Monte Carlo Simulation of Multi-Jet Final States

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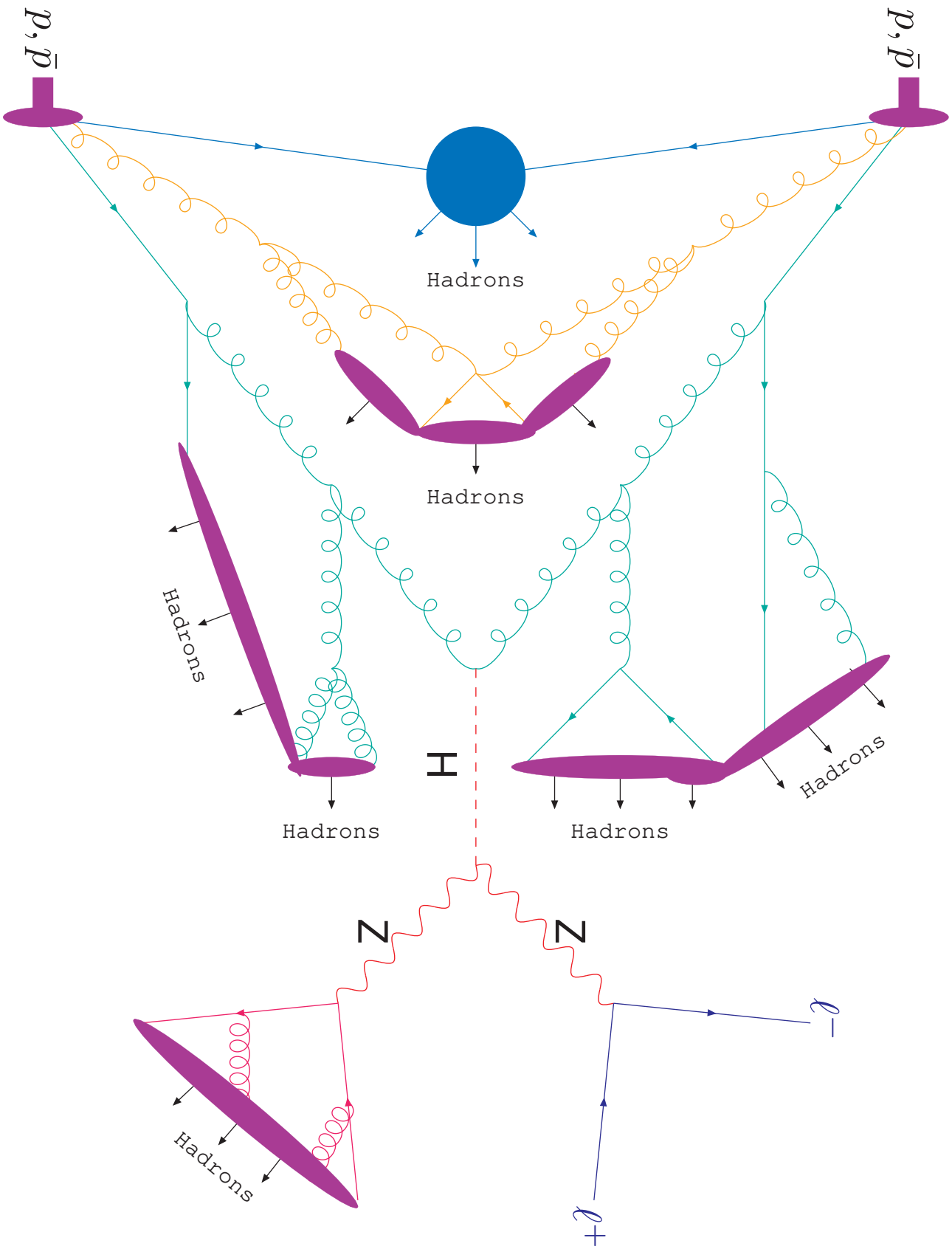
- Introduction
- Monte Carlo Event Generators
- Spin Correlations
- Examples
- Merging Matrix Elements and Parton Showers
- Preliminary Results
- Conclusions

Introduction

- Monte Carlo Simulations are an essential part of most experimental analysis, either of the Standard Model or for searches for new physics.

- There is a lot of interest in the simulation processes involving large numbers of outgoing jets/leptons with these programs as for example:
  - Top quark production including decays;
  - Signal Processes in BSM physics;
  - SM Electroweak gauge boson production with additional jets.

- In this talk I will discuss two sets of recent improvements to the simulation of these processes.
  - The first improves the simulation of processes where the jets come from the decay of heavy particles, *e.g.* the top quark.
  - The second improves the simulation of large number of hard QCD jets, *e.g.* W+jets.


$p, \bar{p}$
$p, \bar{p}$
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
H
Z
Z
ℓ^-
ℓ^+
Hadrons

Monte Carlo Event Generators

- The aim of a Monte Carlo event generator is to provide a simulation of the events seen in experiments based on theoretical predictions.
- In hadron collisions this means the simulation of QCD.
- The simulation is split into a number of phases:
  - Hard Process;
  - Parton Shower;
  - Secondary Decays;
  - Multiple Scattering/Soft Underlying Event;
  - Hadronization;
  - Hadron Decays.
- There are a number of programs available which use different models.
- The most commonly used are:
  - HERWIG;
  - ISAJET;
  - PYTHIA.

Parton Shower

- ISAJET uses the original parton shower algorithm which only resums collinear logarithms.
- HERWIG uses the angular ordered parton shower algorithm which resums both soft and collinear singularities.
- PYTHIA uses the collinear algorithm with an angular veto to try to reproduce the effect of the angular ordered shower.
- The collinear algorithm implemented in ISAJET does not give good agreement for angular ordering variables in hadron collisions or the string effect in e^+e^- collisions.
- HERWIG tends to give the best agreement for the hadron-hadron observables which are sensitive to soft effects.

Hadronization

- ISAJET uses the original independent fragmentation model of Feynman and Field.
- PYTHIA uses the Lund string model.
- HERWIG uses the cluster hadronization model.
- The independent fragmentation model cannot fit the LEP data.
- The cluster model gives good agreement with the LEP data on event shapes but doesn't fit the identified particle spectrum as well.
- The Lund string model give the best agreement with the LEP data.

Underlying Event

- In hadron-hadron collisions in addition to modelling the perturbative physics in the hard collision and parton showers we need to describe the underlying event.

- We can think of this as the interaction of the hadron remnants after the hard collision.

- The different event generators use different models.
  - ISAJET uses an approach based on pomeron exchange.
  - PYTHIA uses a model based on multiple parton collisions.
  - HERWIG uses a soft model based on the UA5 event generator, (there is also a multiple scattering model available.)

- Both the HERWIG and PYTHIA multiple scattering models can be tuned to give good agreement with CDF data.

Heavy Particle Decays

- After the parton shower phase those particles which decay before hadronization, *e.g.* the top quark and SUSY particles, are decayed.
- These decays are usually performed using matrix elements to give the distributions of the decay products.
- However the production and decay of the particles are performed independently.
- This neglects spin correlation effects which may be important.

Spin Correlations

- In the production of heavy particles there are correlations between the production and decay of the particles due to their spin.
- These correlations are known to be important in tau production.
- Run II of the Tevatron should have sufficient statistics to measure the correlations in $\mathrm{t\bar{t}}$ production.
- These correlations are also important for $\mathrm{t\bar{t}}$ production at a future linear collider.
- One potentially important application is in gaugino production at a future linear collider.
  - Here both spin and polarization effects are important.
  - This affects the distribution of the decay products.
  - These distributions can be used to measure the fundamental parameters of the model.

Spin Correlations

- There have been many studies of spin correlations
  - Top production in hadron collisions
    A. Brandenburg Phys. Lett. B338 (1996) 626.
  - Gaugino Production in $\mathrm{e}^+\mathrm{e}^-$ collisions
    G. Moortgat-Pick, *et.al*, Phys. Rev. D59 (1999) 015016, Eur. Phys. J. C7 (1999) 113, Eur. Phys. J. C9 (1999) 521, Eur. Phys. J. C18 (2000) 379.

- Most of these studies used the same formalism.

- In the limit that the width of a particle is small relative to its mass we can neglect the width and keep the particles on mass shell.

- We can write the matrix element for a $2 \to 2$ process, including decays, as

$$\mathcal{M}_{\text{total}} = \frac{1}{m_a\Gamma_a}\frac{1}{m_b\Gamma_b}\sum_{\lambda_c\lambda_d}\mathcal{M}^{\lambda_c\lambda_d}_{ab\to cd}\mathcal{M}^{\lambda_c}_{c\ \text{decay}}\mathcal{M}^{\lambda_d}_{d\ \text{decay}}$$

Spin Correlations

- This means the amplitude squared can be written as

$$|\mathcal{M}_{\text{total}}|^2 = \frac{1}{m_a^2\Gamma_a^2}\frac{1}{m_b^2\Gamma_b^2}\rho^{\lambda_c\lambda_c'\lambda_d\lambda_d'}D_c^{\lambda_c\lambda_c'}D_d^{\lambda_d\lambda_d'}$$

  where

$$\rho_{\text{prod}}^{\lambda_c\lambda_c'\lambda_d\lambda_d'} = \mathcal{M}_{ab\to cd}^{\lambda_c\lambda_d}\mathcal{M}_{ab\to cd}^{*\lambda_c'\lambda_d'},$$

  is the unnormalized spin density matrix, and

$$D_c^{\lambda_c\lambda_c'} = \mathcal{M}_{c\ \text{decay}}^{\lambda_c}\mathcal{M}_{c\ \text{decay}}^{*\lambda_c'},$$

  the unnormalized decay matrix.

- Most authors then chose to evaluate these matrices analytically.

- However it will be easier for our purposes to evaluate the matrix elements using helicity amplitude techniques and then calculate the matrices numerically.

Spin Correlations

- These approaches require the production and decay of the particle to be generated at the same time.

- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.

- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.

Spin Correlations

- HERWIG already includes correlations in and between jets using the algorithm of Collins and Knowles

  J. C. Collins, Nucl. Phys. B304, (1988) 794.

  I. G. Knowles, Nucl. Phys. B304, (1988) 767.

  I. G. Knowles, Nucl. Phys. B310, (1988) 571.

  I. G. Knowles, Comput. Phys. Commum. 58 (1990) 271.

- A basically identical algorithm can be used for the decays.

  P. Richardson, JHEP 0111:029, 2001.

- Instead of presenting the full algorithm I will illustrate it with an example.

- Consider the processes $q\bar{q} \to t\bar{t}$ followed by semi-leptonic decay of the top quarks.

- First we generate the momenta of the top quarks in $q\bar{q} \to t\bar{t}$ using the polarized cross-section

$$\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1\ldots\lambda_n}$$

Spin Correlations

- One of the outgoing quarks is selected at random and a spin density matrix calculated.

$$\rho_{\lambda_j\lambda'_j} = \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j} D^i_{\lambda_i\lambda'_i} \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}$$

  In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
  The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- This normalization is therefore the same as the matrix element used in the previous step.

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

$$\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda_1\ldots\lambda_n}$$

Spin Correlations

- In principle we could now continue down the decay chain, but here all the particles produced are stable.

- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda_0'} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n} D^i_{\lambda_i\lambda_i'}$$

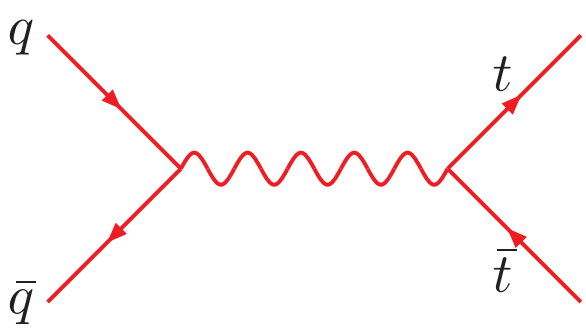
  The normalization is chosen so that $\mathrm{Tr}D = 1$.

- We then select another particle from the hard process. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.

- In this case this means we decay the second quark.

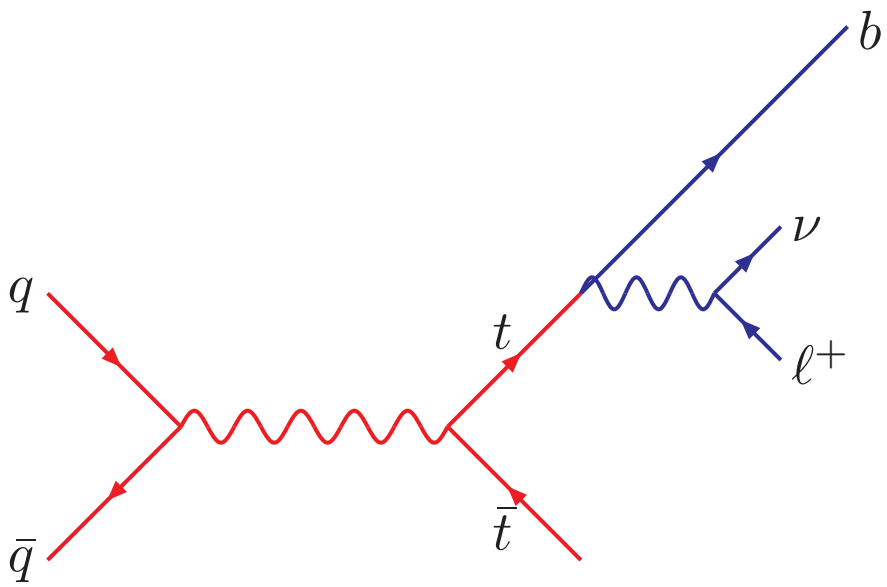
- As before the normalization of the spin density matrix cancels the matrix element used for the previous step.

Top pair Production



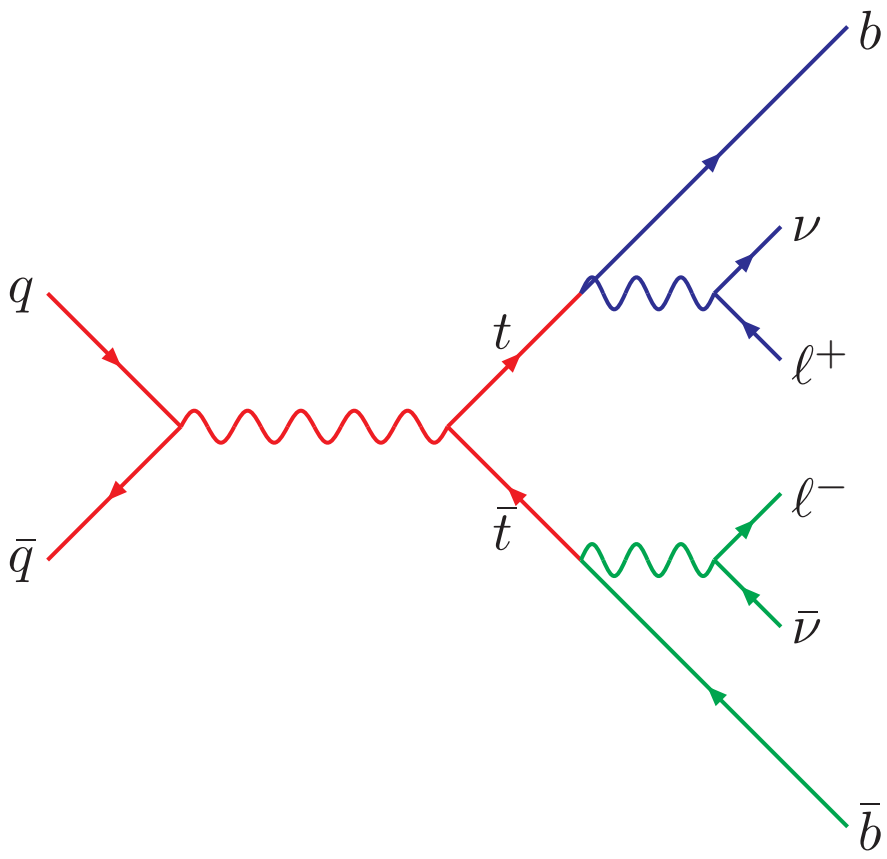
$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t\lambda_{\bar{t}}}$$

Top pair Production



$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}}$$
$$\mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}$$

Top pair Production



$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}'}$$
$$\mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}$$
$$\mathcal{M}^{\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}}\mathcal{M}^{*\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}'}$$

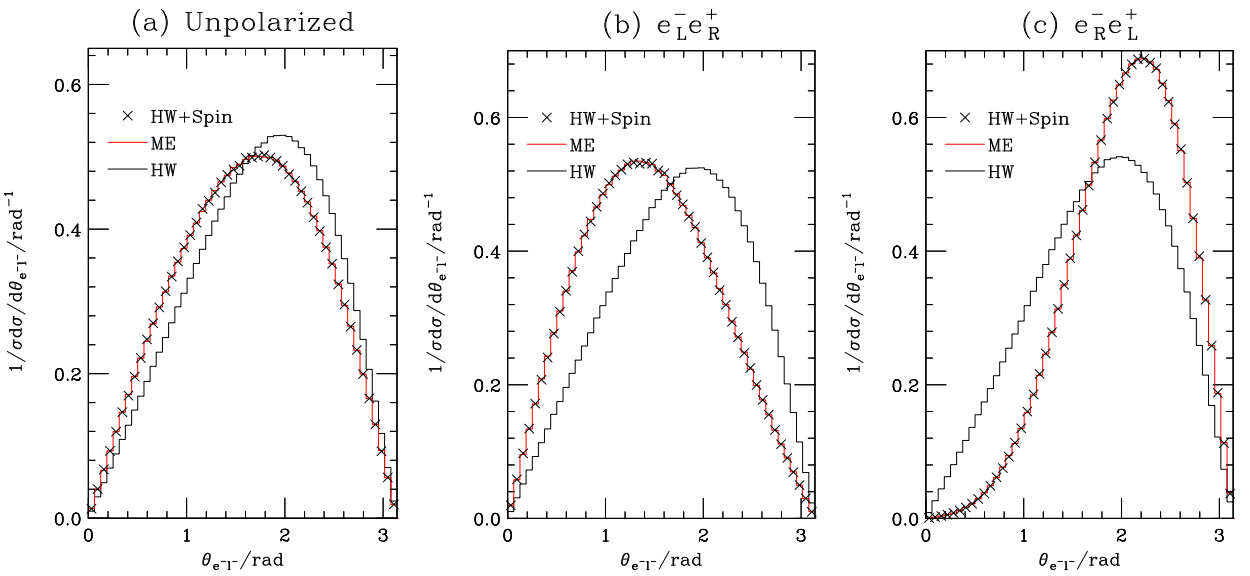
Spin Correlations

- This algorithm can generate the matrix element for an arbitrary process in the narrow width limit.
- Allows us to perform the process step by step.
- In HERWIG this is implemented for SM and SUSY processes in both lepton and hadron collisions.
- Examples
  - Top pair production in e^+e^- collisions.
  - Top pair production in hadron collisions.

