1. Polarisation vector bases for photons

- photon is a massless vector boson
  $\Rightarrow$ 2 degrees of freedom

- ε^i transverse to axis of motion (momentum k parallel to z-axis)

Linear basis

- real valued $(\varepsilon^* = \varepsilon)$

$$\varepsilon^1(\underline{k}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \varepsilon^2(\underline{k}) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Spatial product : $\{i, j\} \in \{1,2,3\}$

$$\sum_{\lambda=1}^{2} \varepsilon^\lambda(k)_i \, \varepsilon^\lambda(k)_j = \varepsilon^1(k)_i \, \varepsilon^1(k)_j + \varepsilon^2(k)_i \, \varepsilon^2(k)_j$$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}_i \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}_j + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}_i \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}_j$$

$$= \begin{cases} \delta_{ij} & i,j \in \{1,2\} \\ 0 & \text{else} \end{cases}$$

$$= \delta_{ij} - \frac{k_i k_j}{|k|^2}$$

n.b

$$\underline{k} = \begin{pmatrix} 0 \\ 0 \\ |\underline{k}| \end{pmatrix}$$

$$= \delta_{ij} - \frac{k_i k_j}{|\underline{k}|^2} \qquad \underline{k} = \begin{pmatrix} 0 \\ 0 \\ |\underline{k}| \end{pmatrix}$$

Circular basis

- complex valued

$$\varepsilon^{L,R}(\underline{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}$$

$$\sum_{\lambda \in \{L,R\}} \varepsilon^{\lambda}(\underline{k})_i^* \, \varepsilon^{\lambda}(\underline{k})_j = \varepsilon^{L}(\underline{k})_i^* \, \varepsilon^{L}(\underline{k})_j + \varepsilon^{R}(\underline{k})_i^* \, \varepsilon^{R}(\underline{k})_j$$

$$= \frac{1}{2} \left[\begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}_i \begin{pmatrix} 0 \\ 1 \\ +i \\ 0 \end{pmatrix}_j + \begin{pmatrix} 0 \\ 1 \\ +i \\ 0 \end{pmatrix}_i \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}_j \right]$$

$$= \begin{cases} \frac{1 \cdot 1 + 1 \cdot 1}{2} = 1 & i,j = 1 \\ \frac{(-i) \cdot i + i \cdot (-i)}{2} = 1 & i,j = 2 \\ \frac{1 \cdot i + 1 \cdot (-i)}{2} = 0 & i = 1,\ j = 2 \\ \frac{(-i) \cdot 1 + i \cdot 1}{2} = 0 & i = 2,\ j = 1 \\ 0 & \text{else} \end{cases}$$

$=$ same as before

$\Rightarrow$ product result independent of choice of basis ✓