Spontaneous $B-L$ Breaking as the Origin of the Hot Early Universe

Valerie Domcke
DESY, Hamburg, Germany

in collaboration with
W. Buchmüller, K. Schmitz, K. Kamada

arxiv[hep-ph]:
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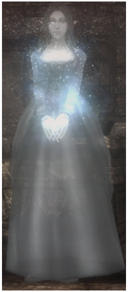

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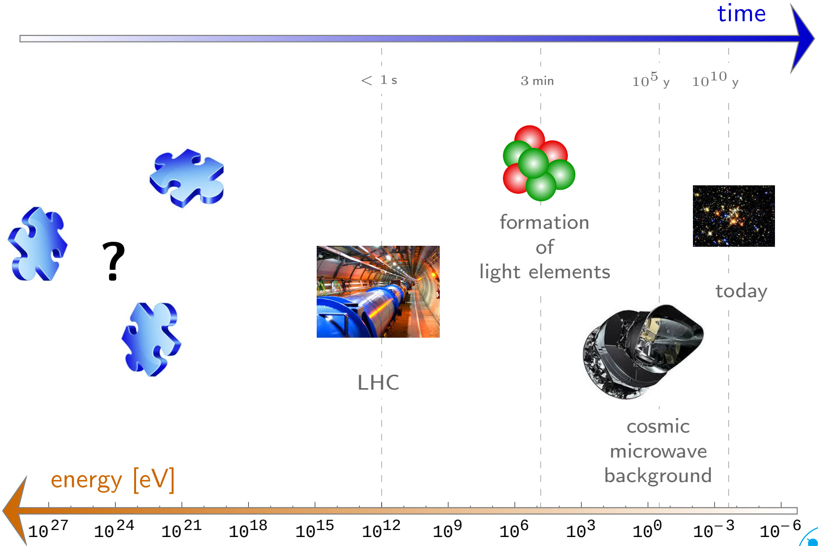
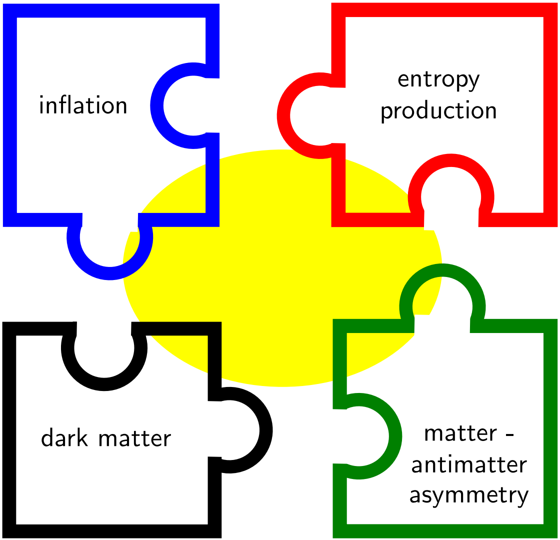
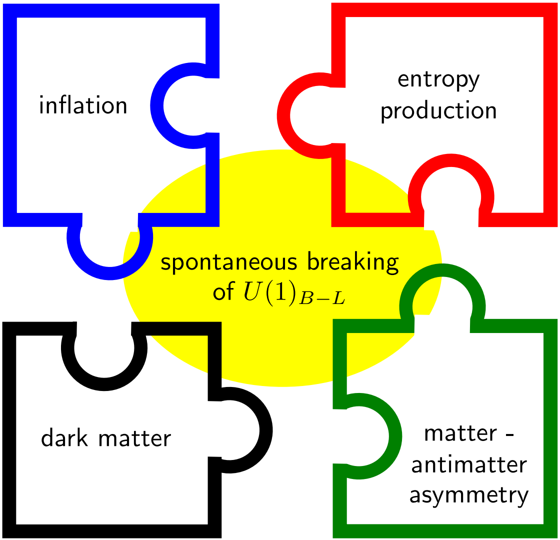
Vanilla Cosmology?



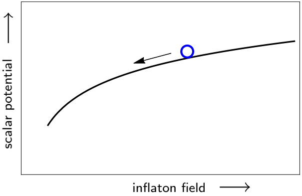
Motivation

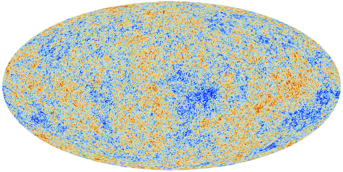

inflation
entropy production
dark matter
matter - antimatter asymmetry

Motivation


inflation
entropy production
spontaneous breaking of $U(1)_{B-L}$
dark matter
matter - antimatter asymmetry

Inflation





[Planck '13]

- exponential expansion driven by slowly rolling scalar field
- 'stretched' quantum fluctuations $\rightarrow$ inhomogeneities of the cosmic microwave background
- more a paradigm than a model

Entropy Production

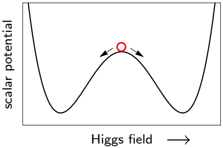
Expanding, cooling universe: Hot thermal plasma as initial state

Reheating:

- generation of the thermal bath through decay of heavy particles
- perturbative process

Preheating:

- rapid, nonperturbative process
- tachyonic preheating: triggered by tachyonic instability, exponential growth of low momentum modes [Felder *et al.* '01]



- large abundance of non-relativistic Higgs bosons, small abundances of particles coupled to it [Garcia-Bellido *et al.* '02]

Matter and Dark Matter

Matter-Antimatter asymmetry

- small, but very significant $B-L$ asymmetry:

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.19 \pm 0.15) \cdot 10^{-10}$$

[Komatsu *et al* '10]

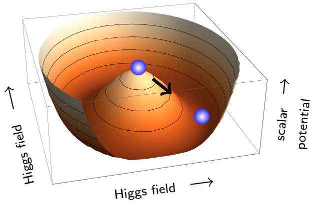
- leptogenesis: generate matter asymmetry dynamically in lepton sector, typically via decay of heavy Majorana neutrino
- transfer to baryon sector via SM processes ($\not{B}+\not{L}$ Sphalerons)

Dark matter

- ... see earlier talks
- here: gravitino or neutralino dark matter

Adding $U(1)_{B-L}$ to the SM gauge group

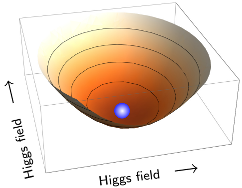
- ... see also Shaaban Khalil's talk
- top-down approach: $U(1)_{B-L}$ as part of GUT group
- bottom-up approach: 'accidental' global symmetry of the SM $\rightarrow$ gauge symmetry
- possible after introduction of right-handed neutrinos for anomaly cancellation
- spontaneously broken at GUT scale

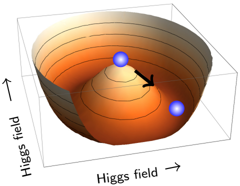


Outline

- Motivation
- Towards a Consistent Cosmological Picture:
  Spontaneous $B\!-\!L$ Breaking
  - qualitative picture:
    linking inflation, leptogenesis and dark matter
  - quantitative description:
    the reheating process in terms of Boltzmann equations
- Phenomenology
- Conclusion

