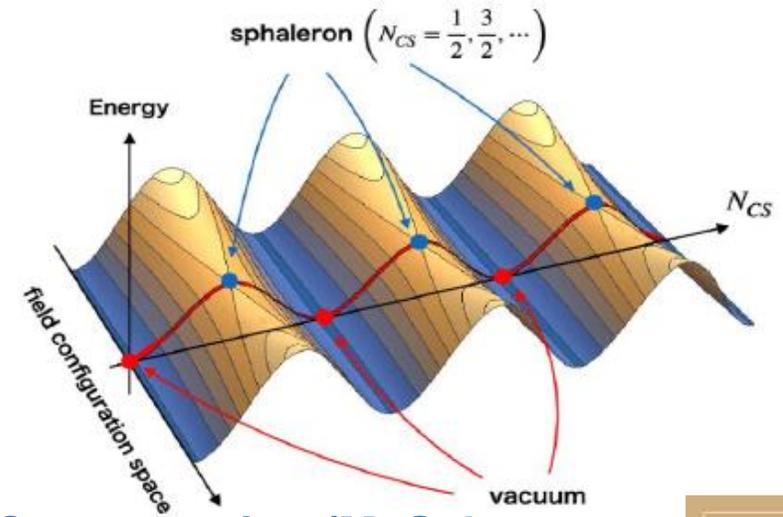
**30 years on**

V. A. Khoze, V. V. Khoze, D. L. Milne and M. G. Ryskin, PRD 104, 05401  
105,03600

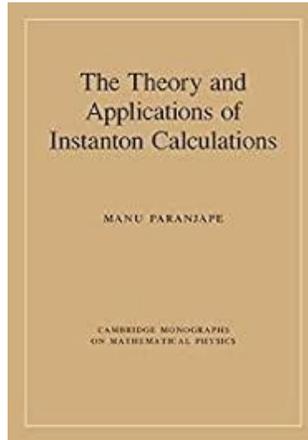
## Currently focus mainly on searches for the BSM phenomena

While the existence of topological effects within the Standard Model, such as QCD instantons or electroweak Sphaleron signatures is well known, it is far from clear, how they can be experimentally observed

Recently –a renewal of interest, initiated by the experimental LHC community (M. Schott et al)



Belavin, Polyakov, Schwarz, Tyupkin, 1975



Instantons describe quantum tunneling between different vacuum sectors of the QCD and are arguably the best motivated yet experimentally unobserved nonperturbative effects predicted by the Standard Model.

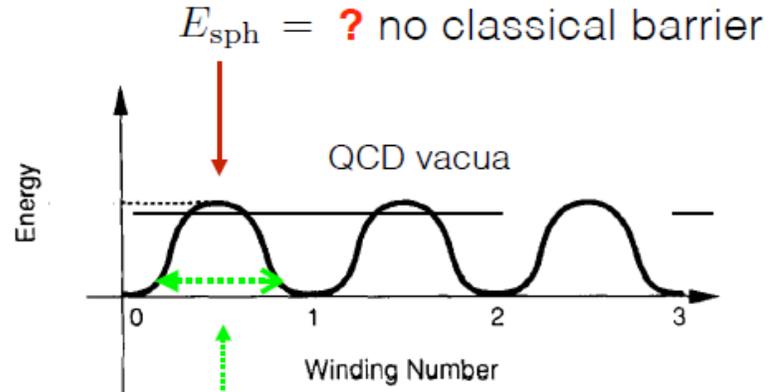
Recent calculations of Instanton-induced processes in pp collisions :

V.V. Khoze, F. Krauss, M. Schott, [1911.09726](#) : JHEP

V.V.Khoze, D.Milne, M. Spannowsky, [2010.02287](#) : PRD

# QCD Instantons

- Yang-Mills vacuum has a nontrivial structure
- *Instantons* are tunnelling solutions between the vacua.
- At the classical level there is no barrier in QCD. The *sphaleron* is a quantum effect
- Transitions between the vacua change chirality (result of the ABJ anomaly).
- All light quark-anti-quark pairs must participate in the reaction
- Not described by perturbation theory.



Sphaleron-transition on top of an energy barrier



$$g + g \rightarrow n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf})$$

Instanton-induced processes with 2 gluons in the initial state:

$$g + g \rightarrow n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf})$$

$\vdots$   
 arbitrary  
 (tends to be large  $\sim 1/\alpha_s$ )

All light flavours of quark-antiquark  
 pairs must be present. Light =>  
 $m_f \leq 1/\rho$ .  
 $\vdots$   
 instanton size

Can also have quark-initiated processes e.g. :

$$u_L + \bar{u}_R \rightarrow n_g \times g + \sum_{f=1}^{N_f-1} (q_{Rf} + \bar{q}_{Lf}),$$

$$u_L + d_L \rightarrow n_g \times g + u_R + d_R + \sum_{f=1}^{N_f-2} (q_{Rf} + \bar{q}_{Lf})$$

Instanton size is cut-off by partonic energy  $\sim \sqrt{s}$   
 this is what sets the  
 effective QCD sphalrenon scale

Quantum corrections  
 due to in-in states  
 interactions

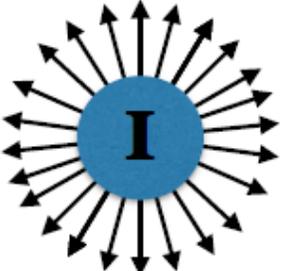
Mueller 1991

Instantons have never been observed **experimentally**, however, they are playing very important role in the **theoretical** models of confinement and chiral symmetry breaking

a possible solution to the axial  $U(1)$  problem

$$\langle 0 | G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle \neq 0$$

Instanton signatures:



one of the biggest challenges for particle physics to date

LO Instanton vertex -> selection on final states at colliders with high sphericity

- large multiplicity  $N_{jet} \sim 1/\alpha_s(\rho_{inst})$   $E_T \sim 1/\rho_{inst}$

'soft bombs' –high-multiplicity spherically symmetric distributions of relatively soft particles

- large 'Sphericity',  $S \rightarrow 1$

- presence of an additional light  $\bar{q}_R q_L$  pairs

(in particular pair of strange (or charm. for the small size instanton) quarks)

**Instanton  $\neq$  the particle ( no peak in  $M_{inst}$ )**

Extended objects in space-time

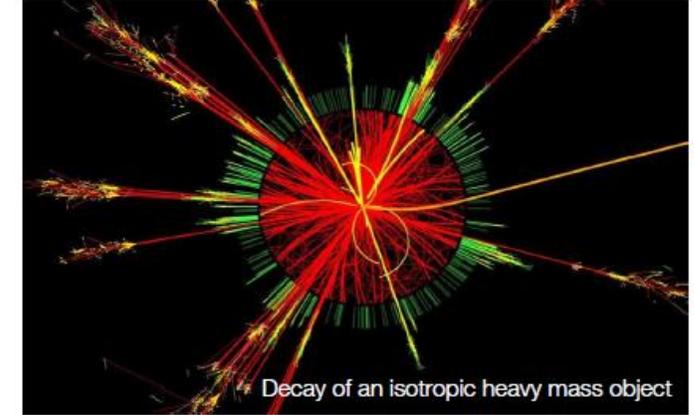
It is a family of objects of different size,  $\rho$ , and orientations in Lorentz and colour spaces

**Effectively –a family of new multiparton vertices in Feynman diagrams**

Elementary  $gg \rightarrow I + \dots$  cross section at  $\sqrt{s'} = M_{inst}$

$\sqrt{s'}$ [GeV]	$1/\rho$ [GeV]	$\alpha_S(1/\rho)$	$\langle n_g \rangle$	$\hat{\sigma}$ [pb]
10.7	0.99	0.416	4.59	$4.922 \cdot 10^9$
15.7	1.31	0.360	5.13	$728.9 \cdot 10^6$
22.9	1.76	0.315	5.44	$85.94 \cdot 10^6$
29.7	2.12	0.293	6.02	$17.25 \cdot 10^6$
40.8	2.72	0.267	6.47	$2.121 \cdot 10^6$
56.1	3.50	0.245	6.92	$229.0 \cdot 10^3$
61.8	3.64	0.223	7.28	$72.97 \cdot 10^3$

