Workshop 2

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(cont. 1)

$$A_\mu \to \mathcal{U}(A_\mu + i\partial_\mu)\mathcal{U}^\dagger \quad , \quad \varphi \to \mathcal{U}\varphi$$

$$\mathcal{D}_\mu = \partial_\mu + a A_\mu$$

$$\mathcal{D}_\mu \varphi \to \partial_\mu(\mathcal{U}\varphi) + a\,\mathcal{U}(A_\mu + i\partial_\mu)\mathcal{U}^\dagger$$

$$= (\partial_\mu \mathcal{U})\varphi + \mathcal{U}(\partial_\mu \varphi) + a\,\mathcal{U}A_\mu\mathcal{U}^\dagger + a i\,\partial_\mu \mathcal{U}^\dagger$$

(b) $$\mathcal{L}_{GG} = \mathrm{tr}\left[(\mathcal{D}_\mu \Phi)(\mathcal{D}^\mu \Phi)\right] - y\,\bar{\Psi}\Phi\Psi$$

$$- \frac{\lambda}{4}\left(\Phi^a \Phi^a - v^2\right)^2 + \mathcal{L}(A_\mu, \bar{\Psi}, \Psi)$$

$$\Phi^a = \left(\Phi^1(x), \Phi^2(x), \Phi^3(x)\right)^a$$

$$\Phi(x) = \Phi^a(x) T^a$$

$$D_\mu \Phi = \partial_\mu \Phi - i[A_\mu, \Phi]$$

$$\Phi(x) \to \mathcal{U}(x)\,\Phi(x)\,\mathcal{U}^\dagger(x) \qquad A_\mu \to \mathcal{U}(A_\mu + i\partial_\mu)\mathcal{U}^\dagger$$

$$\Rightarrow \quad \mathcal{D}_\mu \Phi \to \partial_\mu(\mathcal{U}\Phi\mathcal{U}^\dagger) - i\left[\left(\mathcal{U}(A_\mu + i\partial_\mu)\mathcal{U}^\dagger\right), \mathcal{U}\Phi\mathcal{U}^\dagger\right]$$

$$= (\partial_\mu u)\Phi u^\dagger + u\,\partial_\mu(\Phi u^\dagger) - i\left(u A_\mu u^\dagger + i\,u(\partial_\mu u^\dagger)\right) u\Phi u^\dagger$$

$$+ i\, u\, \Phi\, (A_\mu + i\partial_\mu) u^\dagger$$

$$= \cancel{(\partial_\mu u)\Phi u^\dagger} + u(\partial_\mu \Phi)u^\dagger + \cancel{u\Phi(\partial_\mu u^\dagger)} - i u\, A_\mu \Phi u^\dagger$$

$$\underbrace{+\, \underbrace{u(\partial_\mu u^\dagger)}\, u\Phi u^\dagger} + i\, u\Phi A_\mu u^\dagger - \cancel{u\Phi(\partial_\mu u^\dagger)}$$

$$0 = \partial_\mu(u u^\dagger) = (\partial_\mu u) u^\dagger + u(\partial_\mu u^\dagger)$$

$$= - \cancel{(\partial_\mu u)\Phi u^\dagger}$$

$$= u(\partial_\mu \Phi)u^\dagger - i\, u\,[A_\mu, \Phi]\, u^\dagger$$

$\boxtimes$

$$\mathrm{tr}(\Phi^2) = \Phi^a \Phi^b\, \mathrm{tr}(T^a T^b) \sim \Phi^a \Phi^b \delta^{ab} = \Phi^a \Phi^a$$

$\mathrm{tr}(\Phi^2)$ triv. inv.

$\mathcal{L}_{GG}$ inv? tr cyclic $\rightsquigarrow$ kin. yes.

$$\psi \to u\psi \quad \Rightarrow \quad \bar\psi \to \psi u^\dagger$$

$$\bar\psi \Phi \psi \to \psi u^\dagger u \Phi u^\dagger u \psi = \bar\psi \Phi \psi \quad \checkmark$$

$$\Rightarrow \quad h \quad \checkmark$$

(c) $\underline{\langle \Phi^1 \rangle = \langle \Phi^2 \rangle = 0}$, $\langle \Phi^3 \rangle = v$ — vacuum

$U \in SU(2)$ $\quad \Phi = \Phi^a T^a \quad T^a = \frac{\sigma^a}{2}$

$$[\sigma^i, \sigma^j] = 2i\,\varepsilon^{ijk}\sigma^k \qquad tr(T^a T^b) = \frac{1}{2}\delta^{ab}$$

$$[T^i, T^j] = i\,\varepsilon^{ijk} T^k$$

vacuum: $\langle \Phi \rangle = \langle \Phi^a T^a \rangle = \langle \Phi^1 T^1 + \Phi^2 T^2 + \Phi^3 T^3 \rangle = \cancel{\langle \Phi^1 \rangle} T^1 + \cancel{\langle \Phi^2 \rangle} T^2 + \langle \Phi^3 \rangle T^3$