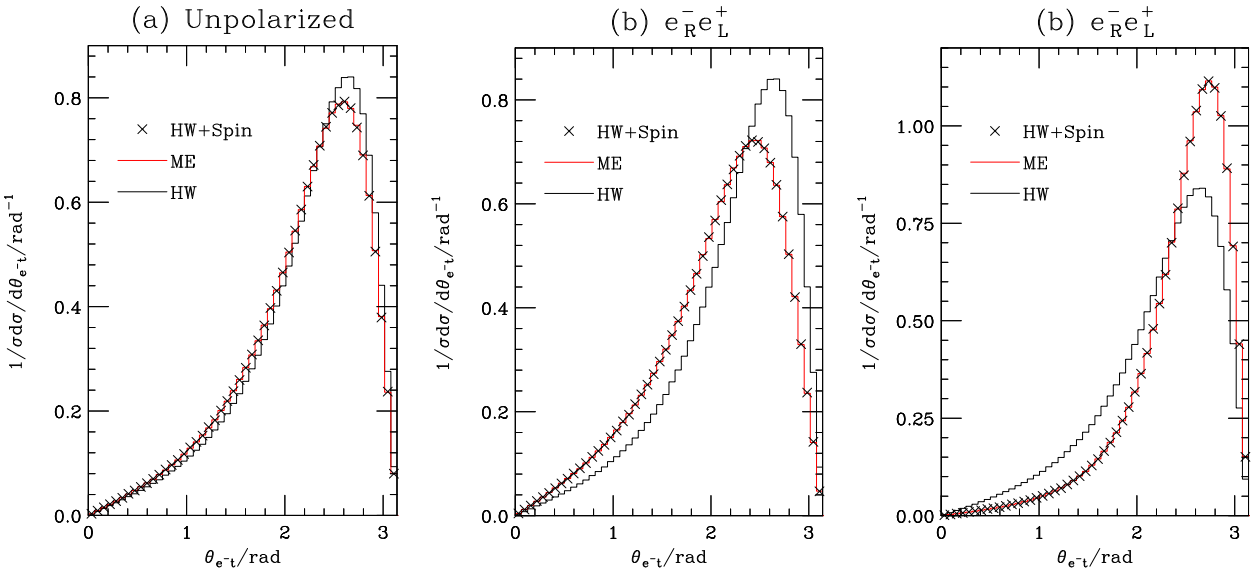
Top pair production in e^+e^- collisions

- Study the semi-leptonic decays of both top quarks and look at the correlations.

- Between the lepton produced in the antitop decay and the beam.

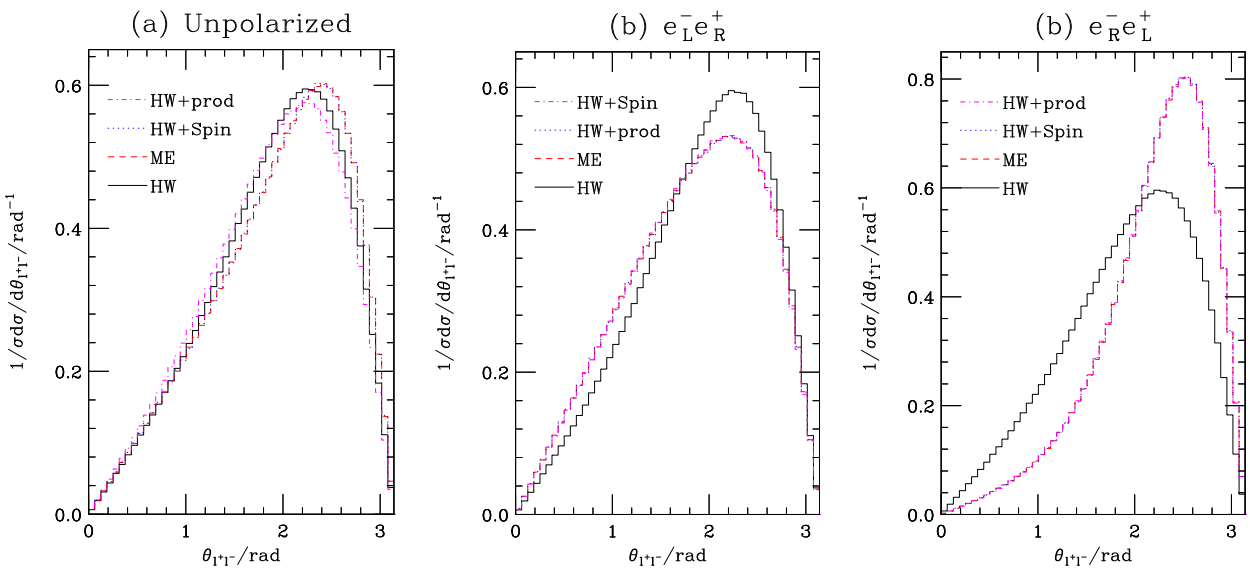


- Between the lepton and the top quark.



Top pair production in e^+e^- collisions

- Between the lepton and antilepton.



- The correlation between the lepton and the beam or top quark is only affected by the production mechanism.

- The correlation between the lepton and antilepton is also effected by the decay of the first quark.

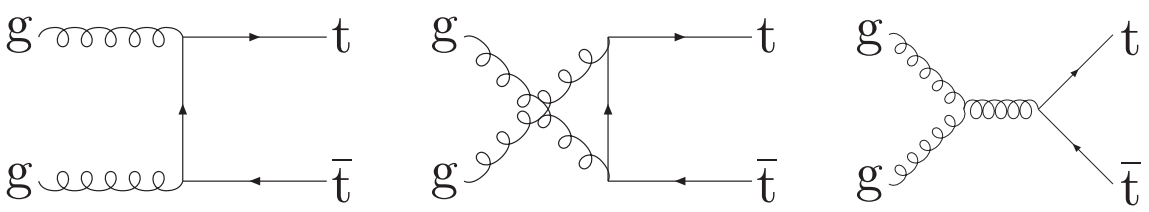
Top quark pair production in hadron collisions

- At leading order there are two processes which contribute

$$\begin{aligned} \mathrm{q\bar{q}} &\longrightarrow \mathrm{t\bar{t}}, \\ \mathrm{gg} &\longrightarrow \mathrm{t\bar{t}}. \end{aligned}$$

- The quark initiated process has the same helicity structure as $\mathrm{e^+e^-} \longrightarrow \mathrm{t\bar{t}}$.

- The gluon initiated process presents complications due to the different colour flows.



- The colour matrices are

$$\begin{aligned} C^{ab}_{t\,c_1c_2} &= t^a_{c_1c'}t^b_{c'c_2} && t-\text{channel}, \\ C^{ab}_{u\,c_1c_2} &= t^b_{c_1c'}t^a_{c'c_2} && u-\text{channel}, \\ C^{ab}_{s\,c_1c_2} &= if^{abc}t^c_{c_1c_2} && s-\text{channel}. \end{aligned}$$

Top quark pair production in hadron collisions

- the s-channel diagram can be written

$$C^{ab}_{sc_1c_2} = C^{ab}_{tc_1c_2} - C^{ab}_{uc_1c_2},$$

  using the definition of f^{abc}.

- This could be a problem if there is more than one colour flow in the decays as well.

- The number of colour flows would then grow exponentially with the number of particles.

- Doesn't happen for the decays we have been considering in the SM or MSSM.

- Therefore we can use

$$\sum_{a,b=1,C_N} C_{ab}\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'} \prod_{i=1,n} D^i_{\lambda_i\lambda_i'},$$

$$\mathcal{M}^a_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{b*}_{\kappa_1'\kappa_2';\lambda_1'\ldots\lambda_n'}$$

to generate the momenta of the particles, etc.

Top quark pair production in hadron collisions

- In some models, e.g RPV SUSY, the decays can have more than one colour flow.
- Then we must select a colour flow in the hard process using

$$C_{aa}\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{a}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{a*}_{\kappa_1'\kappa_2';\lambda_1\ldots\lambda_n}A_a,$$

  where

$$A_a = \frac{\sum_{b,c=1,C_N} C_{bc}\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{b}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{c*}_{\kappa_1'\kappa_2';\lambda_1\ldots\lambda_n}}{\sum_{b=1,C_N} C_{bb}\mathcal{M}^{b}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{b*}_{\kappa_1'\kappa_2';\lambda_1\ldots\lambda_n}}.$$

- or the decay using

$$\rho_{\lambda_0\lambda_0'}C_{aa}\mathcal{M}^{a}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{a*}_{\lambda_0';\lambda_1\ldots\lambda_n}E_a,$$

  where

$$E_a = \frac{\sum_{b,c=1,C_N} C_{bc}\mathcal{M}^{b}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{c*}_{\lambda_0';\lambda_1\ldots\lambda_n}}{\sum_{b=1,C_N} C_{bb}\mathcal{M}^{b}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{b*}_{\lambda_0';\lambda_1\ldots\lambda_n}}.$$

- this stops the exponential growth but neglects some correlations.

Top quark pair production in hadron collisions

- We looked at the correlations using

$$
\begin{aligned}
\mathcal{O}_1 &= \hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{q}}_{\ell^-}; \\
\mathcal{O}_2 &= (\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{p}}_p)\,(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{p}}_p)\,; \\
\mathcal{O}_3 &= \left(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{k}}_t\right)\left(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{k}}_t\right); \\
\mathcal{O}_4 &= \frac{1}{2}\left[(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{p}}_p)\left(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{k}}_t\right) + \left(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{k}}_t\right)(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{p}}_p)\right]; \\
\mathcal{O}_5 &= \operatorname{sgn} y_t \; \mathcal{O}_4; \\
\mathcal{O}_6 &= (\hat{\mathbf{p}}_p \times \hat{\mathbf{q}}_b^*) \cdot (\hat{\mathbf{p}}_p \times \hat{\mathbf{q}}_{\ell^-}) = \mathcal{O}_1 - \mathcal{O}_2.
\end{aligned}
$$

- for the decays $\mathrm{t} \to \mathrm{bW}^+ \to \mathrm{bq\bar{q}}$ and $\bar{\mathrm{t}} \to \bar{\mathrm{b}}\mathrm{W}^- \to \bar{\mathrm{b}}\ell^-\bar{\nu}$

- where
  $\hat{\mathbf{p}}_p$ is the beam direction
  $\hat{\mathbf{q}}_{\ell^-}$ is the momentum of the lepton
  $\hat{\mathbf{q}}_b^*$ is the momentum of the b in the t rest frame
  $\hat{\mathbf{k}}_t$ is the top quark momentum

- and looked at the correlations at the Tevatron and LHC.

Top quark pair production in hadron collisions

- Observables at the Tevatron.

	$\mathcal{O}$/%		
	Full	Spin 1	Spin 2
$\mathcal{O}_1$	2.46 ± 0.04	2.45 ± 0.02	2.41 ± 0.02
$\mathcal{O}_2$	2.94 ± 0.02	2.92 ± 0.01	2.90 ± 0.01
$\mathcal{O}_3$	1.68 ± 0.02	1.67 ± 0.01	1.64 ± 0.01
$\mathcal{O}_4$	0.03 ± 0.02	0.01 ± 0.01	0.01 ± 0.01
$\mathcal{O}_5$	1.95 ± 0.02	1.94 ± 0.01	1.92 ± 0.01
$\mathcal{O}_6$	-0.48 ± 0.03	-0.47 ± 0.02	-0.46 ± 0.02

- Observables at the LHC.

	$\mathcal{O}$/%		
	Full	Spin 1	Spin 2
$\mathcal{O}_1$	-2.01 ± 0.02	-2.04 ± 0.02	-2.04 ± 0.02
$\mathcal{O}_2$	-0.16 ± 0.02	-0.16 ± 0.02	-0.18 ± 0.02
$\mathcal{O}_3$	-0.55 ± 0.02	-0.56 ± 0.02	-0.57 ± 0.01
$\mathcal{O}_4$	0.00 ± 0.02	-0.01 ± 0.01	-0.02 ± 0.01
$\mathcal{O}_5$	-0.23 ± 0.02	-0.25 ± 0.01	-0.25 ± 0.01
$\mathcal{O}_6$	-1.85 ± 0.02	-1.87 ± 0.02	-1.86 ± 0.01

- Here the default result of HERWIG is consistent with zero.

Spin Correlations

- The approach I have outlined allows full correlations to be included in arbitrarily complicated decay chains.
- Available in HERWIG for SM processes, *e.g.* top quark production.
- Main use in SUSY models which have large numbers of unstable heavy particles and long decay chains, also available in HERWIG.
- This approach can be combined with the existing correlations in the parton shower to get the full correlations in an event. This remains to be done.

Merging Matrix Elements and Parton Showers

- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high p_T jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating many high p_T jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, *i.e.*
  - ME gives hard emission,
  - PS gives soft/collinear emission,
  - Smooth matching between the two,
  - No double counting of radiation.

Merging Matrix Elements and Parton Showers

- There have been a number of attempts to do this.
- The first attempts got the hardest emission correct for simple processes, for example
  - $e^+e^- \to q\bar{q}$
  - DIS
  - Drell-Yan
  - Top Decay
- There were problems with this
  - Only the hardest emission was correctly described.
  - The leading-order normalization was retained.

Merging Matrix Elements and Parton Showers

- Recently there has been renewed interest.
- Two types of approach have emerged.
  1. NLO Simulation
     - NLO normalization of the cross section.
     - Gets the hardest emission correct.
     - Only programs by Frixione/Webber and Dobbs for gauge boson pairs.
  2. Multi-jet Leading Order
     - Still leading order.
     - Gets many hard emissions correct.
     - Programs by Krauss et. al. and Lonnblad for e^+e^-.

Merging Matrix Elements and Parton Showers

- I will be looking at the second type of approach.
- The idea is to implement the algorithm of CKKW to produce a multi-jet simulation using existing tools
  - Use the Les accord compliant M.E. generators to calculate matrix elements (MADGRAPH, ALPGEN)
  - Perform the reweighting etc. specified by the algorithm before passing the events to HERWIG.
  - Use the Les accord interface to read the events into HERWIG.
  - Make only minimal changes to the event generator itself.
- Produce a better simulation using existing well tested tools.
- I will first review the algorithm of CKKW and then present some preliminary results.

Procedure

- In order to match the ME and PS we need to separate the phase space:
  - One region should contain the soft/collinear region and be filled by the PS;
  - The other is filled by the matrix element.

- In Catani-Krauss-Kuhn-Webber (CKKW) prescription this is done via the Durham jet algorithm.
  - For all final-state particles compute the resolution variables
    - $d_{kB} \simeq E_k^2 \theta_{kB}^2 \simeq k_{\perp kB}^2, \quad \theta_{kB} \to 0.$
    - $d_{kl} \simeq \min(E_k^2, E_l^2)\theta_{kl}^2, \quad \theta_{kl} \to 0$
  - The smallest of these d_{kB}, d_{kl} is then selected. If d_{ij} is the smallest the two particles are merged, if d_{iB} is the smallest the particle is merged with the beam.
  - This procedure is repeated until the minimum value is above some stopping parameter $d_{\rm cut}$.
  - The remaining particles and pseudo-particles are then the hard jets.

Procedure

1. Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{k=N} \sigma_k}$$

   where σ_n is the n-jet matrix element evaluated at resolution $d_{\rm ini}$ using $d_{\rm ini}$ as the scale for the P.D.F's and α_S, n is the number of additional jets.

