Durham University

Monte Carlo Event Generators

Lecture 1: Basic Principles of Event Generation

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- So the title I was given for these lectures was

‘Event Generator Basics’

- However after talking to some of you yesterday evening I realised it should probably be called

‘Why I Shouldn’t Just Run PYTHIA’

Plan

- Lecture 1: Introduction
  - Basic principles of event generation
  - Monte Carlo integration techniques
  - Matrix Elements
- Lecture 2: Parton Showers
  - Parton Shower Approach
  - Recent advances, CKKW and MC@NLO
- Lecture 3: Hadronization and Underlying Event
  - Hadronization Models
  - Underlying Event Modelling

Plan

- I will concentrate on hadron collisions.
- There are many things I will not have time to cover
  - Heavy Ion physics
  - B Production
  - BSM Physics
  - Diffractive Physics
- I will concentrate on the basic aspects of Monte Carlo simulations which are essential for the Tevatron and LHC.

Plan

- However I will try and present the recent progress in a number of areas which are important for the Tevatron and LHC
  - Matching of matrix elements and parton showers
    - Traditional Matching Techniques
    - CKKW approach
    - MC@NLO
  - Other recent progress
- While I will talk about the physics modelling in the different programs I won't talk about the technical details of running them.

Other Lectures and Information

- Unfortunately there are no good books or review papers on Event Generator physics.
- However many of the other generator authors have given similar lectures to these which are available on the web
- Torbjorn Sjostrand at the Durham Yeti meeting

  http://www.thep.lu.se/~torbjorn and then Talks
- Mike Seymour's CERN training lectures

  http://seymour.home.cern.ch/seymour/slides/CERNlectures.html

Other Lectures and Information

- Steve Mrenna, CTEQ lectures, 2004
  http://www.phys.psu.edu/~cteq/schools/summer04/mrenna/mrenna.pdf
- Bryan Webber, HERWIG lectures for CDF, October 2004
  http://www-cdf.fnal.gov/physics/lectures/herwig_Oct2004.html
- The "Les Houches Guidebook to Monte Carlo Generators for Hadron Collider Physics", hep-ph/0403045
  http://arxiv.org/pdf/hep-ph/0403045
- Often the PYTHIA manual is a good source of information, but it is of course PYTHIA specific.

Lecture 1

Today we will cover

- Monte Carlo Integration Technique
- Basic Idea of Event Generators
- Event Generator Programs
- Matrix Element Calculations

Monte Carlo Integration Technique

- The basis of all Monte Carlo simulations is the Monte Carlo technique for the evaluation of integrals
- Suppose we want to evaluate

$$I = \int_{x_1}^{x_2} f(x) \, \mathrm{d}x = (x_2 - x_1)\langle f(x) \rangle$$

- This can be written as an average.
- The average can be calculated by selecting N values randomly from a uniform distribution

$$I \approx I_N \equiv (x_2 - x_1) \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$

- Often we define a weight

$$W_i = (x_2 - x_1) f(x_i)$$

- In which case the integral is the average of the weight.

Monte Carlo Integration Technique

- We also have an estimate of the error on the integral using the central limit theorem

$$I \approx I_N \pm \sqrt{\frac{V_N}{N}}$$

where

$$I_N = \frac{1}{N}\sum_i W_i$$

$$V_N = \frac{1}{N}\sum_i W_i^{\,2} - \left[\frac{1}{N}\sum_i W_i\right]^2$$

Convergence

- The Monte Carlo technique has an error which converges as $\propto 1/\sqrt{N}$
- Other common techniques converge faster
  - Trapezium rule $\propto 1/N^2$
  - Simpson's rule $\propto 1/N^4$

  However only if the derivatives exist and are finite. Otherwise the convergence of the Trapezium or Simpson's rule will be worse.

Convergence

- In more than 1 dimension
  - Monte Carlo extends trivially to more dimensions and always converges as $\propto 1/\sqrt{N}$
  - Trapezium rule goes as $\propto 1/N^{2/d}$
  - Simpson's rule goes as $\propto 1/N^{4/d}$
- In a typical LHC event we have ~1000 particles so we need to do ~3000 phase space integrals for the momenta.
- Monte Carlo is the only viable option.

Improving Convergence

- The convergence of the integral can be improved by reducing, V_N.
- This is normally called Importance Sampling.
- The basic idea is to perform a Jacobian transform so that the integral is flat in the new integration variable.
- Let's consider the example of a fixed width Breit-Wigner distribution

$$I = \int_{M_{\min}^2}^{M_{\max}^2} dm^2 \frac{1}{(m^2 - M^2)^2 + M^2\Gamma^2}$$

where

  - M is the physical mass of the particle
  - m is the off-shell mass
  - Γ is the width.