$= \langle \Phi^3 \rangle T^3 = v\, T^3$

$v \to e^{i\phi} v$

T^1, T^2 broken : sym: $SU(2) \to U(1)_{T^3}$

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unitary gauge: $\Phi^1 = \Phi^2 = 0$, $\Phi^3(x) = v + \phi(x)$

$$D_\mu \Phi = \partial_\mu \Phi - i[A_\mu, \Phi]$$

$$= T^3 \partial_\mu \phi(x) - i\underbrace{[A_\mu, T^3]}(v + \phi)$$

$\Phi = \Phi^a T^a$
$= (v + \phi) T^3$

$A_\mu = g A_\mu^a T^a$

$= g A_\mu^a [T^a, T^3]$

$= g A_\mu^1 [T^1, T^3] + g A_\mu^2 [T^2, T^3]$

$\varepsilon^{123} = +1$

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$\varepsilon^{123} = +1$

$$= g A_\mu [T^1, T^1] + g A_\mu [T^1, T^1]$$

$$= ig\left(A_\mu^1 \underbrace{\varepsilon^{132}}_{-1} T^2 + A_\mu^2 \underbrace{\varepsilon^{231}}_{+1} T^1\right)$$

231
213
123

$$= ig\left(A_\mu^2 T^1 - A_\mu^1 T^2\right)$$

$$= T^3 \partial_\mu \phi + g\left(A_\mu^2 T^1 - A_\mu^1 T^2\right)(v+\phi)$$

$$\mathcal{L} \ni \mathrm{tr}\left(\mathcal{D}_\mu \Phi\, \mathcal{D}^\mu \Phi\right) \ni g^2 v^2 \underbrace{\left(A_\mu^2 T^1 - A_\mu^1 T^2\right)^2}$$

$$(A_\mu^2)^2 (T^1)^2 + (A_\mu^1)^2 (T^2)^2 - A_\mu^2 A^{\mu 1} \cancel{\underbrace{(T^1 T^2 + T^2 T^1)}_{\{\sigma^a, \sigma^b\} = 2\delta^{ab}\mathbb{1}}} \quad 0$$

$T^a = \frac{\sigma^a}{2}$

$(T^a)^2 = \frac{(\sigma^a)^2}{4} = \frac{\mathbb{1}}{4}$

$\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$$\sim -\frac{1}{2}\left(\frac{g^2 v^2}{\sqrt{2}}\right)\left(A_\mu^2 A^{2\mu} + A_\mu^1 A^{\mu 1}\right)$$

2 massive vector/gauge bosons w/ $m = \frac{g^2 v^2}{\sqrt{2}}$

& 1 massless.

(d) "physical Higgs" boson ; ϕ

$$-\frac{\lambda}{4}\left(\Phi^a \Phi^a - v^2\right)^2 \xrightarrow{\text{vac.}} -\frac{\lambda}{4}\left(\overbrace{(v+\phi)^2}^{(\Phi^3)^2} - v^2\right)^2$$

$$= -\lambda/4 \left(v^2 + \phi^2 + 2v\phi - v^2\right)^2$$

$$= -\lambda/4 \left(\phi^2 + 2v\phi \right)^2$$

$$= -\lambda/4 \; \phi^2 \left(\phi + 2v \right)^2$$

$$= -\lambda/4 \; \phi^2 \; 4v^2 + \dots$$

$$= -\lambda v^2 \phi^2$$

$$= -\frac{1}{2} (2\lambda v^2) \phi^2$$

$$\Rightarrow \quad m_H^2 = 2\lambda v^2 \quad \Rightarrow \quad \underline{m_H = \sqrt{2\lambda}\, v}$$

fermions

$$-y \bar{\Psi} \Phi \Psi \rightarrow -y \bar{\Psi} (v + \phi) T^3 \Psi$$

$$\ni \quad -y \bar{\Psi} v T^3 \Psi$$

$$\ni \quad -yv \frac{\sigma^3}{2}$$

$$= \quad -\frac{1}{2} \left[yv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

also $\bar{\Psi}\Psi$ term $\Rightarrow$ m $\quad (= m \mathbb{1})$

$$\Rightarrow \quad m_1 = \underline{m + yv} \quad , \quad m_2 = \underline{m - yv}$$

fermion doublet splitting