3 Classical fields Q

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Questions from Students.

3.c)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$$

$$\frac{\partial \mathcal{L}}{\partial A^\mu} - \partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\nu A^\mu)} = 0$$

$$\frac{\partial \mathcal{L}}{\partial A^\mu} = \frac{\partial}{\partial A^\mu}\left(\frac{m^2}{2} A_\sigma A^\sigma\right) = m^2 A_\sigma \underbrace{\frac{\partial A^\sigma}{\partial A^\mu}}_{=\delta_\mu{}^\sigma} = m^2 A_\mu$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\nu A^\mu)} = \frac{\partial}{\partial(\partial_\nu A^\mu)}\left(-\frac{1}{4} F_{\sigma\rho} F^{\sigma\rho}\right)$$

n.b.

$$\partial_x f(x)^2 = 2 f(x)\, \partial_x f(x)$$

$$= -\frac{1}{2} F_{\sigma\rho} \frac{\partial F^{\sigma\rho}}{\partial(\partial_\nu A^\mu)}$$

$$= -\frac{1}{2} F_{\sigma\rho}\left(g^{\nu\sigma} \delta_\mu{}^\rho - g^{\nu\rho} \delta_\mu{}^\sigma\right)$$

$$= -\frac{1}{2}\left(F^\nu{}_\mu - F_\mu{}^\nu\right)$$

$$F^{\sigma\rho} = -F^{\rho\sigma} \leftarrow \text{anti symmetric}$$

$$= -\frac{1}{2}\left(F^\nu{}_\mu + F^\nu{}_\mu\right)$$

$$\Rightarrow F^\sigma{}_\rho = g_{\mu\rho} F^{\sigma\mu} = -g_{\mu\rho} F^{\mu\sigma} = -F_\rho{}^\sigma$$

$$= -F^\nu{}_\mu$$

$$= F_\mu{}^\nu$$

I think Frank's sign is wrong here

$$\Rightarrow \quad m^2 A_\mu - \partial_\nu F_\mu{}^\nu = 0$$

$$\Rightarrow \quad \underline{\underline{m^2 A_\mu - \partial_\nu F_\mu{}^{\nu} = 0}}$$

alternatively: $m^2 A_\mu - \partial_\nu \left(\partial_\mu A^\nu - \partial^\nu A_\mu \right) = 0$

$$\Rightarrow \quad m^2 A_\mu - \partial_\nu \partial_\mu A^\nu + \partial_\nu \partial^\nu A_\mu = 0$$

$$\Rightarrow \quad (\Box + m^2) A_\mu - \partial_\nu \partial_\mu A^\nu = 0$$

$$\Rightarrow \quad \underline{\left[g_{\mu\nu} (\Box + m^2) - \partial_\nu \partial_\mu \right] A^\nu = 0}$$

3.d) $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left(\partial_\mu \phi^* + ie A_\mu \phi^* \right) \left(\partial^\mu \phi - ie A^\mu \phi \right) - m^2 |\phi|^2$

$\underline{\underline{A_\mu}}$

$$\frac{\partial \mathcal{L}}{\partial A^\mu} - \partial_\nu \frac{\partial \mathcal{L}}{\partial (\partial_\nu A^\mu)} = 0$$

$$\frac{\partial \mathcal{L}}{\partial A^\mu} = \frac{\partial}{\partial A^\mu} \left[-ie\, \partial_\nu \phi^* A^\nu \phi + ie A_\nu \phi^* \partial^\nu \phi + e^2 A_\nu A^\nu |\phi|^2 \right]$$

$$= ie \left[-(\partial_\mu \phi^*) \phi + \phi^* \partial_\mu \phi \right] + e^2 |\phi|^2 \, 2 A_\mu$$

$$\frac{\partial \mathcal{L}}{\partial (\partial_\nu A^\mu)} = F_\mu{}^{\nu}$$ – same as 3.c)

$$\Rightarrow \quad ie \left[-\phi\, \partial_\mu \phi^* + \phi^* \partial_\mu \phi \right] + 2 e^2 |\phi|^2 A_\mu - \partial_\nu F_\mu{}^{\nu} = 0$$

$$\Rightarrow \quad ie\left[-\phi\,\partial_\mu\phi^* + \phi^*\partial_\mu\phi\right] + 2e^2|\phi|^2 A_\mu - \partial_\nu F_\mu{}^\nu = 0$$

$\underline{\phi}$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{\partial}{\partial \phi}\left[-ie\,\partial_\nu\phi^* A^\nu \phi + ie A_\nu \phi^* \partial^\nu \phi + e^2 A_\nu A^\nu |\phi|^2 + (\partial_\nu\phi^*)(\partial^\nu\phi) - m^2|\phi|^2\right]$$

$$= -ie\,\partial_\nu\phi^* A^\nu + e^2 A_\nu A^\nu \phi^* - m^2\phi^*$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} = \frac{\partial}{\partial(\partial_\nu\phi)}\left[-ie\,\partial_\mu\phi^* A^\mu \phi + ie A_\mu \phi^* \partial^\mu \phi + e^2 A_\mu A^\mu |\phi|^2 + (\partial_\mu\phi^*)(\partial^\mu\phi)\right]$$

$$= ie A^\nu \phi^* + \partial^\nu \phi^*$$

$$\Rightarrow \quad -ie\,\partial_\nu\phi^* A^\nu + e^2 A_\nu A^\nu \phi^* - m^2\phi^* - \partial_\nu\left(ie A^\nu \phi^* + \partial^\nu\phi^*\right) = 0$$

$$\Rightarrow \quad -m^2\phi^* - \Box\phi^* - ie A^\nu\partial_\nu\phi^* + e^2 A^2 \phi^* \underbrace{- ie\,\partial_\nu\left(A^\nu\phi^*\right)}_{= -ie\,\phi^*\partial_\nu A^\nu - ie A^\nu\partial_\nu\phi^*} = 0$$

$$\Rightarrow \quad \left(m^2 + \Box + 2ie A^\nu\partial_\nu + ie(\partial_\nu A^\nu) - e^2 A^2\right)\phi^* = 0$$

and similarly for the Euler-Lagrange equation for ϕ.

Ignore Frank's solutions, they have typos.