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problem-1a

Monday, November 25, 2019 5:10 PM

α: $A = 2$, $Z = 4$ $(N=2)$ – He nucleus

α-decay:

$$E_\alpha = \overset{\text{parent}}{M(A,Z)} - \overset{\text{daughter}}{M(A-4,\, Z-2)} - \overset{\alpha}{M(4,2)}$$

SEMF (= liquid drop): . . .

problem-1b

Monday, November 25, 2019 5:20 PM

$$\left(\frac{A}{2}-2\right)^2 = \left(\frac{A-4}{2}\right)^4$$

$$\underbrace{Z = \frac{A}{2}}, \quad \frac{1}{A} \ll 1$$

$$E_\alpha = -4a_v - a_s\left((A-4)^{2/3} - A^{2/3}\right) - a_c\left(\frac{(Z-2)^2}{(A-4)^{1/3}} - \frac{Z^2}{A^{1/3}}\right)$$

$$- a_a\left(\frac{(A-2Z)^2}{4(A-4)} - \frac{(A-2Z)^2}{4A}\right) + B(4,2)$$

$$= -4a_v - a_s\underbrace{\left((A-4)^{2/3} - A^{2/3}\right)} - a_c\left(\frac{(A-4)^{5/3}}{4} - \frac{A^{5/3}}{4}\right)$$

$$-a_a\Big($$

$$A^{-1/3}\left(A^{1/3}\underbrace{(A-4)^{2/3}} - A\right)$$

$$\left(A - \frac{A4}{A}\right)^{2/3}$$

$$= \left[A\left(1-\frac{4}{A}\right)\right]^{2/3}$$

$$= A^{2/3}\left(1-\frac{4}{A}\right)^{2/3}$$

⇒ Taylor exp. $\left(\frac{1}{A} \ll 1\right)$

$$= A^{2/3}\left(1 - \frac{2}{3}\frac{4}{A}\right)$$

$$= A^{2/3}\left(1 - \frac{8}{3}\frac{1}{A}\right)$$

$$= A^{-1/3}\left(A^{1/3}A^{2/3}\left(1 - 8/3\,\frac{1}{A}\right) - A\right)$$

$$= A^{-1/3}\left(A\left(1 - \frac{8}{3}\frac{1}{A}\right) - A\right)$$

$$= A^{-1/3}\left(\cancel{A} - \frac{8}{3} - \cancel{A}\right)$$

$$= -A^{-1/3}\,\frac{8}{3}$$

& ||ly for others.

problem-1c

Monday, November 25, 2019 5:21 PM

Sub in

problem-1d

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algebra

problem-2

Monday, November 25, 2019 5:21 PM

$^{236}_{92}U$: $R = 1.2 \times 10^{-13} A^{1/3}$

calc. ΔE using $E_{stat} = \frac{3}{5} \frac{Q^2}{4\pi R}$

strikes

Tuesday, February 11, 2020 10:27 AM

Workshops 2 & 3 were missed due to strikes

problem-1a

Tuesday, January 14, 2020 3:16 PM

1//

$$\mathrm{FT}[f(\vec{y})] =: F(\vec{q}) = \int_{-\infty}^{\infty} f(\vec{y})\, e^{i\vec{q}\cdot\vec{y}}\, d^3y \qquad \text{— } (\dagger)$$

3D

$$\mathrm{FT}^{-1}[F(\vec{q})] := f(\vec{y}) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} F(\vec{q})\, e^{-i\vec{q}\cdot\vec{y}}\, d^3q$$

where $\lim_{y\to\pm\infty} f(\vec{y}) = 0.$ — $(*)$

a/

const.

$$\mathrm{FT}[f(\vec{y}+\vec{b})] =: F(\vec{q})$$

$$= \int_{-\infty}^{\infty} f(\vec{y}+\vec{b})\, e^{i\vec{q}\cdot\vec{y}}\, d^3y \qquad \text{by } (\dagger)$$

$\vec{y}\to\vec{y}-\vec{b}$ (dummy integration variable)

$$= \int_{-\infty}^{\infty} f(\vec{y})\, e^{i\vec{q}\cdot(\vec{y}-\vec{b})}\, d^3y$$

$$= \underbrace{\int_{-\infty}^{\infty} f(\vec{y})\, e^{i\vec{q}\cdot y}\, d^3y}\; e^{-i\vec{q}\cdot\vec{b}}$$

$$= F(\vec{q})\, e^{-i\vec{q}\cdot\vec{b}}$$

problem-1b

Tuesday, January 14, 2020 3:28 PM

b/

$a > 0$.

$$\mathrm{FT}\left[f(a\vec{y})\right] = F(\vec{q}) = \int f(a\vec{y})\, e^{i\vec{q}\cdot\vec{y}}\, d^3y$$

$\vec{y} \to \frac{\vec{y}}{a}$

$$= \int f(\vec{y})\, e^{i\vec{q}\cdot\frac{\vec{y}}{a}}\, \frac{d^3y}{a^3}$$

$$= \frac{1}{a^3}\underbrace{\int f(\vec{y})\, e^{i\frac{\vec{q}}{a}\cdot\vec{y}}}_{(4)}$$

$$= \frac{1}{a^3}\, F\left(\frac{\vec{q}}{a}\right)$$

problem-1c

Tuesday, January 14, 2020 3:32 PM

$\frac{\partial}{\partial y_i}$

$$FT\left[\partial_i f(\vec{y})\right] = F(\vec{q})$$

$$= \int \left[\partial_i f(\vec{y})\right] e^{i\vec{q}\cdot\vec{y}}\, d^3y$$

IBP

$$= \underbrace{\left[f(\vec{y})\, e^{i\vec{q}\cdot\vec{y}}\right]_{-\infty}^{\infty}}_{=0 \text{ by } \circledast} - \int f(\vec{y})\, i q_i\, e^{i\vec{q}\cdot\vec{y}}\, d^3y$$

$u = e^{i\vec{q}\cdot\vec{y}}$ $\quad v' = \partial_i f(\vec{y})$

$u' = i q_i u$ $\quad v = f(\vec{y})$

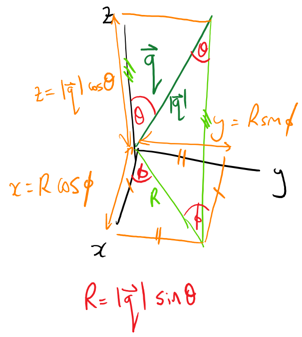
$$\int v' u \, dy = [vu] - \int v u' \, dy$$

$$= -i q_i \underbrace{\int f(\vec{y})\, e^{-i\vec{q}\cdot\vec{y}}\, d^3y}_{= F(\vec{q})}$$

$$= -i q_i F(\vec{q})$$

problem-1d

Tuesday, January 14, 2020 3:38 PM



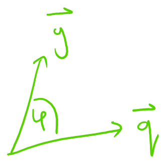
$$f(\vec{y}) = f(r) \qquad \text{spherical sym.}$$

where $|\vec{y}| =: r$

Let's start with $f(\vec{y})$ and impose sym. later.

basis independent expression, eg. in Cartesians, $dV_{\vec{y}} = d^3\vec{y}$.

$$\underbrace{FT[\underbrace{f(\vec{y})}_{\text{function}}]}^{\text{functional}} = \overbrace{F(\vec{q})}^{\text{function}} = \int_{-\infty}^{\infty} dV_{\vec{y}}\, f(\vec{y})\, e^{i\vec{y}\cdot\vec{q}}$$



scalar product rule.

Now, $\vec{y}\cdot\vec{q} = |\vec{y}||\vec{q}|\cos\varphi = |\vec{y}||\vec{q}|\cos\varphi$

$$= \int dV_{\vec{y}}\, f(\vec{y})\, e^{i|\vec{y}||\vec{q}|\cos\varphi}$$

Now choose spherical polar coordinates: $\vec{y} = r\hat{r} + \theta\hat{\theta} + \phi\hat{\phi}$, $dV_{\vec{y}} = r^2\sin\theta\, d\theta\, d\phi\, dr$

$$= \int_0^{\infty} r^2 dr \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)\, f(r,\theta,\phi)\, e^{ir|\vec{q}|\cos(\varphi(\theta,\phi))}$$

Now rotate the co-ord system $(r,\theta,\phi) \to (r',\theta',\phi')$ s.t. $\varphi(\theta,\phi) = \theta'$

$$= \int_0^{\infty} r'^2 dr' \int_0^{2\pi} d\phi' \int_{-1}^{1} d\cos\theta'\, f(r',\theta',\phi')\, e^{ir'|\vec{q}|\cos\theta'}$$

Aside

This is the same as rotating co-ords s.t. $\vec{q} = |\vec{q}|\hat{z}$.

