Merging Matrix Elements and Parton Showers

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- Introduction
- Approach
- CKKW Algorithm
- e^+e^- collisions
- Hadron collisions
- Conclusions

Introduction

- Monte Carlo Simulations are an essential part of most experimental analysis, either of the Standard Model or for searches for new physics.

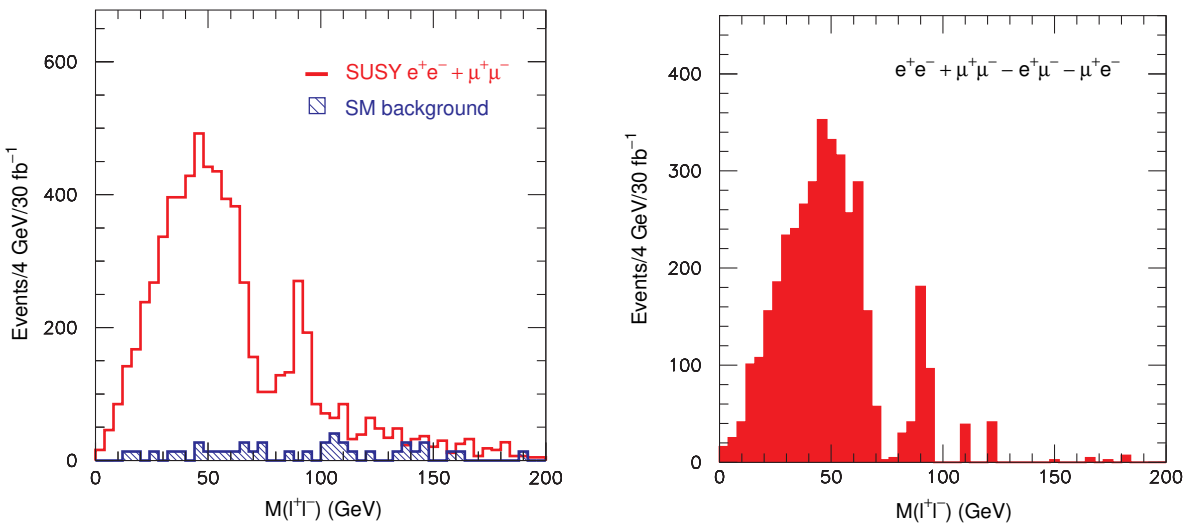
- There is a lot of interest in the simulation processes involving large numbers of outgoing jets/leptons with these programs as for example:

  - Top quark production including decays;
  - Signal Processes in BSM physics;
  - SM Electroweak gauge boson production with additional jets.

- The simulation of the SM multi-jet processes is interesting in its own right, but is essential for the study of backgrounds to top and BSM physics.

Example: LHC SUSY searches

- LHC measurement of $\tilde{\chi}_2^0$ χ_1^0 mass difference.



- Taken from ATLAS TDR.
- Require two leptons $P_{T,1} > 20$ GeV $P_{T,2} > 10$ GeV and $|\eta| < 2.5$, $E_T^{\rm miss} > 200$ GeV.
- Four jets with $p_T > 100, 50, 50, 50$ GeV and $|\eta| < 3.2$.
- Event generators are unlikely to give a reliable estimate of this background.
- Doesn't matter so much here, can do OSDF subtraction but there are cases where it is important.

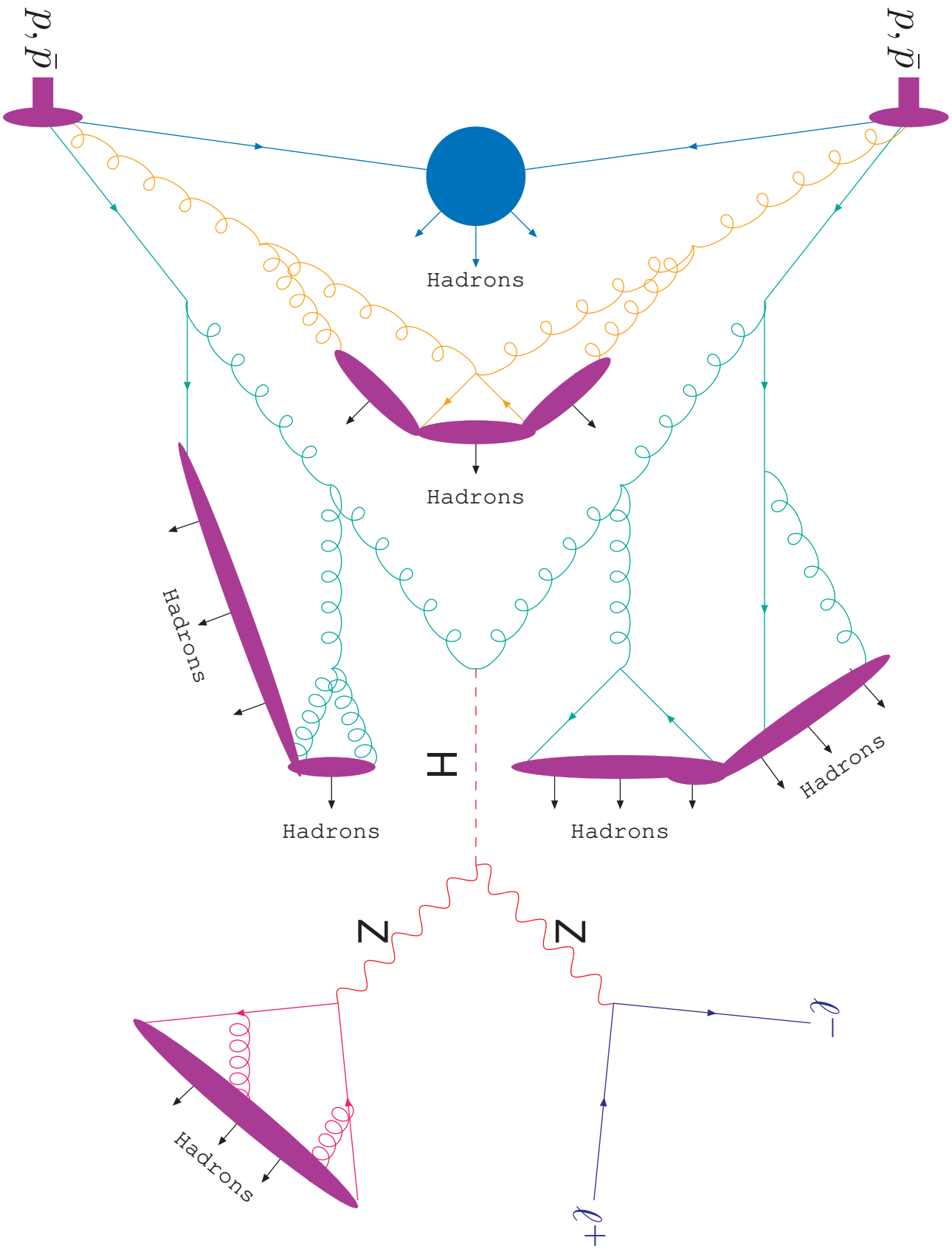
Monte Carlo Event Generators

- The aim of a Monte Carlo event generator is to provide a simulation of the events seen in experiments based on theoretical predictions.

- In hadron collisions this means the simulation of QCD.

- The simulation is split into a number of phases:
  - Hard Process;
  - Parton Shower;
  - Secondary Decays;
  - Multiple Scattering/Soft Underlying Event;
  - Hadronization;
  - Hadron Decays.

- There are a number of programs available which use different models/approximations.


$p, \bar{p}$
$p, \bar{p}$
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
Hadrons
H
Z
Z
ℓ^-
ℓ^+
Hadrons

Parton Shower

- In general the event generators start from a process a $2 \to 2$ scattering process.
- Any additional jets are generated by the parton shower algorithm.
- This is a simulation of QCD in the leading-log approximation, including either:
  - Collinear Logs only (*i.e.* ISAJET);
  - Collinear Logs and Soft Logs via an angular veto (*i.e.* PYTHIA);
  - Collinear Logs and Soft Logs via angular ordering (*i.e.* HERWIG).
- In general the soft logs in HERWIG and PYTHIA are necessary to match the data.
- No parton shower simulation will give the correct rate for large numbers of high P_T jets.

Matching matrix elements and Parton Showers

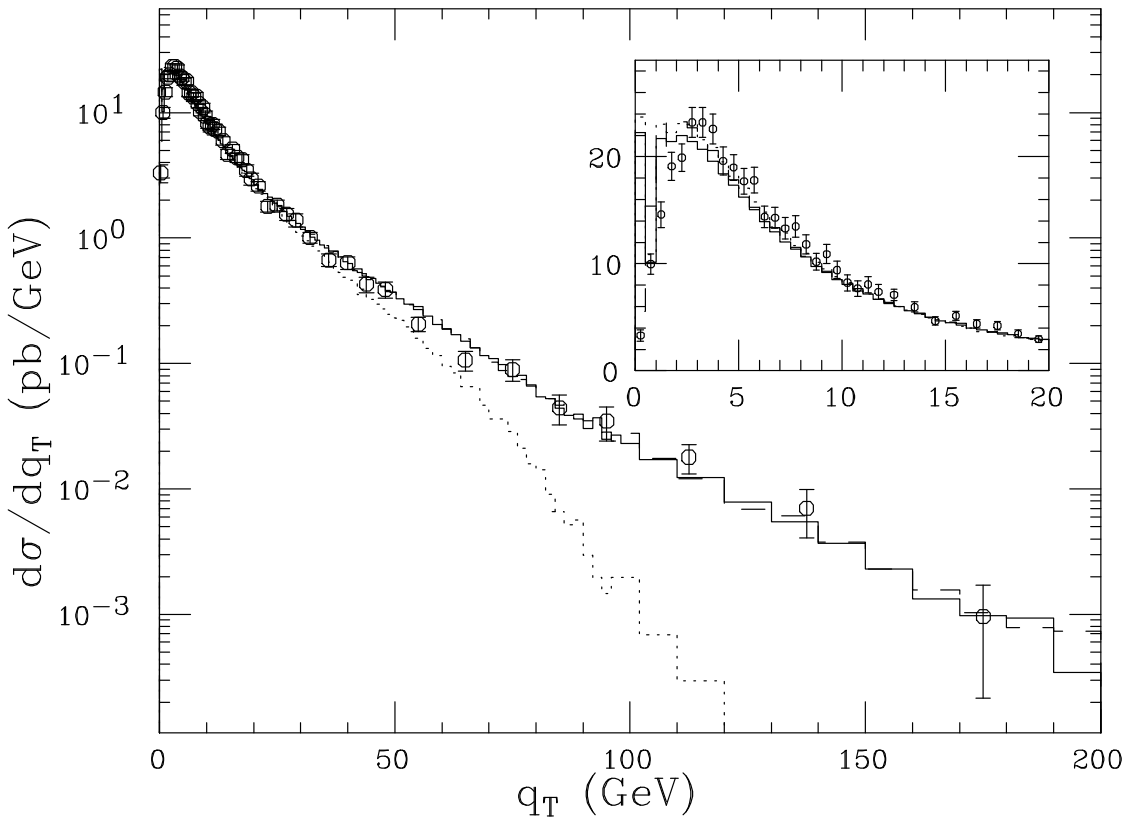
- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high p_T jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating many high p_T jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, *i.e.*
  - ME gives hard emission,
  - PS gives soft/collinear emission,
  - Smooth matching between the two,
  - No double counting of radiation.

Matching matrix elements and Parton Showers

- There have been a number of attempts to do this.
- The first attempts got the hardest emission correct for simple processes, for example:
  - $e^+e^- \to q\bar{q}$;
  - DIS;
  - Drell-Yan;
  - Top Decay.
- There were problems with this:
  - Only the hardest emission was correctly described;
  - The leading-order normalization was retained.

Matching matrix elements and Parton Showers

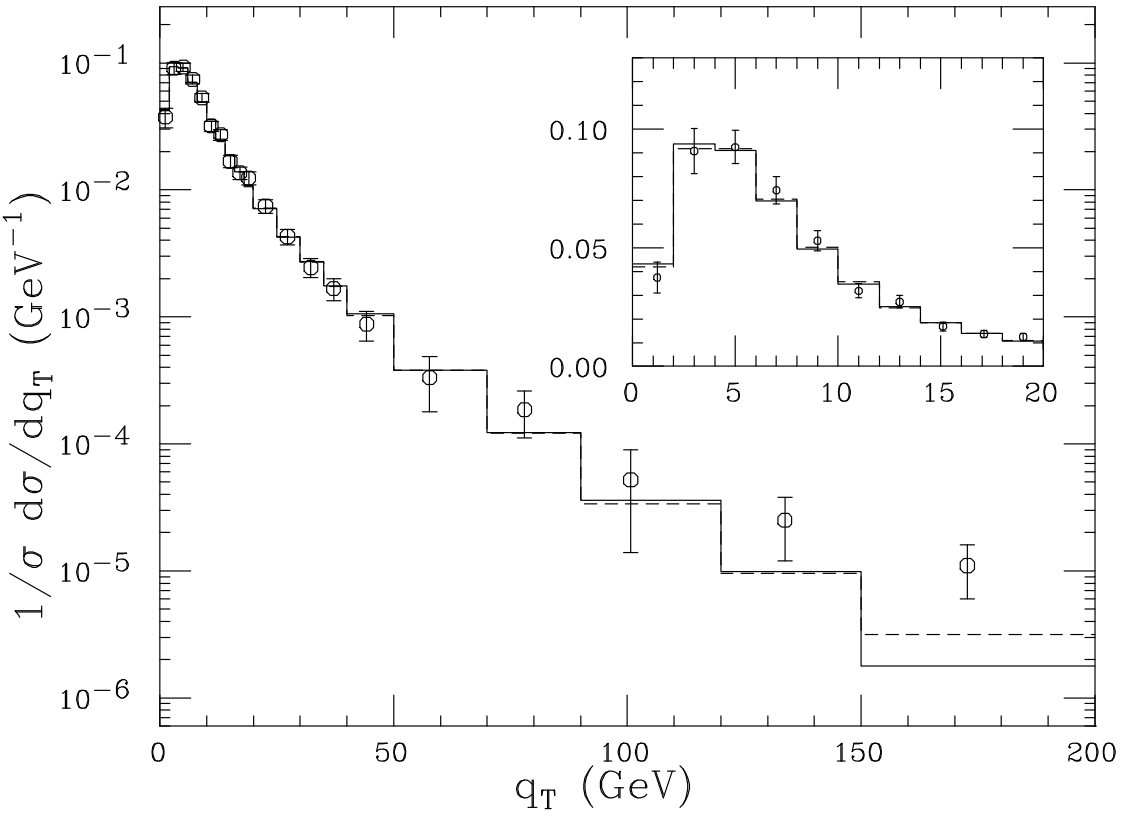
- Transverse momentum distribution of Z at the Tevatron (CDF) compared with HERWIG.



