Matching Matrix Elements and Parton Showers with HERWIG

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- Introduction
- Approach
- CKKW Algorithm
- e^+e^- collisions
- Hadron collisions
- Conclusions

Introduction

- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high p_T jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating many high p_T jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, *i.e.*
  - ME gives hard emission,
  - PS gives soft/collinear emission,
  - Smooth matching between the two,
  - No double counting of radiation.

Introduction

- There have been a number of attempts to do this.
- The first attempts got the hardest emission correct for simple processes, for example
  - $e^+e^- \to q\bar{q}$
  - DIS
  - Drell-Yan
  - Top Decay
- There were problems with this
  - Only the hardest emission was correctly described.
  - The leading-order normalization was retained.

Introduction

- Recently there has been renewed interest.
- Two types of approach have emerged.
  1. NLO Simulation
     - NLO normalization of the cross section.
     - Gets the hardest emission correct.
     - Only programs by Frixione/Webber and Dobbs for gauge boson pairs.
  2. Multi-jet Leading Order
     - Still leading order.
     - Gets many hard emissions correct.
     - Programs by Krauss et. al. and Lonnblad for e^+e^-.

Approach

- I will be looking at the second type of approach.
- The idea is to implement the algorithm of CKKW to produce a multi-jet simulation using existing tools
  - Use the Les accord compliant M.E. generators to calculate matrix elements (MADGRAPH, ALPGEN)
  - Perform the reweighting etc. specified by the algorithm before passing the events to HERWIG.
  - Use the Les accord interface to read the events into HERWIG.
  - Make only minimal changes to the event generator itself.
- Produce a better simulation using existing well tested tools.
- I will first review the algorithm of CKKW and then present some preliminary results.

Procedure

- In order to match the ME and PS we need to separate the phase space:
  - One region should contain the soft/collinear region and be filled by the PS;
  - The other is filled by the matrix element.

- In Catani-Krauss-Kuhn-Webber (CKKW) prescription this is done via the Durham jet algorithm.
  - For all final-state particles compute the resolution variables
    - $d_{kB} \simeq E_k^2 \theta_{kB}^2 \simeq k_{\perp kB}^2, \quad \theta_{kB} \to 0.$
    - $d_{kl} \simeq \min(E_k^2, E_l^2)\theta_{kl}^2, \quad \theta_{kl} \to 0$
  - The smallest of these d_{kB}, d_{kl} is then selected. If d_{ij} is the smallest the two particles are merged, if d_{iB} is the smallest the particle is merged with the beam.
  - This procedure is repeated until the minimum value is above some stopping parameter $d_{\rm cut}$.
  - The remaining particles and pseudo-particles are then the hard jets.

Procedure

1. Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{k=N} \sigma_k}$$

   where σ_n is the n-jet matrix element evaluated at resolution $d_{\rm ini}$ using $d_{\rm ini}$ as the scale for the P.D.F's and α_S, n is the number of additional jets.

2. Distribute the jet momenta according to the ME.

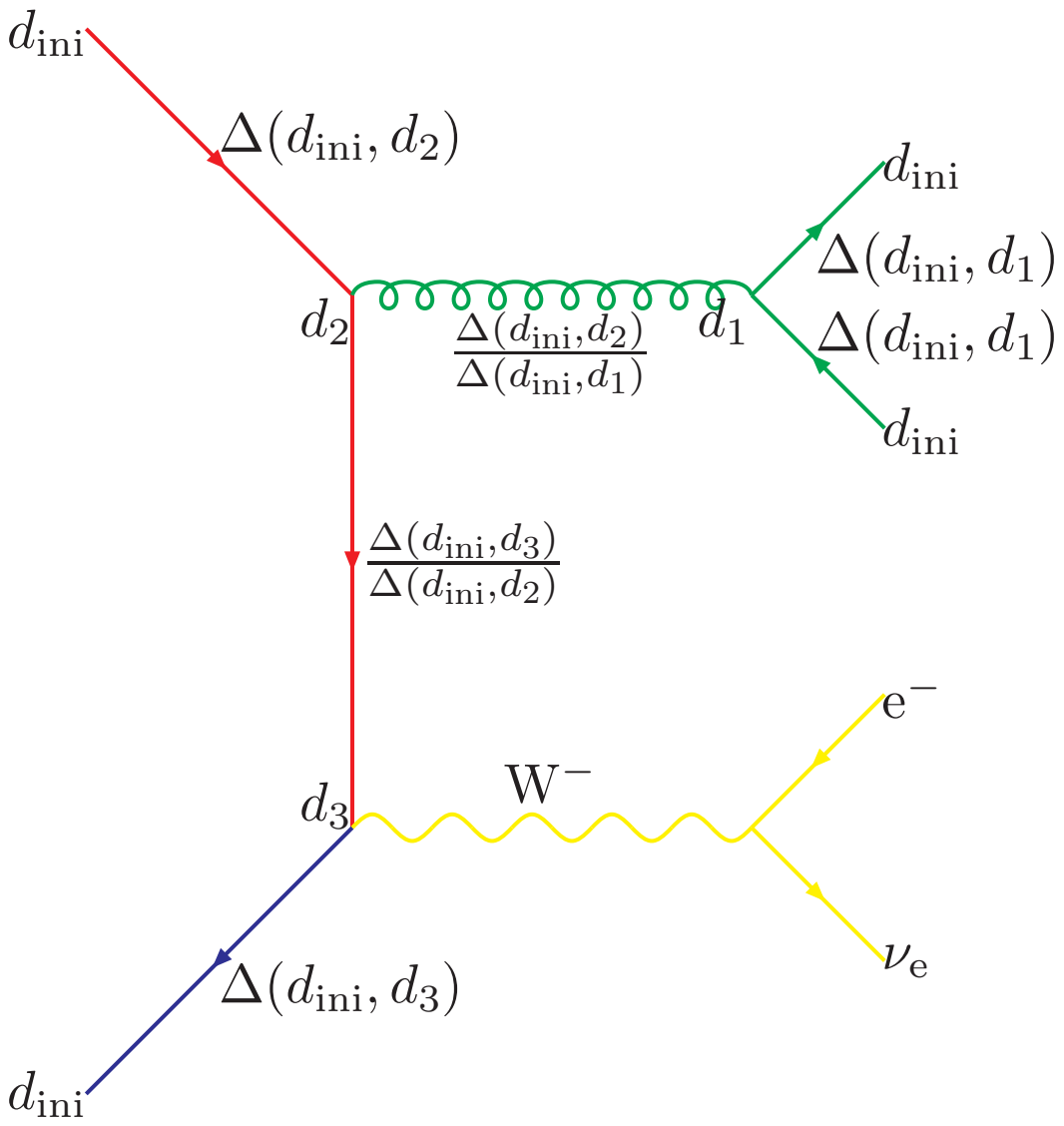
3. Cluster the partons to determine the values at which 1,2,..n-jets are resolved. These give the nodal scales for a tree diagram.

4. Apply a coupling-constant reweighting $\alpha_S(d_1)\alpha_S(d_2)..\alpha_S(d_n)/(\alpha_S(d_{\rm ini})^n \leq 1$.

5. Reweight the lines by a Sudakov factor $\Delta(d_{\rm ini}, d_j)/\Delta(d_{\rm ini}, d_k)$.

6. Accept the configuration if the product of the α_S and Sudakov weight is less the $\mathcal{R} \in [0, 1]$., otherwise return to step 1.

Procedure



7. Generate the parton shower from the event starting the evolution of each parton at the scale it was created and vetoing emission above the scale $d_{\rm ini}$

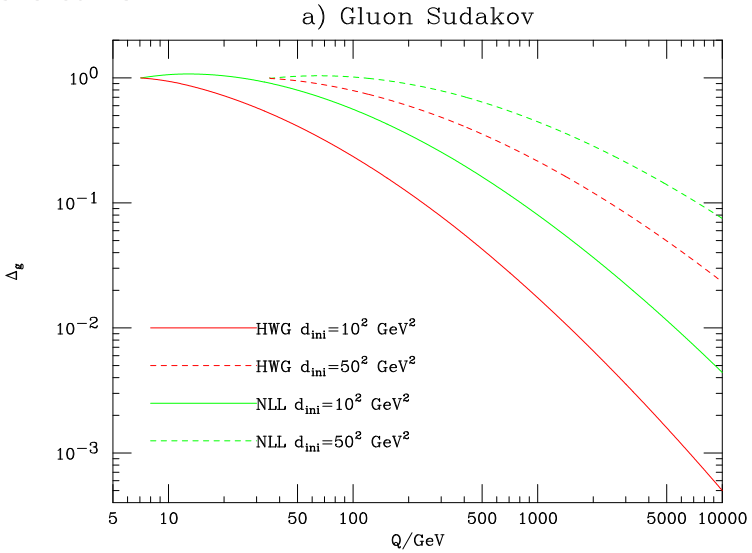
Free Parameters

- Although the algorithm above ensures smooth matching at NLL log level there are a number of free choices
  - Sudakov Form Factors
    - HERWIG or NLL Sudakovs
    - Definition of the scale
    - Prefactor for the scale
  - α_S
    - LO or NLO definition
    - Definition of the scale
    - Prefactor for the scale
  - Treatment of Highest Multiplicity Matrix Element
    - Same as other multiplicities?
    - Different treatment to describe more emissions?
  - Definition of k_T-algorithm away from soft/collinear limit
    - Definition of the k_T-measure.
    - Recombination scheme.

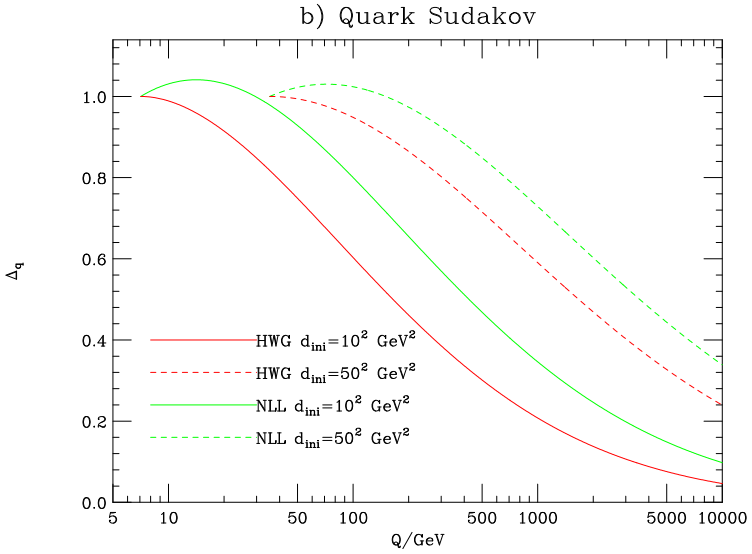
Sudakov Form Factors

- Due to the choice of scale in α_S and regularization the HERWIG Sudakov is always smaller than the NLL one.

- Gluon Sudakov



- Quark Sudakov



Sudakov Form Factors

- To look at the effects of the different Sudakov form factors study e^+e^- collisions at M_Z.
- Look at the differential distributions with respect to y_n where
$$y_n = \frac{d_n}{s},$$
and d_n is the k_T^2 scale where the event changes from n to $n-1$ jets.
- Start with a matching scale of $y = 0.001 \Rightarrow d = 8.3\mathrm{GeV}^2$.
- This is a very low scale which enhances the differences of different choices.
- Start by studying different Sudakovs, different scales in the Sudakov, and different scales in α_S.

Choice of Scale

- There are a number of choices to make.
- First for the Sudakovs

$$d_{\mathrm{SUD}} = \texttt{QFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

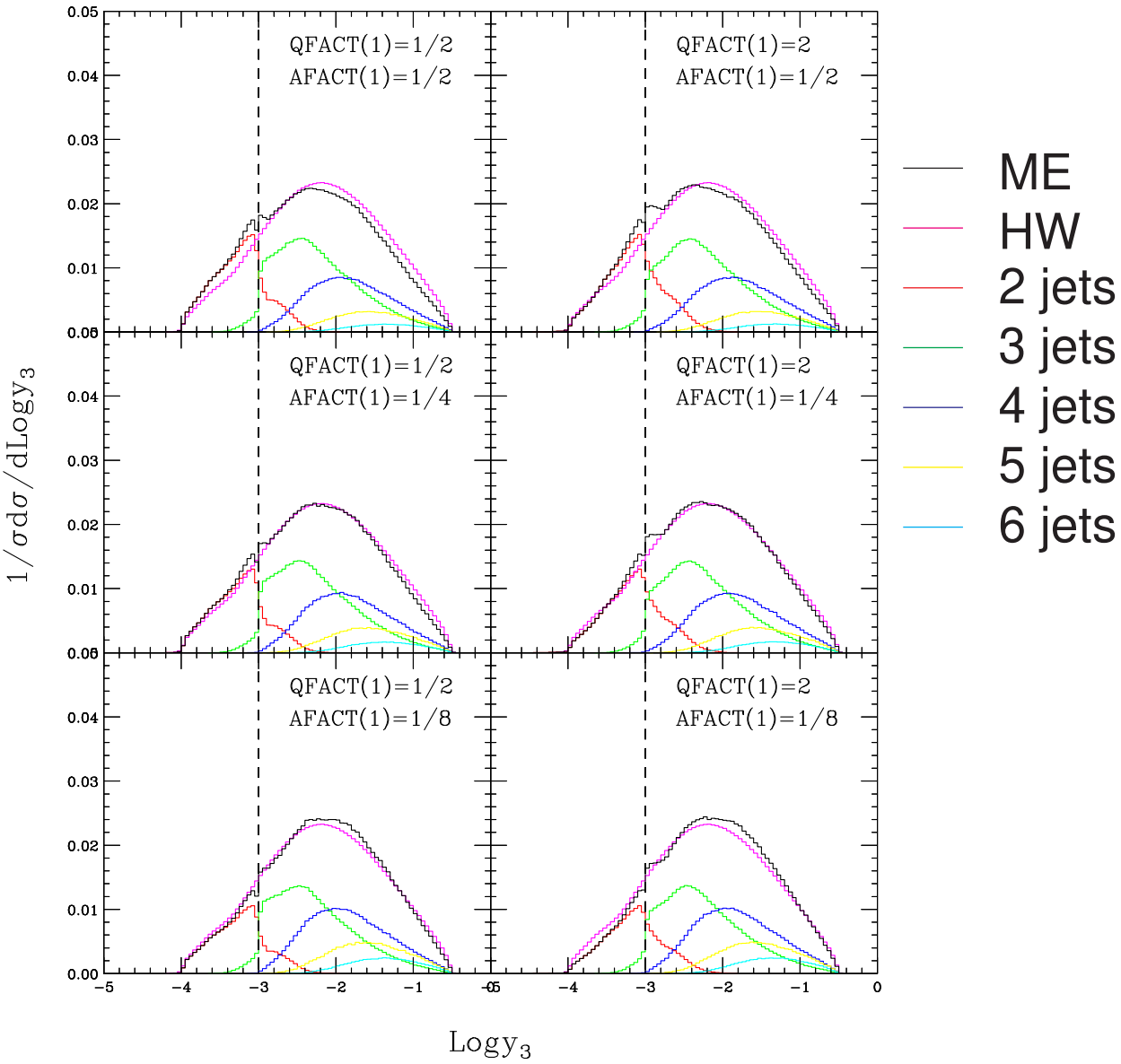
- and for α_S.

$$d_{\alpha_S} = \texttt{AFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

- HERWIG would use `QFACT(1)=1/2` for $e^+e^- \to \mathrm{jets}$ or Drell-Yan.
- However no radiation in shower above $\frac{1}{2}d_{\mathrm{shower}} \Rightarrow$ `QFACT(1)=2`
- HERWIG always has a smaller scale for α_S than in the Sudakov form factor.

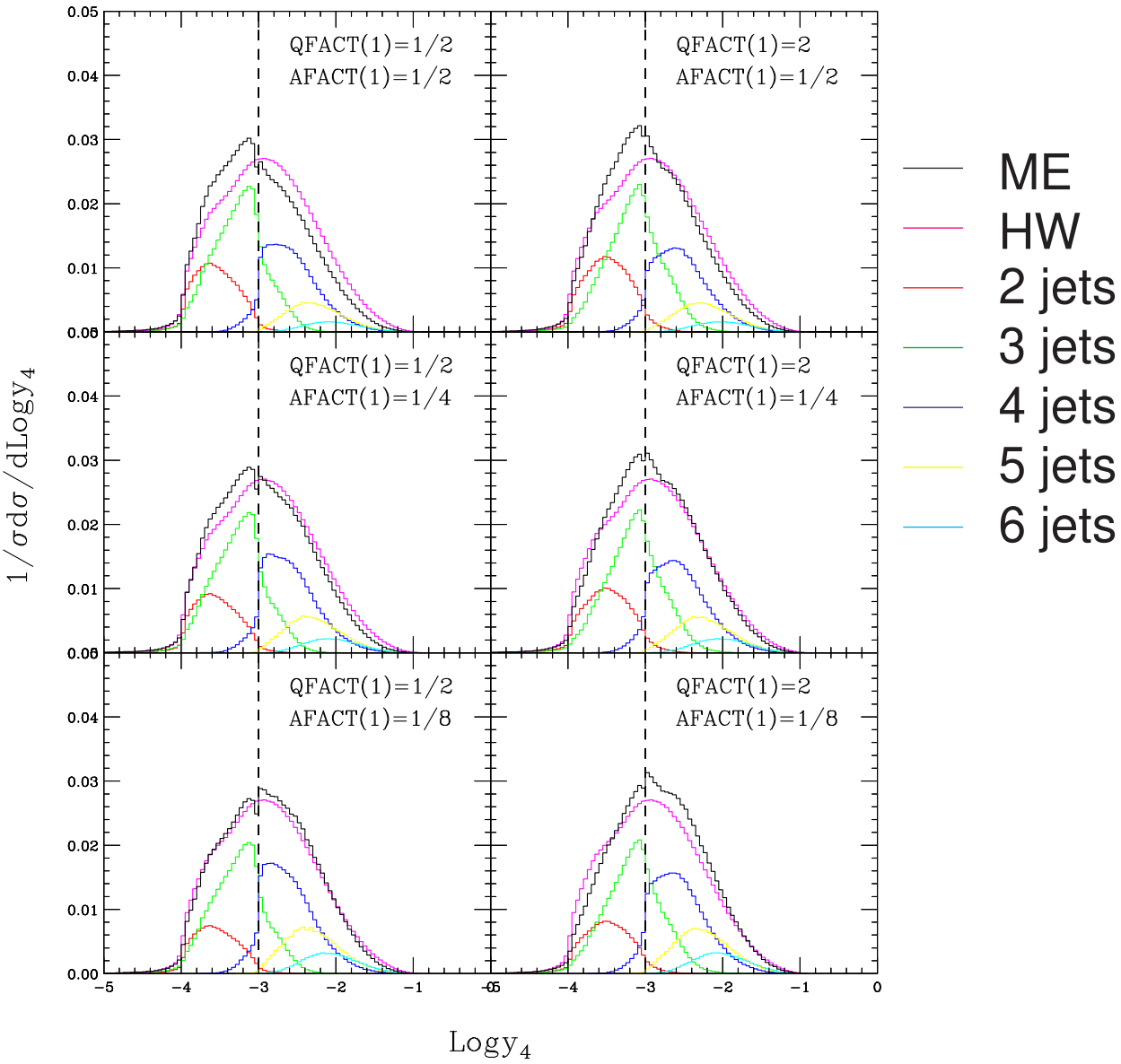
NLL Sudakovs

- NLL Sudakovs with `ISCALE=1`.
- y_3 distribution.



