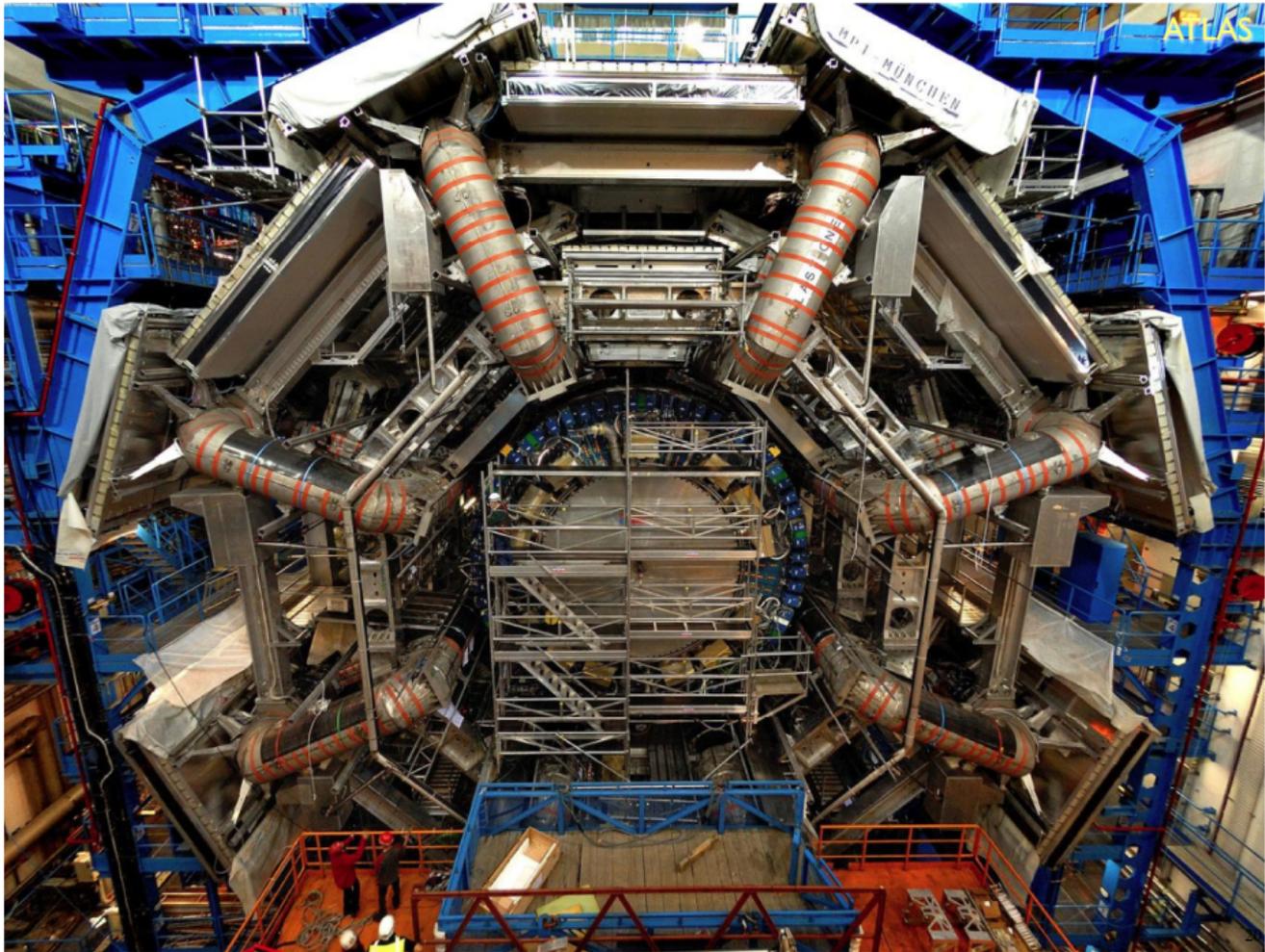
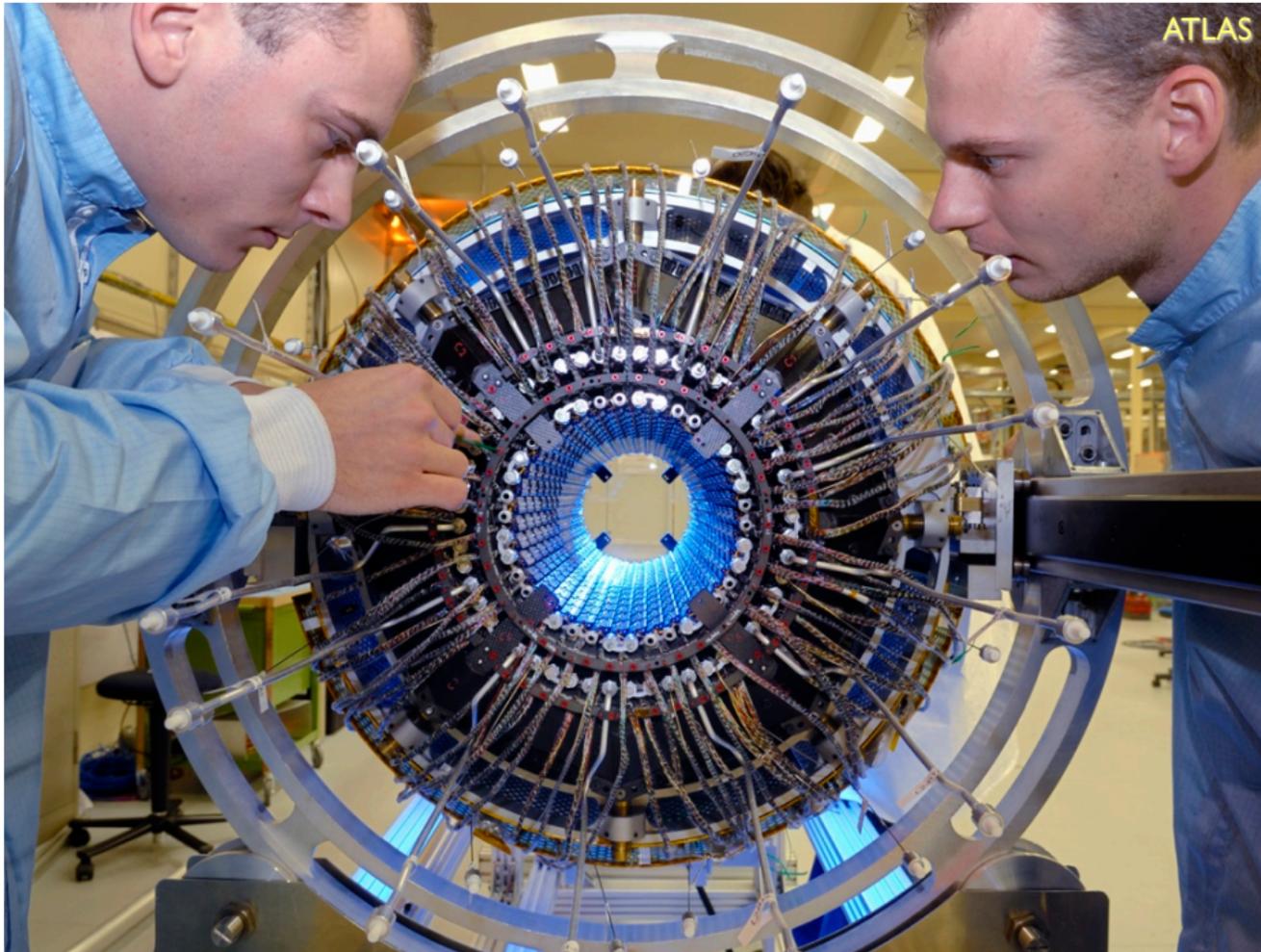