Now in SHERPA



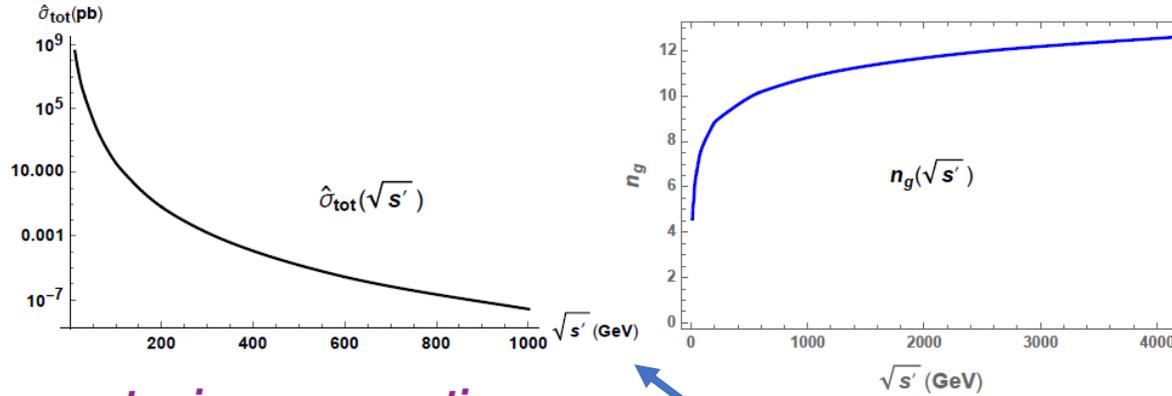
$\sqrt{s'_{min}}$ [GeV]	20	50	100	200	500
$\sigma_{pp \rightarrow I}$	6.32 mb	$40.82 \mu\text{b}$	79.95 nb	105.4 pb	3.54 fb

**Table 2.** Hadronic cross sections for instanton production through initial gluons, at the 13 TeV LHC, using the NNPDF3.1 NNLO set with  $\alpha_s(M_Z) = 0.118$  [67].

V.V. Khoze, F. Krauss, M. Schott, 1911.09726

$$\sigma(pp \rightarrow I) \sim M_{inst}^{-6}$$

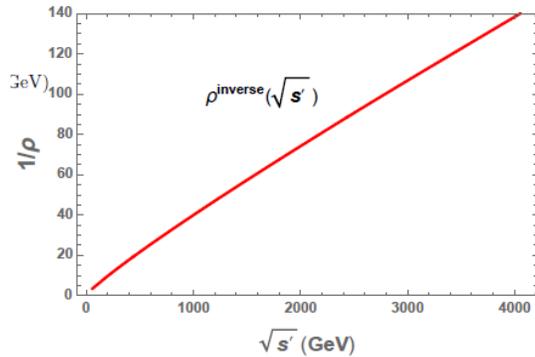
Note infrared divergence at large  $\rho$  (small  $M_{inst}$ )



*partonic cross-sections*

Instanton size is cut-off by partonic energy  $\sim \sqrt{s}$   
 this is what sets the effective QCD sphalrenon scale

Quantum corrections due to in-in states interactions



Small instanton masses-large rates, but difficult to distinguish from soft QCD activity (+ PU 'complications' at high lumi).  
 A large mass-striking multijet signature, but a very low rate.  
 The S/B ratio falls very rapidly with l-mass increasing –a higher chance of observing the signal in the low-mass range.



# Background

1. Multiple parton interactions  
(Double/Triple/... parton scattering)

Large at small  $M_{inst}$

$$\frac{d\sigma}{dE_1^2 \dots dE_n^2} \sim \left( \frac{d\sigma}{\sigma_{eff} dE_1^2} \dots \frac{d\sigma}{\sigma_{eff} dE_n^2} \right) \sigma_{eff}$$

$$d\sigma/d\vec{E}_i^2 \sim \pi\alpha_s^2/E_i^4 \quad \sigma_{eff} \sim 10 \text{ mb}$$

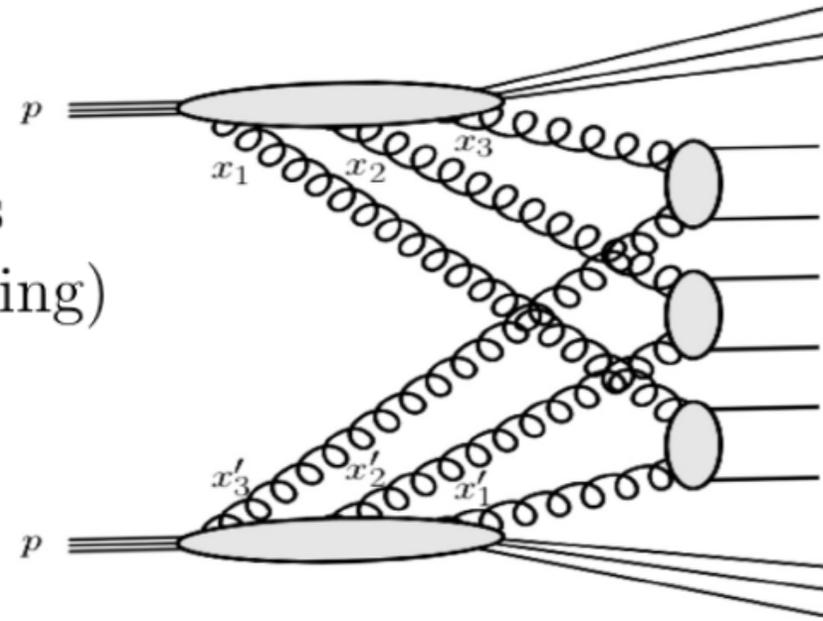
$E_i$  denotes the transverse energy of a jet in the  $i$  dijet system,

$$S^2 \leq 0.1$$

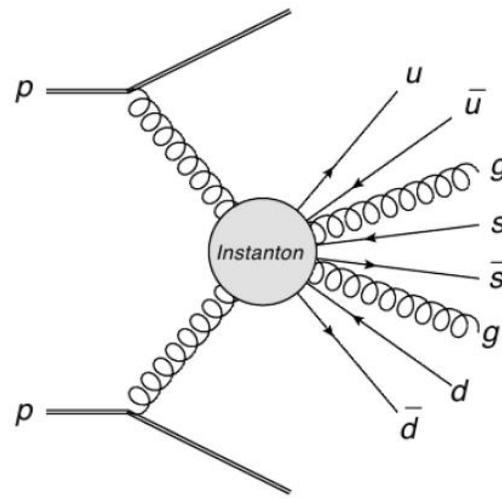
Thus the probability to observe  $n$  additional branches

in LRG events is suppressed by the factor  $(S^2)^n$ .

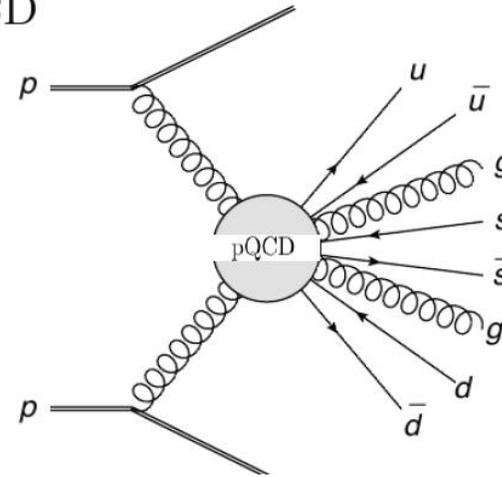
events with an LRG mainly occur at large value of  $b_t$ ,