A Phase Transition in the Early Universe





$$W = \frac{\sqrt{\lambda}}{2}\,\Phi\,(v_{B-L}^2 - 2\,S_1 S_2) + \frac{1}{\sqrt{2}} h_i^n n_i^c n_i^c S_1 + h_{ij}^\nu n_i^c \mathbf{5}_j^* H_u + W_{MSSM}$$

Before

- hybrid inflation

[Dvali *et al.* '94]

Phase transition

- tachyonic preheating
- cosmic strings

After

- reheating
- leptogenesis
- dark matter

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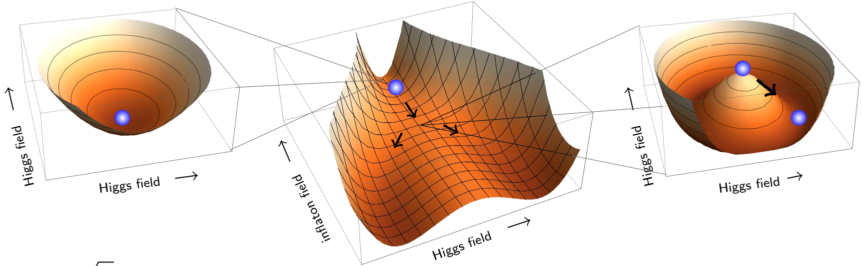

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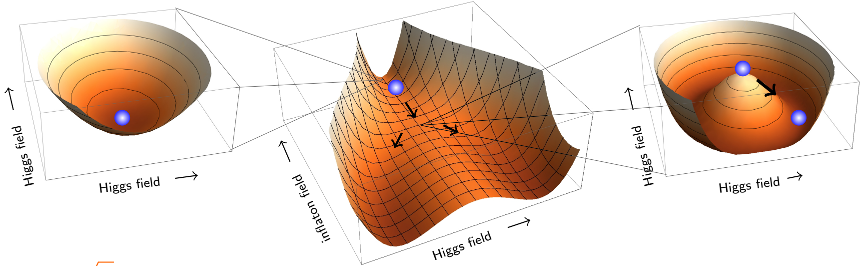

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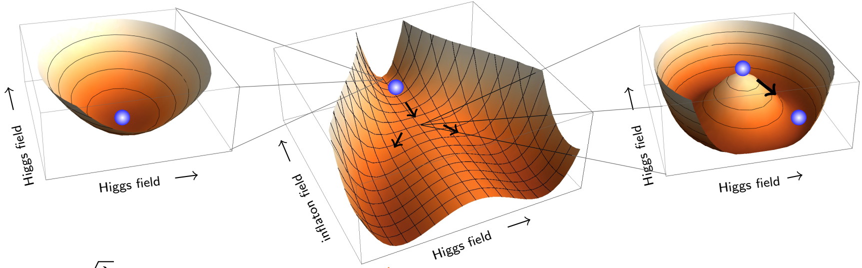

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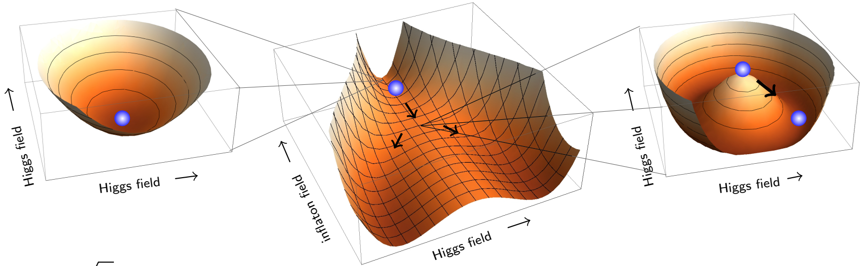

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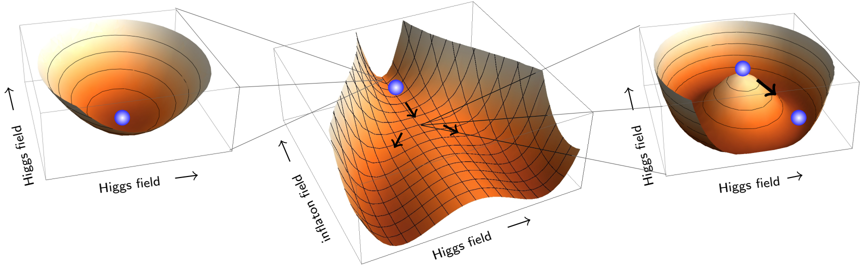

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SSB of $B-L$ links inflation, (p)reheating, leptogenesis and DM

A Useful Tool: Boltzmann Equations

- evolution of the phase space density $f_X(t,p)$:

$$\hat{\mathcal{L}} f_X(t,p) = \left(\frac{\partial}{\partial t} - \frac{\dot{a}}{a}\, p\, \frac{\partial}{\partial p} \right) f_X(t,p) = \sum C_X$$

- collision operator: $C_X(Xab.. \leftrightarrow ij..) =$

$$\frac{1}{2 g_X E_X} \int d\Pi(X|a,b,..;i,j,..)(2\pi)^4 \delta^{(4)}(P_{\text{out}} - P_{\text{in}})$$
$$\times \left[f_i f_j .. |\mathcal{M}(ij.. \to Xab..)|^2 - f_X f_a f_b .. |\mathcal{M}(Xab.. \to ij..)|^2 \right]$$

- Friedmann equation: $3 M_P^2 \left(\frac{\dot{a}}{a}\right)^2 = \rho_{\text{tot}}$

Calculating the time evolution of phase space densities

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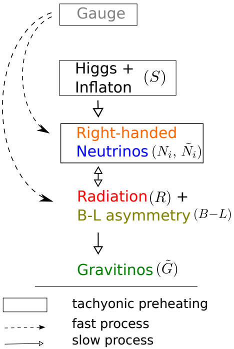
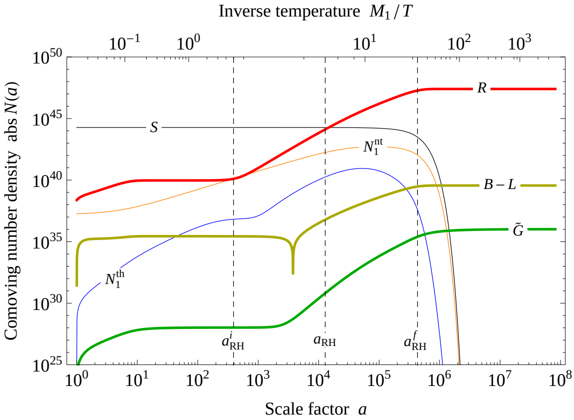
Calculating the time evolution of phase space densities