In that case, we could use Cartesians:

$$F(\vec{q}) = \int d^3\vec{y}\, f(y_1,y_2,y_3)\, e^{i(y_1q_1 + y_2q_2 + y_3q_3)}$$

... $\vec{q} = (0, 0, |\vec{q}|)$ and recall $y = \begin{pmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{pmatrix}$

$$f(\vec{q}) = \int d^3\vec{y}\, f(y_1,y_2,y_3)\, e$$

then rotate coord axes s.t. $\vec{q} = (0,0,|\vec{q}|)$ and recall $y = \begin{pmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{pmatrix}$

here

$$= \int d^3\vec{y}\, f(y_1,y_2,y_3)\, e^{ir\cos\theta|\vec{q}|}$$

then switch to sphericals, with identification of r,θ already as those of co-ord axes (of integration) to get (#)

Note that we could also have chosen, e.g. $\vec{q} = |\vec{q}|\,\hat{x}$, then

$\cos\varphi = \sin\theta'\cos\phi'$, but above choice gives simplest integrand.

End aside

Then change dummy integration variables because I'm ~~too~~ lazy to carry the primes:

$$= \int_0^\infty r^2 dr \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta\, f(r,\theta,\phi)\; e^{ir|\vec{q}|\cos\theta} \quad \text{— (\#)}$$

Now impose spherical symmetry of $f(\vec{y})$: $f(r,\theta,\phi) = f(r)$

$$= \int_0^\infty r^2 dr \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \int_{-1}^{1} d\cos\theta\, f(r)\; e^{ir|\vec{q}|\cos\theta}$$

$$= 2\pi \int r^2 f(r)\, dr \underbrace{\int_{-1}^{1} d\cos\theta\; e^{ir|\vec{q}|\cos\theta}}$$

$$= -i\left[\frac{e^{-ir|\vec{q}|\cos\theta}}{r|\vec{q}|}\right]_{\cos\theta=-1}^{1}$$

$$= \frac{-i}{r|\vec{q}|}\left(e^{ir|\vec{q}|} - e^{-ir|\vec{q}|}\right)$$

$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$= \frac{2}{r|\vec{q}|} \sin\left(r|\vec{q}|\right)$$

$$= \frac{4\pi}{|\vec{q}|} \int_0^\infty dr \; r \; f(r) \; \sin\left(|\vec{q}|r\right)$$

problem-1e

Tuesday, January 14, 2020 4:11 PM

$FT[e^{-r^2/\sigma^2}] \Rightarrow f(r) = e^{-r^2/\sigma^2}$ - spherically symmetric $\rightarrow$ use (d)

$$\boxed{FT[f(r)] = \frac{4\pi}{|\vec{q}|}\int_0^\infty dr\; r\, f(r)\, \sin(|\vec{q}|r)}$$

So: $FT[e^{-r^2}]$

$q := |\vec{q}|$

$$= \frac{4\pi}{q}\underbrace{\int_0^\infty dr\; r\, e^{-r^2}\sin(qr)}$$

$\rangle$ $\sin(x) = \mathrm{Im}[e^{+ix}]$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\mathrm{Im}(e^{i\theta}) = \sin\theta$

$$= \int dr\; r\, e^{-r^2}\,\mathrm{Im}[e^{+iqr}]$$

$$= \mathrm{Im}\Big[\underbrace{\int dr\; r\, e^{-r^2+iqr}}\Big]$$

$$= e^{-\frac{q^2}{4}}\int dr\; r\,\underbrace{e^{-r^2+iqr+\frac{q^2}{4}}}_{e^{-(r-\frac{iq}{2})^2}}$$

$$\underbrace{\qquad\qquad\qquad}$$

$$\int dr\; r\, e^{-(r-\frac{iq}{2})^2}$$

$\rangle$ $r \to \left(r + \frac{iq}{2}\right)$; $dr \to dr$ (q const.)

$$\int dr\, e^{-r^2} + \frac{iq}{2} \int dr\, e^{-r^2}$$

$\left) r \to \left(r + \frac{iq}{2}\right);\ dr \to dr \right.$ (q const.)

$$= \frac{\sqrt{\pi}}{2} \qquad \int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

$$= \frac{\sqrt{\pi}}{2}\left(1 + \frac{iq}{2}\right)$$

$$= \frac{4\pi}{q}\, \mathrm{Im}\left[e^{-q^2/4} \frac{\sqrt{\pi}}{2}\left(1 + i\frac{q}{2}\right)\right]$$

$$= \frac{\cancel{4}\pi}{\cancel{q}}\, e^{-q^2/4} \frac{\sqrt{\pi}}{\cancel{2}}\left(+\frac{\cancel{q}}{\cancel{2}}\right)$$

$$= \pi^{3/2}\, e^{-q^2/4}$$

$$\Rightarrow \boxed{FT\left[e^{-r^2}\right] = F(\vec{q}) = \pi^{3/2}\, e^{-q^2/4}}$$

Now, (b):

$$\boxed{FT\left[f(a\vec{y})\right] = \frac{1}{a^3} F\left(\frac{\vec{q}}{a}\right)}$$

Shows scaling behavior, so:

Since $f(r) = e^{-r^2} \Rightarrow e^{-r^2/b^2} = f(r/b)$

$$FT\left[e^{-r^2/b^2} \right] = b^3 \, F(b\vec{q})$$

$$= b^3 \pi^{3/2} e^{-b^2 q^2/4}$$

problem-1f

Tuesday, January 14, 2020 4:32 PM

$$\lim_{b\to 0} \underbrace{\frac{1}{b^3 \pi^{3/2}} e^{-r^2/b^2}}_{\text{Gaussian}} =: \delta^{(3)}(\vec{r})$$

$$\Rightarrow \mathrm{FT}\left[\delta(\vec{r})\right] \propto \lim_{b\to 0} \underbrace{\mathrm{FT}\left[e^{-r^2/b^2}\right]}_{= b^3 \pi^{3/2} e^{-q^2 b^2/4}}$$

$$\Rightarrow \mathrm{FT}\left[\delta(\vec{r})\right] = \lim_{b\to 0} e^{-q^2 b^2/4} \to 1 .$$

problem-1g

Tuesday, January 14, 2020 4:34 PM

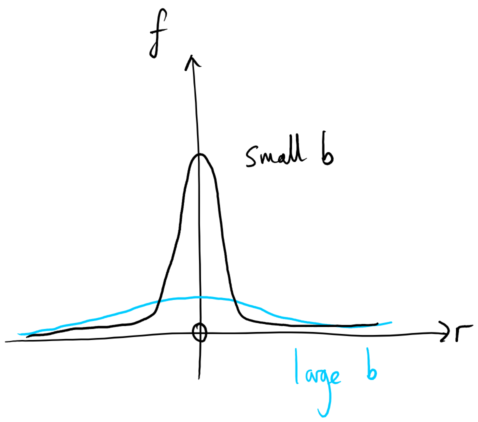
(e): $\boxed{FT\left[e^{-r^2/b^2}\right] = b^3 \pi^{3/2} e^{-b^2q^2/4}}$

$\Rightarrow$ for $f(r) = \frac{1}{b^3\pi^{3/2}} e^{-r^2/b^2}$; $FT[f(r)] = e^{-b^2q^2/4} =: g(q)$

<u>Sketch</u>

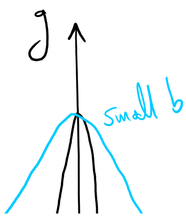
$\rightarrow$ $f(r) = \underbrace{\frac{1}{b^3\pi^{3/2}}}_{\text{amplitude}} \underbrace{e^{-r^2/b^2}}_{\text{sharpness}}$

b: amplitude sharpness

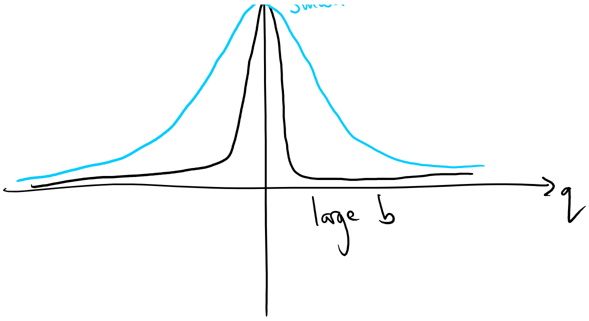


So $\lim_{b\to 0} f(r)$ will look like $\delta(r)$

$\rightarrow$ FT: $g(q) = \underbrace{e^{-b^2q^2/4}}_{b:\ \text{sharpness}}$



So $\lim_{b\to 0} g(q)$ will look flat.