- Taken from Nucl. Phys. B565:227-244,2000, G. Corcella and M. H. Seymour.

Matching matrix elements and Parton Showers

- Transverse momentum distribution of W at the Tevatron (D0) compared with HERWIG.



- Taken from Nucl. Phys. B565:227-244,2000, G. Corcella and M. H. Seymour.

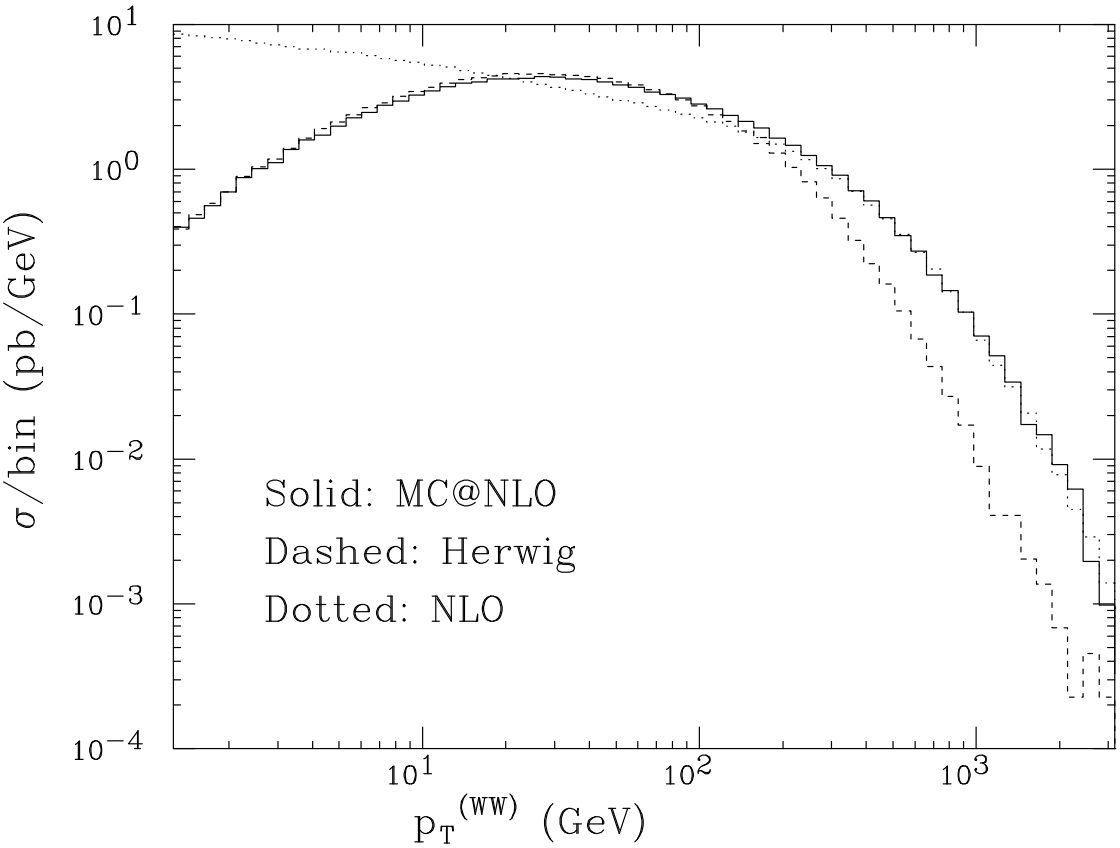
Matching matrix elements and Parton Showers

- Recently there has been renewed interest.
- Two types of approach have emerged.
  1. NLO Simulation
     - NLO normalization of the cross section.
     - Gets the hardest emission correct.
     - Only programs by Frixione/Webber and Dobbs.
  2. Multi-jet Leading Order
     - Still leading order.
     - Gets many hard emissions correct.
     - Programs by Krauss et. al. and Lonnblad for e^+e^-.

NLO Simulation

- Bryan Webber gave a talk on this recently. I will just one plot to illustrate the results.

- Transverse momentum of W^+W^- pair at the LHC.



- Taken from JHEP 0206:029, 2002, S. Frixione and B R. Webber.

Multi-Jet Leading Order Simulation

- While the NLO approach is good for one hard additional jet and the overall normalization can not be used to give many extra hard jets, at least at the moment.

- Therefore to simulate these processes use matching at leading order to get many hard emissions correct.

Multi-Jet Leading Order Simulation

- The idea is to implement the algorithm of CKKW to produce a multi-jet simulation using existing tools
  - Use the Les accord compliant M.E. generators to calculate matrix elements (MADGRAPH, ALPGEN)
  - Perform the reweighting etc. specified by the algorithm before passing the events to HERWIG.
  - Use the Les accord interface to read the events into HERWIG.
  - Make only minimal changes to the event generator itself.
- Produce a better simulation using existing well tested tools.
- I will first review the algorithm of CKKW and then present some preliminary results.

Procedure

- In order to match the ME and PS we need to separate the phase space:
  - One region should contain the soft/collinear region and be filled by the PS;
  - The other is filled by the matrix element.

- In Catani-Krauss-Kuhn-Webber (CKKW) prescription this is done via the Durham jet algorithm.
  - For all final-state particles compute the resolution variables
    - $d_{kB} \simeq E_k^2 \theta_{kB}^2 \simeq k_{\perp kB}^2, \quad \theta_{kB} \to 0.$
    - $d_{kl} \simeq \min(E_k^2, E_l^2)\theta_{kl}^2, \quad \theta_{kl} \to 0$
  - The smallest of these d_{kB}, d_{kl} is then selected. If d_{ij} is the smallest the two particles are merged, if d_{iB} is the smallest the particle is merged with the beam.
  - This procedure is repeated until the minimum value is above some stopping parameter $d_{\rm cut}$.
  - The remaining particles and pseudo-particles are then the hard jets.

Procedure

1. Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{k=N} \sigma_k}$$

   where σ_n is the n-jet matrix element evaluated at resolution $d_{\rm ini}$ using $d_{\rm ini}$ as the scale for the P.D.F's and α_S, n is the number of additional jets.

2. Distribute the jet momenta according to the ME.

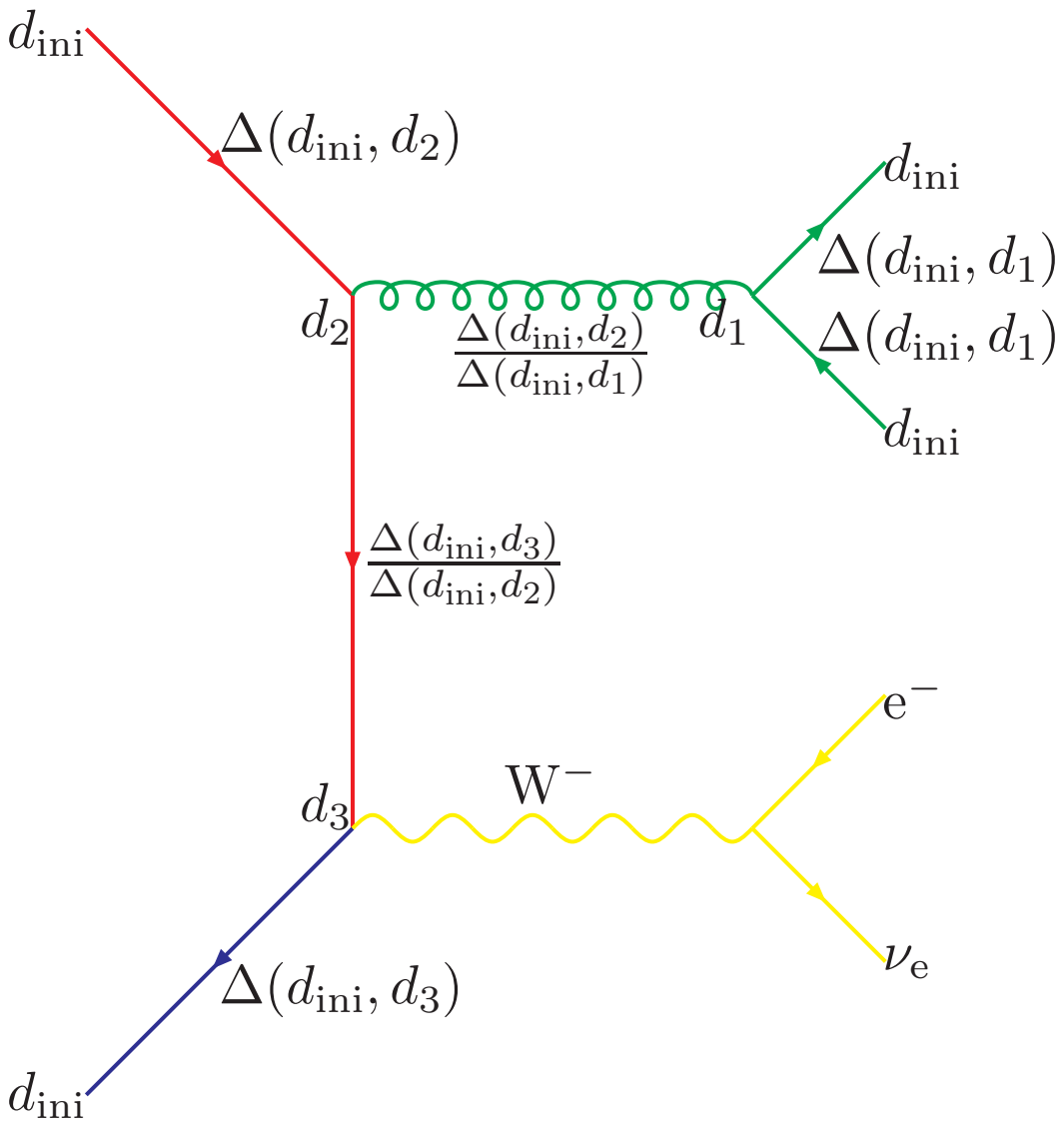
3. Cluster the partons to determine the values at which 1,2,..n-jets are resolved. These give the nodal scales for a tree diagram.

4. Apply a coupling-constant reweighting $\alpha_S(d_1)\alpha_S(d_2)..\alpha_S(d_n)/(\alpha_S(d_{\rm ini})^n \leq 1$.

5. Reweight the lines by a Sudakov factor $\Delta(d_{\rm ini}, d_j)/\Delta(d_{\rm ini}, d_k)$.

6. Accept the configuration if the product of the α_S and Sudakov weight is less the $\mathcal{R} \in [0, 1]$., otherwise return to step 1.

Procedure



7. Generate the parton shower from the event starting the evolution of each parton at the scale it was created and vetoing emission above the scale $d_{\rm ini}$

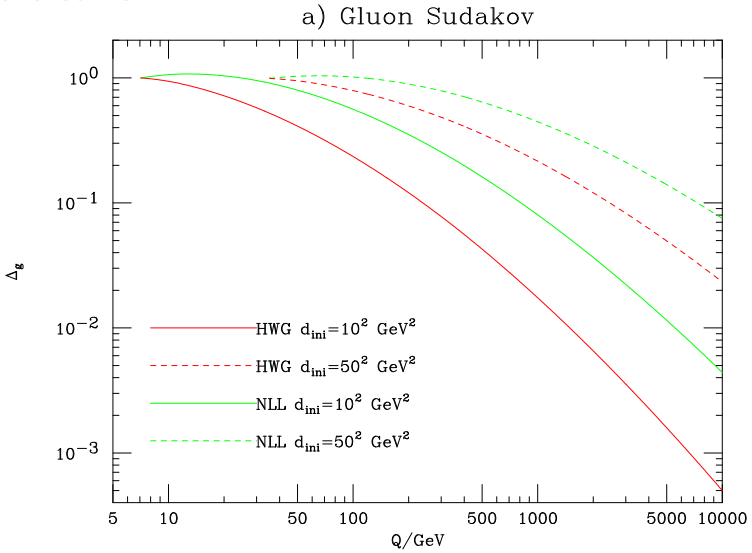
Free Parameters

- Although the algorithm above ensures smooth matching at NLL log level there are a number of free choices
  - Sudakov Form Factors
    - HERWIG or NLL Sudakovs
    - Definition of the scale
    - Prefactor for the scale
  - α_S
    - LO or NLO definition
    - Definition of the scale
    - Prefactor for the scale
  - Treatment of Highest Multiplicity Matrix Element
    - Same as other multiplicities?
    - Different treatment to describe more emissions?
  - Definition of k_T-algorithm away from soft/collinear limit
    - Definition of the k_T-measure.
    - Recombination scheme.

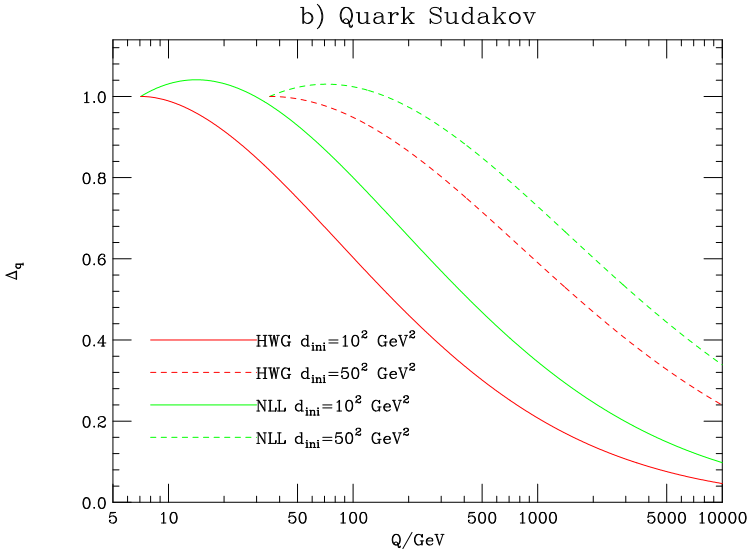
Sudakov Form Factors

- Due to the choice of scale in α_S and regularization the HERWIG Sudakov is always smaller than the NLL one.

