Simulation of High Multiplicity Jet and Lepton Final-States

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- Introduction
- Basics of Monte Carlo Simulations
- Spin Correlations
- Matrix Element Corrections
- Conclusions

Introduction

- Monte Carlo simulations are an essential part of most experimental analysis, either of the Standard Model or for searches for new physics.

- They are often treated as black boxes by experimentalists and not understood by theorists.

- It's important to understand the assumptions and approximations involved in these simulations.

- Experimental physicists need to be able to answer the following questions:
  - Is the effect I'm seeing due to different models, or approximations, or is it a bug?
  - I'm I measuring a fundamental quantity or merely a parameter in the simulation code?

- Theorists need to understand enough to be able to understand problems due to the simulations used in getting experimental results.

Introduction

- There is a lot of interest in the simulation of processes involving large numbers of outgoing jets/leptons:
  - Top quark production including decays;
  - Signal Processes in BSM physics;
  - SM Electroweak gauge boson production with additional jets.
- In this talk I will discuss two sets of recent improvements to the simulation of these processes.
  - One to the simulation of processes where the jets/leptons come from the decay of heavy particles.
  - The second improves the simulation of large numbers of hard QCD jets, *e.g.* W+jets.
- These are important for physics at both the Tevatron and LHC.

Basics of Monte Carlo Generators

- All event generators split the simulation up into a number of parts:
  - Hard Process;
  - Parton Shower;
  - Secondary Decays;
  - Multiple Scattering/Soft Underlying Event;
  - Hadronization;
  - Hadron Decays.
- The general approach is the same in different programs but the models and approximations used are different.
- I will try and discuss the various models and approximations used in the different programs and compare them.
- I will try and give a fair objective comparison, but bear in mind I'm one of the authors of HERWIG.

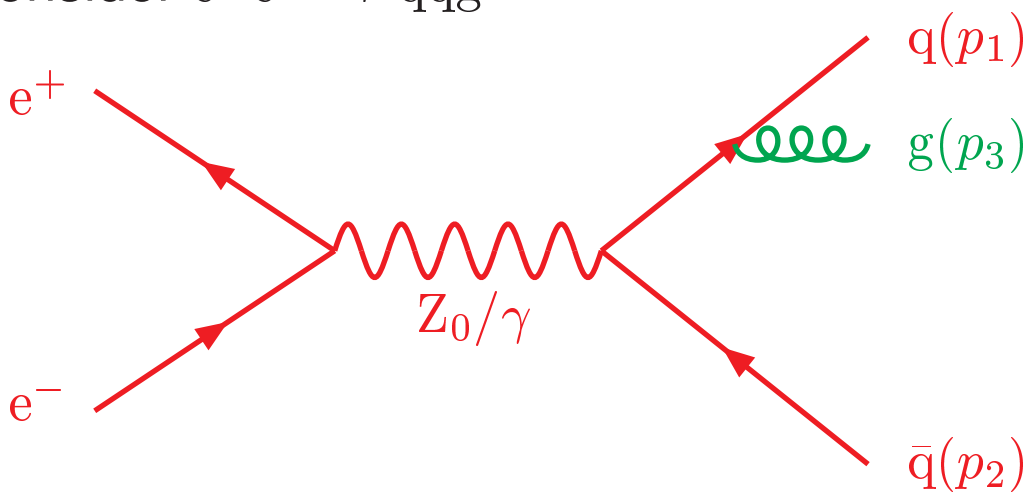
$p,\bar{p}$
$p,\bar{p}$
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
u
$\bar{p}$
$W+$
t
b
$\bar{t}$
$\bar{b}$
$W-$
Hadrons
Hadrons

QCD Radiation

- It is impossible to calculate and integrate the matrix elements for processes involving large numbers of partons.

- Instead we treat the regions where the emission of QCD radiation is enhanced.

- Consider $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}\mathrm{g}$



- The cross section is given by

$$\frac{d\sigma_{q\bar{q}g}}{dx_1 dx_2} = N_c \sigma_0 Q_q^2 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- This diverges as $x_{1,2} \to 1$ where $x_i = 2E_i/\sqrt{s}$.

QCD Radiation

- Physically this happens when

1. The gluon and one of the quarks are collinear

$$(1 - x_1) = \frac{E_2 E_3}{Q^2}(1 - \cos\theta_{2g})$$

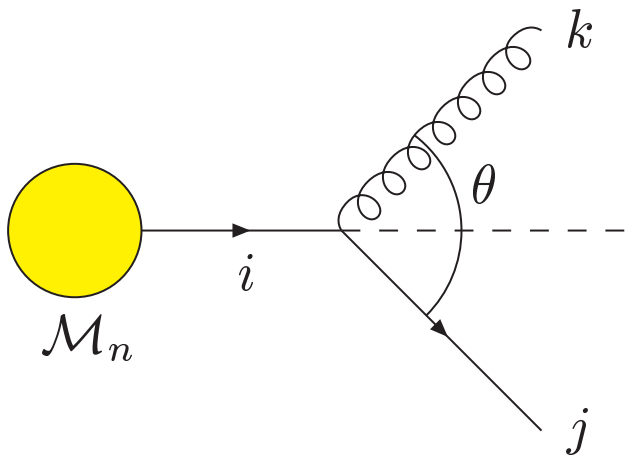
Collinear singularity

2. The energy of the gluon goes to zero

$$x_1 + x_2 + x_3 = 2, x_1, x_2 \to 1 \Rightarrow x_3 \to 0$$

Soft Singularity

Collinear Singularities



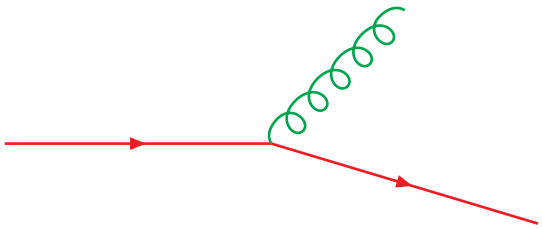
- In the collinear limit the cross section for a process factorizes

$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2}\, dz \frac{\alpha_s}{2\pi} P_{ji}(z),$$

- P_{ji} is the DGLAP splitting function.
- This is singular as $\theta \to 0$.
- What is a parton? (difference between collinear pair and a parton)

Collinear Singularities

- Introduce resolution criterion *e.g.* $k_T > Q_0$.
- Combine virtual corrections & unresolvable emission.



Resolvable Emission

Finite

Unresolvable Emission

Finite

- Unitary: Unresolved + Resolved = 1.

Monte Carlo Procedure

- Probability of Emission between $q^2 + dq^2$ and q^2

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2)$$

- Define probability of no emission between Q^2 and q^2 $\Delta(Q^2, q^2)$

$$\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2, q^2) = \exp\left[-\int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2)\right]$$

- $\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ The Sudakov Form Factor probability of emitting no resolvable radiation.

Monte Carlo Procedure

- We can implement this in the following way:
  - Generate a random number $0 < \rho < 1$;
  - If $\rho < \Delta(Q^2)$ no resolvable emission;
  - Otherwise solve

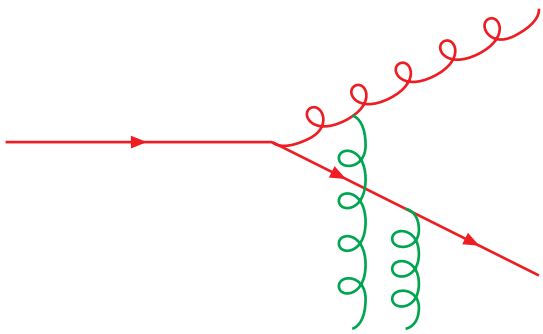
$$\rho = \Delta(Q^2, q^2)$$

    to give the emission scale q^2.

- A number of formally free choices
  - Evolution scale: $q^2/k_\perp/\theta$
  - z: Energy, light-cone momentum.

Soft Emission

- There are two regions where the emission of QCD radiation is enhanced.
- The soft limit also factorizes but at the amplitude level.

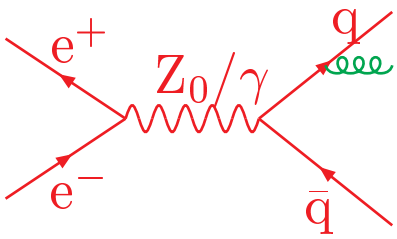


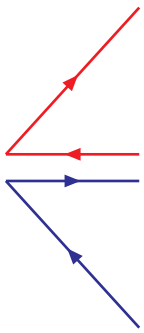
- Soft gluons come from everywhere in the event.
- There is quantum interference between these gluons.
- Does this spoil the parton shower picture?

Angular Ordering

- HERWIG uses the angular-ordered parton shower to resum both the collinear and soft logarithms.

- *e.g.* $e^+e^- \longrightarrow q\bar{q}g$ with radiation of an extra gluon

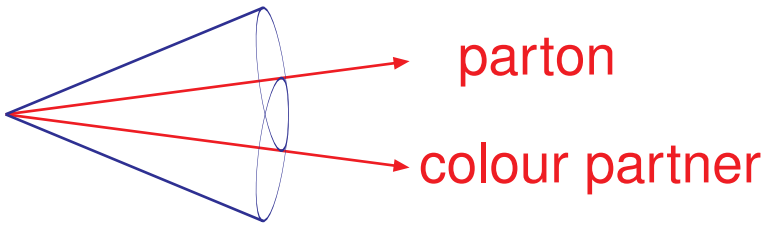




Colour Flow

- We then get that
  - q and the g are colour connected.
  - $\bar{q}$ and the g are anticolour connected.

- QCD radiation only occurs in a cone up to the direction of the colour partner.
(Marchesini & Webber)



Parton Shower

- ISAJET uses the original parton shower algorithm which only resums collinear logarithms.
- HERWIG uses the angular ordered parton shower algorithm which resums both soft and collinear singularities.
- PYTHIA use the collinear algorithm with an angular veto to try to reproduce the effect of the angular ordered shower.
- ARIADNE uses the colour dipole model.
- The collinear algorithm implemented in ISAJET does not give good agreement for angular ordering variables in hadron collisions or the string effect in e^+e^- collisions.
- HERWIG tends to give the best agreement for the hadron-hadron observables which are sensitive to soft effects.

Hadronization

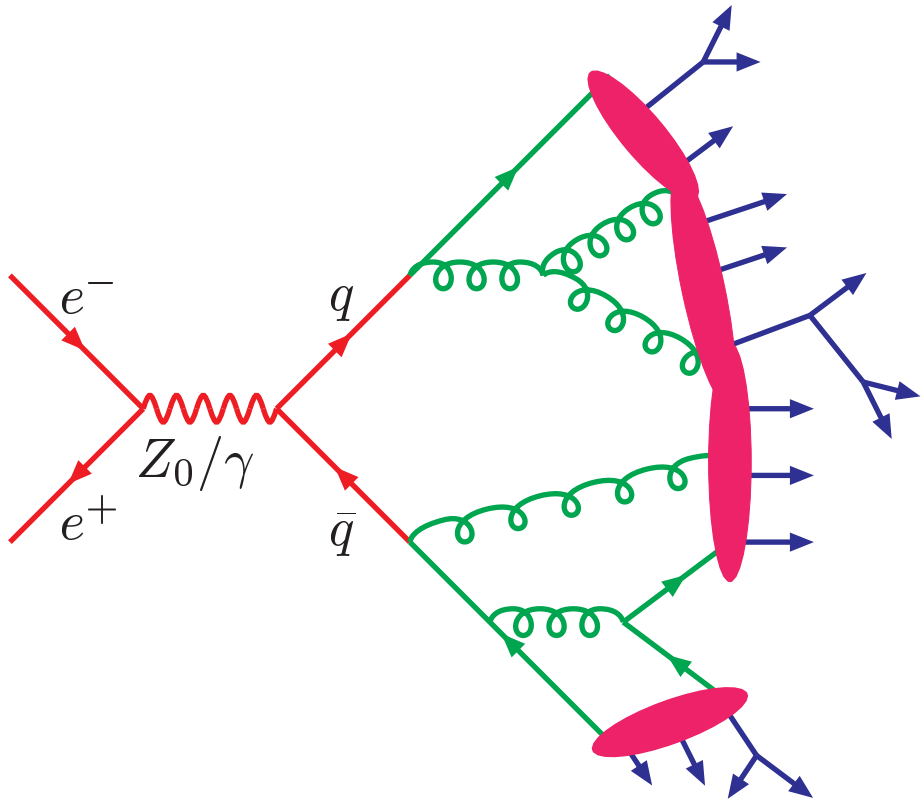
- ISAJET uses the original independent fragmentation model of Feynman and Field.
- PYTHIA uses the Lund string model.
- HERWIG uses the cluster hadronization model.
- ARIADNE uses the Lund string model.
- The independent fragmentation model cannot fit the LEP data.
- The cluster model gives good agreement with the LEP data on event shapes but doesn't fit the identified particle spectrum as well.
- The Lund string model give the best agreement with the LEP data.

Independent Fragmentation Model

- Original hadronization model, Feynman and Field
- Recursively split $\mathrm{q} \to \mathrm{q}' + \mathrm{hadron}$.
- Hook up the soft q and $\bar{\mathrm{q}}$ at the end.
- Strongly frame dependent.
- Doesn't agree with the data.

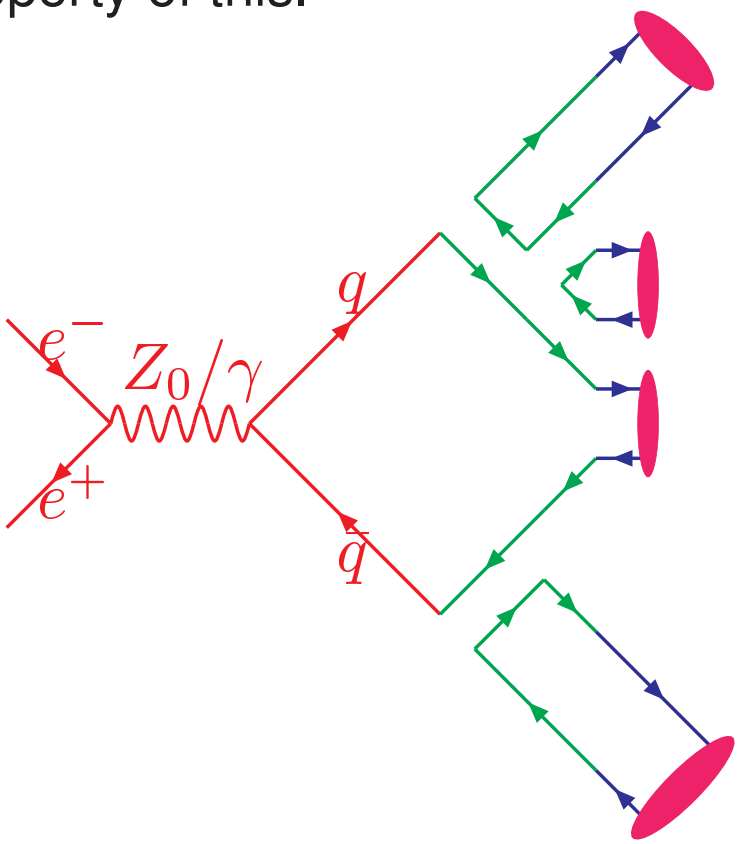
LUND String Model

- Assume quark and antiquark produced in $\mathrm{e}^+\mathrm{e}^-$ collisions joined by a string.
- As the move apart quark-antiquark pairs created by tunneling.
- String breaks up into hadrons.
- Very successful fit to data, however many parameters.