Improving Convergence

- A useful transformation is

$$m^2 = M\Gamma \tan\rho + M^2$$
$$dm^2 = M\Gamma \sec^2\rho \, \mathrm{d}\rho$$

which gives

$$I = \int_{M^2_{\mathrm{min}}}^{M^2_{\mathrm{max}}} \mathrm{d}m^2 \frac{1}{(m^2 - M^2)^2 + M^2\Gamma^2} = \int_{\rho_{\mathrm{min}}}^{\rho_{\mathrm{max}}} \mathrm{d}\rho \mathrm{M}\Gamma \sec^2\rho \frac{1}{M^2\Gamma^2 \tan^2\rho + M^2\Gamma^2}$$

$$= \frac{1}{M\Gamma} \int_{\rho_{\mathrm{min}}}^{\rho_{\mathrm{max}}} \mathrm{d}\rho$$

- So we have in fact reduced the error to zero.

Improving Convergence

- In practice few of the cases we need to deal with in real examples can be exactly integrated.
- In these cases we try and pick a function that approximates the behaviour of the function we want to integrate.
- For example suppose we have a spin-1 meson decaying to two scalar mesons which are much lighter, consider the example of the ρ decaying to massless pions.
- In this case the width

$$\Gamma(m) = \frac{\Gamma_0 M}{m}\left(\frac{p(m)}{p(M)}\right)^3 = \frac{\Gamma_0 M}{m}\left(\frac{m}{M}\right)^{3/2} = \Gamma_0\left(\frac{m}{M}\right)^{1/2}$$

Improving Convergence

- If we were just to generate flat in m^2 then the weight would be

$$W_i = \frac{m_{\max}^2 - m_{\min}^2}{(m^2 - M^2)^2 + \frac{\Gamma_0^2 m^3}{M}}$$

- If we perform a jacobian transformation the integral becomes

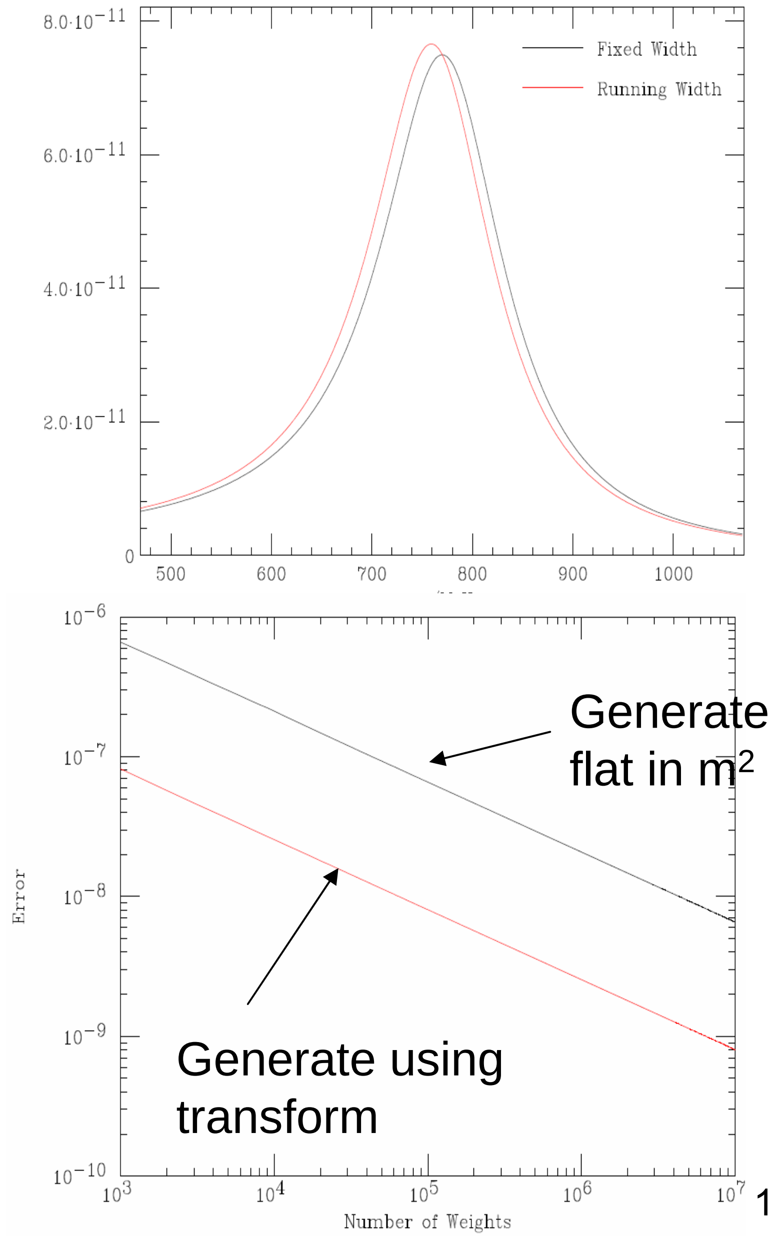
$$I = \int_{M_{\min}^2}^{M_{\max}^2} \mathrm{d}m^2 \frac{1}{(m^2 - M^2)^2 + \frac{\Gamma_0^2 m^3}{M}} = \frac{1}{M\Gamma_0} \int_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho \frac{(m^2 - M^2)^2 + M^2\Gamma_0^2}{(m^2 - M^2)^2 + \frac{\Gamma_0^2 m^3}{M}}$$