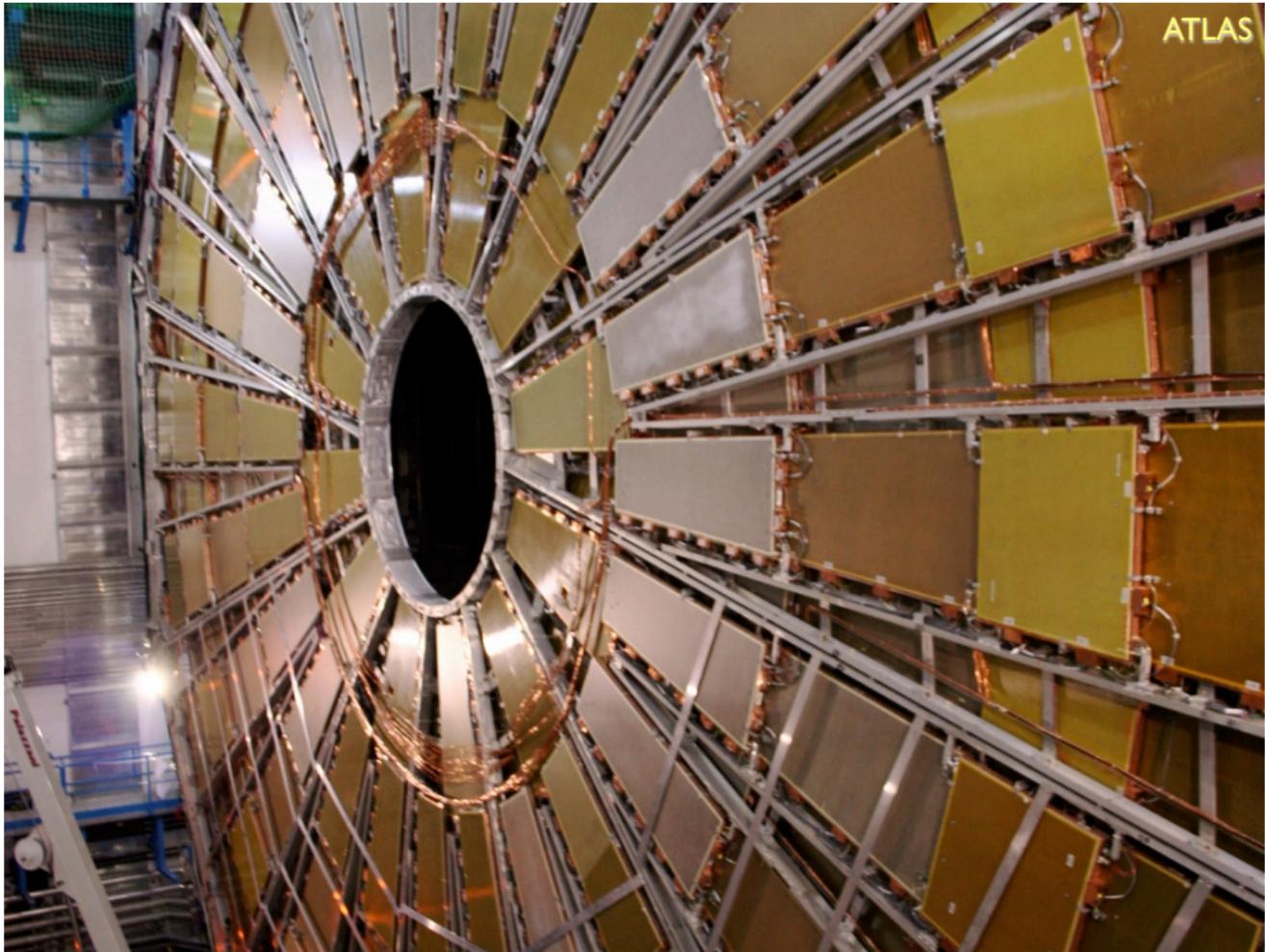
# Physics of the Large Hadron Collider

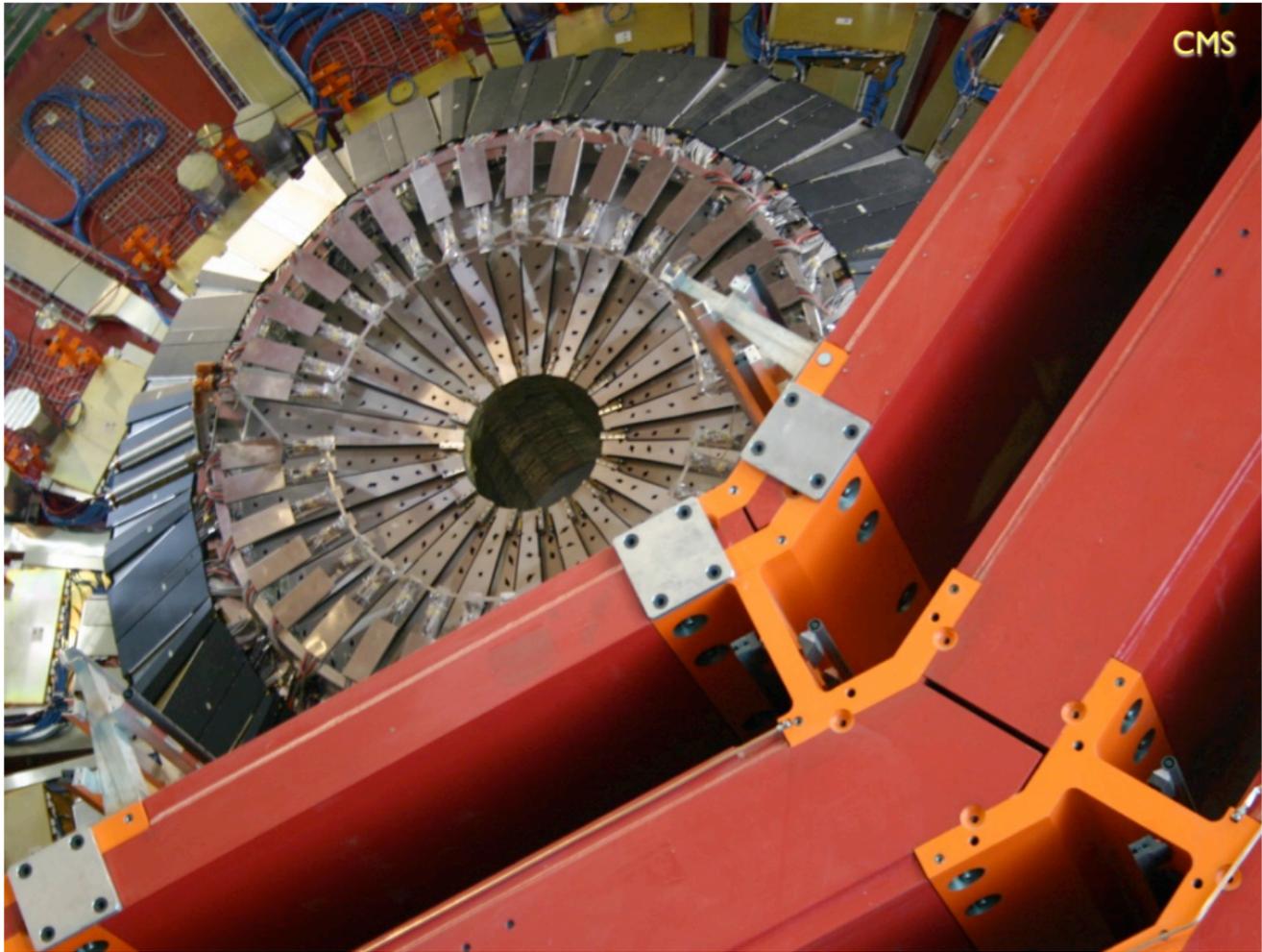
Chris Quigg

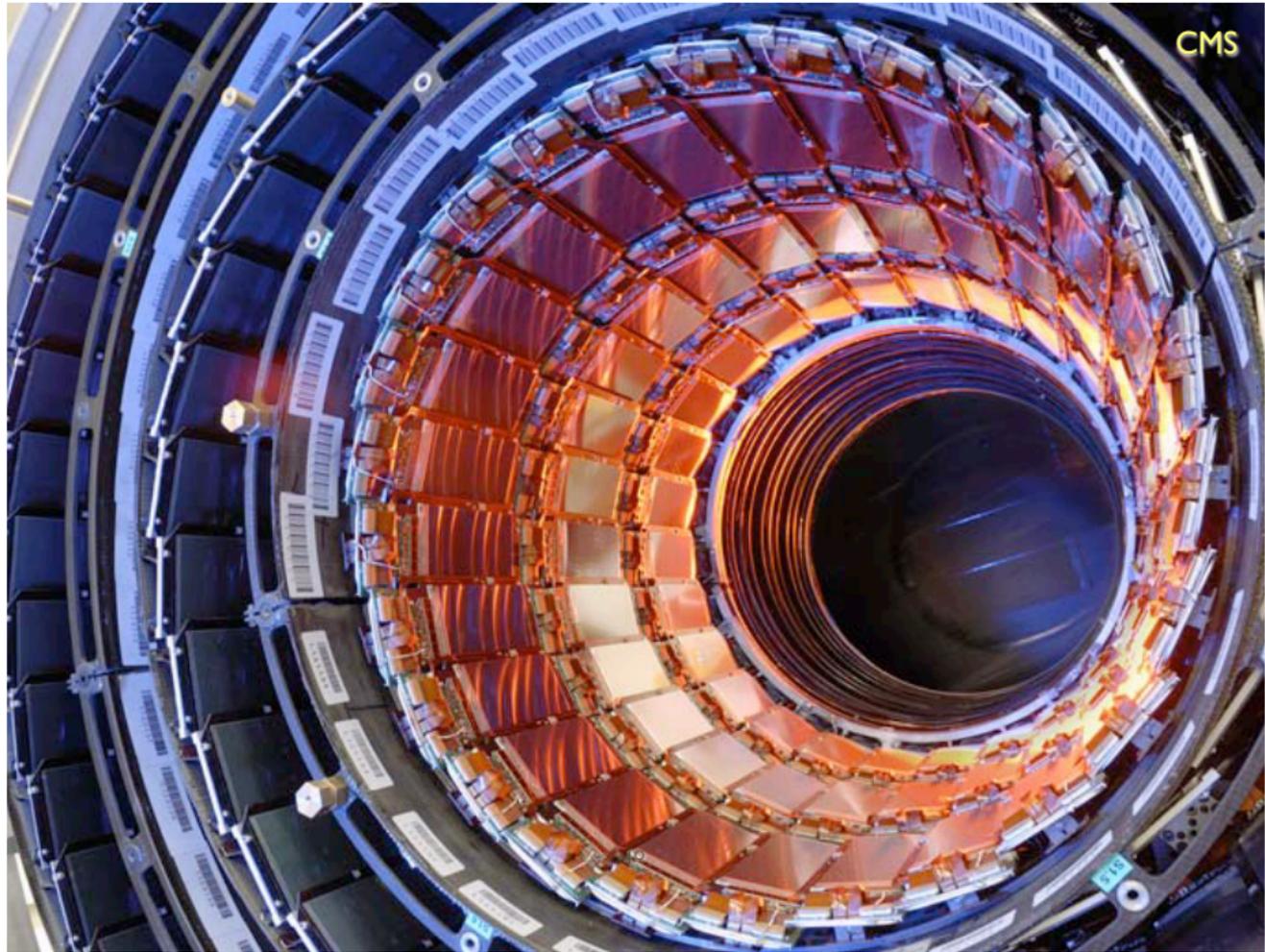
*Fermilab*

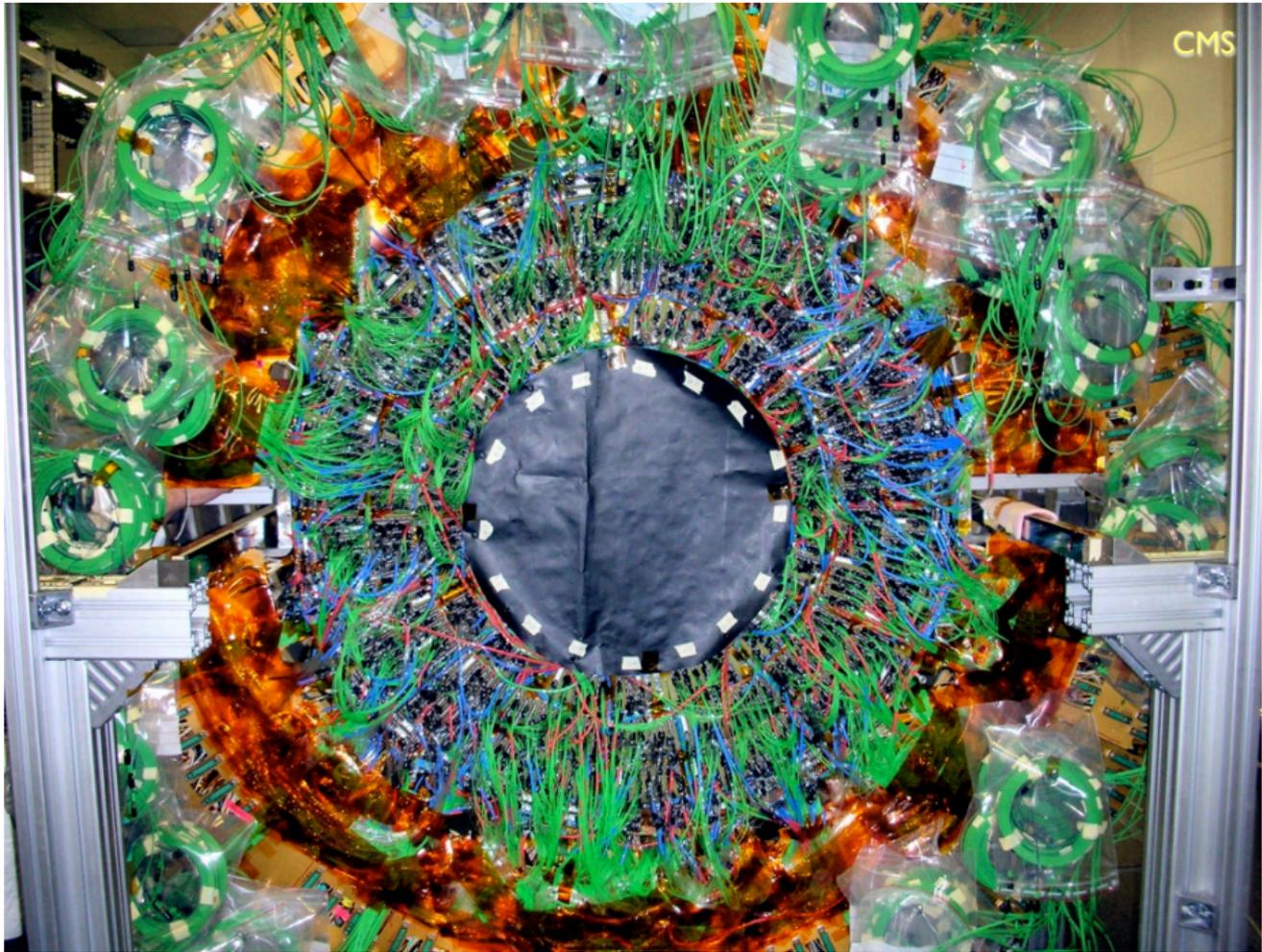












Muon Spectrometer

Hadronic Calorimeter

Electromagnetic Calorimeter

Solenoid magnet

Tracking

Transition Radiation Tracker  
Pixel/SCT detector

Proton

Neutrino

Muon

Neutron

Electron

Photon

The dashed tracks are invisible to the detector

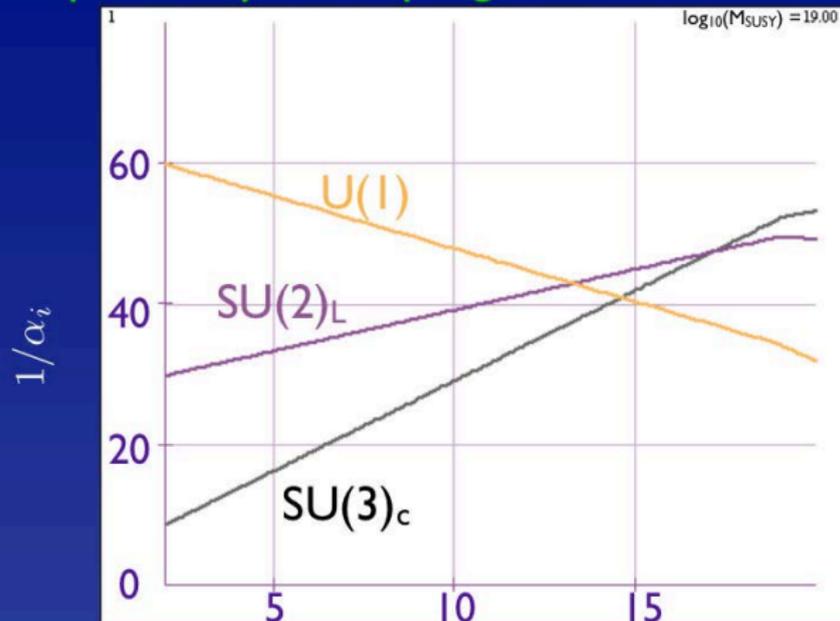
## Exercise 5

Explain the response of the ATLAS detector to different particle species, as shown in the graphic on the [preceding page](#).

An [interactive slice](#) through the CMS detector animates the response to five particle types.

# Coupling-constant Unification

Different running of  $U(1)_Y$ ,  $SU(2)_L$ ,  $SU(3)_c$  gives possibility of coupling constant unification

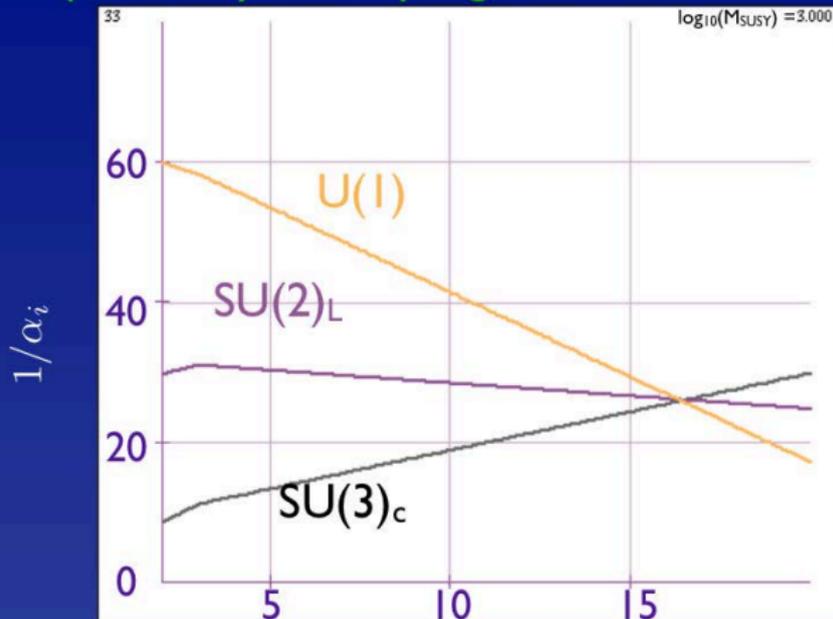


$$\alpha^{-1} = \frac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$$

$$\log_{10}(E[\text{GeV}])$$

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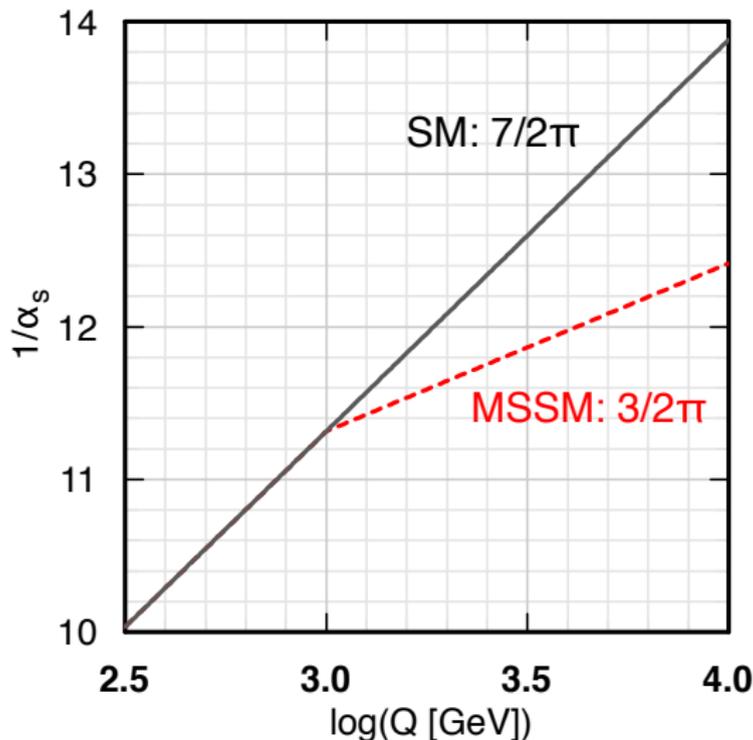


$$\alpha^{-1} = \frac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$$

$$\log_{10}(E[\text{GeV}])$$

# Can LHC See Change in Evolution?

Sensitive to new colored particles



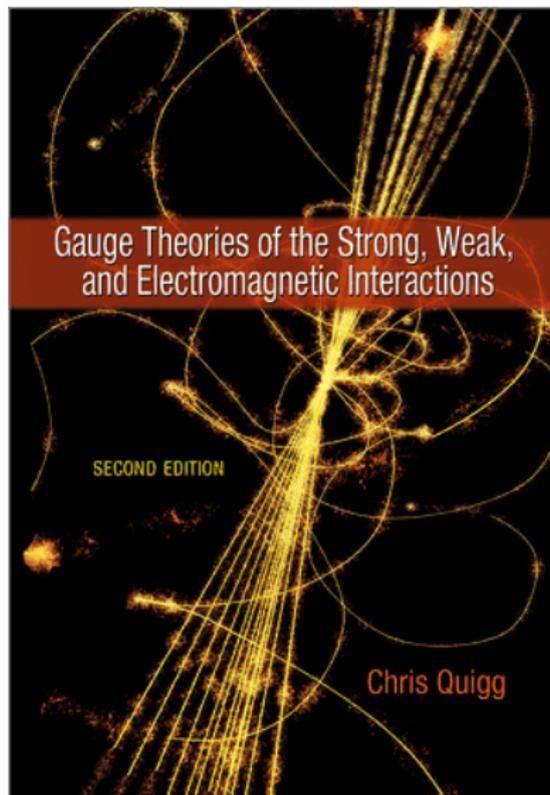
(sharp threshold illustrated)

... also for  $\sin^2 \theta_W$

# Anyone unfamiliar with electroweak theory?

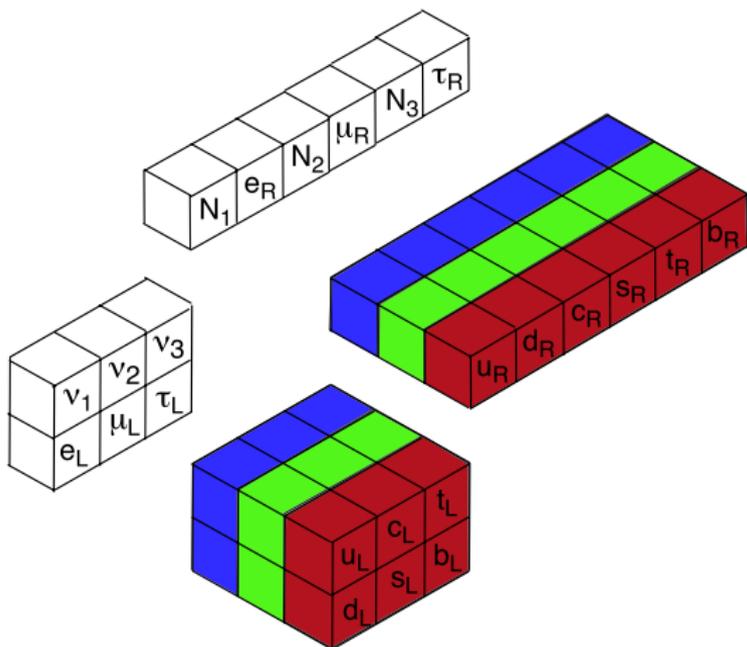