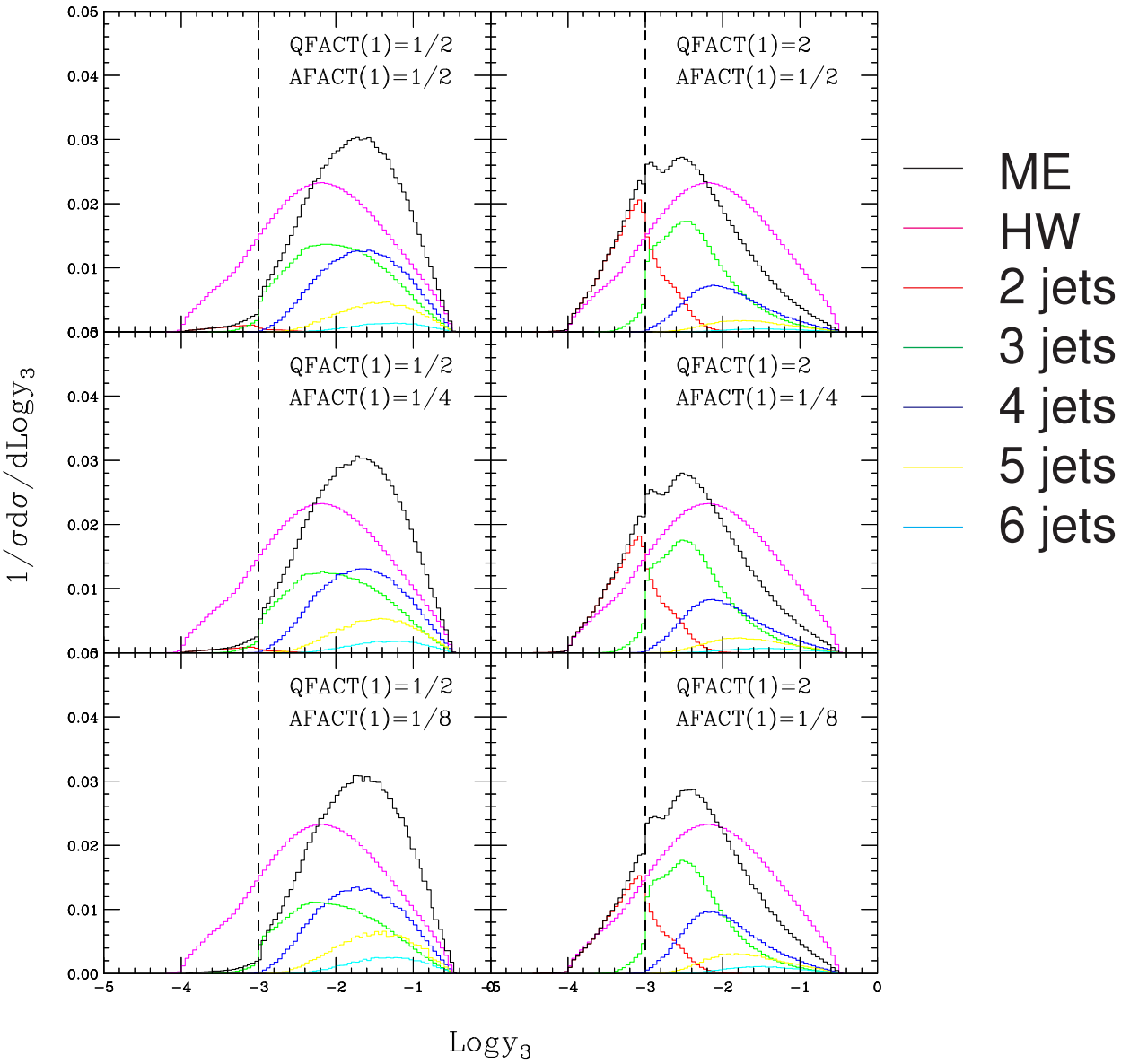
NLL Sudakovs

- NLL Sudakovs with `ISCALE=1.`
- y_4 distribution



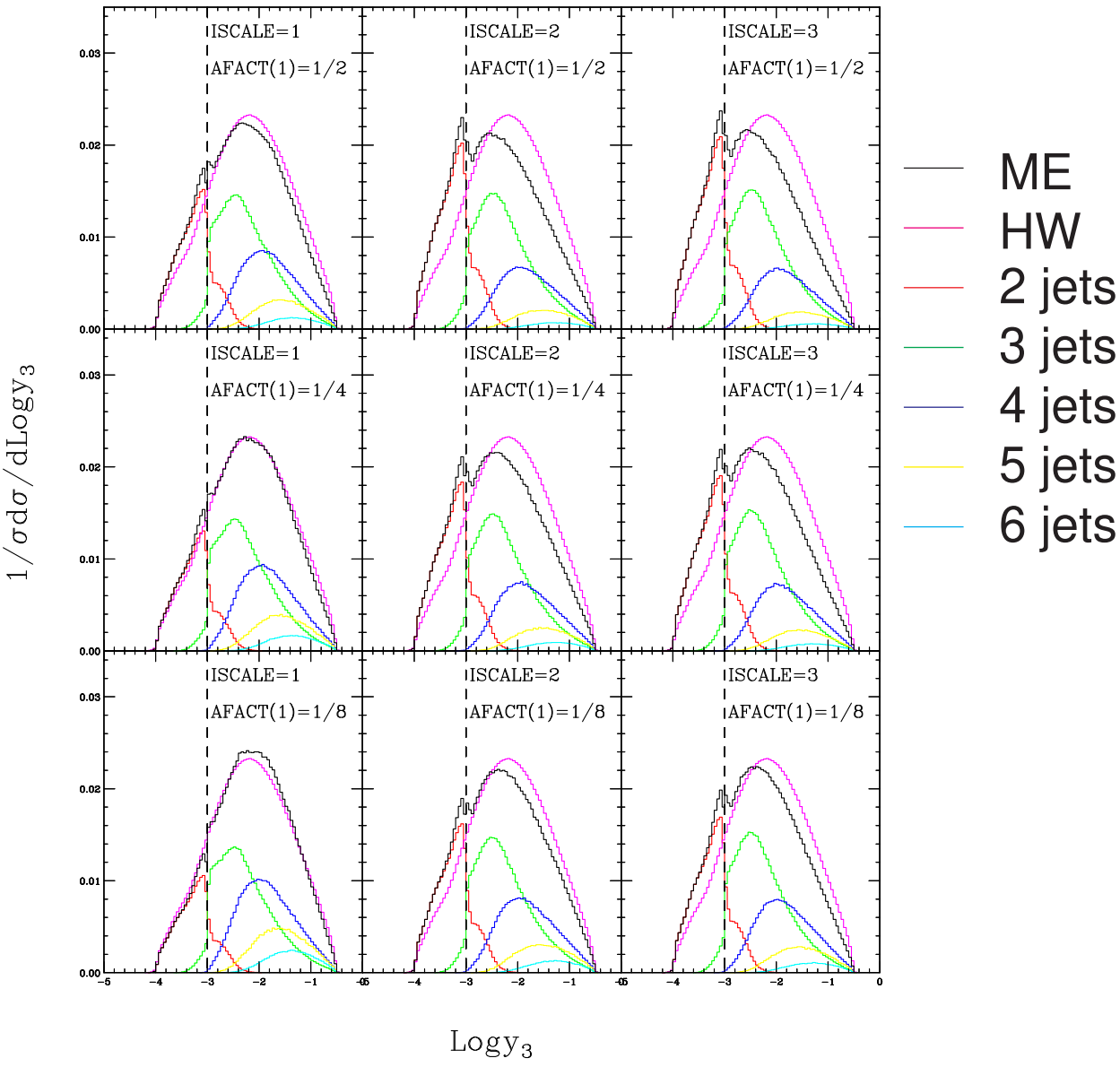
HERWIG Sudakovs

- HERWIG Sudakovs with `ISCALE=1`.
- y_3 distribution.



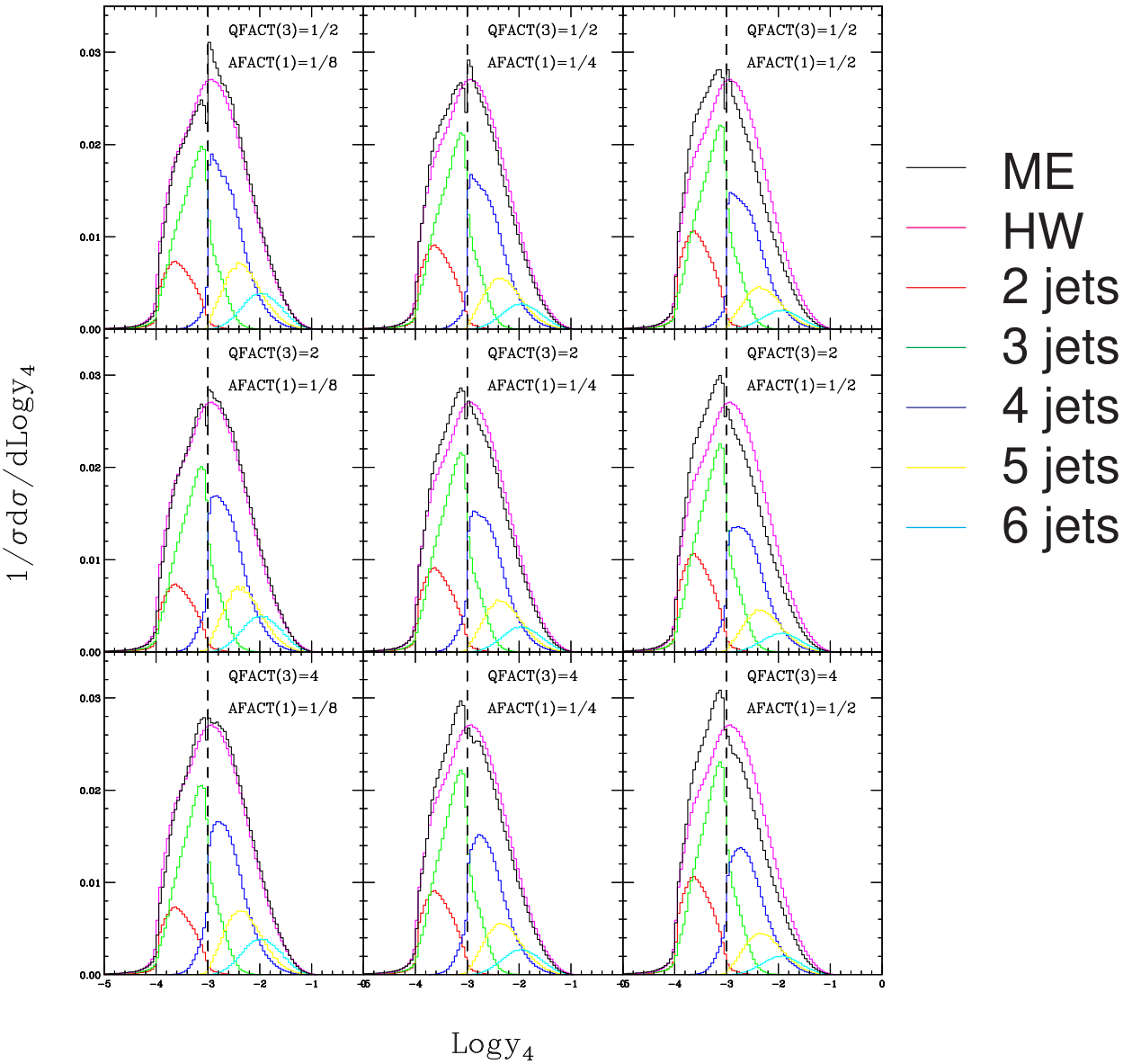
NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- y_3 distribution.



NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- Variation of `QFACT(3)` which is the minimum scale starting scale for the HERWIG shower.
- y_4 distribution.

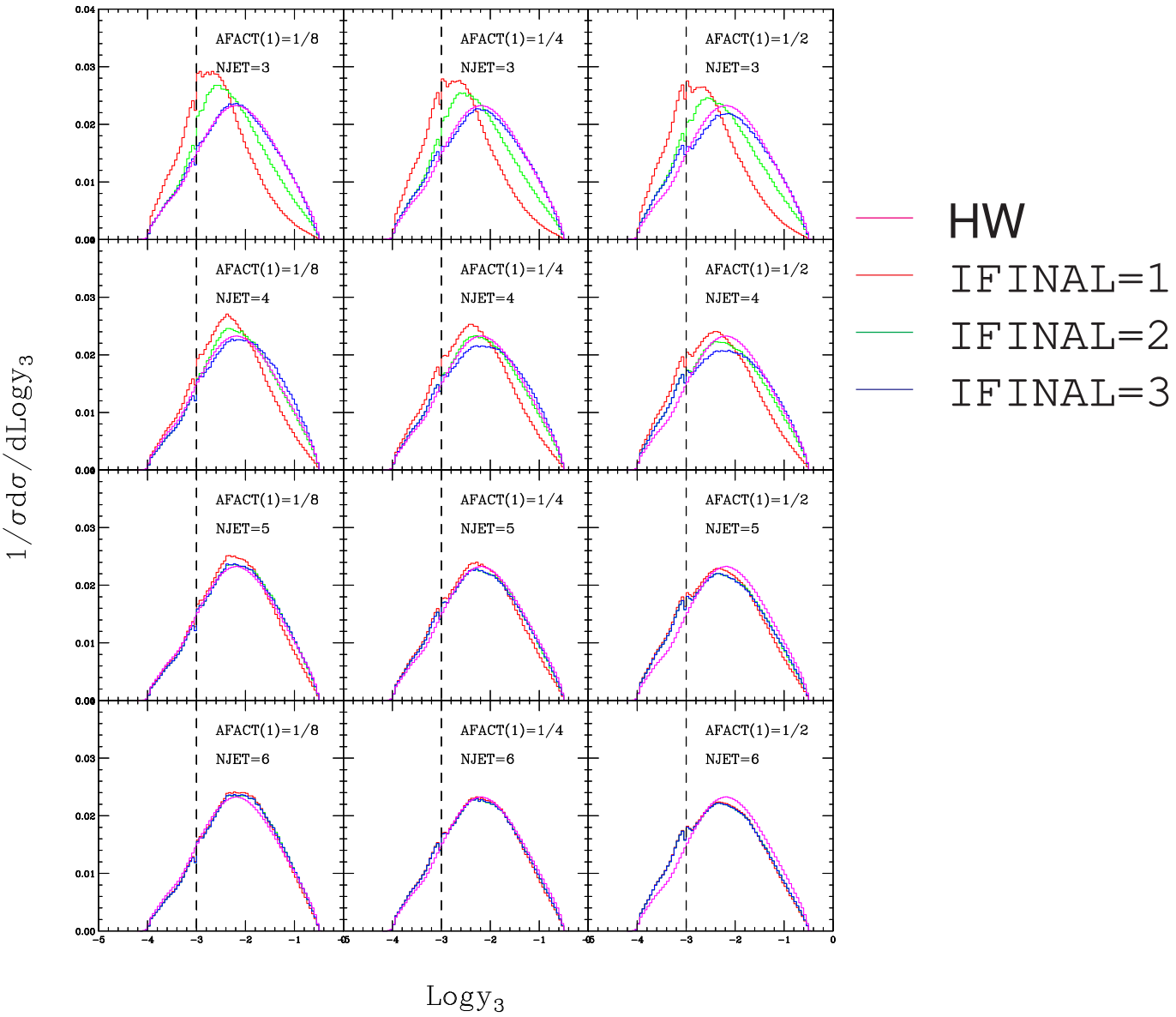


Treatment of Highest Multiplicity

- In the original CKKW algorithm all jet multiplicities were treated in the same way, `IFINAL=1`.
- Not appropriate for highest multiplicity.
- Instead only apply α_S reweighting and Sudakovs for internal lines.
- Two options for the shower
  - Apply no veto and start at normal HERWIG scale, `IFINAL=2`
  - Start at same scale as other multiplicities but veto above scale of last emission in ME. `IFINAL=3`
- `IFINAL=2` is easier to implement but has some double counting problems
- `IFINAL=3` is the best theoretical choice.

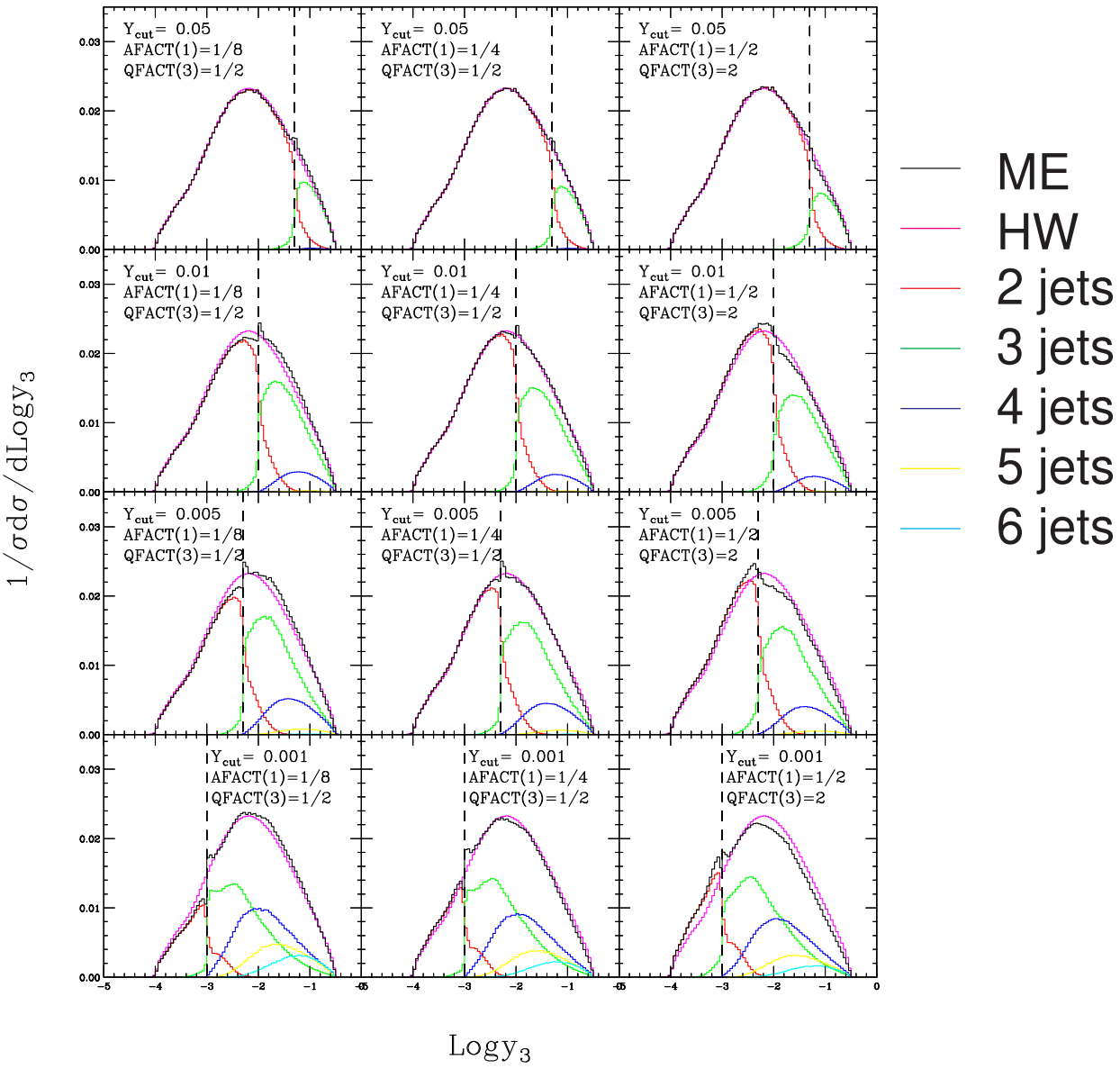
Treatment of Highest Multiplicity

- NLL Sudakovs with `QFACT(1)=1/2`.
- y_3 distribution.