and the weight is

$$W_i = \frac{1}{M\Gamma_0}(\rho_{\max} - \rho_{\min}) \frac{(m^2 - M^2)^2 + M^2\Gamma_0^2}{(m^2 - M^2)^2 + \frac{\Gamma_0^2 m^3}{M}}$$

Improving Convergence

- For this case if we perform the integral using m^2 the error is ~100 times larger for the same number of evaluations.
- When we come to event generation this would be a factor of 100 slower.

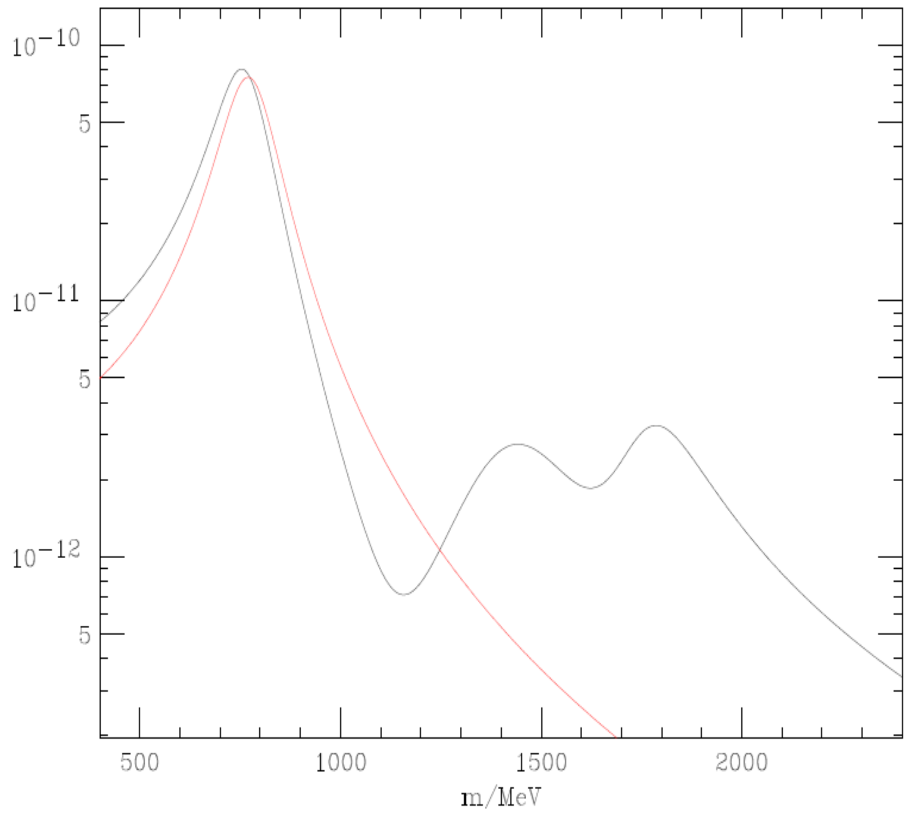


Improving Convergence

- Using a Jacobian transformation is always the best way of improving the convergence.
- There are automatic approaches (e.g. VEGAS) but they are never as good.

Multi-Channel approaches

- Suppose instead of having one peak we have an integral with lots of peaks, say from the inclusion of excited ρ resonances in some process.
- Can't just use one Breit-Wigner. The error becomes large.