Solving the Boltzmann Equations

Schematic view





Tracking the evolution of the number densities of all species

Outline

- Motivation
- Towards a Consistent Cosmological Picture:
  Spontaneous $B{-}L$ Breaking
- Phenomenology
  - investigating the parameter space
  - gravitational waves
- Conclusion

Model Parameters

$$W = \frac{\sqrt{\lambda}}{2}\,\Phi\,(v_{B-L}^2 - 2\,S_1 S_2) + \frac{1}{\sqrt{2}} h_i^n n_i^c n_i^c S_1 + h_{ij}^\nu n_i^c \mathbf{5}_j^* H_u + W_{MSSM}$$

- parameters of the superpotential: λ, h_{ij}, v_{B-L}
- re-parametrize assuming hierarchical right-handed neutrinos

$$\Rightarrow \quad v_{B-L}\,, \quad M_1\,, \quad \widetilde{m}_1$$

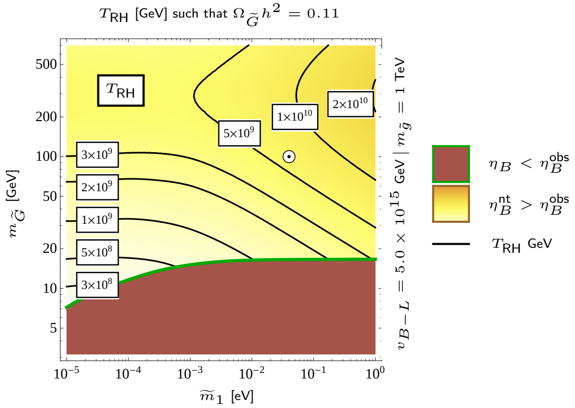
with $v_{B-L} = 5 \times 10^{15}$ GeV fixed by hybrid inflation + cosmic strings

- plus sparticle masses $m_{\tilde{g}} \equiv 1$ TeV, $m_{\tilde{G}}$

Model parameters: $v_{B-L}, M_1, \tilde{m}_1, m_{\tilde{G}}$

Baryon Asymmetry and $\widetilde{G}$ Dark Matter

- requiring $\Omega_{\rm DM} = 0.11$ eliminates M_1
- insert maximal CP-asymmetry $\Rightarrow \eta_B \overset{!}{>} \eta_B^{\rm obs}$



Consistency of inflation, $\widetilde{G}$-DM and leptogenesis for $m_{\widetilde{G}} \gtrsim 10$ GeV

Sources for Gravitational Waves

> tensor perturbations of homogeneous background metric
> sourced by strong gravitational fields

Inflation [Rubakov *et al.* '82, Turner *et al.* '93]

- quantum tensor perturbations become classical during inflation
- governed by evolution of scale-factor after horizon re-entry

Preheating [Garcia-Bellido *et al.* '07, Dufaux *et al.* '09, Dufaux *et al.* '10]

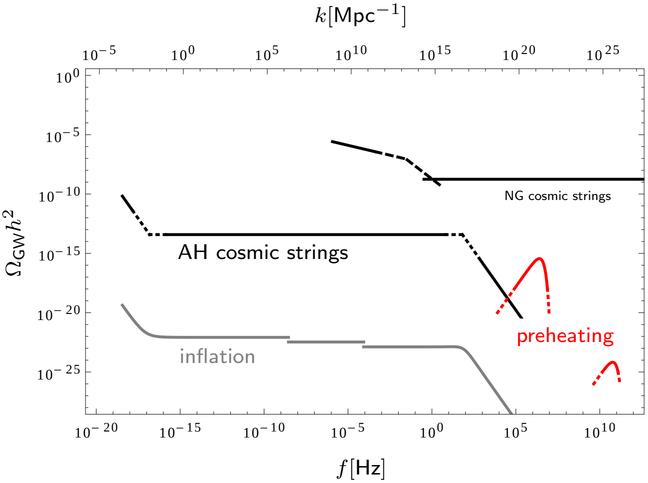
- from bubble-collisions during phase transition
- at high frequencies, governed by small length-scales at preheating

Cosmic strings [Vilenkin '81, Hindmarsh *et al.* '12]

- high energy density along cosmic strings
- huge range of scales $\rightarrow$ large theoretical uncertainty (AH vs NG)

Consistent picture at hand $\rightarrow$ calculate complete spectrum

Observational Prospects



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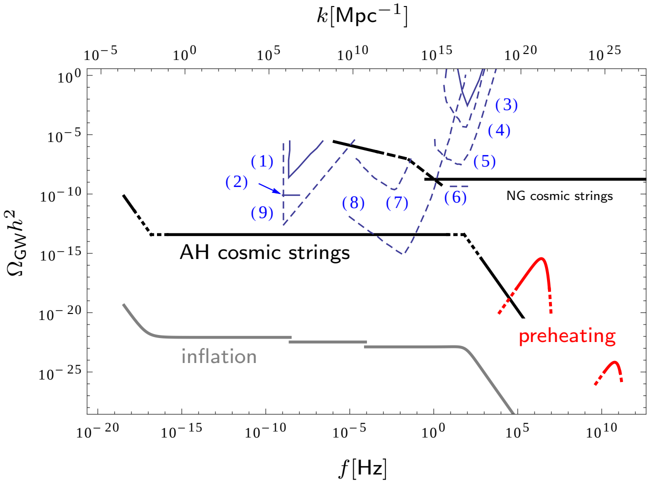
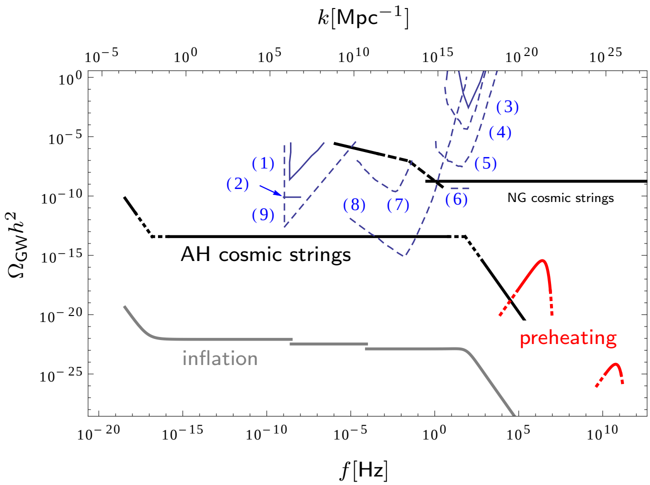


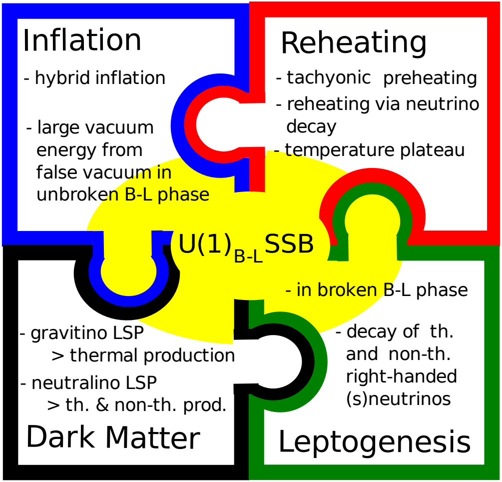
(1) msPT, (2) EPTA, (3) LIGO, (4) KAGRA, (5) ET, (6) adv. LIGO, (7) eLISA, (8) BBO/DECIGO, (9) SKA

Observational Prospects



$U(1)_{B-L}$ phase transition within reach of upcoming experiments

Conclusion



→ relations between superparticle and neutrino mass spectrum

→ predictions for GW spectrum

backup slides

Outlook: Superconformal D-term inflation

Motivation: F-term hybrid inflation in supergravity requires fine-tuning $\rightarrow$ search for viable alternatives.