## 2. pQCD



$$\sigma(pp \rightarrow I) \sim 1/M_{inst}^7$$

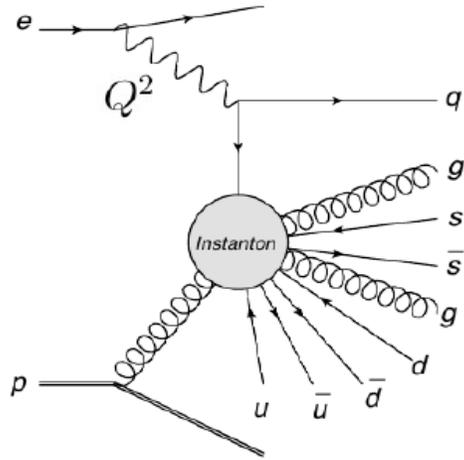


$$\sigma(gg \rightarrow N \cdot jets) \sim (16\pi/M_{inst}^2)\alpha_s^N$$

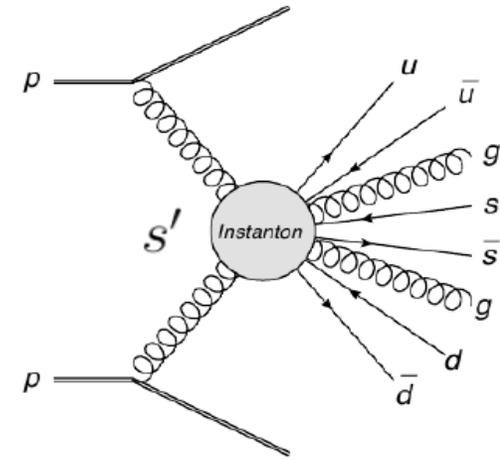
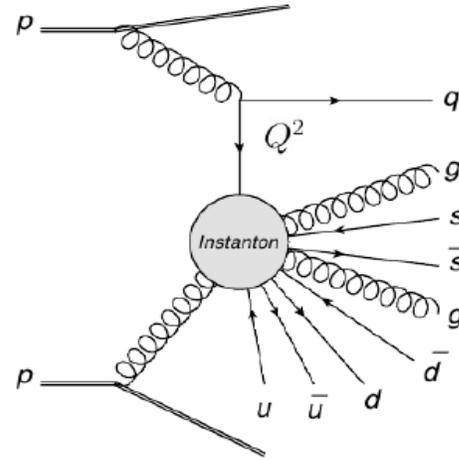
(hedgehog-like)

$$\sigma(gg \rightarrow N jets) \sim \sigma(gg \rightarrow I) \text{ at } M_{inst} > 200 \text{ GeV}$$

Thus, at sufficiently large values of  $M_{inst}$  the instanton signal will become negligible relative to the purely perturbative QCD background.



**Figure 2.** Depiction of a QCD Instanton processes in electron-proton (left) and proton-proton (right) collisions, where an external scale parameter  $Q'$  is required.



**Figure 3.** Depiction of a QCD Instanton processes in proton-proton (right) collisions.

a) To select  $Q^2$  in DIS (or  $q_{T,jet}$ )  
 (A. Ringwald, F.Schrempp, PL B438 (1998) 217)

b) To select events with  $\sum_i E_{T,i} > E_{cut}$

*If the instanton is recoiled by a high  $p_T$  jet emitted from one of the initial state gluons => hadronic cross-section is tiny*



- Instanton event – large  $N_{ch}$  (due to  $N_{jets}$ ) but not too large  $\Sigma E_{T,i}$

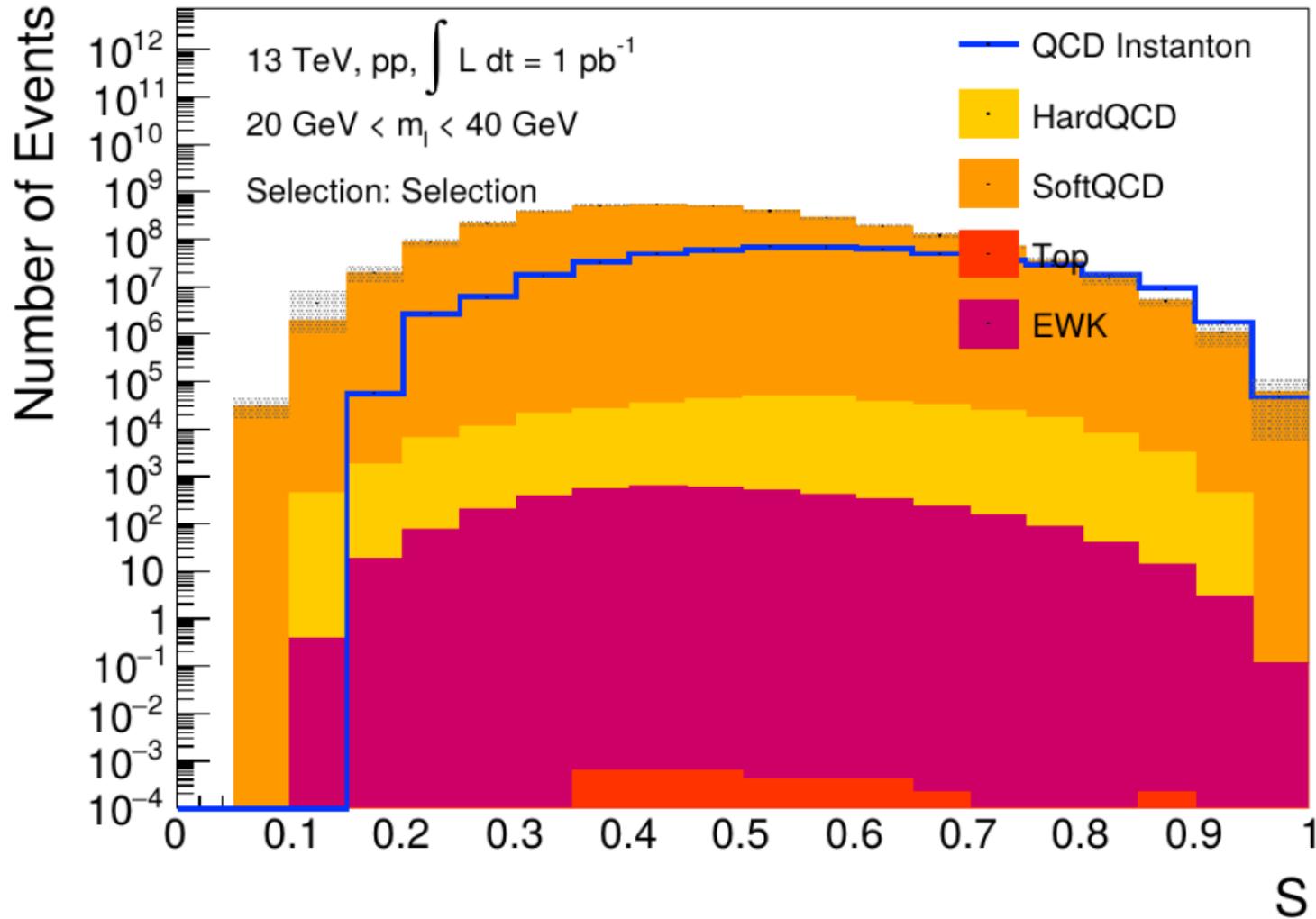
- Sphericity  $S = (3/2)(\lambda_2 + \lambda_3)$  close to 1  
 $\lambda_1 > \lambda_2 > \lambda_3$  are the eigenvalues of  $S^{\alpha\beta}$

(dijet events lead to sphericity of  $S = 0$ )

$$S^{\alpha\beta} = \frac{\Sigma p_i^\alpha p_i^\beta}{\Sigma |\vec{p}_i|^2}$$

- extra  $(\bar{s}s)$  pair of strange particles

$$g + g \rightarrow n_g \times g + \sum_{f=1}^{N_f} (q_{Rf} + \bar{q}_{Lf})$$



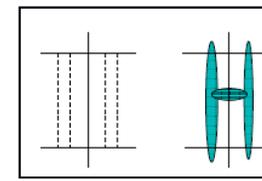
Instanton signal samples have been produced with a modified version of the SHERPA

BG *softQCD* processes in the PYTHIA used as baseline. simulated through the Delphes framework no PU



Simone Amoroso<sup>a</sup> Deepak Kar<sup>b</sup> Matthias Schott<sup>c</sup> 2012.09120

First limits based on existing LHC Minimum Bias data



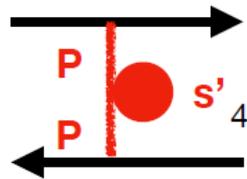
# UA8 and double-Pomeron production

A Study of Inclusive  
Double-Pomeron-Exchange  
in  $p\bar{p} \rightarrow pX\bar{p}$  at  $\sqrt{s} = 630$  GeV

A. Brandt<sup>1</sup>, S. Erhan<sup>2</sup>, A. Kuzucu<sup>2</sup>, M. Medinnis<sup>3</sup>,  
N. Ozdes<sup>2,4</sup>, P.E. Schlein<sup>5</sup>, M.T. Zeyrek<sup>5</sup>, J.G. Zweizig<sup>6</sup>  
University of California<sup>1</sup>, Los Angeles, California 90024, U.S.A.