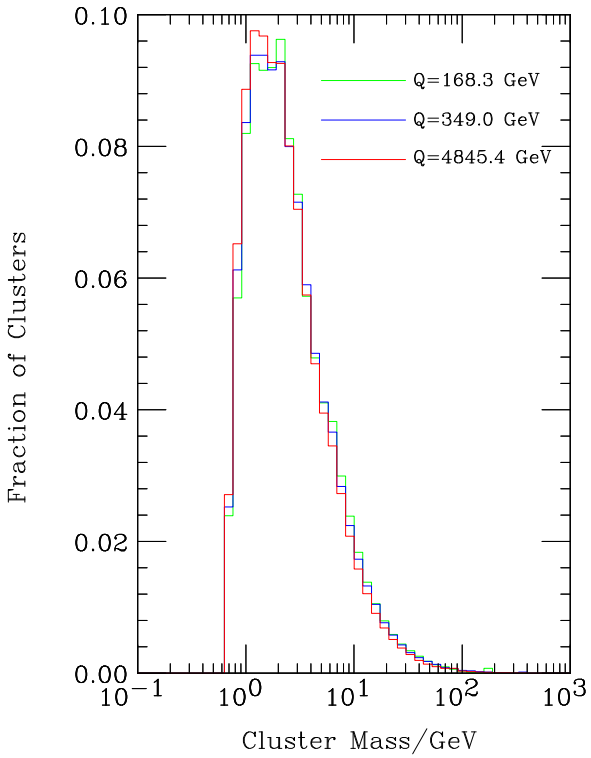
Cluster Hadronization Model

- HERWIG includes angular-ordered parton showers.
- The hadronization model is based on one property of this.



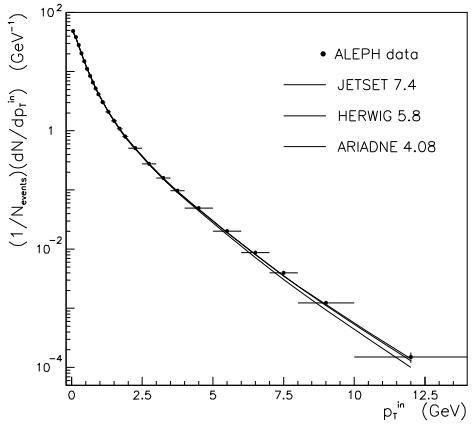
- First split the gluons into $q\bar{q}$ pairs.
- Each quark is then uniquely paired with an antiquark in a colour singlet.
- Form these into colour singlet clusters.

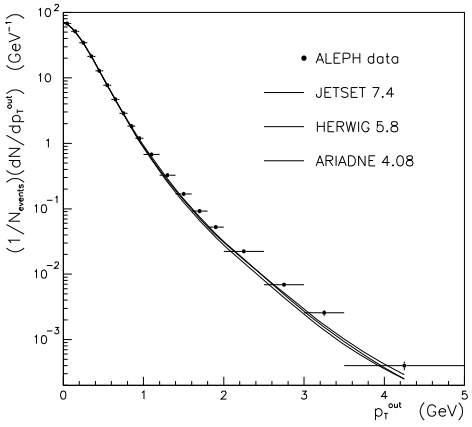
Cluster Hadronization Model

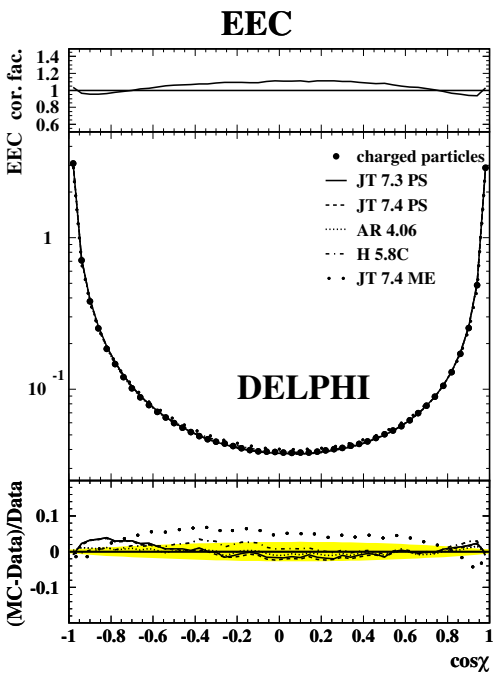


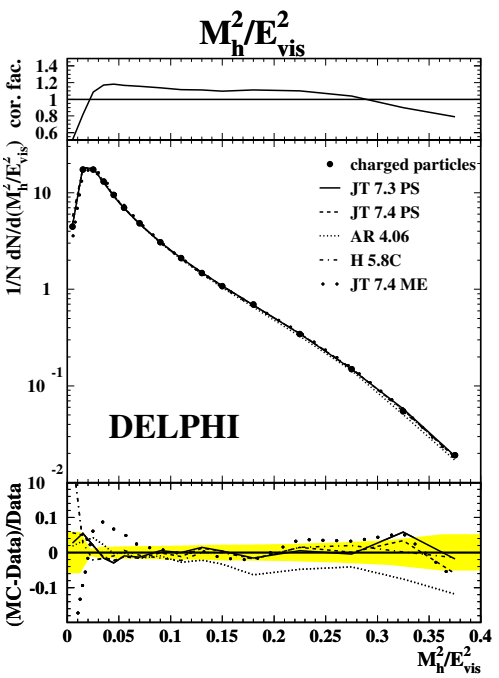
- The mass spectrum of pairs of colour singlet clusters after gluon splitting.
- Called colour 'pre-confinement'.
- Assume these clusters are superpositions of resonances.
- Decay according to phase space to hadrons.
- The hadrons decays are then performed using phase space and the measured branching ratios.

Event Shapes at M_Z



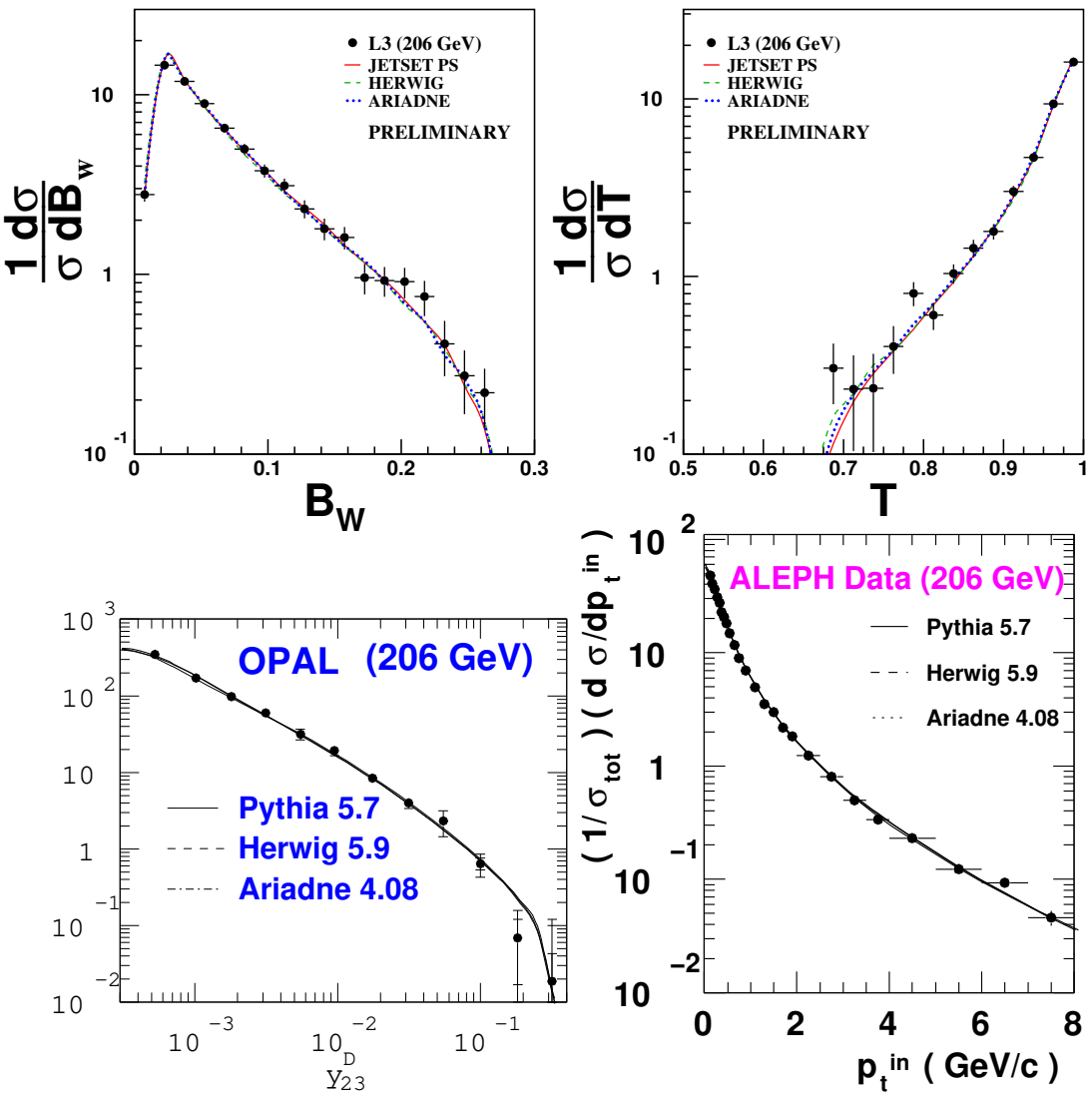






- Generally good agreement

Event Shapes at 206 GeV



- Remember only tuned at M_Z

Agreement with data

- The various generators are tuned by the LEP collaborations to data.
- Generally this is to data on event shapes and particle spectra.
- The basic message is
  - HERWIG, PYTHIA and ARIADNE all give good fits to event shapes.
  - PYTHIA and ARIADNE give the best fit to particle spectra.
  - ARIADNE gives the best overall fit.
- Conclusions from this
  - The perturbative QCD is well modelled by all the generators which include coherence.
  - HERWIG and ARIADNE are the most sophisticated for QCD.
  - The LUND string model is the best hadronization model.
  - Baryon production is a problem.
  - Heavy quark production is also problematic.

High p_T jet production

- We are now interested in processes which produce may high p_T jets, for example:
  - Top pair production including the decay products of the top;
  - BSM processes with high p_T jets from the decay of new heavy particles.
- The backgrounds to these processes are typically the production of electroweak gauge bosons with additional QCD radiation.
- We need better simulation of both the signal type processes and the backgrounds.
- To do this we need
  - Correlations between the decay products in the signal processes.
  - Better treatment of the emission of hard radiation in the parton shower.

Spin Correlations

- In the production of heavy particles there are correlations between the production and decay of the particles due to their spin.
- These correlations are known to be important in tau production.
- Run II of the Tevatron should have sufficient statistics to measure the correlations in $\mathrm{t\bar{t}}$ production.
- These correlations are also important for $\mathrm{t\bar{t}}$ production at a future linear collider.
- One potentially important application is in gaugino production at a future linear collider.
- Traditionally these processes have been calculated as $2 \to n$ processes including all the decay products.

Spin Correlations

- This requires the production and decay of the particle to be generated at the same time.

- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.

- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.

- Approaches have been developed which satisfy these requirements and are now implemented in HERWIG giving the correct correlation effects for a wide range of processes.

Spin Correlations

- HERWIG already includes correlations in and between jets using the algorithm of Collins and Knowles

  J. C. Collins, Nucl. Phys. B304, (1988) 794.

  I. G. Knowles, Nucl. Phys. B304, (1988) 767.

  I. G. Knowles, Nucl. Phys. B310, (1988) 571.

  I. G. Knowles, Comput. Phys. Commum. 58 (1990) 271.

- A basically identical algorithm can be used for the decays.

  P. Richardson, JHEP 0111:029, 2001.

- Instead of presenting the full algorithm I will illustrate it with an example.

- Consider the processes $q\bar{q} \to t\bar{t}$ followed by semi-leptonic decay of the top quarks.

- First we generate the momenta of the top quarks in $q\bar{q} \to t\bar{t}$ using the polarized cross-section

$$\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1\ldots\lambda_n}$$

Spin Correlations

- One of the outgoing quarks is selected at random and a spin density matrix calculated.

$$\rho_{\lambda_j\lambda'_j} = \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j} D^i_{\lambda_i\lambda'_i} \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}$$

  In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
  The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- This normalization is therefore the same as the matrix element used in the previous step.

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

$$\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda_1\ldots\lambda_n}$$

Spin Correlations

- In principle we could now continue down the decay chain, but here all the particles produced are stable.

- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda_0'} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n} D^i_{\lambda_i\lambda_i'}$$

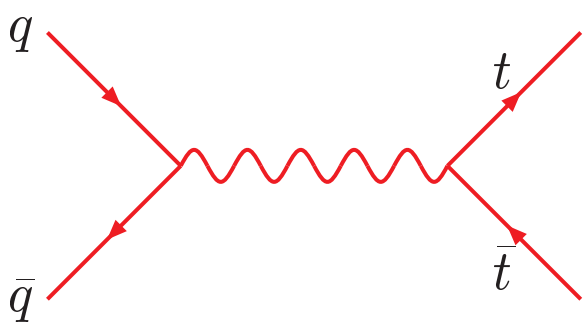
  The normalization is chosen so that $\mathrm{Tr}D = 1$.

- We then select another particle from the hard process. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.

- In this case this means we decay the second quark.