See my [five lectures](#), “The Standard Model—Its Magic and Its Shortcomings,” at the São Paulo school, *Particle Physics in the LHC Era*, April 2013.

Coming soon . . .



# Our Picture of Matter

Pointlike constituents ( $r < 10^{-18}$  m)



$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{em}$$

# Unbroken $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ Theory:

As in QED, massless gauge bosons  
*but weak interaction is short-range*

In contrast to QED, massless fermions  
Mass term  $\mathcal{L}_e = -m_e \bar{e}e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R)$   
*violates local gauge invariance*

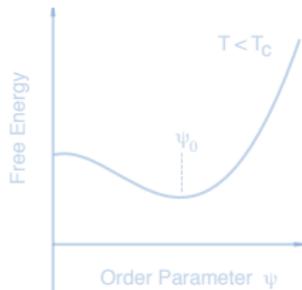
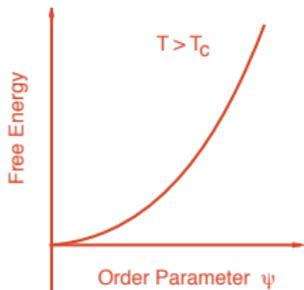
# Massive Gauge Boson? *Hiding Symmetry*

Recall **2** miracles of superconductivity:

- No resistance ... .. Meissner effect (exclusion of **B**)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers ... .. + superconducting charge carriers



$$\mathbf{B} = 0: \quad G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0$$

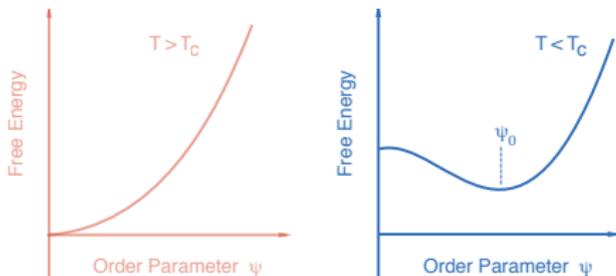
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$$T < T_c: \quad \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0$$

# In a nonzero magnetic field ...

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi \right|^2$$

$$\left. \begin{array}{l} e^* = -2 \\ m^* \end{array} \right\} \text{ of superconducting carriers}$$

Weak, slowly varying field:  $\psi \approx \psi_0 \neq 0$ ,  $\nabla\psi \approx 0$