J.B. Cheze, J. Zembery  
Centre d'Etudes Nucleaires-Saclay, 91191 Gif-sur-Yvette, France.

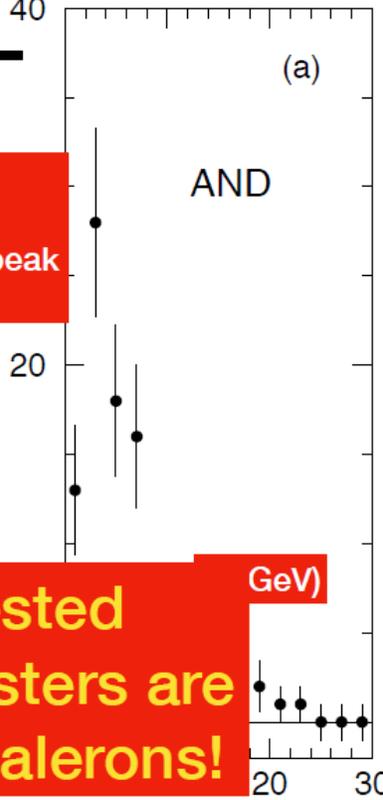
ep-ex/0205037v3 21 Jul 2002



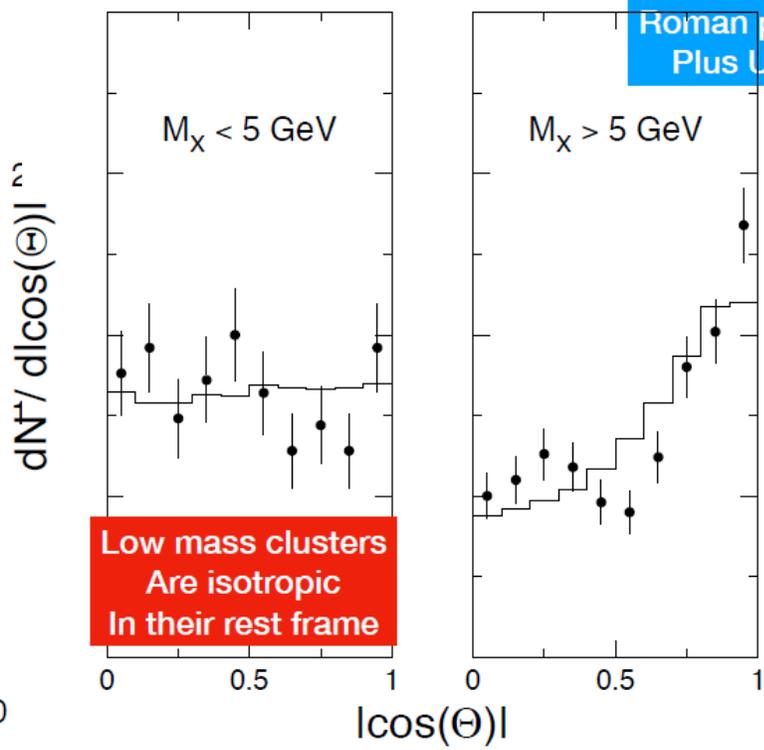
$$\frac{d^6\sigma_{DPE}}{d\xi_1 d\xi_2 dt_1 dt_2 d\phi_1 d\phi_2} = F_{P/p}(t_1, \xi_1) \cdot F_{P/p}(t_2, \xi_2) \cdot \sigma_{PP}^{tot}(s').$$

$s'=M^2$  is the cluster mass squared

Mass Distribution  
Has unpredicted peak  
At 2-8 GeV



We suggested  
 $M < 5$  GeV clusters are  
The QCD sphalerons!



Low mass clusters  
Are isotropic  
In their rest frame

Roman pots on both sides (AND)  
Plus UA2 central calorimeter

Small and large M  
Production is different  
In magnitude and  
Angular distributions

## Instanton in diffractive events

V. A. Khoze, V. V. Khoze, D. L. Milne and M. G. Ryskin,

PRD 104, 054013  
105,036008

**Lower background since  
No multiparton interactions**

**Event selection::**

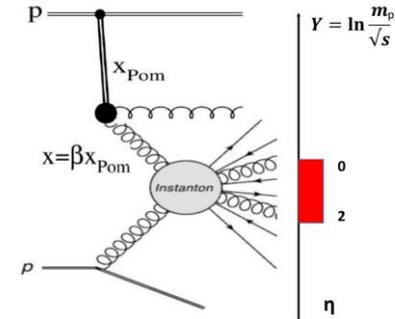
$$N_{ch} > 20 \quad \sum_i E_{T,i} > 15 \text{ GeV}$$
$$(0 < \eta < 2 \quad p_{T,i} > 0.5 \text{ GeV})$$

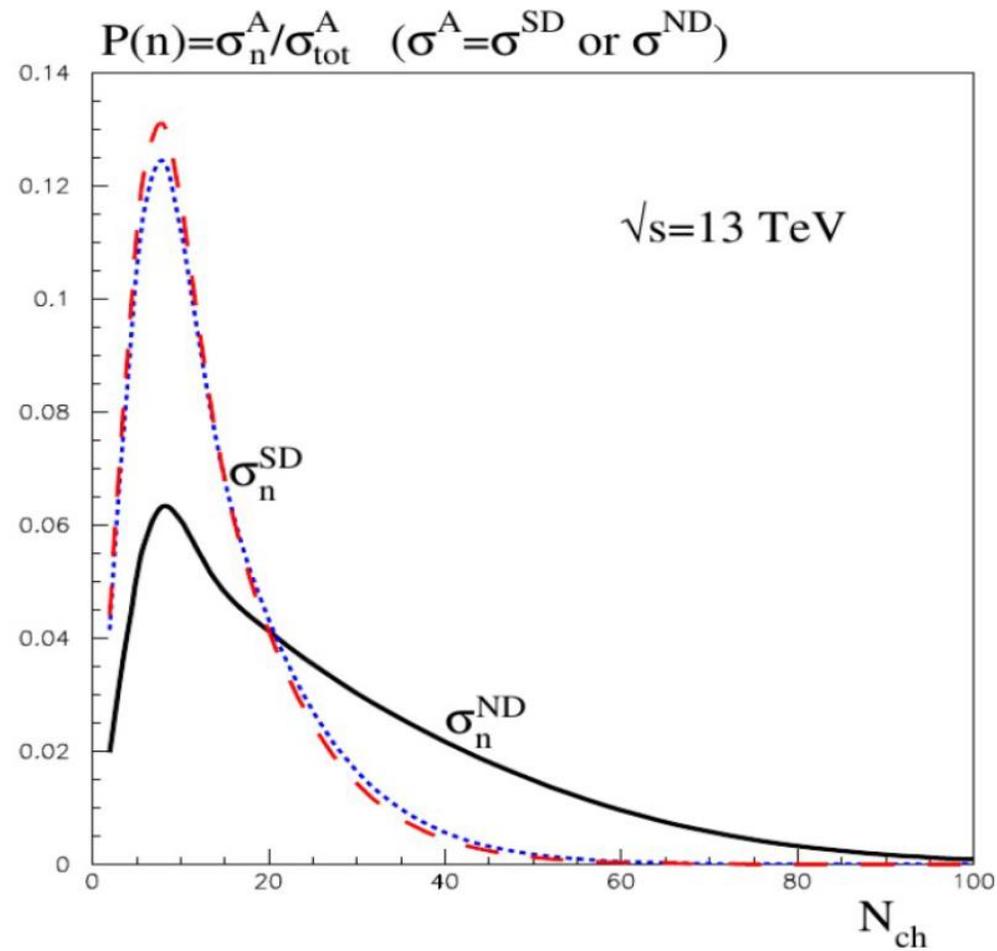
**No charged at  $-2 < \eta < 2$  with  $p_T > 2 \text{ GeV}$   
(to exclude high  $E_T$  jets)**

use low luminosity runs to avoid problems with large pile-up

**Searching for the Instanton as a multiparticle cluster/fireball with a mass  $\sim 20\text{-}60 \text{ GeV}$  in events with an LRG**

LRG can be detected either by detecting the leading forward proton with beam momentum fraction,  $x_L = 1 - \xi$ , very close to 1 ( $\xi = x_{Pom} \leq 0.01$ ), or by observing no hadron activity in the forward calorimeters.