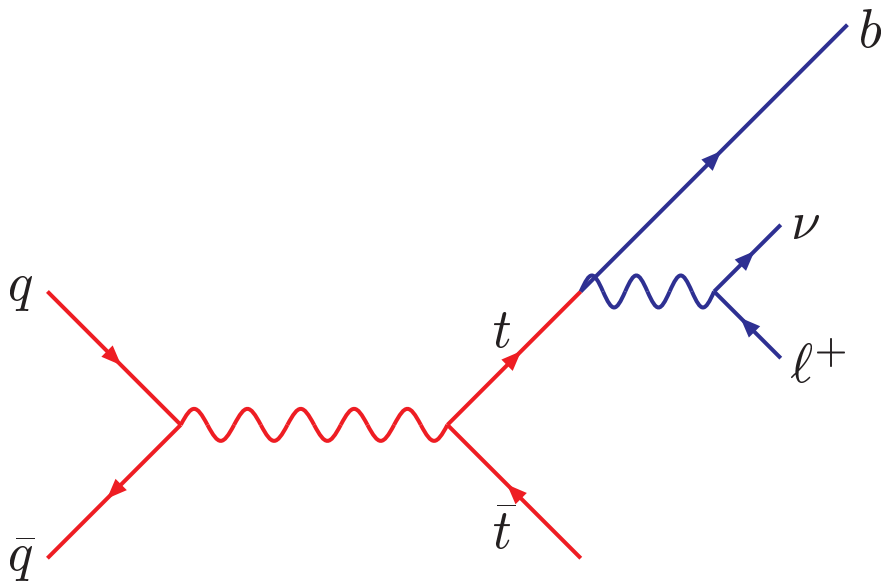
- As before the normalization of the spin density matrix cancels the matrix element used for the previous step.

Top pair Production



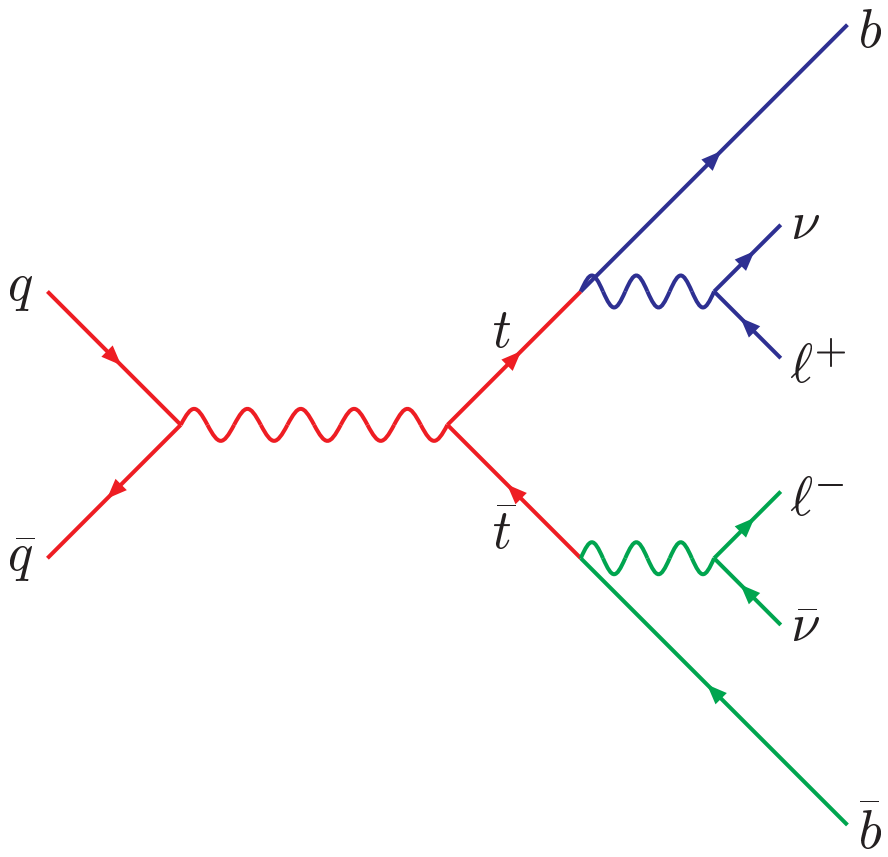
$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t\lambda_{\bar{t}}}$$

Top pair Production



$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}}\mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}$$

Top pair Production

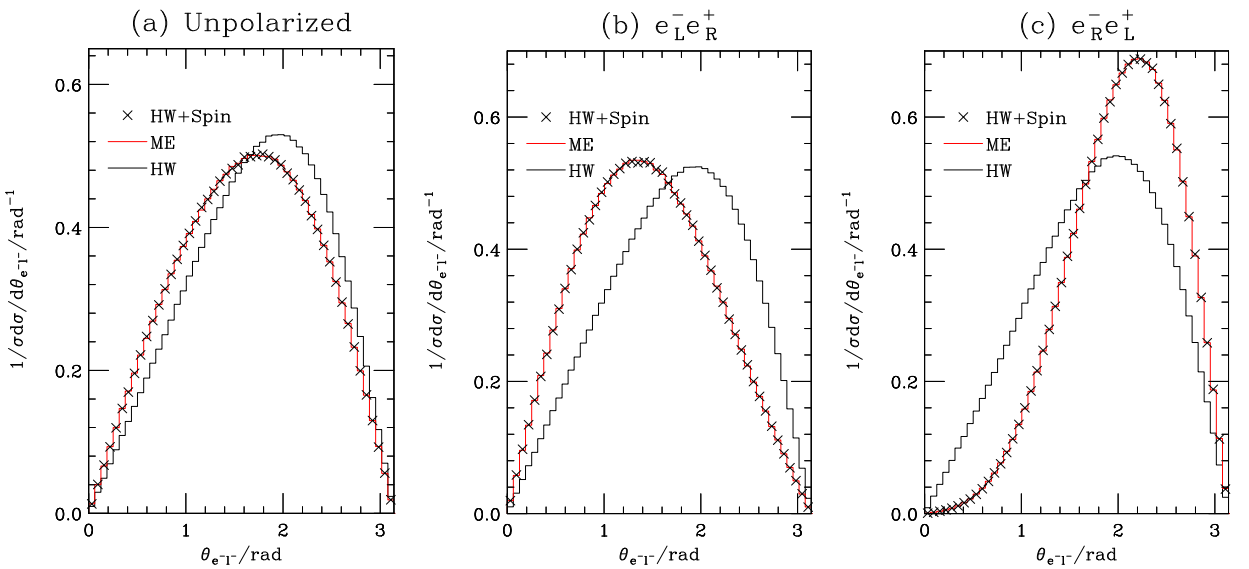


$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{q\bar{q}\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*q\bar{q}\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}'}$$
$$\mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}$$
$$\mathcal{M}^{\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}}\mathcal{M}^{*\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}'}$$

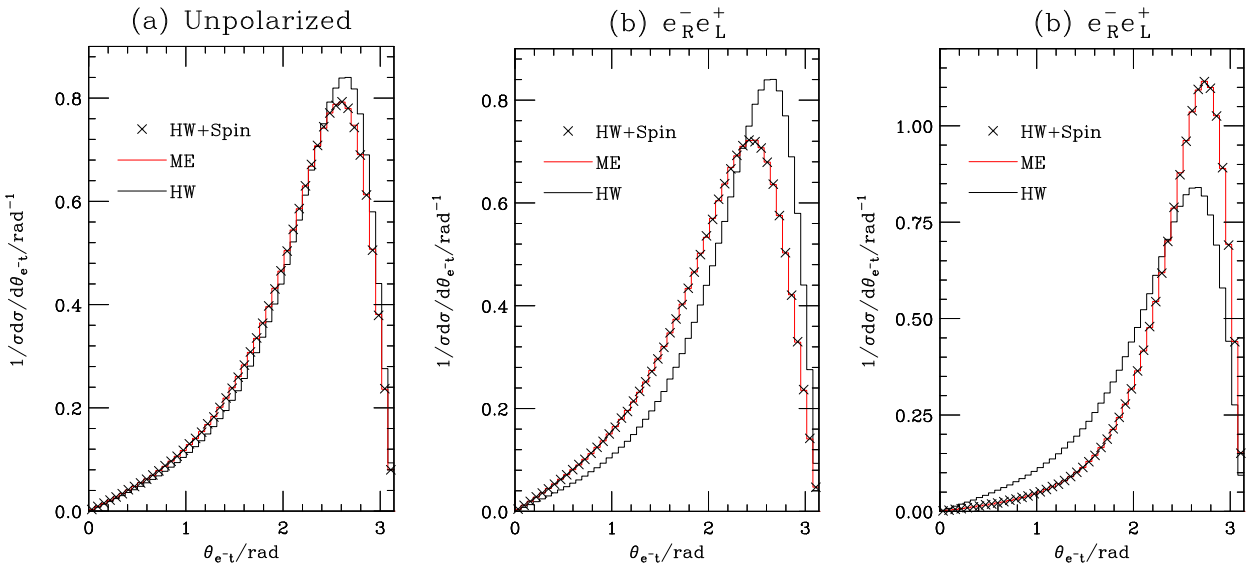
Top pair production in e^+e^- collisions

- Study the semi-leptonic decays of both top quarks and look at the correlations.

- Between the lepton produced in the antitop decay and the beam.



- Between the lepton and the top quark.



Matching matrix elements and Parton Showers

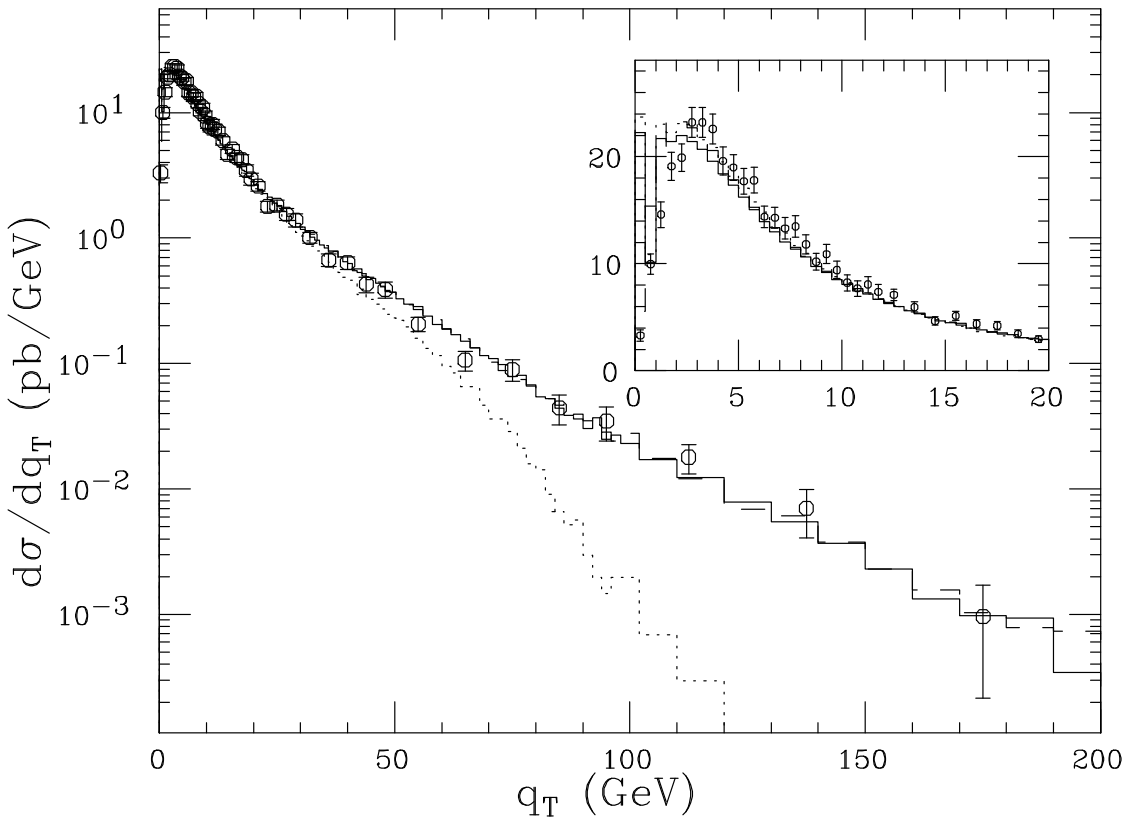
- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high p_T jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating many high p_T jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, *i.e.*
  - ME gives hard emission,
  - PS gives soft/collinear emission,
  - Smooth matching between the two,
  - No double counting of radiation.

Matching matrix elements and Parton Showers

- There have been a number of attempts to do this.
- The first attempts got the hardest emission correct for simple processes, for example:
  - $e^+e^- \to q\bar{q}$;
  - DIS;
  - Drell-Yan;
  - Top Decay.
- There were problems with this:
  - Only the hardest emission was correctly described;
  - The leading-order normalization was retained.

Matching matrix elements and Parton Showers

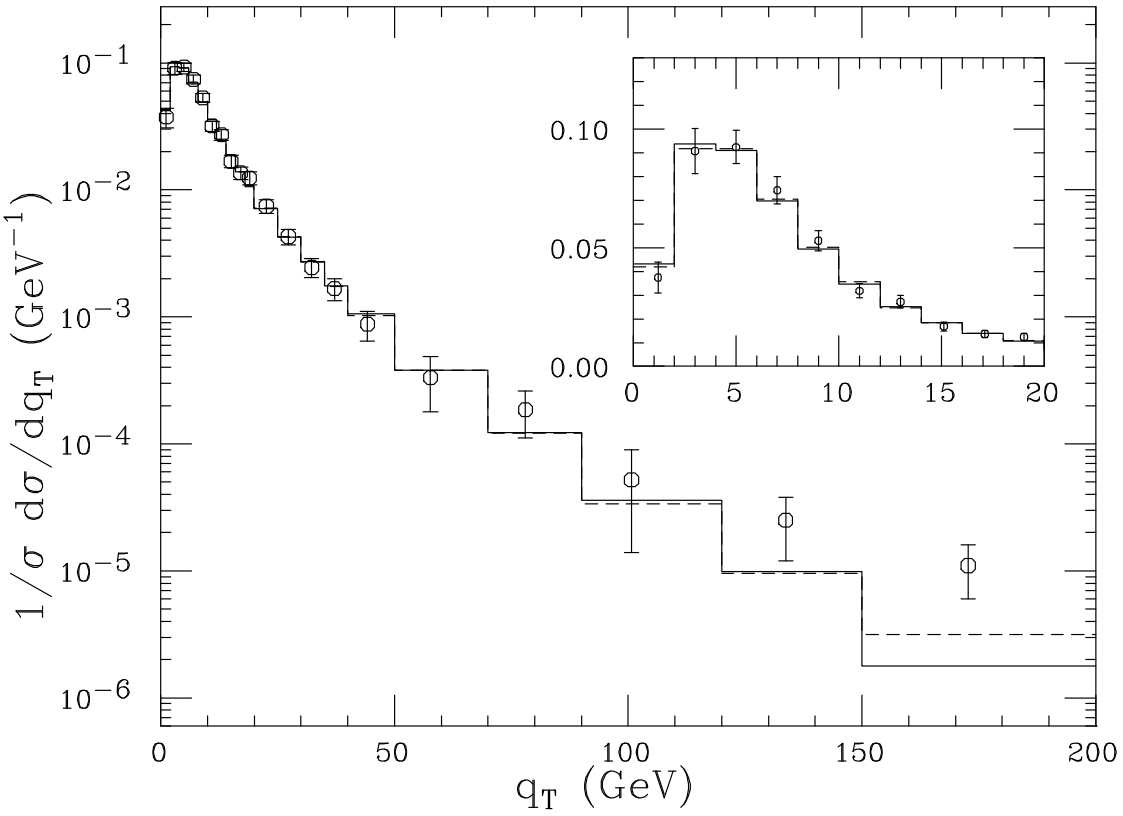
- Transverse momentum distribution of Z at the Tevatron (CDF) compared with HERWIG.