large b
q

problem-1a

Saturday, January 18, 2020 2:18 PM

$$\frac{d\sigma}{d\theta} = \frac{\pi}{2} R^2 \sin\theta$$

$$\sigma = \iint d^2\sigma = \iint \frac{d^2\sigma}{d^2\Omega} d^2\Omega$$

$$= \iint \frac{d^2\sigma(\theta)}{\cancel{\sin\theta}\, d\theta\, \cancel{d\phi}} \cancel{\sin\theta}\, d\theta\, \cancel{d\phi}$$

$$= \int \frac{d\sigma}{d\theta} d\theta$$

$$= \frac{\pi}{2} R^2 \underbrace{\int_0^{\pi} \sin\theta \, d\theta}_{= -\cos\pi + \cos 0 = +1 + 1 = 2}$$

$$= \pi R^2$$

problem-1b

Saturday, January 18, 2020 2:29 PM

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

$$= \frac{e^4 Z^2 z^2}{4E^2} \int \frac{2\pi \sin\theta \, d\theta}{\sin^4(\theta/2)}$$

$$= \frac{e^4 Z^2 \pi z^2}{2E^2} \underbrace{\int_0^{2\pi} d\theta \frac{\sin\theta}{\sin^4(\theta/2)}}$$

$$= \left[\frac{4}{\cos\theta - 1} \right]_0^{2\pi}$$

$$= \frac{4}{1-1} - \frac{4}{1-1}$$

$$= 4\left(\frac{1}{0} - \frac{1}{0} \right)$$

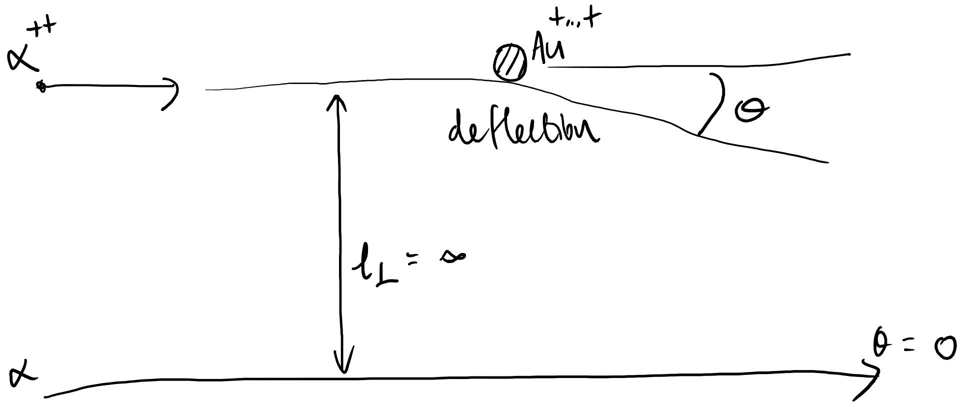
$\Rightarrow$ divergence at $\cos\theta = 1$

$\Rightarrow$ $\theta = n2\pi$, $n \in \mathbb{W}$

physically, this is just $\theta = 0$

$\Rightarrow$ forward scattering

divergence arises since EM force has infinite range, so we include all particles that never actually scatter (to measurable degree)



and there is infinite space away from atom

$\Rightarrow$ divergence.

problem-2a

Saturday, January 18, 2020 3:05 PM

$$\phi(\underline{x}) = \frac{Ze}{|\underline{x}-\underline{x}_0|} e^{-M|\underline{x}-\underline{x}_0|} = \frac{Ze}{r} e^{-Mr}$$

New Coulomb potential

$$[-Mr] = \emptyset \quad (\text{dim}^{\text{n}}\text{-less since exp. arg.})$$

$$\Rightarrow [M] = [r]^{-1} = [E] \quad \text{in nat. units}$$

$$E = \frac{hc}{\lambda}$$

problem-2b

Saturday, January 18, 2020 3:11 PM

$$\lim_{M\to 0} \phi(r) = \frac{Ze}{r} e^{-Mr}\Big|_{M\to 0} = \frac{Ze}{r} =: \phi_c$$

$=$ Coulomb pot. $\left(\vec{F} = q\vec{\nabla}\phi = -q\frac{Ze}{r^2}\hat{r}\right)$

$$\left.\begin{aligned} Mr < 1 &: 0.4 < e^{-Mr} < 1 \Rightarrow e^{-Mr} < 1 \\ Mr \geq 1 &: -\infty < e^{-Mr} < 0.4 \Rightarrow e^{-Mr} \ll 1 \end{aligned}\right] \quad \phi \searrow$$

So for M const.,

r small gives $\phi \approx \phi_c$

but r large $\Rightarrow$ $\phi \ll \phi_c$ - supression

$\left(r \geq \frac{1}{M}\right)$

problem-2c

Saturday, January 18, 2020 3:26 PM

$$M_{fi} = \frac{4\pi e^2 z Z}{V} e^{i\vec{q}\cdot\vec{x}_0} \left(|\vec{q}|^2 + M^2\right)^{-1}$$

$M \gg |\vec{q}|$

$$\Rightarrow \left(|\vec{q}|^2 + M^2\right)^{-1} = \left[M^2\left(1 + \frac{|\vec{q}|^2}{M^2}\right)\right]^{-1}$$

$$\rightarrow \frac{1}{M^2}\left(1 - \frac{|\vec{q}|^2}{M^2}\right)$$

$$\rightarrow M_{fi} \rightarrow \frac{4\pi e^2 z Z}{V} \frac{e^{i\vec{q}\cdot\vec{x}_0}}{M^2} + O\left(\frac{|\vec{q}|^2}{M^4}\right)$$

problem-2d

Saturday, January 18, 2020 3:31 PM

$$\frac{d\sigma}{d\Omega} = \frac{4e^2 z^2 Z^2 E'^2}{M^4}$$

σ unchanged by M_{fi} change

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

$$= \frac{4e^2 z^2 Z^2 E'^2}{M^4} \int d\Omega$$

$$= \frac{16\pi e^2 z^2 Z^2 E'^2}{M^4}$$

$$= \pi R_{eff} \quad \text{with } R_{eff} := \frac{16 e^2 z^2 Z^2 E'^2}{M^4}$$

limit $|\vec{q}| \ll M$ (small mom. $\Leftrightarrow$ large λ)

$\vec{q}$: momentum transfer (small r_{min} = large q)

$\vec{q}$: momentum transfer (small r_{min} = large $\vec{q}$)

$\Rightarrow$ $r \gg r_{min}$

ie no probing for $r < \frac{1}{M}$ (Coulomb pot.)

only for $r > \frac{1}{M}$ (exp. decay pot.)

(large λ)

So low $|\vec{q}|$ projectile doesn't see charged target. Sees no target, or see unchaged target.

problem-2e

Saturday, January 18, 2020 3:43 PM

$\hbar c = 200$ MeV fm.

$M = 100$ MeV

Coulomb behaviour for $r < \frac{1}{M}$

Need small λ: $\lambda < r$

limit: $\lambda_{max} = \frac{1}{M} = \frac{1}{100\text{ MeV}}$ (natural units)

$= \frac{\hbar c}{100\text{ MeV}}$ (SI units)

$= \frac{200\text{ MeV fm}}{100\text{ MeV}}$

$= 2\text{ fm} = 2 \times 10^{-15}\text{ m}.$

n.b.

de Broglie λ: $\lambda = \frac{h}{p}$

$E = pc \Rightarrow p = \frac{E}{c}$

$\Rightarrow \lambda = \frac{hc}{E}$

problem-1a

Friday, January 24, 2020 10:41 AM

$$\Delta\phi(r) = -\rho(r) \qquad \rho(r) = 4\pi\,\delta(r) \qquad \Rightarrow \qquad \boxed{\Delta\phi(r) = -4\pi\,\delta(r)}$$

Ansatz: $\phi(r) = \frac{1}{r}$

Assume $r \neq 0$:

Insert in LHS: $\Delta\frac{1}{r} = \nabla^2\frac{1}{r} = \vec{\nabla}\cdot\vec{\nabla}\frac{1}{r} = \vec{\nabla}\cdot\underbrace{\left(\vec{\nabla}\frac{1}{r}\right)}$

$(\vec{r} = \vec{x})$

Cartesians: $r = |\vec{x}|$, $\vec{x} = (x_1, x_2, x_3)$, $|\vec{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2} = \left(\sum_{j=1}^{3} x_j\right)^{1/2}$

& $\vec{\nabla} = (\partial_1, \partial_2, \partial_3)$ where $\partial_j = \frac{\partial}{\partial x_j}$

$$\Rightarrow \nabla_i \frac{1}{r} = \frac{1}{\sqrt{\sum_{j=1}^{3} x_j^2}} = \partial_i \left(\sum_j x_j^2\right)^{-1/2} = -\frac{1}{\cancel{2}}\left(\sum_j x_j^2\right)^{-3/2}(\cancel{2}x_i)$$

$$= -\frac{x_i}{|\vec{x}|^3}$$

$$\Rightarrow \vec{\nabla}\frac{1}{r} = -\frac{\vec{x}}{|\vec{x}|^3} = -\frac{\vec{r}}{r^3} \qquad - (\dagger)$$