; the instanton of mass 30 GeV produces about 17 jets (9 gluons plus 4 light  $\bar{q}q$  pairs). The energy of each jet  $E_{T_i} \sim 1/\rho \sim 2$  GeV. After hadronization in such an event we expect about 40-60 particles. The large multiplicity can be used as the main (or additional) trigger to select the events of interest.

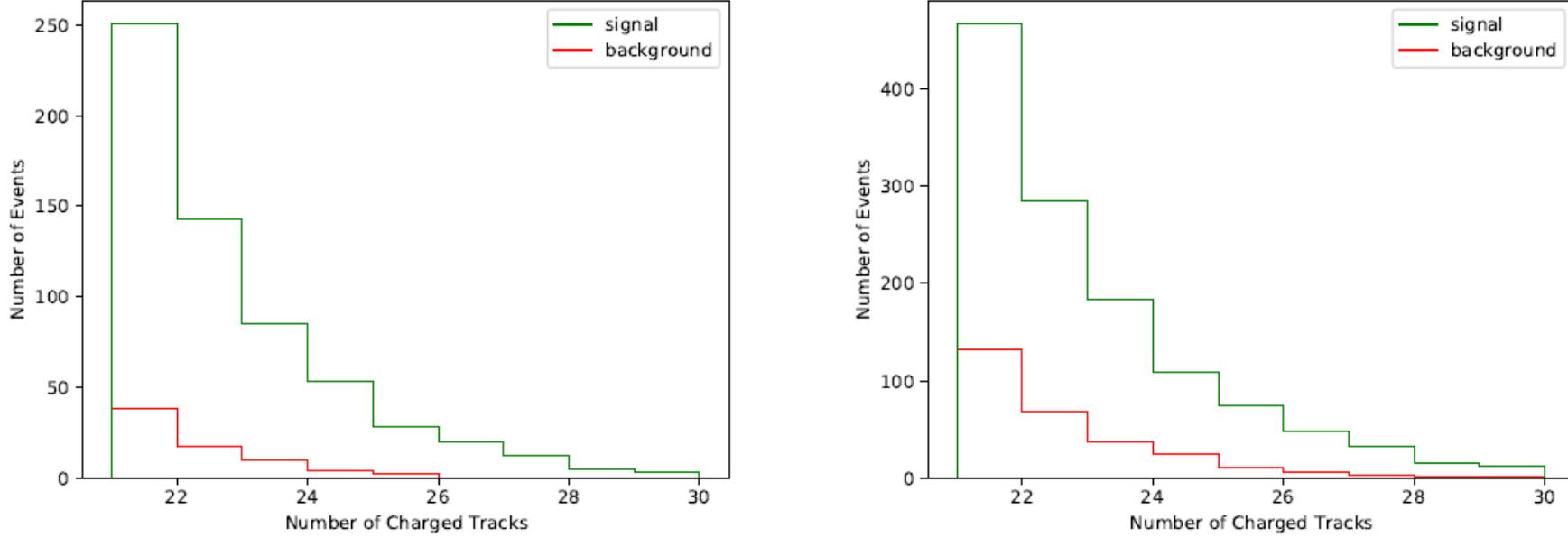


Figure 3: Multiplicity distribution of charged hadrons produced in the events with the instanton (green) in comparison with the expected background (red). The number of events is normalised to the integrated luminosity  $L = 1 \text{ pb}^{-1}$  and  $\Delta \ln(x_{Pom}) = 1$  interval. We required events to have  $\sum_i E_{T,i} > 15 \text{ GeV}$  and  $N_{ch} > 20$ , summing only over charged particles in the region  $0 < \eta < 2$  with  $p_T > 0.5 \text{ GeV}$ , with an additional constraint that there is no charged particle in this region with  $p_T > 2 \text{ GeV}$  (left figure), or no charged particle in the region  $-2 < \eta < 2$  with  $p_T > 2.5 \text{ GeV}$  (right figure).

## Low Luminosity run (no PU)

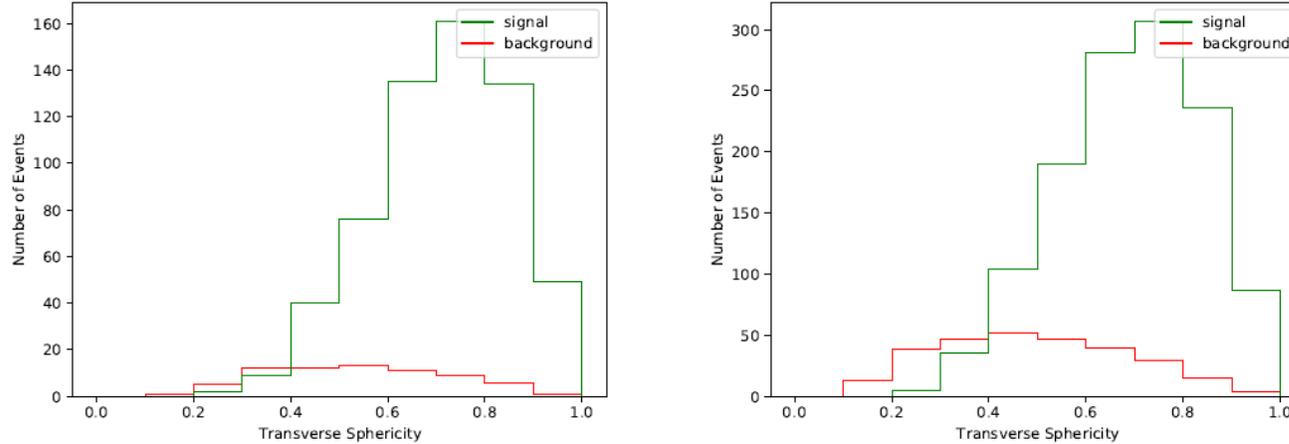


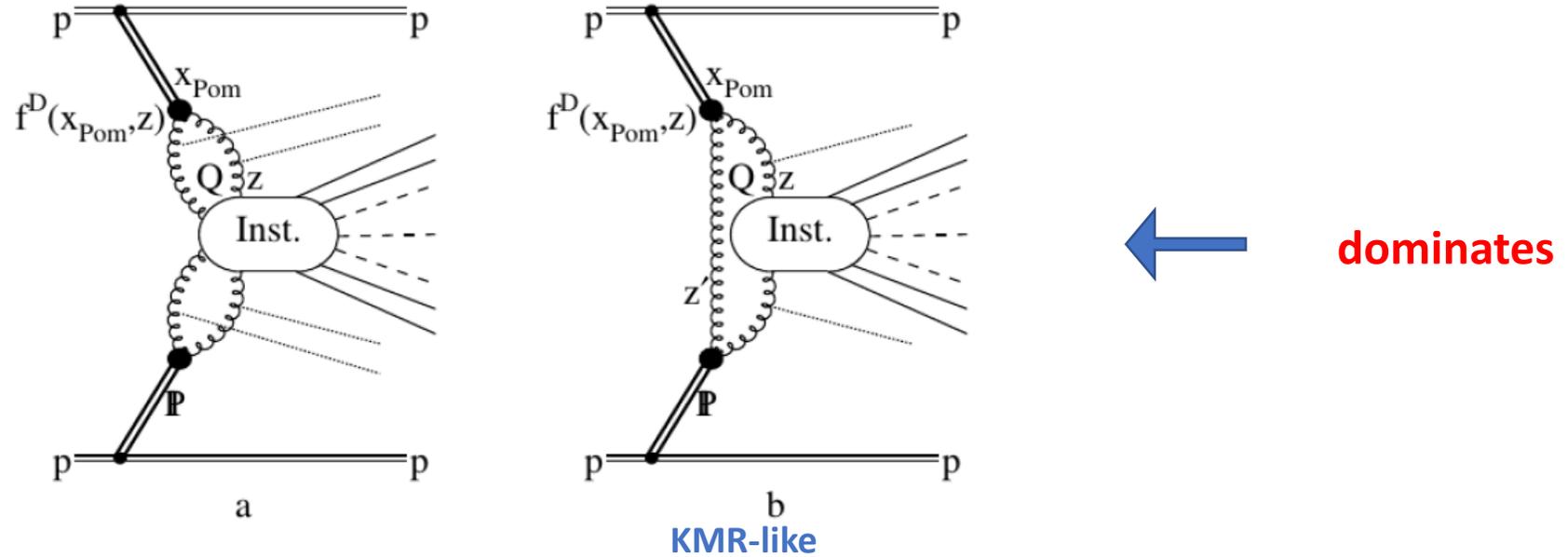
Figure 4: Distribution over the transverse sphericity  $S_T$ , Eq. (8), of the charged hadrons produced in the events with the instanton (green) in comparison with the expected background (red). The selection criteria used are the same as those described in Fig. 3