- Gluon Sudakov



- Quark Sudakov



Sudakov Form Factors

- To look at the effects of the different Sudakov form factors study e^+e^- collisions at M_Z.

- Look at the differential distributions with respect to y_n where

$$y_n = \frac{d_n}{s},$$

  and d_n is the k_T^2 scale where the event changes from n to $n-1$ jets.

- Start with a matching scale of $y = 0.001 \Rightarrow d = 8.3\mathrm{GeV}^2$.

- This is a very low scale which enhances the differences of different choices.

- Start by studying different Sudakovs, different scales in the Sudakov, and different scales in α_S.

Choice of Scale

- There are a number of choices to make.
- First for the Sudakovs

$$d_{\mathrm{SUD}} = \texttt{QFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

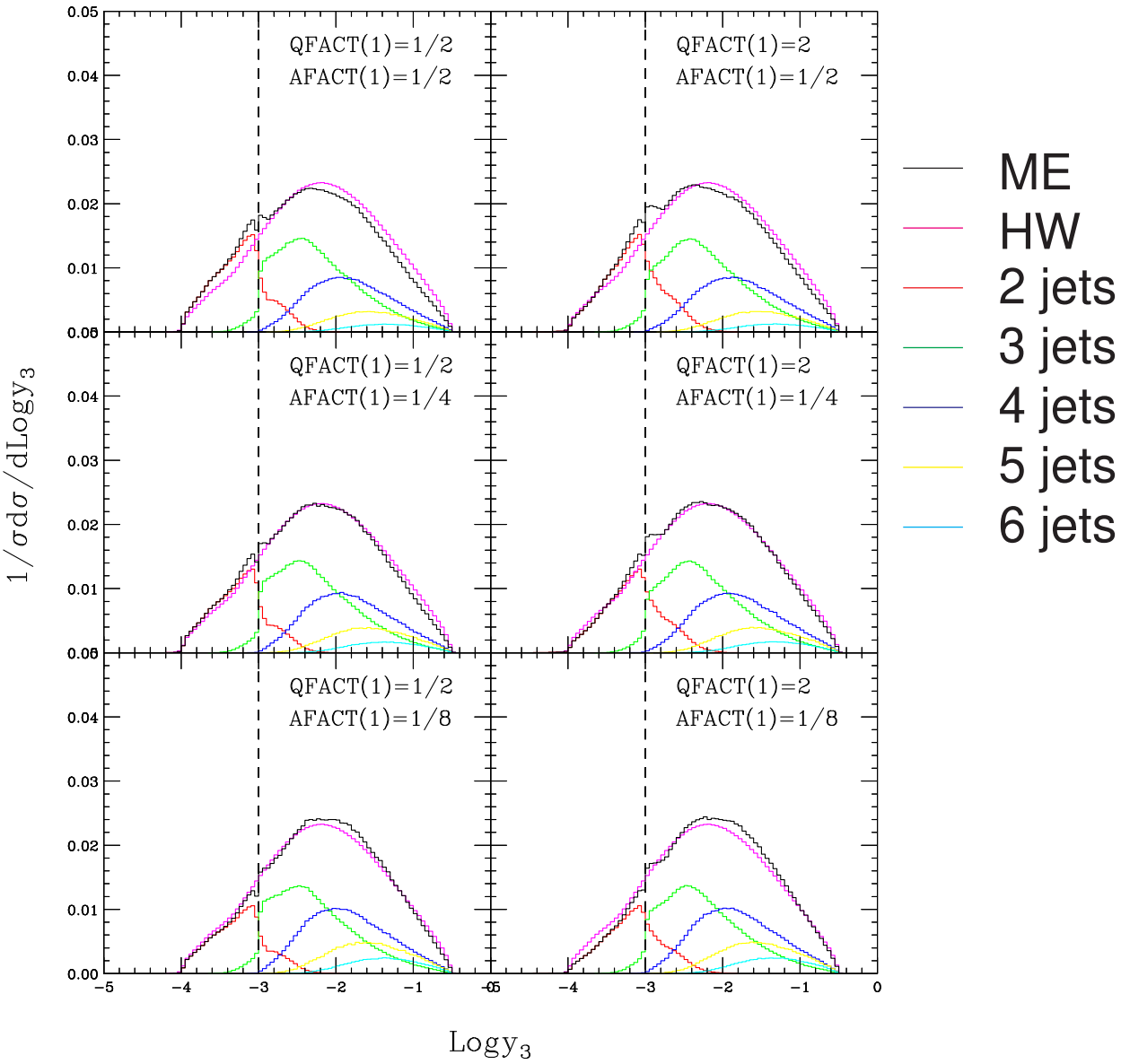
- and for α_S.

$$d_{\alpha_S} = \texttt{AFACT}(1) \begin{cases} k_T^2 & \texttt{ISCALE} = 1, \\ 2p_i \cdot p_j & \texttt{ISCALE} = 2, \\ Q^2 & \texttt{ISCALE} = 3, \end{cases}$$

- HERWIG would use `QFACT(1)=1/2` for $e^+e^- \to \mathrm{jets}$ or Drell-Yan.
- However no radiation in shower above $\frac{1}{2}d_{\mathrm{shower}} \Rightarrow$ `QFACT(1)=2`
- HERWIG always has a smaller scale for α_S than in the Sudakov form factor.

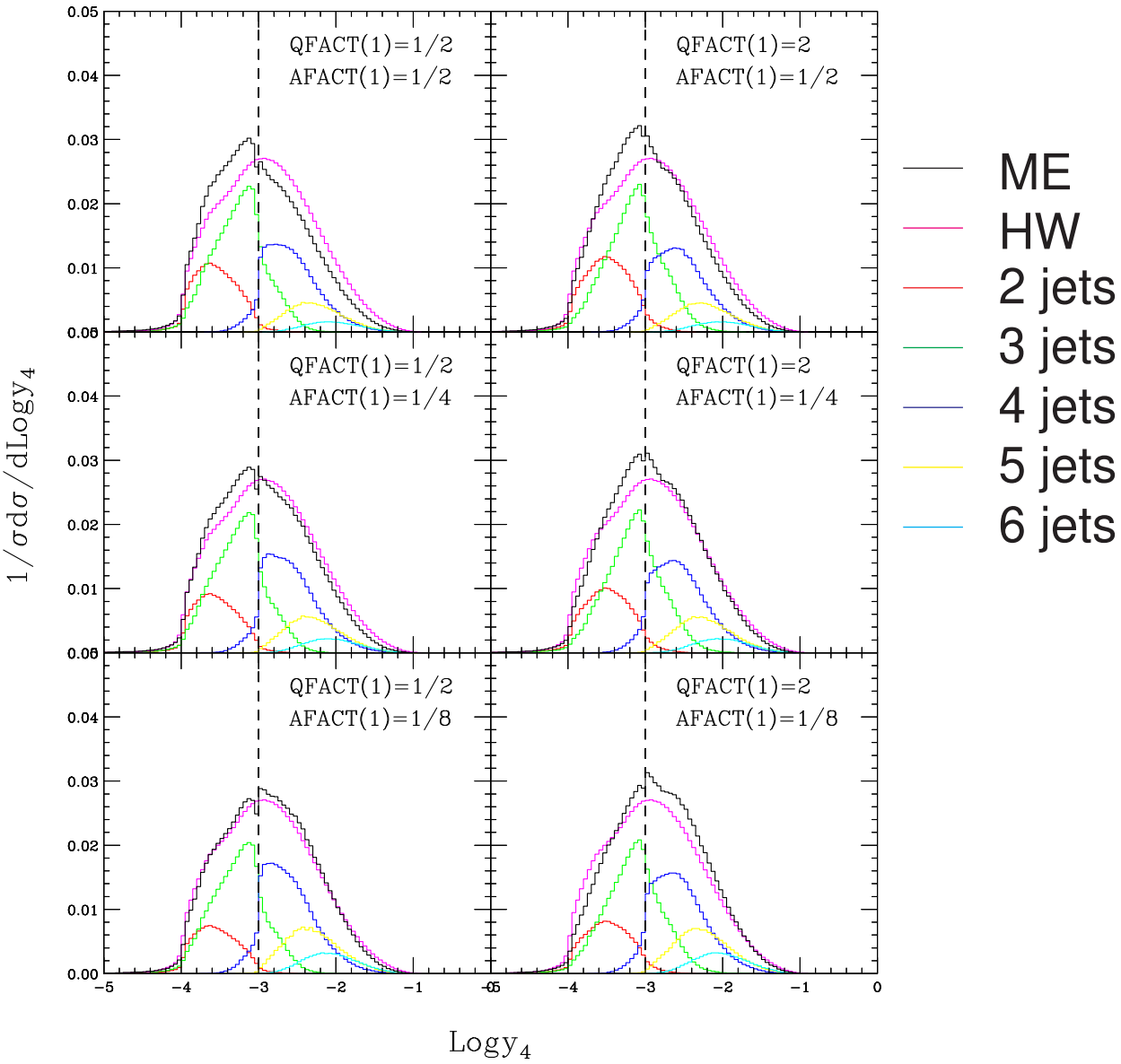
NLL Sudakovs

- NLL Sudakovs with `ISCALE=1`.
- y_3 distribution.