2. Distribute the jet momenta according to the ME.

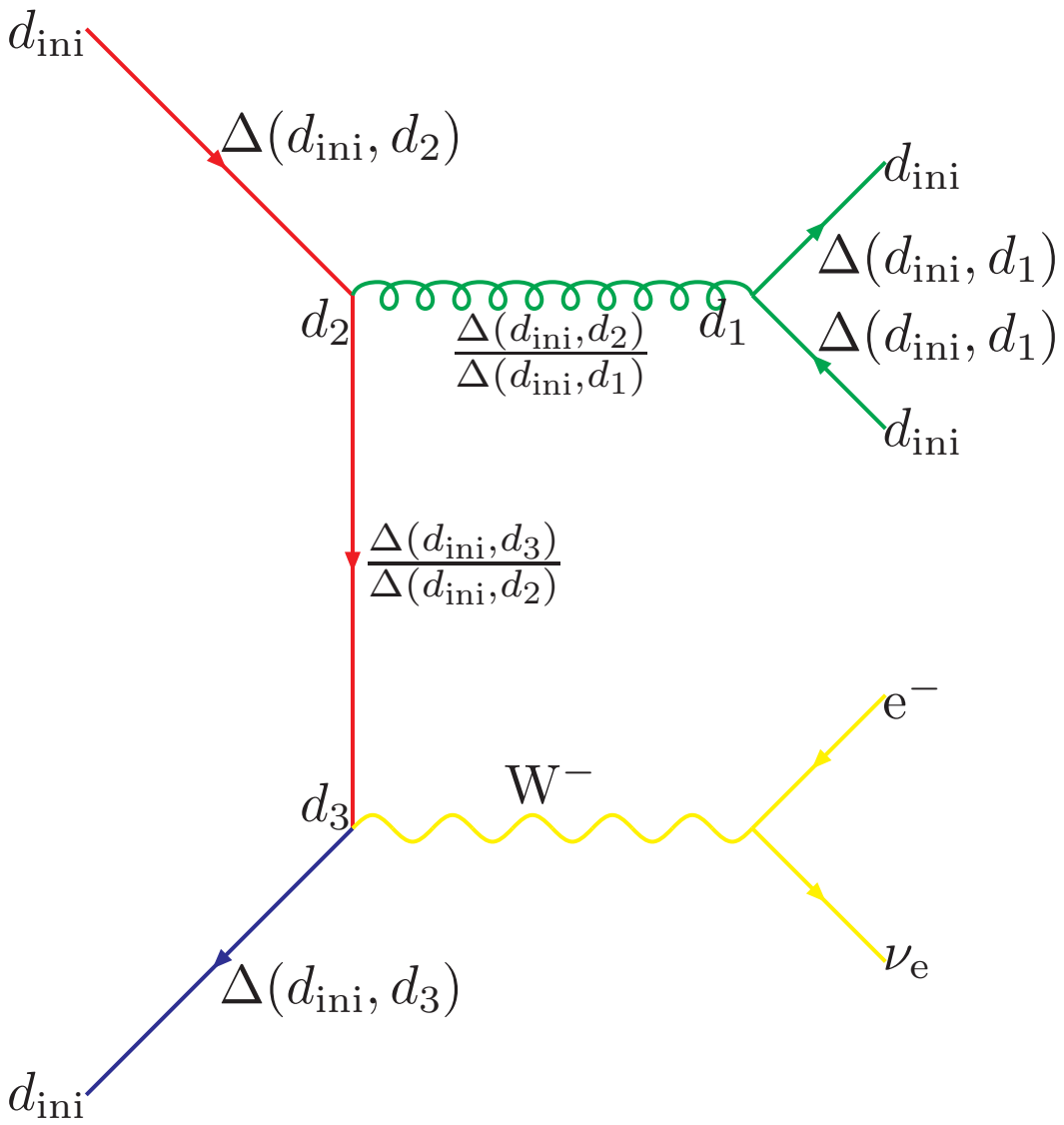
3. Cluster the partons to determine the values at which 1,2,..n-jets are resolved. These give the nodal scales for a tree diagram.

4. Apply a coupling-constant reweighting $\alpha_S(d_1)\alpha_S(d_2)..\alpha_S(d_n)/(\alpha_S(d_{\rm ini})^n \leq 1$.

5. Reweight the lines by a Sudakov factor $\Delta(d_{\rm ini}, d_j)/\Delta(d_{\rm ini}, d_k)$.

6. Accept the configuration if the product of the α_S and Sudakov weight is less the $\mathcal{R} \in [0, 1]$., otherwise return to step 1.

Procedure



7. Generate the parton shower from the event starting the evolution of each parton at the scale it was created and vetoing emission above the scale $d_{\rm ini}$

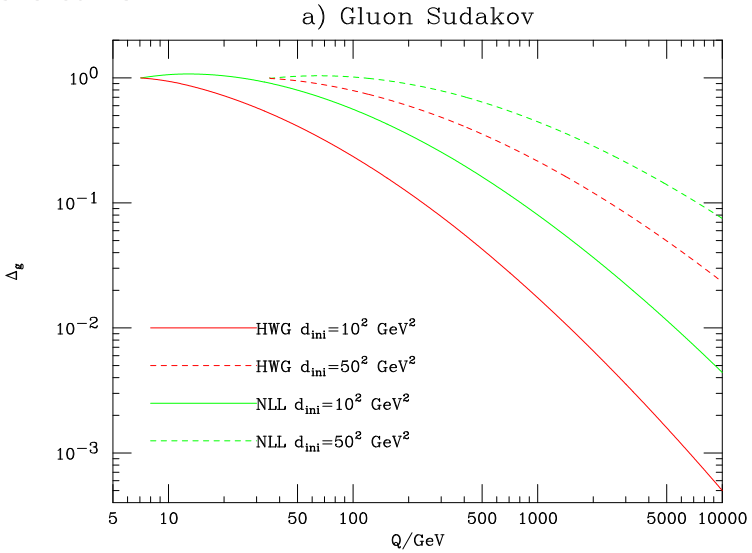
Free Parameters

- Although the algorithm above ensures smooth matching at NLL log level there are a number of free choices
  - Sudakov Form Factors
    - HERWIG or NLL Sudakovs
    - Definition of the scale
    - Prefactor for the scale
  - α_S
    - LO or NLO definition
    - Definition of the scale
    - Prefactor for the scale
  - Treatment of Highest Multiplicity Matrix Element
    - Same as other multiplicities?
    - Different treatment to describe more emissions?
  - Definition of k_T-algorithm away from soft/collinear limit
    - Definition of the k_T-measure.
    - Recombination scheme.

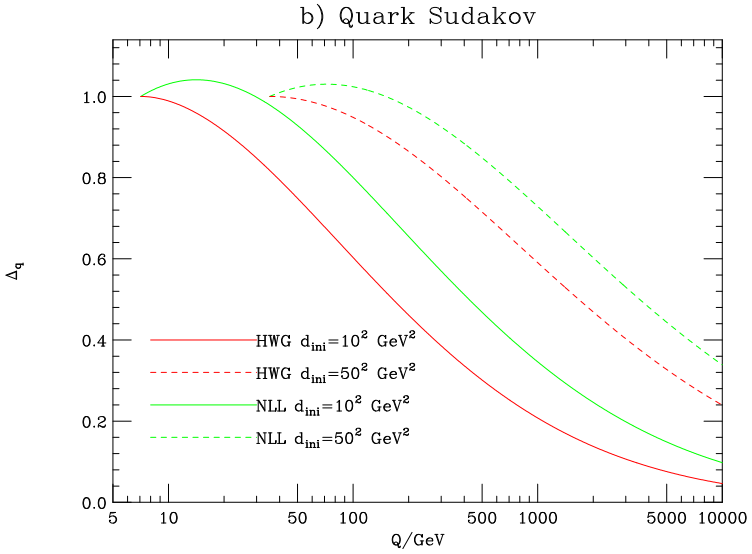
Sudakov Form Factors

- Due to the choice of scale in α_S and regularization the HERWIG Sudakov is always smaller than the NLL one.

- Gluon Sudakov



- Quark Sudakov



Sudakov Form Factors

- To look at the effects of the different Sudakov form factors study e^+e^- collisions at M_Z.
- Look at the differential distributions with respect to y_n where
$$y_n = \frac{d_n}{s},$$
and d_n is the k_T^2 scale where the event changes from n to $n-1$ jets.
- Start with a matching scale of $y = 0.001 \Rightarrow d = 8.3\mathrm{GeV}^2$.
- This is a very low scale which enhances the differences of different choices.
- Start by studying different Sudakovs, different scales in the Sudakov, and different scales in α_S.

Choice of Scale

- There are a number of choices to make.
- First for the Sudakovs

$$d_{\mathrm{SUD}} = \texttt{QFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

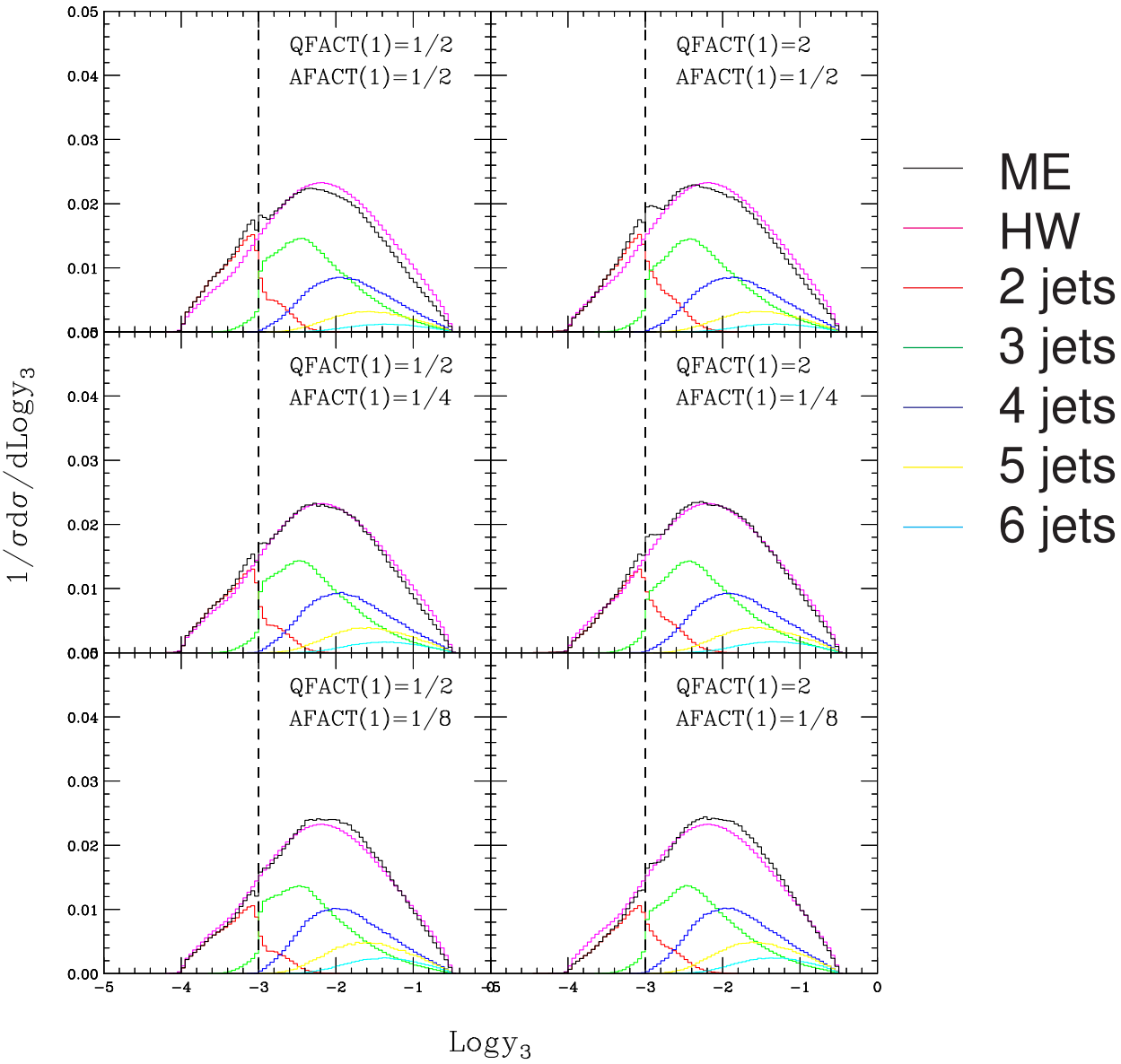
- and for α_S.

$$d_{\alpha_S} = \texttt{AFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

- HERWIG would use `QFACT(1)=1/2` for $e^+e^- \to \mathrm{jets}$ or Drell-Yan.
- However no radiation in shower above $\frac{1}{2}d_{\mathrm{shower}} \Rightarrow$ `QFACT(1)=2`
- HERWIG always has a smaller scale for α_S than in the Sudakov form factor.

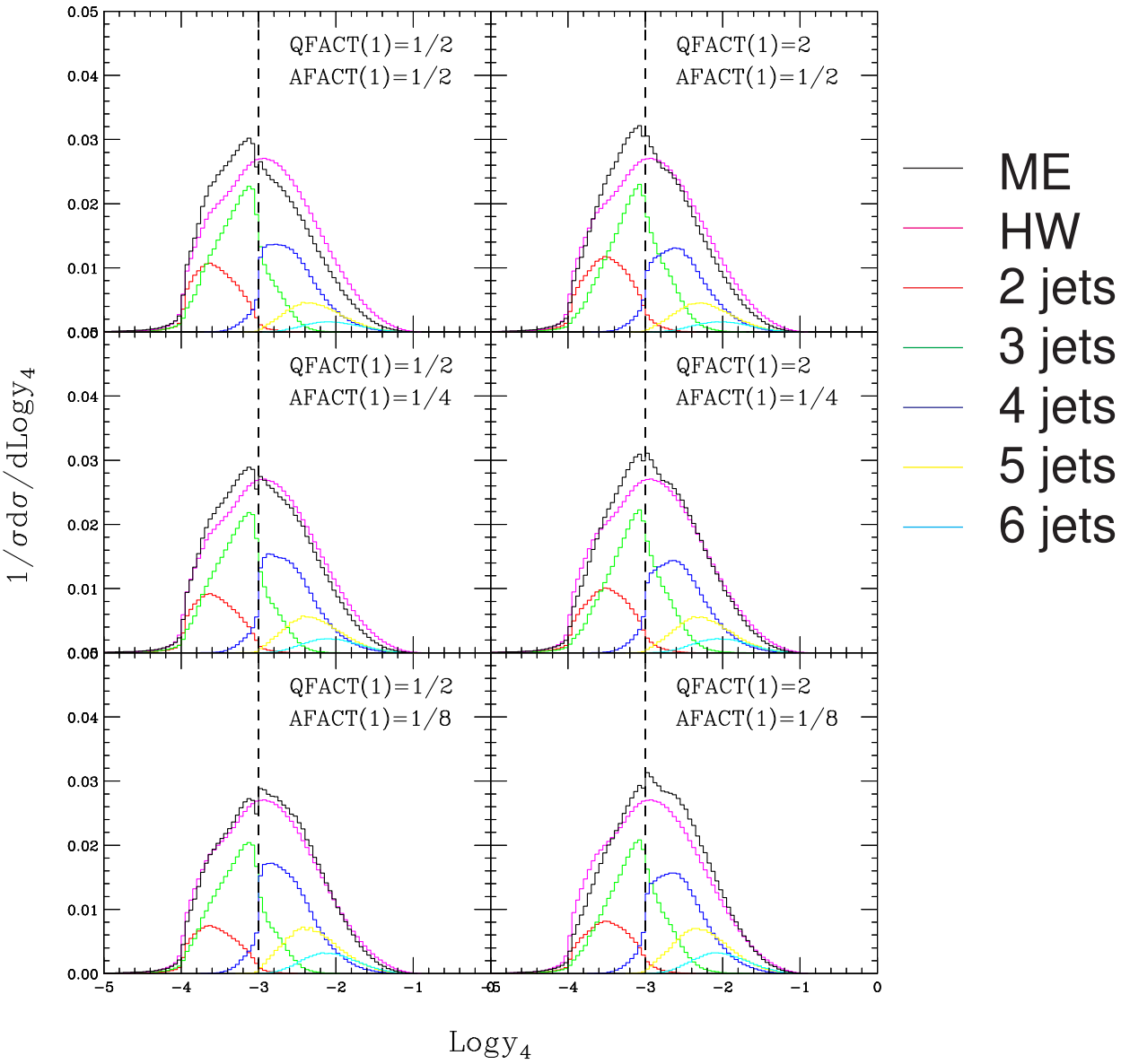
NLL Sudakovs

- NLL Sudakovs with `ISCALE=1`.
- y_3 distribution.