- It is shown that by imposing appropriate cuts on final states we can select the kinematical region where the I-signal exceeds BG by a factor of at least 2.5. At  $\sum_i E_{T,i} > 15$  GeV,  $N_{\text{ch}} > 20$  measured within the  $0 < \eta < 2$  the rate is expected to be large enough to measure Instanton production in the events with LRG at low luminosity
- Even with these rather strong cuts in place, the expected instanton cross-section remains sufficiently large ( $\sim 1$  nb) to effectively produce and probe QCD instantons at the LHC, at low luminosity runs, avoiding pile-up problems.

# Central Instanton Production

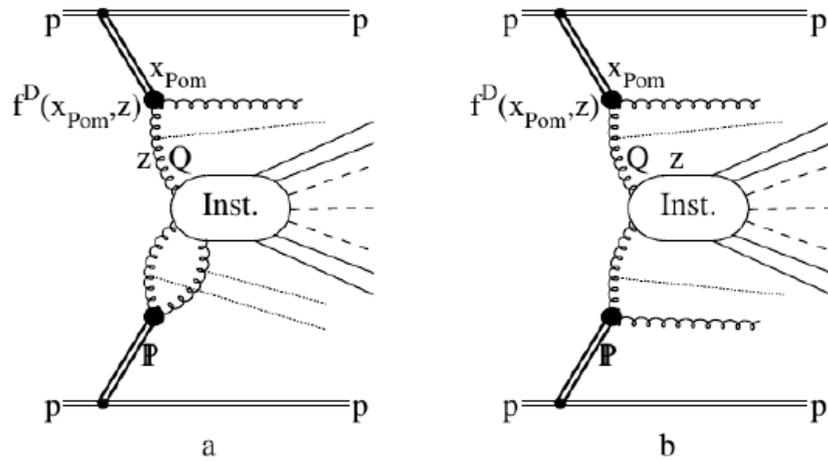
$$pp \rightarrow p + \mathbb{P} + \mathbb{P} + p \rightarrow p + X + p.$$

Fig.1



- ★ Detecting two outgoing protons would allow placing an upper limit on Instanton mass.
- ★ only a small part of the finally produced hadron state will avoid detection ('hermiticity')

Fig.2

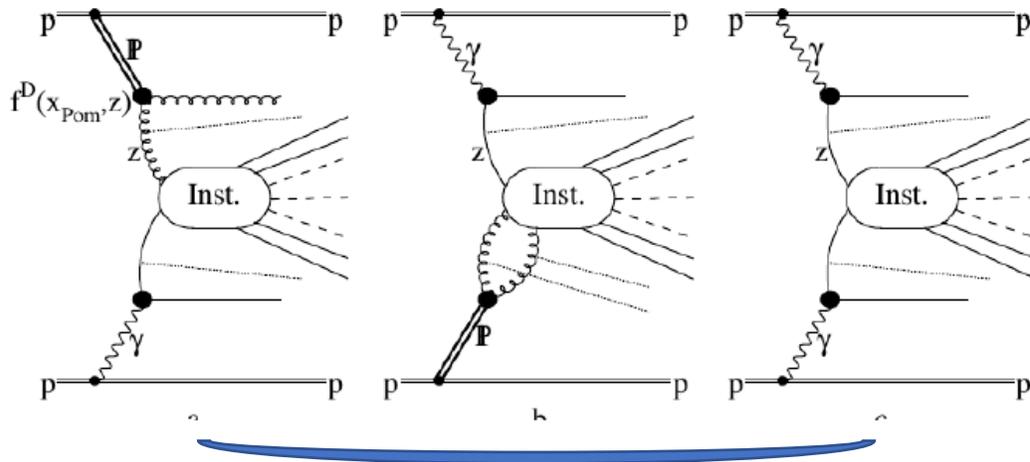


**Dominant**

For a reasonable instanton mass  $M_{inst} \gtrsim 50$  GeV

$$\sigma_{pp \rightarrow I}^{(2b)} \sim \text{hundreds of pb}$$

Fig.3



**UPC-no PU nightmare**

L. Harland-Lang et al

$$\alpha_{em}^2 \sim 10^{-4}$$

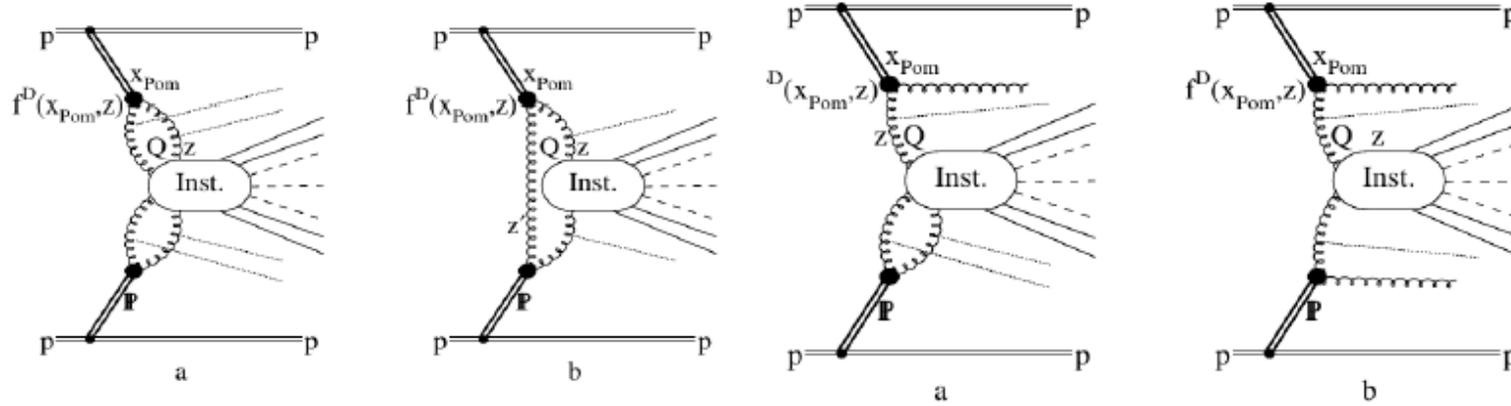
**HIC**

Full set of analytical formulas for the partonic processes, calculated for the first time

$M_{inst}$ [GeV]	$d\sigma_{pp}^{(1a)}$ [pb]	$d\sigma_{pp}^{(1b)}$ [pb]	$d\sigma_{pp}^{(2a)}$ [pb]	$d\sigma_{pp}^{(2b)}$ [pb]	$d\sigma_{pp}^{(2b)}, Q_t > 20\text{GeV}$
15	13.3	$4.56 \cdot 10^4$	$3.72 \cdot 10^3$	$1.83 \cdot 10^5$	-
35	$6 \cdot 10^{-3}$	$1.69 \cdot 10^2$	8.10	$2.28 \cdot 10^3$	$1.99 \cdot 10^{-3}$
55	$3.82 \cdot 10^{-5}$	3.27	$1.19 \cdot 10^{-1}$	$8.96 \cdot 10^1$	$2.95 \cdot 10^{-3}$
75	$8.8 \cdot 10^{-7}$	$1.61 \cdot 10^{-1}$	$4.72 \cdot 10^{-3}$	7.06	$1.70 \cdot 10^{-3}$
95	$4.27 \cdot 10^{-8}$	$1.38 \cdot 10^{-2}$	$3.42 \cdot 10^{-4}$	$8.58 \cdot 10^{-1}$	$7.26 \cdot 10^{-4}$
115	$3.37 \cdot 10^{-9}$	$1.74 \cdot 10^{-3}$	$3.68 \cdot 10^{-5}$	$1.39 \cdot 10^{-1}$	$2.80 \cdot 10^{-4}$
135	$3.77 \cdot 10^{-10}$	$2.86 \cdot 10^{-4}$	$5.29 \cdot 10^{-6}$	$2.75 \cdot 10^{-2}$	$1.04 \cdot 10^{-4}$