Multi-Channel approaches

- If we want to smooth out many peaks pick a function

$$f(m^2) = \sum_i \alpha_i g_i(m^2) = \sum_i \alpha_i \frac{1}{(m^2 - M_i^2)^2 + M_i^2\Gamma_i^2}$$

where α_i is the weight for a given term such that $\Sigma\alpha_i$=1.

- We can then rewrite the integral of a function *I(m²)* as

$$\int_{M_{\min}^2}^{M_{\max}^2} \mathrm{d}m^2 I(m^2) = \int_{M_{\min}^2}^{M_{\max}^2} \mathrm{d}m^2 I(m^2) \frac{f(m^2)}{f(m^2)}$$

$$= \int_{M_{\min}^2}^{M_{\max}^2} \mathrm{d}m^2 \sum_i \alpha_i g_i(m^2) \frac{I(m^2)}{f(m^2)} = \sum_i \alpha_i \int_{M_{\min}^2}^{M_{\max}^2} \mathrm{d}m^2 g_i(m^2) \frac{I(m^2)}{f(m^2)}$$

Multi-Channel approaches

- We can then perform a separate Jacobian transform for each of the integrals in the sum

$$\sum_i \alpha_i \int_{M^2_{\min}}^{M^2_{\max}} \mathrm{d}m^2 g_i(m^2) \frac{I(m^2)}{f(m^2)} = \sum_i \alpha_i \int_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho_i \frac{I(m^2)}{f(m^2)}$$

- This is then easy to implement numerically by picking one of the integrals (channels) with probability α_i and then calculating the weight as before.
- This is called the Multi-Channel procedure and is used in the most sophisticated programs for integrating matrix elements in particle physics.
- There are methods to automatically optimise the choice of the channel weights, α_i.

Monte Carlo Integration Technique

- In addition to calculating the integral we often also want to select values of x at random according to *f(x).*
- This is easy provided that we know the maximum value of the function in the region we are integrating over.
- Then we randomly generate values of x in the integration region and keep them with probability

$$P = \frac{f(x)}{f_{\max}} \geq R$$

which is easy to implement by generating a random number between 0 and 1 and keeping the value of x if the random number is less than the probability.

- This is called unweighting.

Summary

- Disadvantages of Monte Carlo
  - Slow convergence in few dimensions
- Advantages of Monte Carlo
  - Fast convergence in many dimensions
  - Arbitrarily complex integration regions
  - Few points needed to get first estimate
  - Each additional point improves the accuracy
  - Easy error estimate
  - More than one quantity can be evaluated at once.

Phase Space

- Cross section $\sigma = \frac{1}{2s} \int |M|^2 d\Phi_n(\sqrt{s})$

- And decay rates $\Gamma = \frac{1}{2M} \int |M|^2 d\Phi_n(M)$

- Can be calculated as the integral of the matrix element over the Lorentz invariant phase space.

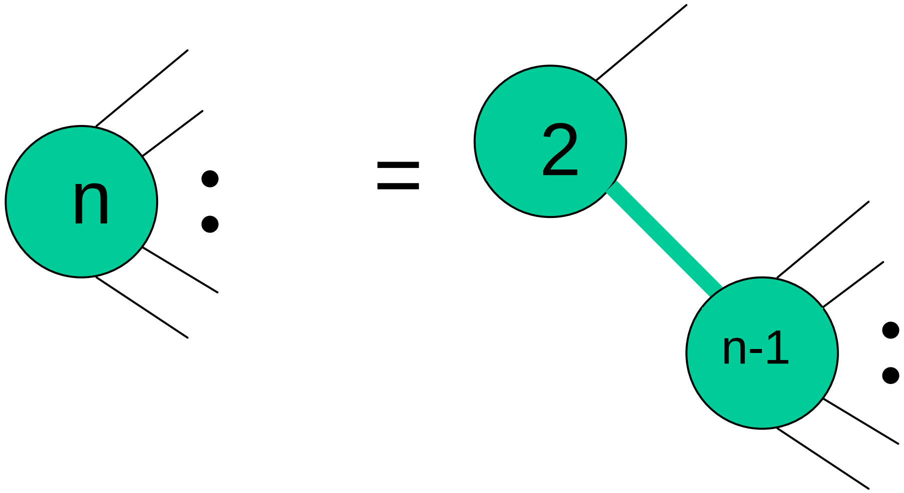
$$d\Phi_n(M) = \left[\prod_{i=1}^{n} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} \right] (2\pi)^4 \delta^4 \left(p_0 - \sum_{i=1}^{n} p_i \right)$$