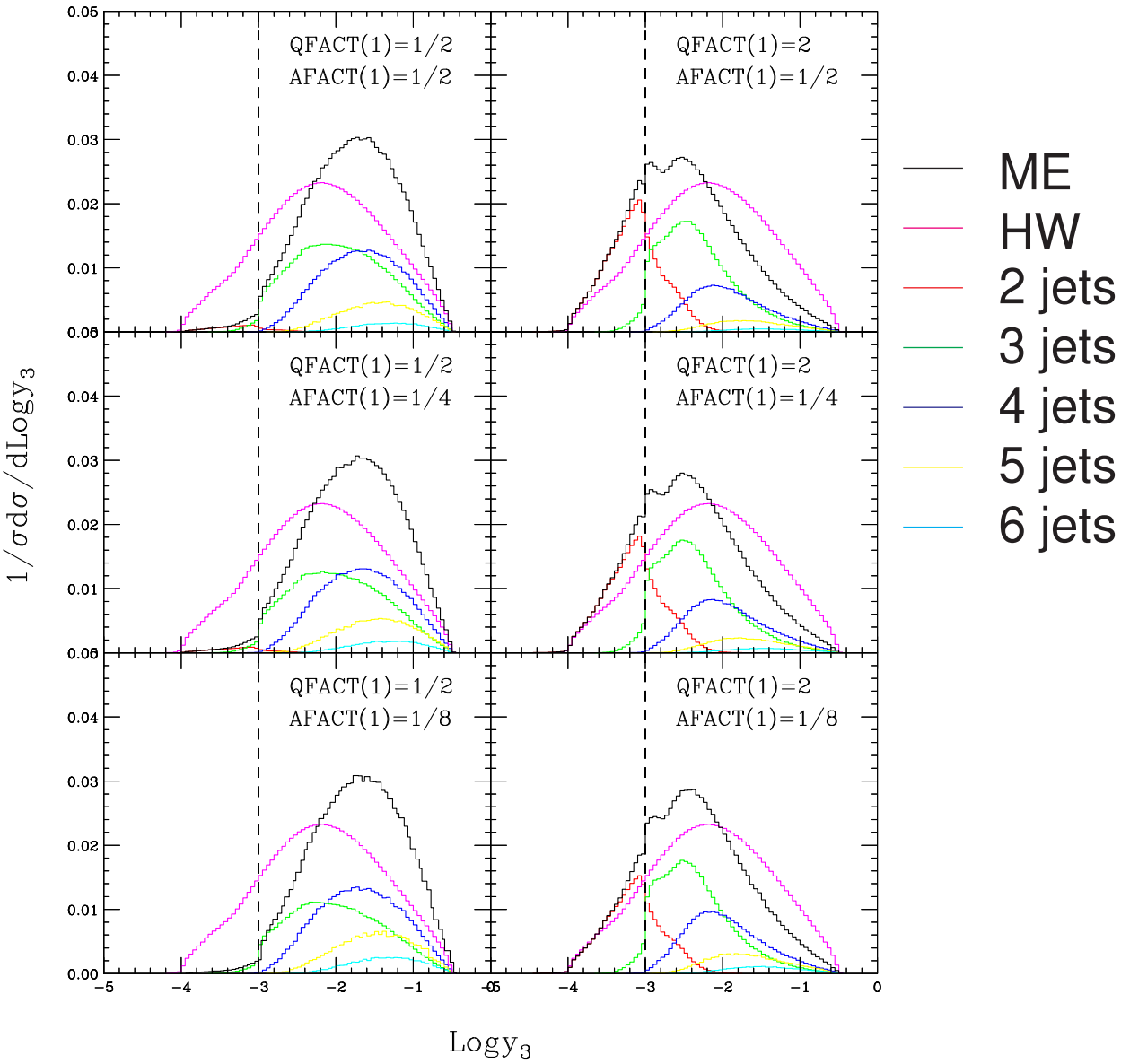
NLL Sudakovs

- NLL Sudakovs with `ISCALE=1.`
- y_4 distribution



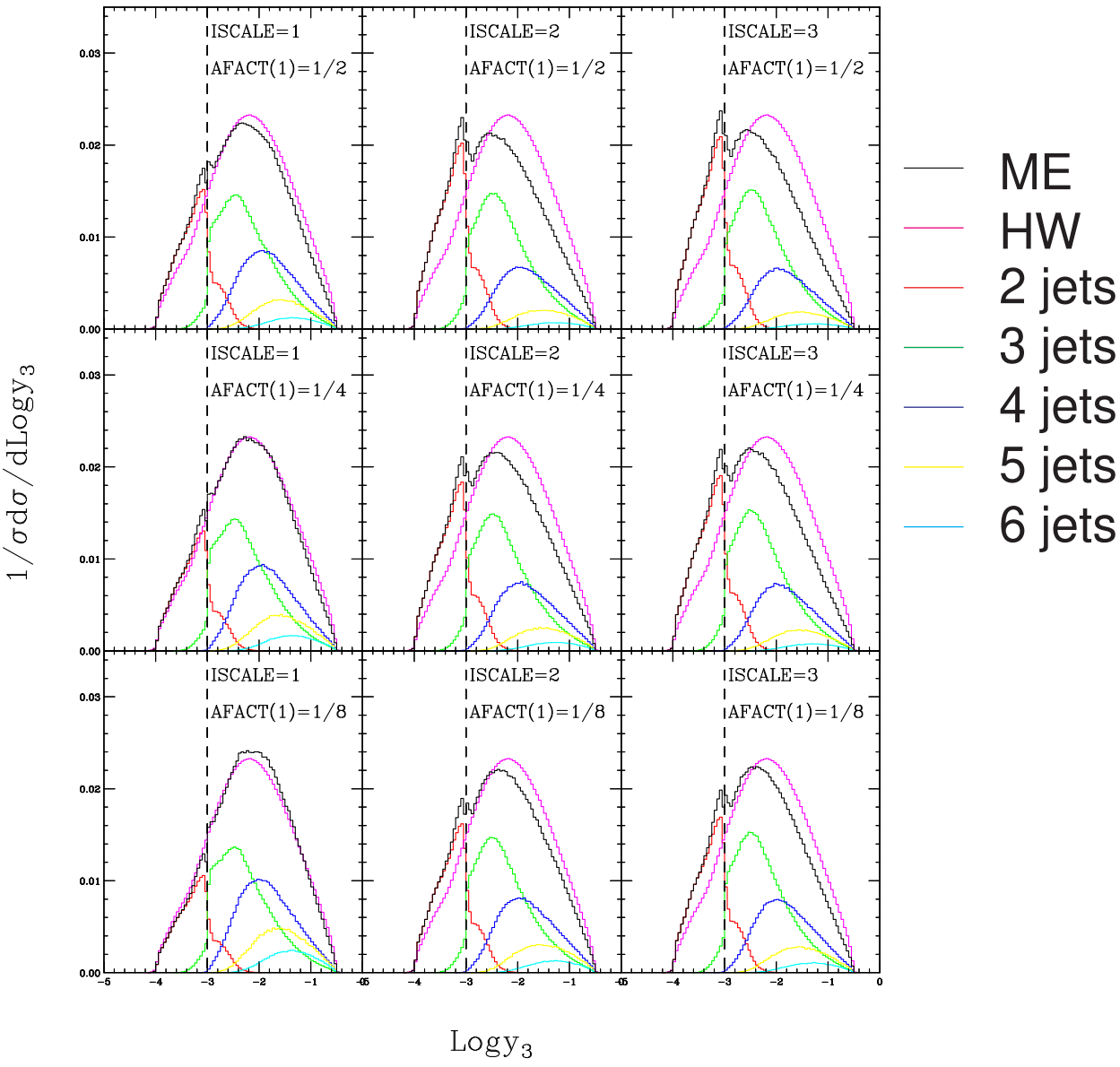
HERWIG Sudakovs

- HERWIG Sudakovs with `ISCALE=1`.
- y_3 distribution.



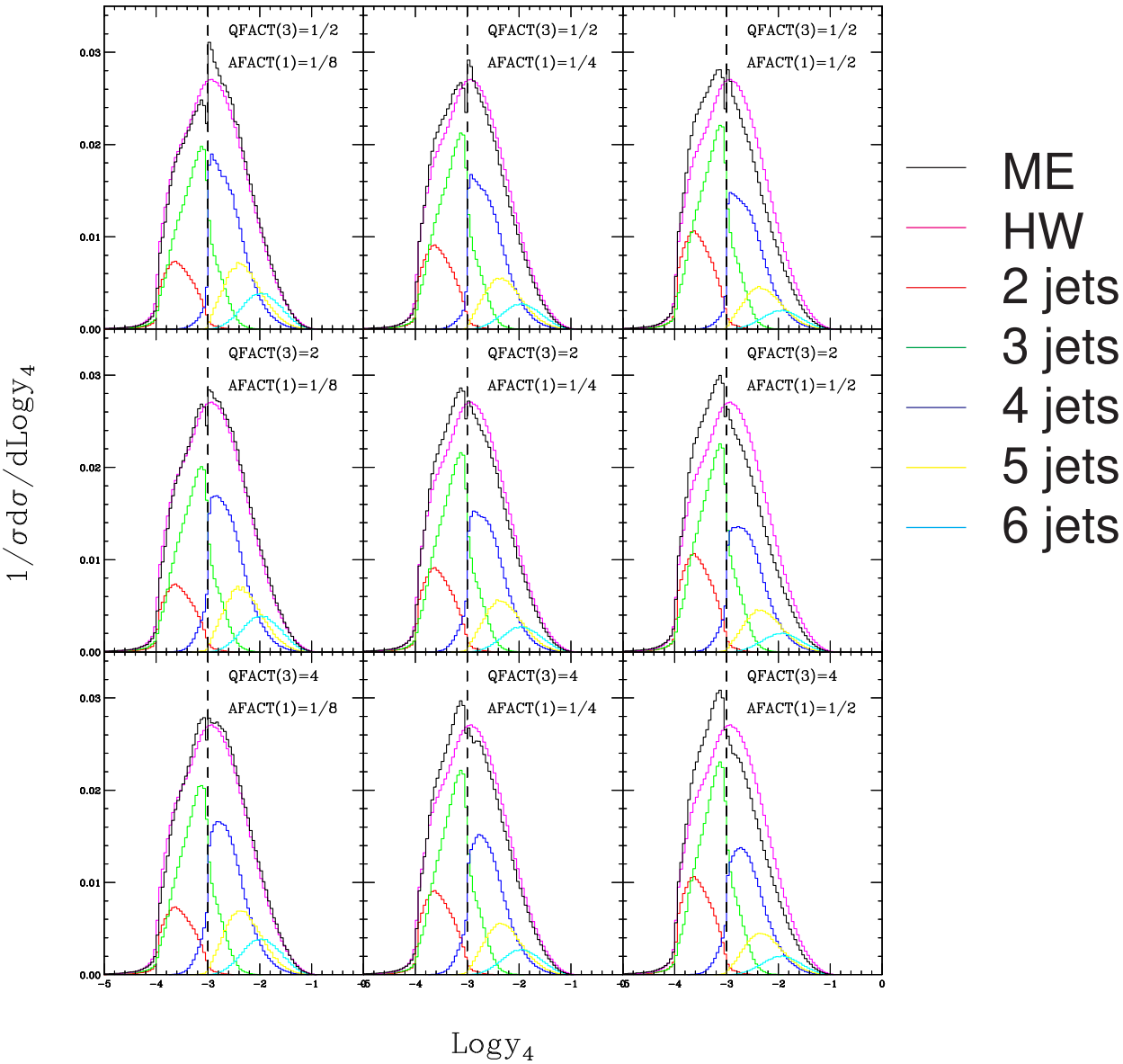
NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- y_3 distribution.



NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- Variation of `QFACT(3)` which is the minimum scale starting scale for the HERWIG shower.
- y_4 distribution.

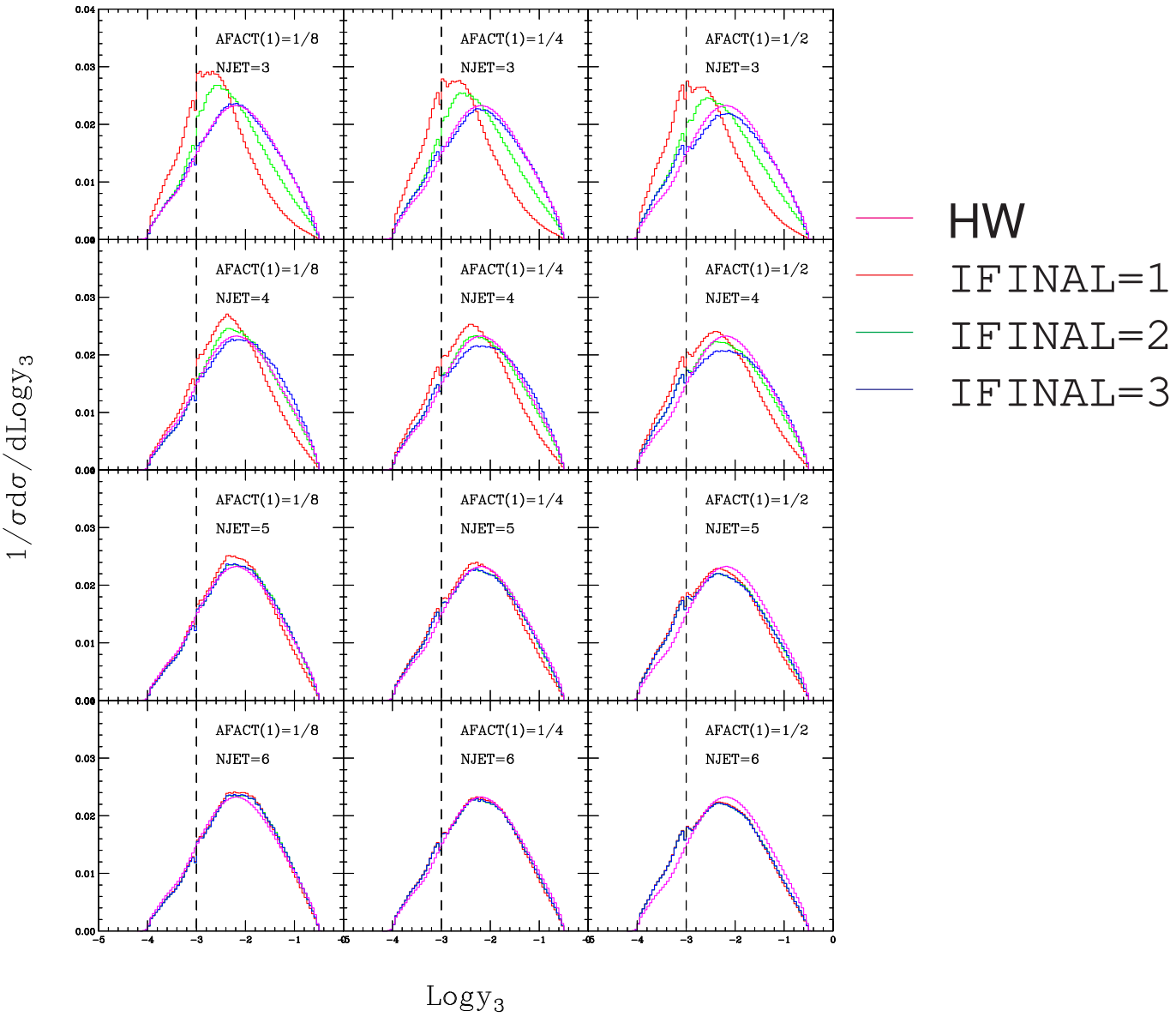


Treatment of Highest Multiplicity

- In the original CKKW algorithm all jet multiplicities were treated in the same way, `IFINAL=1`.
- Not appropriate for highest multiplicity.
- Instead only apply α_S reweighting and Sudakovs for internal lines.
- Two options for the shower
  - Apply no veto and start at normal HERWIG scale, `IFINAL=2`
  - Start at same scale as other multiplicities but veto above scale of last emission in ME. `IFINAL=3`
- `IFINAL=2` is easier to implement but has some double counting problems
- `IFINAL=3` is the best theoretical choice.

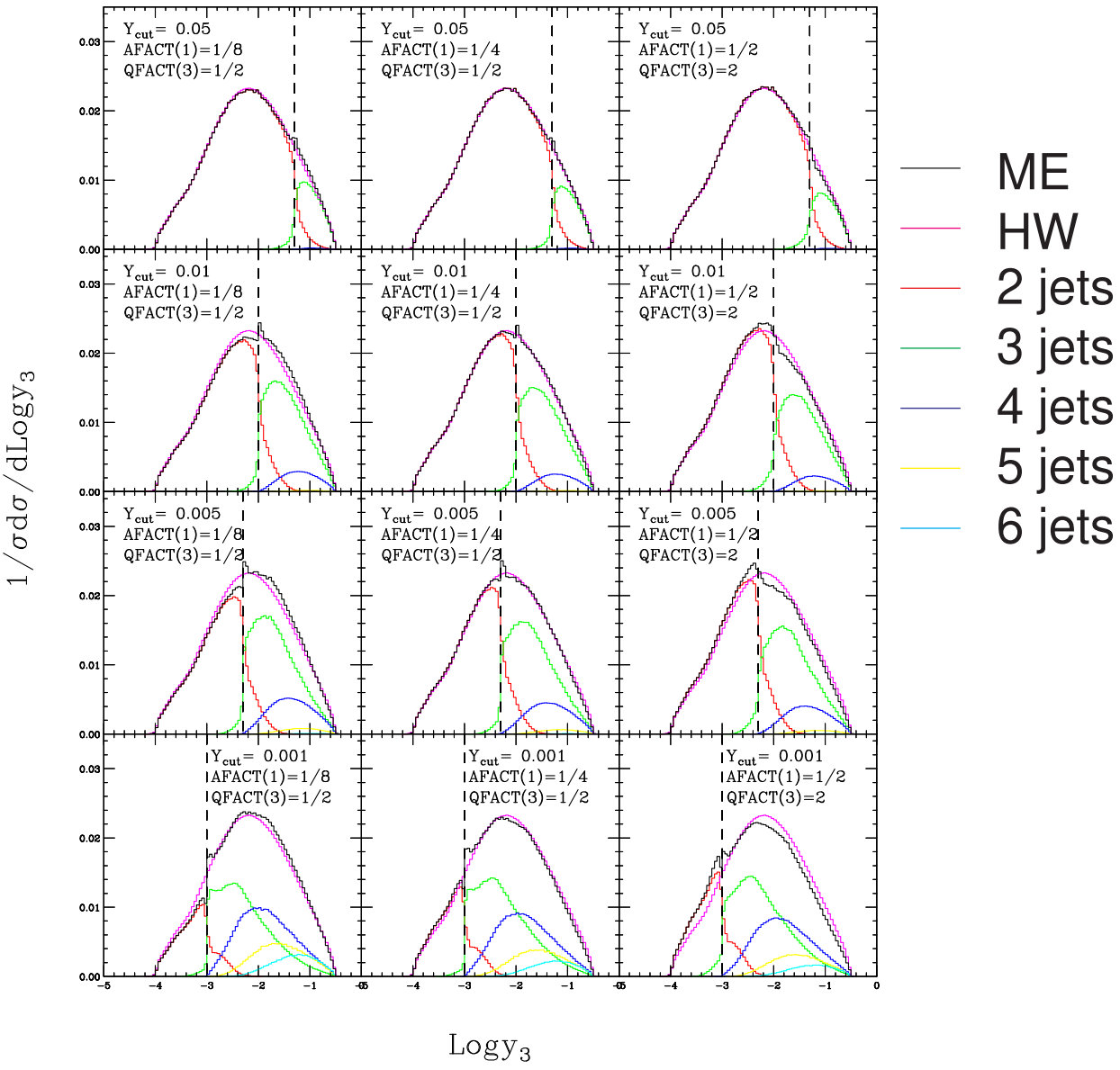
Treatment of Highest Multiplicity

- NLL Sudakovs with `QFACT(1)=1/2`.
- y_3 distribution.