Variational analysis  $\rightsquigarrow$

$$\nabla^2 \mathbf{A} - \frac{4\pi e^{*2}}{m^* c^2} |\psi_0|^2 \mathbf{A} = 0$$

wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass within superconductor

origin of Meissner effect

In gauge theory: Brout, Englert, Higgs, Guralnik, Hagen, Kibble

## Hide EW Symmetry in Analogy to Ginzburg–Landau

- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass  $M_W^2 = \pi\alpha / G_F \sqrt{2} \sin^2 \theta_W$ ,
- Mediator of (new!) neutral-current weak interaction acquires mass  $M_Z^2 = M_W^2 / \cos^2 \theta_W$ ;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine  $\sin^2 \theta_W$  to predict  $M_W, M_Z$

# A theory of leptons

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad R \equiv e_R$$

weak hypercharges  $Y_L = -1$ ,  $Y_R = -2$

Gell-Mann–Nishijima connection,  $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$  gauge group  $\Rightarrow$  gauge fields:

- weak isovector  $\vec{b}_\mu$ , coupling  $g$

$$b_\mu^\ell = b_\mu^\ell - \varepsilon_{jkl} \alpha^j b_\mu^k - (1/g) \partial_\mu \alpha^\ell$$

- weak isoscalar  $\mathcal{A}_\mu$ , coupling  $g'/2$

$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \partial_\mu \alpha$$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g \varepsilon_{jkl} b_\mu^j b_\nu^k, \text{SU}(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu, \text{U}(1)_Y$$

# Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^{\ell} F^{\ell\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

$$\begin{aligned} \mathcal{L}_{\text{leptons}} &= \bar{R} i\gamma^{\mu} \left( \partial_{\mu} + i\frac{g'}{2} \mathcal{A}_{\mu} Y \right) R \\ &+ \bar{L} i\gamma^{\mu} \left( \partial_{\mu} + i\frac{g'}{2} \mathcal{A}_{\mu} Y + i\frac{g}{2} \vec{\tau} \cdot \vec{b}_{\mu} \right) L. \end{aligned}$$

Mass term  $\mathcal{L}_e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R) = -m_e \bar{e} e$  violates local gauge inv.

Theory: 4 massless gauge bosons ( $\mathcal{A}_{\mu}$   $b_{\mu}^1$   $b_{\mu}^2$   $b_{\mu}^3$ ); Nature: 1 ( $\gamma$ )

# Hiding EW Symmetry

*Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition*

- Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- Add to  $\mathcal{L}$  (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

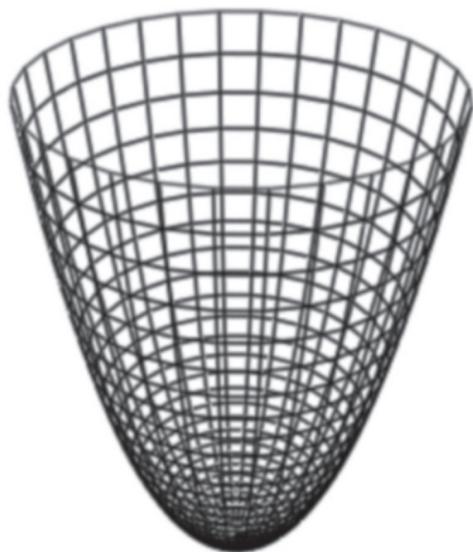
where  $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$  and

$$V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|(\phi^\dagger \phi)^2$$

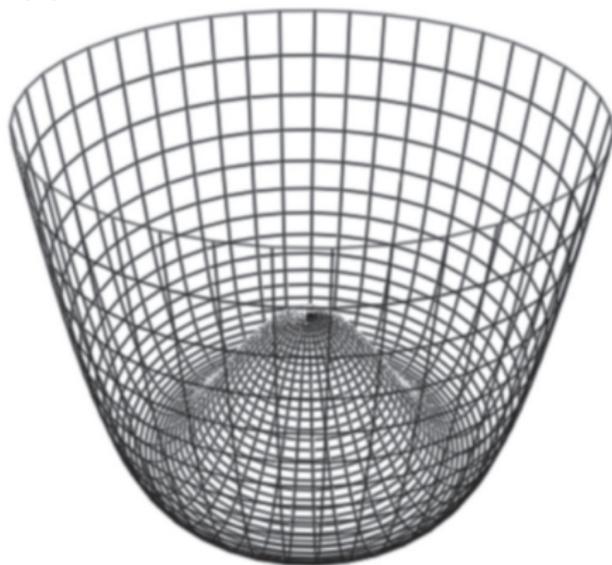
- Add a Yukawa interaction  $\mathcal{L}_{\text{Yukawa}} = -\zeta_e [\bar{R}(\phi^\dagger L) + (\bar{L}\phi)R]$

# Unique and degenerate vacuum states

(a)



(b)



# Origin of Fermion Masses

By decree, Weinberg & Salam add interactions between fermions and scalars that give rise to quark and lepton masses.

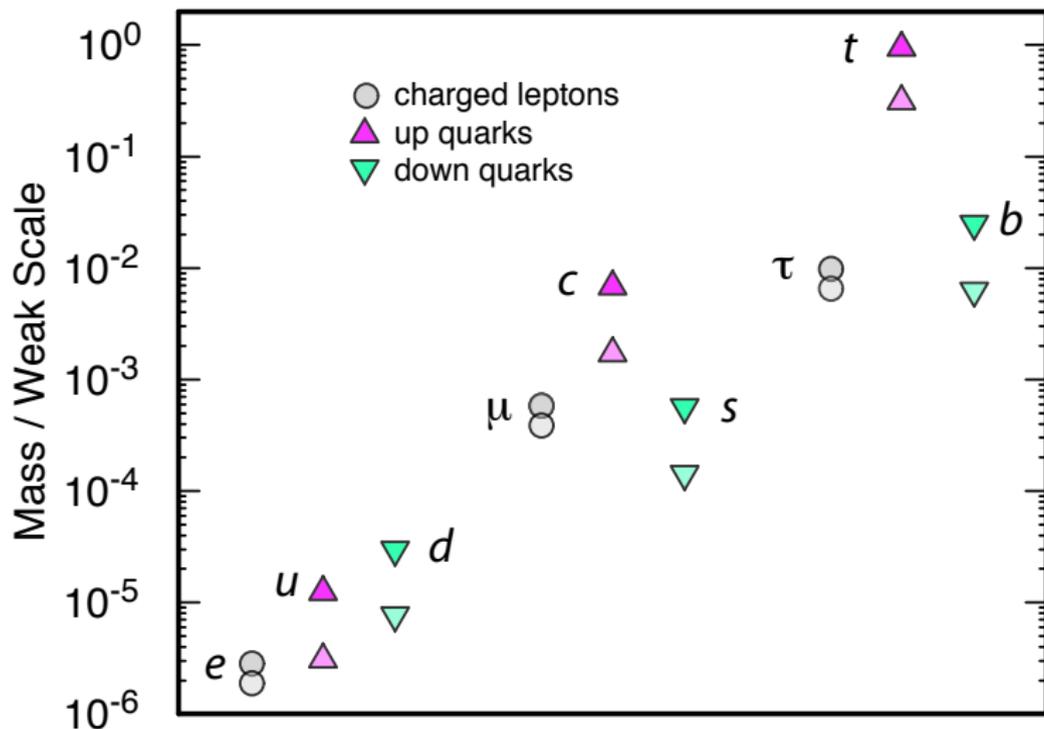
$$\zeta_e [(\bar{e}_L \Phi) e_R + \bar{e}_R (\Phi^\dagger e_L)] \rightsquigarrow m_e = \zeta_e v / \sqrt{2}$$