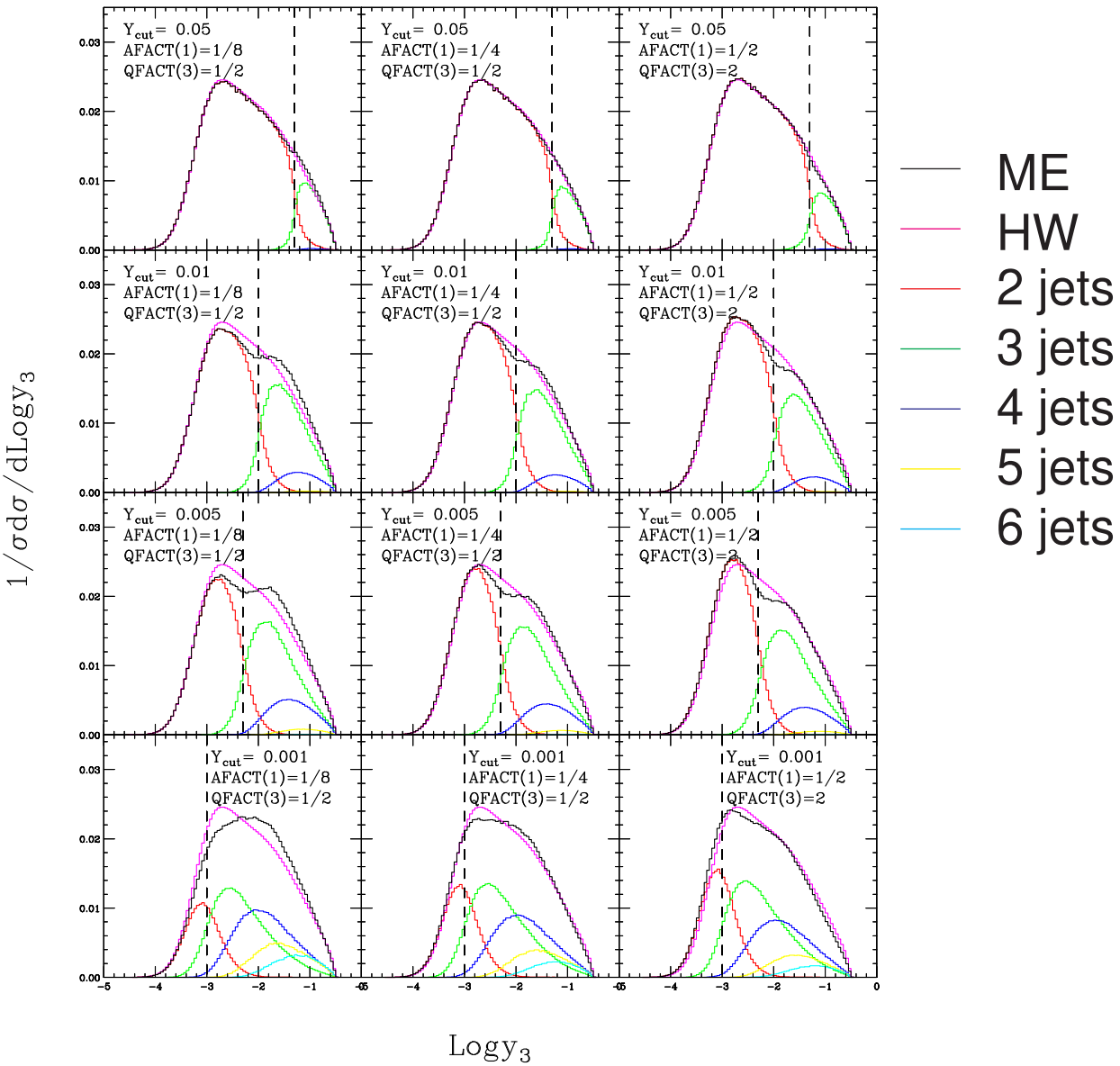
Effect of Varying the cut-off at M_Z

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3.`
- Parton-level y_3 distribution.



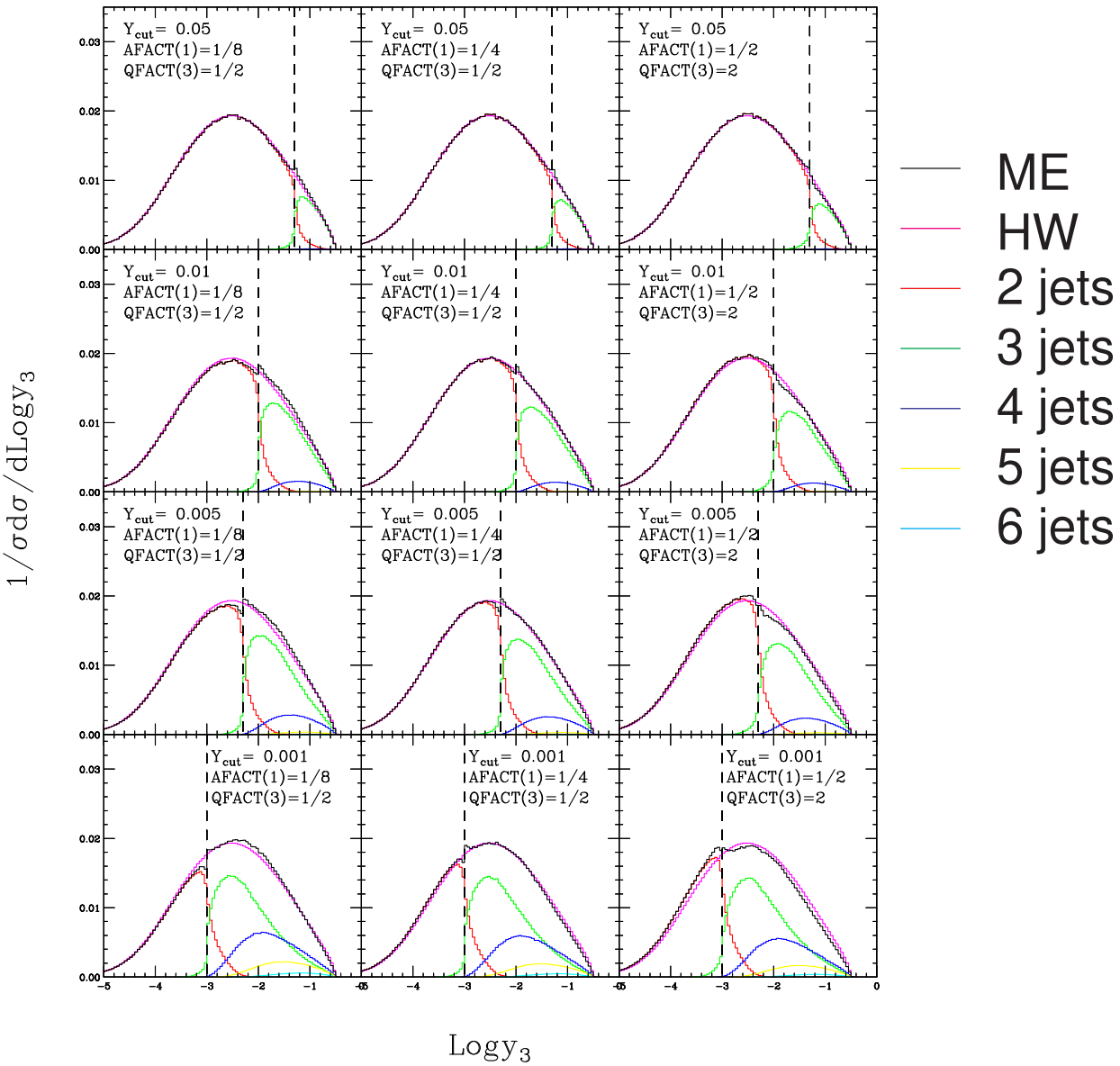
Effect of Varying the cut-off at M_Z

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Hadron-level y_3 distribution.
- Problems with hadronization model which remain to be solved.



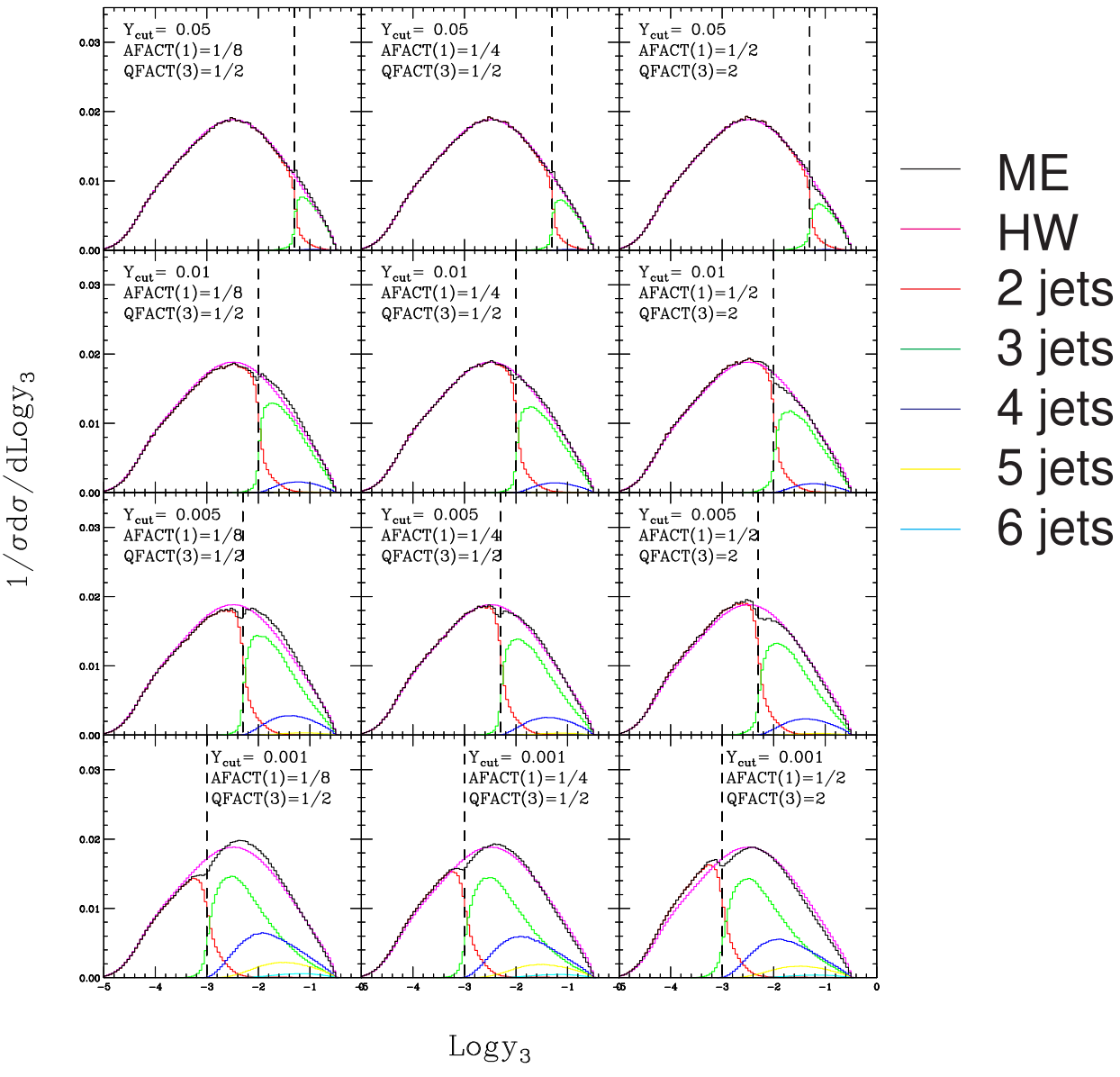
Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Parton-level y_3 distribution.



Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Hadron-level y_3 distribution.
- Problems with hadronization model reduced.



Hadron Collisions

- In hadron collisions in addition to final-state radiation we have the problems of initial-state radiation.
- I will consider W and Z production in hadron collisions
- All the events were generated with MADGRAPH including the leptonic decays of the gauge bosons.
- For the W samples no cuts were applied on the leptons
- For Z production the photon diagrams were included and a cut on the invariant mass of the lepton pair imposed, $m_{\ell\ell} > 20$ GeV.

Hadron Collisions

- We also have a greater choice of the clustering algorithm.
- I will consider three choices.
  - E-scheme

$$\begin{aligned} d_{iB} &= 2E_i^2(1-\cos\theta_{iB}), \\ d_{ij} &= 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij}). \end{aligned}$$

    Combine 4-momenta to get pseudoparticles
  - Covariant E-scheme

$$\begin{aligned} d_{iB} &= p_{ti}^2, \\ d_{ij} &= \min(p_{ti}^2, p_{tj}^2)R_{ij}^2, \end{aligned}$$

    Combine 4-momenta to get pseudoparticles
  - Covariant monotonic p_T^2 scheme
    Same definition of d as Covariant E-scheme but define p_t and R for pseudoparticles as

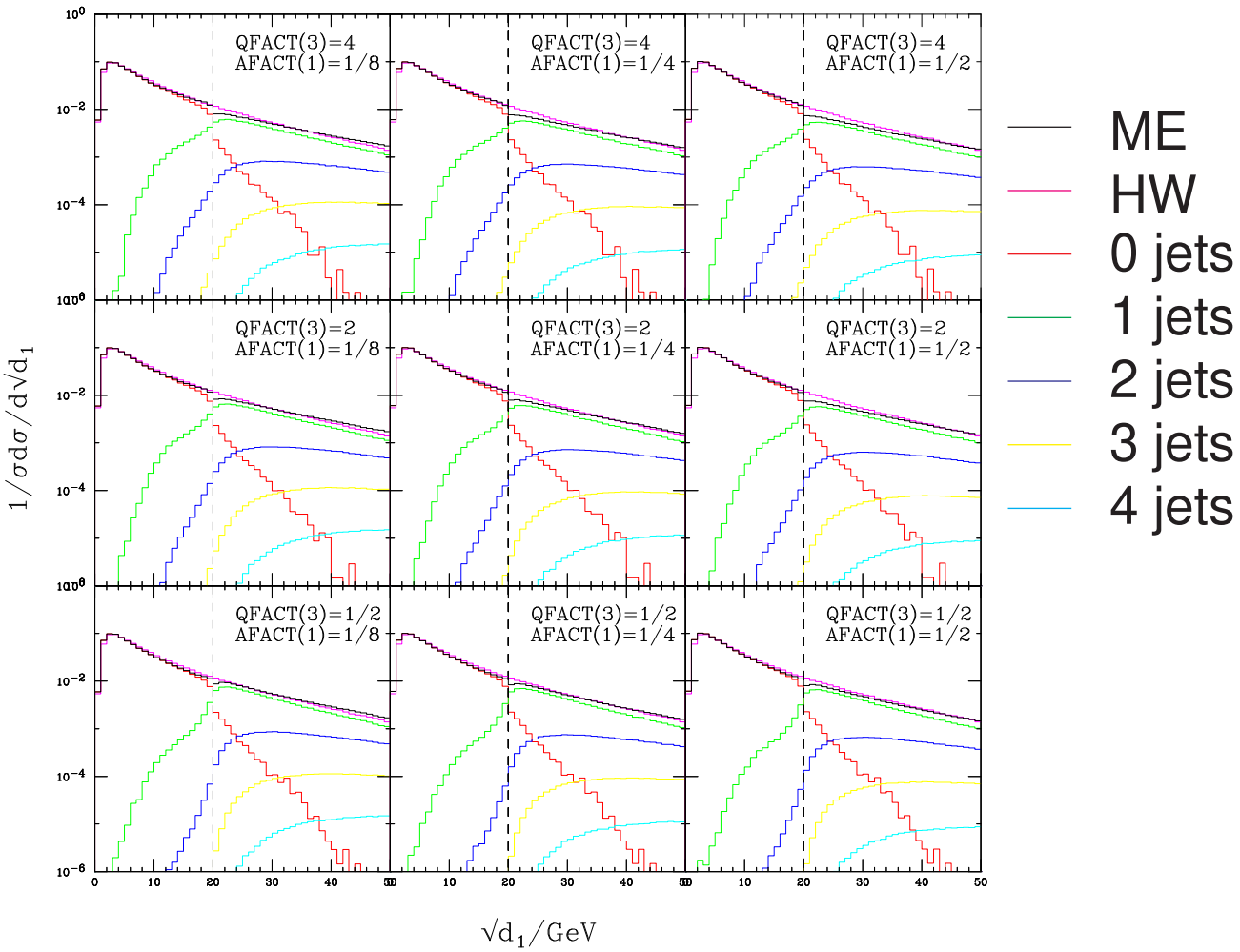
$$\begin{aligned} p_{t(ij)} &= p_{ti} + p_{tj}, \\ R_{(ij)k}^2 &= \frac{p_{ti}^2 R_{ik}^2 + p_{tj}^2 R_{jk}^2}{p_{ti}^2 + p_{tj}^2}. \end{aligned}$$

Hadron Collisions

- Start by considering W and Z production at the Tevatron.
- Use the Covariant monotonic p_T^2 scheme.
- and a matching scale of $400\ \mathrm{GeV}^2$.
- Look at the differential cross section with respect to $\sqrt{d_n}$ where d_n is the scale at which n jets are resolved.
- This is like the E_T in a cone algorithm.