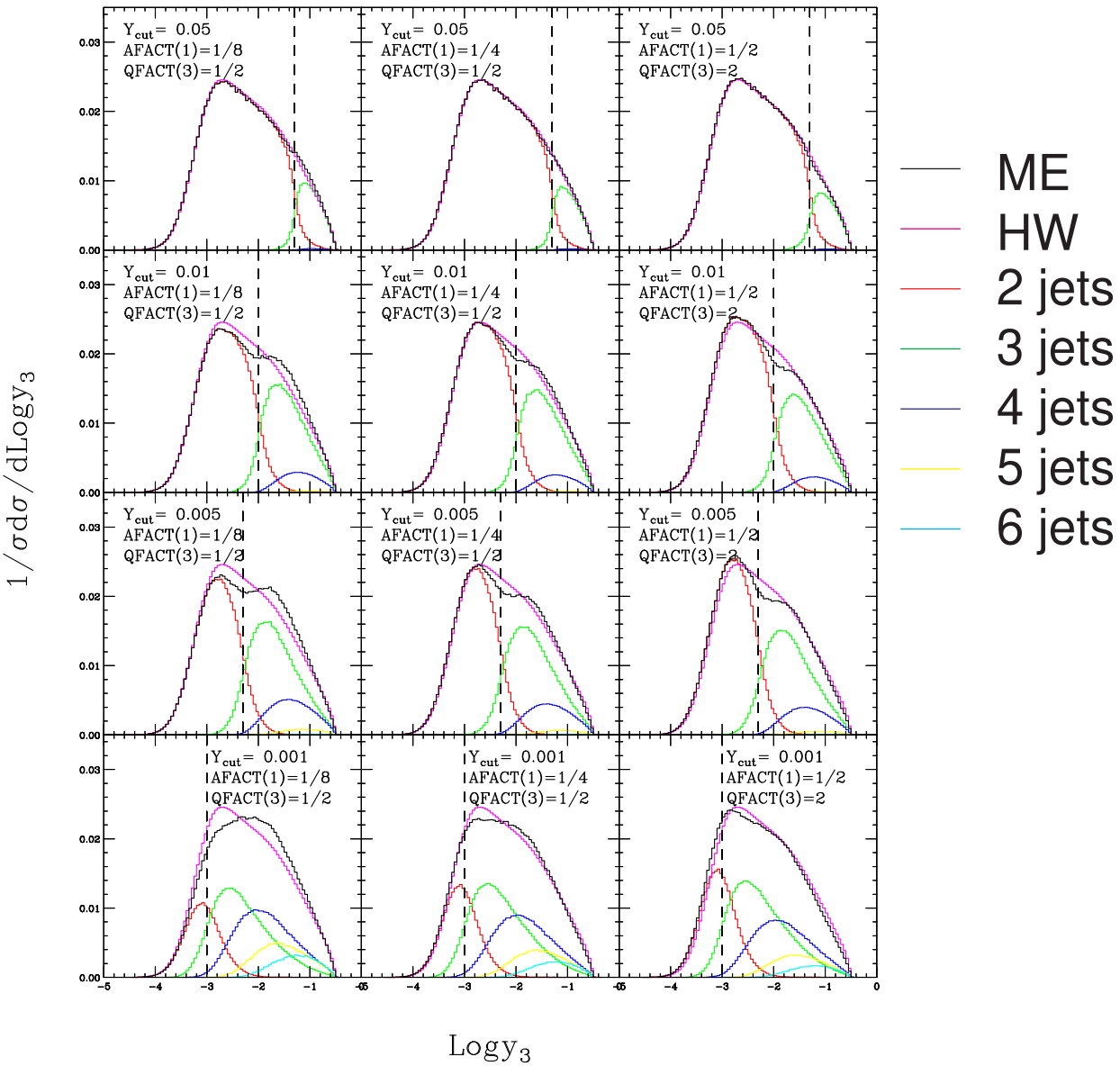
Effect of Varying the cut-off at M_Z

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Parton-level y_3 distribution.



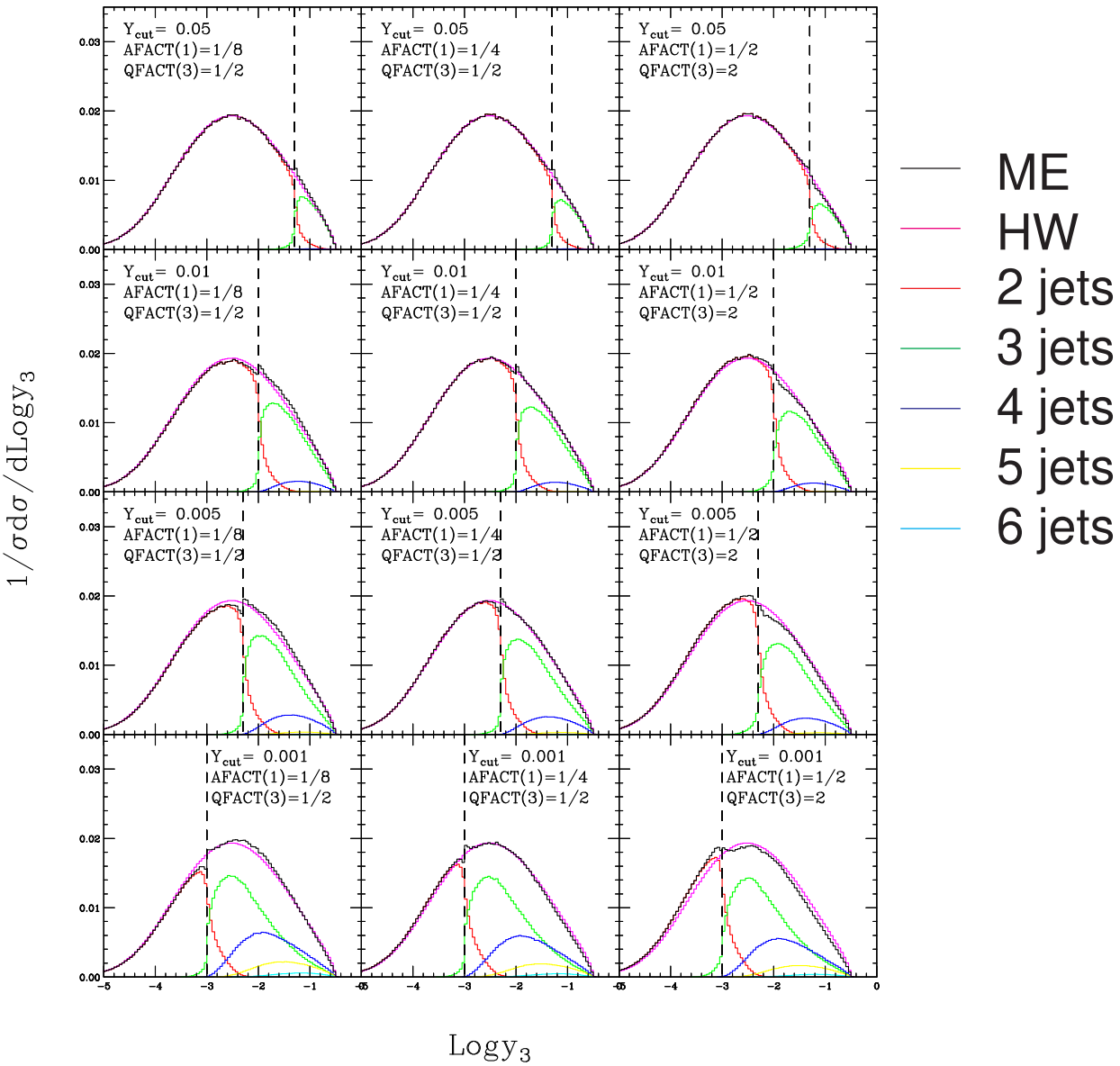
Effect of Varying the cut-off at M_Z

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Hadron-level y_3 distribution.
- Problems with hadronization model which remain to be solved.



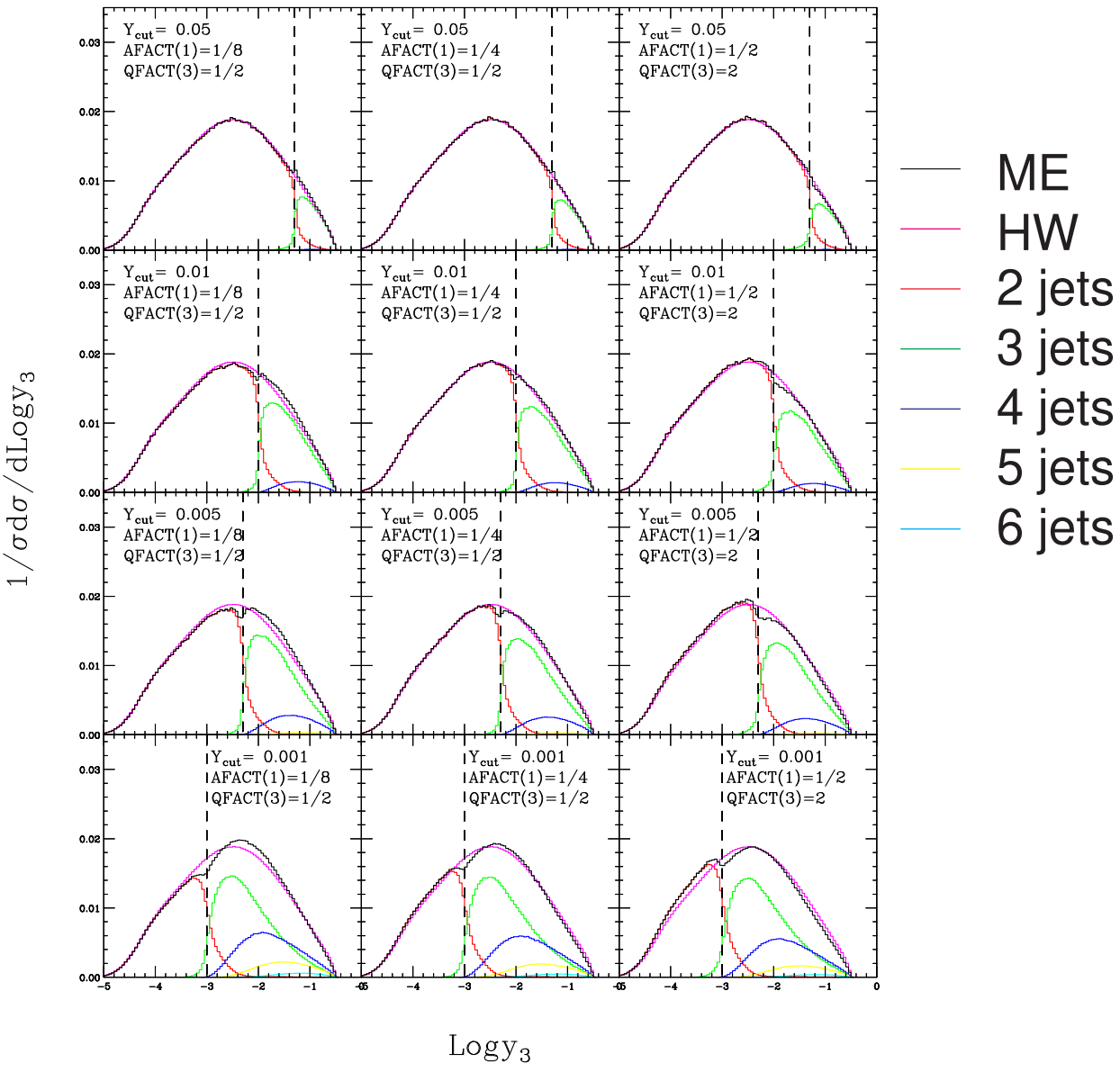
Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Parton-level y_3 distribution.



Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2`, `IFINAL=3`.
- Hadron-level y_3 distribution.
- Problems with hadronization model reduced.



Hadron Collisions

- In hadron collisions in addition to final-state radiation we have the problems of initial-state radiation.
- I will consider W and Z production in hadron collisions
- All the events were generated with MADGRAPH including the leptonic decays of the gauge bosons.
- For the W samples no cuts were applied on the leptons
- For Z production the photon diagrams were included and a cut on the invariant mass of the lepton pair imposed, $m_{\ell\ell} > 20$ GeV.

Hadron Collisions

- We also have a greater choice of the clustering algorithm.
- I will consider three choices.
  - E-scheme

$$\begin{aligned} d_{iB} &= 2E_i^2(1-\cos\theta_{iB}), \\ d_{ij} &= 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij}). \end{aligned}$$

    Combine 4-momenta to get pseudoparticles
  - Covariant E-scheme

$$\begin{aligned} d_{iB} &= p_{ti}^2, \\ d_{ij} &= \min(p_{ti}^2, p_{tj}^2)R_{ij}^2, \end{aligned}$$

    Combine 4-momenta to get pseudoparticles
  - Covariant monotonic p_T^2 scheme
    Same definition of d as Covariant E-scheme but define p_t and R for pseudoparticles as

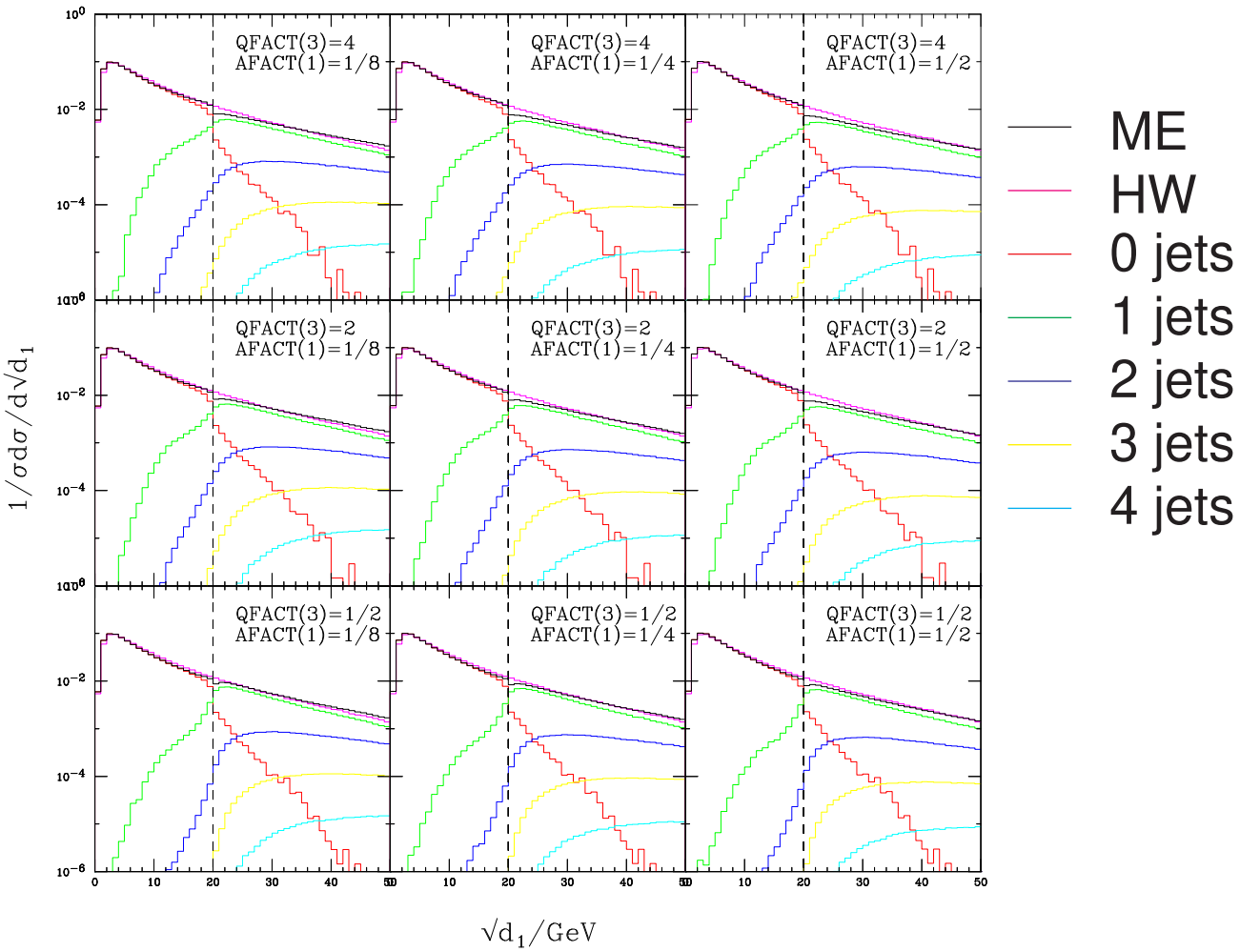
$$\begin{aligned} p_{t(ij)} &= p_{ti} + p_{tj}, \\ R_{(ij)k}^2 &= \frac{p_{ti}^2 R_{ik}^2 + p_{tj}^2 R_{jk}^2}{p_{ti}^2 + p_{tj}^2}. \end{aligned}$$

Hadron Collisions

- Start by considering W and Z production at the Tevatron.
- Use the Covariant monotonic p_T^2 scheme.
- and a matching scale of $400\ \mathrm{GeV}^2$.
- Look at the differential cross section with respect to $\sqrt{d_n}$ where d_n is the scale at which n jets are resolved.
- This is like the E_T in a cone algorithm.