- Taken from Nucl. Phys. B565:227-244,2000, G. Corcella and M. H. Seymour.

Matching matrix elements and Parton Showers

- Transverse momentum distribution of W at the Tevatron (D0) compared with HERWIG.



- Taken from Nucl. Phys. B565:227-244,2000, G. Corcella and M. H. Seymour.

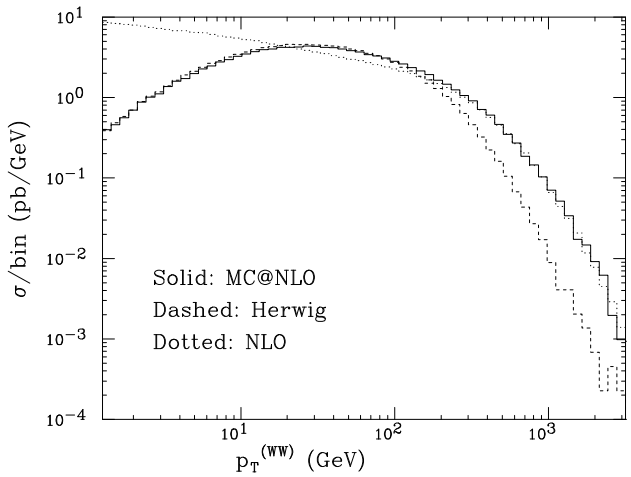
Matching matrix elements and Parton Showers

- Recently there has been renewed interest.
- Two types of approach have emerged.
  1. NLO Simulation
     - NLO normalization of the cross section.
     - Gets the hardest emission correct.
     - Only programs by Frixione/Webber and Dobbs.
  2. Multi-jet Leading Order
     - Still leading order.
     - Gets many hard emissions correct.
     - Programs by Krauss et. al. and Lonnblad for e^+e^-.

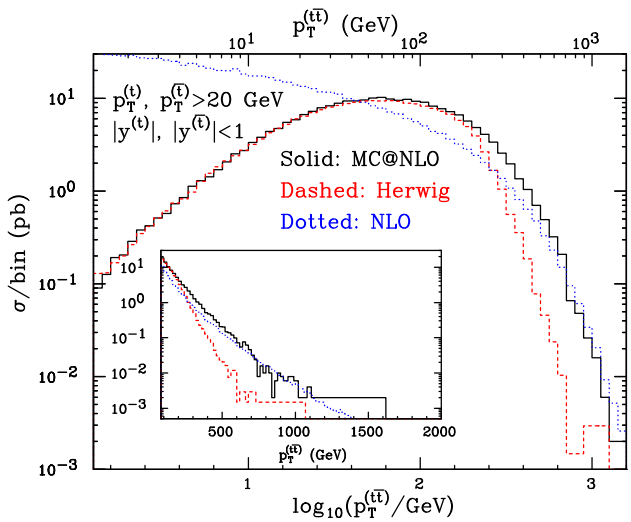
NLO Simulation

- These simulations are now available for a range of processes using the method of Frixione and Webber.

- Transverse momentum of W^+W^- pair at the LHC.



- Transverse momentum of $t\bar{t}$ pairs at the LHC.



Multi-Jet Leading Order Simulation

- While the NLO approach is good for one hard additional jet and the overall normalization can not be used to give many extra hard jets, at least at the moment.

- Therefore to simulate these processes use matching at leading order to get many hard emissions correct.

- I will briefly review the general idea behind this approach and then show some results from work I have been doing implementing it using MADGRAPH and HERWIG.

Procedure

- In order to match the ME and PS we need to separate the phase space:
  - One region should contain the soft/collinear region and be filled by the PS;
  - The other is filled by the matrix element.

- In Catani-Krauss-Kuhn-Webber (CKKW) prescription this is done via the Durham jet algorithm.
  - For all final-state particles compute the resolution variables
    * $d_{kB} \simeq E_k^2 \theta_{kB}^2 \simeq k_{\perp kB}^2, \quad \theta_{kB} \to 0.$
    * $d_{kl} \simeq \min(E_k^2, E_l^2)\theta_{kl}^2, \quad \theta_{kl} \to 0$
  - The smallest of these d_{kB}, d_{kl} is then selected. If d_{ij} is the smallest the two particles are merged, if d_{iB} is the smallest the particle is merged with the beam.
  - This procedure is repeated until the minimum value is above some stopping parameter $d_{\rm cut}$.
  - The remaining particles and pseudo-particles are then the hard jets.

Procedure

1. Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{k=N} \sigma_k}$$

   where σ_n is the n-jet matrix element evaluated at resolution $d_{\rm ini}$ using $d_{\rm ini}$ as the scale for the P.D.F's and α_S, n is the number of additional jets.

2. Distribute the jet momenta according to the ME.

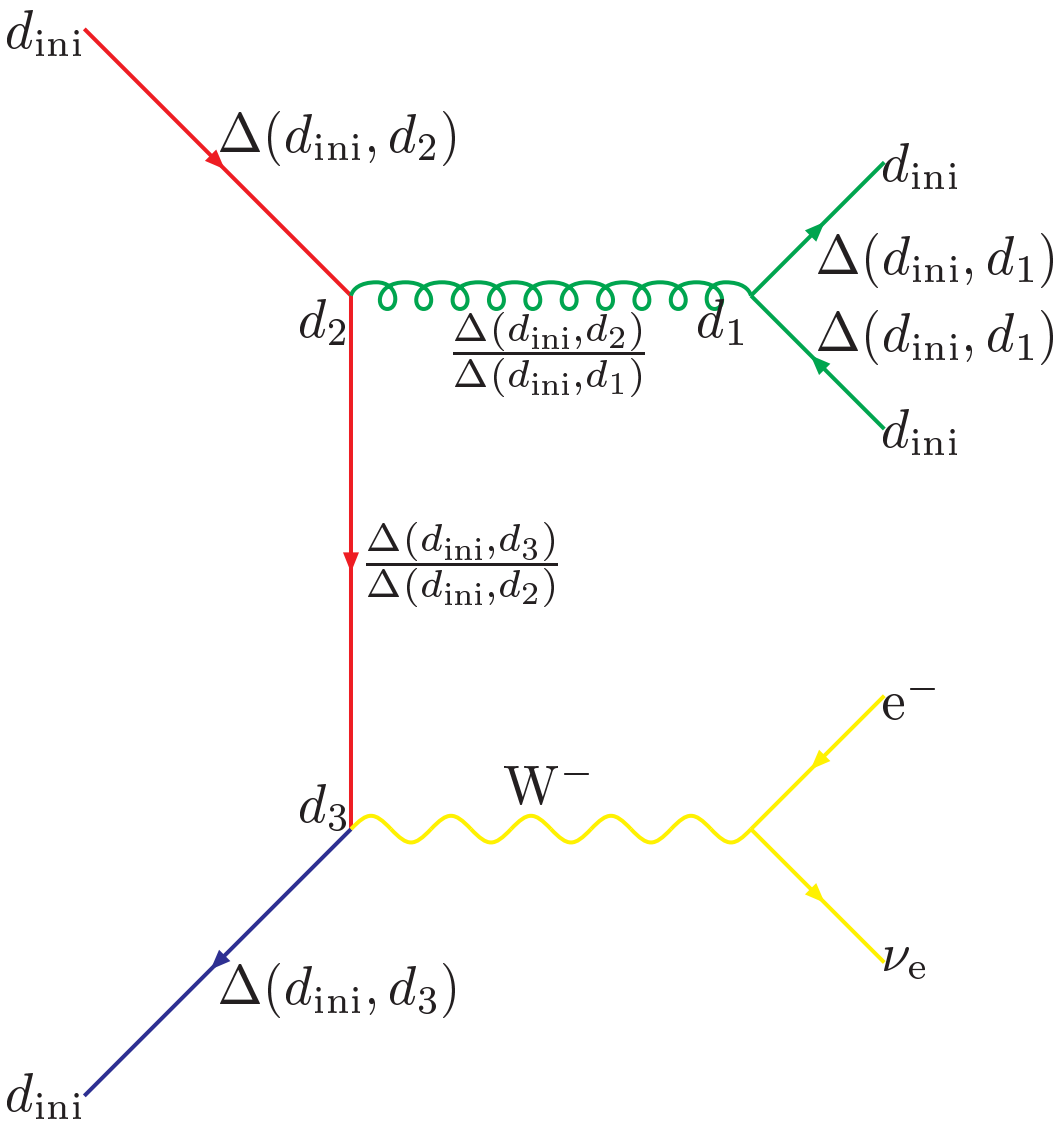
3. Cluster the partons to determine the values at which 1,2,..n-jets are resolved. These give the nodal scales for a tree diagram.

4. Apply a coupling-constant reweighting $\alpha_S(d_1)\alpha_S(d_2)..\alpha_S(d_n)/(\alpha_S(d_{\rm ini})^n \leq 1$.

5. Reweight the lines by a Sudakov factor $\Delta(d_{\rm ini}, d_j)/\Delta(d_{\rm ini}, d_k)$.

6. Accept the configuration if the product of the α_S and Sudakov weight is less the $\mathcal{R} \in [0, 1]$., otherwise return to step 1.

Procedure



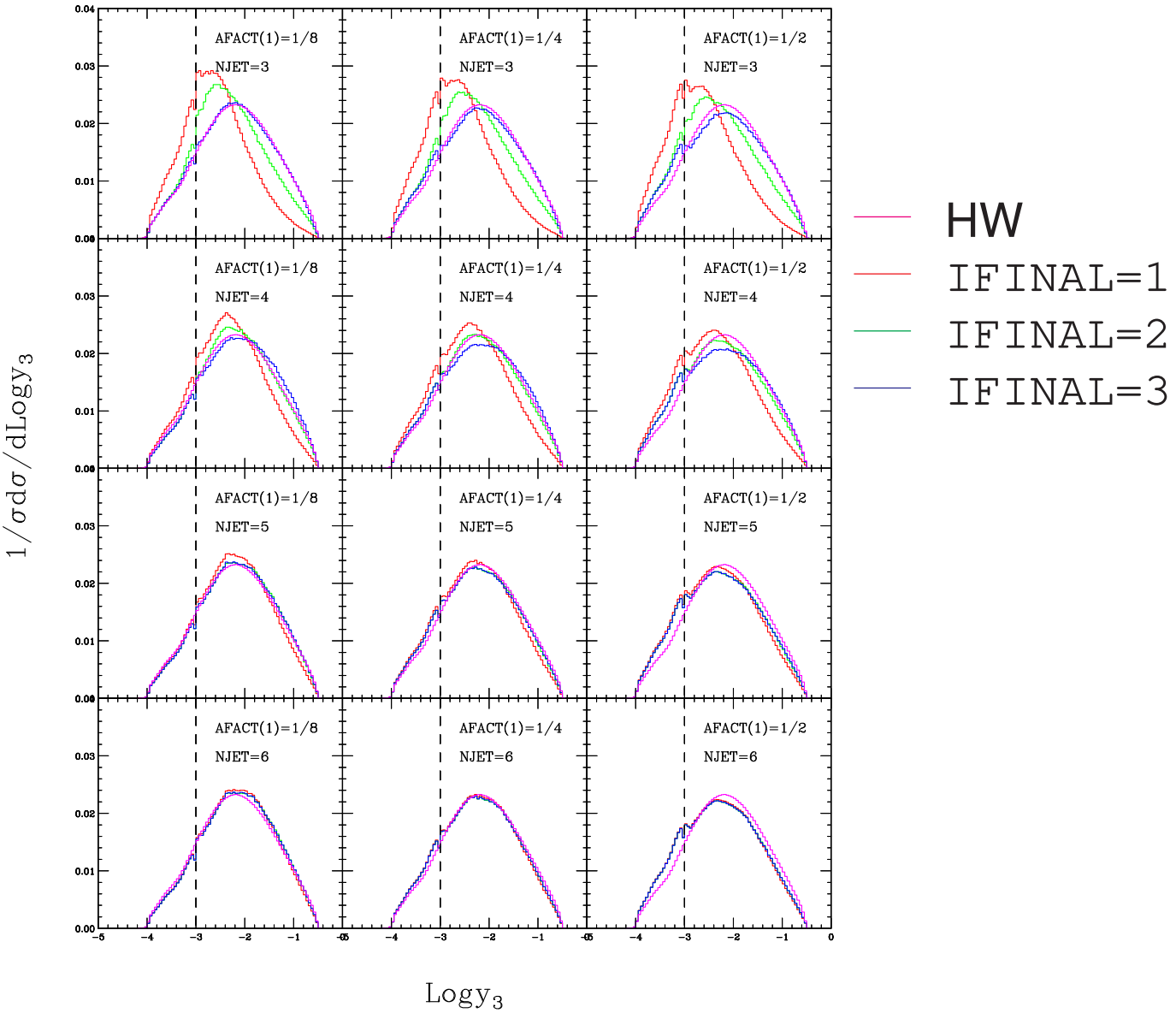
7. Generate the parton shower from the event starting the evolution of each parton at the scale it was created and vetoing emission above the scale $d_{\rm ini}$

Free Parameters

- Although the algorithm above ensures smooth matching at NLL log level there are a number of free choices
  - Sudakov Form Factors
    - HERWIG or NLL Sudakovs
    - Definition of the scale
    - Prefactor for the scale
  - α_S
    - LO or NLO definition
    - Definition of the scale
    - Prefactor for the scale
  - Treatment of Highest Multiplicity Matrix Element
    - Same as other multiplicities?
    - Different treatment to describe more emissions?
  - Definition of k_T-algorithm away from soft/collinear limit
    - Definition of the k_T-measure.
    - Recombination scheme.