$\zeta_e$  is picked to give right mass, not predicted

Fermion mass implies physics beyond standard model

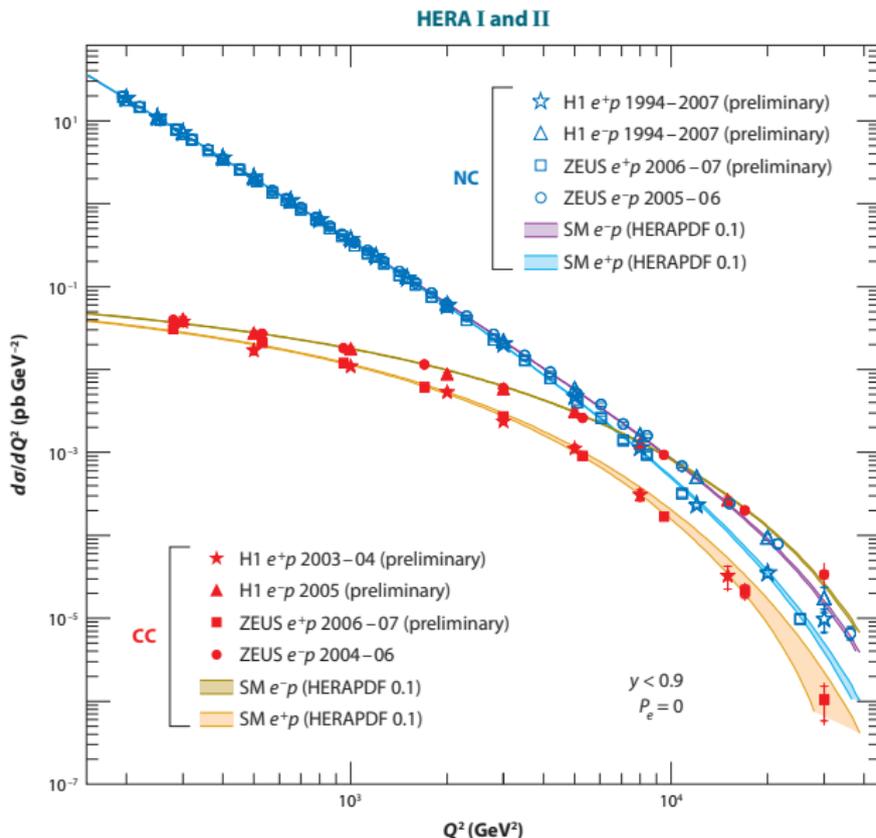
*Highly economical, but is it true?*

# Charged Lepton and Quark Masses

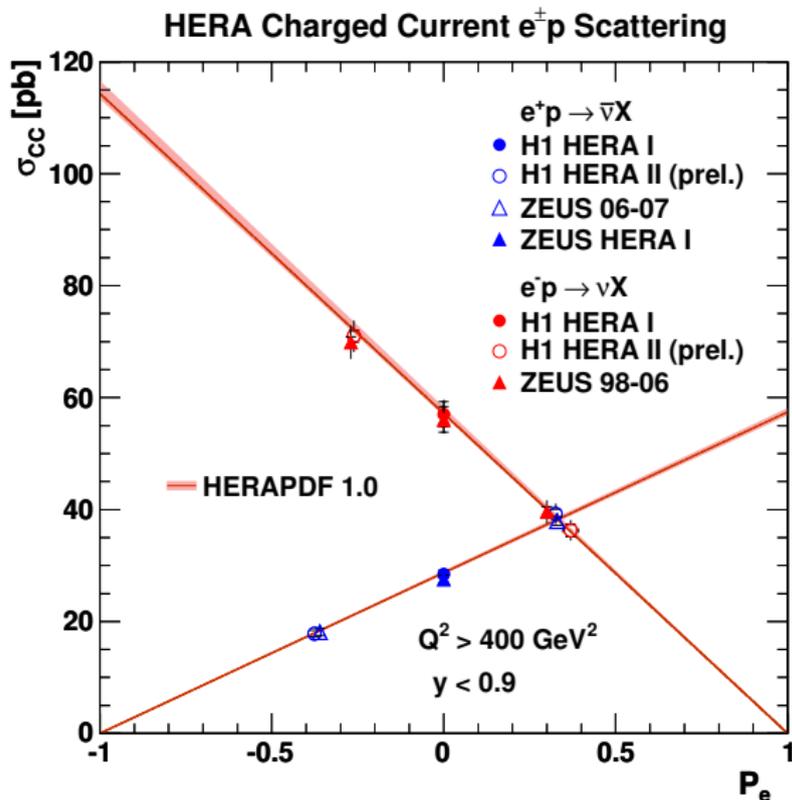


Running mass  $m(m) \dots m(U)$

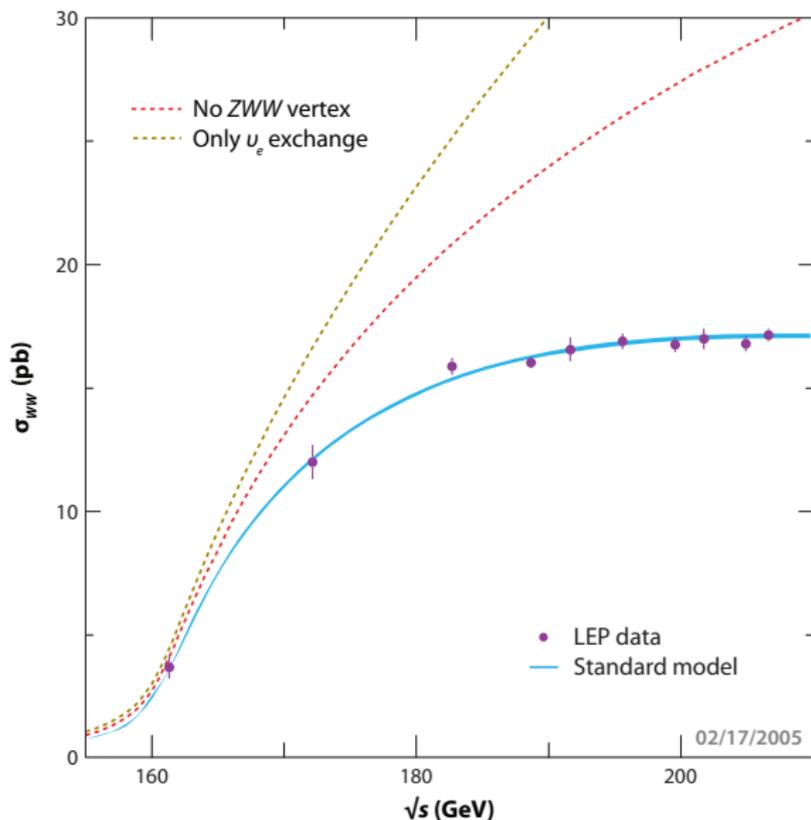
# Electroweak theory tests: tree level



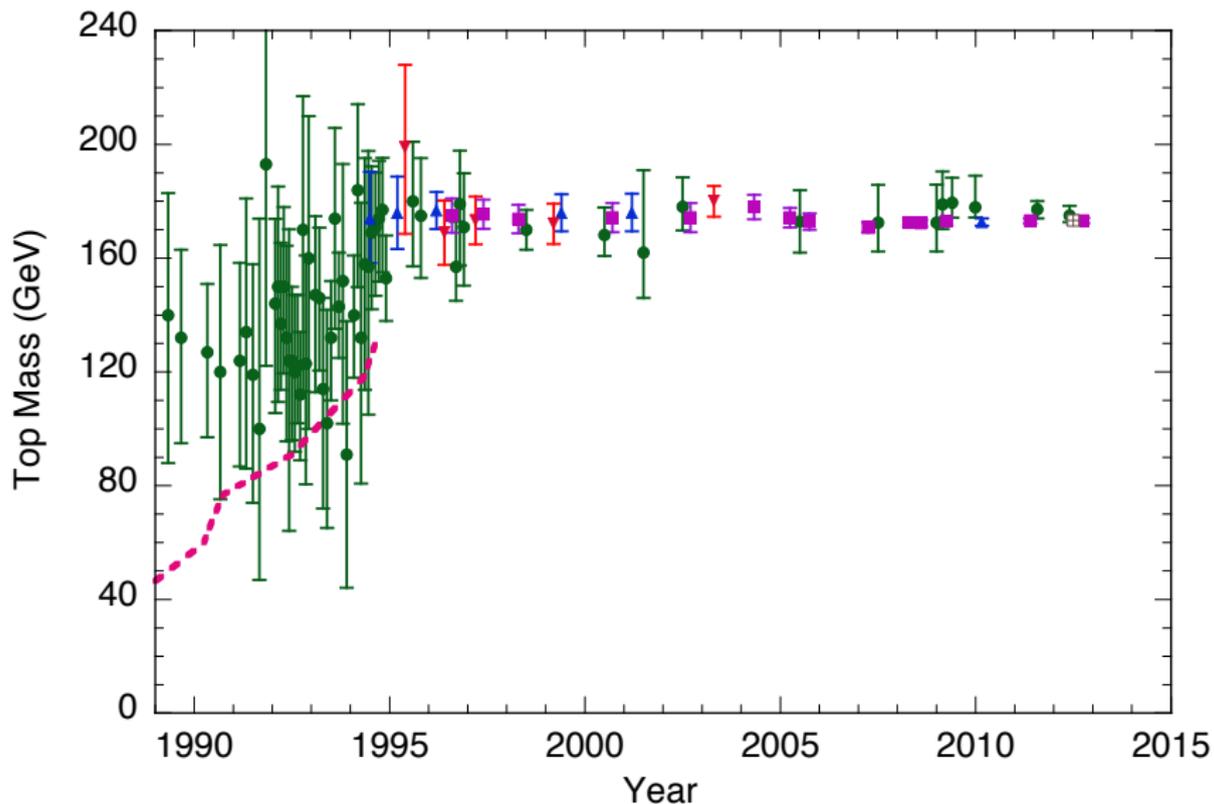
# Electroweak theory tests: tree level (no RHCC)



# Electroweak theory tests: tree level

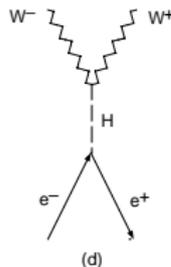
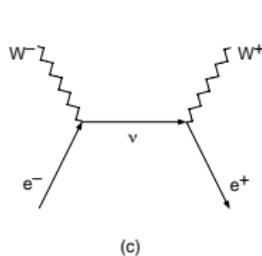
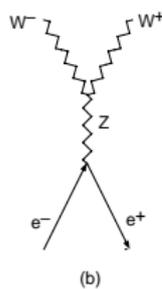
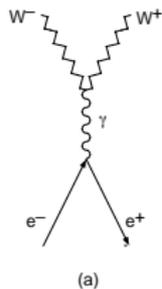


# Electroweak theory tests: loop level



# Why a Higgs boson must exist

$S$ -matrix analysis of  $e^+e^- \rightarrow W^+W^-$



Individual  $J = 1$  partial-wave amplitudes  $\mathcal{M}_\gamma^{(1)}$ ,  $\mathcal{M}_Z^{(1)}$ ,  $\mathcal{M}_\nu^{(1)}$  have unacceptable high-energy behavior ( $\propto s$ )

... but sum is well-behaved

“Gauge cancellation” observed at LEP2 (Tevatron)

$J = 0$  amplitude exists because electrons have mass, and can be found in “wrong” helicity state

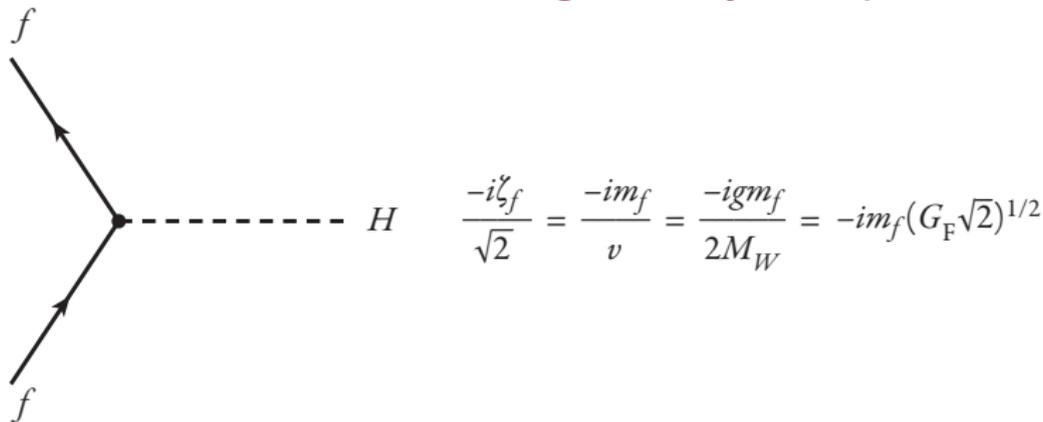
$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior}$$

(no contributions from  $\gamma$  and  $Z$ )

This divergence is canceled by the Higgs-boson contribution

⇒  $He\bar{e}$  coupling must be  $\propto m_e$ ,

because “wrong-helicity” amplitudes  $\propto m_e$



*If the Higgs boson did not exist, something else would have to cure divergent behavior*

If gauge symmetry were unbroken ...

- no Higgs boson; no longitudinal gauge bosons
- no extreme divergences; no wrong-helicity amplitudes

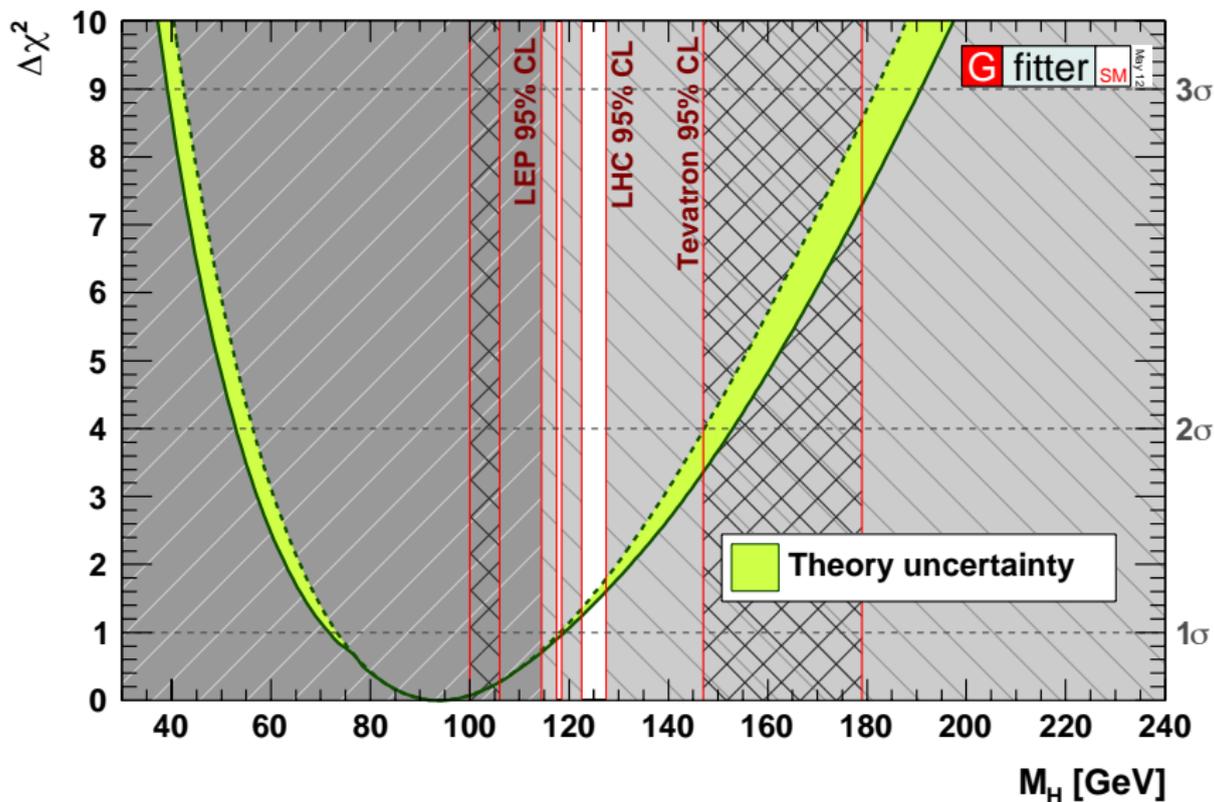
... and no viable low-energy phenomenology

In spontaneously broken theory ...