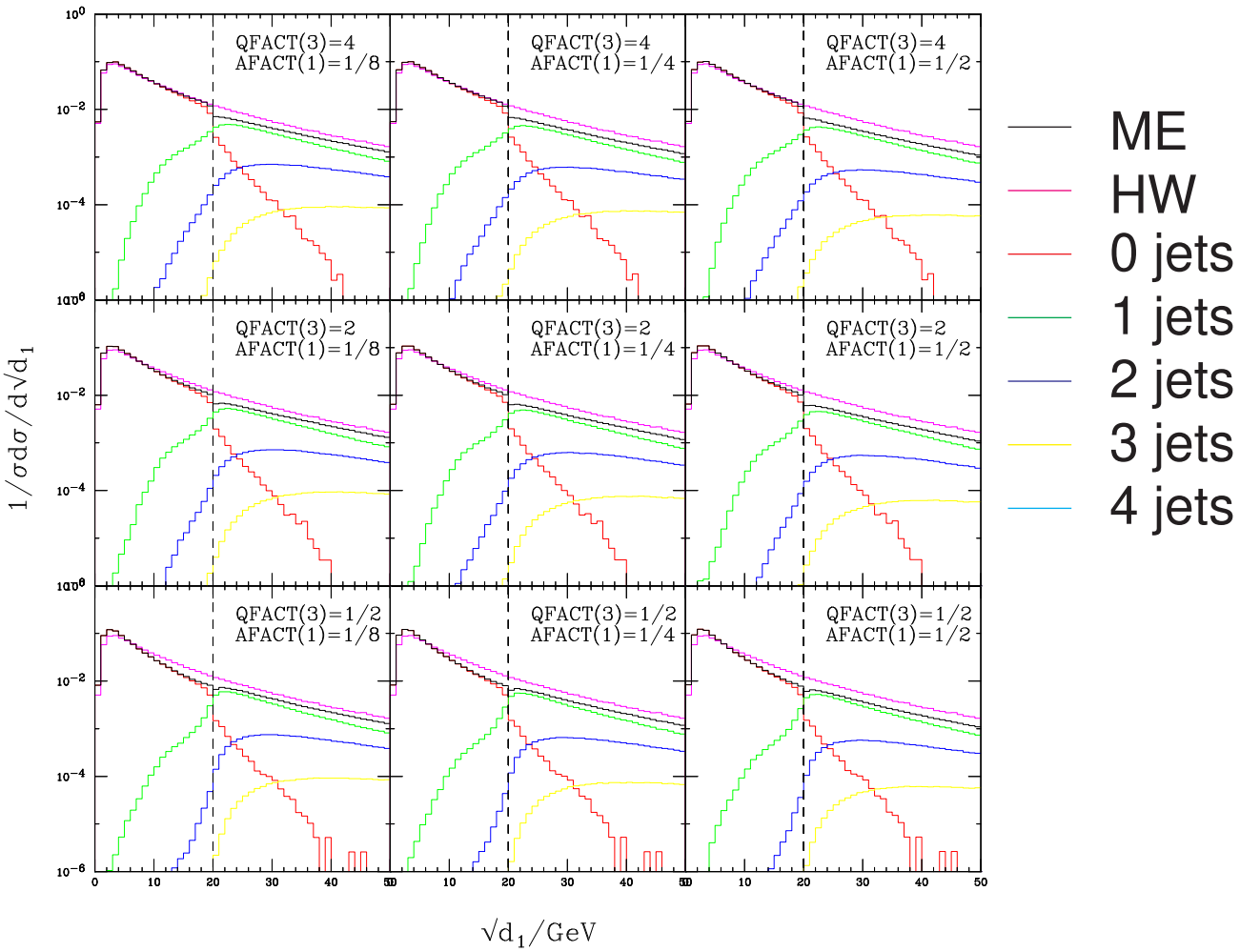
W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_1 distribution at the Tevatron



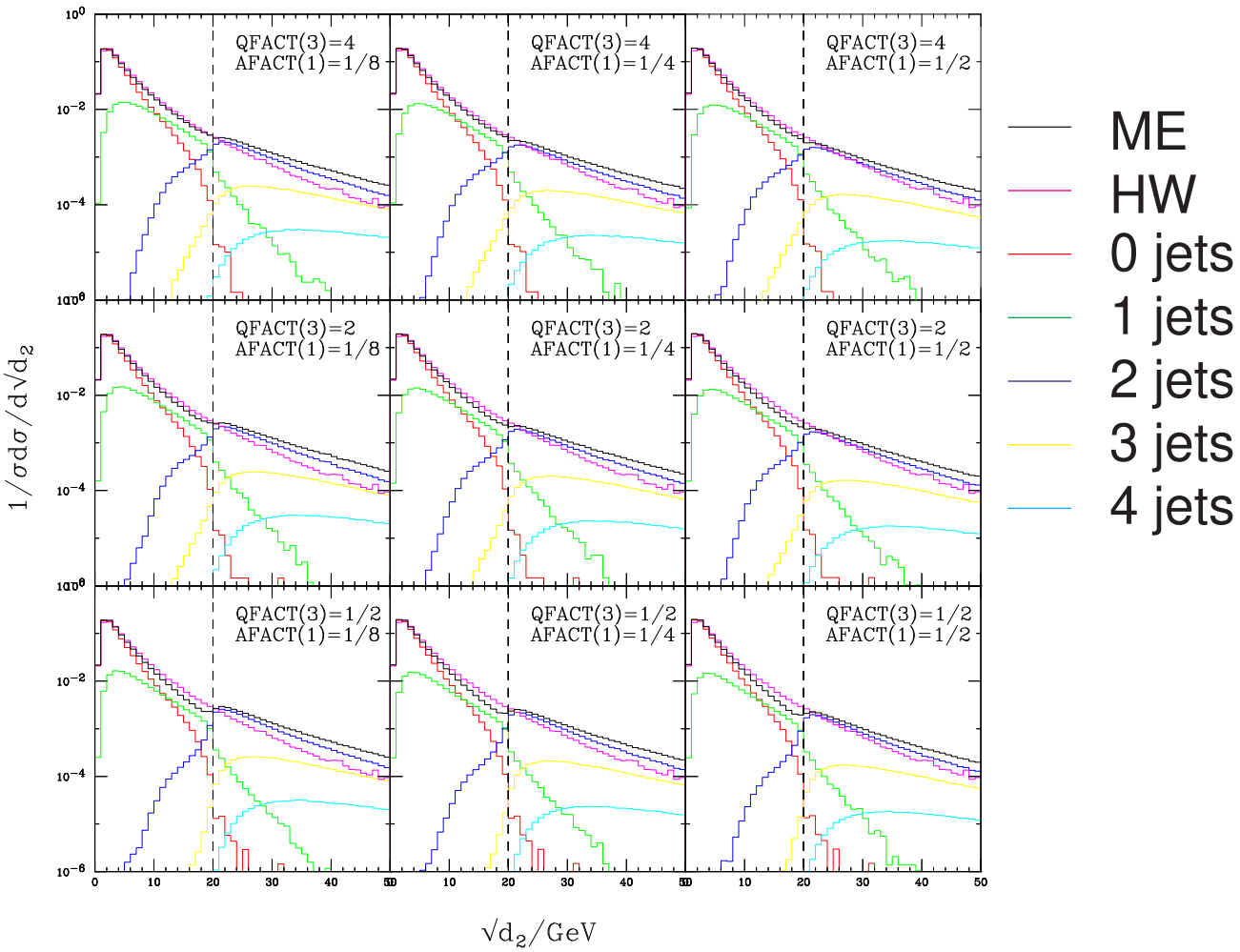
Z Production

- Variation of results with scale of α_S and starting scale of shower.
- d_1 distribution at the Tevatron



W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_2 distribution at the Tevatron

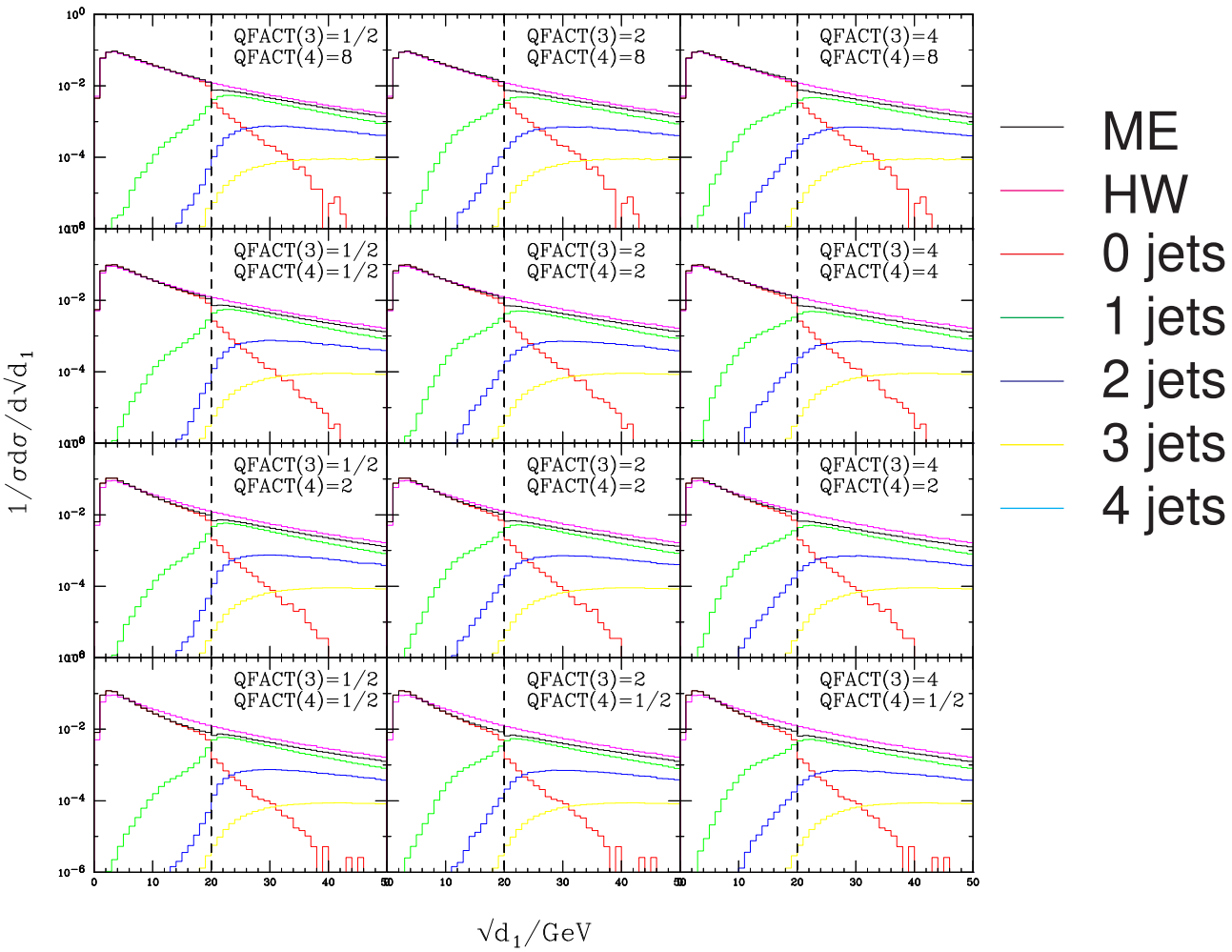


Hadron Collisions

- There is a radiation dip at the matching scale $\Rightarrow$ high `QFACT(3)`.
- However there is also a problem with events migrating below the matching scale $\Rightarrow$ low `QFACT(3)`.
- Solution is to decouple the scale for the initial-state shower `QFACT(4)` from the scale for the final-state shower `QFACT(3)`.

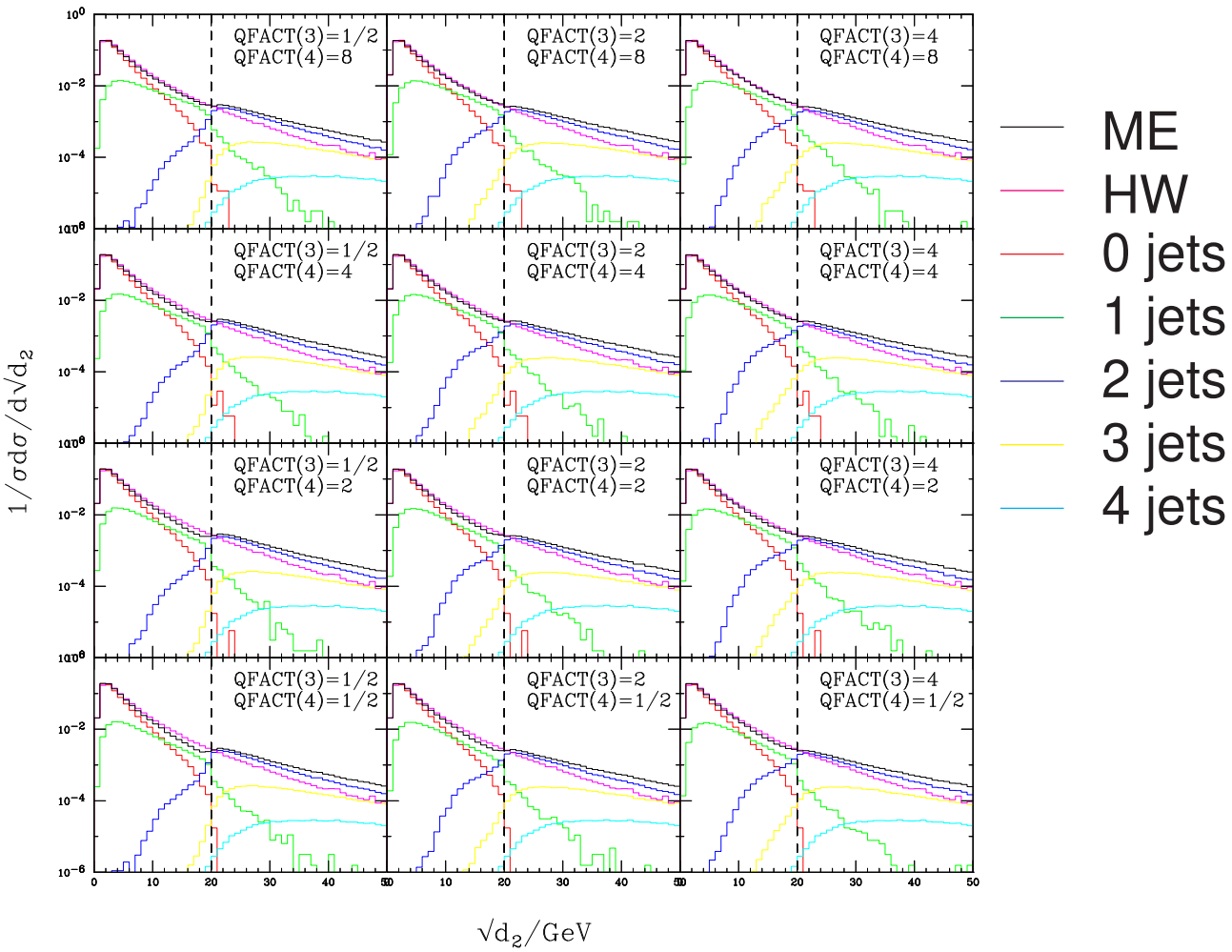
Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- d_1 distribution at the Tevatron



