

Q's

$$4.a) \quad :\hat{p}^\mu: = \int \frac{d^3k}{(2\pi)^3 2k_0} k^\mu \hat{a}^\dagger(\underline{k}) \hat{a}(\underline{k})$$

Start with:
$$:\hat{p}^\mu: = \int d^3x : \hat{\pi}(x) \partial^\mu \hat{\phi}(x) :$$

Insert (142):
$$\begin{cases} \hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^3 2k_0} \left[\hat{a}(\underline{k}) e^{-ik \cdot x} + \hat{a}^\dagger(\underline{k}) e^{ik \cdot x} \right] \\ \hat{\pi}(x) = \int \frac{d^3k}{(2\pi)^3 2k_0} \left[-ik_0 \hat{a}(\underline{k}) e^{-ik \cdot x} + ik_0 \hat{a}^\dagger(\underline{k}) e^{ik \cdot x} \right] \end{cases}$$

n.b.
$$\begin{aligned} \partial^\mu \hat{\phi}(x) &= \int \frac{d^3k}{(2\pi)^3 2k_0} \frac{\partial}{\partial x^\mu} \left[\hat{a}(\underline{k}) e^{-ik^\nu x_\nu} + \hat{a}^\dagger(\underline{k}) e^{ik^\nu x_\nu} \right] \\ &= \int \frac{d^3k}{(2\pi)^3 2k_0} \left(-ik^\mu \hat{a}(\underline{k}) e^{-ik \cdot x} + ik^\mu \hat{a}^\dagger(\underline{k}) e^{ik \cdot x} \right) \end{aligned}$$

$$\Rightarrow \quad :\hat{p}^\mu: = \int d^3x \frac{d^3k}{(2\pi)^3 2k_0} \frac{d^3q}{(2\pi)^3 2q_0} \left[-ik_0 (\hat{a}(\underline{k}) e^{-ik \cdot x} - \hat{a}^\dagger(\underline{k}) e^{ik \cdot x}) \right. \\ \left. \times iq^\mu (-\hat{a}(\underline{q}) e^{-iq \cdot x} + \hat{a}^\dagger(\underline{q}) e^{iq \cdot x}) \right]$$