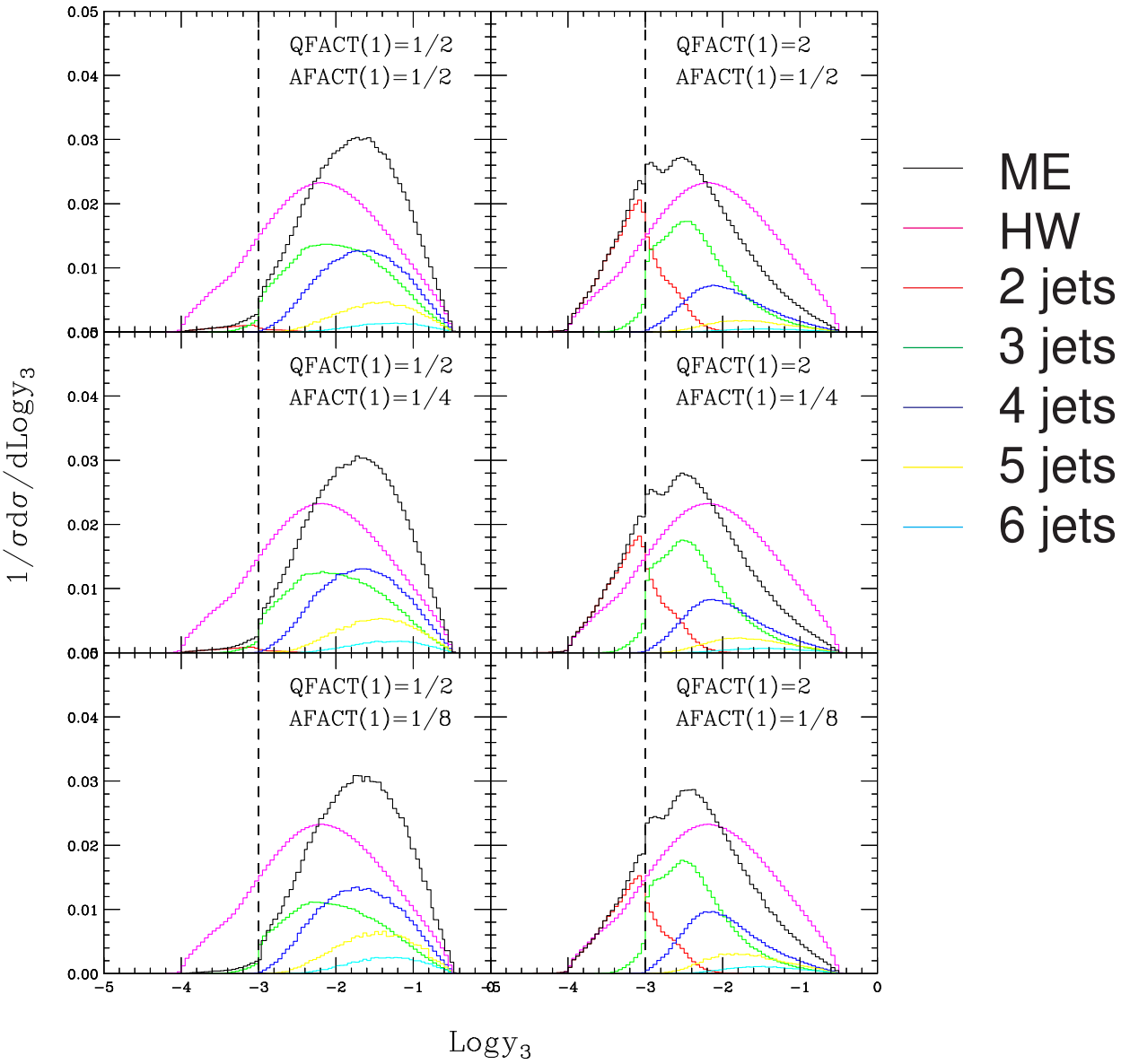
NLL Sudakovs

- NLL Sudakovs with `ISCALE=1.`
- y_4 distribution



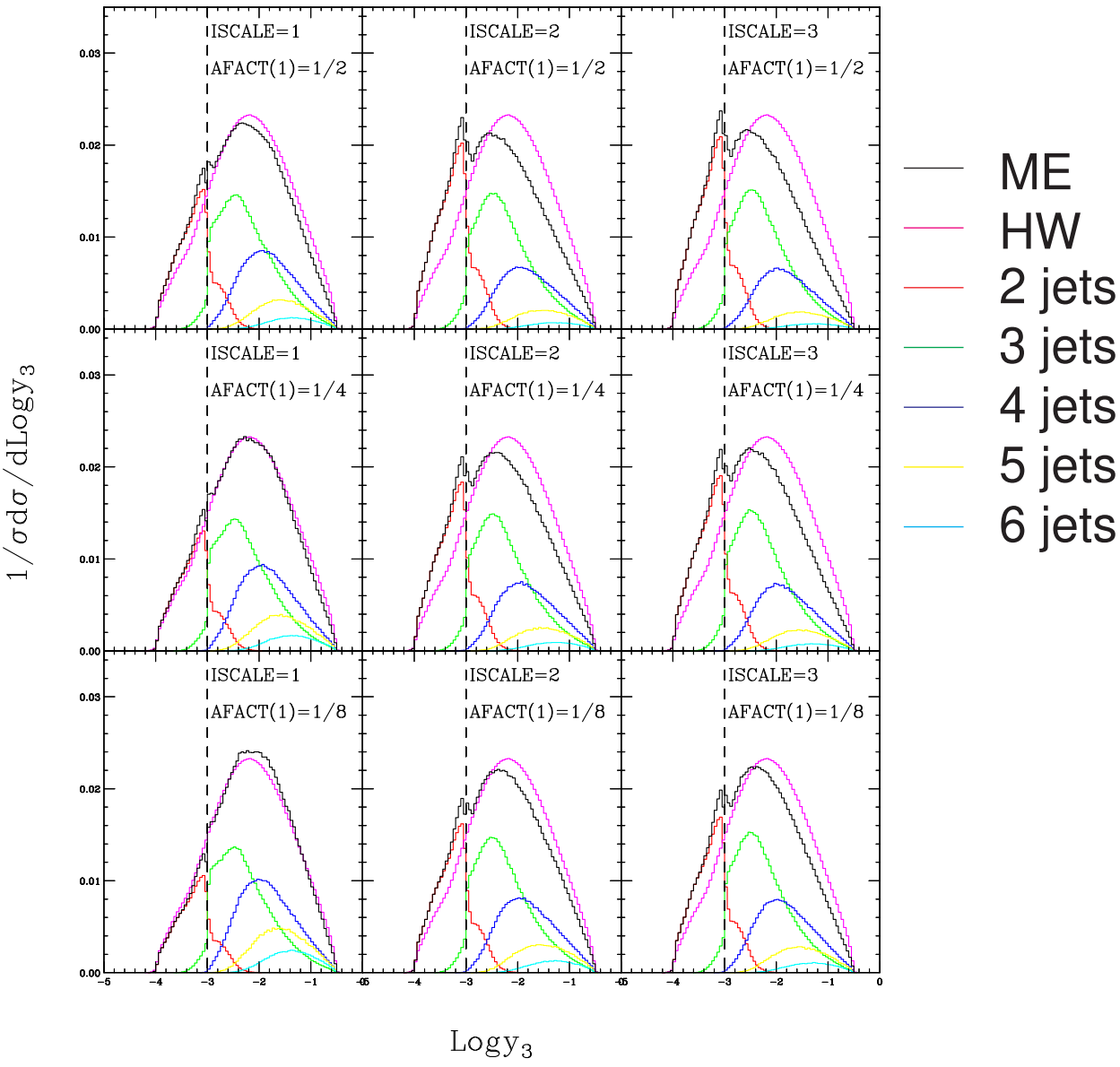
HERWIG Sudakovs

- HERWIG Sudakovs with `ISCALE=1`.
- y_3 distribution.



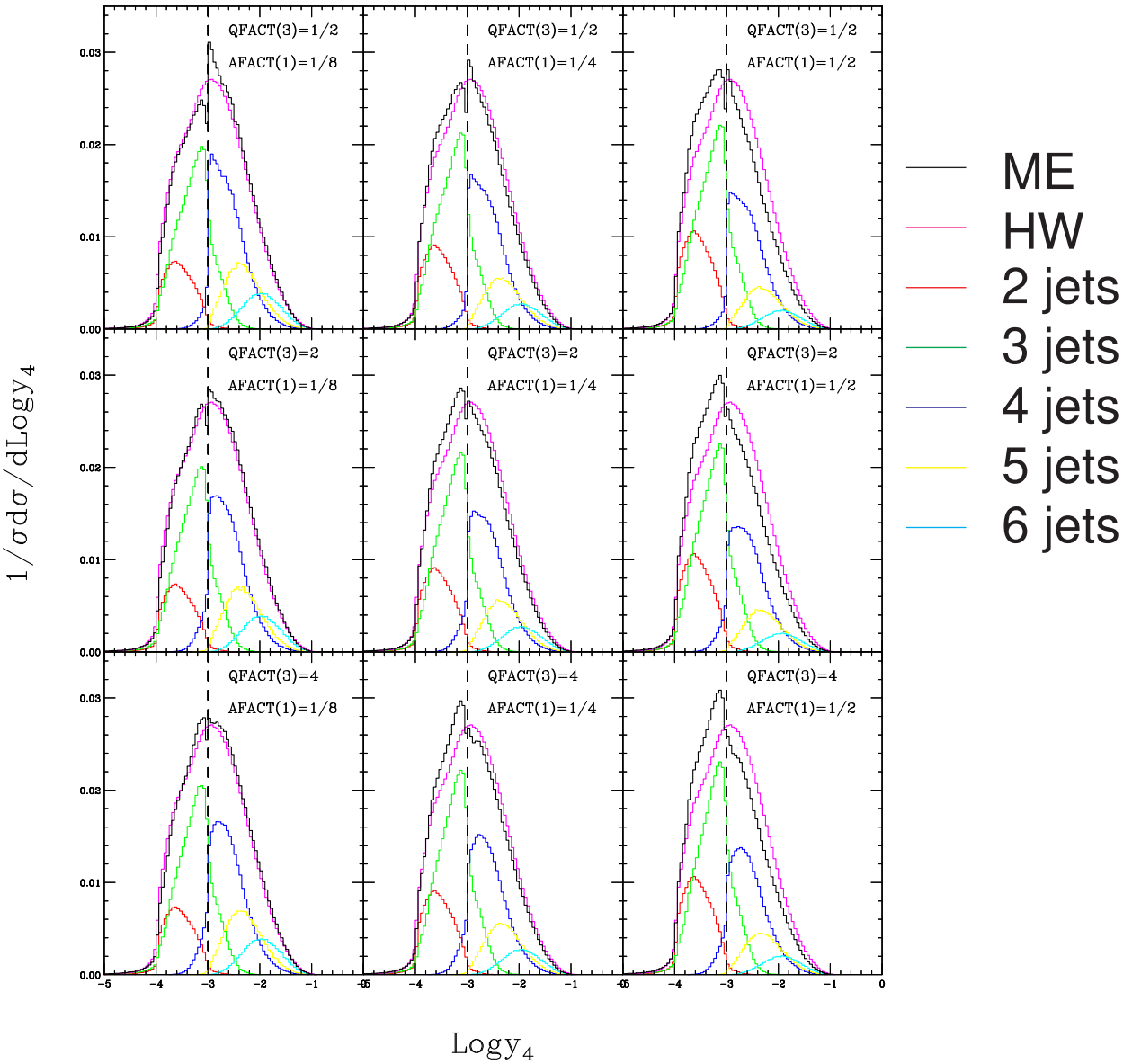
NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- y_3 distribution.



NLL Sudakovs

- NLL Sudakovs with `QFACT(1)=1/2`.
- Variation of `QFACT(3)` which is the minimum scale starting scale for the HERWIG shower.
- y_4 distribution.

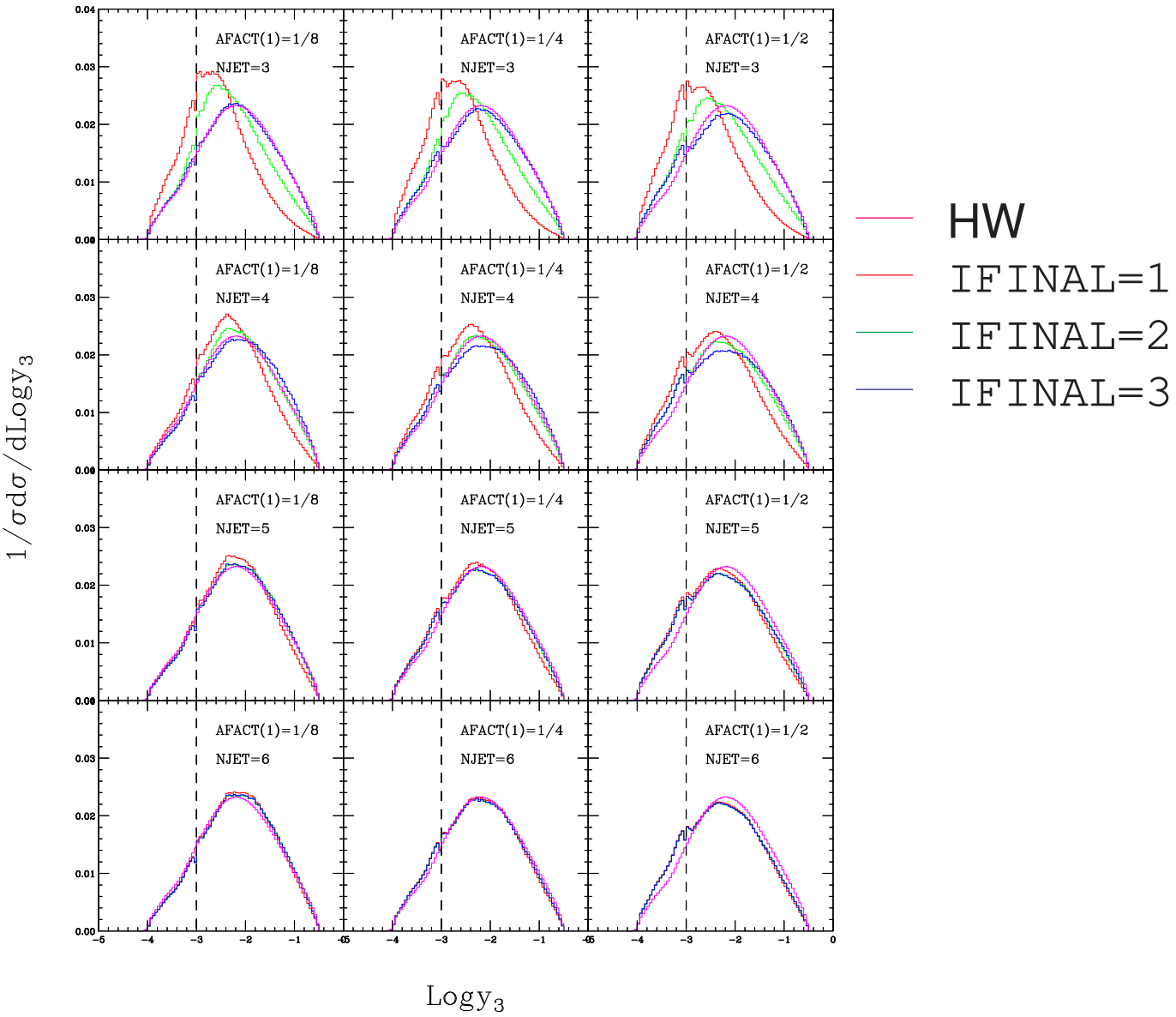


Treatment of Highest Multiplicity

- In the original CKKW algorithm all jet multiplicities were treated in the same way, `IFINAL=1`.
- Not appropriate for highest multiplicity.
- Instead only apply α_S reweighting and Sudakovs for internal lines.
- Two options for the shower
  - Apply no veto and start at normal HERWIG scale, `IFINAL=2`
  - Start at same scale as other multiplicities but veto above scale of last emission in ME. `IFINAL=3`
- `IFINAL=2` is easier to implement but has some double counting problems
- `IFINAL=3` is the best theoretical choice.

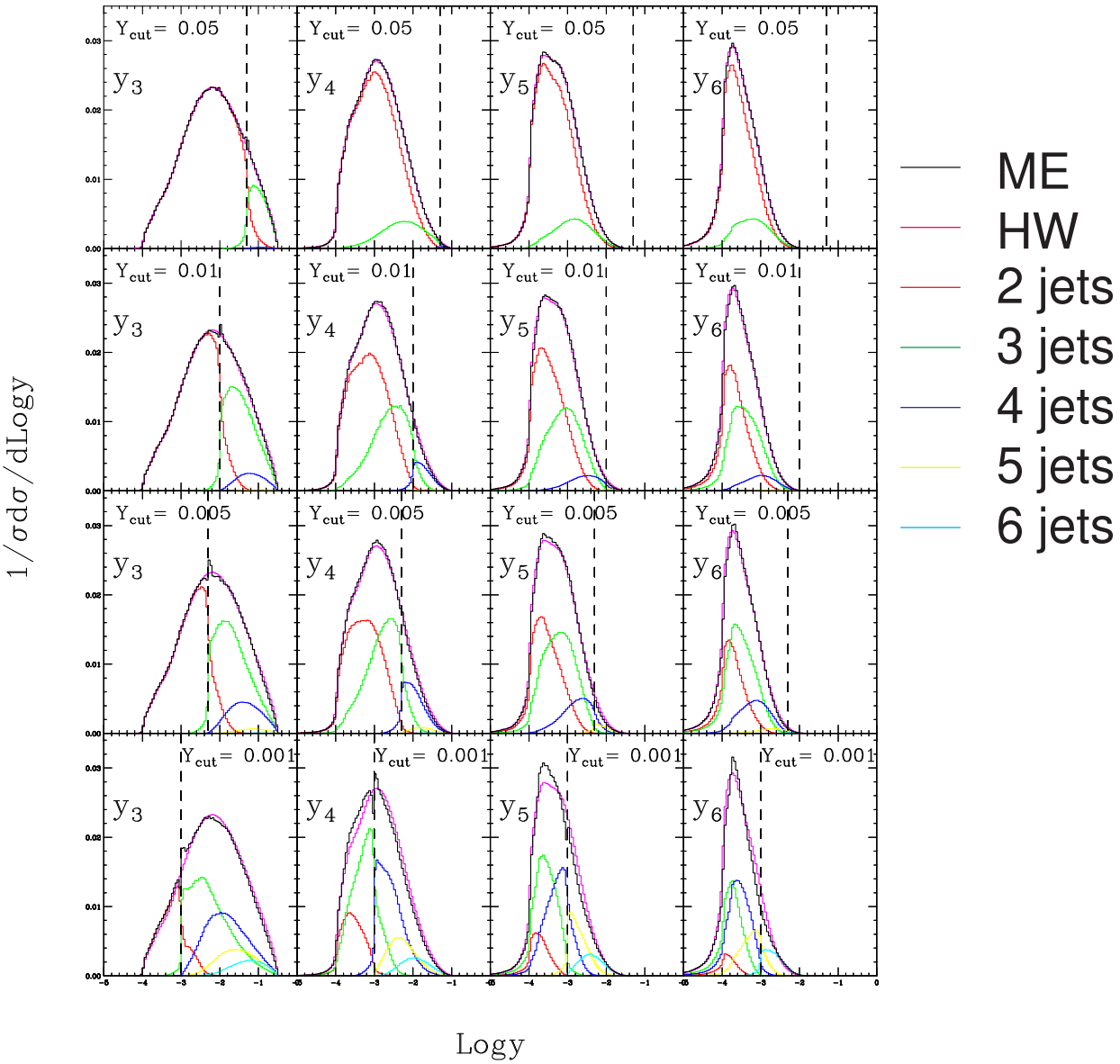
Treatment of Highest Multiplicity

- NLL Sudakovs with `QFACT(1)=1/2`.
- y_3 distribution.