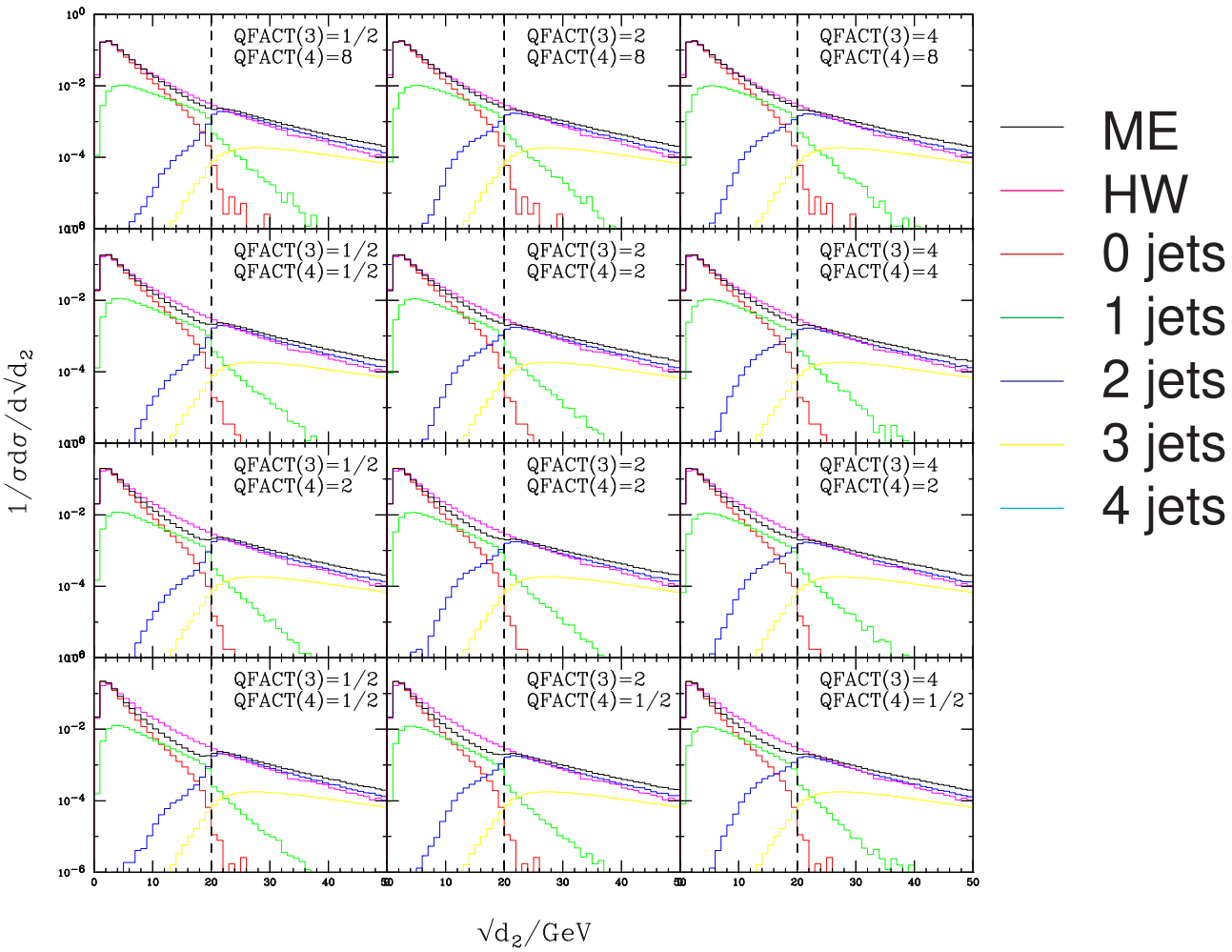
W Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- d_2 distribution at the Tevatron



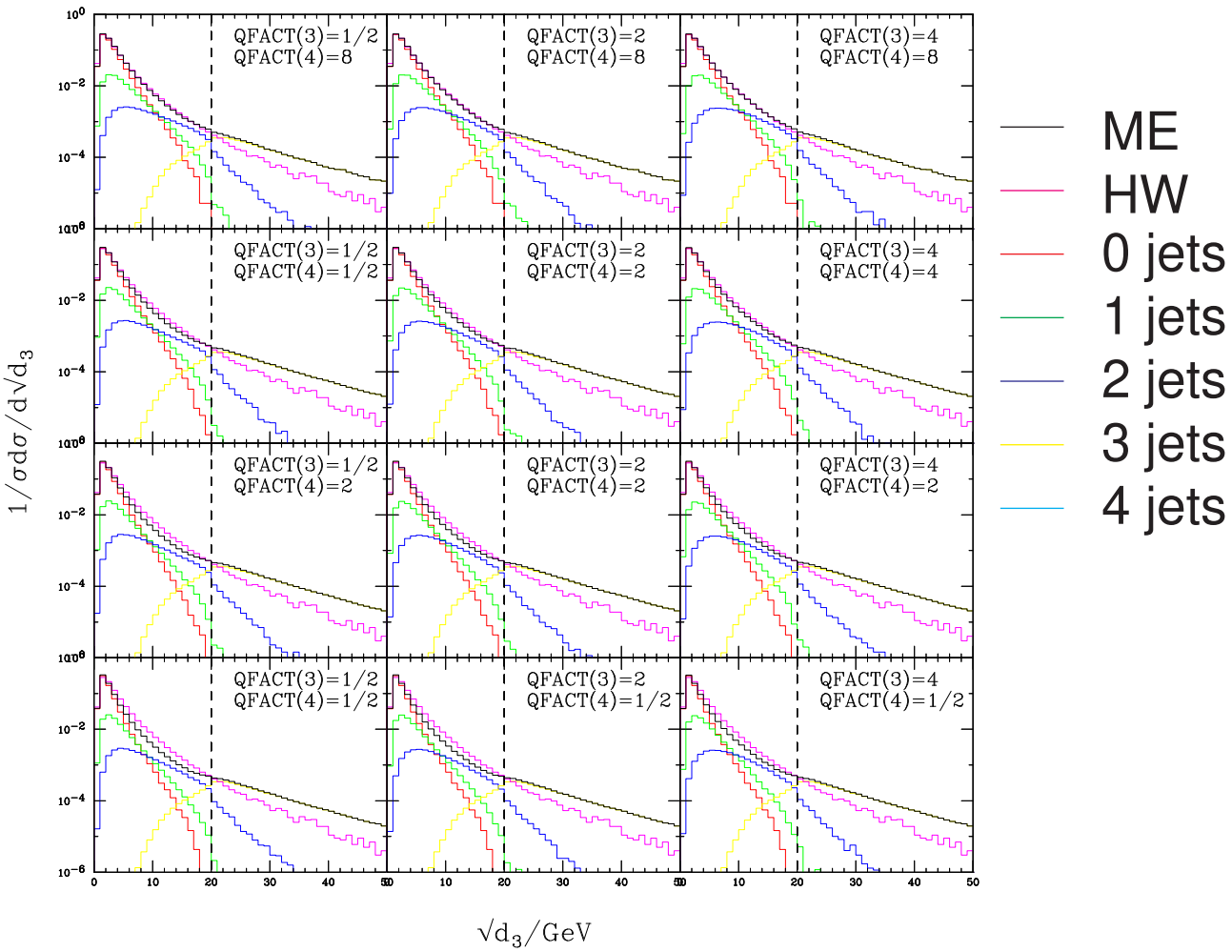
Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- d_2 distribution at the Tevatron



Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- d_3 distribution at the Tevatron

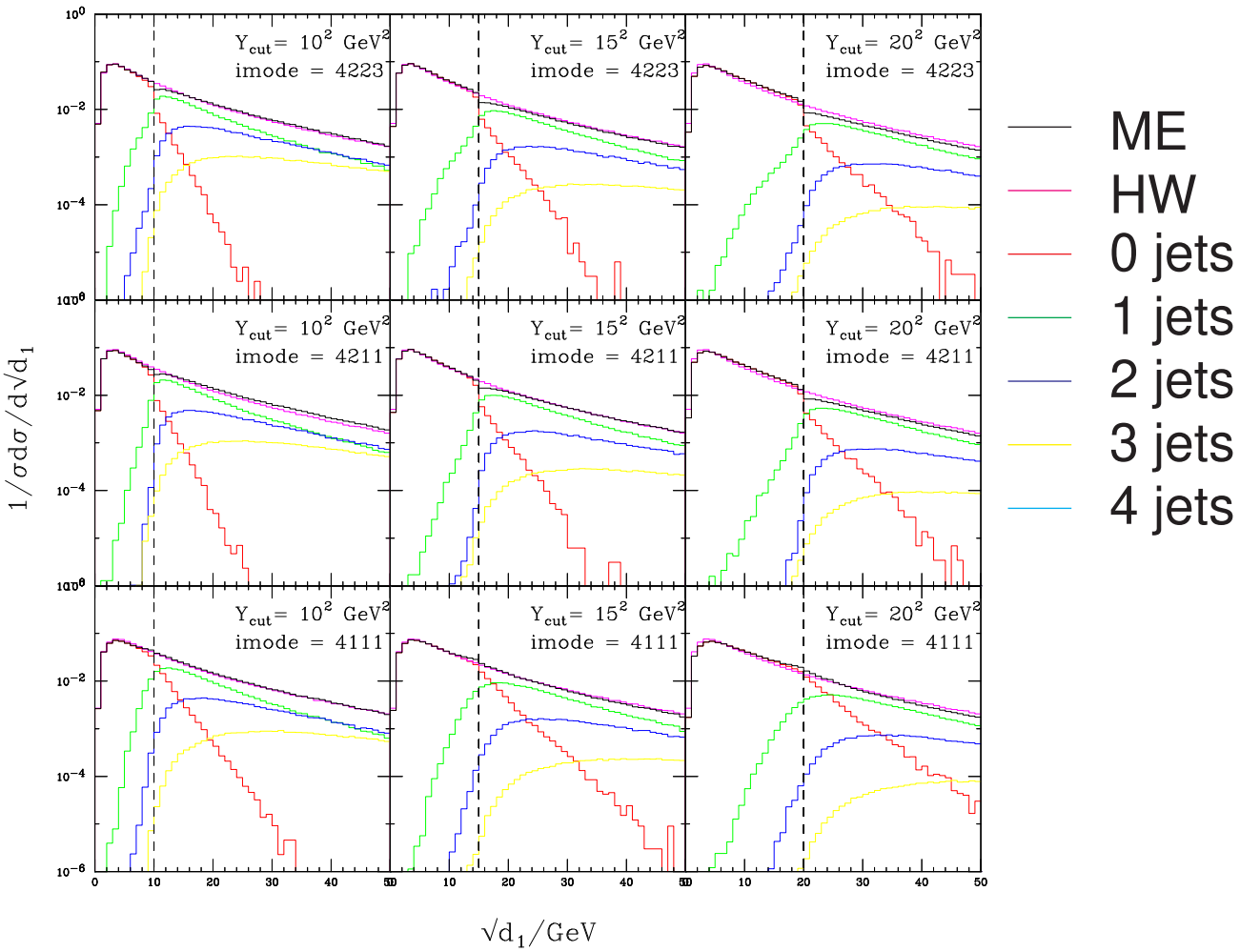


Hadron Collisions

- Also need to look at the variation with matching scale and algorithm
- Look at three scales
  - 10^2 GeV^2
  - 15^2 GeV^2
  - 20^2 GeV^2
- and three algorithms
  - E-scheme, `IMODE=4111`
  - Covariant E-scheme, `IMODE=4211`
  - Covariant monotonic p_T^2 scheme, `IMODE=4223`.

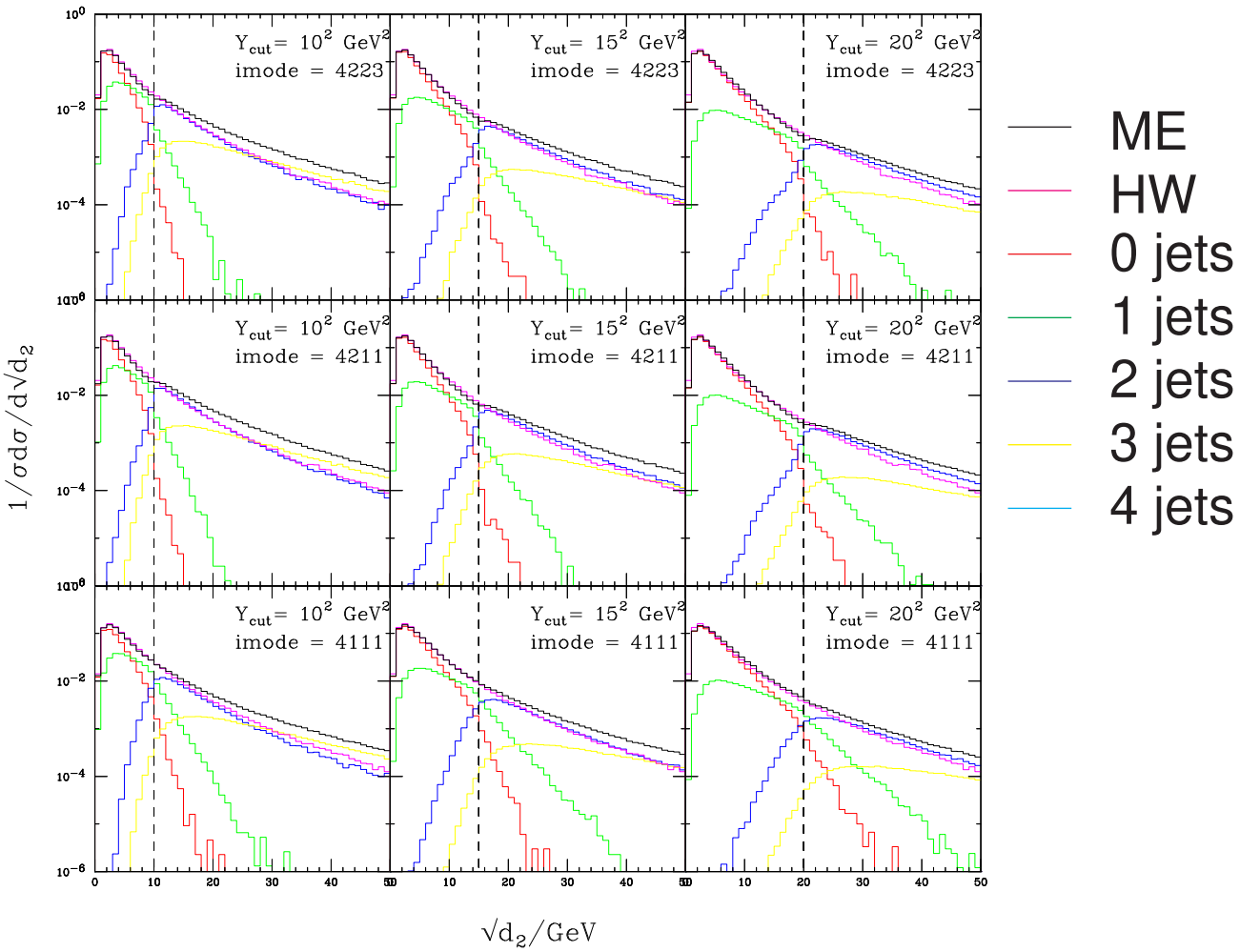
Z Production

- Variation of results with matching scale and algorithm
- d_1 distribution at the Tevatron



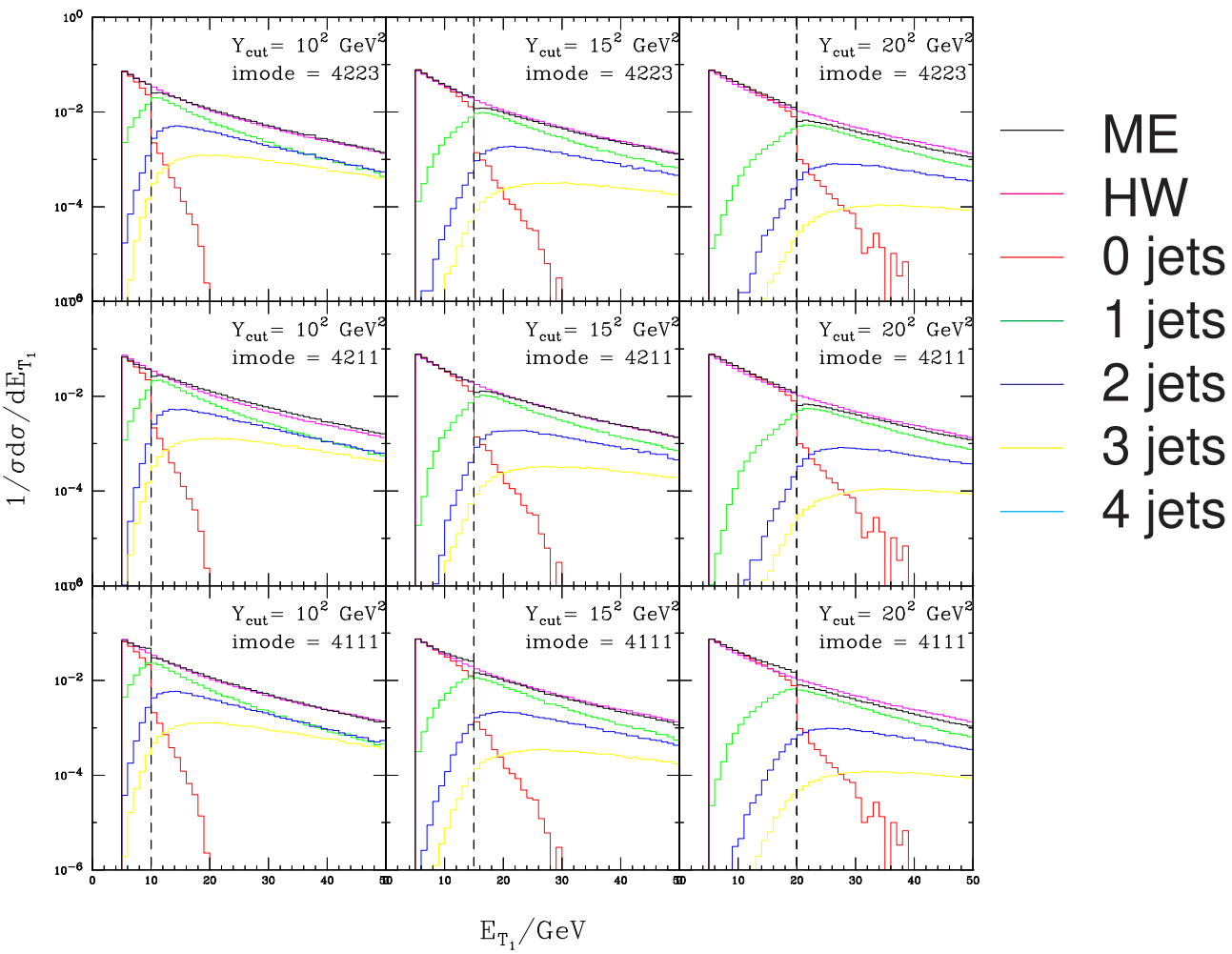
Z Production

- Variation of results with matching scale and algorithm
- d_2 distribution at the Tevatron