Table 1: Instanton cross-sections at the 14 TeV LHC. The differential cross-sections for the process in Figs.1a, 1b and 2a, 2b, given by Eqs. (5.1) and (5.2), are computed for a range of instanton masses  $M_{inst}$ .

$$x_{P1} = x_{P2} = 0.03 \text{ integrated over } z.$$



## CENTRAL PRODUCTION “PROPONENT’S” SUMMARY



- ★ It is shown that for a instanton mass  $M_{inst} \geq 50$  GeV the expected central production cross sections for the instanton-induced processes are of the order of picobarns in the pure exclusive case and increase up to hundreds of pb when the emission of spectator jets is allowed.
- ★ The x- sections are encouragingly large and under favourable background conditions there is a tantalising chance that QCD instanton effects can either be seen or ruled out.
- ★ The expected experimental signature for the instanton-induced process in the central detector is a large multiplicity and transverse energy ( $\sum_i ET_i$ ) in relatively small rapidity interval ( $\delta y \simeq 2 - 3$  and large sphericity  $S > 0.8$  of the event.

# CONCLUSION



- The direct experimental observation of Instanton-induced processes would be a real breakthrough in particle physics.
- QCD instanton cross-sections can be very large at hadron colliders (lower end of partonic energies 20-80 GeV).
- An existing lack of evidence by no means leads to the conclusion that QCD instanton “does not exist”, but rather that their actual production rate is on the low end of predictions.
- Potential for large sources of theoretical uncertainties covering orders of magnitude. A practical point for future progress is to test theory normalization of predicted instanton rate with data.
- Searches for the signal in non-diffractive events are very challenging: modeling the detailed final state, background suppression, and separation from the possible SM and BSM sources of the “hedgehog-like ” events. Events with a large  $p_T$  signature are too rare.
- Diffraction (single tag, CEP) promises some attractive advantages: cleaner signal, suppressed ‘standard’ backgrounds (MPI).
- By imposing appropriate cuts on final states in SD we can select the kinematical region where the I-signal exceeds BG by a factor of at least 2.5. The rate is expected to be large enough to measure Instanton production in the events with a single proton tag.
- Searches for Instantons with  $M_{inst} \geq 50$  GeV in the CEP mode would require measurements with 420m stations (PPS, AFP)



Strong need for enthusiastic experimental experts to join the efforts, addressing such issues as detector effects, PU at high luminosity, and timing resolution.....

(Marek Tasevsky et al)

**The main obstacle currently – PU at high luminosity**



# Conclusion



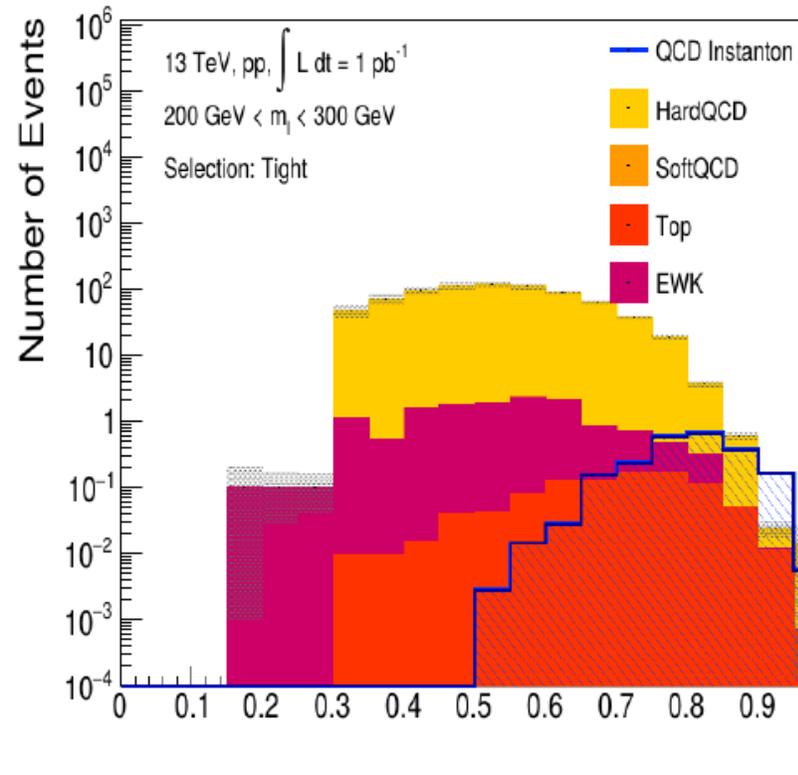
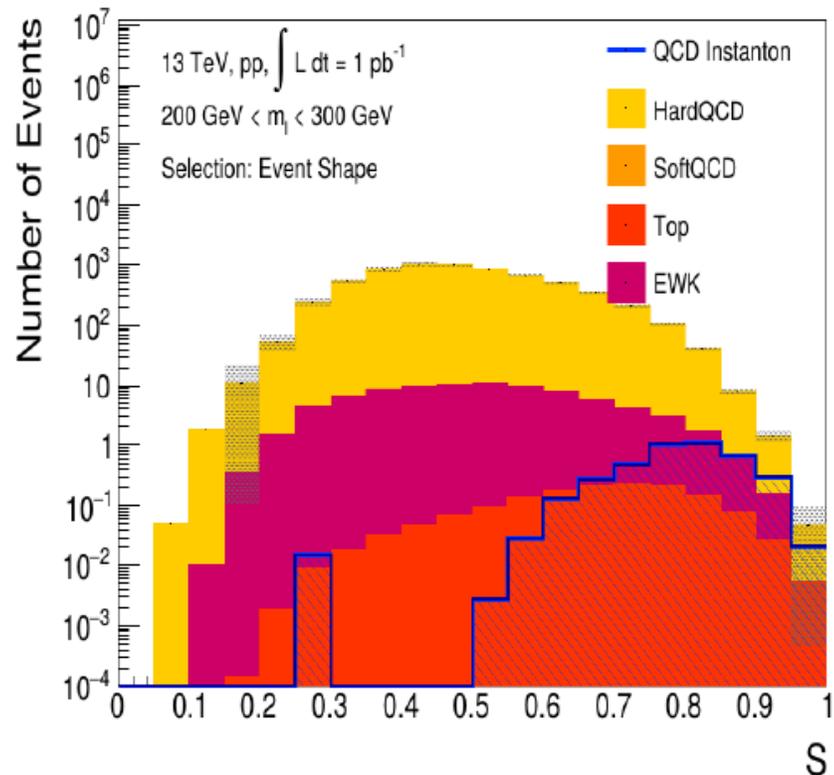


BACKUP

	Signal Region			Control Region	
	Standard	Event-Shape	Tight	A	B
Invariant mass of rec. tracks (Instanton Mass), $m_I$	$200 \text{ GeV} < m_I < 300 \text{ GeV}$				
Selection Requirements					
Number of rec. tracks, $N_{\text{Trk}}$	>80	>80	>80	>80	>80
Number of rec. tracks/Instanton mass, $m_I/N_{\text{Trk}}$	<3.0	<3.0	<3.0	>3.0	<3.0
Number of Jets, $N_{\text{Jets}}$	3-6	3-6	3-6	3-6	3-6
Broadening, $\mathcal{B}_{\text{Tracks}}$		>0.3	>0.3	>0.3	>0.3
Thrust, $\mathcal{T}_{\text{Tracks}}$		>0.3	>0.3	>0.3	>0.3
Number of displaced vertices, $N_{\text{Displaced}}$			>15		<10
Results					
Expected Events for $\int Ldt = 1 \text{ pb}^{-1}$ in the Signal Region ( $\mathcal{S} > 0.85$ )					
$N_{\text{Signal}}$	5.6	1.0	0.54	0.04	0.21
$N_{\text{Background}}$	1900	9.6	0.64	200	1100