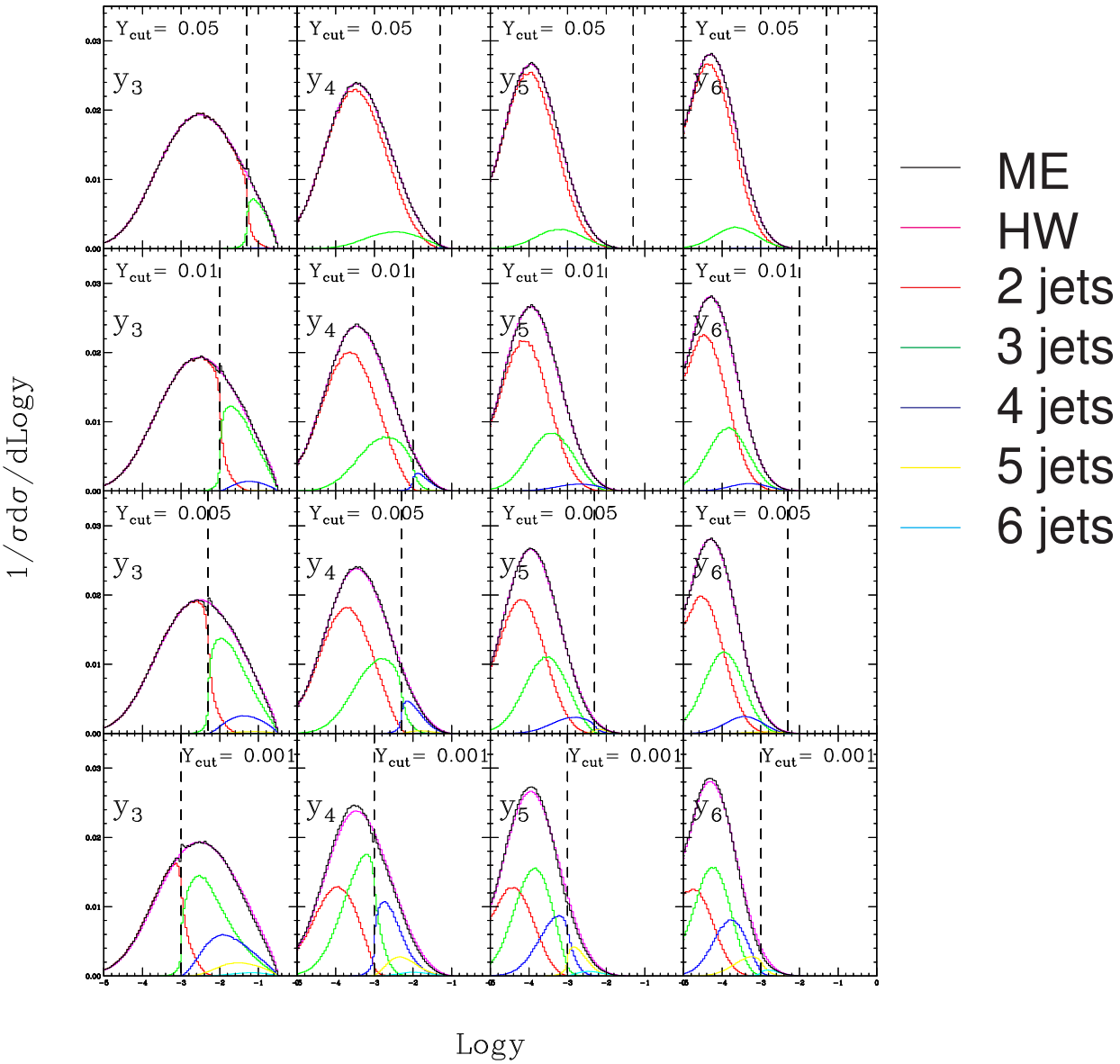
Effect of Varying the cut-off at M_Z

- NLL Sudakovs with `QFACT(1,2,3)=1/2, AFACT(1,2,3)=1, IFINAL=3`.
- Parton-level distributions.



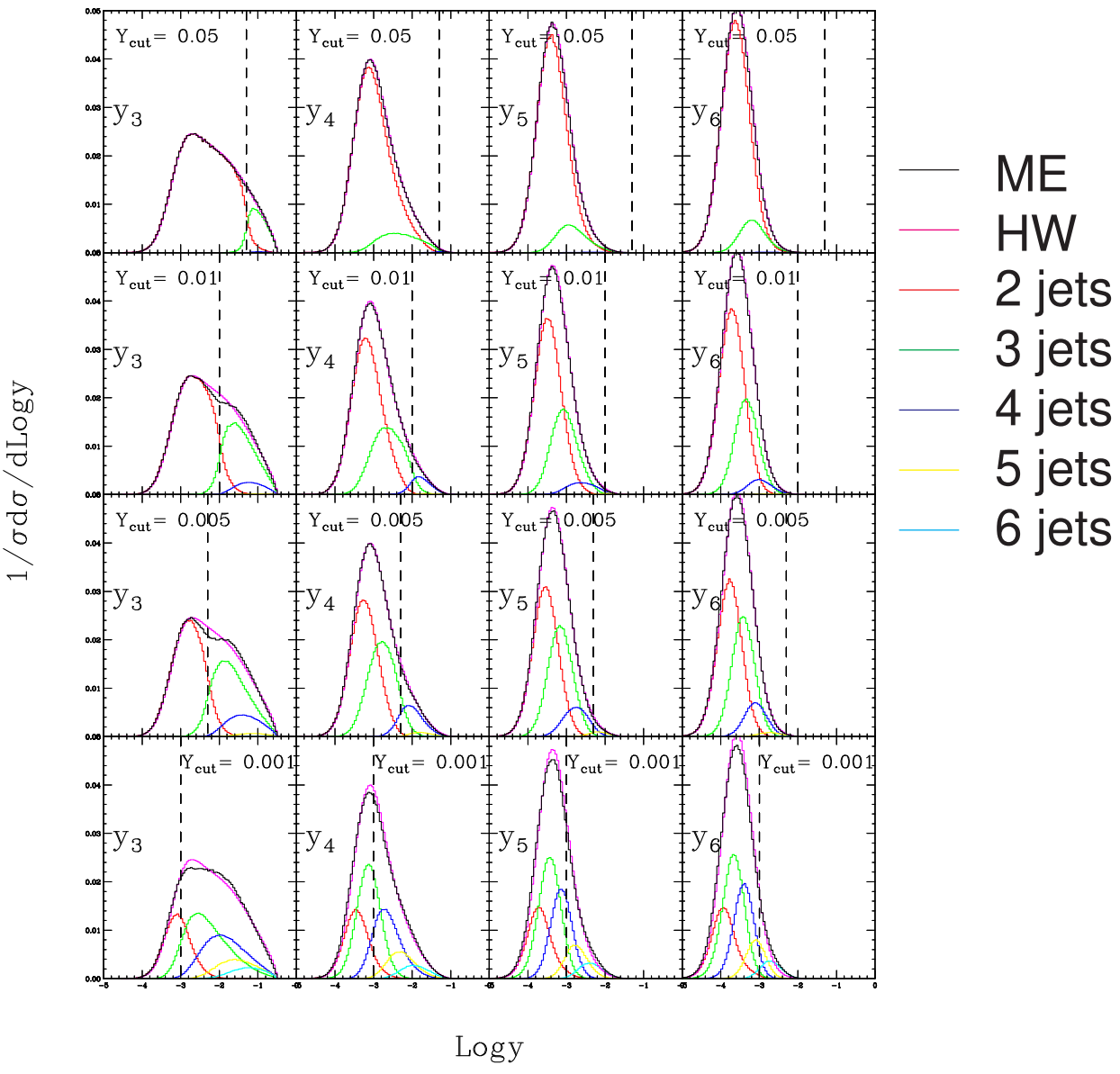
Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Parton-level distributions.



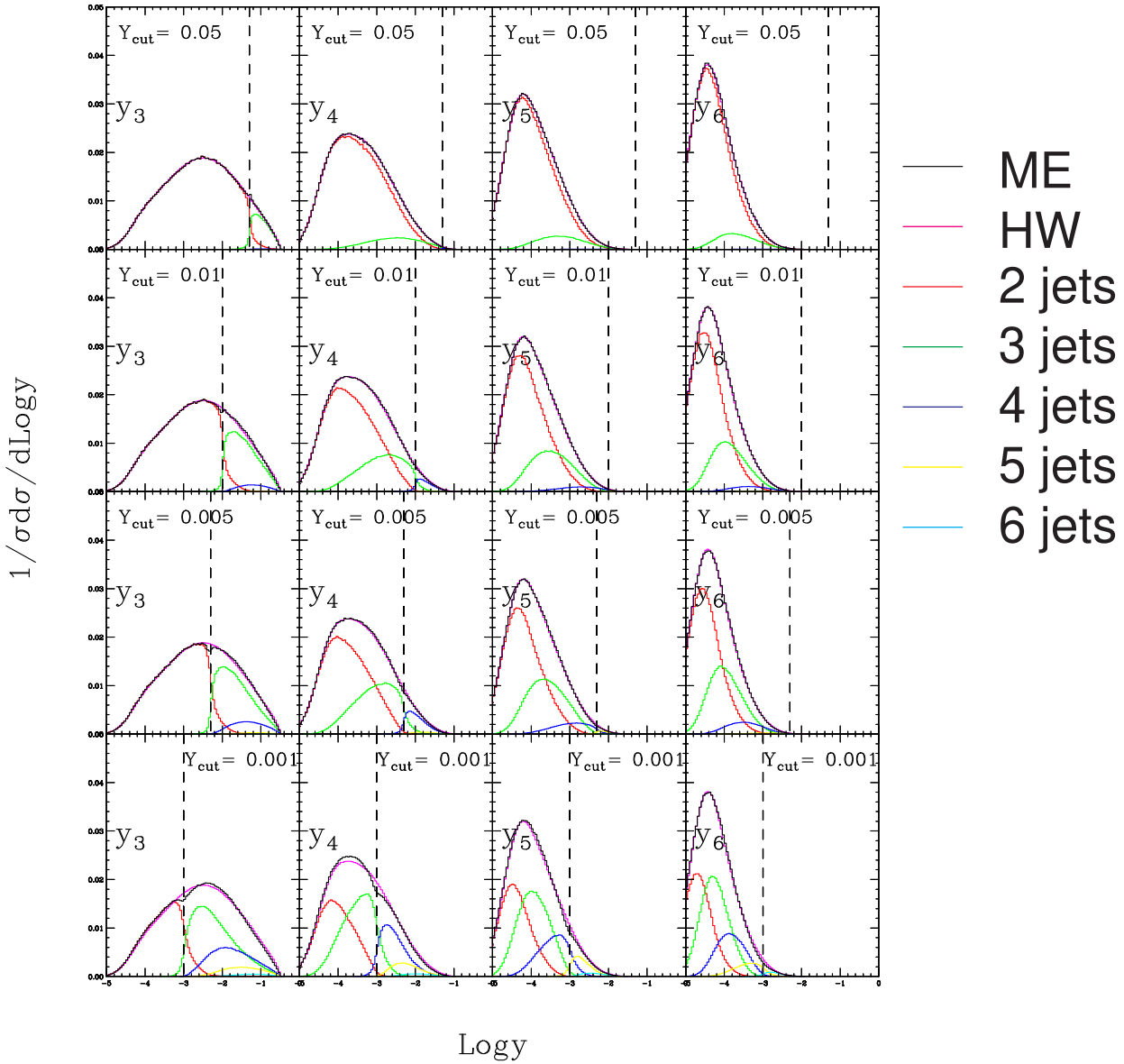
Effect of Varying the cut-off at M_Z

- NLL Sudakovs with `QFACT(1,2,3)=1/2, AFACT(1,2,3)=1/2, IFINAL=3`.
- Hadron-level distributions.
- Problems with hadronization model which remain to be solved.



Effect of Varying the cut-off at $\sqrt{s} = 500$ GeV

- NLL Sudakovs with `QFACT(1)=1/2, IFINAL=3`.
- Hadron-level y_3 distribution.
- Problems with hadronization model reduced.



Hadron Collisions

- In hadron collisions in addition to final-state radiation we have the problems of initial-state radiation.
- I will consider W production in hadron collisions
- All the events were generated with MADGRAPH including the leptonic decays of the gauge bosons.
- For the W samples no cuts were applied on the leptons.