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass ... due to SSB
- SSB provides its own cure—the Higgs boson

Similar interplay and compensation *must exist* in any acceptable theory

# Electroweak theory tests: Higgs influence



# The importance of the 1-TeV scale ...

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity

Compute amplitudes  $\mathcal{M}$  for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta)$$

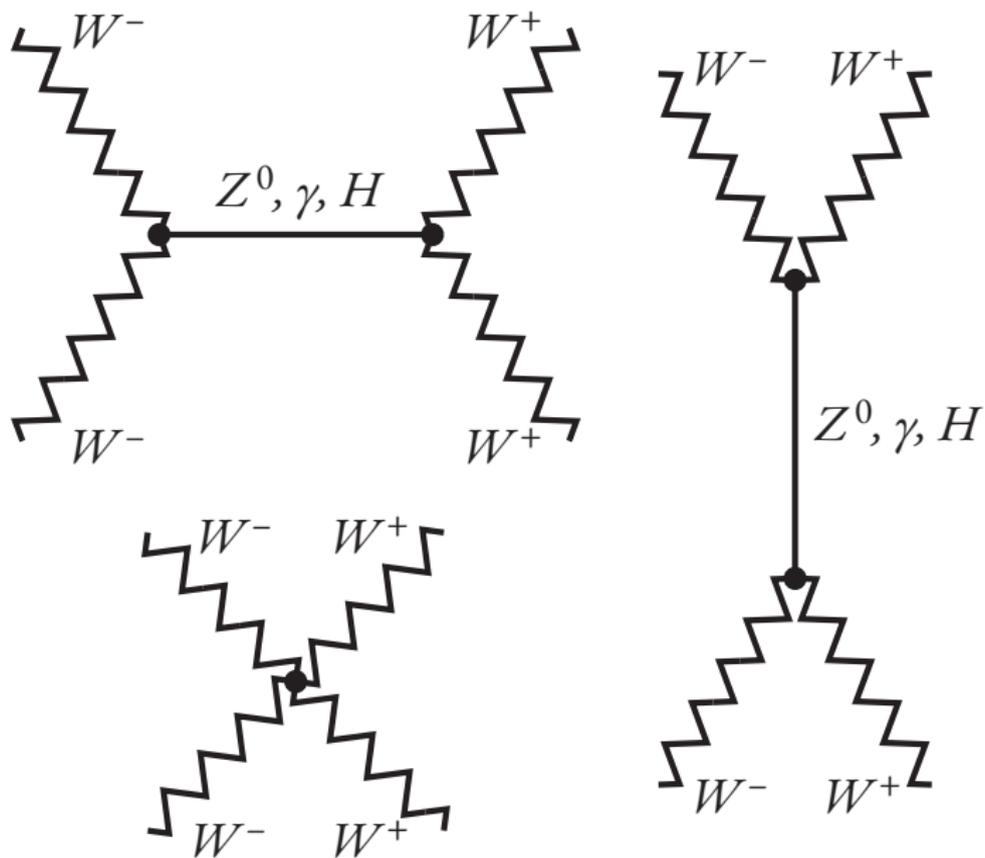
Most channels decouple – pw amplitudes are small at “all” energies –  $\forall M_H$ .

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad HH / \sqrt{2} \quad HZ_L^0$$

$L$ : longitudinal,  $1/\sqrt{2}$  for identical particles

# The importance of the 1-TeV scale ...



# The importance of the 1-TeV scale . .

In HE limit,  $s$ -wave amplitudes  $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect partial-wave unitarity condition  $|a_0| \leq 1$

$$\Rightarrow M_H \leq \left( \frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} \approx 1 \text{ TeV}$$

condition for perturbative unitarity

# The importance of the 1-TeV scale . . .

If the bound is respected

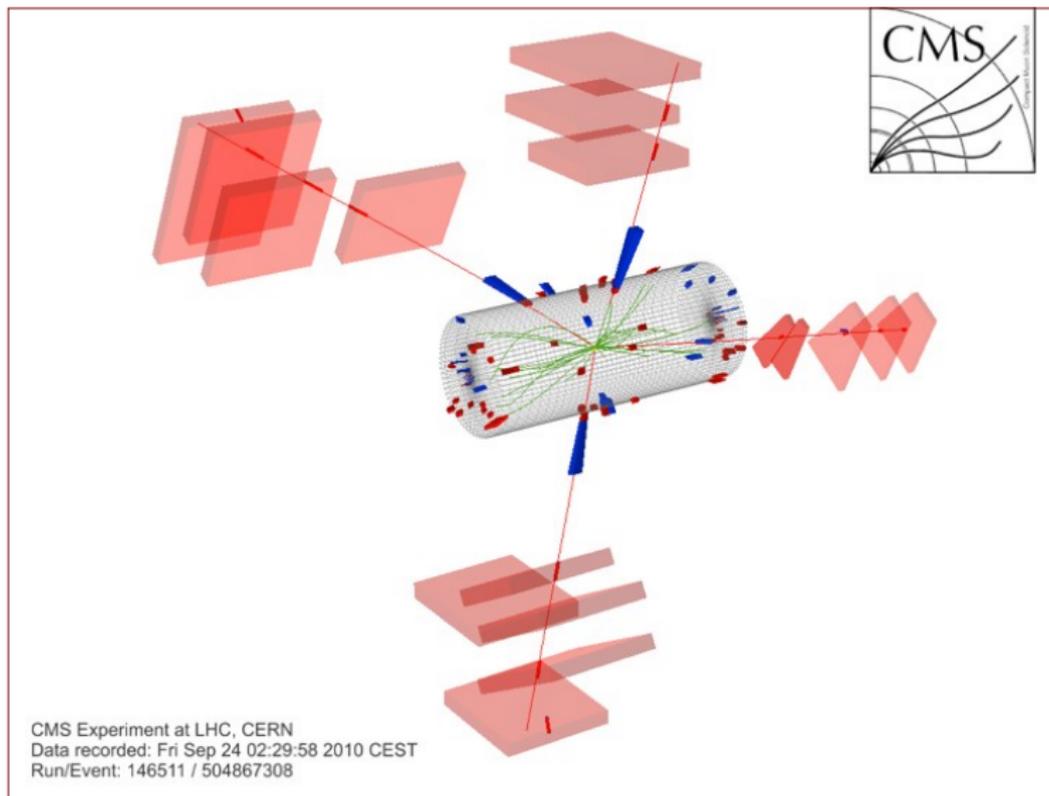
- weak interactions remain weak at all energies
- perturbation theory is everywhere reliable

If the bound is violated

- perturbation theory breaks down
- weak interactions among  $W^\pm$ ,  $Z$ ,  $H$  become strong on 1-TeV scale

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

# Heavy Higgs Signature: $ZZ \rightarrow \mu^+ \mu^- \mu^+ \mu^-$



# What the Higgs Field Accomplishes

Hides the electroweak symmetry

Gives masses to  $W$  and  $Z$   
(provides longitudinal components)

Generates fermion masses and mixings  
through mysterious Yukawa terms

Makes electroweak theory behave at high energies

# Search for the Standard-Model Higgs Boson

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$  in the limit of large Higgs mass;  $\propto \beta^3$  for scalar

$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2) \quad x \equiv 4M_W^2/M_H^2$$

$$\Gamma(H \rightarrow Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically  $\propto M_H^3$  and  $\frac{1}{2}M_H^3$ , respectively

$2x^2$  and  $2x'^2$  terms  $\Leftrightarrow$  decays into transverse gauge bosons

Dominant decays for large  $M_H$ : pairs of longitudinal weak bosons

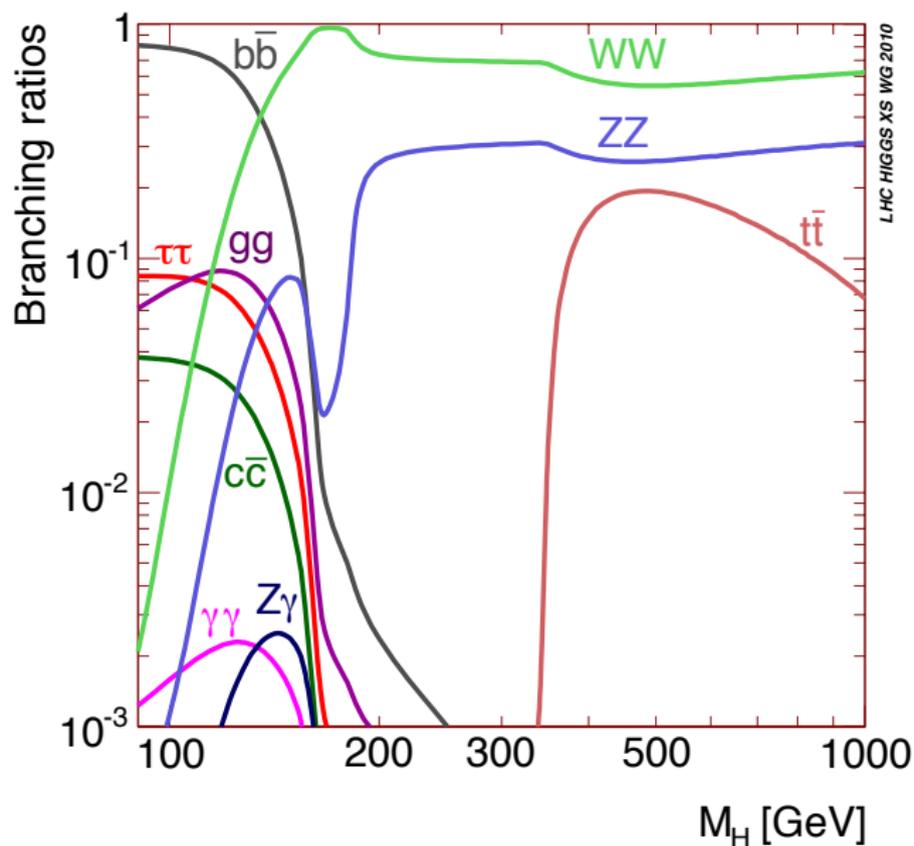
## Exercise 6

Compute the decay rate for the Higgs boson into fermion pairs given on [the previous page](#),

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2},$$

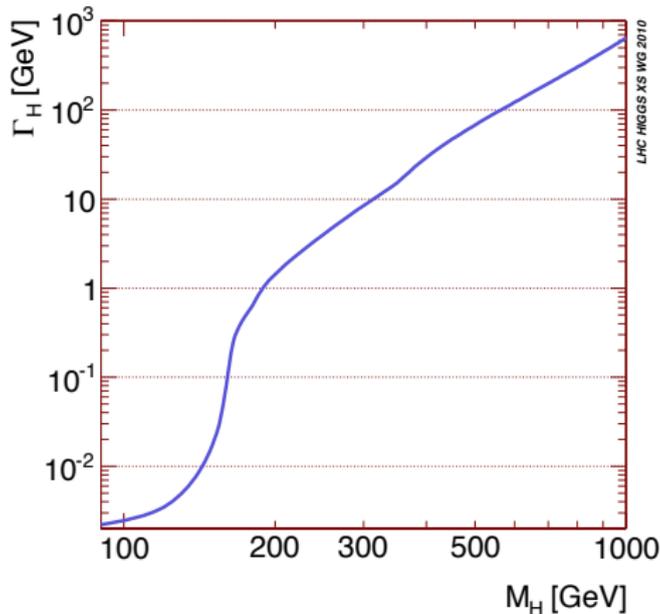
using the Feynman rule given [above](#).