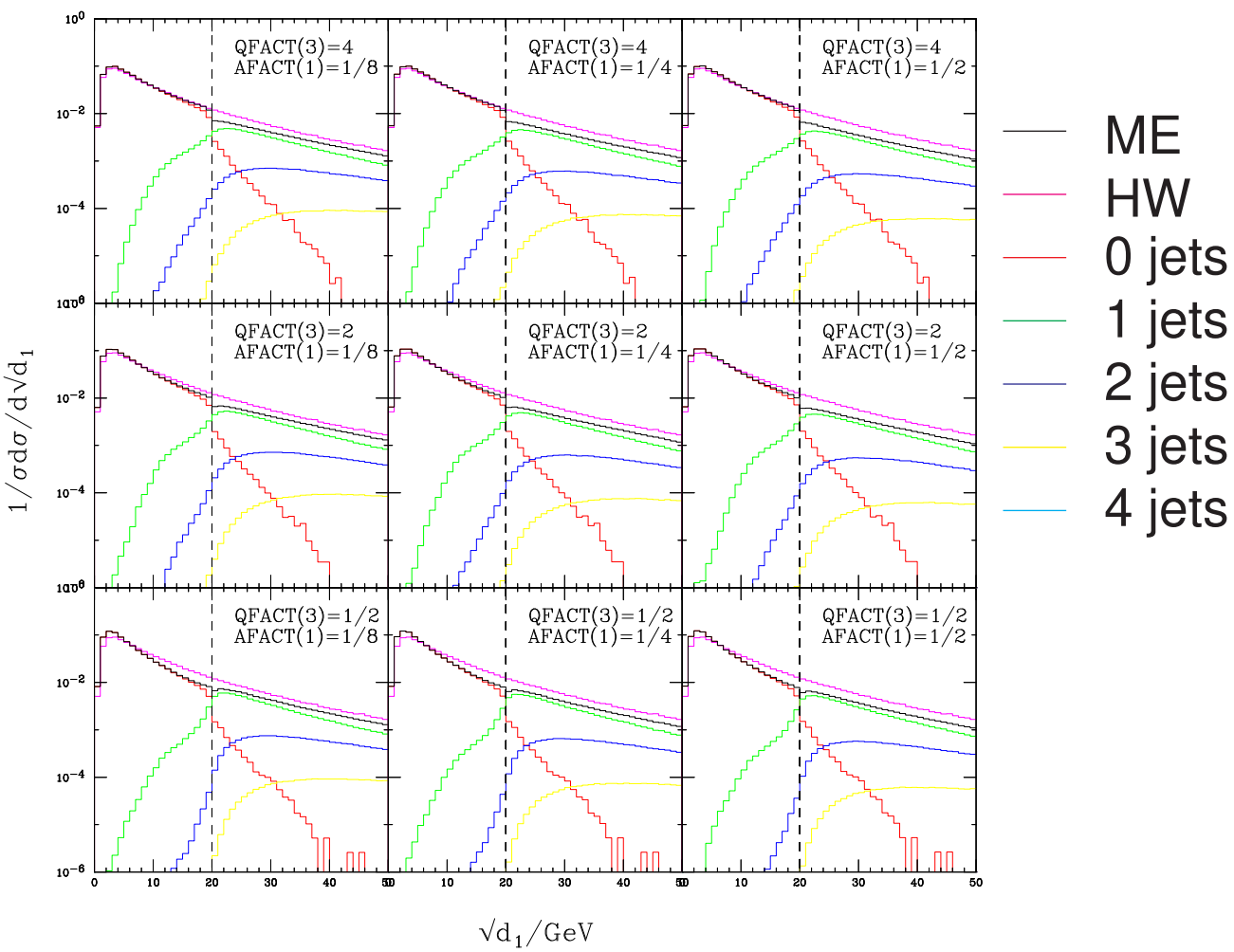
W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_1 distribution at the Tevatron



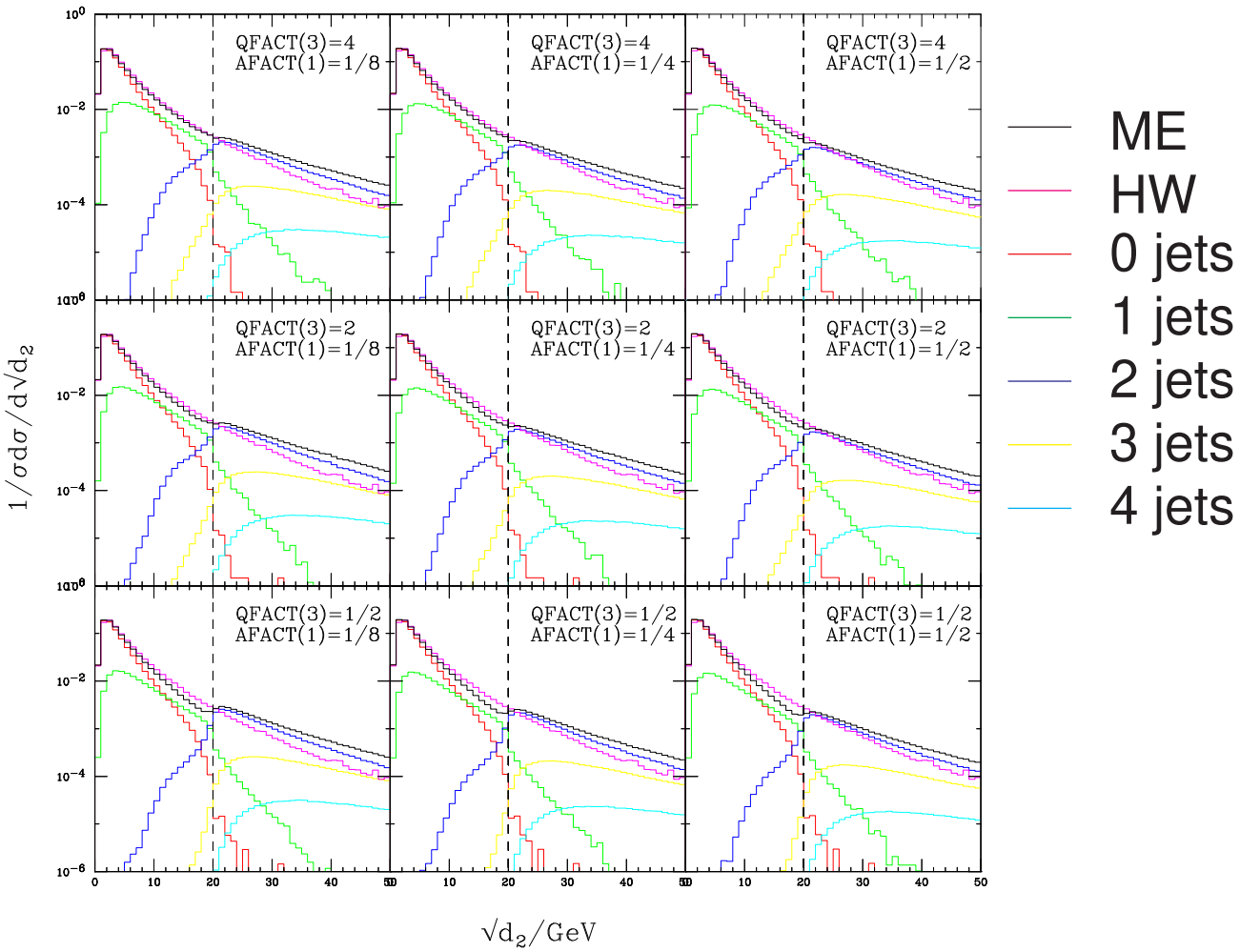
Z Production

- Variation of results with scale of α_S and starting scale of shower.
- d_1 distribution at the Tevatron



W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_2 distribution at the Tevatron

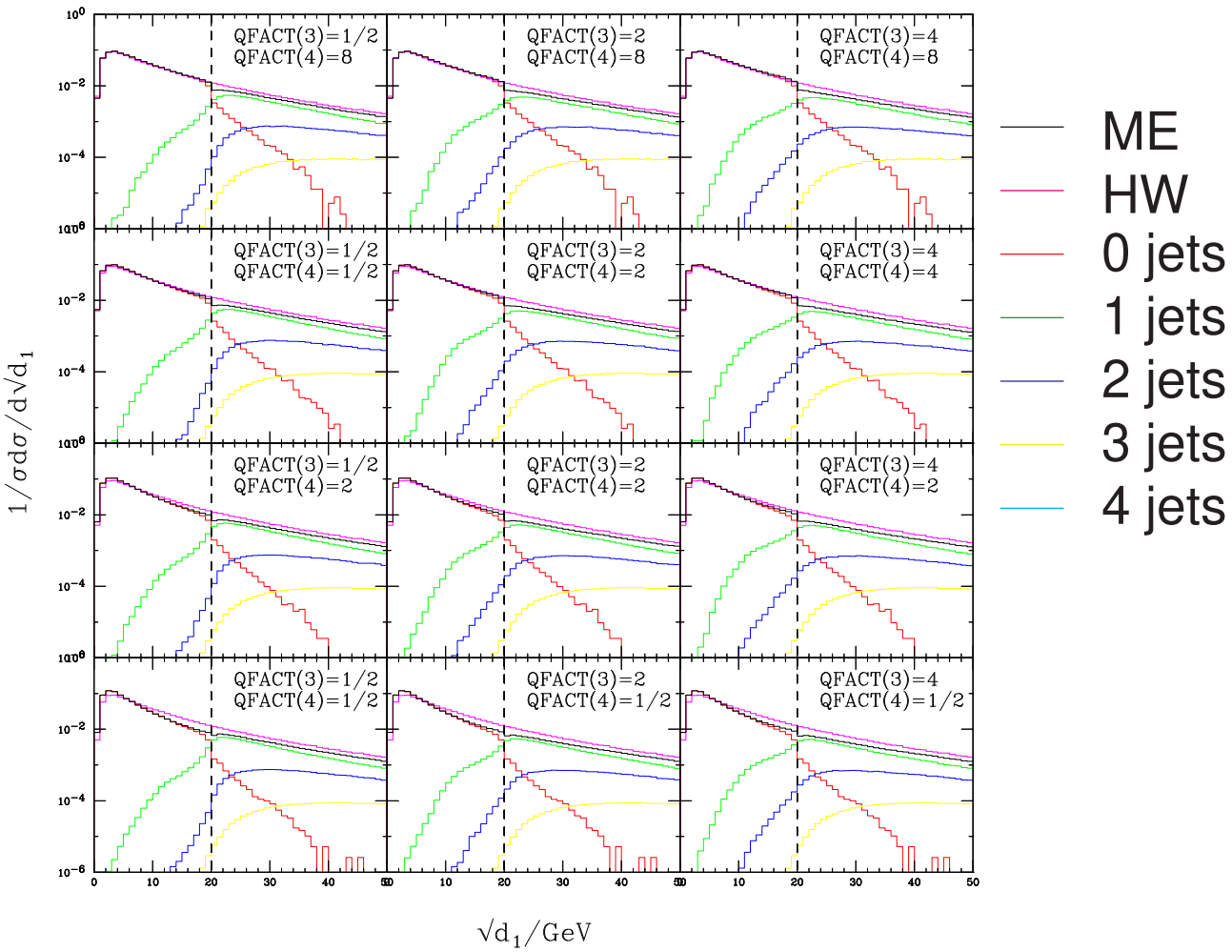


Hadron Collisions

- There is a radiation dip at the matching scale $\Rightarrow$ high `QFACT(3)`.
- However there is also a problem with events migrating below the matching scale $\Rightarrow$ low `QFACT(3)`.
- Solution is to decouple the scale for the initial-state shower `QFACT(4)` from the scale for the final-state shower `QFACT(3)`.

Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- d_1 distribution at the Tevatron



W Production

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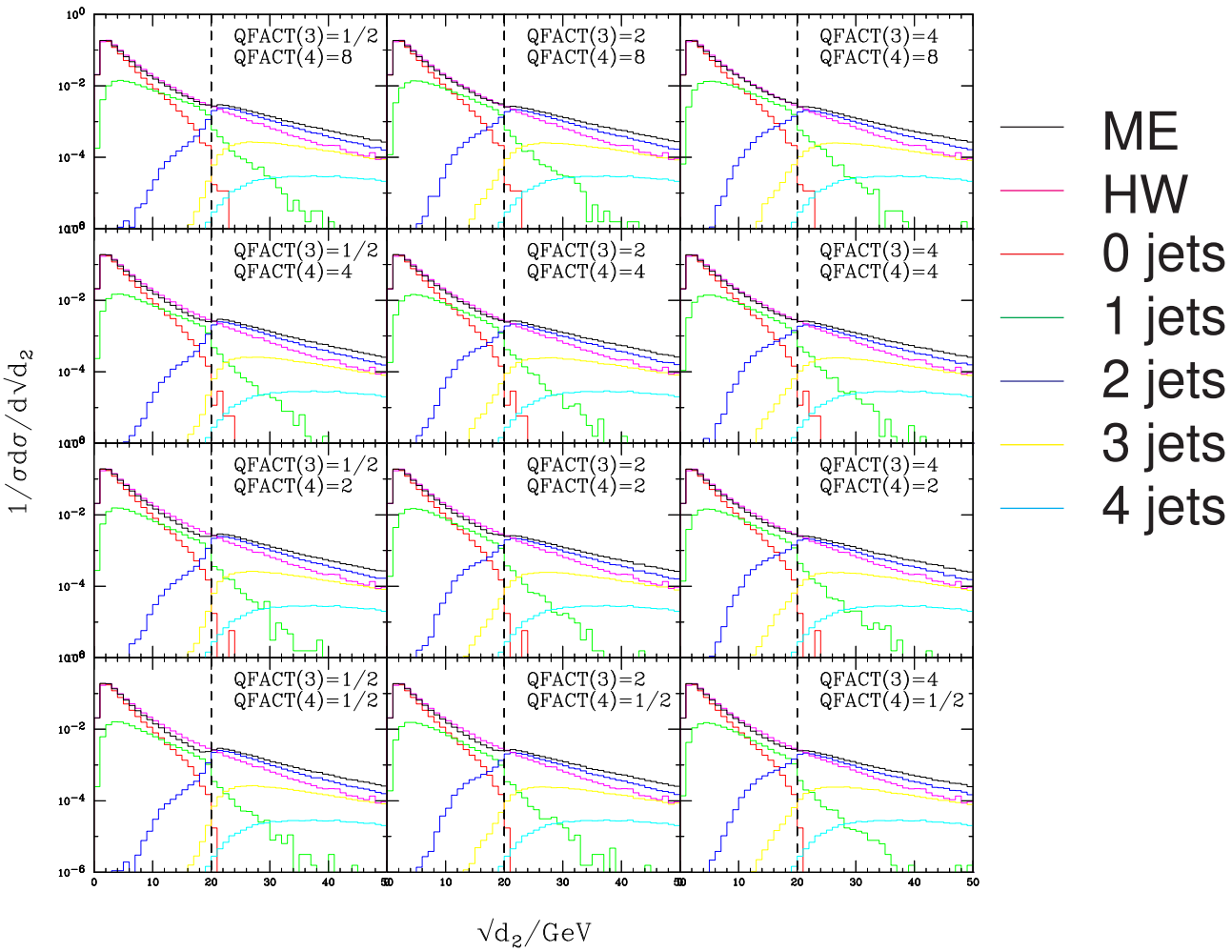
- d_2 distribution at the Tevatron



Z Production

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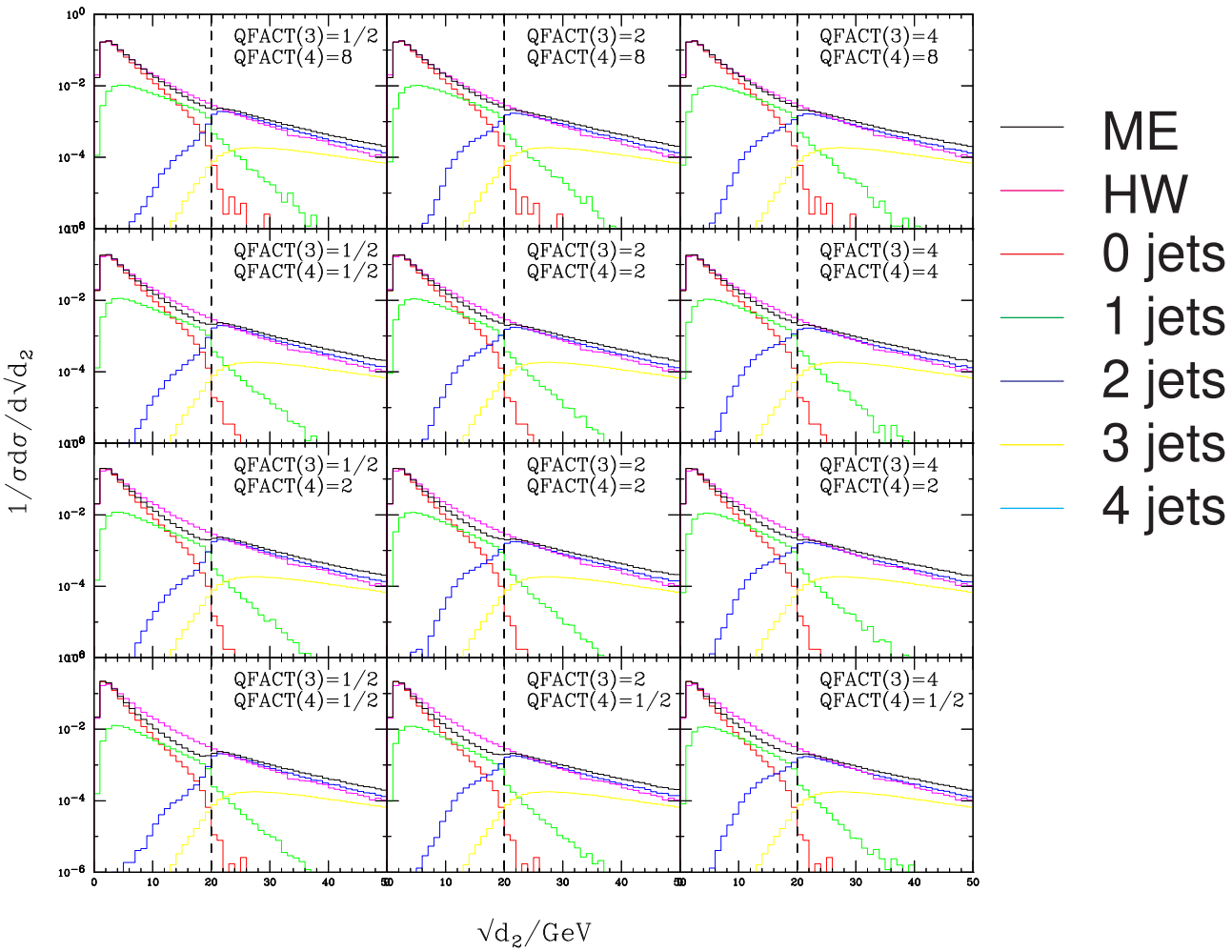
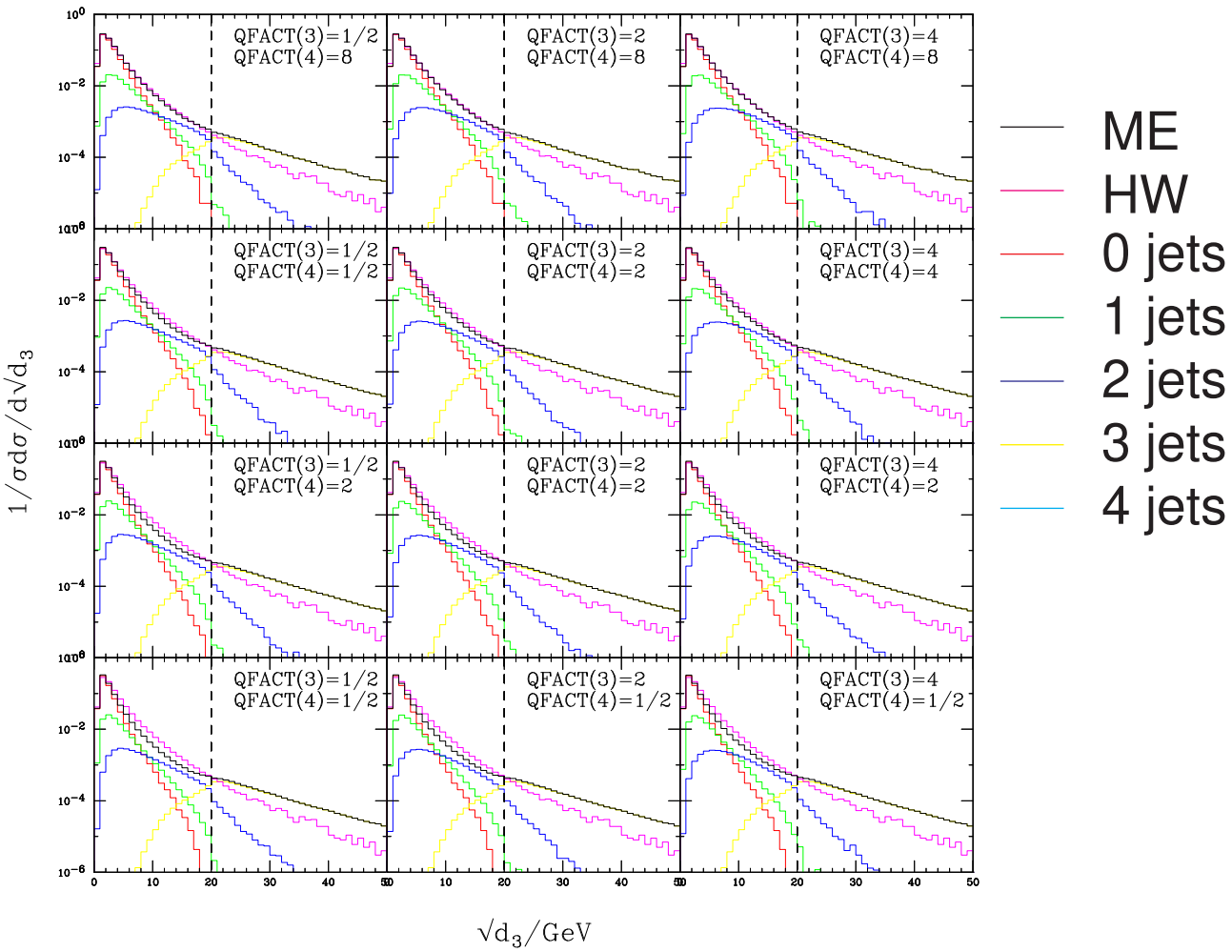