Hadron Collisions

- We also have a greater choice of the clustering algorithm.
- I will only show one choice here.
  - Covariant monotonic p_T^2 scheme

$$\begin{aligned} d_{iB} &= p_{ti}^2, \\ d_{ij} &= \min(p_{ti}^2, p_{tj}^2) R_{ij}^2, \end{aligned}$$

Define p_t and R for pseudoparticles as

$$\begin{aligned} p_{t(ij)} &= p_{ti} + p_{tj}, \\ R_{(ij)k}^2 &= \frac{p_{ti}^2 R_{ik}^2 + p_{tj}^2 R_{jk}^2}{p_{ti}^2 + p_{tj}^2}. \end{aligned}$$

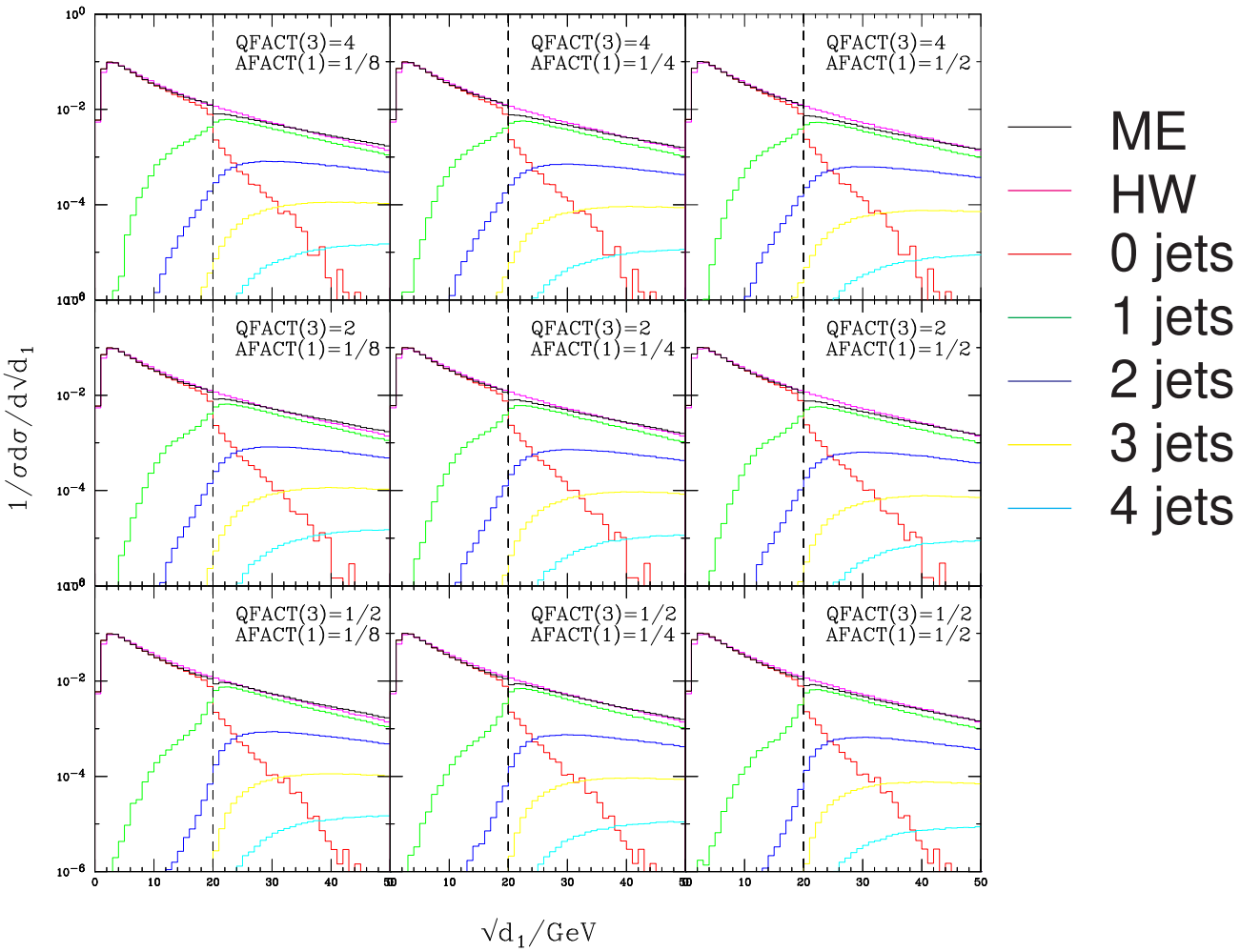
- Other schemes give similar results, (provided they are longitudinally boost invariant.)

Hadron Collisions

- Start by considering W production at the Tevatron.
- Use the Covariant monotonic p_T^2 scheme.
- and a matching scale of $400\ \mathrm{GeV}^2$.
- Look at the differential cross section with respect to $\sqrt{d_n}$ where d_n is the scale at which n jets are resolved.
- This is like the E_T in a cone algorithm.

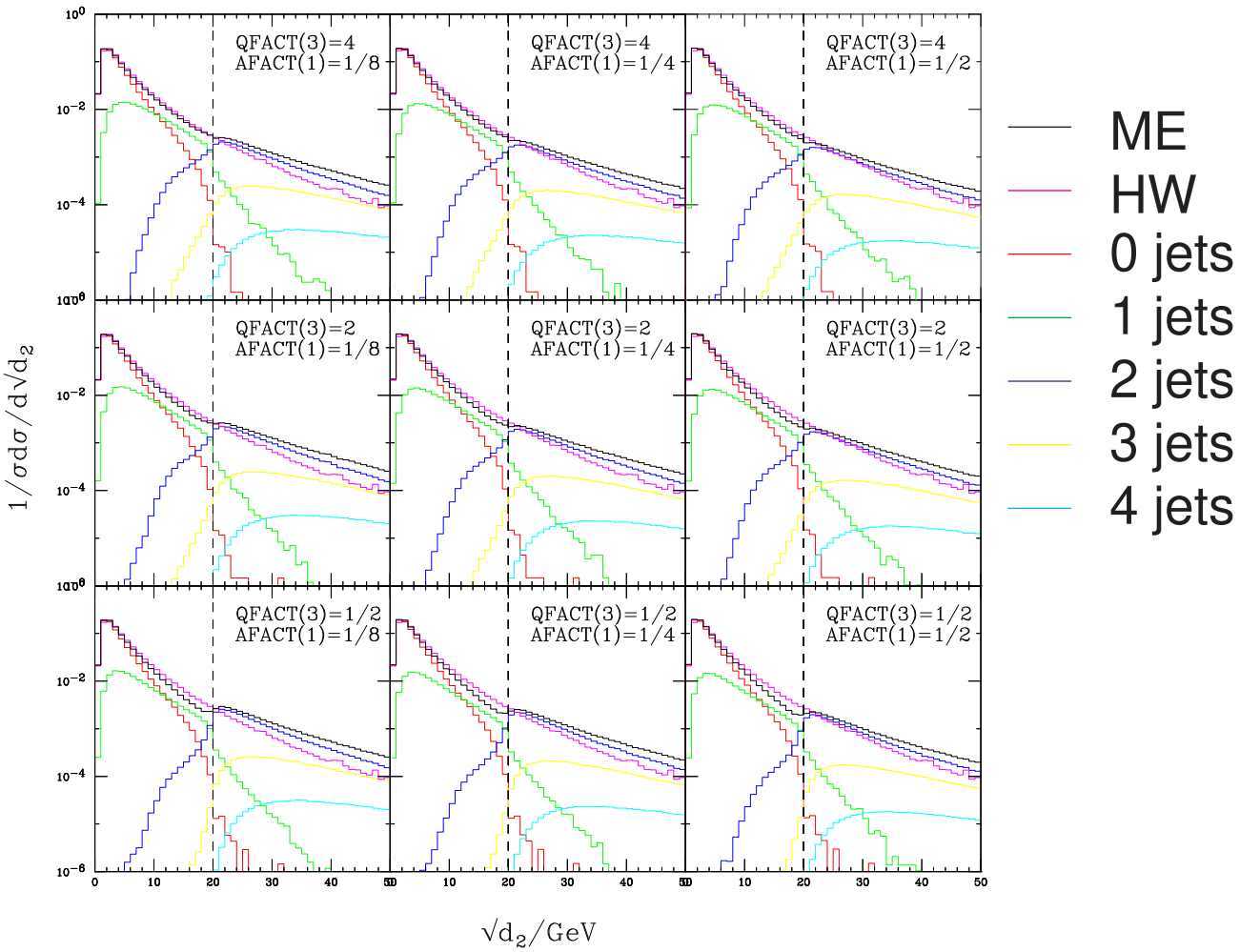
W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_1 distribution at the Tevatron



W Production

- Variation of results with scale of α_S and starting scale of shower.
- d_2 distribution at the Tevatron

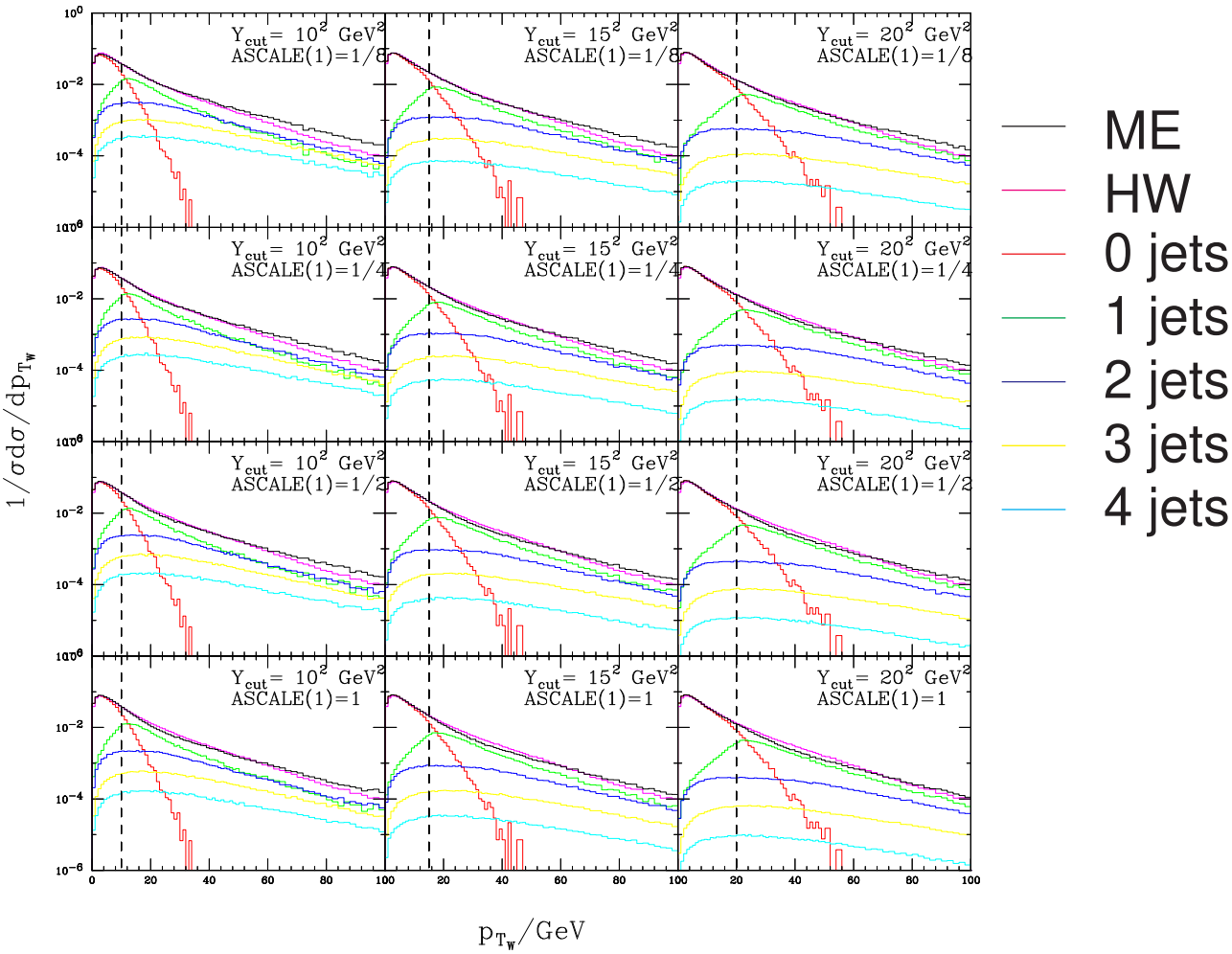


Hadron Collisions

- There is a radiation dip at the matching scale.
- There is also a problem with events migrating below the matching scale.
- Hard to get the HERWIG shower to produce more radiation than before below the matching scale.
- Solution is to decouple the scale used in the Sudakovs and that used to start the shower. Use a choice closer to the HERWIG variables to start the shower.

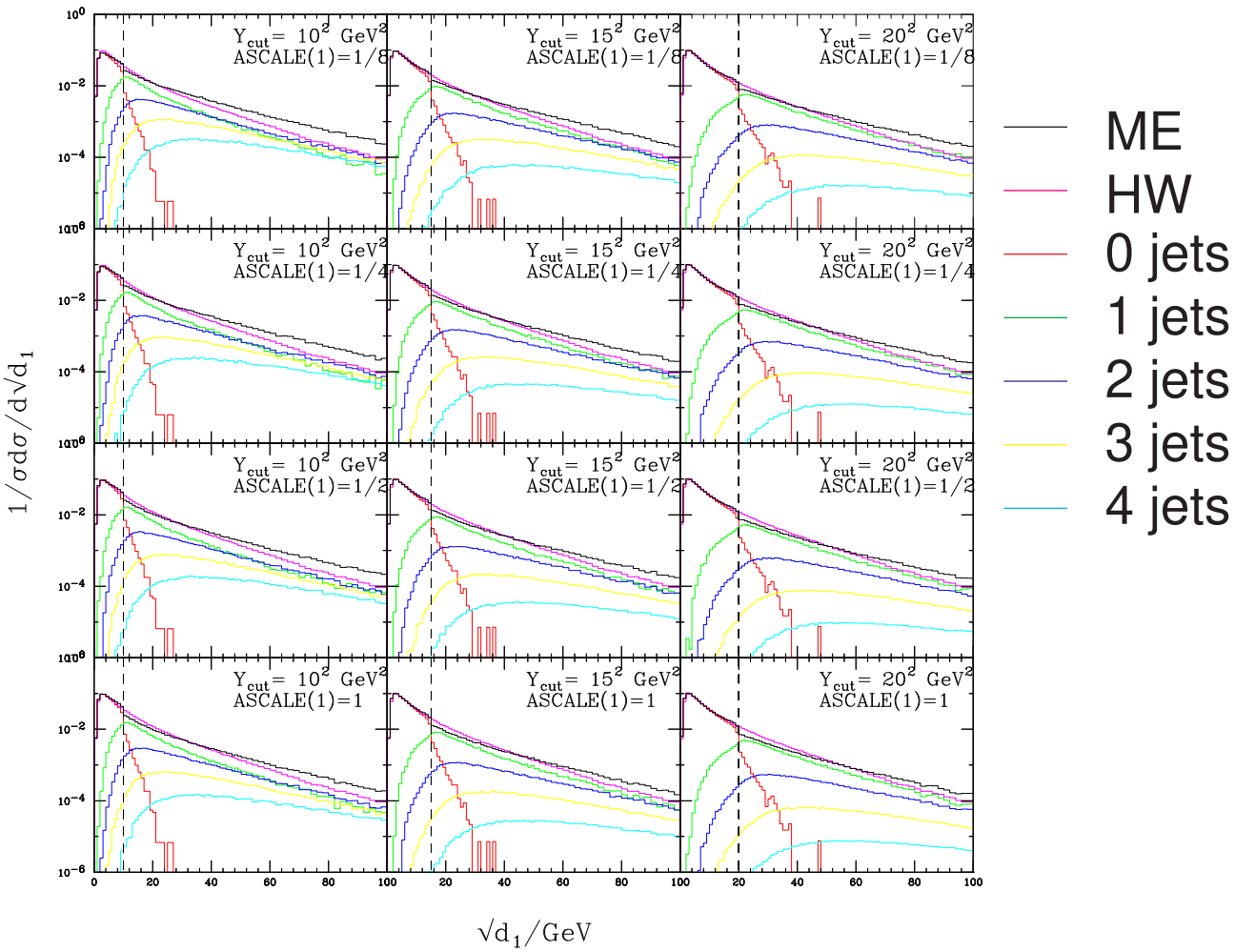
W Production

- Variation of results with scale of α_S and matching scale.
- p_T of the W at the Tevatron