Symmetry-inspired approach: mildly broken superconformal symmetry [Ferrara *et al.* '11] + D-term hybrid inflation

- Jordan frame: theory closely resembles global supersymmetry

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = \underbrace{\frac{1}{2} M_P^2 \left(R(g_J) + 6\mathcal{A}_\mu^2 \right)}_{\text{pure supergravity}} - \underbrace{\frac{1}{6} R(g_J)|z|^2 - \delta_{\alpha\bar{\beta}}\, g_J^{\mu\nu}\, (\widetilde{\nabla}_\mu z^\alpha)(\widetilde{\nabla}_\nu \bar{z}^{\bar{\beta}}) - V_J}_{\text{superconformal matter}}$$

- Einstein frame: symmetry well hidden

$$W = \lambda \phi S_+ S_- \,, \quad K = -3M_P^2 \ln \left(\frac{-\Phi(z,\bar{z})}{3M_P^2} \right), \quad \Phi(z,\bar{z}) = -3M_P^2 + |z|^2$$

$\Rightarrow n_s \sim 0.96$ possible, but tension with cosmic string bound.

B-L breaking

Unitary gauge:

$$S_{1,2} = \frac{1}{\sqrt{2}} S' \exp(\pm iT) \,, \qquad V = Z + \frac{i}{2gq_S}(T - T^*) \,.$$

- Supermultiplet T drops out of W and K ('eaten up')
- Gauge supermultiplet becomes massive

Time dependent mass spectrum:

$$\begin{aligned}
m_\sigma^2 &= \frac{1}{2}\lambda(3v^2(t) - v_{B-L}^2) \,, \qquad m_\tau^2 = \frac{1}{2}\lambda(v_{B-L}^2 + v^2(t)) \,, \\
m_\phi^2 &= \lambda v^2(t) \,, \qquad m_\psi^2 = \lambda v^2(t) \,, \\
m_Z^2 &= 8g^2 v^2(t) \,, \\
M_i^2 &= (h_i^n)^2 v^2(t) \,.
\end{aligned}$$

Model parameters

$$W = \frac{\sqrt{\lambda}}{2}\,\Phi\,(v_{B-L}^2 - 2\,S_1 S_2) + \frac{1}{\sqrt{2}} h_i^n n_i^c n_i^c S_1 + h_{ij}^\nu n_i^c L_j H_u + W_{MSSM}$$

- Parameters of the superpotential: λ, h_{ij}, v_{B-L}
- Froggatt-Nielsen parametrization: [Froggatt, Nielsen '79]

$$Y_{ijk} \sim \eta^{Q_i+Q_j+Q_k}\,, \quad \eta \simeq 1/\sqrt{300}$$

$+$ SM mass hierarchies, hierarchical right-handed neutrinos
$\Rightarrow$ Froggatt - Nielsen charge assignments.

ψ_i	$\mathbf{10}_3$	$\mathbf{10}_2$	$\mathbf{10}_1$	$\mathbf{5}^*_3$	$\mathbf{5}^*_2$	$\mathbf{5}^*_1$	n_3^c	n_2^c	n_1^c	Φ
Q_i	0	1	2	a	a	$a+1$	$d-1$	$d-1$	d	$2(d-1)$

Model parameters

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$+$ SM mass hierarchies, hierarchical right-handed neutrinos

- Physical parameters:

$$v_{B-L} \sim \eta^{2a}\frac{v_{EW}^2}{\overline{m}_\nu}\,, \quad M_1 \sim \eta^{2d} v_{B-L}\,, \quad \widetilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1} \sim \eta^{2a}\frac{v_{EW}^2}{v_{B-L}}$$

- ... with $v_{B-L} \simeq 5 \times 10^{15}$ GeV from inflation & cosmic strings

Model parameters

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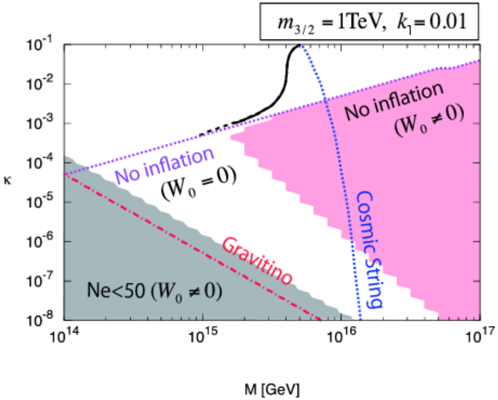
- plus sparticle masses $m_{\tilde{g}} \equiv 1$ TeV, $m_{\tilde{G}}$

Model parameters: $v_{B-L}, M_1, \tilde{m}_1, m_{\tilde{G}}$

Hybrid inflation

Bounds from successful supersymmetric hybrid inflation and cosmic string production:



$\kappa \sim \sqrt{\lambda}$, $M \sim v_{B-L}$

[Nakayama, Takahashi, Yanagida '10]

Tachyonic preheating

Higgs boson

- acquires tachyonic mass term $-m^2 < 0$
- long wavelength modes grow exponentially [Felder *et al.* '01]

$$\langle \delta\sigma^2 \rangle = \frac{1}{4\pi^2} \int_0^m dk\, k\, e^{2t(\sqrt{m^2-k^2})}$$

- can be treated as classical background field

Particles coupled to the Higgs boson

- particle production due to dynamical background field:

$$n_B(\alpha) \simeq 1 \times 10^{-3} g_s m_S^3 f(\alpha, 1.3)/\alpha$$
$$n_F(\alpha) \simeq 3.6 \times 10^{-4} g_s m_S^3 f(\alpha, 0.8)/\alpha$$
$$\text{with } f(\alpha, \gamma) = \sqrt{\alpha^2 + \gamma^2} - \gamma$$

[Garcia-Bellido, Morales '02]

A Simple Example: $B-L$ Higgs Bosons σ

- Boltzmann equation:

$$\hat{\mathcal{L}} f_\sigma = -C_\sigma(\sigma \to N_1 N_1) - C_\sigma(\sigma \to \tilde{N}_1 \tilde{N}_1)$$

