<u>Q's</u>

7.a) Prove $2m\bar{u}_1\gamma_\mu u_2 = \bar{u}_1\left[(p_1+p_2)_\mu + i\sigma_{\mu\nu}(p_1-p_2)^\nu\right]u_2$

Dirac: $\bar{u}_1(\not{p}_1 - m) = (\not{p}_1 - m)u_1 = 0$

$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$

$\bar{u}_1\left[i\sigma_{\mu\nu}(p_1-p_2)^\nu\right]u_2$ $\quad\quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$

$= -\frac{1}{2}\bar{u}_1\left\{[\gamma_\mu, \gamma_\nu](p_1-p_2)^\nu\right\}u_2$

$= -\frac{1}{2}\bar{u}_1\left[\gamma_\mu(\not{p}_1-\not{p}_2) - (\not{p}_1-\not{p}_2)\gamma_\mu\right]u_2$

Dirac:

$\not{p}_1 u_1 = m u_1$

$\bar{u}_1 \not{p}_1 = \bar{u}_1 m$

$= -\frac{1}{2}\bar{u}_1\left[\gamma_\mu(\not{p}_1 - m) - (m - \not{p}_2)\gamma_\mu\right]u_2$

$= -\frac{1}{2}\left[\bar{u}_1\gamma_\mu\not{p}_1 u_2 - \bar{u}_1\gamma_\mu m u_2 - \bar{u}_1 m\gamma_\mu u_2 + \bar{u}_1\not{p}_2\gamma_\mu u_2\right]$

$= -\frac{1}{2}\left[\bar{u}_1\gamma_\mu\not{p}_1 u_2 - 2m\bar{u}_1\gamma_\mu u_2 + \bar{u}_1\not{p}_2\gamma_\mu u_2\right]$

$= m\bar{u}_1\gamma_\mu u_2 - \frac{1}{2}\bar{u}_1\left(\gamma_\mu\not{p}_1 + \not{p}_2\gamma_\mu\right)u_2$

$= m\bar{u}_1\gamma_\mu u_2 - \frac{1}{2}\bar{u}_1\left(p_1^\nu\gamma_\mu\gamma_\nu + p_2^\nu\gamma_\nu\gamma_\mu\right)u_2$

$$\left(p_1^{\nu}\, \underbrace{\gamma_\mu \gamma_\nu} + p_2^{\nu}\, \underbrace{\gamma_\nu \gamma_\mu}_{\text{Similarly}} \right) u_2$$

$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$$

$$\Rightarrow \gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 g_{\mu\nu}$$

$$\Rightarrow \gamma_\mu \gamma_\nu = 2 g_{\mu\nu} - \gamma_\nu \gamma_\mu$$

$$= m \bar{u}_1 \gamma_\mu u_2 - \frac{1}{2} \bar{u}_1 \left(p_1^{\nu}\, 2 g_{\mu\nu} - p_1^{\nu} \gamma_\nu \gamma_\mu + p_2^{\nu}\, 2 g_{\nu\mu} - p_2^{\nu} \gamma_\mu \gamma_\nu \right) u_2$$

$$= m \bar{u}_1 \gamma_\mu u_2 - \frac{1}{2} \bar{u}_1 \left(2 p_{1\mu} - \not{p}_1 \gamma_\mu + 2 p_{2\mu} - \gamma_\mu \not{p}_2 \right) u_2$$

now can use Dirac

$$= m \bar{u}_1 \gamma_\mu u_2 - \frac{1}{2} \bar{u}_1 \left(2 (p_1 + p_2)_\mu - m \gamma_\mu - m \gamma_\mu \right) u_2$$

$$= m \bar{u}_1 \gamma_\mu u_2 - \bar{u}_1 (p_1 + p_2)_\mu u_2 + m \bar{u}_1 \gamma_\mu u_2$$

$$= \bar{u}_1 \left[2m \gamma_\mu - (p_1 + p_2)_\mu \right] u_2$$

which cancels terms on insertion, proving Gordon identity.

7.b) $$\bar{u}_1 \left[\sigma_{\mu\nu} (p_1 + p_2)^{\nu} \right] u_2$$

similarly to 7.a)

$$\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$$

$$= \frac{i}{2} \bar{u}_1 \left[\gamma_\mu (\not{p}_1 + \not{p}_2) - (\not{p}_1 + \not{p}_2) \gamma_\mu \right] u_2$$

$$\bar{u}_1 \not{p}_1 = \bar{u}_1 m$$

$$\not{p}_2 u_2 = m u_2$$

$$= \frac{i}{2} \bar{u}_1 \left[\gamma_\mu (\not{p}_1 + \cancel{m}) - (\cancel{m} + \not{p}_2) \gamma_\mu \right] u_2$$

$$= \frac{i}{2}\,\bar{u}_1\left[\gamma_\mu \not{p}_1 - \not{p}_2\gamma_\mu\right]u_2$$

$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}$

$$= \frac{i}{2}\,\bar{u}_1\left[2\not{p}_{1\mu} - \not{p}_1\gamma_\mu + 2\not{p}_{2\mu} - \gamma_\mu\not{p}_2\right]u_2$$

Dirac

$$= \frac{i}{2}\,\bar{u}_1\left[2(\not{p}_1+\not{p}_2)_\mu - 2m\gamma_\mu\right]u_2$$

$$= i\,\bar{u}_1(\not{p}_1+\not{p}_2)_\mu u_2 - im\,\bar{u}_1\gamma_\mu u_2$$

I don't get Frank's answer

7. c) $\not{p}_1\gamma_\mu\not{p}_2$

$\not{p}_1 = p_1^\nu\gamma_\nu$

$$= \left(p_1^\nu\gamma_\nu\gamma_\mu\right)\not{p}_2$$

$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}$

$$= \left[p_1^\nu\left(2g_{\nu\mu} - \gamma_\mu\gamma_\nu\right)\right]\not{p}_2$$

$$= \left(2p_1^\nu g_{\mu\nu} - \gamma_\mu\not{p}_1\right)\not{p}_2$$