**Table 5.** Overview of the standard and tight signal selection as well as the definition of two control regions aiming at very low Instanton masses ( $200 \text{ GeV} < m_I < 300 \text{ GeV}$ )

**Simone Amoroso<sup>a</sup> Deepak Kar<sup>b</sup> Matthias Schott 2012.09120**



$$N_{displ} > 15$$

First limits based on existing LHC Minimum Bias data

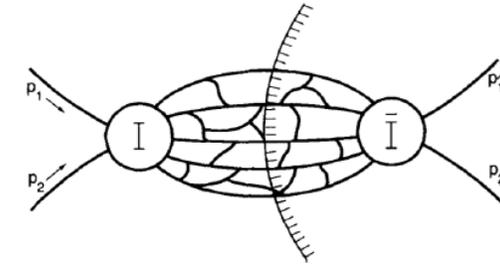
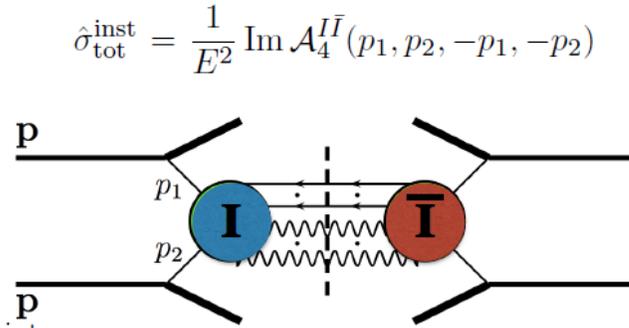
Simone Amoroso<sup>a</sup> Deepak Kar<sup>b</sup> Matthias Schott<sup>c</sup> 2012.0912

Hedgehog-like QCD background not accounted for (yet)



## The Optical Theorem approach: to include final state interactions

- Cross-section is obtained by [squaring] the instanton amplitude.
- Final states have been instrumental in combatting the exp. suppression.
- Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.



V.V.Khoze, A.Ringwald-1991

**Total hadronic cross-sections for instanton processes are large**

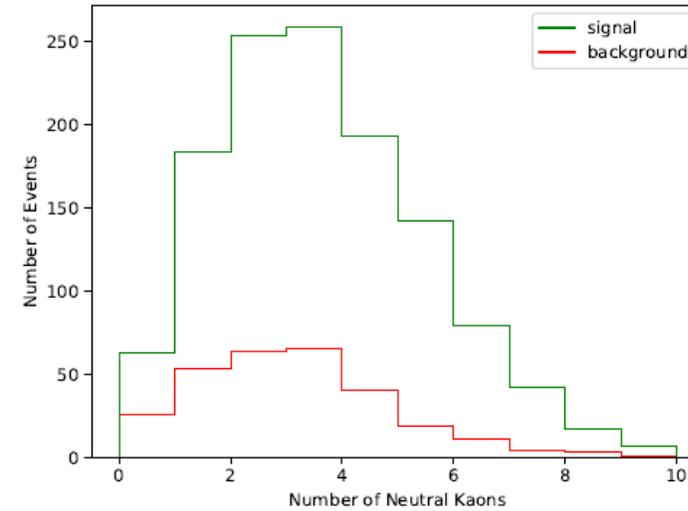
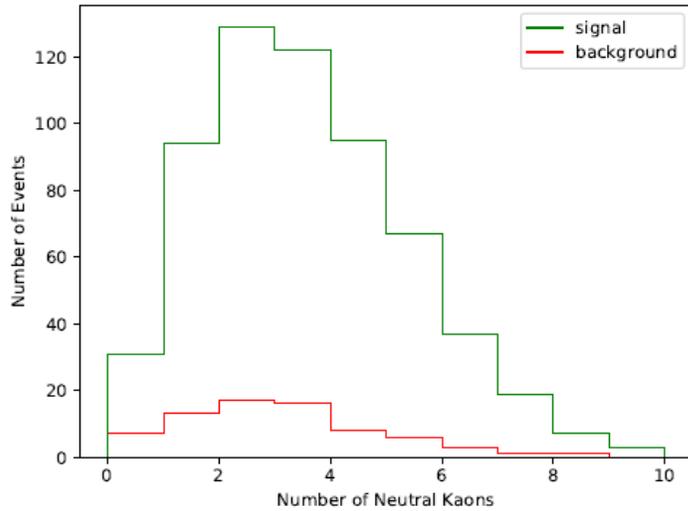
$$\sigma_{pp \rightarrow I}(\hat{s} > \hat{s}_{\min}) = \int_{\hat{s}_{\min}}^{s_{pp}} dx_1 dx_2 f(x_1, Q^2) f(x_2, Q^2) \hat{\sigma}(\hat{s} = x_1 x_2 s_{pp})$$

V.V.Khoze, F.Krauss, M.Schott-1911.0977

V.V.Khoze, D. Milne, M.Spannowsky-2010.02287

practical approach: vary minimal E

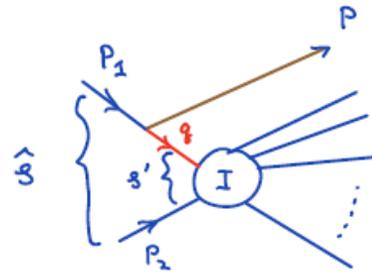
$E_{\min}$ [GeV]	50	100	150	200	300	400	500
$\sigma_{p\bar{p} \rightarrow I}$ $\sqrt{s_{p\bar{p}}} = 1.96$ TeV	2.62 $\mu\text{b}$	2.61 nb	29.6 pb	1.59 pb	6.94 fb	105 ab	3.06 ab
$\sigma_{pp \rightarrow I}$ $\sqrt{s_{pp}} = 14$ TeV	58.19 $\mu\text{b}$	129.70 nb	2.769 nb	270.61 pb	3.04 pb	114.04 fb	8.293 fb
$\sigma_{pp \rightarrow I}$ $\sqrt{s_{pp}} = 30$ TeV	211.0 $\mu\text{b}$	400.9 nb	9.51 nb	1.02 nb	13.3 pb	559.3 fb	46.3 fb
$\sigma_{pp \rightarrow I}$ $\sqrt{s_{pp}} = 100$ TeV	771.0 $\mu\text{b}$	2.12 $\mu\text{b}$	48.3 nb	5.65 nb	88.3 pb	4.42 pb	395.0 fb



- It is shown that by imposing appropriate cuts on final states we can select the kinematical region where the I-signal exceeds BG by a factor of at least 2.5. At  $\sum_i E_{T,i} > 15$  GeV,  $N_{ch} > 20$  measured within the  $0 < \eta < 2$  the rate is expected to be large enough to measure Instanton production in the events with LRG at low luminosity

- Even with these rather strong cuts in place, the expected instanton cross-section remains sufficiently large ( $\sim 1$  nb) to effectively produce and probe QCD instantons at the LHC, at low luminosity runs, avoiding pile-up problems.

**HOWEVER:** If the instanton is recoiled by a high  $p_T$  jet emitted from one of the initial state gluons  $\Rightarrow$  hadronic cross-section is tiny



$$Q^2 = -q^2 = \sqrt{3} p_T$$

$$s' = (q+p_2)^2 = \hat{s} - 2Q^2$$

A virtual log

$$\Rightarrow e^{-Qq} \leftarrow \text{formfactor.}$$

↑ cuts-off  
low-energy range.

Mueller corr.s  
cuts-off high-energy  
range (as before.)

$$\exp(-Q(\rho + \bar{\rho})) = \exp\left(-\frac{Q}{E} \sqrt{y(x+1/x+2)}\right)$$

$\sqrt{\hat{s}}$ [GeV]	310	350	375	400	450	500
$\hat{\sigma}_{\text{tot}}^{\text{inst}}$ [pb]	$3.42 \times 10^{-23}$	$1.35 \times 10^{-18}$	$1.06 \times 10^{-17}$	$1.13 \times 10^{-16}$	$9.23 \times 10^{-16}$	$3.10 \times 10^{-15}$

**Table 3.** The instanton partonic cross-section recoiled against a hard jet with  $p_T = 150$  GeV emitted from an initial state and calculated using Eq. (3.7). Results for the cross-section are shown for a range of partonic C.o.M. energies  $\sqrt{\hat{s}}$ .