(Alternatively in spherical polars: $\nabla\frac{1}{r} = \frac{\partial}{\partial r}\frac{1}{r}\hat{r} = -\frac{1}{r^2}\hat{r}$)

angle coords vanish

$$\Rightarrow \Delta\frac{1}{r} = \vec{\nabla}\cdot\left(-\frac{\vec{x}}{r^3}\right)$$

$$= -\frac{1}{r^3}\underbrace{\left(\vec{\nabla}\cdot\vec{x}\right)} - \underbrace{\left(\underbrace{\vec{\nabla}\frac{1}{r^3}}\right)\cdot\vec{x}}$$

$$\nabla_i \frac{1}{r^3} = \partial_i \left(\sum_j x_j^2\right)^{-3/2} = -\frac{3}{\cancel{2}}\left(\sum_j x_j^2\right)^{-5/2}\cancel{2}x_i$$

$$\Rightarrow \vec{\nabla}\frac{1}{r^3} = -3\,\frac{\vec{x}}{r^5}$$

$$\vec{\nabla}\cdot\vec{x} = \sum_i \nabla_i x_i = \sum_i 1 = 3$$

$$+3\,\frac{\vec{x}}{r^5}\cdot\vec{x} = 3\,\frac{|\vec{x}|^2}{r^5} = 3/r^3$$

$$\Rightarrow \quad \Delta\frac{1}{r} = -\frac{3}{r^3} + \frac{3}{r^3} = 0 \quad //$$

agrees with $\delta(r)$

Assume $r=0$: $\quad \frac{1}{r} \to \infty$: seems sensible for $\delta(r)$.

problem-1b

Friday, January 24, 2020 12:30 PM

$$\int \Delta \frac{1}{r} \, dV$$

$$= - \int \vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3} \right) dV$$

divergence theorem:

$$\int_V \vec{\nabla} \cdot \vec{F} \, dV = \oint_{\partial V} \vec{F} \cdot d\vec{S}$$

$$= - \int \frac{\vec{r}}{r^3} \cdot d\vec{S}$$

in spherical polar co-ordinates, we need surface of sphere

$$\Rightarrow d\vec{S} = r^2 \, d\cos\theta \, d\phi$$

$$= - \int \frac{\vec{r}}{r^3} \cdot \hat{r} \, r^2 \, d\cos\theta \, d\phi$$

normal vector to surface

$$= -4\pi \quad //$$

correct volume : $\int -4\pi \delta(r) \, dV = -4\pi$

problem-2a

Friday, January 24, 2020 1:51 PM

$a =$ Gaussian width

$$f(r) = f_0 \left[1 + \frac{(Z-2) r^2 a^2}{6} \right] e^{-\frac{a^2 r^2}{2}} \quad \text{w/ } f_0 \text{ s.t. } 4\pi \int_0^\infty f(r)\, r^2\, dr = 1$$

$$\Rightarrow 4\pi f_0 \int \left[1 + \frac{(Z-2) r^2 a^2}{6} \right] e^{-\frac{a^2 r^2}{2}} r^2\, dr = 1$$

$$\Rightarrow \frac{1}{f_0} = 4\pi \left\{ \int dr\, r^2 e^{-a^2 r^2/2} + \frac{Z-2}{6} a^2 \int dr\, r^4 e^{-\frac{a^2}{2} r^2} \right\}$$

$$= 4\pi \left\{ \frac{\sqrt{2\pi}}{2a^3} + \frac{Z-2}{6} a^2 \frac{3\sqrt{2\pi}}{2a^5} \right\}$$

$$= 2 \frac{\cancel{4}\pi \sqrt{2\pi}}{\cancel{2} a^3} \left\{ 1 + \frac{Z-2}{\cancel{6}\,2} \cancel{3} \right\}$$

$$= \frac{(2\pi)^{3/2}}{a^3} \left[1 + \frac{Z-2}{2} \right]$$

$$= \frac{Z}{2} \left(\frac{\sqrt{2\pi}}{a} \right)^3$$

$$= \frac{2^{3/2} \pi^{3/2}}{2 a^3} Z$$

$$= \sqrt{2\pi^3} \frac{Z}{a^3} \qquad \Rightarrow f_0 = \frac{a^3}{Z} \frac{1}{\sqrt{2\pi^3}}.$$

$$= \sqrt{2\pi^3}\,\frac{Z}{a^3} \qquad \Rightarrow \qquad f_0 = \frac{a}{Z}\sqrt{2\pi^3}$$

$$\Rightarrow \quad f(r) = \frac{a^3}{Z}\,\frac{1}{\sqrt{2\pi^3}}\left[1 + \frac{(Z-2)r^2a^2}{6}\right]e^{-\frac{a^2r^2}{2}}$$

problem-2b

Friday, January 24, 2020 1:56 PM

$$F(|\underset{\sim}{q}|^2) = 4\pi \int f(r) \frac{\sin(|\underset{\sim}{q}|r)}{|\underset{\sim}{q}|r} r^2\, dr \left[1 + \frac{(Z-2)r^2a^2}{6}\right] e^{-a^2r^2/2}$$

$$= \frac{a^3}{Z} \frac{4\pi}{\sqrt{2\pi^3}} \frac{1}{|\underset{\sim}{q}|} \int \sin(|\underset{\sim}{q}|r)\, r\, dr \left[1 + \frac{(Z-2)r^2a^2}{6}\right] e^{-a^2r^2/2}$$

$$\frac{2^2}{2^{1/2}} \frac{\pi}{\pi^{3/2}}$$

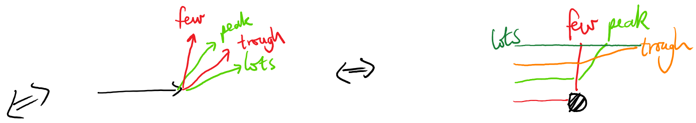
$$= \frac{a^3}{|\underset{\sim}{q}|Z} \frac{2^{3/2}}{\pi^{1/2}} \left[\int \sin(|\underset{\sim}{q}|r)\, r\, e^{-\frac{a^2}{2}r^2} dr + \frac{(Z-2)a^2}{6} \int \sin(|\underset{\sim}{q}|r)\, r^3 e^{-a^2r^2/2} dr\right]$$

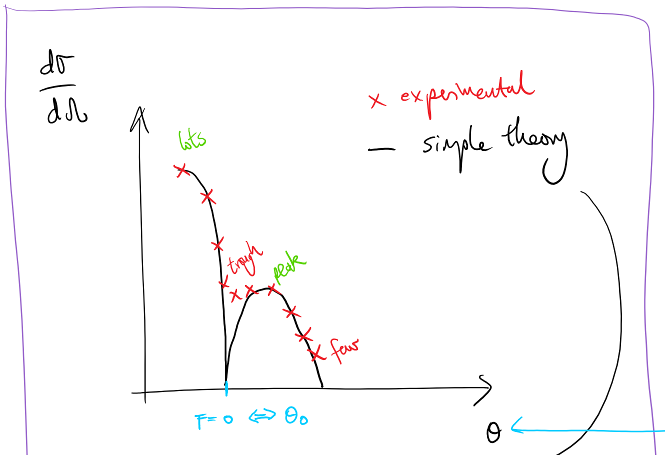
$$= \frac{\cancel{a^3}}{\cancel{|\underset{\sim}{q}|}Z} \frac{2^{\cancel{3/2}\,\cancel{1/2}\,1}}{\cancel{\pi^{1/2}}} \left[\frac{\cancel{|\underset{\sim}{q}|}\sqrt{\cancel{2}\cancel{\pi}}}{\cancel{2}\cancel{a^3}} e^{-\frac{|\underset{\sim}{q}|^2}{2a^2}} + \frac{(Z-2)\cancel{a^2}}{6} \frac{\cancel{|\underset{\sim}{q}|}\sqrt{\cancel{2}\cancel{\pi}}}{\cancel{2}a^{\cancel{7}\cancel{4}2}} (3a^2 - |\underset{\sim}{q}|^2)\, e^{-\frac{|\underset{\sim}{q}|^2}{2a^2}}\right]$$

$$= \frac{2}{Z} e^{-|\underset{\sim}{q}|^2/2a^2} \left[1 + \frac{(Z-2)}{6a^2}(3a^2 - |\underset{\sim}{q}|^2)\right]$$

problem-2c

Friday, January 24, 2020 1:56 PM



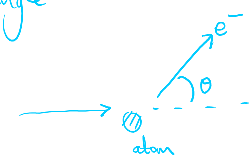


$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\text{point-like}}}{d\Omega} \cdot |F|^2$$

We're picking the $F=0$ point in parameter space purely for convenience to fix the equation, not for physical reasons!

ie we know $F=0 \Leftrightarrow$ first min. of $\frac{d\sigma}{d\Omega} \Rightarrow \theta_0$

scattering angle

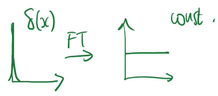


$$F \overset{!}{=} 0 \quad \Rightarrow \quad 1 + \frac{(Z-2)}{6a^2}\left(3a^2 - |\vec{q}|^2\right) = 0$$

form factor

$$\Rightarrow \quad 3a^2 - |\vec{q}|^2 = -\frac{6a^2}{Z-2}$$

c.f.



$$\Rightarrow \quad -|\vec{q}|^2 = \left(-\frac{6}{Z-2} - 3\right)a^2$$

$$\rightarrow \quad |\vec{q}|^2 = 3\underbrace{\left(\frac{2}{Z-2} + 1\right)}_{\frac{Z}{Z-2}} a^2$$

$$\Rightarrow \quad a^2 = \frac{Z-2}{3Z}|\vec{q}|^2$$

a is "characteristic radius" of atom

$$\Rightarrow \quad a = \sqrt{\frac{Z-2}{3Z}}\,|\vec{q}| \qquad (Z>2)$$

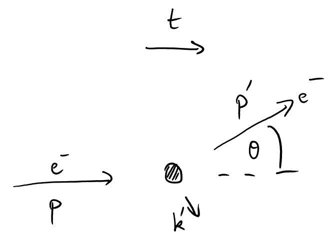
mass: 0.5 MeV

limit

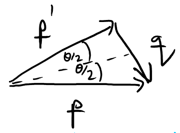
electron mass: 0.5 MeV
so this is high energy limit
($m_e << E$)

Consider scattering of 374.5 MeV electrons of some nucleus:

momentum transfer: $q = p - p'$



ie.



energy of e^- unchanged by scattering

assume: elastic scattering $\Rightarrow E = E'$ (for e^-).

e^- vs. nucleus.