Z Production

- Variation of results with `QFACT(3,4)` from `AFACT(1,2)=1/8`.
- d_3 distribution at the Tevatron



Hadron Collisions

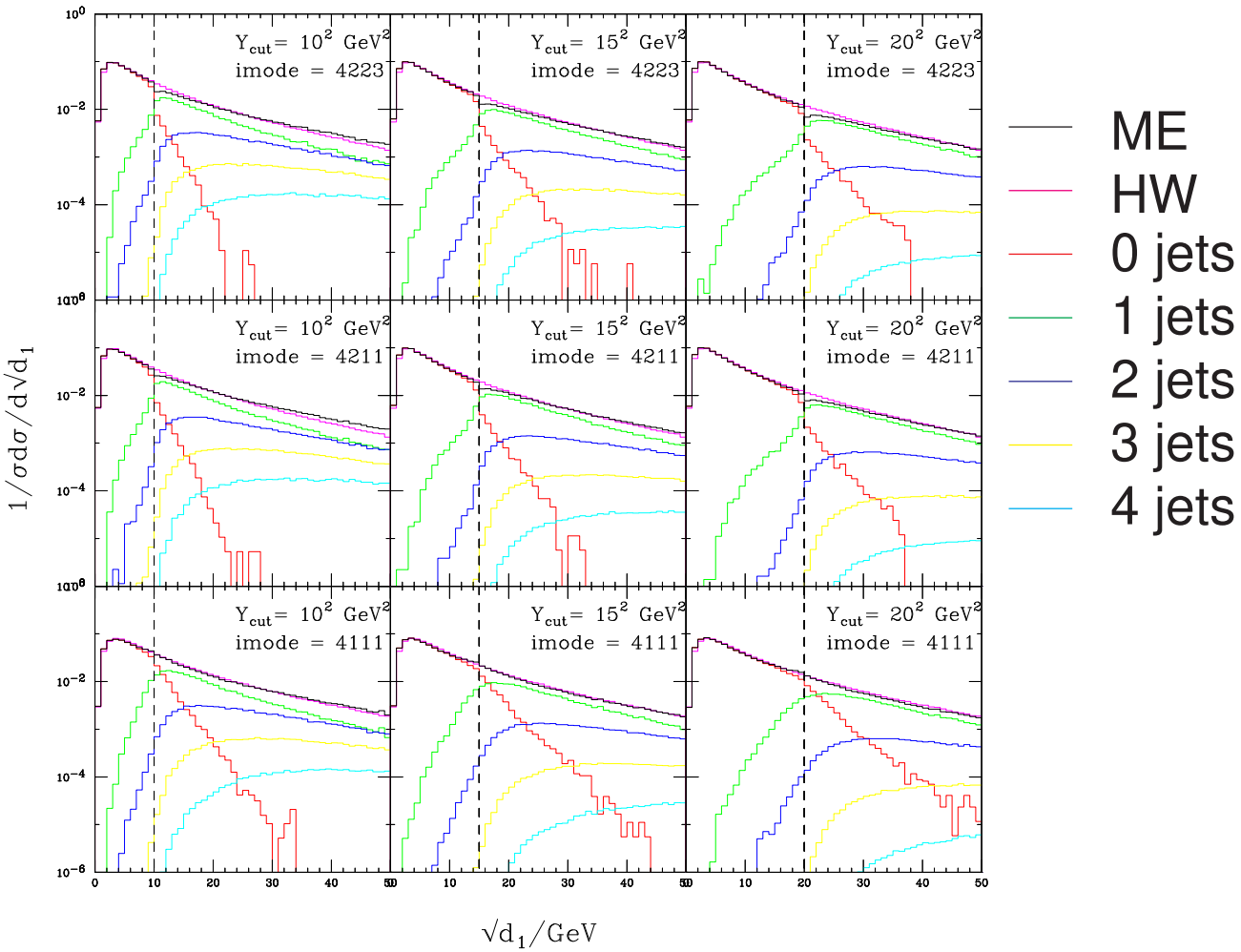
- Hard to get the HERWIG shower to produce more radiation than before below the matching scale.
- Hopefully there is some solution to this.
- Otherwise we will have to use low values of the matching scale.
- *i.e.* a region where the PS is already giving the right answer.
- One other option which helps is to always start the initial state shower at the highest scale in the process.

Hadron Collisions

- Look at the variation with matching scale and algorithm
- Look at three scales
  - 10^2 GeV^2
  - 15^2 GeV^2
  - 20^2 GeV^2
- and three algorithms
  - E-scheme, `IMODE=4111`
  - Covariant E-scheme, `IMODE=4211`
  - Covariant monotonic p_T^2 scheme, `IMODE=4223`.

W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- d_1 distribution at the Tevatron



W Production

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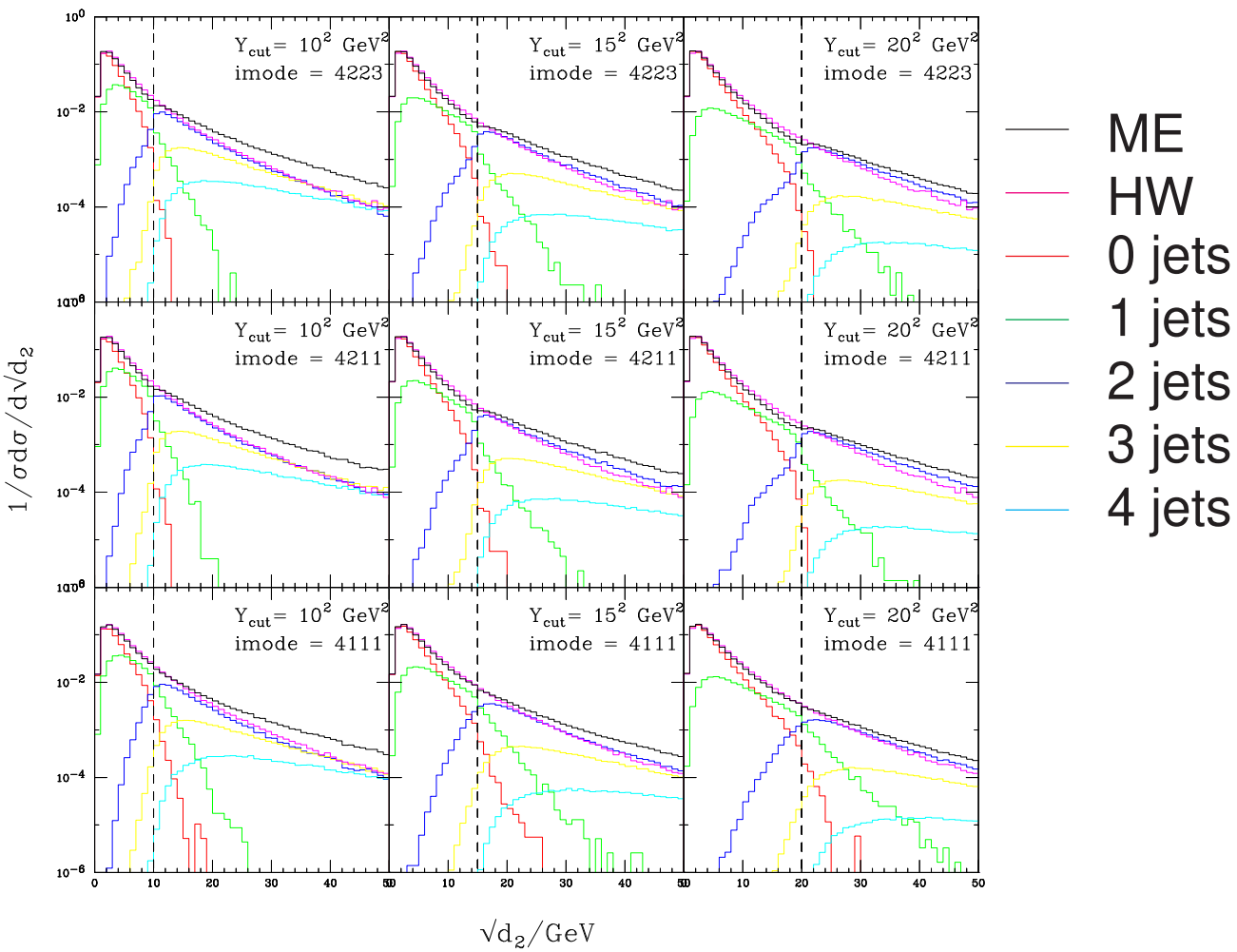
- Normal starting scale.
- d_2 distribution at the Tevatron



W Production

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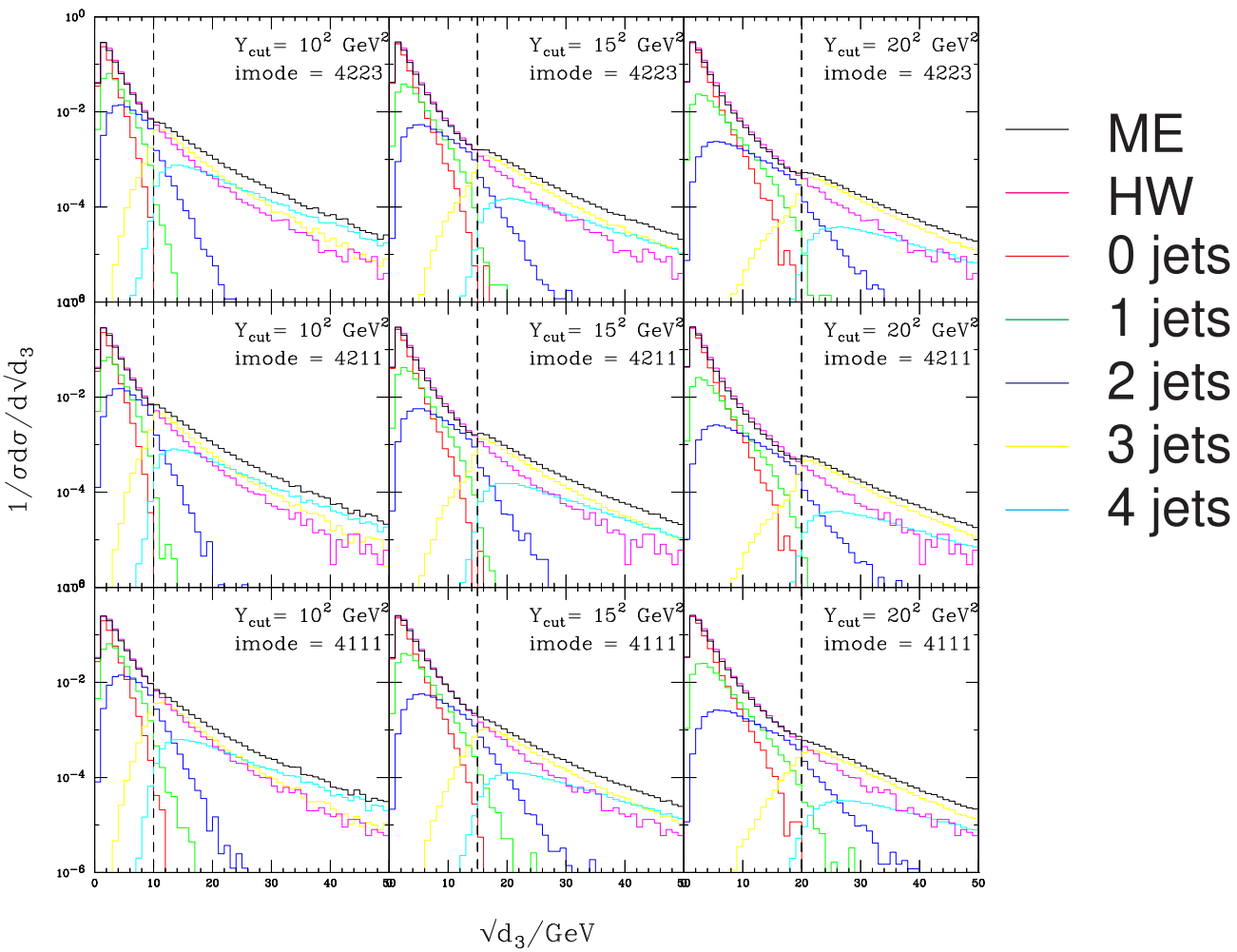
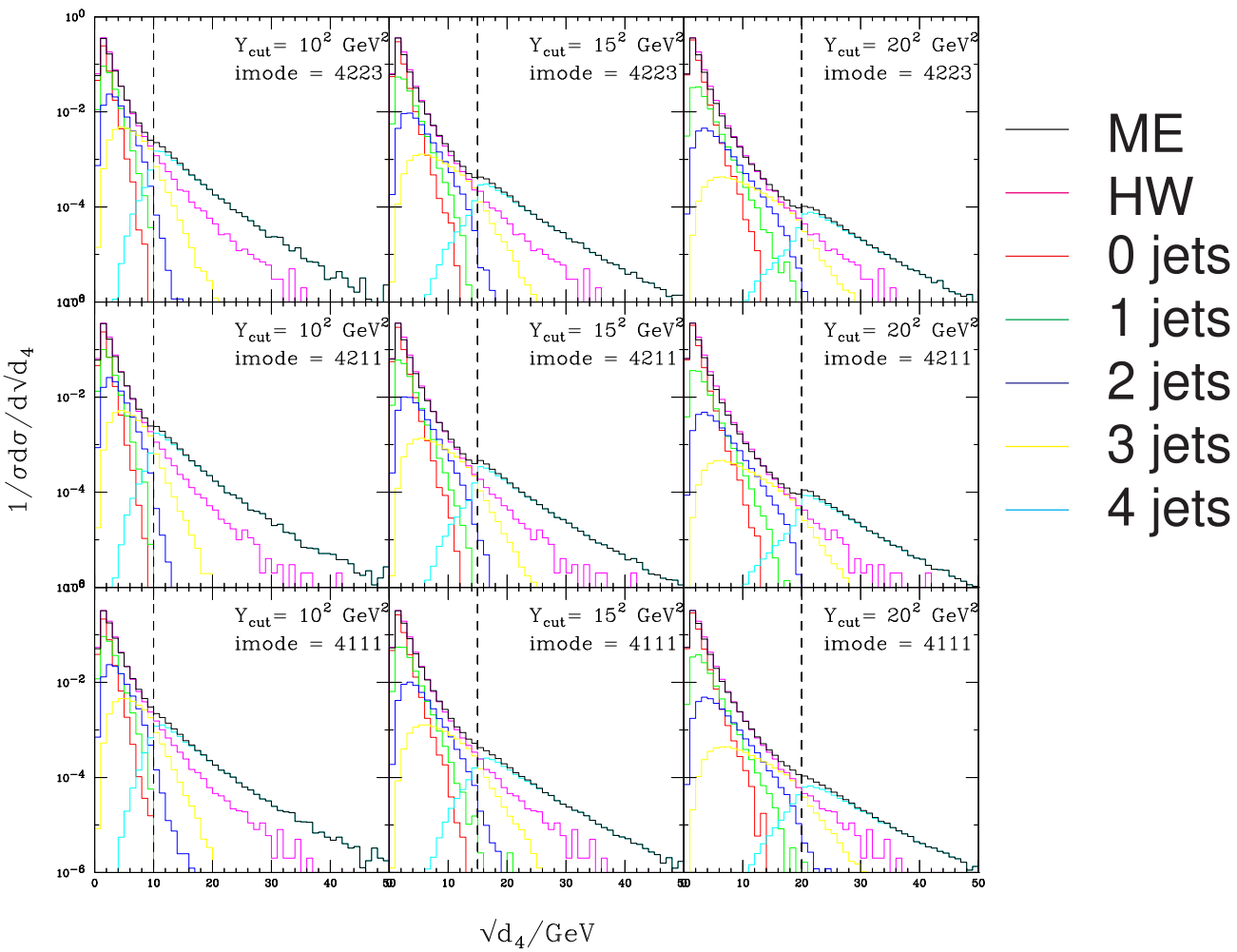
- Normal starting scale.
- d_3 distribution at the Tevatron



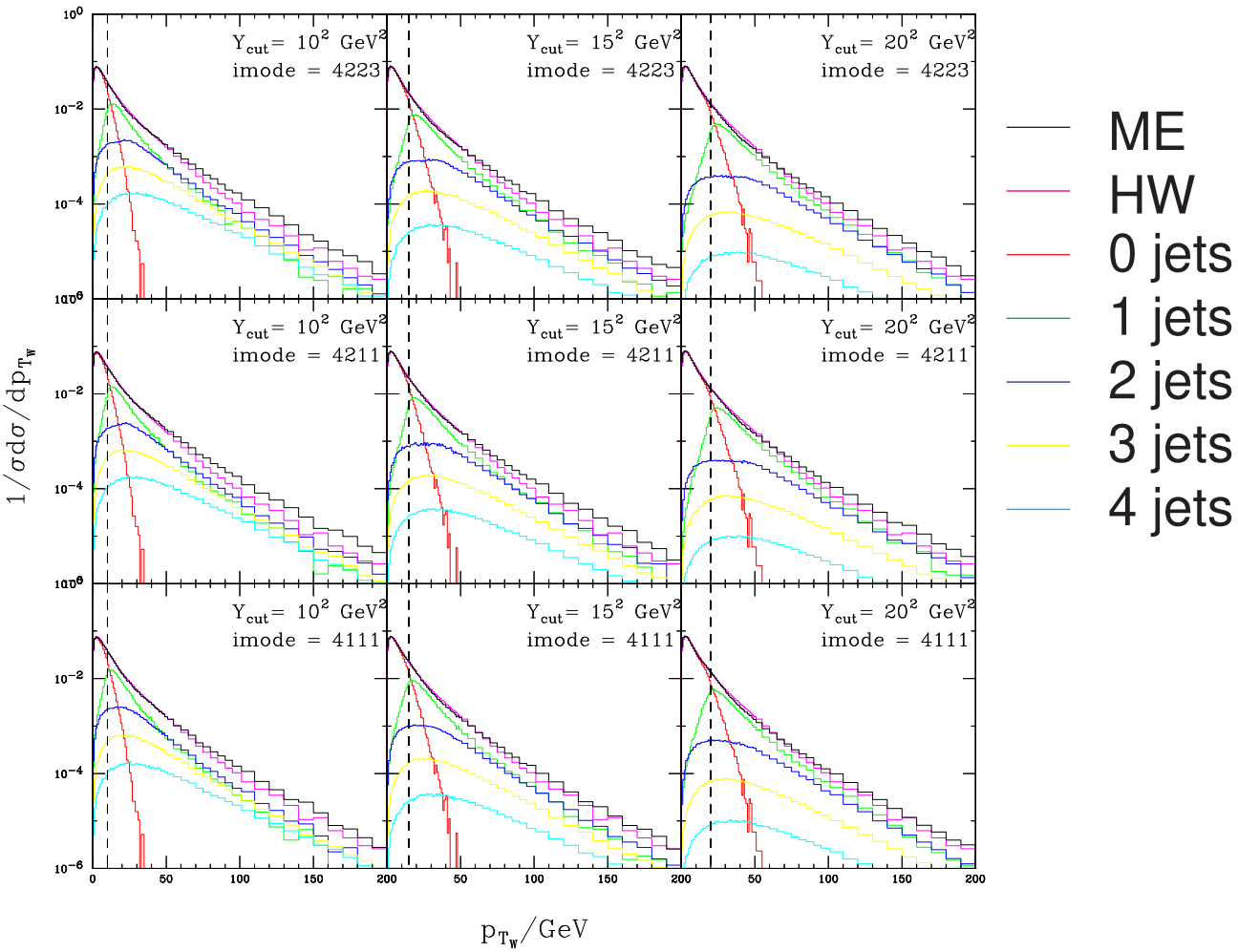
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- d_4 distribution at the Tevatron