# SM Higgs Boson Branching Fractions



# Total width of the standard-model Higgs boson

$$\Gamma_H(M_H = 126 \text{ GeV}) = 4.2 \text{ MeV}$$



Below  $W^+W^-$  threshold,  $\Gamma_H \lesssim 1 \text{ GeV}$

Far above  $W^+W^-$  threshold,  $\Gamma_H \propto M_H^3$

## A few words on Higgs production ...

$e^+e^- \rightarrow H$ : hopelessly small

$\mu^+\mu^- \rightarrow H$ : scaled by  $(m_\mu/m_e)^2 \approx 40\,000$

$e^+e^- \rightarrow HZ$ : prime channel

Hadron colliders:

$gg \rightarrow H \rightarrow b\bar{b}$ : background ?!

$gg \rightarrow H \rightarrow \tau\tau, \gamma\gamma$ : rate ?!

$gg \rightarrow H \rightarrow W^+W^-$ : best Tevatron sensitivity

$\bar{p}p \rightarrow H(W, Z)$ : prime Tevatron channel for light Higgs

At the LHC:

Many channels accessible, search sensitive up to 1 TeV

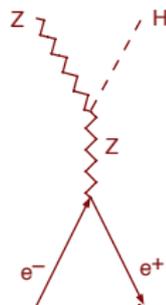
# Higgs search in $e^+e^-$ collisions

$\sigma(e^+e^- \rightarrow H \rightarrow \text{all})$  is *minute*,  $\propto m_e^2$

Even narrowness of low-mass  $H$  is not enough to make it visible ... Sets aside a traditional strength of  $e^+e^-$  machines—*pole physics*

Most promising:

associated production  $e^+e^- \rightarrow HZ$   
(has no small couplings)

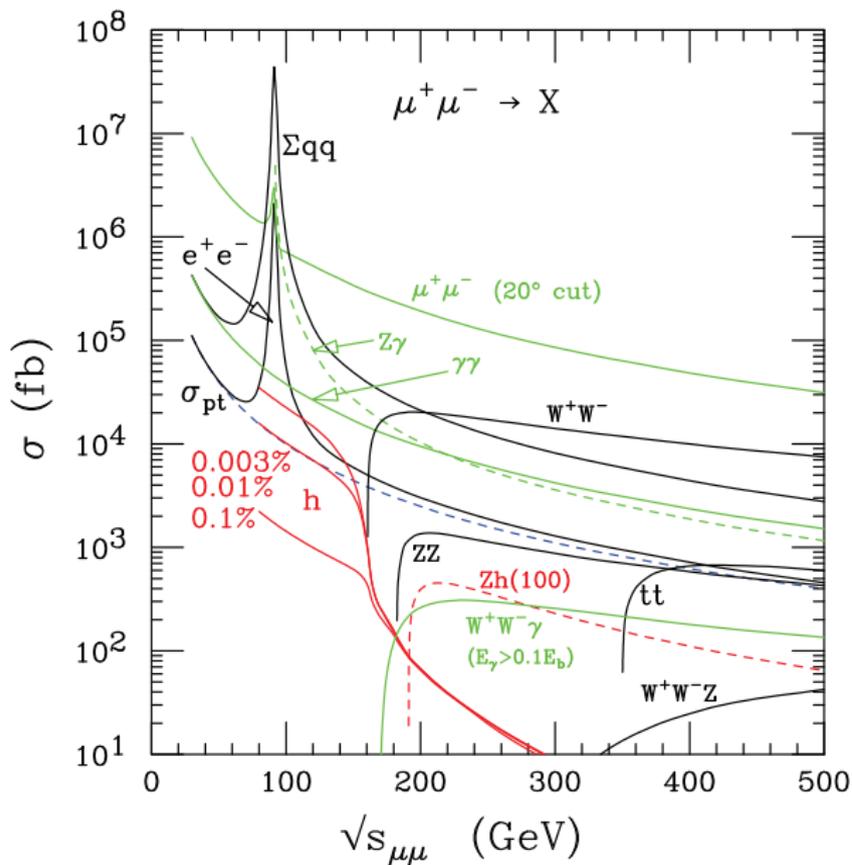


$$\sigma = \frac{\pi\alpha^2}{24\sqrt{s}} \frac{K(K^2 + 3M_Z^2)[1 + (1 - 4x_W)^2]}{(s - M_Z^2)^2 x_W^2(1 - x_W)^2}$$

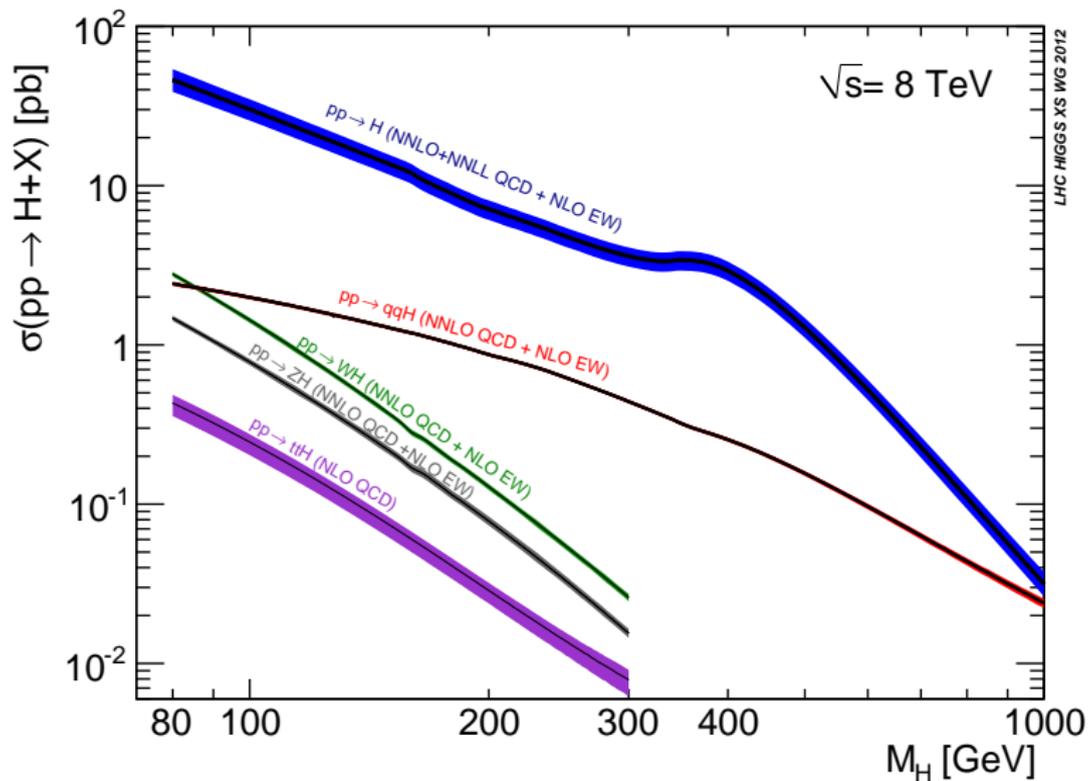
$K$ : c.m. momentum of  $H$

$x_W \equiv \sin^2 \theta_W$

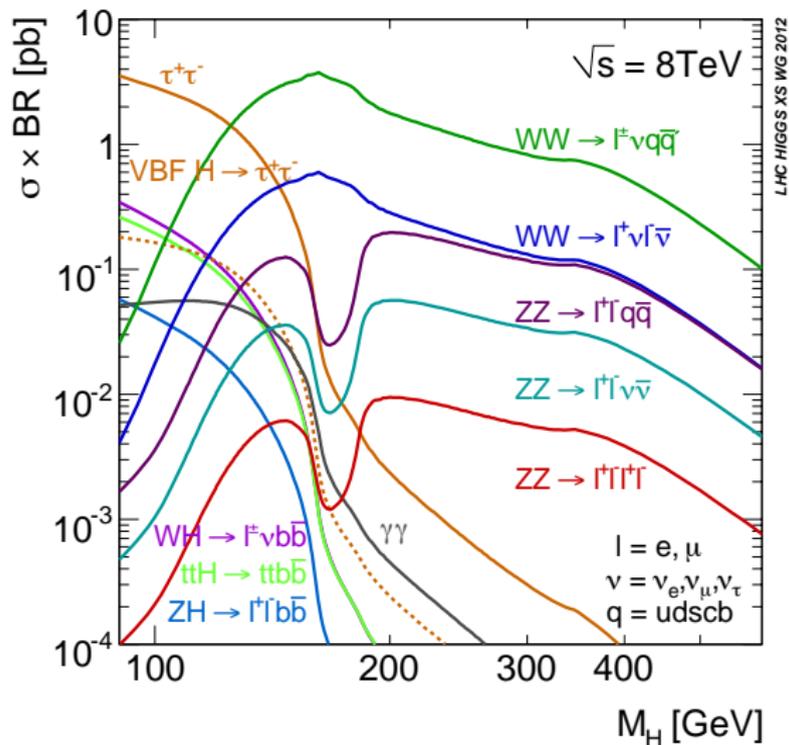
$$l^+l^- \rightarrow X \dots$$



# Higgs-boson production at the LHC: 8 TeV



# Higgs-boson production and decay: 8 TeV





# ATLAS $\gamma\gamma$ signal evolution

# CMS $4\mu$ signal evolution

# Evolution of evidence at the LHC

Evidence is developing as it would for  
a “standard-model” Higgs boson

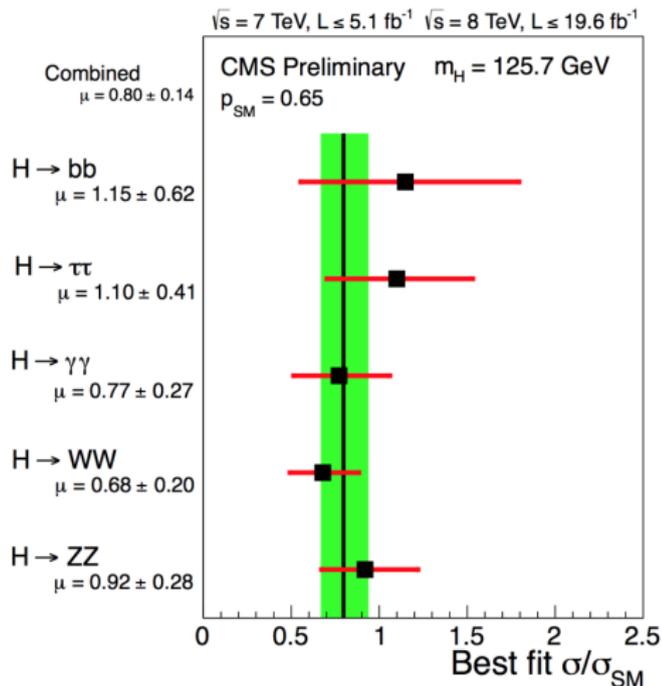
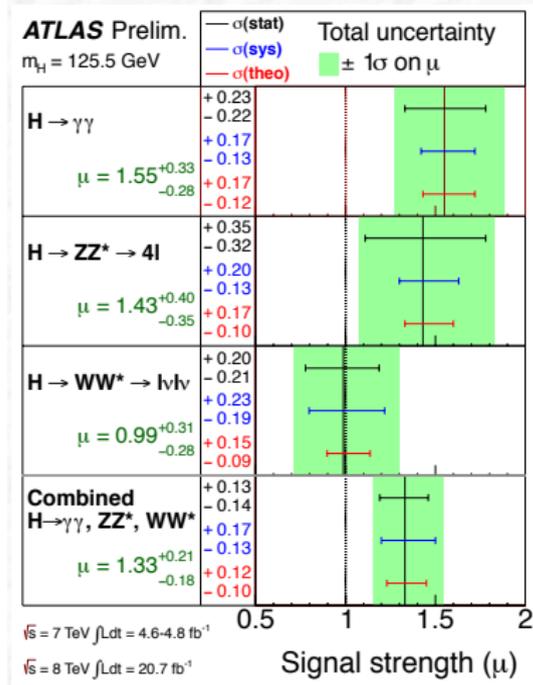
Unstable neutral particle near 126 GeV

decays to  $\gamma\gamma$ ,  $W^+W^-$ ,  $ZZ$   
likely (dominantly) spin-parity  $0^+$   
evidence for  $\tau^+\tau^-$ ,  $b\bar{b}$

# Consistency with Standard-Model Higgs

ATLAS twiki

CMS twiki



# Why Electroweak Symmetry Breaking Matters

*What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?*

(No EWSB agent at  $v \approx 246$  GeV)

Consider effects of **all** SM interactions!