($E >> m \Rightarrow |p| = E$ $(= \sqrt{\cancel{m^2} + |p|^2})$)

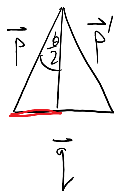
relativistic e^-'s

& similarly, $|p'| = E'$

assume no nuclear recoil since $m_N >> m_{e^-}$

$\Rightarrow |p| = |p'|$

$\Rightarrow$



$\Rightarrow \frac{|q|}{2} = |p| \sin(\theta/2)$

$\Rightarrow |q| = 2|p| \sin(\theta/2)$

$$\boxed{|q| = 2E \sin(\theta/2)}$$

We take the θ for the $\frac{d\sigma}{d\Omega}$ minimum, θ_0.

as we'll find a for $F = 0$: form factor F ($F = 1$ for point-like & reduced otherwise) (Fourier tr. of $\rho(\underline{r})$)

So $F = 0 \Leftrightarrow$ min $d\sigma$. ie. since $d\sigma = d\sigma_{\text{point-like}} \cdot |F|^2$ so $F = 0 \Leftrightarrow$ "$d\sigma = 0$" $\rightsquigarrow$ min. $d\sigma$
(never really $F = 0$ experimentally, just because we have a simple model of the $\rho(\underline{r})$ of the nucleus & $F = F.T.(\rho)$.

Graph:

	${}^{12}_{6}C$	${}^{16}_{8}O$
$\frac{d\sigma}{d\Omega}$ min. at: $\theta_0 =$	$58°$	$50°$

dΩ

$$|\underline{q}| = 2E \sin(\theta/2) =$$

$$2 \cdot 374.5 \cdot \sin(29^\circ)$$

$$= 363.1 \text{ MeV}$$

$$2 \cdot 374.5 \cdot \sin(25^\circ)$$

$$= 316.5 \text{ MeV}$$

Z

6

8

$$\underline{a} = \sqrt{\left(1 - \frac{2}{Z}\right)\frac{1}{3}}\, |\vec{\underline{q}}| =$$

$$\frac{\sqrt{2}}{3}\, 363.1 = \underline{171.2 \text{ MeV}}$$

$$\frac{1}{2}\, 316.5 = \underline{\underline{158.3 \text{ MeV}}}$$

problem-2d

Tuesday, 13 November, 2018 20:05

$$\langle r^2 \rangle = 4\pi \int r^2 f(r)\, r^2\, dr$$

$$= 4\pi \frac{a^3}{Z} \frac{1}{\sqrt{2\pi^3}} \int dr\, r^4 \left[1 + \frac{(Z-2) r^2 a^2}{6} \right] e^{-a^2 r^2/2}$$

$$= \frac{a^3}{Z} \frac{2^{3/2}}{\pi^{1/2}} \left[\int dr\, r^4 e^{-\frac{a^2}{2} r^2} + \frac{(Z-2)a^2}{6} \int r^6 e^{-\frac{a^2}{2} r^2} dr \right]$$

$$= \frac{a^3}{Z} \frac{2^{3/2}}{\cancel{\pi}^{1/2}} \left[\frac{3\sqrt{2\cancel{\pi}}}{2a^5} + \frac{(Z-2)a^2}{6} \frac{15\sqrt{2\cancel{\pi}}}{2a^7} \right]$$

$$= \frac{2}{Z} \left[\frac{3}{a^2} + \frac{(Z-2)15}{6a^2} \right]$$

$$= \frac{6}{a^2 Z} \left[1 + (Z-2)\frac{5}{6} \right]$$

$$= \frac{6}{a^2 Z} \left[1 + \frac{5}{6} Z - \overset{5/3}{\cancel{\frac{10}{6}}} \right]$$

$$= \frac{6}{a^2 Z} \left[\frac{5}{6} Z - \frac{2}{3} \right]$$

$$= \frac{5Z - 4}{a^2 Z}$$

n.b.// de Broglie wavelength: (also E-eq for photon)

$$\lambda = \frac{h}{p} = \frac{hc}{E}$$

so convert by $\times hc$

use $\hbar c$ (why?) w/ $\hbar c = 1.973269804 \times 10^2$ MeV fm

E was ν?

$^{12}_{6}C$: $Z = 6$,

$$\rightarrow \langle r^2 \rangle = \frac{13}{\ } \frac{1}{\ } = 1.479 \times 10^{-4}\ \text{MeV}^{-2} = 5.758\ \text{fm}^2$$

$$\Rightarrow \quad \langle r^2\rangle_C = \frac{13}{3}\frac{1}{\underbrace{171.2^2}_{a \text{ in MeV}}} = 1.479 \times 10^{-4}\ \text{MeV}^{-2} \xrightarrow{\times(\hbar c)^2} 5.758\ \text{fm}^{-2}$$

$$\Rightarrow \quad \sqrt{\langle r^2\rangle_C} = 1.216 \times 10^{-2}\ \text{MeV}^{-1} \xrightarrow{\times \hbar c} 2.400\ \text{fm}$$

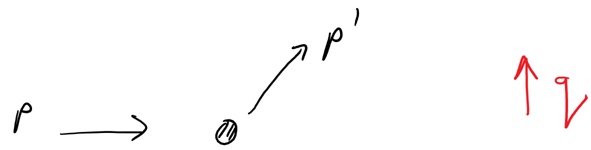
$$\ell\ \text{MeV}^{-1} \rightarrow \ell\ \hbar c$$

$${}^{16}_{8}O : \quad \langle r^2\rangle_O = \frac{9}{2}\frac{1}{158.3^2} = 1.796 \times 10^{-4}\ \text{MeV}^{-2} \xrightarrow{\times(\hbar c)^2} 6.995\ \text{fm}^2$$

$$\Rightarrow \quad \sqrt{\langle r^2\rangle_O} = 1.340 \times 10^{-2}\ \text{MeV}^{-1} \xrightarrow{\times \hbar c} 2.645\ \text{fm}$$

problem-1a

Monday, February 3, 2020 12:42 PM



$$Q^2 = -q^2$$

$$= -(p-p')^2$$

$$= -\begin{pmatrix} E-E' \\ 0 \\ -E'\sin\theta \\ E - E'\cos\theta \end{pmatrix}^2 \quad \downarrow \text{ Minkowski product}$$

$$= -\left[(E-E')^2 - E'^2\sin^2\theta - (E-E'\cos\theta)^2\right]$$

$$= -\left[E^2 + E'^2 - 2EE' - E'^2\sin^2\theta - E^2 + 2EE'\cos\theta - E'^2\cos^2\theta\right]$$

$$= -\left[\qquad -2EE'(1-\cos\theta)\right]$$

$$= 2EE'(1-\cos\theta)$$

$$= 2EE'\left(1 - 1 + 2\sin^2\theta/2\right)$$

$$= 4EE'\sin^2(\theta/2)$$

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

$$\Rightarrow \frac{1}{2}\cos 2\theta = \frac{1}{2} - \sin^2\theta$$

$$\Rightarrow \cos 2\theta = 1 - 2\sin^2\theta$$

problem-1b

Monday, February 3, 2020 12:52 PM

Mom cons. $p + k = p' + k'$

$\Rightarrow p - p' = k' - k$

Now, $p - p' = q \Rightarrow q = k' - k$

$\Rightarrow k' = q + k$

Then $k'^2 = (q+k)^2 = q^2 + k^2 + 2q.k$

But $k^2 = E_k^2 = M_p^2$, & $k'^2 = E_k'^2 - |\vec{k}'|^2 = M_p^2$

$\Rightarrow \cancel{M_p^2} = q^2 + \cancel{M_p^2} + 2q.k$

$\Rightarrow -q^2 = 2q.k$

$\Rightarrow Q^2 = 2q.k$

$$\Rightarrow \qquad Q^2 = 2q \cdot k$$

Also:

$$p - p' = \begin{pmatrix} E - E' \\ 0 \\ -E'\sin\theta \\ E - E'\cos\theta \end{pmatrix} \qquad k = \begin{pmatrix} E_k \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad E_k = M_p$$

$$q \cdot k = (p - p') \cdot k = (E - E')\, M_p$$

$$\Rightarrow \qquad Q^2 = 2\,(E - E')\, M_p$$

$$\Rightarrow \qquad \frac{Q^2}{2M_p} = E - E'$$

problem-1c

Monday, February 3, 2020 2:37 PM

Have $Q^2 = 4EE' \sin^2(\theta/2)$ — (a)