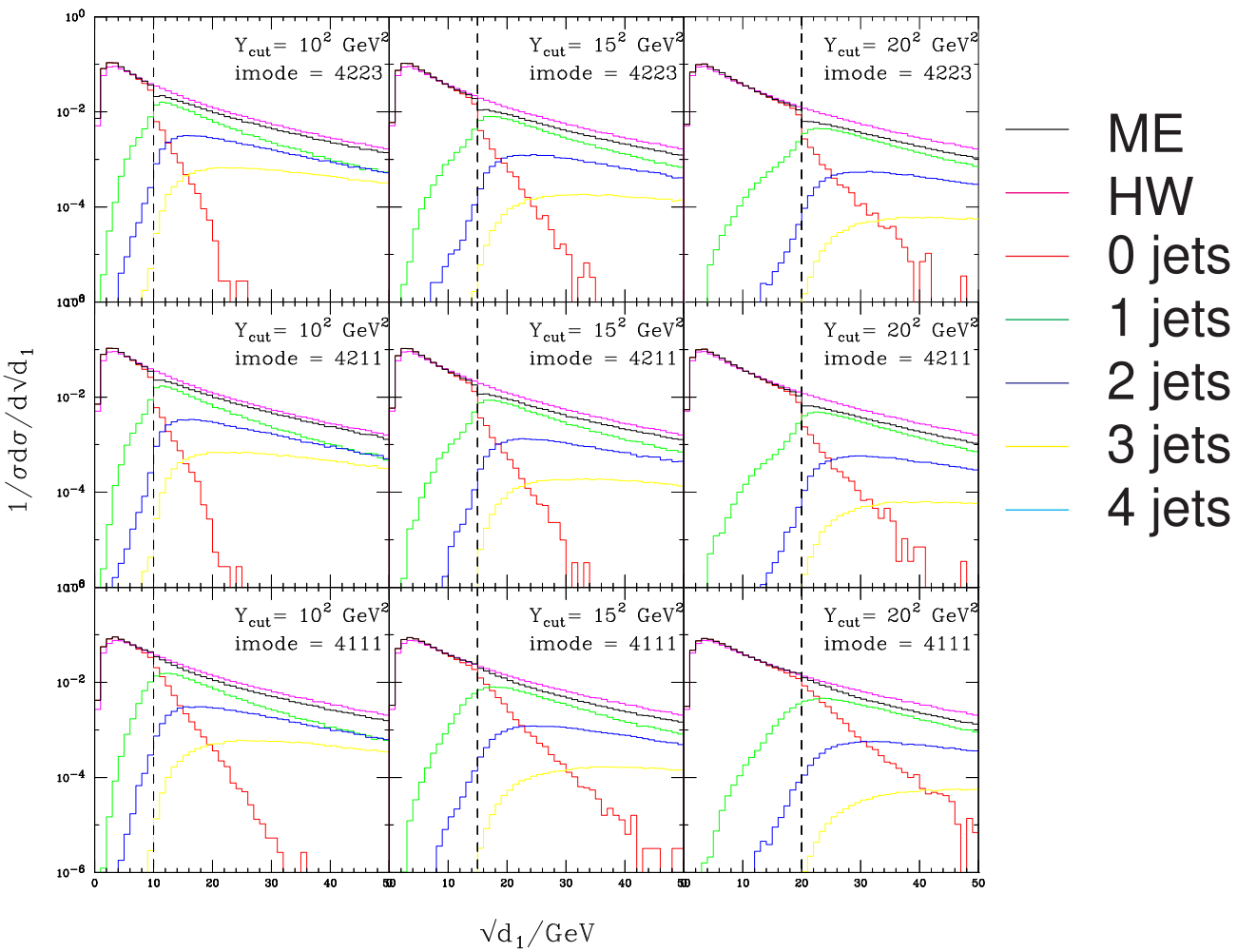
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- W p_T distribution at the Tevatron



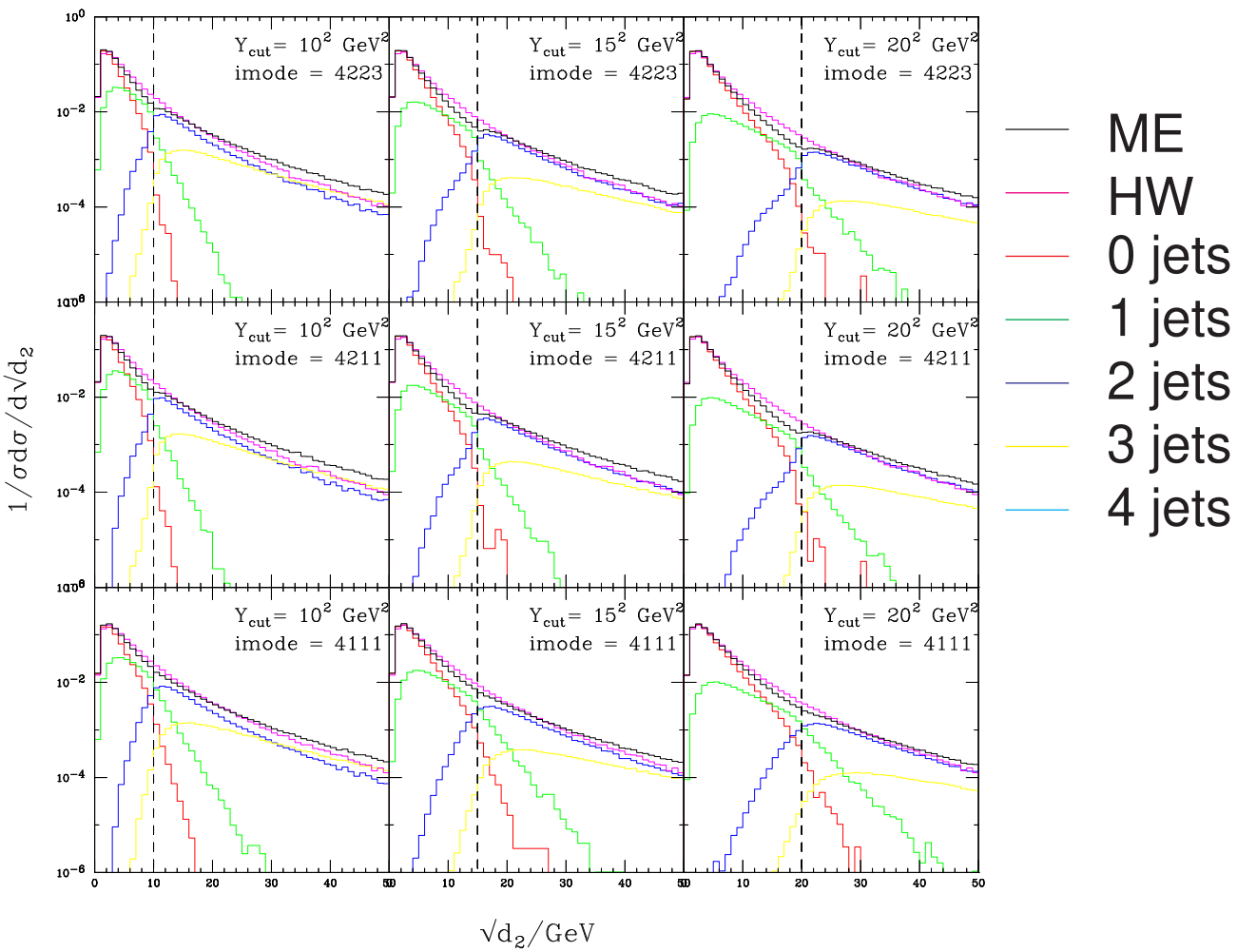
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- d_1 distribution at the Tevatron



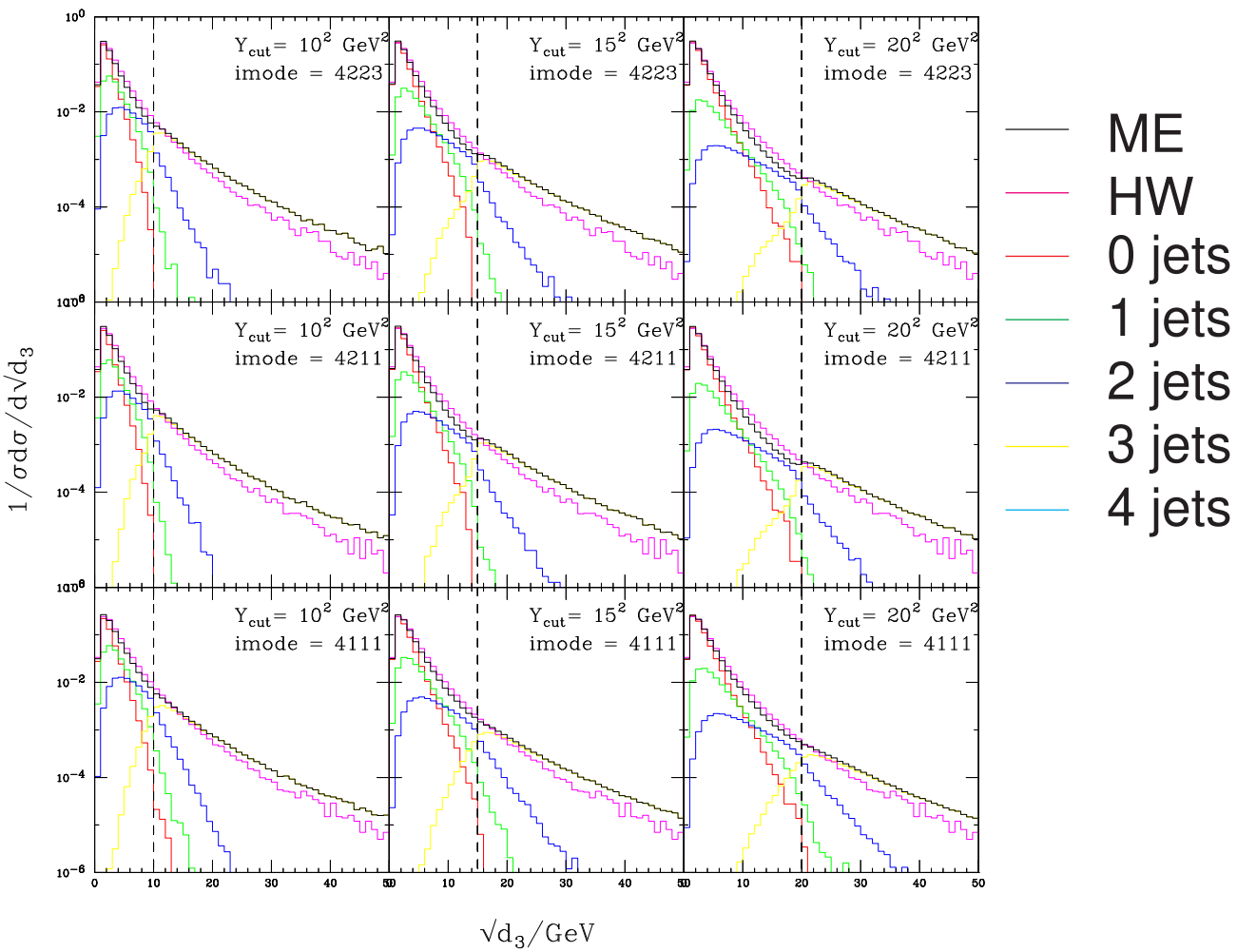
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- d_2 distribution at the Tevatron



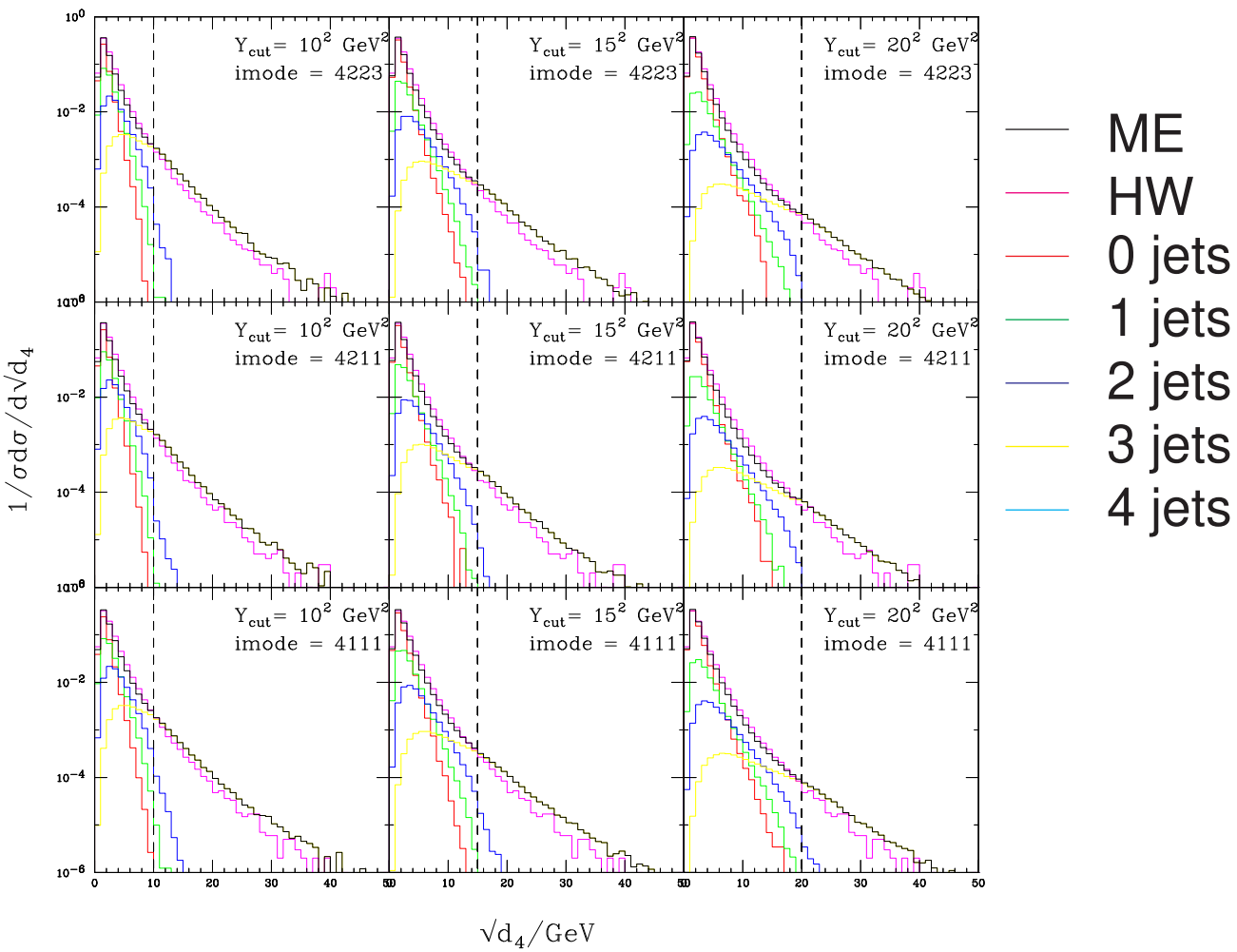
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- d_3 distribution at the Tevatron



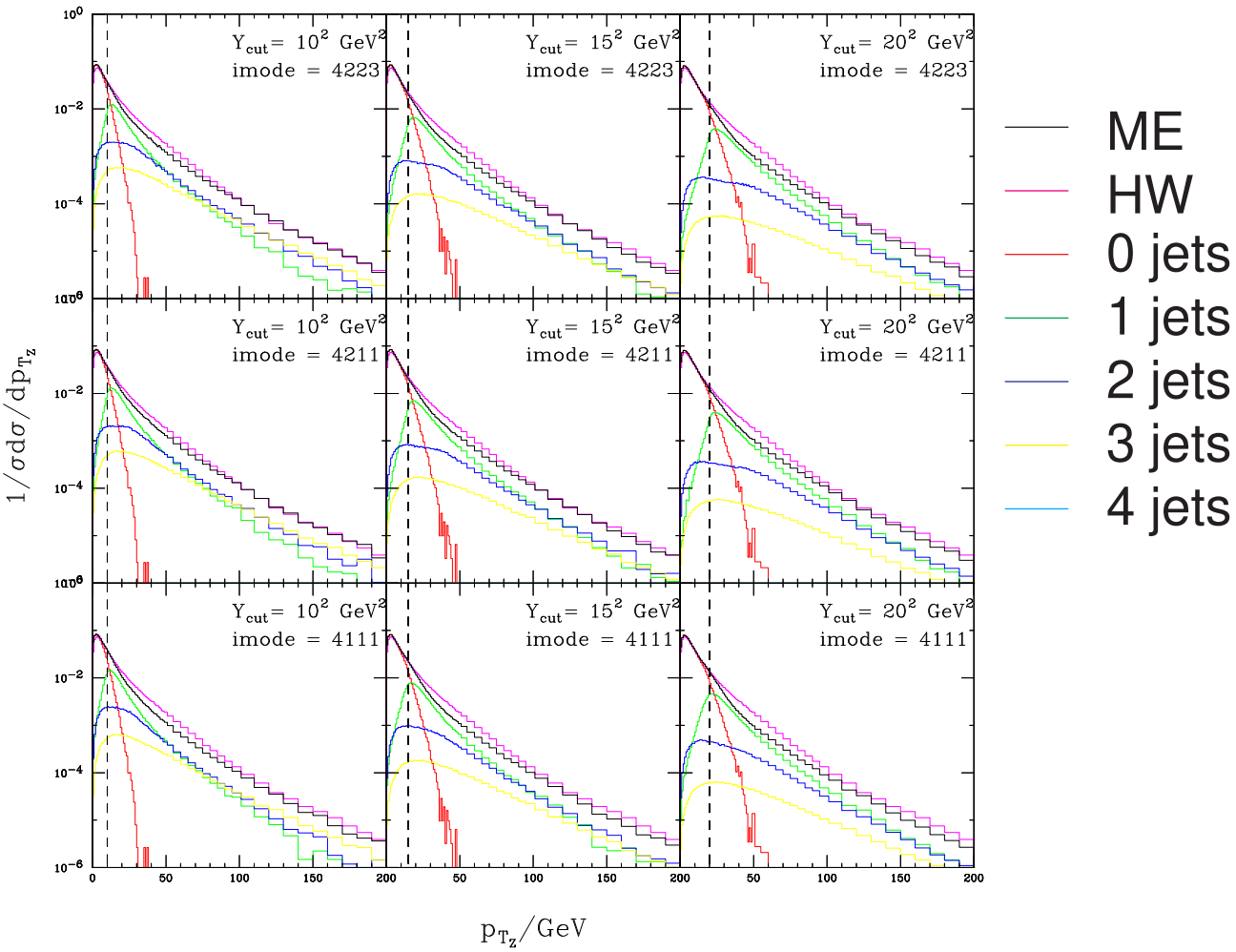
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- d_4 distribution at the Tevatron