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

# Without a Higgs Mechanism ...

Electron and quarks would have no mass

QCD would confine quarks into protons, etc.

Nucleon mass little changed

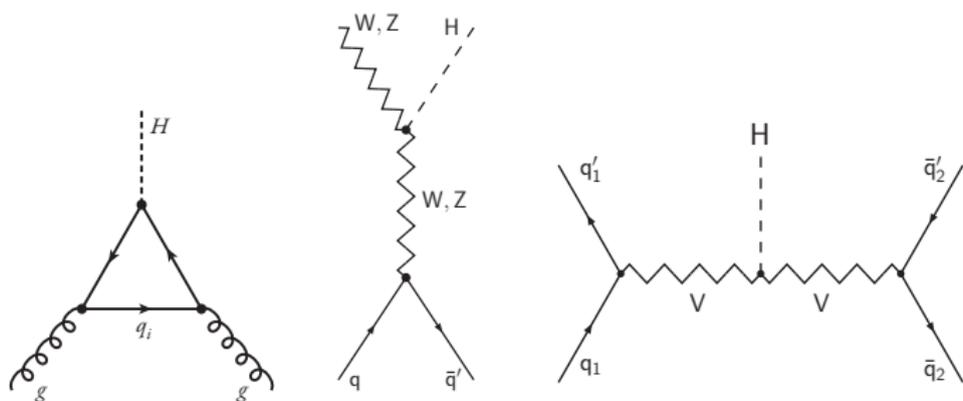
Surprise: QCD would hide EW symmetry,  
give tiny masses to  $W, Z$

Massless electron: atoms lose integrity

No atoms means no chemistry, no stable composite  
structures like liquids, solids, ...

# LHC: Multiple looks at the new boson

3 production mechanisms,  $\geq 5$  decay modes



$\gamma\gamma$ ,  $WW^*$ ,  $ZZ^*$ ,  $b\bar{b}$ ,  $\tau^+\tau^-$ ,  $Z\gamma(?)$

# Questions for ATLAS and CMS

Fully accounts for EWSB ( $W, Z$  couplings)?

Couples to fermions?

Top from production, need direct observation for  $b, \tau$

Accounts for fermion masses?

Fermion couplings  $\propto$  masses?

Are there others?

Quantum numbers?

SM branching fractions to gauge bosons?

Decays to new particles? via new forces?

All production modes as expected?

Implications of  $M_H \approx 126$  GeV?

Any sign of new strong dynamics?

# Standard-model shortcomings

- No explanation of Higgs potential
- No prediction for  $M_H$
- Doesn't predict fermion masses & mixings
- $M_H$  unstable to quantum corrections
- No explanation of charge quantization
- Doesn't account for three generations
- Vacuum energy problem
- Beyond scope: dark matter, matter asymmetry, etc.

~> imagine more complete, predictive extensions

## Beyond the Standard Model

### *More physics on the TeV scale?*

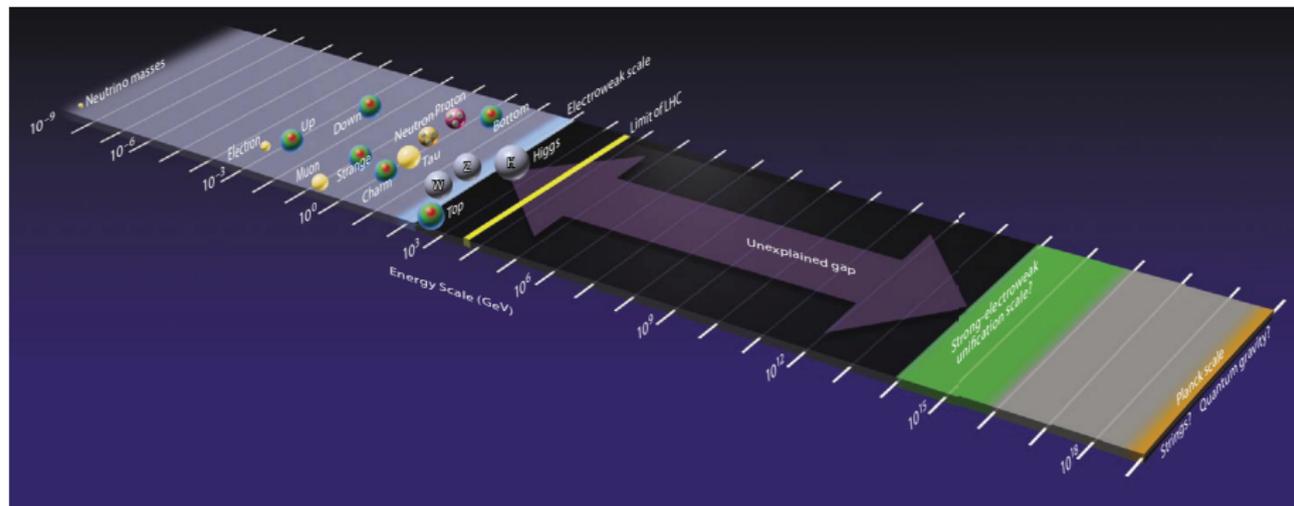
Partial-wave unitarity analysis of  $WW$  scattering argues for new physics on the TeV scale.

In SM: a Higgs boson or strongly interacting gauge sector  
In general, something new on the TeV scale

At the level of suggestion, rather than theorem ...

- The hierarchy problem: if light  $H$ , new physics implicated on the TeV scale
- WIMPs as dark matter: reproduce relic density for masses 0.1–1 TeV

# The Hierarchy Problem



How to keep the distant scales from mixing in the face of quantum corrections? *OR*

How to stabilize the mass of the Higgs boson on the electroweak scale? *OR*

Why is the electroweak scale small?

# The Hierarchy Problem

## *Possible paths*

- Fine tuning
- A new symmetry (supersymmetry)  
fermion, boson loops contribute with opposite sign
- Composite “Higgs boson” (technicolor ... )  
form factor damps integrand
- Little Higgs models, etc.
- Low-scale gravity (shortens range of integration)

All but first require new physics near the TeV scale

But Where is the New Physics?

# The unreasonable effectiveness of the Standard Model

# More Electroweak Questions for the LHC

- What is the agent that hides electroweak symmetry?
- Is the “Higgs boson” elementary or composite? How does the Higgs boson interact with itself? What triggers electroweak symmetry breaking?
- New physics in pattern of Higgs-boson decays?
- Will (unexpected or rare) decays of  $H$  reveal new kinds of matter?
- What would discovery of  $> 1$  Higgs boson imply?
- What stabilizes  $M_H$  below 1 TeV?
- How can a light  $H$  coexist with absence of new phenomena?
- Is EWSB related to gravity via extra dimensions?

# More Electroweak Questions for the LHC<sup>bis</sup>

- Is EWSB emergent, connected with strong dynamics?
- If new strong dynamics, how can we diagnose? What takes place of  $H$ ?
- Does the Higgs boson give mass to fermions, or only to the weak bosons? What sets the masses and mixings of the quarks and leptons?
- Does the different behavior of left-handed and right-handed fermions with respect to charged-current weak interactions reflect a fundamental asymmetry in the laws of nature?

# More Electroweak Questions for the LHC<sup>ter</sup>

- What will be the next symmetry recognized in Nature? Is Nature supersymmetric? Is the electroweak theory part of some larger edifice?
- Are there additional generations of quarks and leptons?
- What resolves the vacuum energy problem?
- What lessons does electroweak symmetry breaking hold for unified theories of the strong, weak, and electromagnetic interactions?

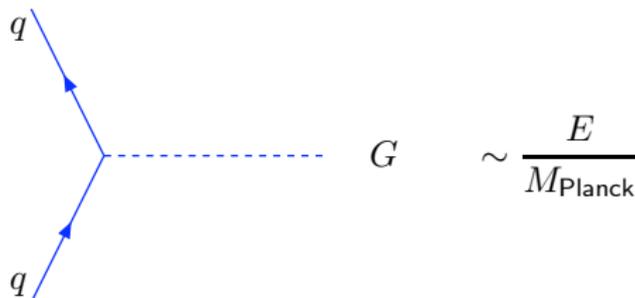
Thanks and good luck!

# Why is empty space so nearly massless?

Natural to neglect gravity in particle physics ...

Gravitational  $ep$  interaction  $\approx 10^{-41} \times$  EM

$$G_{\text{Newton}} \text{ small} \iff M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{\frac{1}{2}} \approx 1.22 \times 10^{19} \text{ GeV large}$$



300 years after Newton: Why **is** gravity weak?

# But gravity is not always negligible ...

*The vacuum energy problem*

$$\text{Higgs potential } V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|(\varphi^\dagger\varphi)^2$$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.$$

$$\text{Identify } M_H^2 = -2\mu^2$$

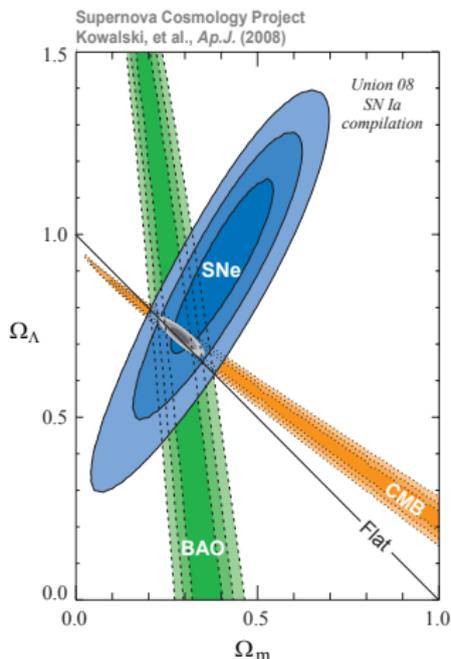
$V \neq 0$  contributes position-independent vacuum energy density

$$\rho_H \equiv \frac{M_H^2 v^2}{8} \geq 10^8 \text{ GeV}^4 \approx 10^{24} \text{ g cm}^{-3}$$

Adding vacuum energy density  $\rho_{\text{vac}} \Leftrightarrow$  adding cosmological constant  $\Lambda$  to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \quad \Lambda = \frac{8\pi G_N}{c^4} \rho_{\text{vac}}$$

Observed  $\rho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4$



$\rho_H \gtrsim 10^8 \text{ GeV}^4$ : mismatch by  $10^{54}$

A chronic dull headache for thirty years ...