$+$ initial conditions: tachyonic preheating

- solution:

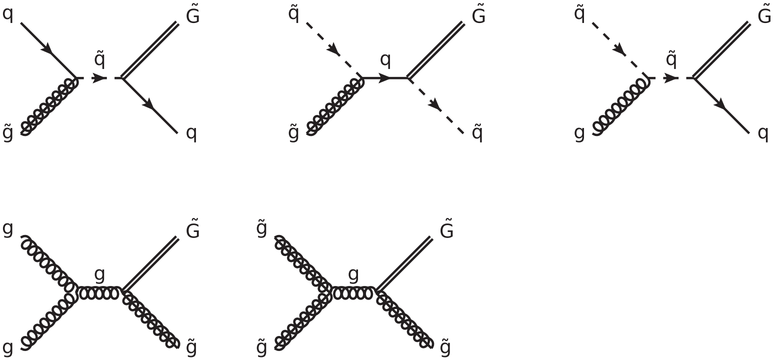
$$f_\sigma(t,p) = \frac{2\pi^2}{g_\sigma} N_\sigma(t_{\mathsf{PH}}) \frac{\delta(ap)}{(ap)^2} \exp[-\Gamma^0_\sigma (t - t_{\mathsf{PH}})]$$

$$\Rightarrow \quad N_\sigma = \left(\frac{a(t)}{\mathsf{GeV}}\right)^3 g_\sigma \int \frac{d^3p}{(2\pi)^3} f_\sigma(t,p) , \quad \rho_\sigma = g_\sigma \int \frac{d^3p}{(2\pi)^3} E_\sigma(p) f_\sigma(t,p)$$

$+$ require self-consistency of Friedmann equation

Boltzmann equation for σ solved analytically

Thermal gravitino production



$$\gamma_{\tilde{G}}(T) = \left(1 + \frac{m_{\tilde{g}}^2(T)}{3m_{\tilde{G}}^2}\right) \frac{54\,\zeta(3)\,g_s^2(T)}{\pi^2 M_P^2} T^6 \left[\ln\left(\frac{T^2}{m_g^2(T)}\right) + 0.8846\right]$$

[Bolz, Brandenburg, Buchmüller '01]

A representative parameter point

Input parameters:

$$M_1 = 5.4 \times 10^{10}\,\mathrm{GeV}\,, \qquad \widetilde{m}_1 = 4.0 \times 10^{-2}\,\mathrm{eV}\,,$$
$$m_{\widetilde{G}} = 100\,\mathrm{GeV}\,, \qquad m_{\tilde{g}} = 1\,\mathrm{TeV}\,,$$
$$\left(\alpha \,=\, 10^{-12}\right)\,.$$

Resulting parameter values:

$$m_S = 1.6 \times 10^{13}\,\mathrm{GeV}\,, \quad M_{2,3} = 1.6 \times 10^{13}\,\mathrm{GeV}\,,$$
$$\Gamma_S^0 = 1.9 \times 10\,\mathrm{GeV}\,, \quad \Gamma_{N_{2,3}}^0 = 2.1 \times 10^{10}\,\mathrm{GeV}\,, \quad \Gamma_{N_1}^0 = 3.0 \times 10^5\,\mathrm{GeV}\,,$$
$$\lambda = 1.0 \times 10^{-5}\,, \quad \epsilon_{2,3} = -1.6 \times 10^{-3}\,, \quad \epsilon_1 = 5.3 \times 10^{-6}\,,$$
$$G\mu = 2 \times 10^{-7}\,.$$

An Example: Nonthermal N_1 Neutrinos

- Boltzmann equation:
  $\hat{\mathcal{L}} f_{N_1} = 2\, C_{N_1}(\phi_S \to N_1 N_1) + C_{N_1}(\psi_S \to \tilde{N}_1^* N_1) + C_{N_1}(N_1 \leftrightarrow \text{MSSM})$
  $+$ initial conditions: tachyonic preheating, $B-L$ gauge boson decay

- solution: comoving number density of *nonthermal* neutrinos

$$
\begin{aligned}
N_{N_1}^{\text{nt}}(t) &= N_{N_1}^{\text{PH}}(t) + N_{N_1}^{G}(t) + N_{N_1}^{S}(t) \\
&= N_{N_1}^{\text{PH}}(t_{\text{PH}})\, e^{-\Gamma_{N_1}^0 (t - t_{\text{PH}})} + N_{N_1}^{G}(t_G)\, \exp\left[-\int_{t_G}^{t} dt'\, \frac{M_1\, \Gamma_{N_1}^0}{\mathcal{E}_{N_1}(m_G/2; t_G, t')}\right] \\
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with

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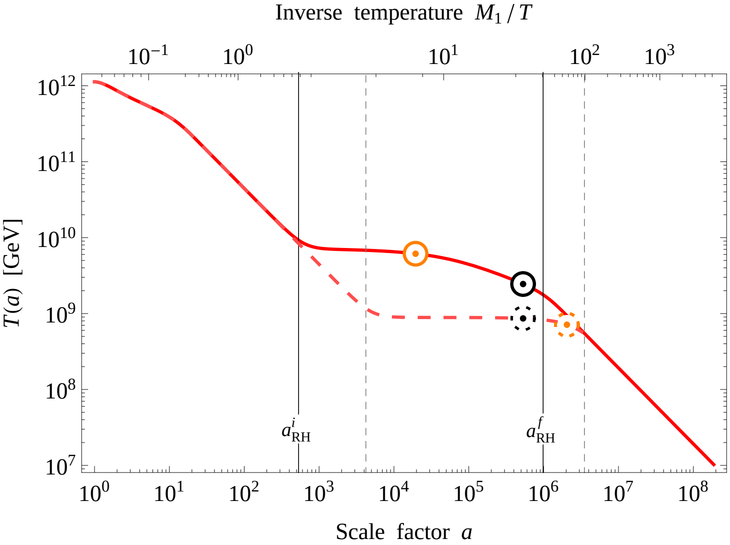
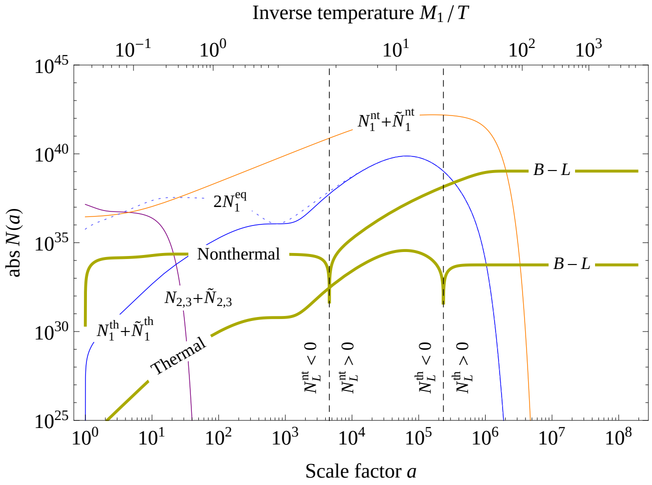
\end{aligned}
$$