- Easy to evaluate for the two body case

$$d\Phi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{\mathrm{d}\Omega}{4\pi}$$

Phase Space

- More complicated cases can then be handled recursively

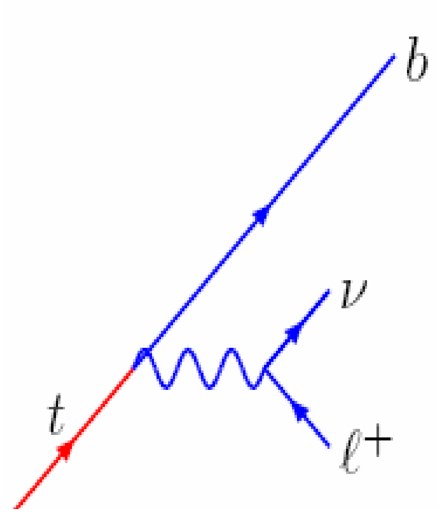


- Given by

$$d\Phi_n(M) = \frac{1}{2\pi} \int_{\left(\sum_{i=2}^{n} m_i\right)^2}^{(M-m_1)^2} \mathrm{d}M_X^2 \, d\Phi_2(M) \, d\Phi_{n-1}(M_X)$$

- There are other approaches (RAMBO/MAMBO) which are better if the matrix element is flat, but this is rarely true in practice.

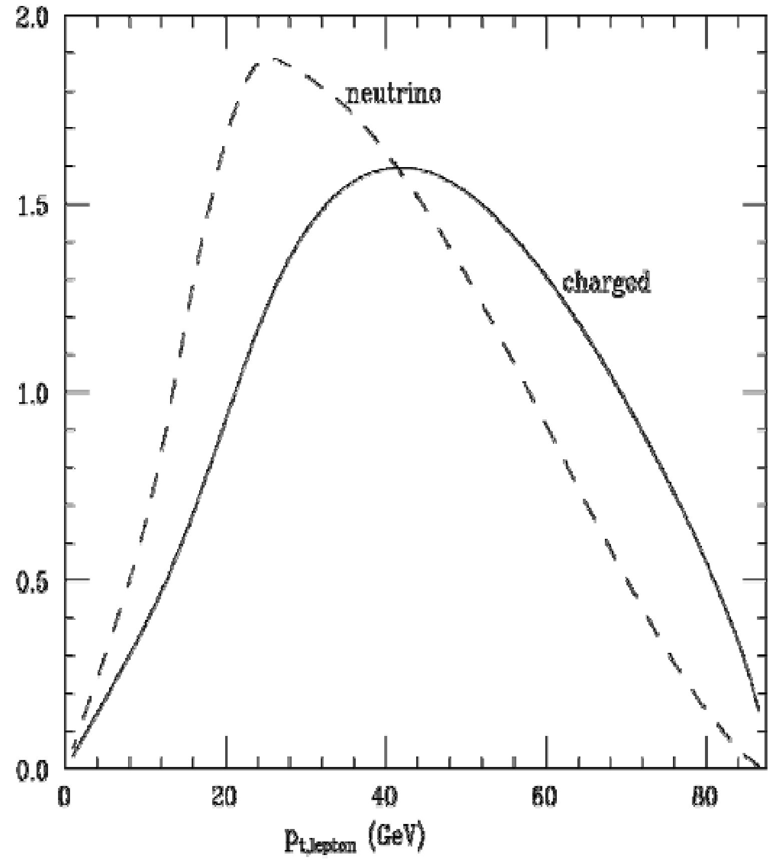
Particle Decays



- If we consider the example of top quark decay $t \rightarrow bW^+ \rightarrow bl^+\nu_l$

$$|M|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_w}\right)^2 \frac{p_t \cdot p_\upsilon p_b \cdot p_l}{(m_w^2 - M_w^2)^2 + \Gamma_W^2 M_w^2}$$

- The Breit-Wigner peak of the W is very strong and must be removed using a Jacobian factor.
- Illustrates another big advantage of Monte Carlo.
- Can just histogram any quantities we are interested in. Other techniques require a new integration for each observable.