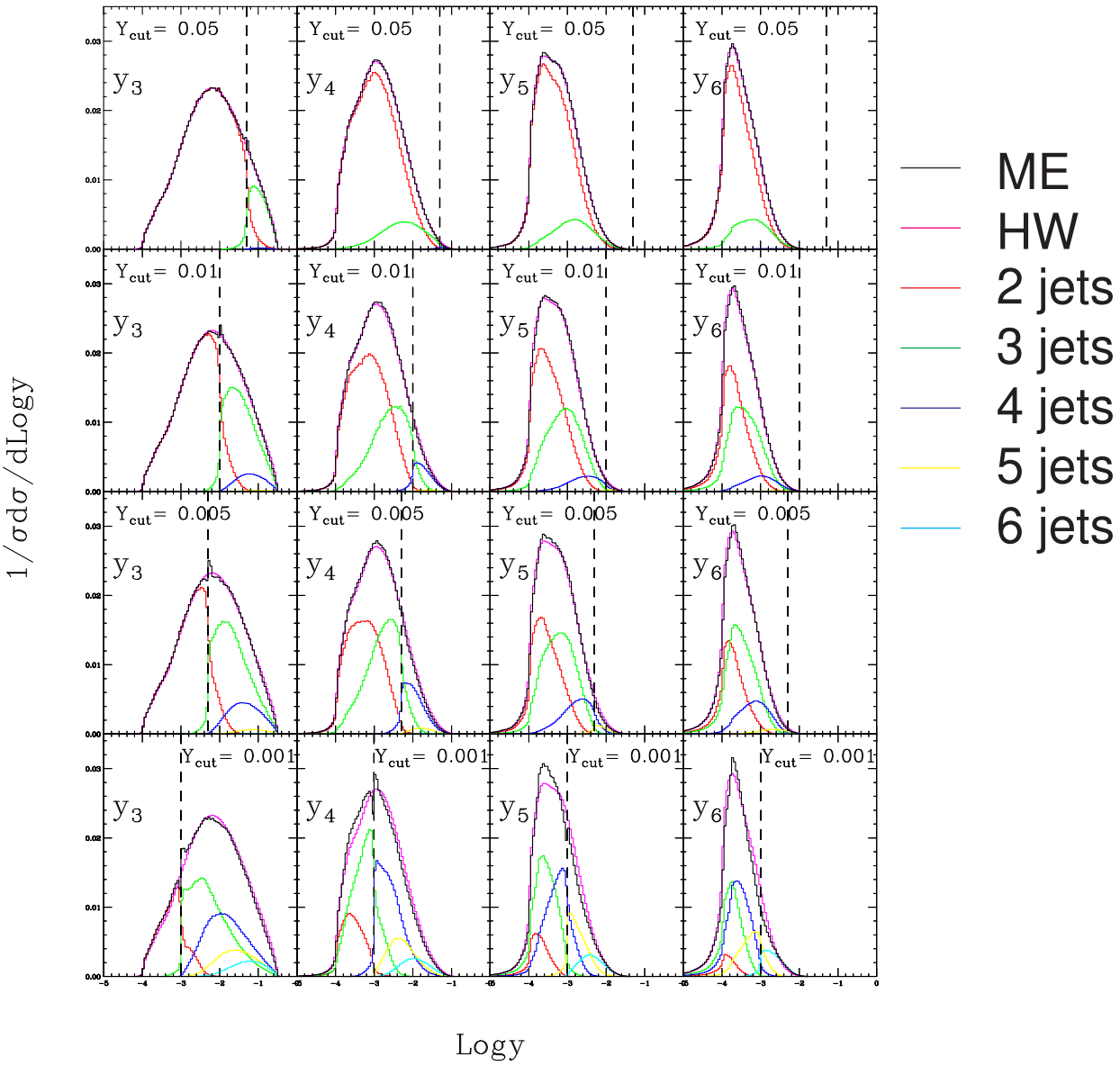
Treatment of Highest Multiplicity

- y_3 distribution.



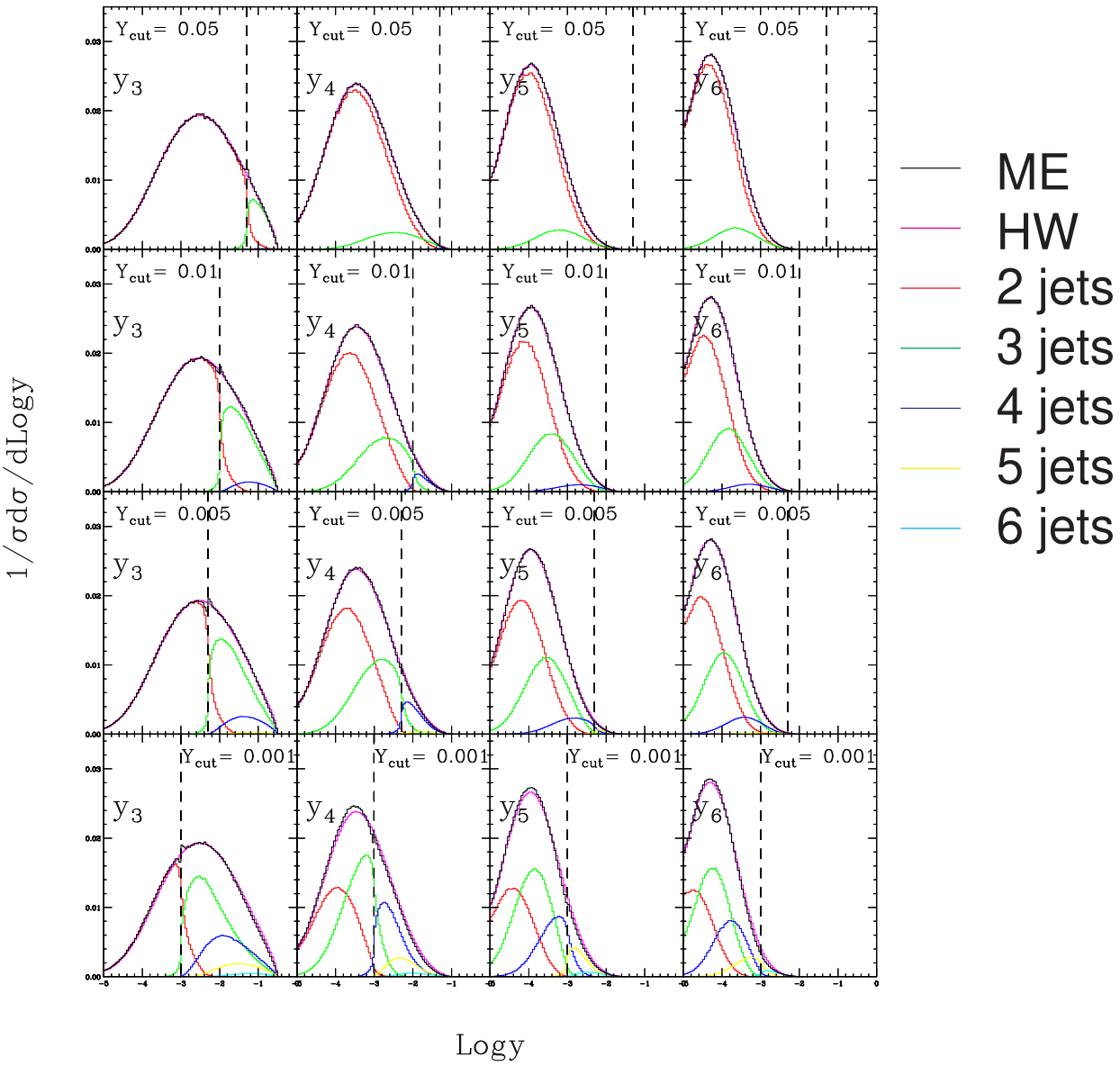
Effect of Varying the cut-off at M_Z

- Parton-level distributions.



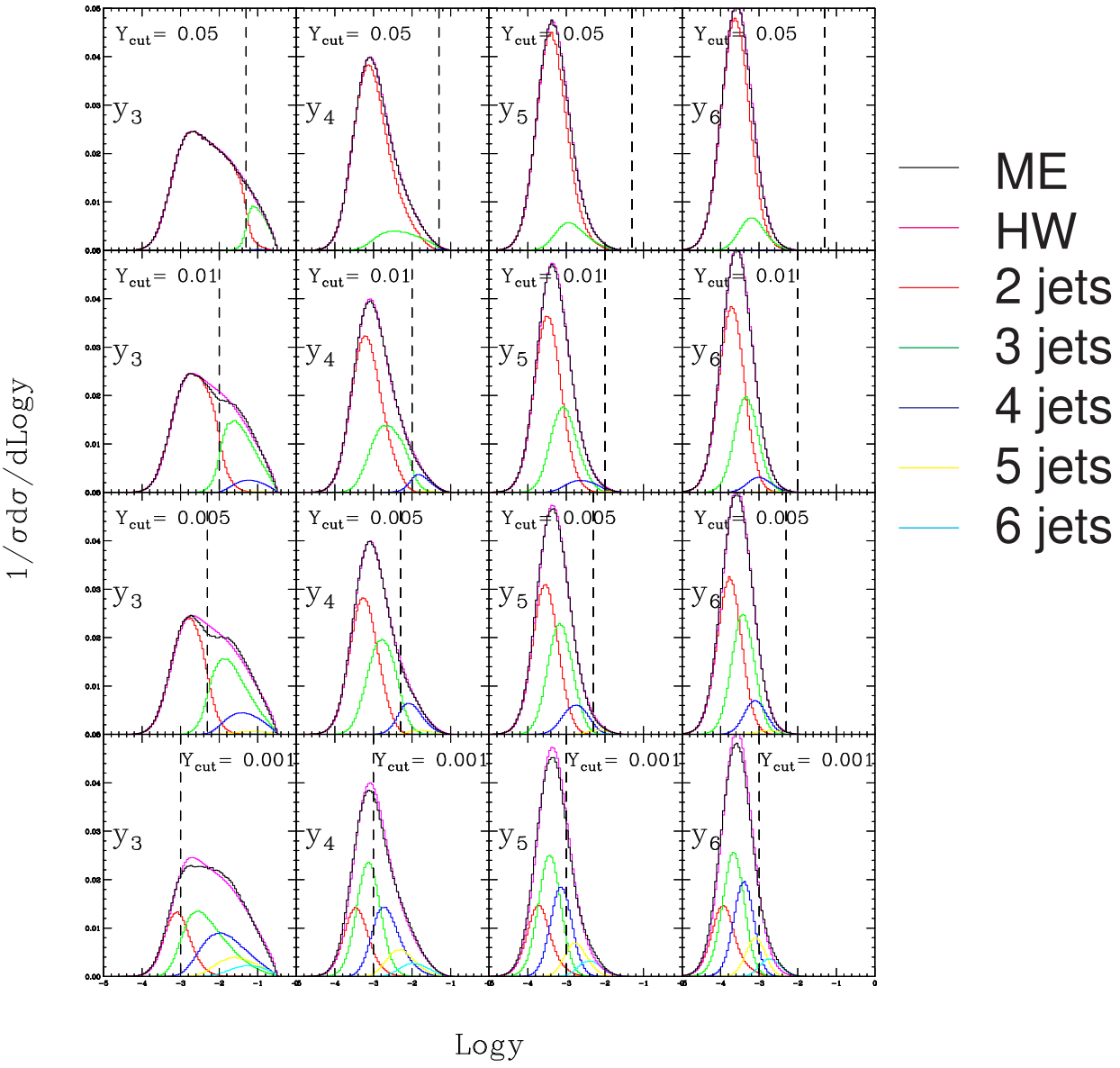
Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- Parton-level distributions.



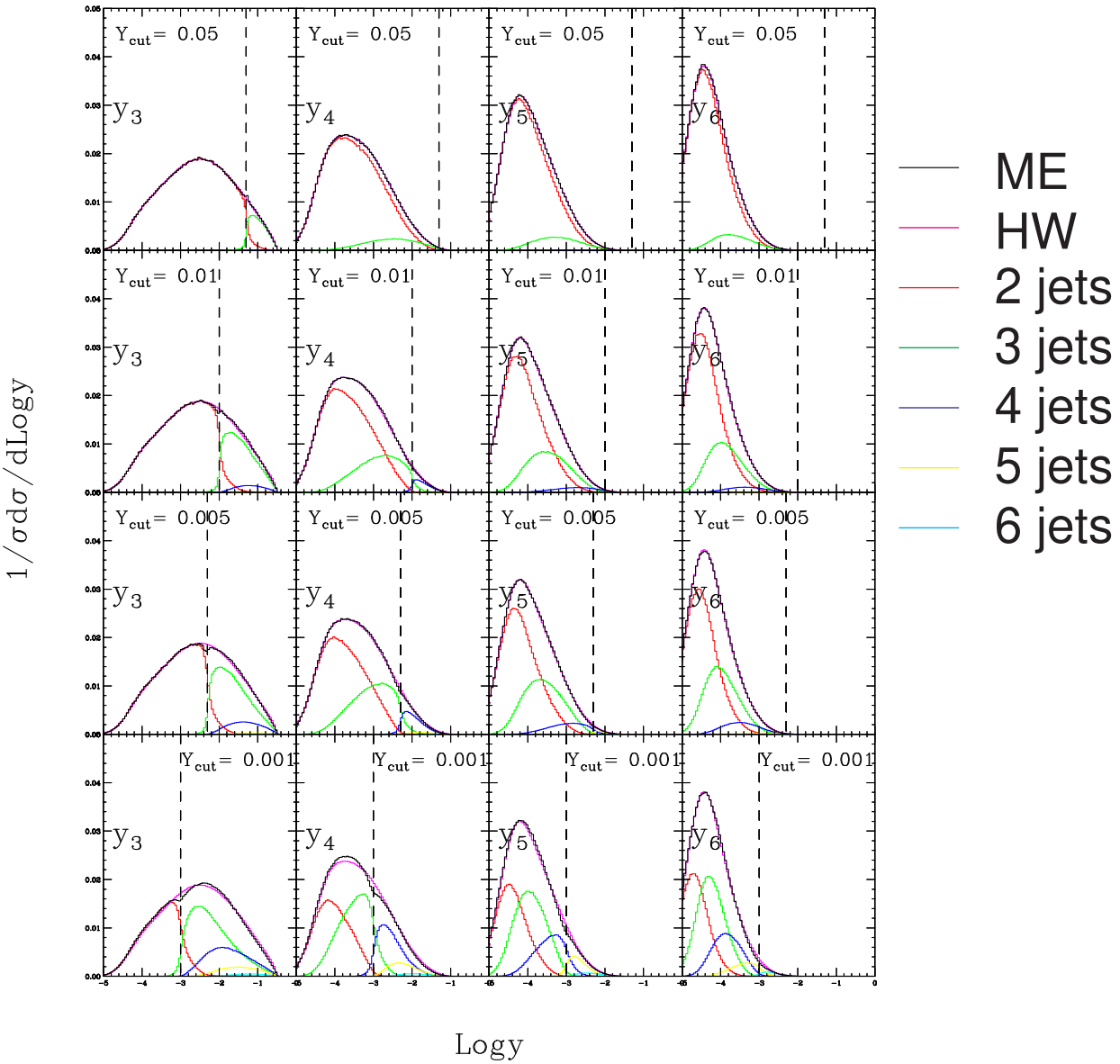
Effect of Varying the cut-off at M_Z

- Hadron-level distributions.
- Problems with hadronization model which remain to be solved.



Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- Hadron-level y_3 distribution.
- Problems with hadronization model reduced.



Hadron Collisions

- In hadron collisions in addition to final-state radiation we have the problems of initial-state radiation.
- I will consider W production in hadron collisions
- All the events were generated with MADGRAPH including the leptonic decays of the gauge bosons.
- For the W samples no cuts were applied on the leptons.

Hadron Collisions

- We also have a greater choice of the clustering algorithm.
- I will only show one choice here.
  - Covariant monotonic p_T^2 scheme

$$\begin{aligned} d_{iB} &= p_{ti}^2, \\ d_{ij} &= \min(p_{ti}^2, p_{tj}^2) R_{ij}^2, \end{aligned}$$

Define p_t and R for pseudoparticles as

$$\begin{aligned} p_{t(ij)} &= p_{ti} + p_{tj}, \\ R_{(ij)k}^2 &= \frac{p_{ti}^2 R_{ik}^2 + p_{tj}^2 R_{jk}^2}{p_{ti}^2 + p_{tj}^2}. \end{aligned}$$

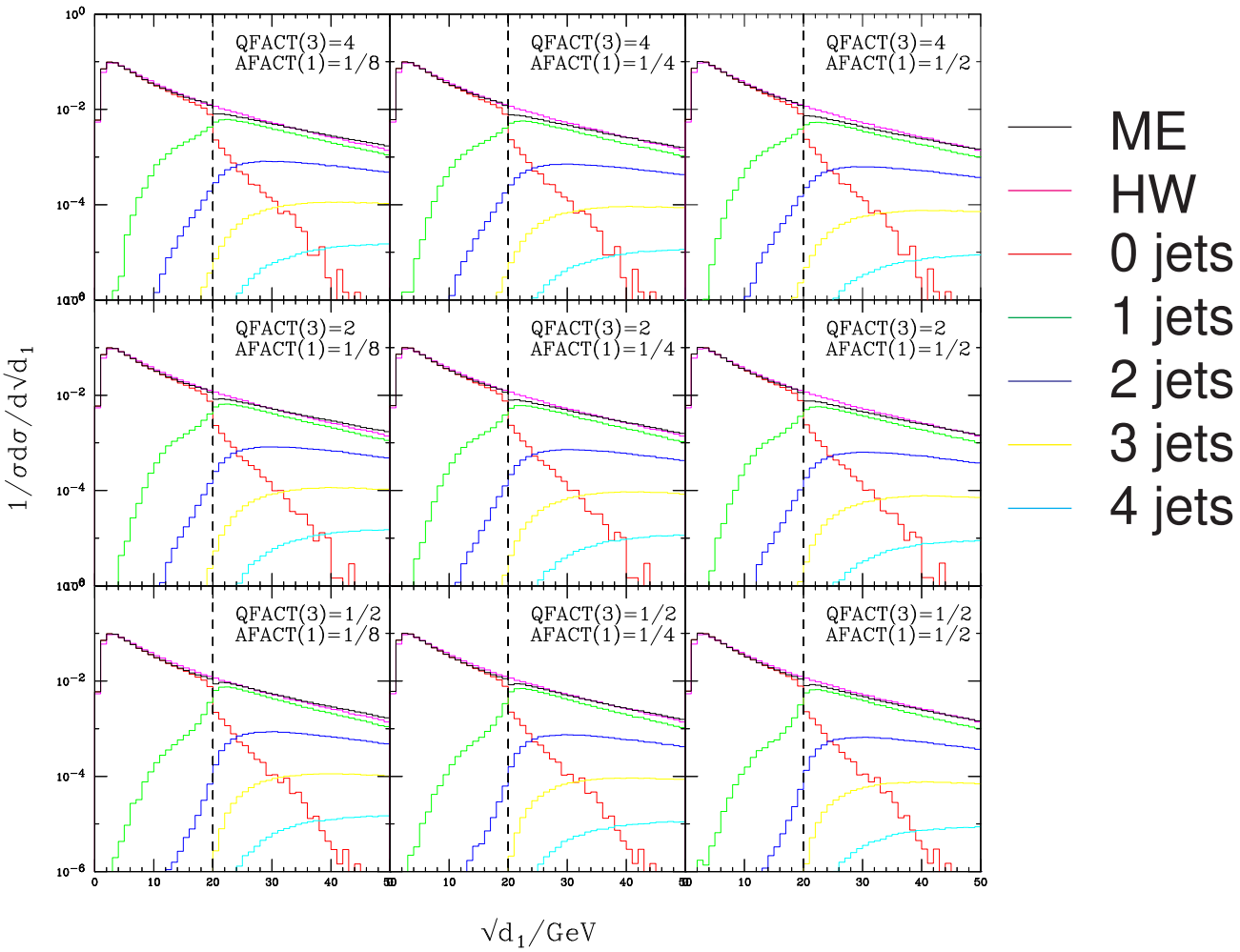
- Other schemes give similar results, (provided they are longitudinally boost invariant.)

Hadron Collisions

- Start by considering W production at the Tevatron.
- Use the Covariant monotonic p_T^2 scheme.
- and a matching scale of $400\ \mathrm{GeV}^2$.
- Look at the differential cross section with respect to $\sqrt{d_n}$ where d_n is the scale at which n jets are resolved.
- This is like the E_T in a cone algorithm.

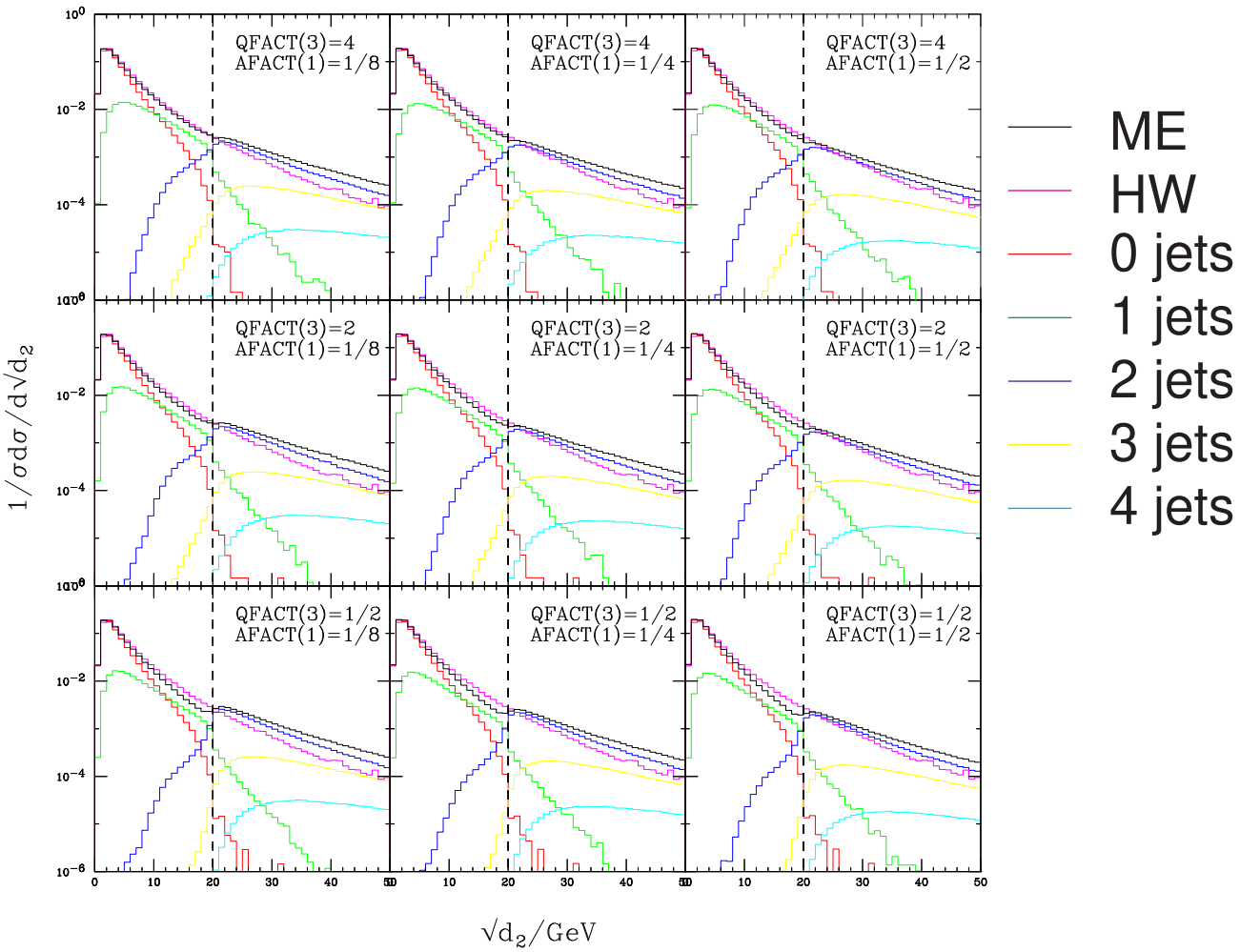
W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_1 distribution at the Tevatron



W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_2 distribution at the Tevatron

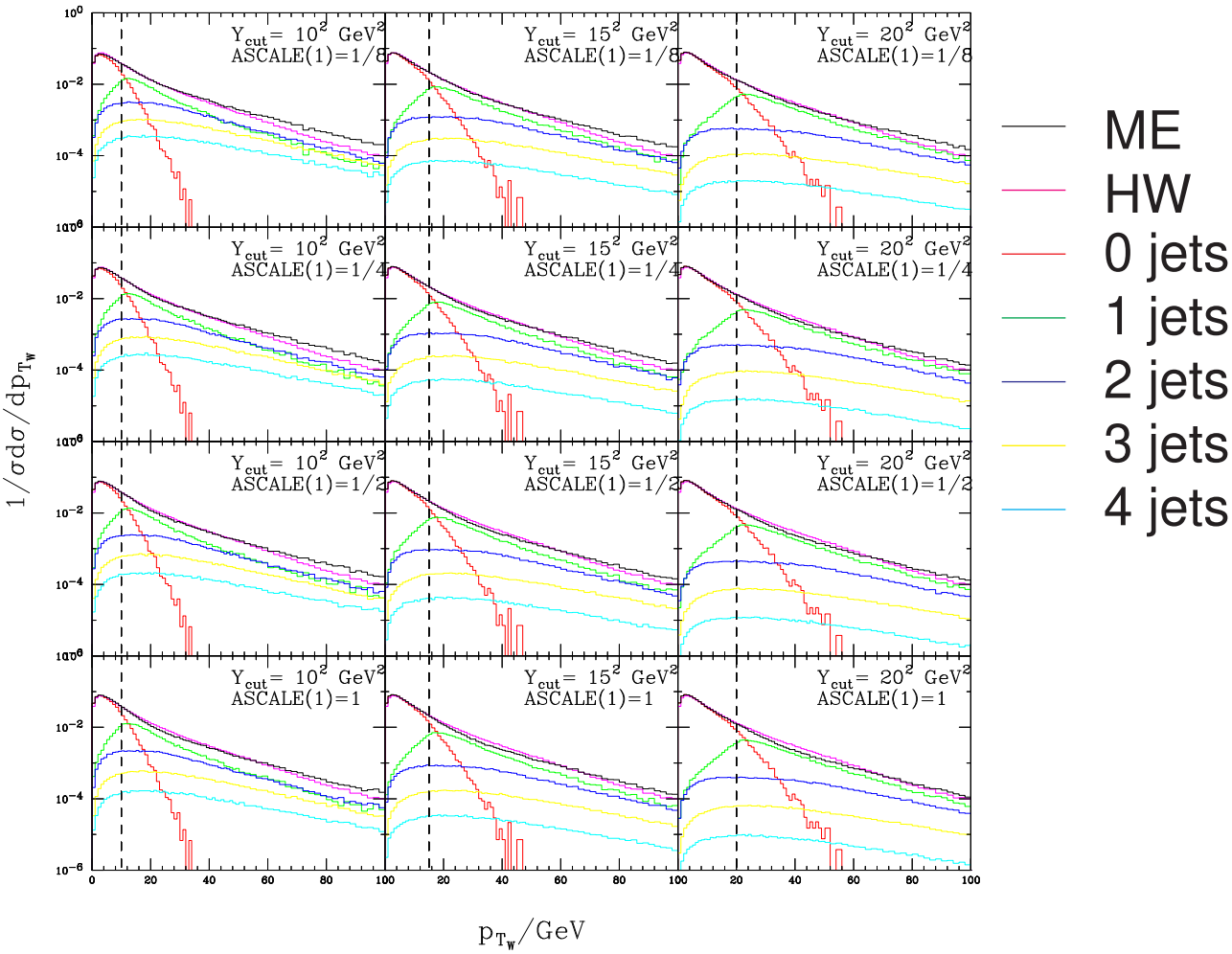


Hadron Collisions

- There is a radiation dip at the matching scale.
- There is also a problem with events migrating below the matching scale.
- Hard to get the HERWIG shower to produce more radiation than before below the matching scale.
- Solution is to decouple the scale used in the Sudakovs and that used to start the shower. Use a choice closer to the HERWIG variables to start the shower.