Boltzmann equation for N_1 solved semi-analytically

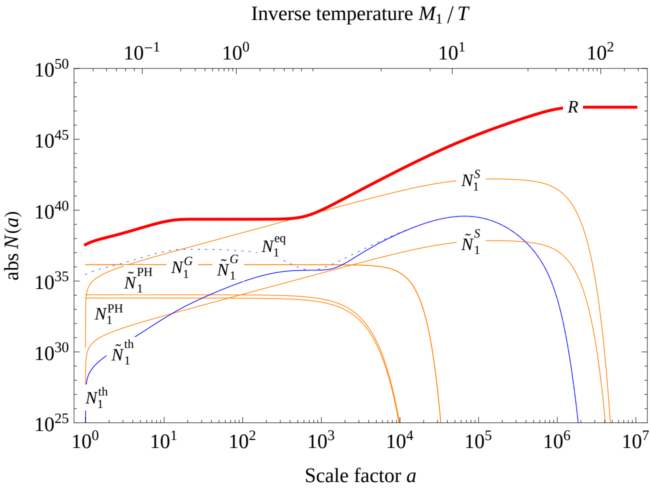
Evolution of the reheating temperature



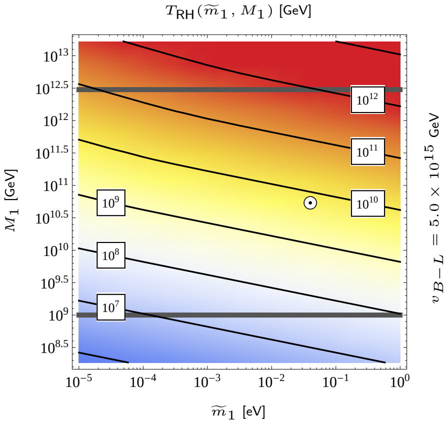
Thermal and nonthermal $B-L$ asymmetry

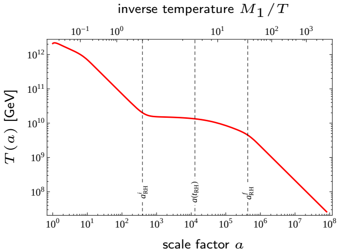


First generation (s)neutrino population



Reheating Temperature

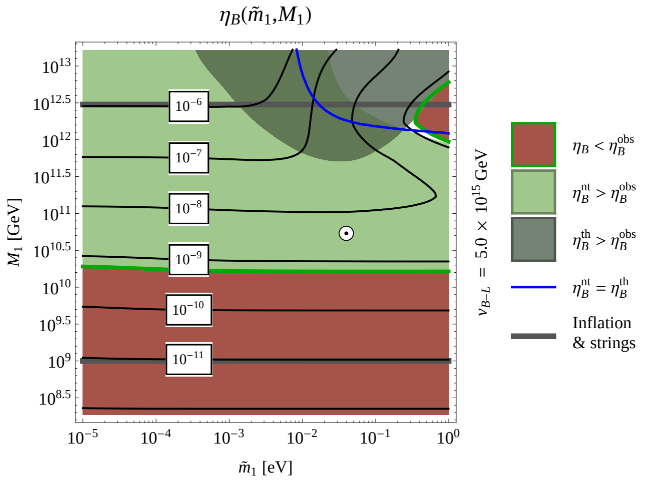




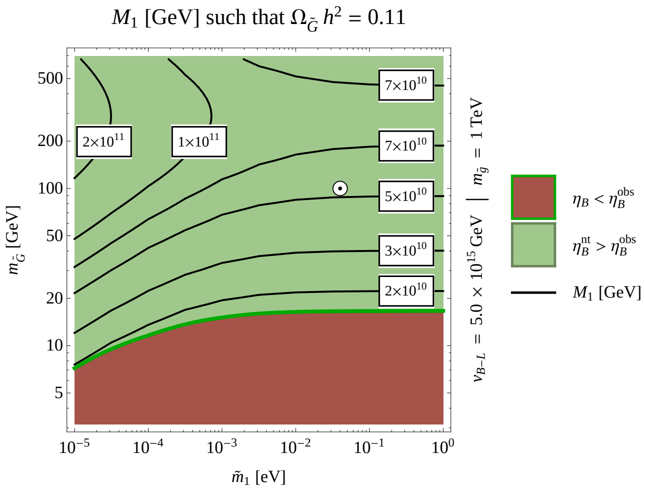
- plateau in temperature evolution
- controlled by neutrino physics parameters

T_{RH} plays a key role for both DM production and leptogenesis

Scan of the parameter space: lepton asymmetry



Scan of the parameter space: neutrino mass



Semi-analytical formulas - temperature

$$T_{\mathrm{RH}} = \alpha^{-1}\beta^{-1/2}\gamma^{-1/2}\left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\sqrt{\Gamma^0_{N_1} M_P}$$
$$= 1.3 \times 10^{10}\ \mathrm{GeV}\left(\frac{\widetilde{m}_1}{0.04\ \mathrm{eV}}\right)^{1/4}\left(\frac{M_1}{10^{11}\ \mathrm{GeV}}\right)^{5/4}$$

- γ: relativistic time-dilatiation
- β: numerical imprecision when solving the Friedmann equation
- $\alpha = \rho_{\mathrm{tot}}(a_{\mathrm{RH}})/\rho_R(a_{\mathrm{RH}})$, function of $\widetilde{m}_1$ and M_1

Semi-analytica formulas - lepton asymmetry

- weak wash-out regime, instantaneous N_1 decay

$$\eta_B \simeq \eta_B^{\mathsf{nt}} \simeq 6.7 \times 10^{-9} \left(\frac{M_1}{10^{11}\ \mathsf{GeV}} \right)^{3/2}$$

- thermal regime, wash-out important

$$\eta_B \simeq \eta_B^{\mathsf{th}} \simeq 7.0 \times 10^{-1} \left(\frac{0.1\ \mathsf{eV}}{\widetilde{m}_1} \right)^{1.1} \left(\frac{M_1}{10^{12}\ \mathsf{GeV}} \right)$$

Semi-analytical formulas - dark matter

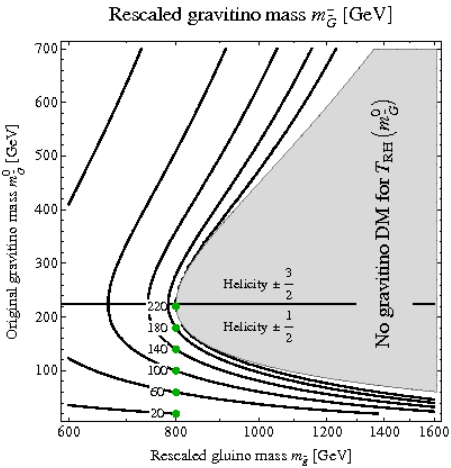
Thermal abundance controlled by T_{RH}

$$\Omega_{\widetilde{G}}h^2 = \varepsilon\, C_1 \left(\frac{T_{\text{RH}}}{10^{10}\ \text{GeV}}\right) \left[C_2 \left(\frac{m_{\widetilde{G}}}{100\ \text{GeV}}\right) + \left(\frac{100\ \text{GeV}}{m_{\widetilde{G}}}\right) \left(\frac{m_{\tilde{g}}}{1\ \text{TeV}}\right)^2\right]$$