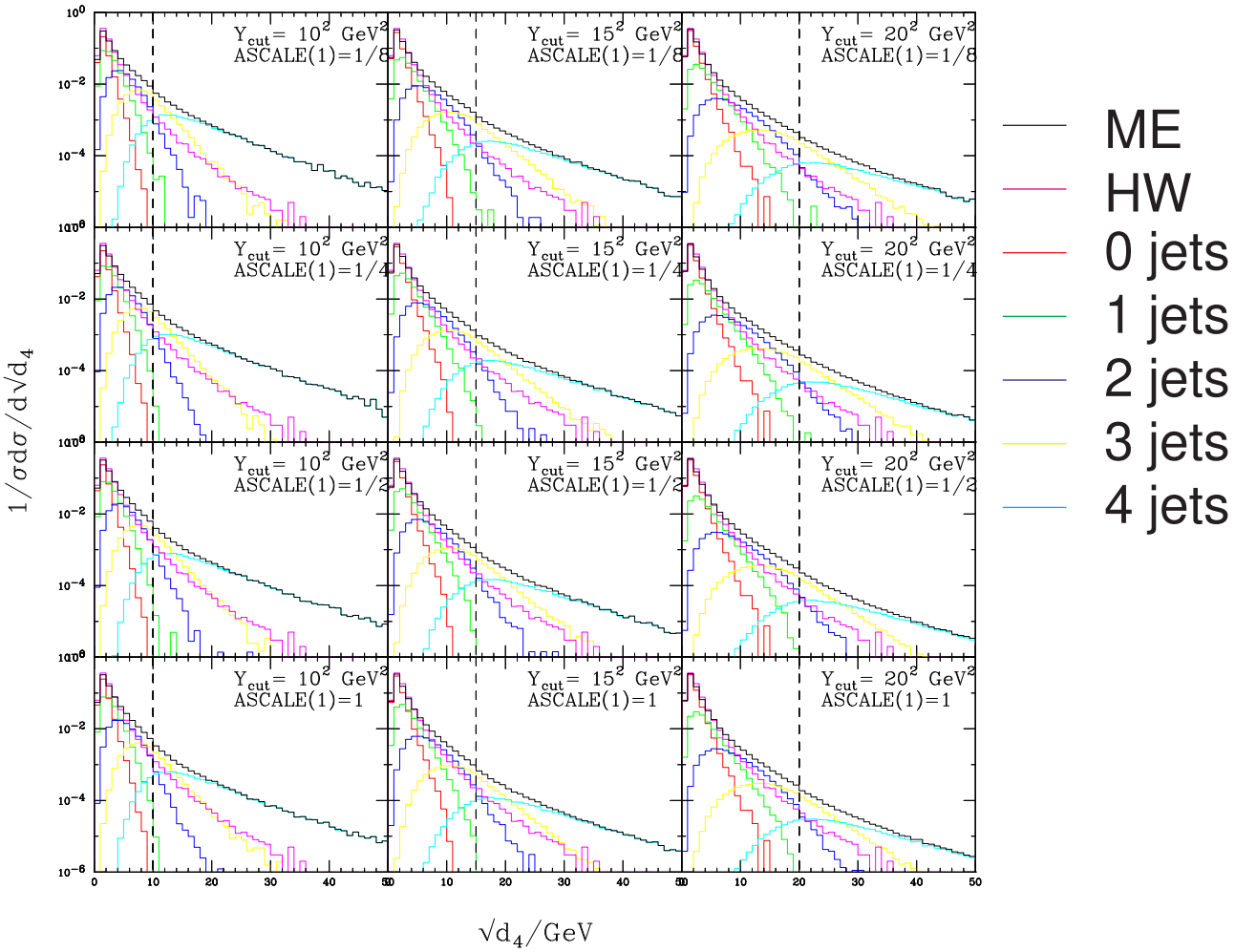
W Production

- Variation of results with scale of α_S and matching scale.
- d_1 distribution at the Tevatron



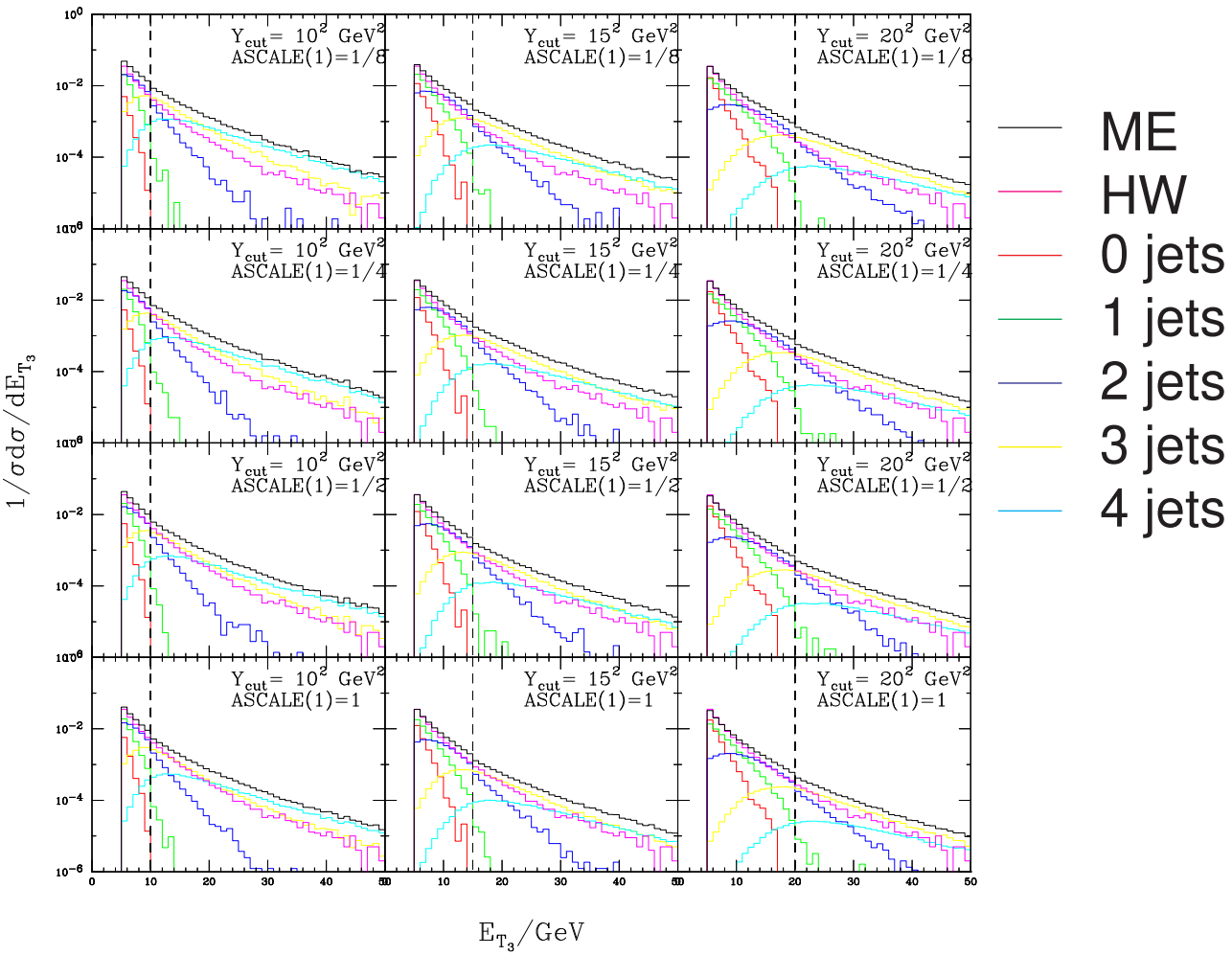
W Production

- Variation of results with scale of α_S and matching scale.
- d_4 distribution at the Tevatron



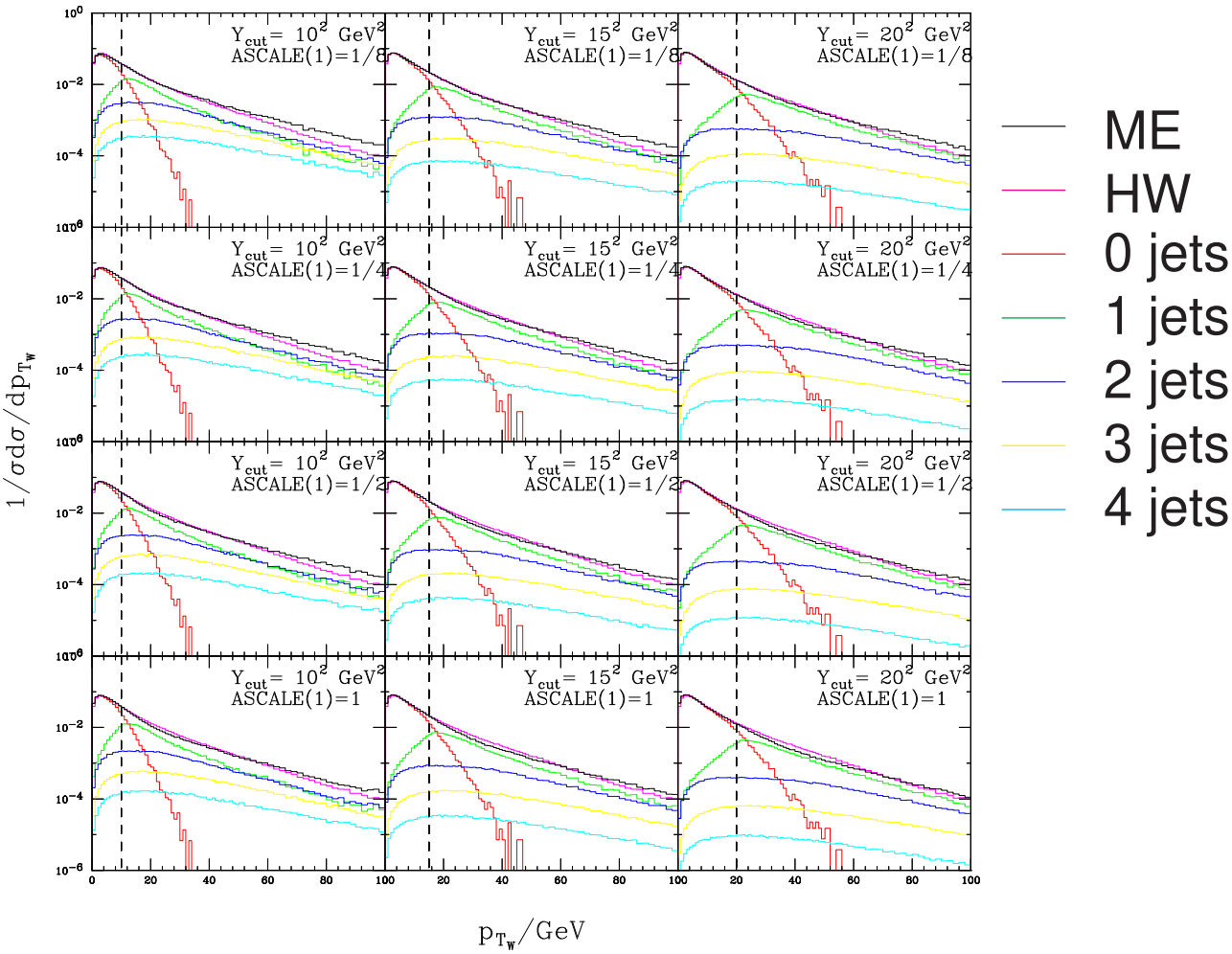
W Production

- Variation of results with scale of α_S and matching scale.
- E_T of the third jet at the Tevatron with cone $R = 0.7$.