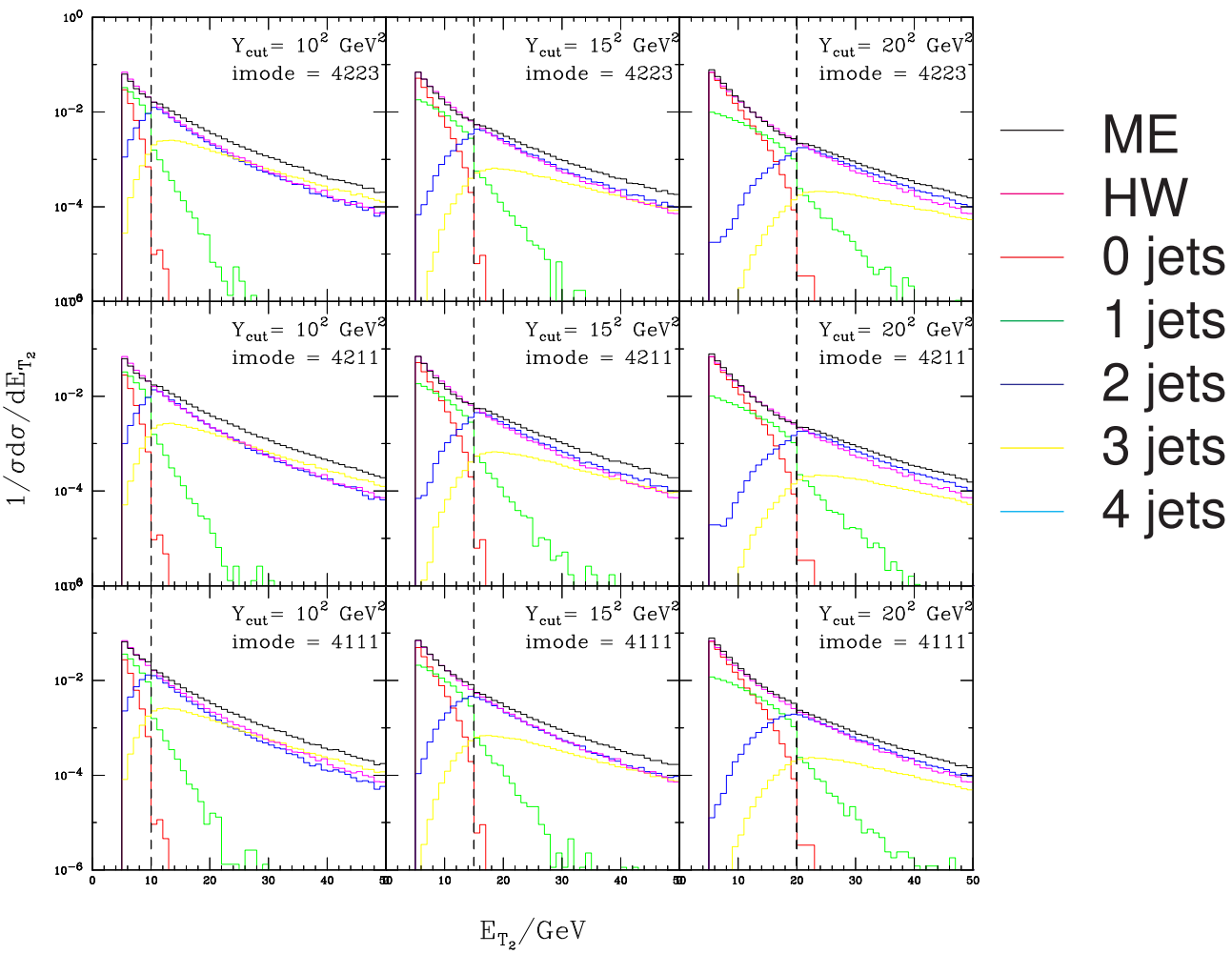
Z Production

- Variation of results with matching scale and algorithm
- E_{T_1} distribution at the Tevatron for cone R=0.4.



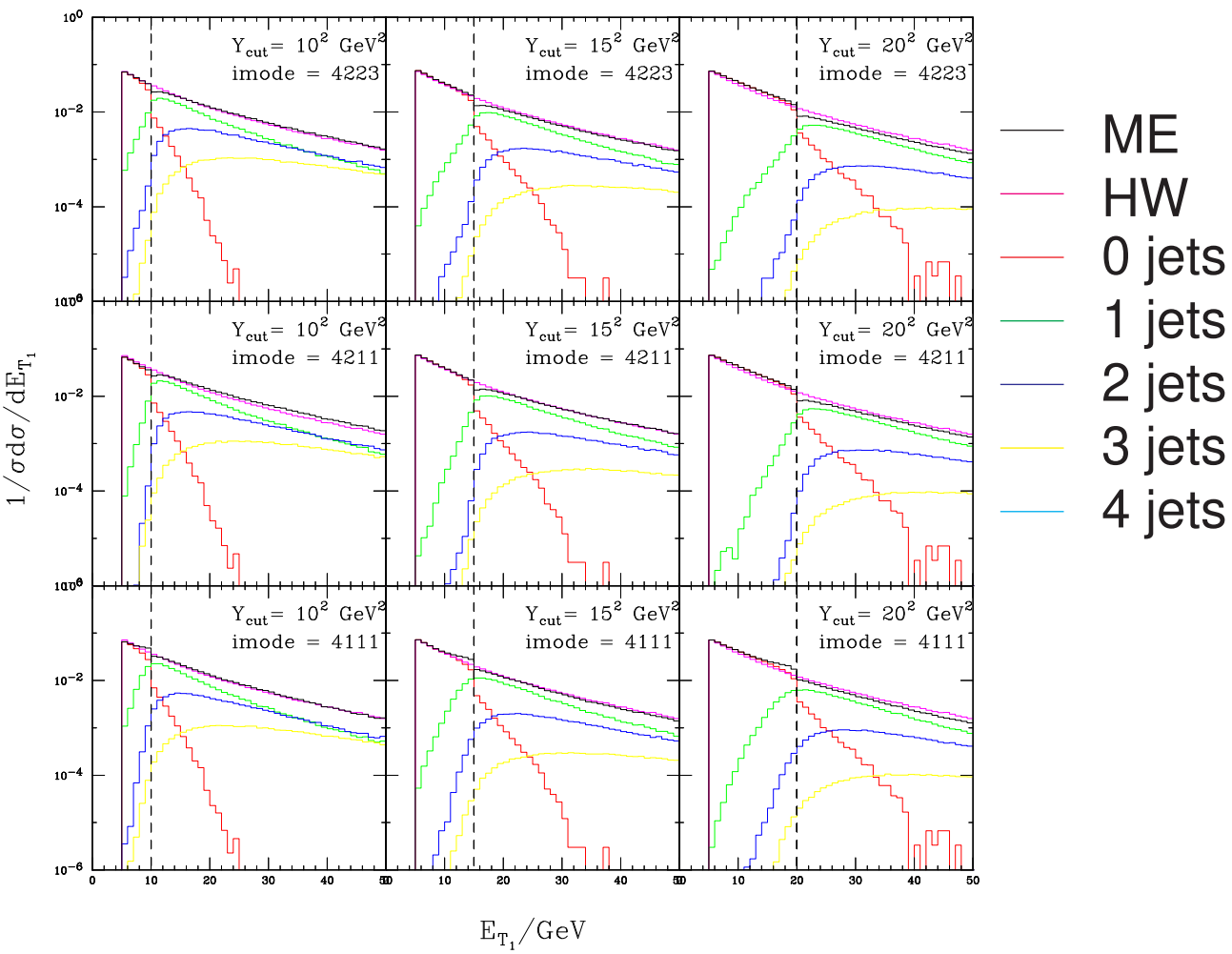
Z Production

- Variation of results with matching scale and algorithm
- E_{T_2} distribution at the Tevatron for cone R=0.4.



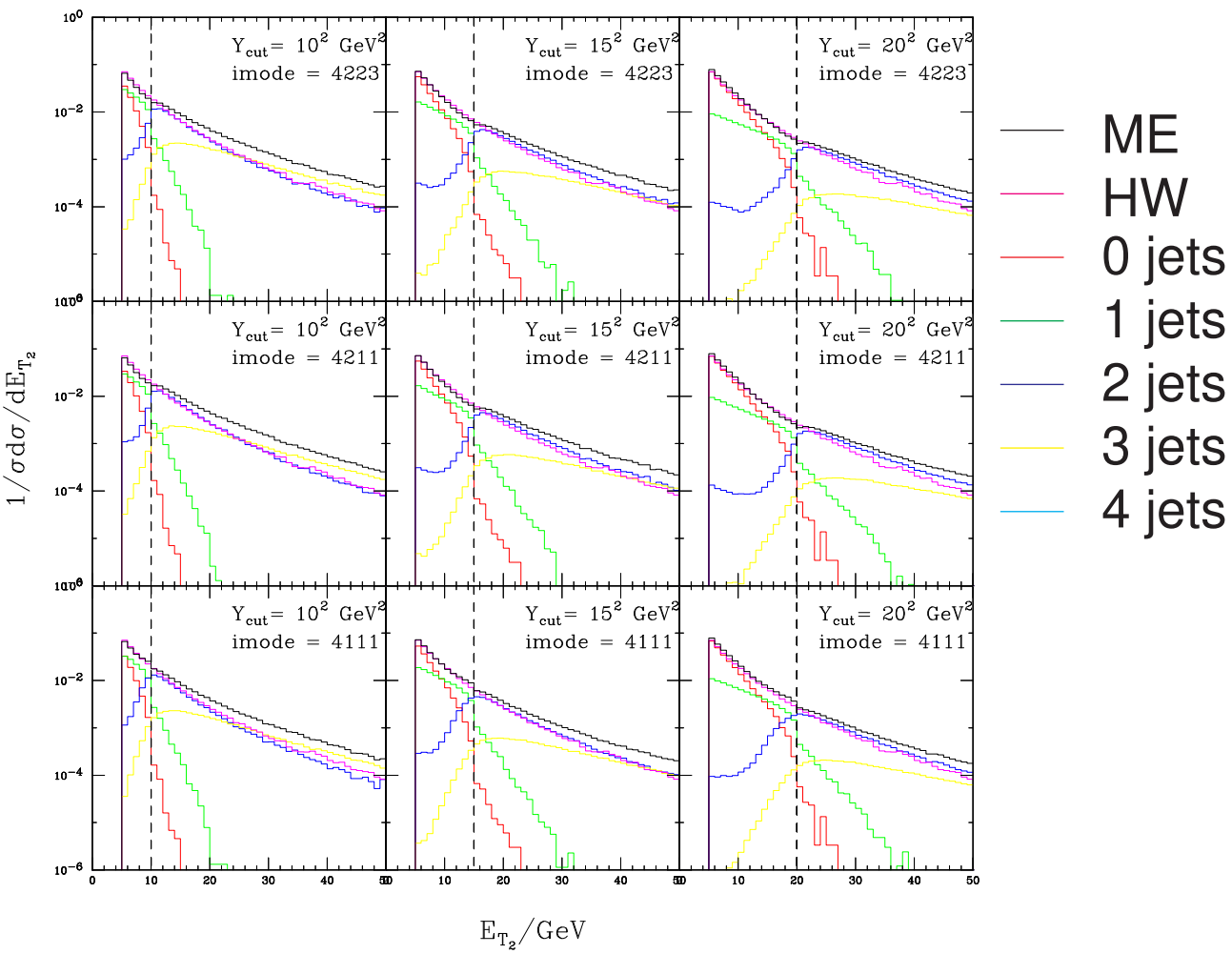
Z Production

- Variation of results with matching scale and algorithm
- E_{T_1} distribution at the Tevatron for cone R=0.7.



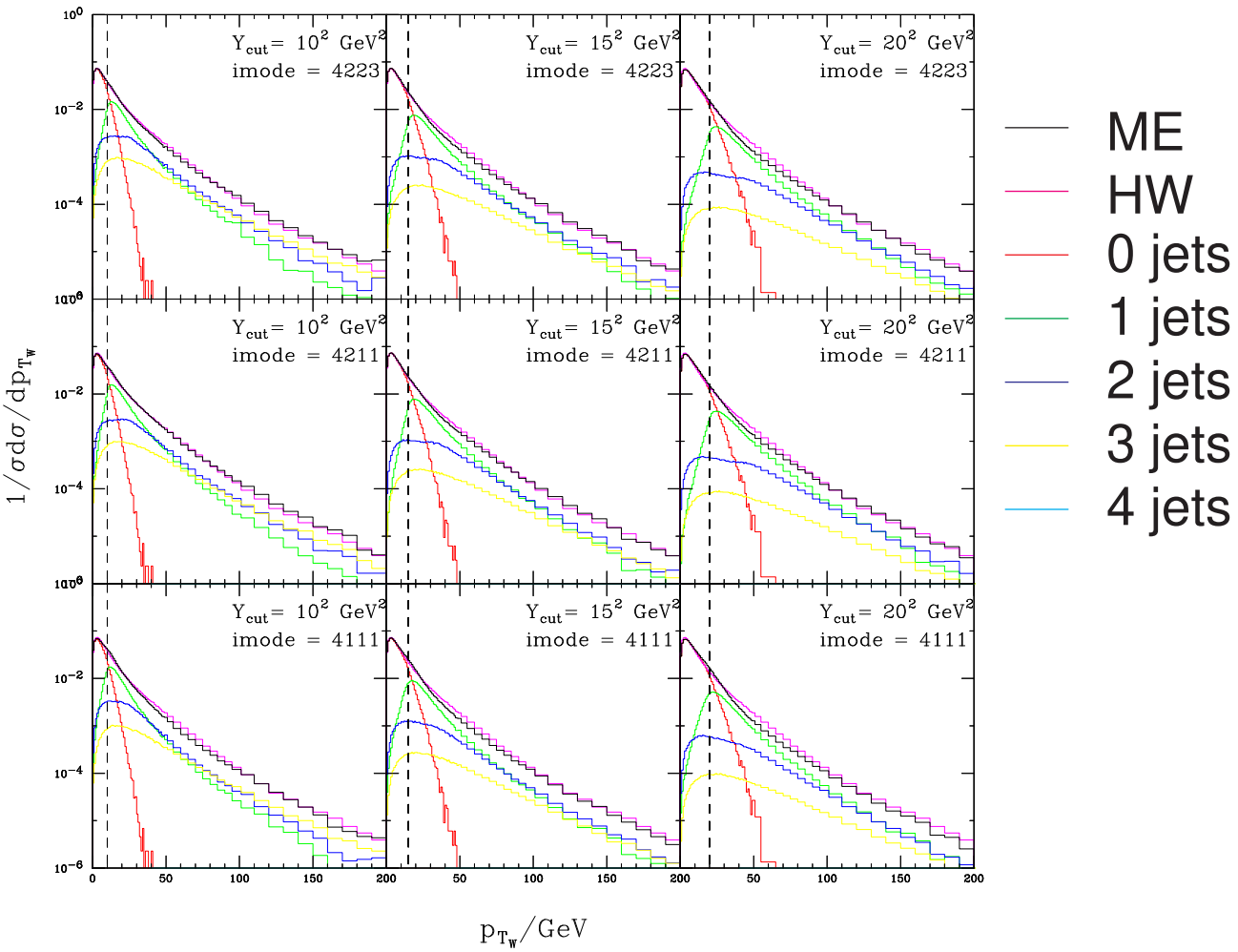
Z Production

- Variation of results with matching scale and algorithm
- E_{T_2} distribution at the Tevatron for cone R=0.7.



Z Production

- Variation of results with matching scale and algorithm
- p_T distribution of the Z at the Tevatron



Hadron Collisions

- Variation of the Z results with scale.
- Problem as the matching scale approaches the minimum Z mass.
- Due to the scales in the Sudakovs.
- For a one jet event at the boundary scale in Sudakov is $k^2_{T,\mathrm{match}} + M^2_Z$.
- For a zero jet event it is M^2_Z
- This is fine provided $M_Z \gg k_{T,\mathrm{match}}$
- Solution replace the scale M^2_Z with $k^2_{T,\mathrm{match}} + M^2_Z$ in the Sudakovs for the zero jet case.
- This will reduce this component with respect to the rest.

Conclusions

- I have presented two sets of recent improvements to Monte Carlo event generators.
- These give us a better description of multi-jet/lepton final state via either
  - Heavy particle decay
  - Hard QCD jet production
- Spin correlations in heavy particle decays are fully implemented.
- Some work remains to be done for hard QCD jet production.