- $C_1(T_{\text{RH}}) \simeq 0.26$ analytically
- $C_2(T_{\text{RH}}) \simeq 0.13$ analytically
- ε: entropy and gravitino production after a_{RH}

$$\varepsilon \simeq 1.2 \left(\frac{10^{-3}\ \text{eV}}{\widetilde{m}_1}\right)^c , \quad c = \begin{cases} -0.01 & \text{for } \widetilde{m}_1 \lesssim 10^{-3}\ \text{eV} \\ 0.21 & \text{for } \widetilde{m}_1 \gtrsim 10^{-3}\ \text{eV} \end{cases}$$

Rescaling the gluino mass



[Buchmüller, Schmitz, Vertongen '11]

Gravitational Waves: Overview

Described by linearised Einstein equation (Fourier space, TT gauge)

$$\tilde{h}''_{ij}(\boldsymbol{k},\tau) + \left(k^2 - \frac{a''}{a}\right) \underbrace{\tilde{h}_{ij}(\boldsymbol{k},\tau)}_{a\,h_{ij}} = 16\pi\, G\, a\, \underbrace{\Pi_{ij}(\boldsymbol{k},\tau)}_{\text{FT of TT part of anisotropic stress tensor}}$$

Solved by Greens function $\mathcal{G} = \sin(k(\tau - \tau'))/k$ for $k\tau \gg 1$

$$h_{ij}(\boldsymbol{k},\tau) = 16\pi\, G \frac{1}{a(\tau)} \int_{\tau_i}^{\tau} d\tau' a(\tau') \mathcal{G}(k,\tau,\tau') \Pi_{ij}(\boldsymbol{k},\tau)$$

Observed quantity:

$$\Omega_{\mathrm{GW}} = \frac{1}{\rho_c} \frac{\partial \rho_{\mathrm{GW}}}{\partial \ln k}\,, \quad \text{with } \rho_{\mathrm{GW}} = \frac{1}{32\pi G} \langle \dot{h}_{ij}(\boldsymbol{x},\tau)\, \dot{h}^{ij}(\boldsymbol{x},\tau) \rangle$$

Goal: calculate spectrum $\Omega_{\mathrm{GW}}(f)$

For Example: AH Cosmic Strings

- translational invariance and isotropy of the source
- scaling (self-similar) regime [Durrer *et al.* '99]

$$\Rightarrow \langle \Pi_{ij}(\boldsymbol{k},\tau)\,\Pi^{ij}(\boldsymbol{k}',\tau')\rangle \approx (2\pi)^3 \frac{4v_{B-L}^2}{\sqrt{\tau\tau'}}\delta(\boldsymbol{k}+\boldsymbol{k}')\delta(x-x') \underbrace{\widetilde{C}(x)}_{\text{falls of rapidly for } x \gg 1}$$

with $x = k\tau > 1$ on sub-horizon scales.

- resulting spectrum:

$$\Rightarrow \Omega_{\text{GW}} = \frac{k^2}{3\pi^2 H_0^2 a_0^2}\left(\frac{v_{B-L}}{M_P}\right)^4 \underbrace{\int_{x_i}^{x_0} \frac{\overbrace{a^2(x/k)}^{\text{extract } k\text{-dep.}}}{a_0^2\, x}\,\widetilde{C}(x)\,dx}_{\rightarrow\text{const., dominated by lower boundary } x_i \,=\, \mathcal{O}(1)}$$

$x_i \in$ radiation dom.: $a \propto \tau \propto k^{-1} \rightarrow \Omega_{\text{GW}} \propto k^0$ [Hindmarsh *et al.* '12]

$x_i \in$ matter dom.: $a \propto \tau^2 \propto k^{-2} \rightarrow \Omega_{\text{GW}} \propto k^{-2}$

Plateau during radiation, k^{-2} during matter dom. and reheating

GWs from inflation

Shape of the spectrum governed by transfer function:

$$\Omega_{\rm GW}(k,\tau) = \frac{A_t}{12}\,\frac{k^2}{a_0^2 H_0^2}\,T_k^2(\tau)\,, \quad T_k(\tau) = \frac{a(\tau_k)}{a(\tau)} \text{ with } k = a(\tau_k)H(\tau_k)$$

Consider frequency intervalls $[k_0, k_{\rm eq}),\ [k_{\rm eq}, k_{\rm RH}),\ [k_{\rm RH}, k_{\rm PH})$:

$$f_{\rm eq} = 1.57 \times 10^{-17}\,{\rm Hz}\left(\frac{\Omega_m h^2}{0.14}\right)\,, \qquad f_{\rm RH} = 4.25 \times 10^{-1}\,{\rm Hz}\left(\frac{T_*}{10^7\,{\rm GeV}}\right)$$

$$f_{\rm PH} = 1.93 \times 10^4\,{\rm Hz}\left(\frac{\lambda}{10^{-4}}\right)^{1/6}\left(\frac{10^{-15}\,v_{B-L}}{5\,{\rm GeV}}\right)^{2/3}\left(\frac{T_*}{10^7\,{\rm GeV}}\right)^{1/3}$$

$$\Rightarrow T_k \simeq {\Omega_r}^{1/2}\left(\frac{g_*^k}{g_*^0}\right)^{1/2}\left(\frac{g_{*,s}^0}{g_{*,s}^k}\right)^{2/3}\frac{k_0}{k} \times \begin{cases} \frac{1}{\sqrt{2}}\,k_{\rm eq}/k\,, & k_0 \ll k \ll k_{\rm eq} \\ 1\,, & k_{\rm eq} \ll k \ll k_{\rm RH} \\ \sqrt{2}\,R^{1/2}\,C_{\rm RH}^3\,k_{\rm RH}/k\,, & k_{\rm RH} \ll k \ll k_{\rm PH} \end{cases}$$