& $Q^2 = 2(E-E')M_p$ — (b)

$$\Rightarrow \quad 2(E-E')M_p = 4EE' \sin^2(\theta/2)$$

$$\Rightarrow \quad 2EM_p - 2E'M_p - 4EE' \sin^2\theta/2 = 0$$

$$\Rightarrow \quad E'\left(-2M_p - 4E\sin^2\theta/2\right) = -2EM_p$$

$$\Rightarrow \quad \frac{E'}{E} = \frac{M_p}{M_p + 2E\sin^2\theta/2}$$

$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$\Rightarrow 2\sin^2\theta/2 = 1 - \cos\theta$

$$\Rightarrow \quad \frac{E'}{E} = \frac{M_p}{M_p + E(1-\cos\theta)}$$

$E \ll M_p$: $\frac{E'}{E} \approx 1$. $\iff$ no recoil

e^- negligible c.f. proton

e^- en.

$\theta \to 0$: $\frac{E'}{E} \approx 1$ $\iff$ forward scattering (no recoil because no scattering)

problem-1d

Monday, February 3, 2020 2:44 PM

$$q = \begin{pmatrix} E - E' \\ \vec{q} \end{pmatrix} \qquad q^2 = (E-E')^2 - |\vec{q}|^2 = -Q^2$$

Recall: $\dfrac{Q^2}{2M_p} = E - E' \Rightarrow (E-E')^2 = \dfrac{Q^4}{4M_p^2}$

$$\Rightarrow \quad -Q^2 = \frac{Q^4}{4M_p^2} - |\vec{q}|^2$$

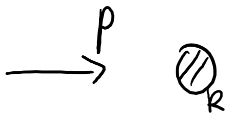
$$\Rightarrow \quad |\vec{q}|^2 = Q^2\left(1 + \frac{Q^2}{4M_p^2}\right)$$

$$\Rightarrow \quad |\vec{q}|^2 = Q^2\left[1 + \left(\frac{Q}{2M_p}\right)^2\right]$$

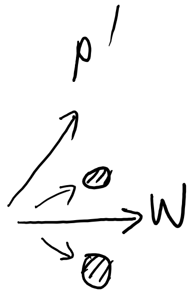
$$Q \ll 2M_p \Rightarrow |\vec{q}|^2 \approx Q^2$$

problem-2a

Monday, February 3, 2020 3:15 PM



then



a) uncharged: $q = p - p'$ ✓

b) charged: $W \neq k'$, $W^2 \neq M_p^2$

then $k'^2 \to W^2$ - diff.

problem-2b

Monday, February 3, 2020 3:25 PM

$$x = \frac{Q^2}{2M_p\nu} \qquad \nu = E - E'$$

$$\frac{Q^2}{2M_p} = \nu x \quad , \qquad Q^2 = 2EE'(1-\cos\theta)$$

elastic: $x = 1$

inelastic: $0 < x < 1$.

ϕ-ind. $\Rightarrow$ $d\Omega = 2\pi d\cos\theta$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2\sin^4\theta/2}\left[W_2\cos^2\theta/2 + 2W_1\sin^2\theta/2\right] \quad — (\dagger)$$

$$dy = \left|\frac{dy}{dx}\right| dx$$

$$\begin{vmatrix} \frac{dy_1}{dx_1} & \frac{dy_2}{dx_1} \\ \frac{dy_1}{dx_2} & \frac{dy_2}{dx_2} \end{vmatrix}$$

$$\frac{d^2\sigma}{d\Omega\, dE'} = \underbrace{\left|\frac{dQ^2 d\nu}{d\Omega\, dE'}\right| \left|\frac{dx}{d\nu}\right|}_{}\frac{d^2\sigma}{dQ^2\, dx}$$

changing a measure generates a Jacobian.

$$\left|\frac{dQ^2 d\nu}{d\Omega\, dE'}\right| = \left|\frac{dQ^2 d\nu}{(2\pi\, d\cos\theta)dE'}\right| = \begin{vmatrix} \frac{1}{2\pi}\frac{dQ^2(E',\theta)}{d\cos\theta} & \frac{1}{2\pi}\frac{d\nu(E',\theta)}{d\cos\theta} \\ \frac{dQ^2(E',\theta)}{dE'} & \frac{d\nu(E',\theta)}{dE'} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{2\pi}\frac{d}{d\cos\theta}\left[2EE'(1-\cos\theta)\right] & \frac{1}{2\pi}\frac{d}{d\cos\theta}\left[E-E'\right] \\ \frac{d}{dE'}\left[2EE'(1-\cos\theta)\right] & \frac{d}{dE'}\left[E-E'\right] \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{1}{2\pi}2EE' & 0 \\ & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{EE'}{\pi} & 0 \\ & -1 \end{vmatrix} = \frac{EE'}{\pi}$$

$$\& \quad \frac{dx}{d\nu} = \frac{d}{d\nu}\left[\frac{Q^2}{2M_p\nu}\right] = -\frac{Q^2}{2M_p\nu^2} = -\frac{Q^2}{2M_p}\frac{4M_p^2x^2}{Q^4} = -\frac{2M_px^2}{Q^2}$$

$$\nu = \frac{Q^2}{2M_px}$$

$$\Rightarrow \left|\frac{dx}{d\nu}\right| = \frac{2M_px^2}{Q^2}$$

$$\Rightarrow \frac{d^2\sigma}{d\Omega\, dE'} = \frac{EE'}{\pi}\frac{2M_px^2}{Q^2}\frac{d^2\sigma}{dQ^2\,dx}$$

problem-2c

Monday, February 3, 2020 4:03 PM

$$y := \frac{k\cdot q}{k\cdot p} = \frac{\cancel{M_p}(E-E')}{\cancel{M_p}E} \qquad q = \begin{pmatrix} E-E' \\ 0 \\ -E'\sin\theta \\ E-E'\cos\theta \end{pmatrix} \quad p = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \quad k = \begin{pmatrix} M_p \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= 1 - \frac{E'}{E}$$

$$\Rightarrow \frac{E'}{E}\sin^2\theta/2 = \frac{\cancel{E'}}{E}\frac{Q^2}{4E\cancel{E'}} = \left(\frac{Q}{2E}\right)^2 = \frac{2M_p(E-E')x}{4E^2} = \frac{2M_p x}{4E}y$$

$$s^2\theta/2 = \frac{Q^2}{4EE'} \qquad Q^2 = 2M_p(E-E')x$$

$$= \frac{\cancel{2}M_p x}{\cancel{4}E}y\,\frac{\cancel{2}M_p(E-E')x}{Q^2} = \frac{M_p^2x^2y^2}{Q^2}$$

$$\frac{E-E'}{E} = y$$

$$\frac{E'}{E}\cos^2\theta/2 = \frac{E'}{E}\left(1-\sin^2\theta/2\right) = \frac{E'}{E}\left(1-\frac{Q^2}{4EE'}\right) = \frac{E'}{E} - \frac{M_p^2x^2y^2}{Q^2}$$

$$= 1 - y - \frac{M_p^2x^2y^2}{Q^2}$$

problem-2d

Monday, February 3, 2020 4:14 PM

$$F_1(Q^2,x) = \nu W_1(Q^2,\nu)$$

$$F_2(Q^2,x) = M_p W_1(Q^2,\nu)$$

(b)$^{-1}$:

$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{Q^2}{2M_p x^2}\frac{\pi}{E'E}\frac{d^2\sigma}{d\Omega\,dE'}$$

(+)

$$= \frac{\pi Q^2}{2M_p x^2 E' E}\frac{\alpha^2}{4E^2 \sin^4\theta/2}\left(W_2\, c^2\theta/2 + 2W_1\, s^2\theta/2\right)$$

$$= \left(\frac{E'}{E}\right)^2 \frac{\pi Q^2}{2M_p x^2 E' E}\frac{\alpha^2}{4E^2\left(\frac{E'}{E}\sin^2\theta/2\right)^2}\frac{E}{E'}\left[W_2\left(\frac{E'}{E}c^2\theta/2\right) + 2W_1\left(\frac{E}{E'}s^2\theta/2\right)\right]$$