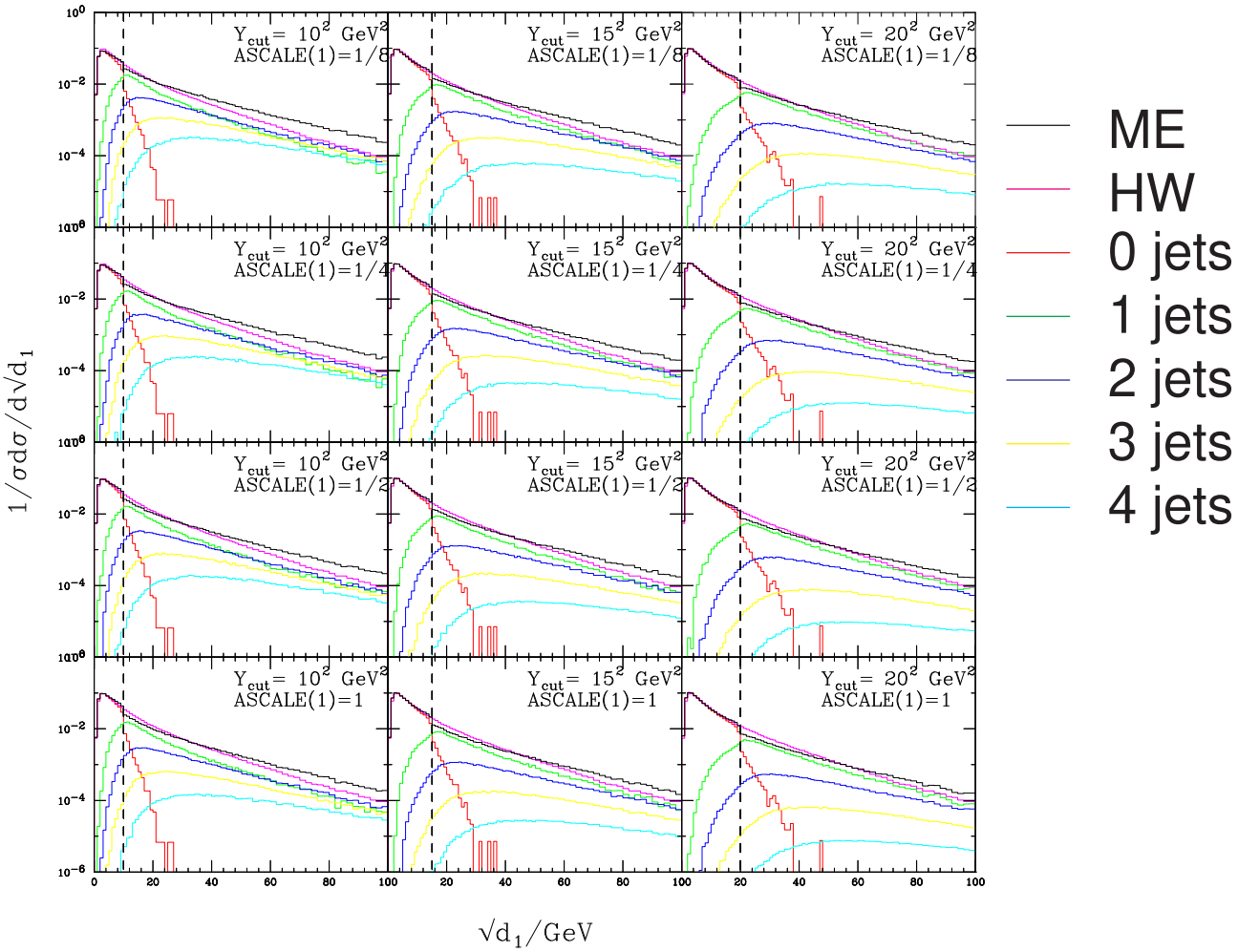
W Production

- Variation of results with scale of α_S and matching scale.
- p_T of the W at the Tevatron



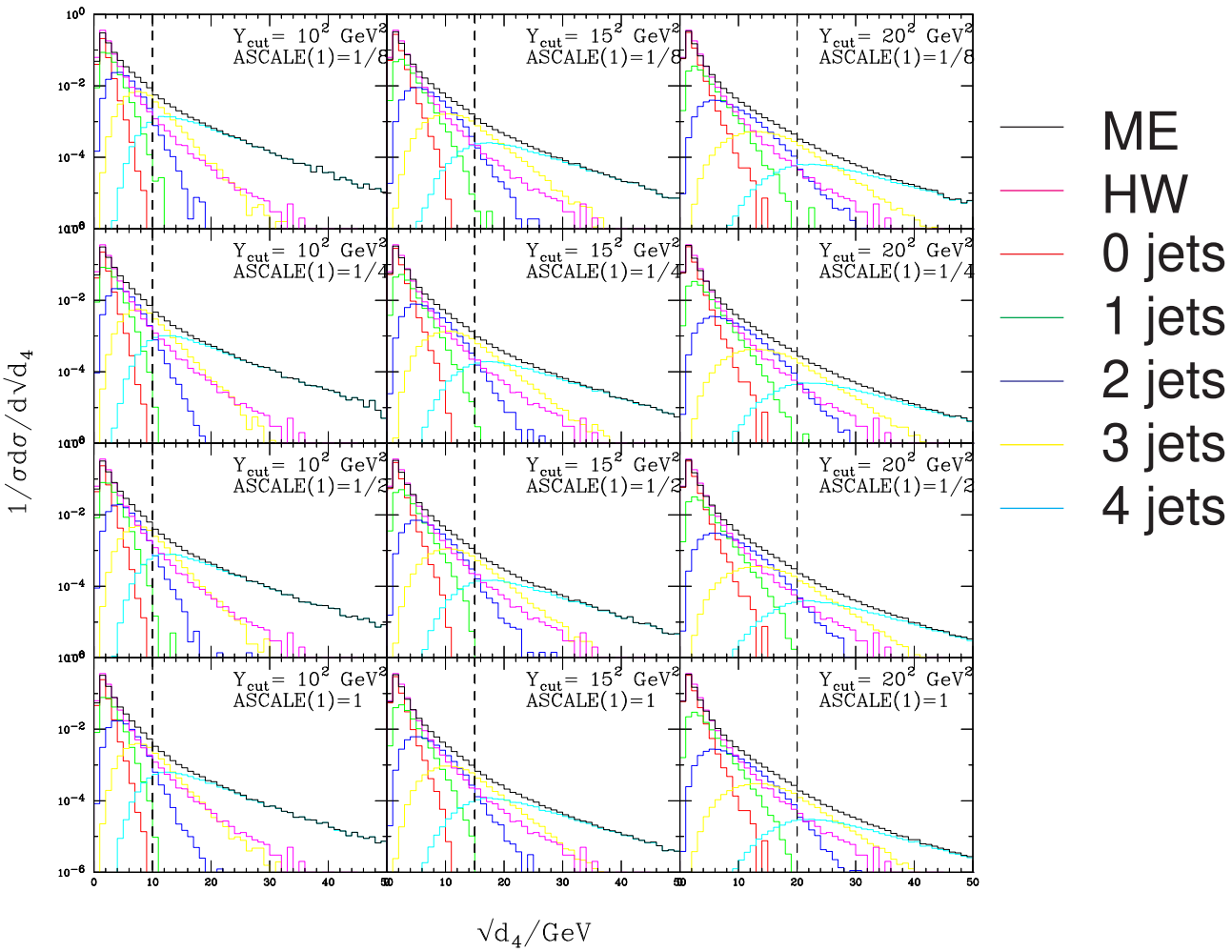
W Production

- Variation of results with scale of α_S and matching scale.
- d_1 distribution at the Tevatron



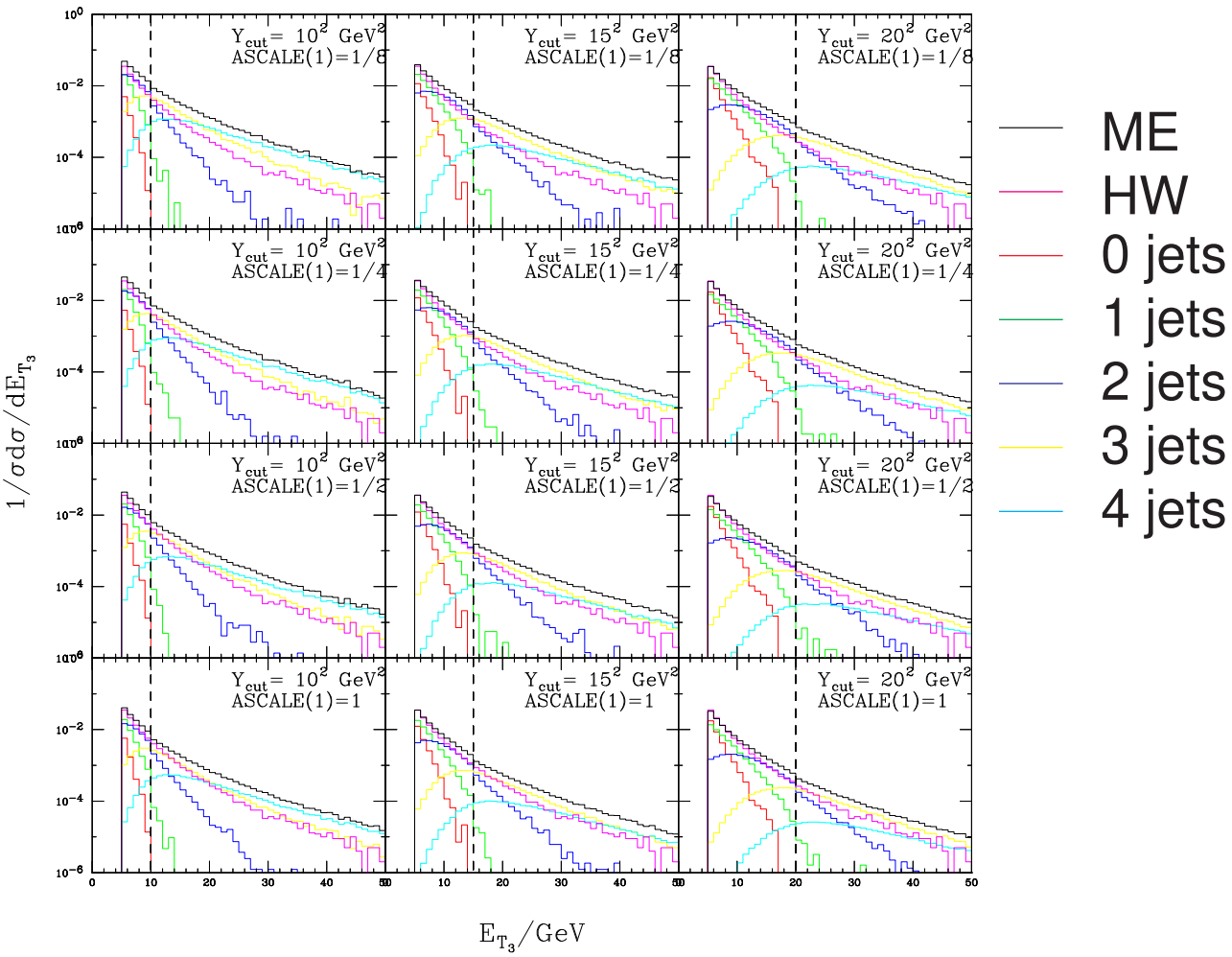
W Production

- Variation of results with scale of α_S and matching scale.
- d_4 distribution at the Tevatron



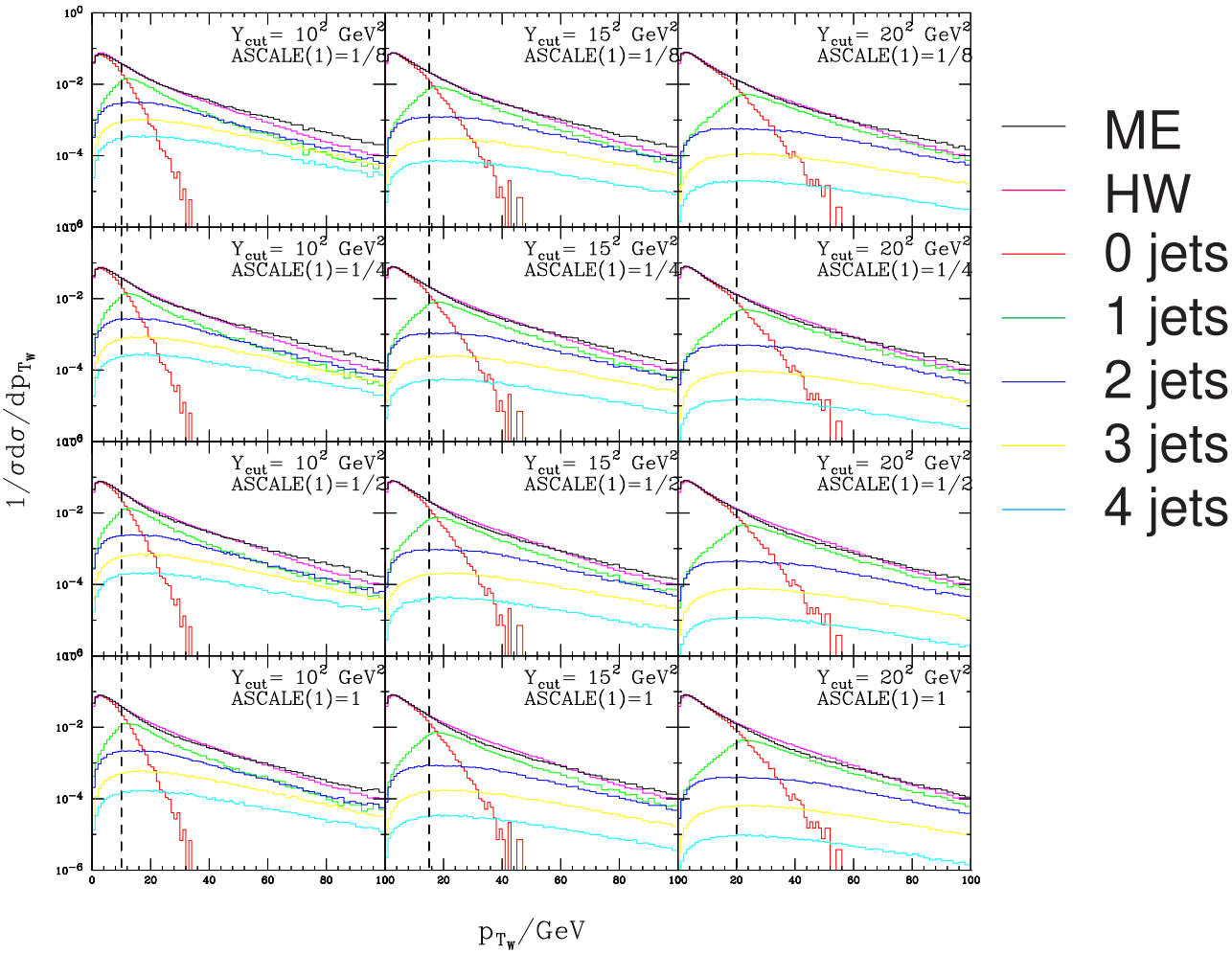
W Production

- Variation of results with scale of α_S and matching scale.
- E_T of the third jet at the Tevatron with cone $R = 0.7$.