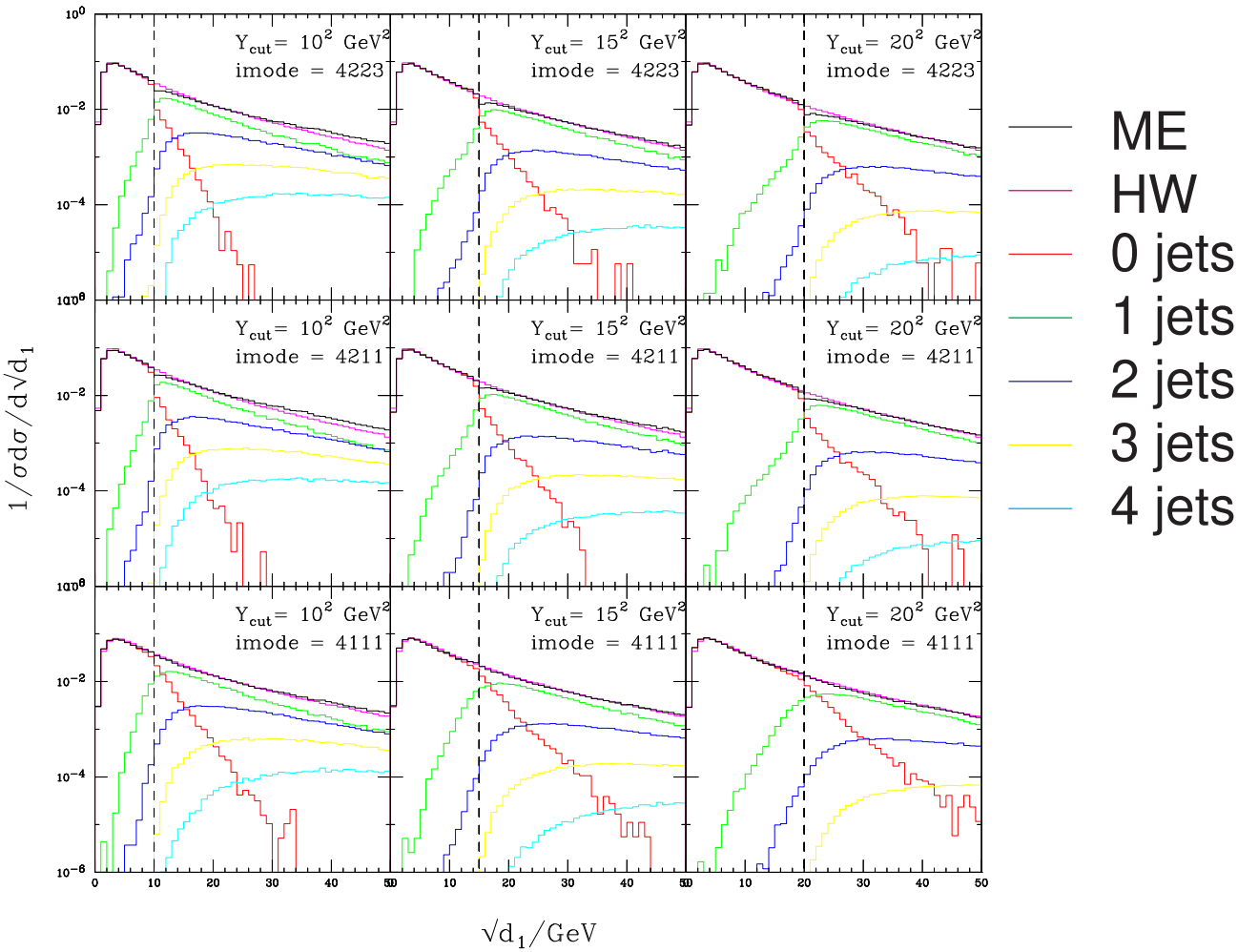
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Normal starting scale.
- Z p_T distribution at the Tevatron



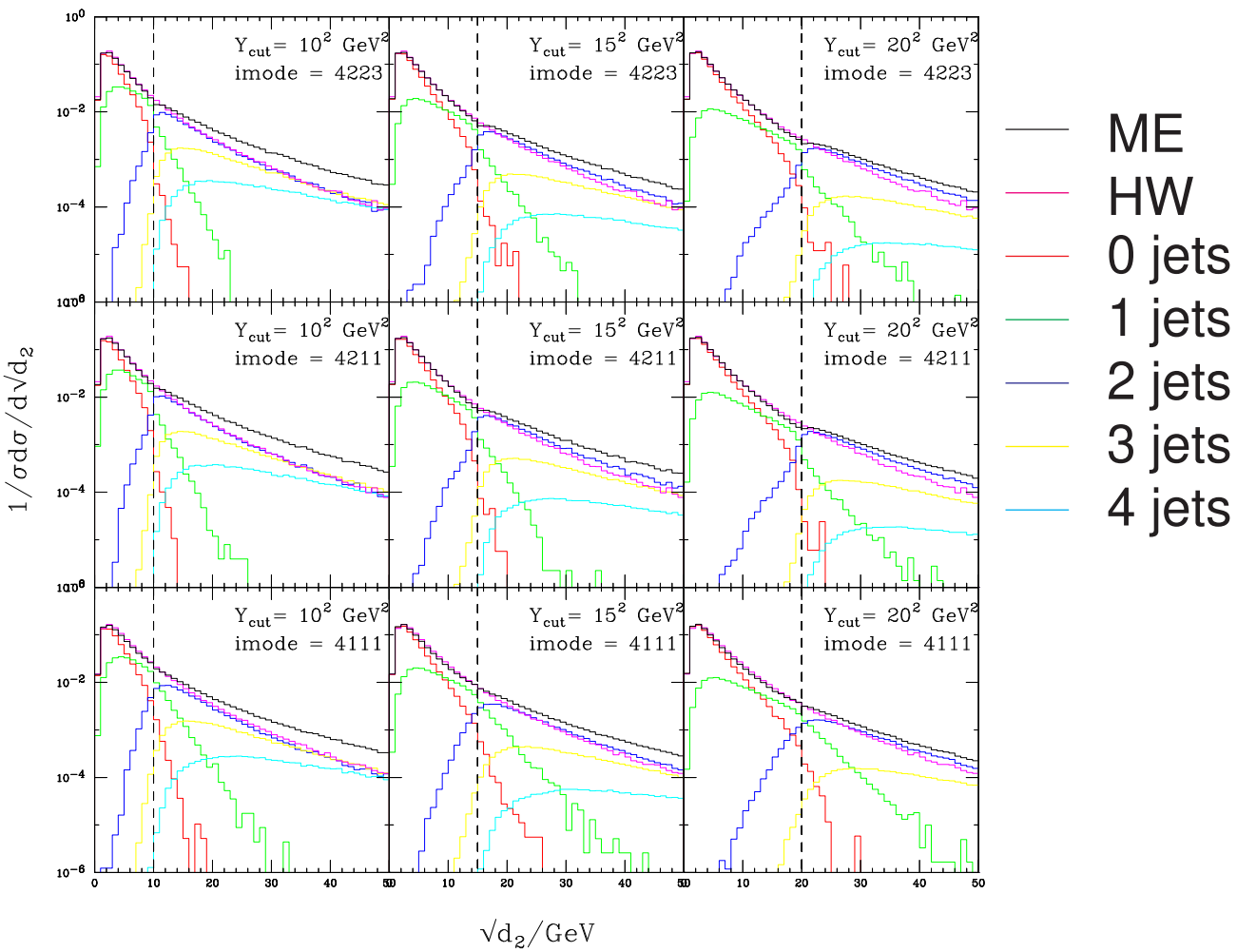
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_1 distribution at the Tevatron



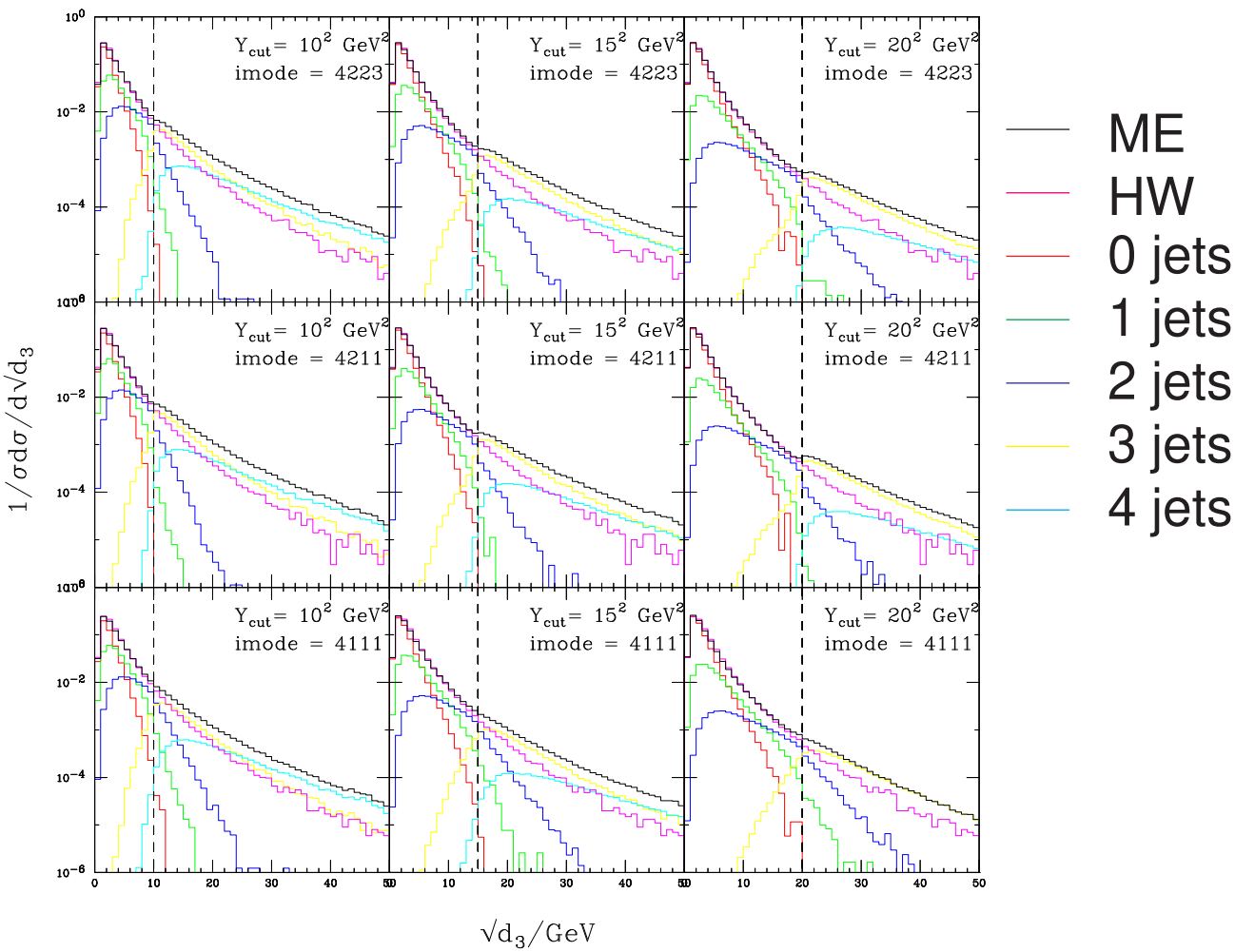
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_2 distribution at the Tevatron



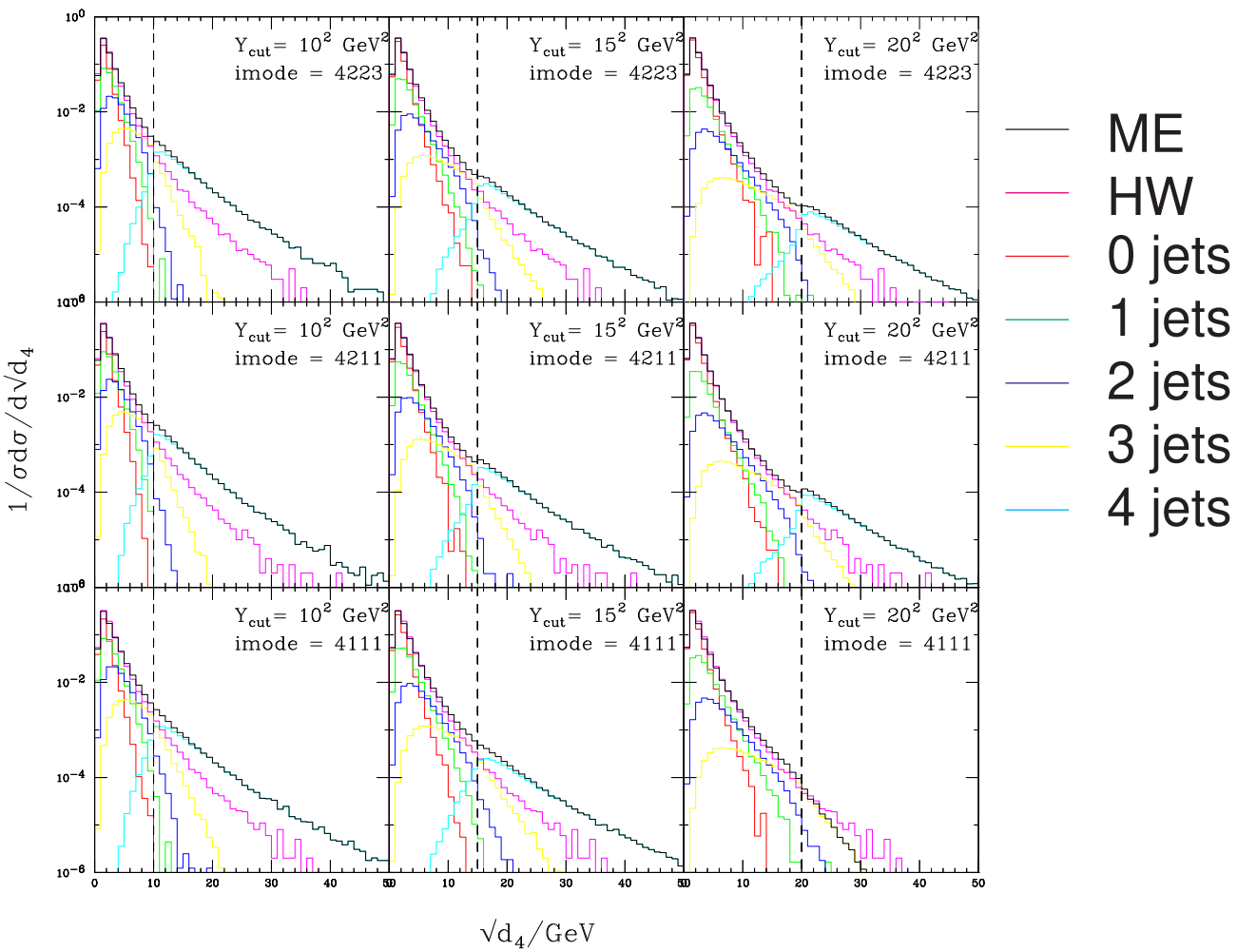
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_3 distribution at the Tevatron



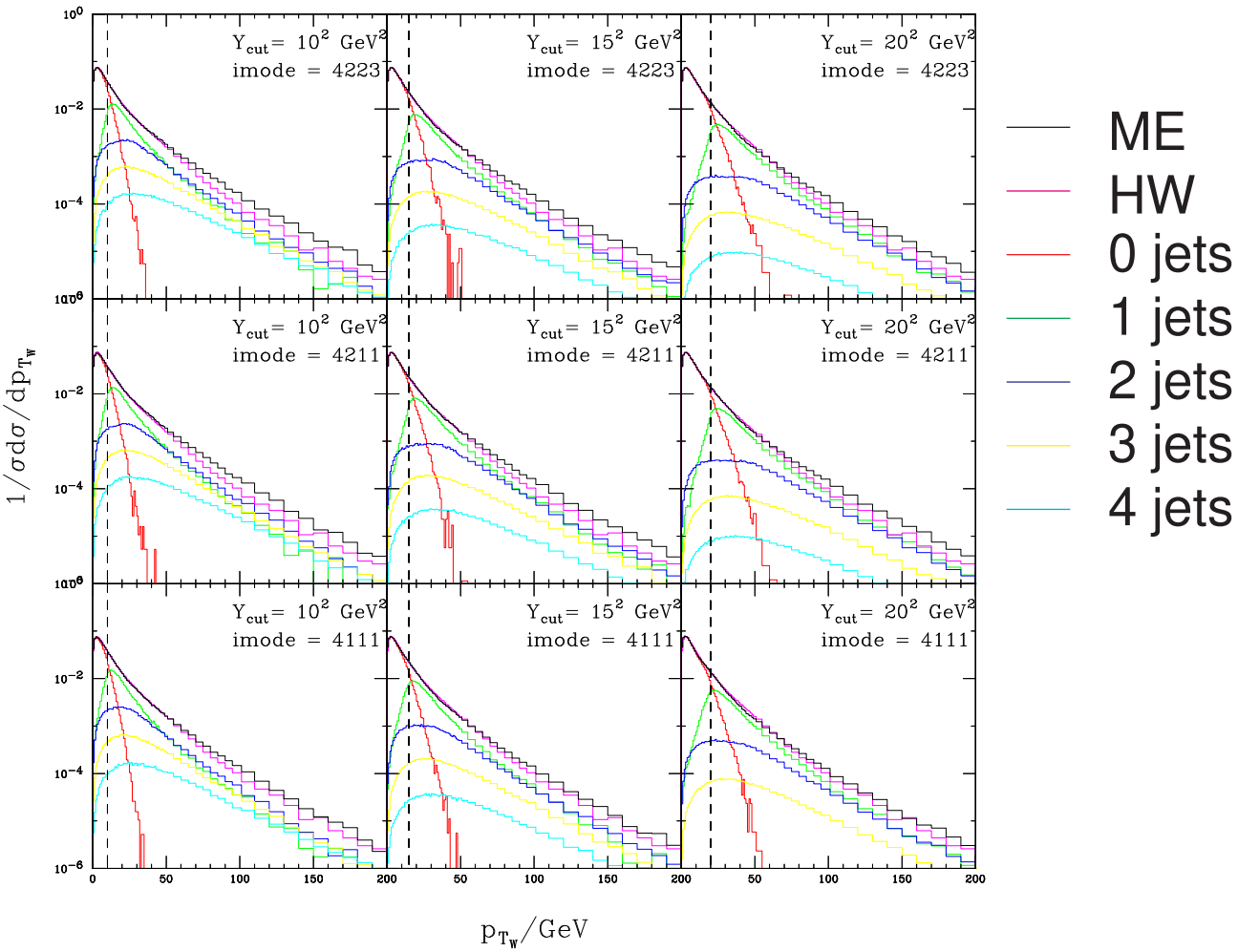
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_4 distribution at the Tevatron