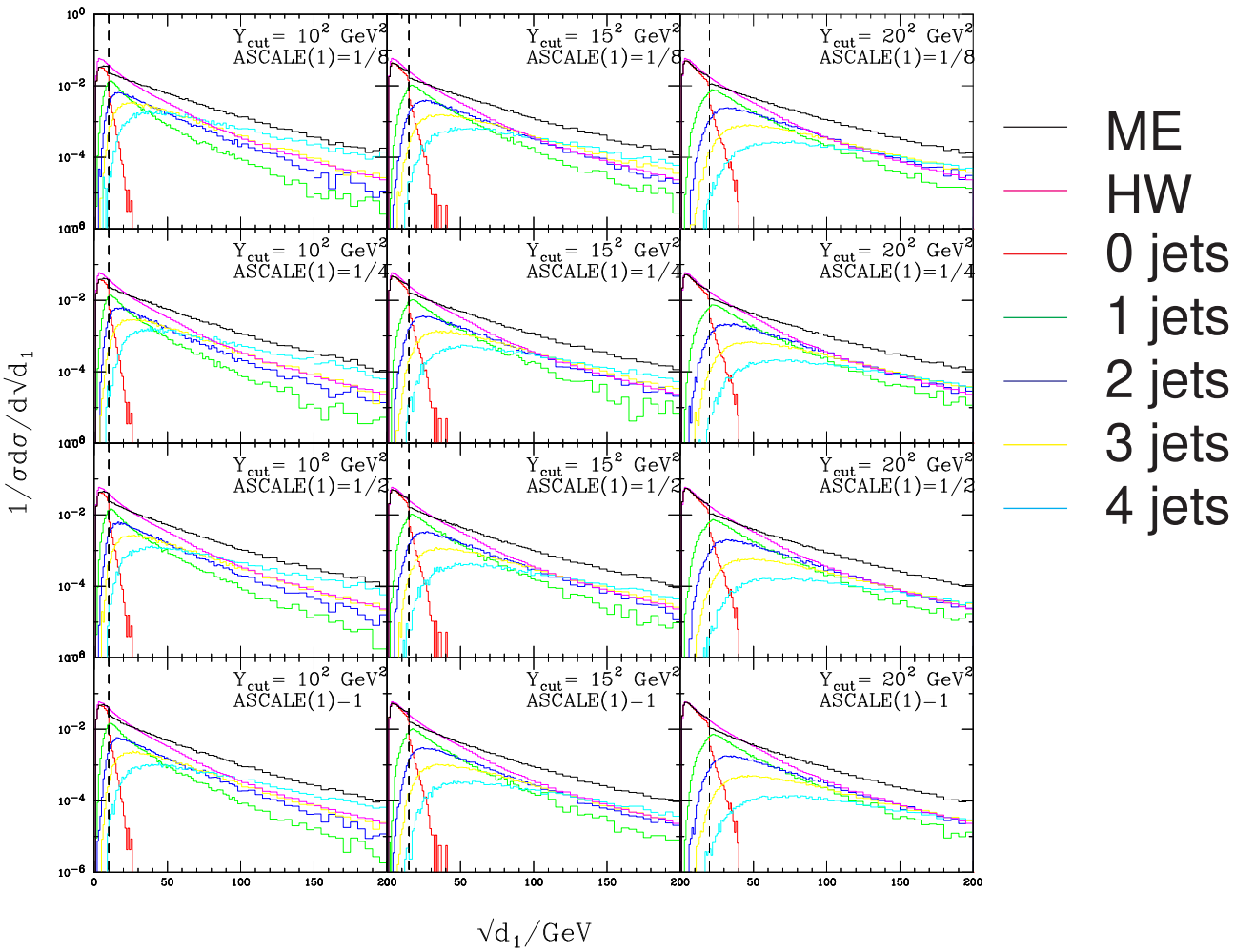
W Production

- Variation of results with scale of α_S and matching scale.
- p_T of the W at the LHC.



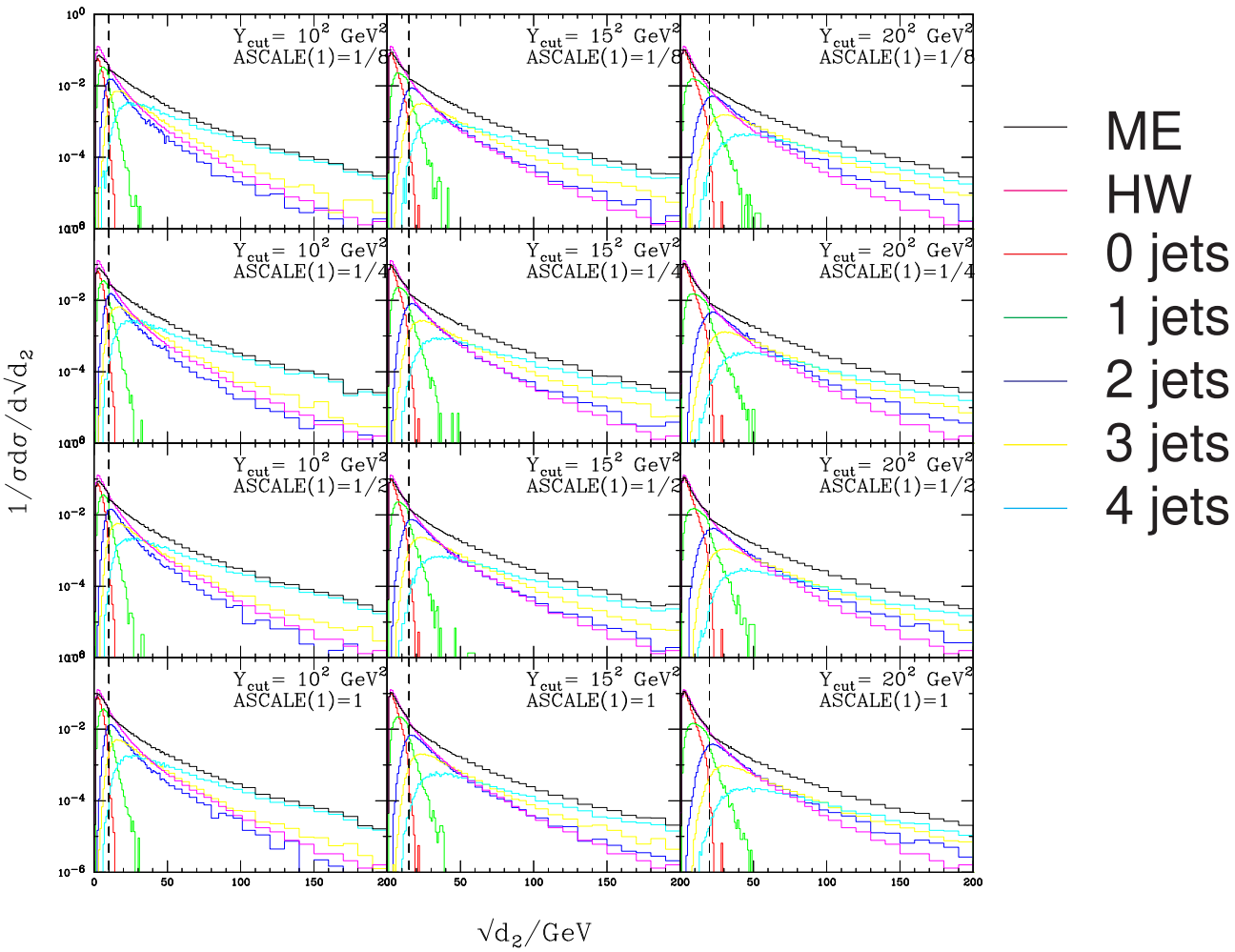
W Production

- Variation of results with scale of α_S and matching scale.
- d_1 distribution at the LHC.



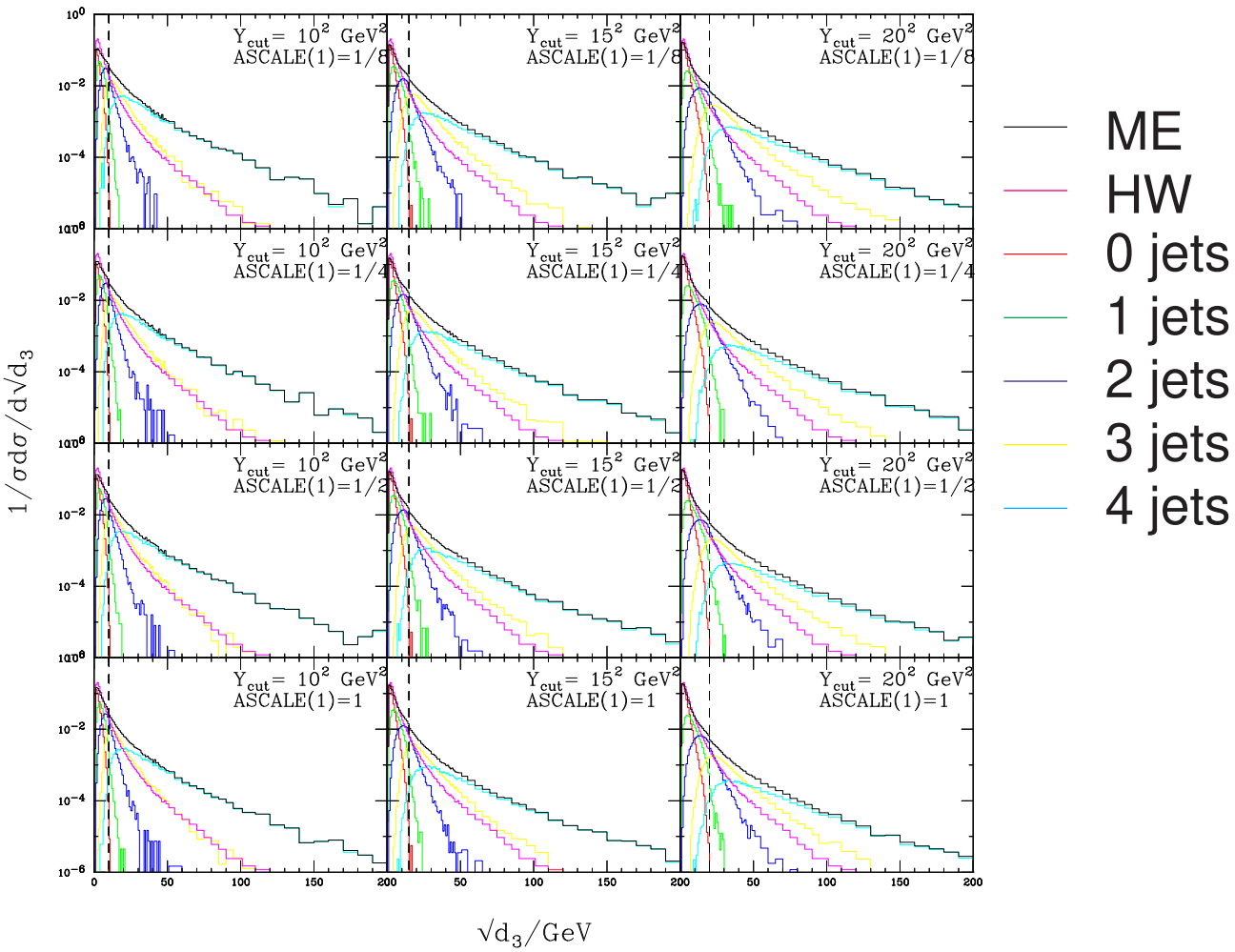
W Production

- Variation of results with scale of α_S and matching scale.
- d_2 distribution at the LHC.



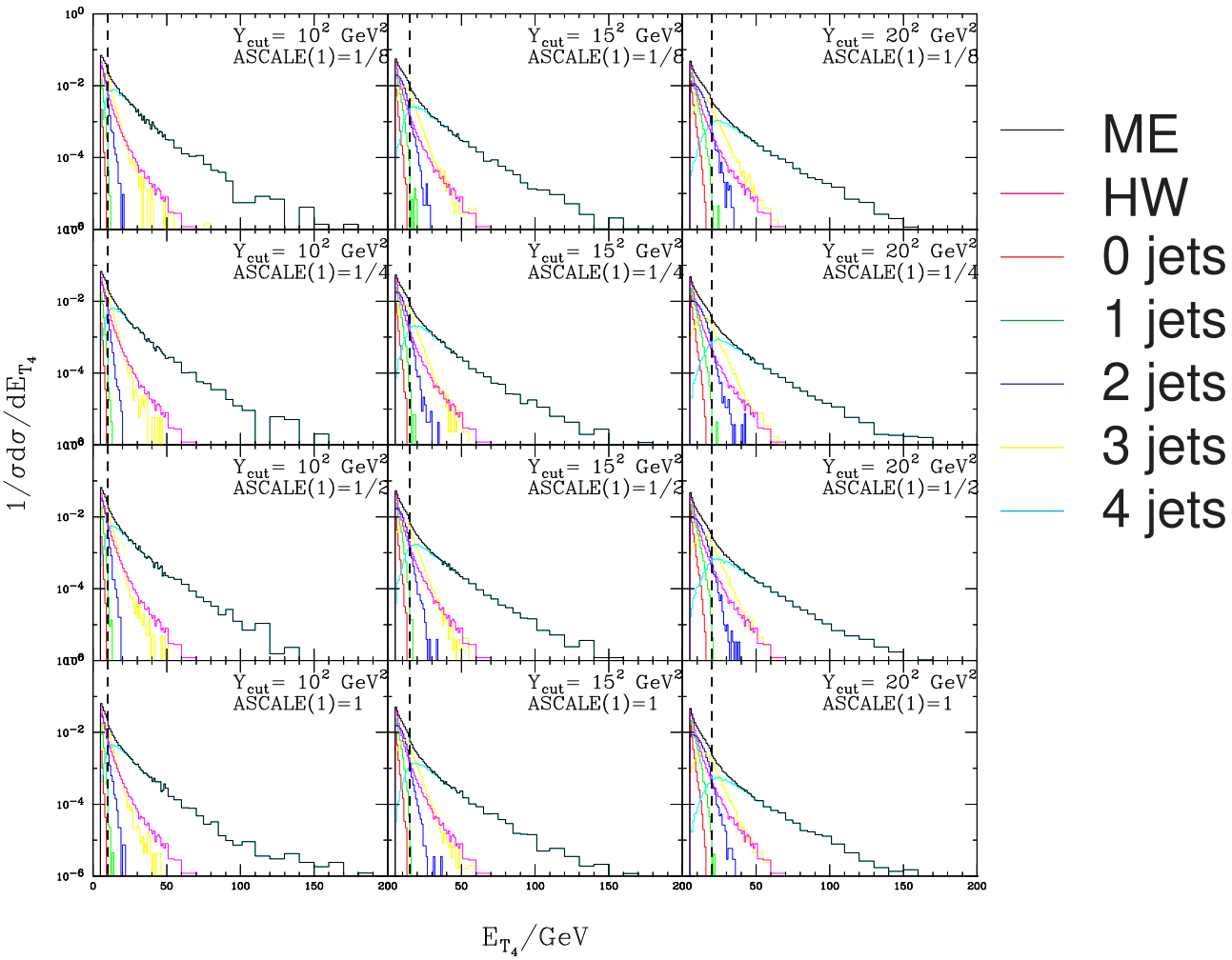
W Production

- Variation of results with scale of α_S and matching scale.
- d_3 distribution at the LHC.



W Production

- Variation of results with scale of α_S and matching scale.
- E_T of the fourth jet at the LHC with cone $R = 0.4$.



Conclusions

- I have presented results for the CKKW algorithm with HERWIG.
- We now have good results for e^+e^- collisions
- Still some problems to iron-out in hadron collisions.
- Hopefully will be available soon.