(c)

$$= \frac{\pi Q^2}{8M_p x^2 E'}\frac{\alpha^2}{E^3}\left(\frac{E'}{E}\right)^2\left(\frac{M_p^2x^2y^2}{Q^2}\right)^{-2}$$

$$\times\frac{E}{E'}\left[W_2\left(1-y-\frac{M_p^2x^2y^2}{Q^2}\right) + 2W_1\frac{M_p^2x^2y^2}{Q^2}\right]$$

$$= \frac{\pi Q^2\alpha^2}{8M_p x^2 \cancel{E'^2} E^2}\frac{\cancel{E'^2}}{E^2}\frac{Q^4}{M_p^4x^4y^4}$$

$$\times \frac{E}{E'}\left[W_2\left(1-y-\frac{M_p^2x^2y^2}{Q^2}\right)+2W_1\frac{M_p^2x^2y^2}{Q^2}\right]$$

$$=\underbrace{\frac{\pi Q^6\alpha^2}{8M_p^5x^6E^3y^4E'}}\left[W_2\left(1-y-\frac{M_p^2x^2y^2}{Q^2}\right)+2W_1\frac{M_p^2x^2y^2}{Q^2}\right]$$

$$Q^2=2M_p(E-E')x=2M_pEyx \quad -(*)$$

$$=\frac{\pi Q^4\alpha^2}{4M_p^4x^5E^2y^3E'}=\frac{\pi Q^2\alpha^2}{2M_p^3x^4Ey^2E'}=\frac{\pi\alpha^2}{M_p^2x^3yE'}$$

$$=\frac{4\pi\alpha^2}{Q^4}\underbrace{\frac{Q^4}{4}\frac{1}{M_p^2x^3yE'}}$$

$$\frac{Q^2E}{2M_px^2E'}=\frac{yE^2}{xE'}=\frac{E-E'}{E}\frac{E^2}{xE'}=\frac{E-E'}{x}\frac{E}{E'}$$

$$=\frac{\nu}{x}\frac{E}{E'}$$

$$=\frac{4\pi\alpha^2}{Q^4}\frac{E}{E'}\left[\frac{\nu}{x}W_2\left(1-y-\frac{M_p^2x^2y^2}{Q^2}\right)+\underbrace{\cancel{2}W_1\frac{M_p^{\cancel{2}}\cancel{x^2}y^{\cancel{2}}}{\cancel{Q^2}}\frac{Q^{\cancel{4}\,2}}{\cancel{2}\cancel{4}}\frac{1}{\cancel{M_p^2}x^{3}\cancel{y}E}}\right]$$

$$W_1\frac{yQ^2}{2xE}$$

$$\downarrow(*)$$

$$=W_1yM_p\frac{E-E'}{E}$$

$$\begin{cases}\nu W_2=:F_2\\ M_pW_1=:F_1\end{cases}$$

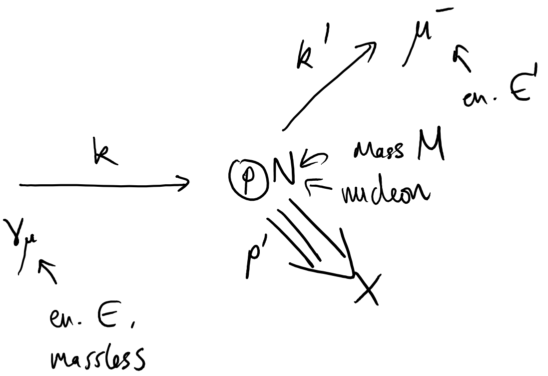
$= W_1 \, y^2 M_p$

$$= \frac{4\pi\alpha^2}{Q^4} \frac{E}{E'} \left[\frac{1}{x} F_2 \left(1 - y - \frac{M_p^2 x^2 y^2}{Q^2} \right) + y^2 F_1 \right]$$

problem-1a

1.

DIS



$s = (p+k)^2$

$q = k - k'$

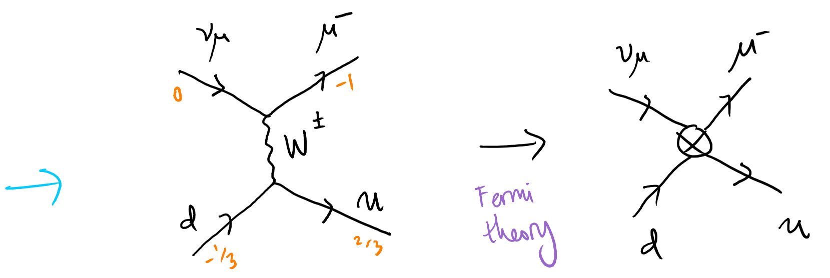
$Q^2 = -q^2$

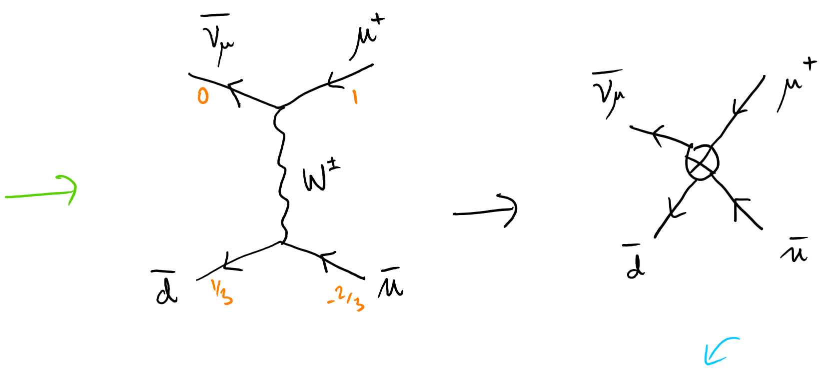
$x = Q^2 / 2P \cdot q$

init. $\quad p = \begin{pmatrix} M \\ \vec{0} \end{pmatrix} \qquad k = \begin{pmatrix} E \\ \vec{k} \end{pmatrix}, \quad |\vec{k}|^2 = E$

fin. $\quad p' = \begin{pmatrix} ? \end{pmatrix} \qquad k' = \begin{pmatrix} \epsilon' \\ \vec{k}' \end{pmatrix} \quad |\vec{k}'| \neq \epsilon'$

a) $\quad \sigma(\nu_\mu d) = \sigma(\bar{\nu}_\mu \bar{d}) = \dfrac{G_F^2 \, x \, s}{\pi} \qquad$ — (*).1





$$\sigma(\nu_\mu \bar{u}) = \sigma(\bar{\nu}_\mu u) = \frac{\sigma(\nu_\mu d)}{3} \quad — \quad (*).2$$

other combinations vanish.

$$\sigma(\nu_\mu N) = \int_0^1 dx \sum_{q \in \{u,d\}} \left[q_N(x)\, \sigma(\nu_\mu q) + \bar{q}_N(x)\, \sigma(\nu_\mu \bar{q}) \right]$$

$$= \int_0^1 dx \left[d_N(x)\, \sigma(\nu_\mu d) + \bar{u}_N(x)\, \sigma(\nu_\mu \bar{u}) \right]$$

average over nucleons, $N \in \{p, n\}$

$$= \frac{1}{2} \int_0^1 dx \left[\left(d_p(x) + d_n(x) \right) \sigma(\nu_\mu d) + \underbrace{\left(\bar{u}_p(x) + \bar{u}_n(x) \right)}_{\text{PDFs}} \underbrace{\sigma(\nu_\mu \bar{u})}_{\text{partonic cross-section}} \right]$$

problem-1b

Tuesday, February 11, 2020 10:42 AM

parton model:

isospin sym. $d_p(x) = u_n(x)$ ie $p = uud$, $n = udd$

$u_p(x) = d_n(x)$ & "ly for a.p.'s.

$$\Rightarrow \quad \sigma(\nu_\mu N) = \frac{1}{2}\int_0^1 dx \left[\left(d_p(x) + u_p(x)\right)\sigma(\nu_\mu d) + \left(\bar{u}_p(x) + \bar{d}_p(x)\right)\sigma(\nu_\mu \bar{u}) \right]$$

$\downarrow (*)$

$$\Rightarrow \quad \sigma(\nu_\mu N) = \frac{G_F^2 s}{2\pi} \underbrace{\int_0^1 dx \left[x(d_p + u_p)\right.}_{\substack{\text{average momentum}\\ \text{fraction carried by}\\ u \,\&\, d \text{ quarks}}} \left. + \frac{1}{3} x\left(\bar{u}_p + \bar{d}_p\right) \right]$$

ie. $\int dx \, x \, d_p := f_d$, etc.

$$\Rightarrow \quad \sigma(\nu_\mu N) = \frac{G_F^2 s}{2\pi} \left[\underbrace{(f_d + f_u)}_{= f_q} + \frac{1}{3} \underbrace{\left(\bar{f}_u + \bar{f}_d\right)}_{= f_{\bar{q}}} \right]$$

$$\Rightarrow \quad \sigma(\nu_\mu N) = \frac{G_F^2 s}{2\pi} \left(f_q + \frac{1}{3} f_{\bar{q}}\right)$$

& "ly for $\sigma(\bar{\nu}_\mu N)$.