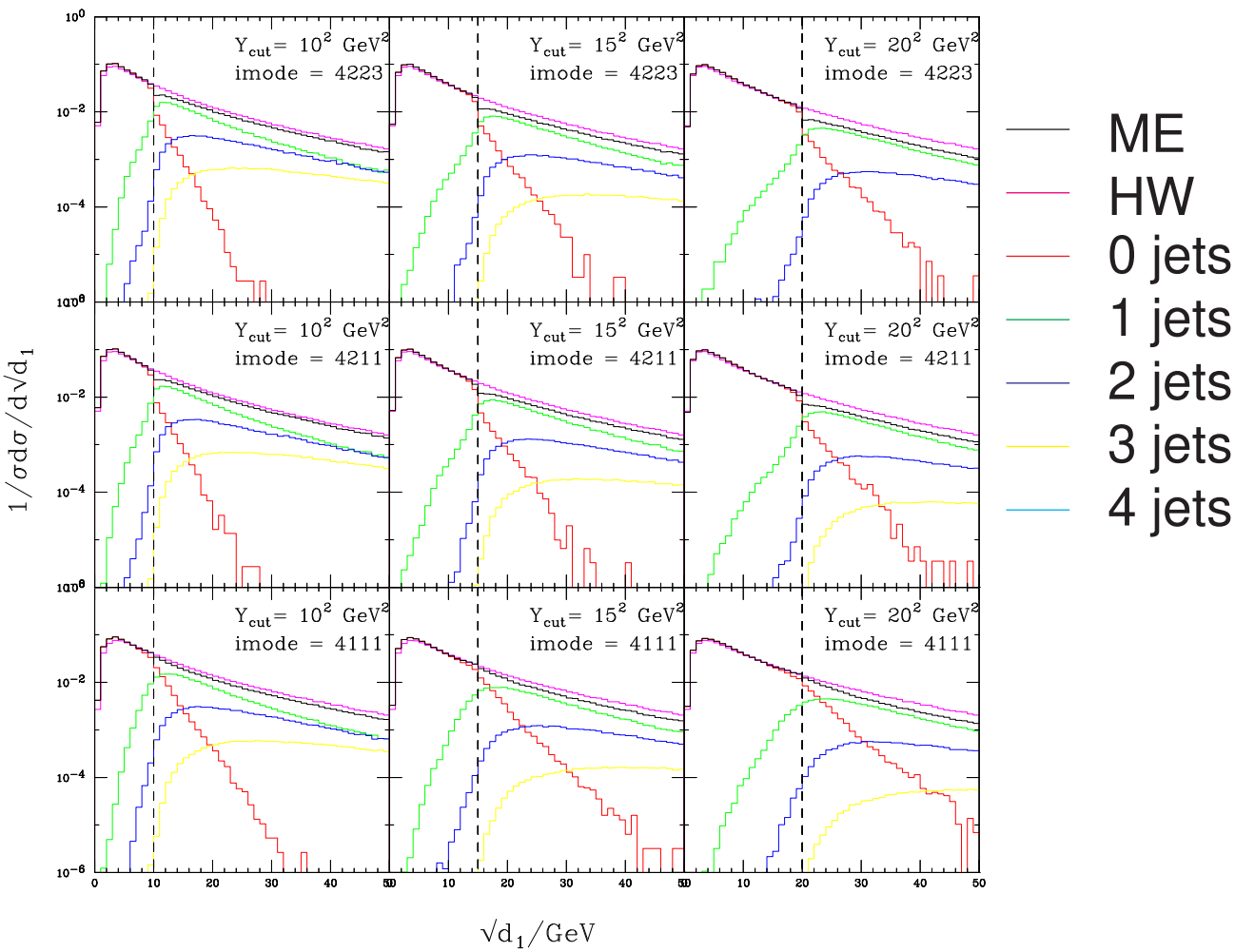
W Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- W p_T distribution at the Tevatron



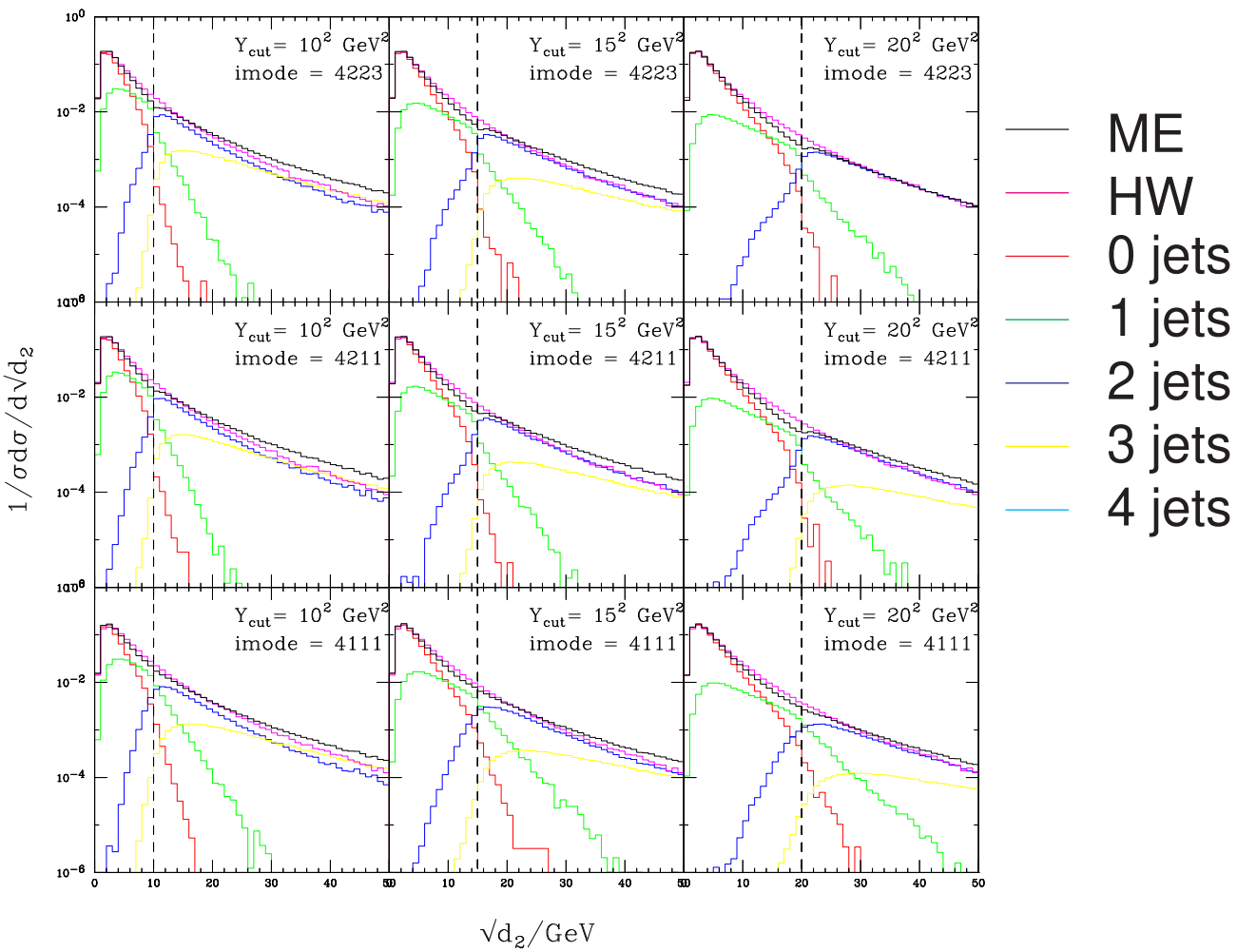
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_1 distribution at the Tevatron



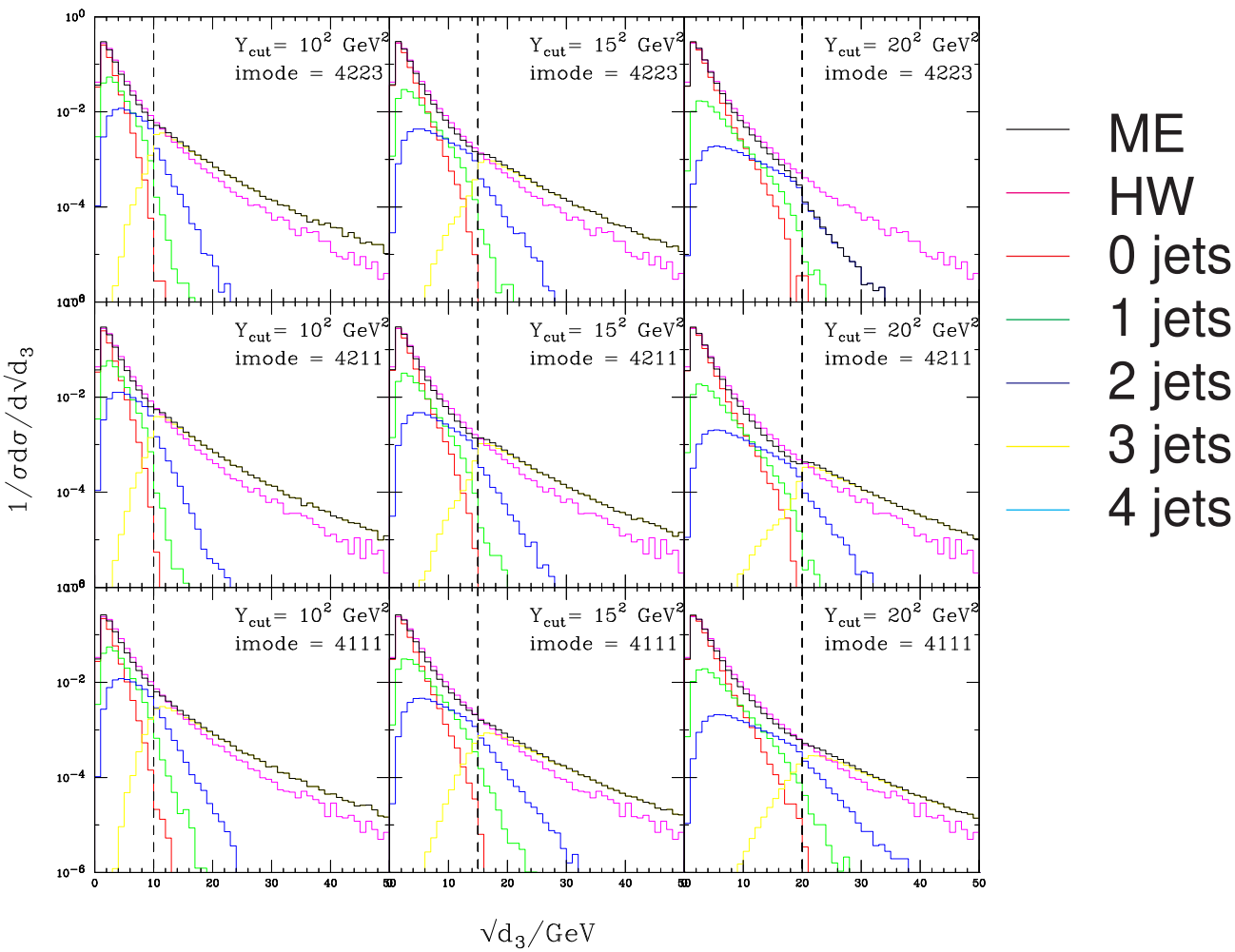
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_2 distribution at the Tevatron



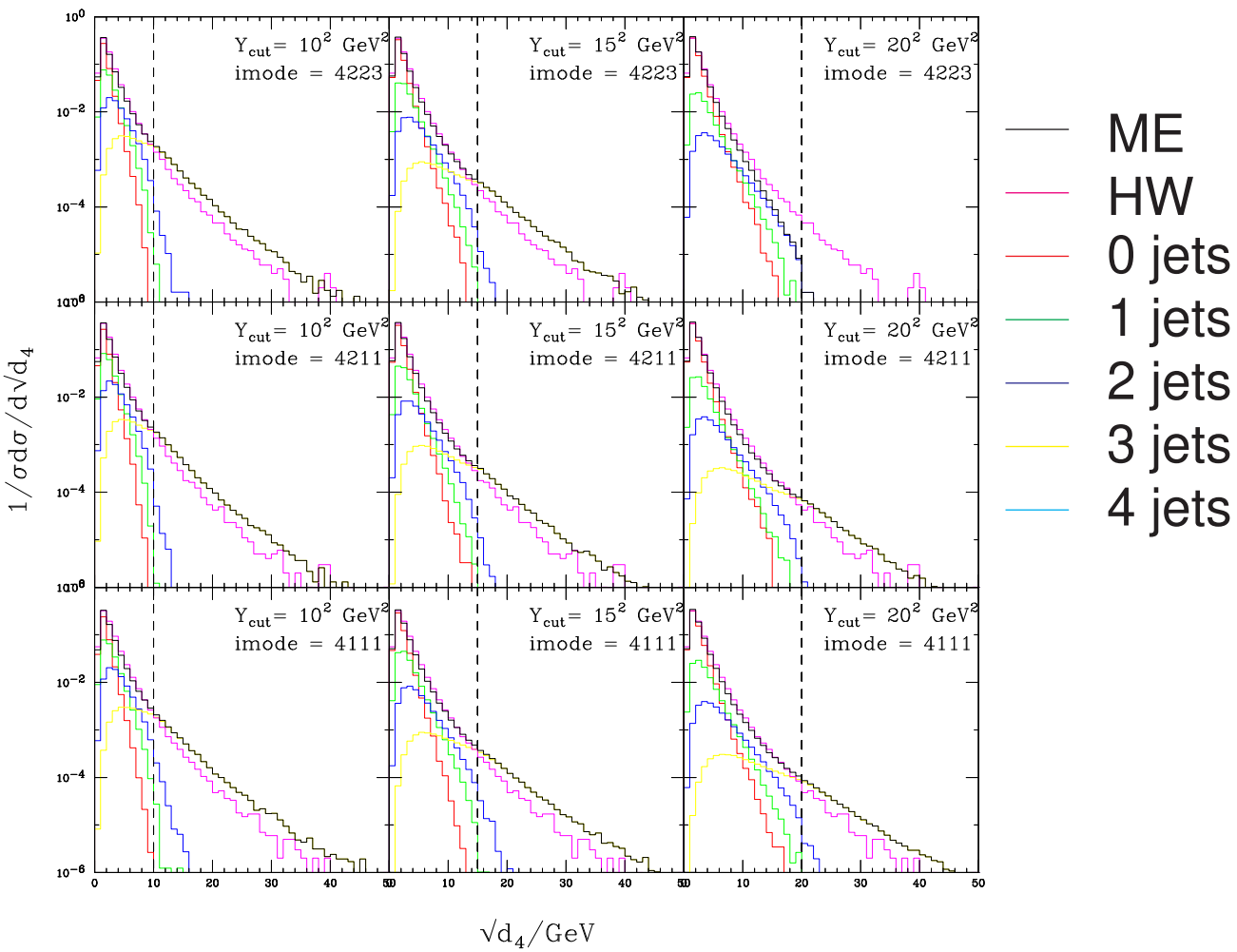
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_3 distribution at the Tevatron



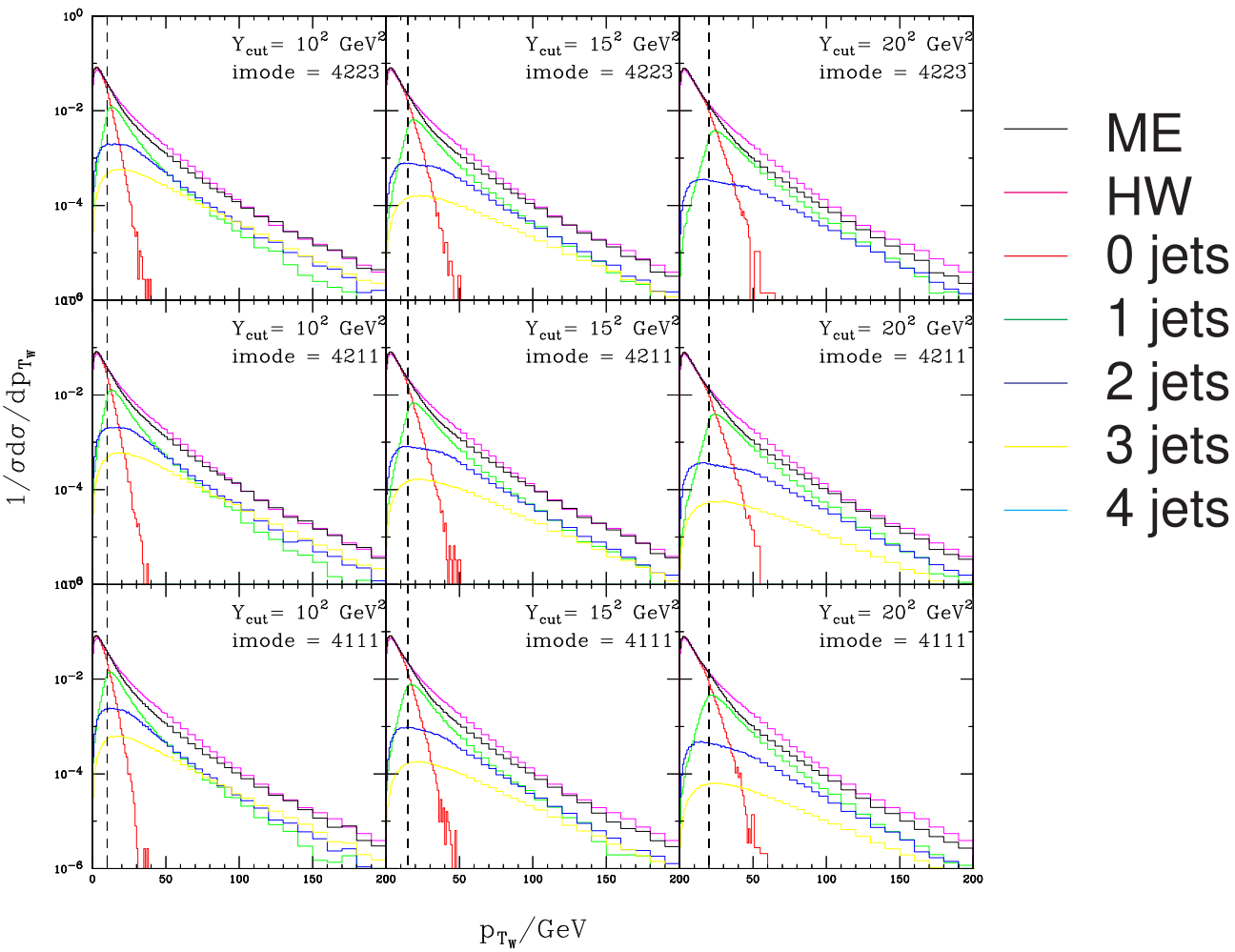
Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- d_4 distribution at the Tevatron



Z Production

- Variation of results with matching scale for `AFACT(1,2)=1/2`, `QFACT(3)=1/2` and `QFACT(4)=2`.
- Starting initial state shower at highest scale
- Z p_T distribution at the Tevatron



Program

- I have written a program to implement the CKKW algorithm with HERWIG.
- Uses the Les Houches interface to HERWIG to pass events from MADGRAPH/ALPGEN.
- Performs the clustering, reweighting and works out stsrt scale for the shower.
- Should be available soon

Conclusions

- I have presented results for the CKKW algorithm with HERWIG.
- We now have good results for e^+e^- collisions
- Still some problems to iron-out in hadron collisions.
- Hopefully the code will be available soon.