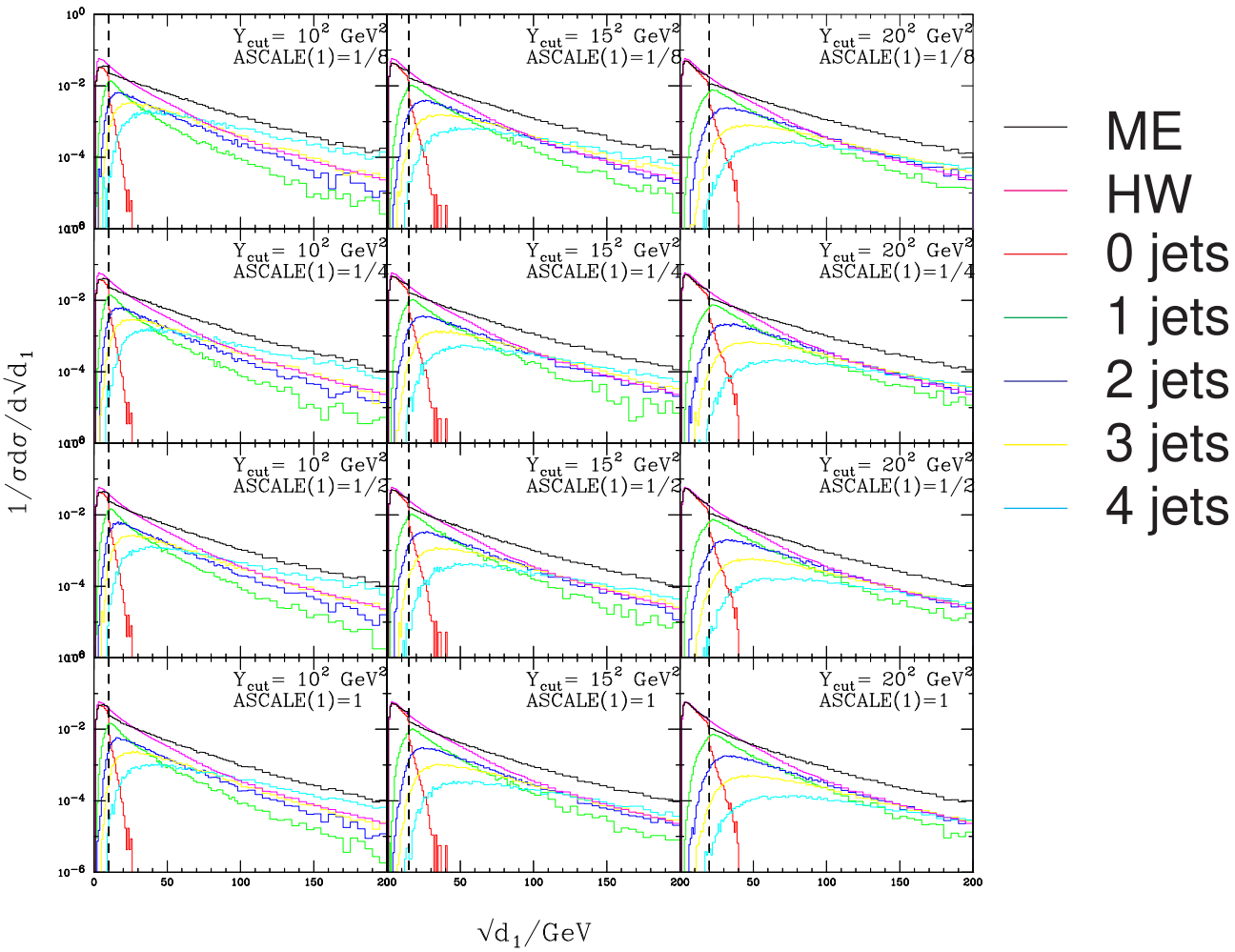
W Production

- Variation of results with scale of α_S and matching scale.
- p_T of the W at the LHC.



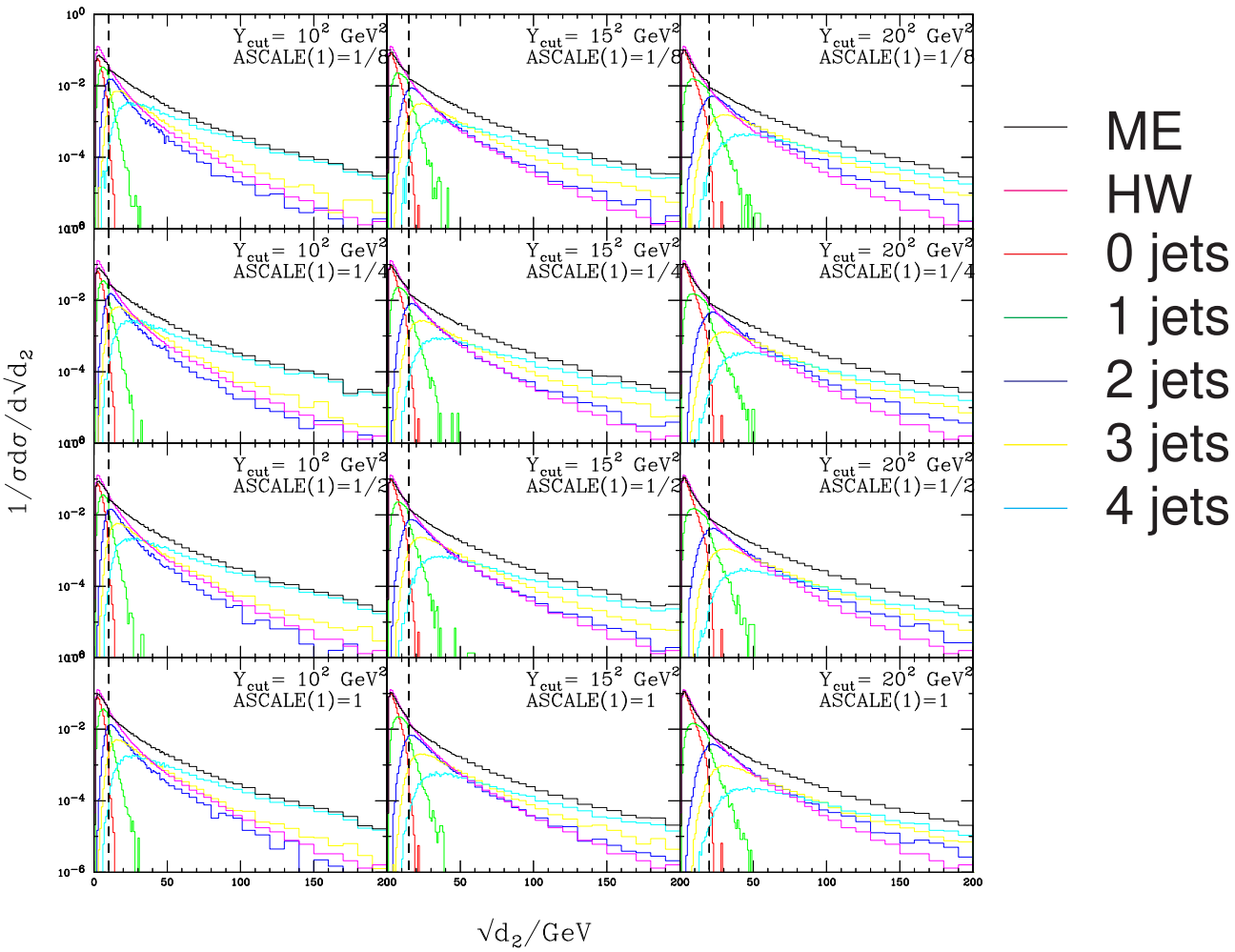
W Production

- Variation of results with scale of α_S and matching scale.
- d_1 distribution at the LHC.



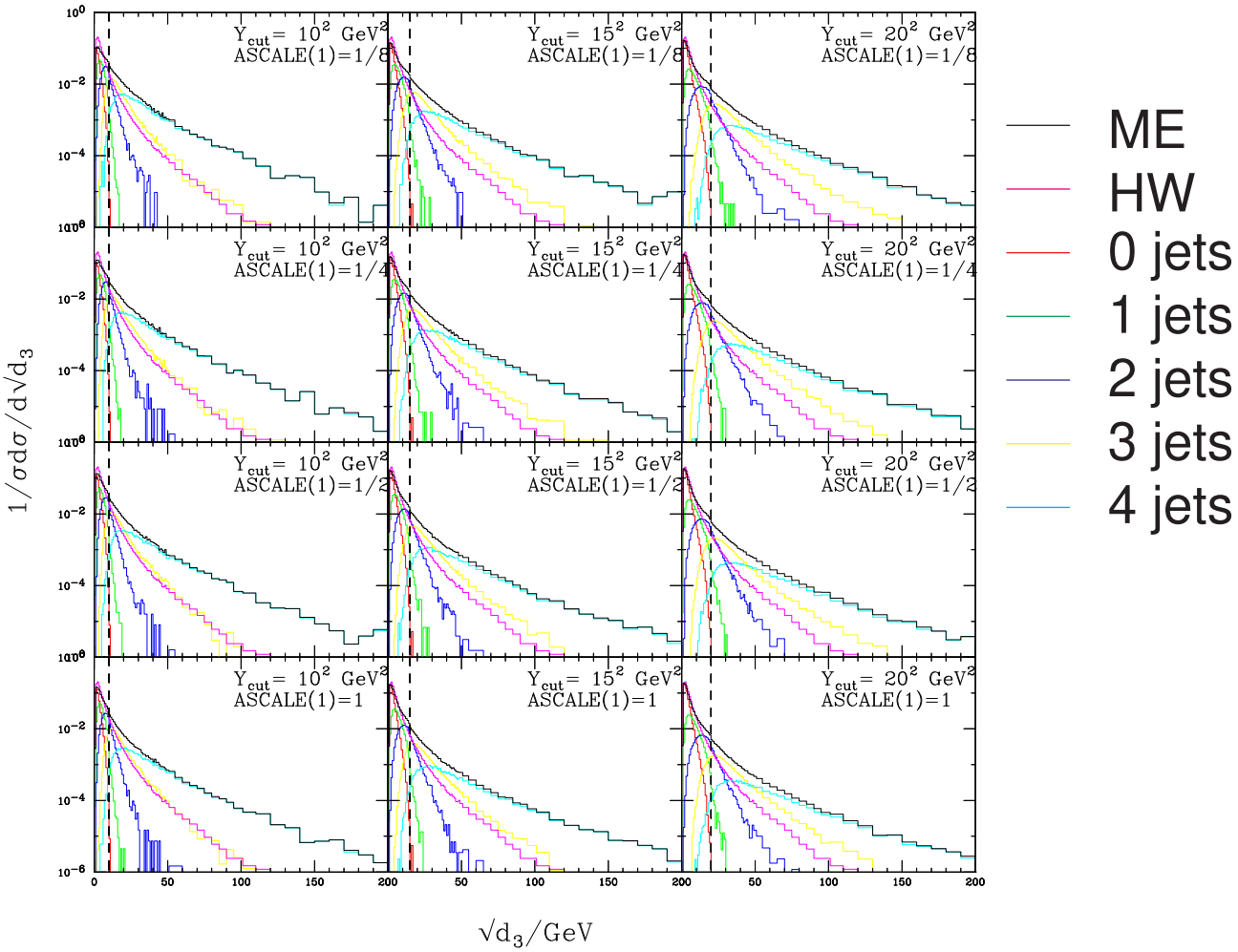
W Production

- Variation of results with scale of α_S and matching scale.
- d_2 distribution at the LHC.



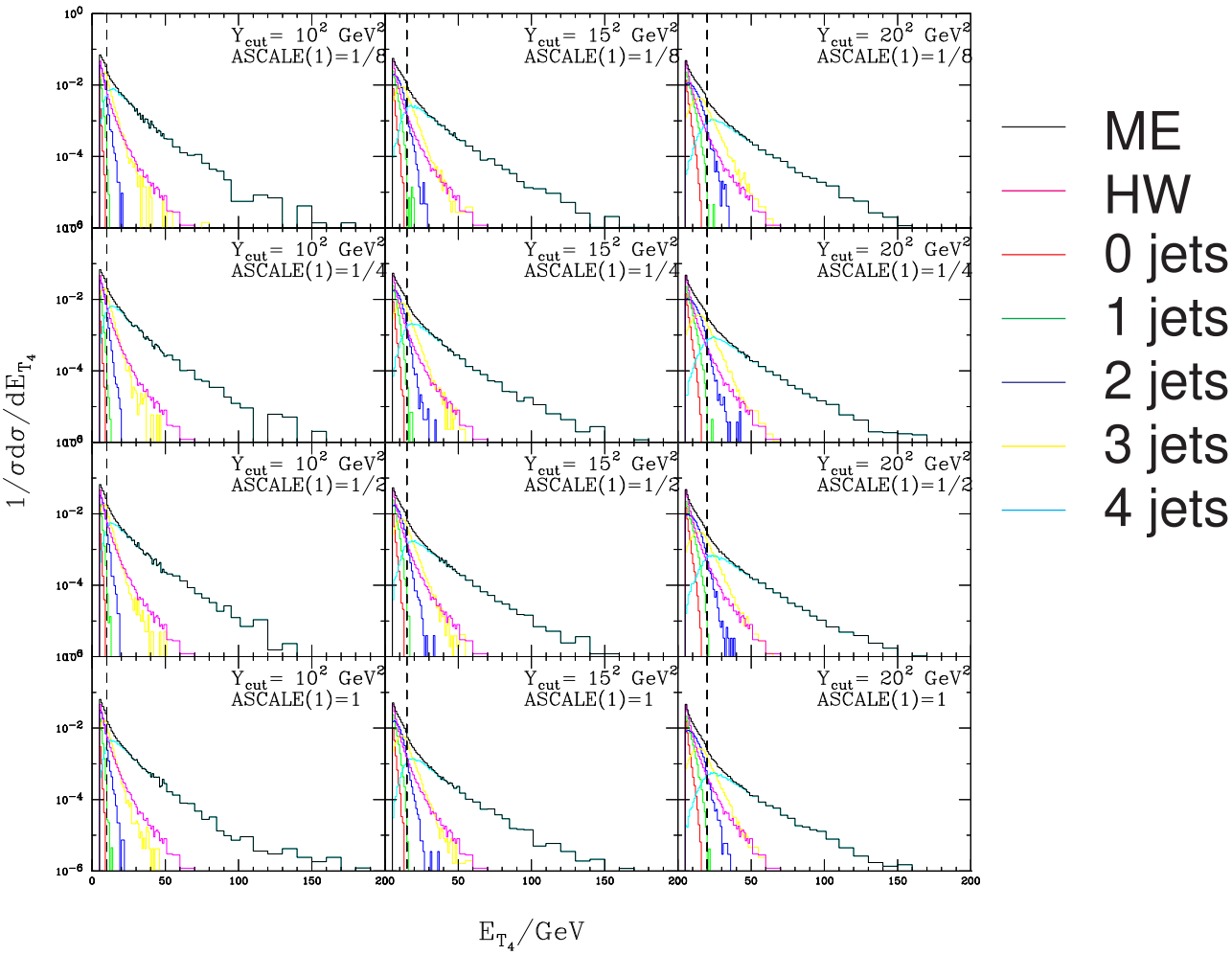
W Production

- Variation of results with scale of α_S and matching scale.
- d_3 distribution at the LHC.



W Production

- Variation of results with scale of α_S and matching scale.
- E_T of the fourth jet at the LHC with cone $R = 0.4$.



Conclusions

- I have presented two recent developments in Monte Carlo event generation to allow the simulation of high multiplicity final-states.
- A lot of progress has been made recently in this area.
- Some of this is already available for studies of Tevatron physics
- Hopefully all of it will be available together in time for the LHC.