problem-1c

Tuesday, February 11, 2020 10:50 AM

$$s = (p + k)^2$$

$$= \left[\begin{pmatrix} M \\ 0 \end{pmatrix} + \begin{pmatrix} E \\ \vec{k} \end{pmatrix} \right]^2$$

$$= \begin{pmatrix} M + E \\ \vec{k} \end{pmatrix}^2$$

$$= (M + E)^2 - \underbrace{|\vec{k}|^2}_{= E^2}$$

$$= M^2 + 2ME$$

$$\Rightarrow \quad s = 2EM + M^2$$

problem-1d

Tuesday, February 11, 2020 10:52 AM

$E >> M$:
$$\frac{\sigma(\nu_\mu N)}{E} \simeq 0.43 \frac{G_F^2 M}{\pi}$$
$\uparrow (= E_\nu)$

recall:
$$\sigma(\nu_\mu N) = \frac{G_F^2 s}{2\pi}\left(f_q + \frac{1}{3} f_{\bar{q}}\right)$$

$$\Rightarrow \quad E \; 0.43 \; M \; 2 = s \underbrace{\left(f_q + \frac{1}{3} f_{\bar{q}}\right)}_{=: f}$$

$$\Rightarrow \quad 0.86 \, EM = (2EM + M^2) f \qquad (1)$$

also:
$$\frac{\sigma(\bar{\nu}_\mu N)}{E} \simeq 0.21 \frac{G_F^2 M}{\pi}$$
$\uparrow (= E_{\bar{\nu}})$

&
$$\sigma(\bar{\nu}_\mu N) = \frac{G_F^2 s}{2\pi} \underbrace{\left(\frac{1}{3} f_q + f_{\bar{q}}\right)}_{=: \bar{f}}$$

$$\Rightarrow \quad E\, 0.42\, M = (2\epsilon M + M^2)\, \bar{f} \qquad — (2)$$

So,

$$\begin{cases} 0.86\ \epsilon M = (2\epsilon M + M^2)\left(f_q + \frac{1}{3} f_{\bar{q}}\right) & (1) \\ 0.42\ \epsilon M = (2\epsilon M + M^2)\left(\frac{1}{3} f_q + f_{\bar{q}}\right) & (2) \end{cases}$$

(1) − 3 (2) :

$$(0.86 - 1.26)\, \epsilon M = s\left(\frac{1}{3} - 3\right) f_{\bar{q}}$$

$$\underbrace{0.4}_{4/10}\ \epsilon M = s\ 8/3\ f_{\bar{q}}$$

$$f_{\bar{q}} = \frac{\epsilon M}{s}\ \frac{12}{80}$$

$$f_{\bar{q}} = 3/20\ \frac{\epsilon M}{s}$$

3 (1) − (2) :

$$(2.58 - 0.42)\, \epsilon M = s\left(3 - \frac{1}{3}\right) f_q$$

$$2.16\ \epsilon M = s\ 8/3\ f_q$$

$$f_q = \frac{\epsilon M}{s}\ \frac{3}{8} \cdot \underbrace{\frac{216}{100}}$$

$$\frac{108}{50} = 54/25$$

$$\frac{162}{200} = \frac{81}{100}$$

$$\Rightarrow \quad f_q = \frac{81}{100} \frac{EM}{S}$$

So $\quad f_q = 0.81 \frac{EM}{S} \quad ; \quad f_{\bar{q}} = 0.15 \frac{EM}{S}$

but $\quad E \gg M \Rightarrow S = M^2 + 2ME \rightarrow S \approx 2ME$

$$\Rightarrow \quad f_q = 0.405 \quad \& \quad f_{\bar{q}} = 0.075$$

problem-1e

Tuesday, February 11, 2020 11:07 AM

$$f_q + f_{\bar{q}} = 0.48 < 1.$$

rest of momentum carried by partons not interacting with the γ_μ

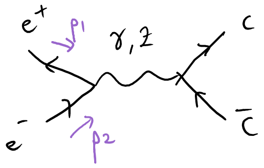
$\longrightarrow$ gluons.

problem-one

Monday, February 17, 2020 3:53 PM

a)

$e^+e^- \to c\bar{c}$:



b) $s = 100\ \text{GeV}^2$ $D = \frac{1}{s-m^2}$

$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

$g_Z = 1/10$

$\gamma : Q_c e^2 \frac{1}{s} = Q_c \frac{4\pi\alpha}{s} \approx \frac{2}{3} \frac{4\pi}{137} \frac{1}{100} \approx 10^{-3}$ ← dominant

$Z : g_Z^2 \frac{1}{s - M_Z^2} \approx \frac{1}{100} \frac{1}{100 - 91^2} \approx -10^{-6}$

c) J/ψ: spin-1 $c\bar{c}$ resonance (meson)

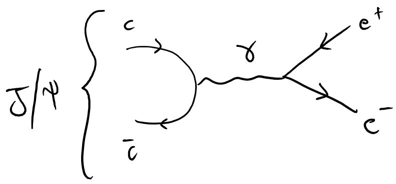
$m = 3096$ MeV

$J^P = 1^-$

order of descending prob.

- $J/\psi \to e^+e^-$

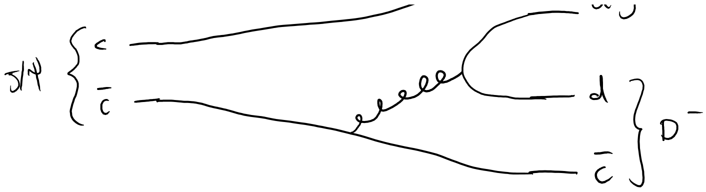
① EM int.



- $J/\psi \to D^+D^-$, $D^+ = c\bar{d}$, $D^- = \bar{c}d$ } $m = 1869$ MeV

u	c	t
d	s	b

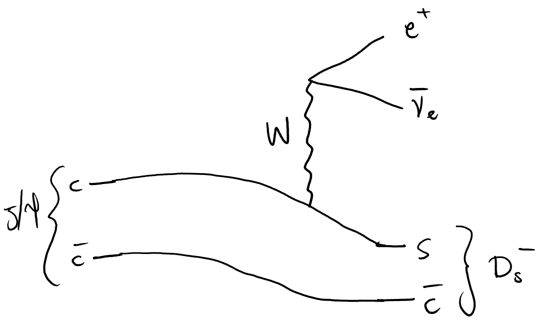
③ should be most likely since strong int.
but $m_F = 3738$ MeV $> m_I = 3096$ MeV



but $m_F = 3738$ MeV $> m_I$ 3096 MeV

• $J/\psi \to D_s^- \, e^+ \bar{\nu}_e$; $D_s^- = s\bar{c}$, $m = 1968$ MeV

$m_\mu = 105$ MeV



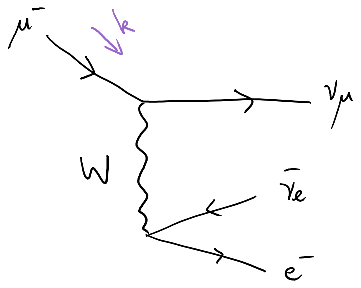
u c t
d s b

(2) weak int.

problem-two

Monday, February 17, 2020 4:14 PM

a)



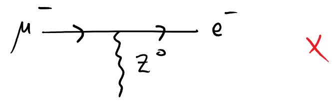
$m_\mu = 105$ MeV

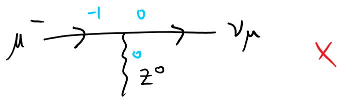
$$\mathcal{M} \propto \frac{1}{s - m_W}$$

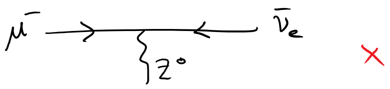
b) ν : no charge : no coupling to γ

leptons : no colour charge : " " " g.

need charge change : no Z^0, γ, g.







c) $\Gamma_\mu \propto |\mathcal{M}|^2$

$M_W \approx 80$ GeV

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$$\tau = \frac{\hbar c}{\Gamma_\mu} \approx 2.2 \times 10^{-6} \text{ s}$$

if $m_W = 800$ GeV, W prop. : $\dfrac{1}{s - 80^2} \to \dfrac{1}{s - 800^2}$

CoM : $k = 105$ MeV $\Rightarrow s = (105 \text{ MeV})^2 \ll (80 \text{ GeV})^2$

$$\Rightarrow \quad |\text{prop.}| \rightarrow \frac{1}{80^2} \rightarrow \frac{1}{800^2}$$

$$\text{ie.} \quad M \rightarrow \frac{M}{10^2}$$

$$\Rightarrow \quad \Gamma_\mu \rightarrow \frac{\Gamma_\mu}{10^4}$$

$$\Rightarrow \quad \tau \rightarrow 10^4 \, \tau. \qquad \text{ie} \quad \underline{2.2\times10^{-2}\ s}$$

d) $m_\tau = 1777 \text{ MeV} \Rightarrow m_I \gg m_F$

$$\Gamma_\tau(\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e) : \Gamma_\tau(\tau^- \rightarrow \mu^- \nu_\tau \bar{\nu}_\mu) : \Gamma_\tau(\tau^- \rightarrow \nu_\tau + \text{hadrons}) \approx 1:1:\underline{\underline{3}}$$