GWs from tachyonic preheating

- f_{PH}: typical scale of reheating, redshifted
- $\Omega_{\mathrm{GW}}^{(\mathrm{max})}$: redshift of $\Omega_{\mathrm{GW}}^{\mathrm{PH}}(k_{\mathrm{PH}}) \simeq\ c_{\mathrm{PH}}\,(d_{\mathrm{PH}} H_{\mathrm{PH}})^2$

$$f_{\mathrm{PH}}^{(s)} \simeq 6.3 \times 10^6 \text{ Hz} \left(\frac{M_1}{10^{11} \text{ GeV}}\right)^{1/3} \left(\frac{5\times 10^{15} \text{ GeV}}{v_{B-L}}\right)^2 \left(\frac{m_S}{3\times 10^{13} \text{ GeV}}\right)^{7/6}$$

$$\Omega_{\mathrm{GW}}^{(s,\mathrm{max})} h^2 \simeq 3.6 \times 10^{-16} \frac{c_{\mathrm{PH}}}{0.05} \left(\frac{M_1}{10^{11} \text{ GeV}}\right)^{\frac{4}{3}} \left(\frac{5\times 10^{15} \text{ GeV}}{v_{B-L}}\right)^{-2} \left(\frac{m_S}{3\times 10^{13} \text{ GeV}}\right)^{-\frac{4}{3}}$$

$$f_{\mathrm{PH}}^{(v)} \simeq 7.5 \times 10^{10} \text{ Hz}\ g \left(\frac{M_1}{10^{11} \text{ GeV}}\right)^{1/3} \left(\frac{m_S}{3\times 10^{13} \text{ GeV}}\right)^{-1/2}$$

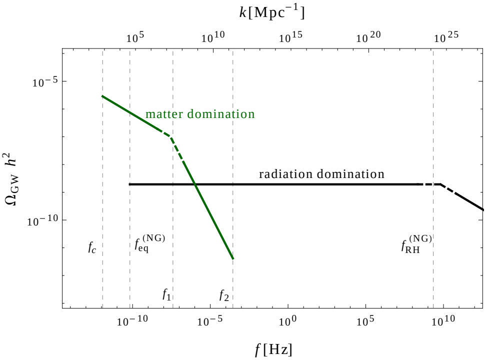
$$\Omega_{\mathrm{GW}}^{(v,\mathrm{max})} h^2 \simeq 2.6 \times 10^{-24} \frac{1}{g^2} \frac{c_{\mathrm{PH}}}{0.05} \left(\frac{M_1}{10^{11} \text{ GeV}}\right)^{\frac{4}{3}} \left(\frac{5\times 10^{15} \text{ GeV}}{v_{B-L}}\right)^2 \left(\frac{m_S}{3\times 10^{13} \text{ GeV}}\right)^2$$

GWs from NG cosmic strings

Integrate over all redshifts and loop sizes ($\rightarrow h$)

$$\Omega_{\mathsf{GW}}(f) = \frac{2\pi^2 f^3}{3H_0^2} \int_0^{h_*} dh \int_0^{z_{\mathsf{PH}}} dz\, h^2 \frac{d^2 R}{dz dh}$$

$\rightarrow$ resulting spectrum:



WIMP dark matter: neutralino production

Setup:

- $m_{\text{LSP}} \ll m_{\text{squark, slepton}} \ll m_{\tilde{G}}$
- LSP = 'pure' wino or higgsino

Thermal production ($m_{\text{LSP}} \gtrsim 1$ TeV): [Arkani-Hamed *et al.* '10]

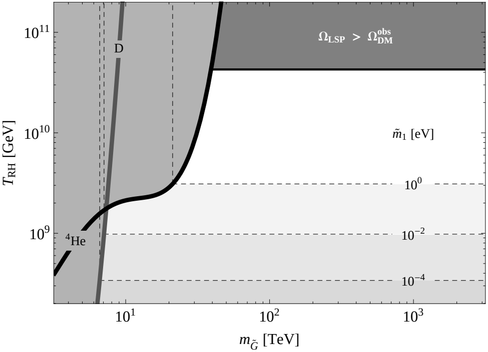
$$\Omega_{\text{LSP}}^{\text{th}} h^2 = c_{\tilde{w},\tilde{h}} \left(\frac{m_{\text{LSP}}}{1 \text{ TeV}} \right)^2 , \quad c_{\tilde{w}} = 0.014,\ c_{\tilde{h}} = 0.10$$

Production from heavy gravitino decay:

$$\Omega_{\text{LSP}}^{\widetilde{G}} h^2 = \frac{m_{\text{LSP}}}{m_{\widetilde{G}}} \Omega_{\widetilde{G}} h^2 \simeq 2.7 \times 10^{-2} \left(\frac{m_{\text{LSP}}}{100 \text{ GeV}} \right) \left(\frac{T_{\text{RH}}(M_1, \widetilde{m}_1)}{10^{10} \text{ GeV}} \right)$$

Thermal and non-thermal neutralino production

Bounds on the reheating temperature

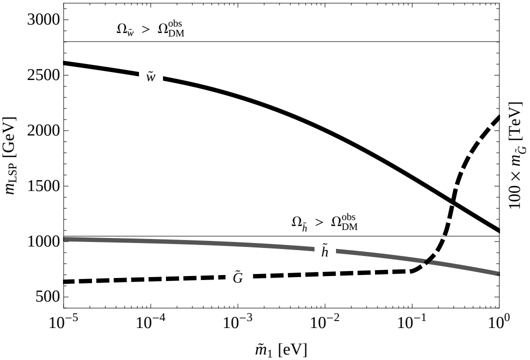


Leptogenesis, BBN [Matsumoto *et al.* '05] & observed DM abundance: bounds on T_{RH} as function of $m_{\tilde{G}}$ and $\tilde{m}_1$

T_{RH} as a link between Leptogenesis, BBN and DM

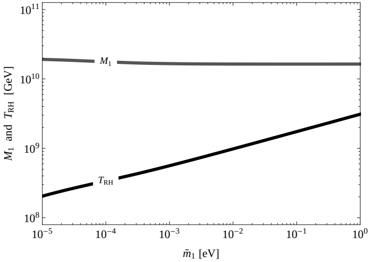
Bounds on neutralino mass

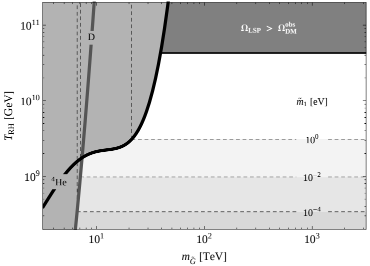
$$\Omega^{\text{th}}_{\text{LSP}} h^2 + \Omega^{\tilde{G}}_{\text{LSP}} h^2 \overset{!}{=} 0.11$$



Consistency between inflation, leptogenesis, BBN and DM for *e.g.*
$m_1 = 0.05$ eV, $m_{\tilde{h}} < 900$ GeV, $m_{\tilde{G}} > 10$ TeV

Bounds on the reheating temperature





Successful leptogenesis: lower bounds on M_1 and T_{RH}, as function of effective neutrino mass $\tilde{m}_1$

Leptogenesis, BBN [Matsumoto *et al.* '05] & observed DM abundance: bounds on T_{RH} as function of $m_{\tilde{G}}$

T_{RH} as a link between Leptogenesis, BBN and DM

$m_{LSP} - m_{\tilde{G}}$ bounds