Cross Sections

- In hadron collisions we have additional integrations over the incoming parton densities

$$\sigma(s) = \int_0^1 \mathrm{d}x_1\, f_1(x_1) \int_0^1 \mathrm{d}x_2\, f_2(x_2) \hat{\sigma}(x_1 x_2 s)$$

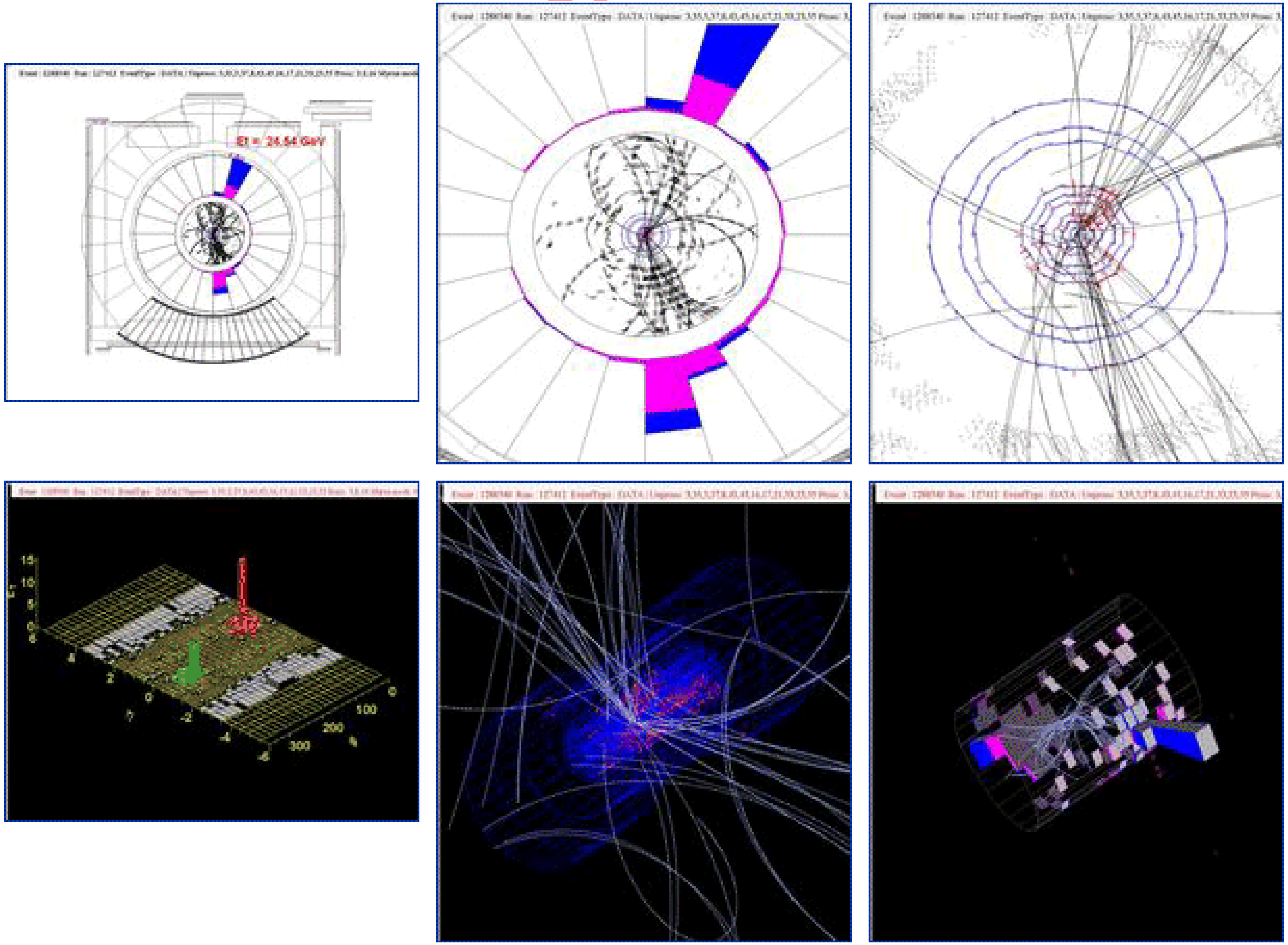
$$\sigma(s) = \int_0^1 \frac{\mathrm{d}\tau}{\tau} \hat{\sigma}(\tau s) \int_\tau^1 \frac{\mathrm{d}x_1}{x_1} x_1 f_1(x_1) \frac{\tau}{x_1} f_2\left(\frac{\tau}{x_1}\right)$$

- The parton level cross section can have strong peaks, e.g. the Z Breit-Wigner which need to be smoothed using a Jacobian transformation.

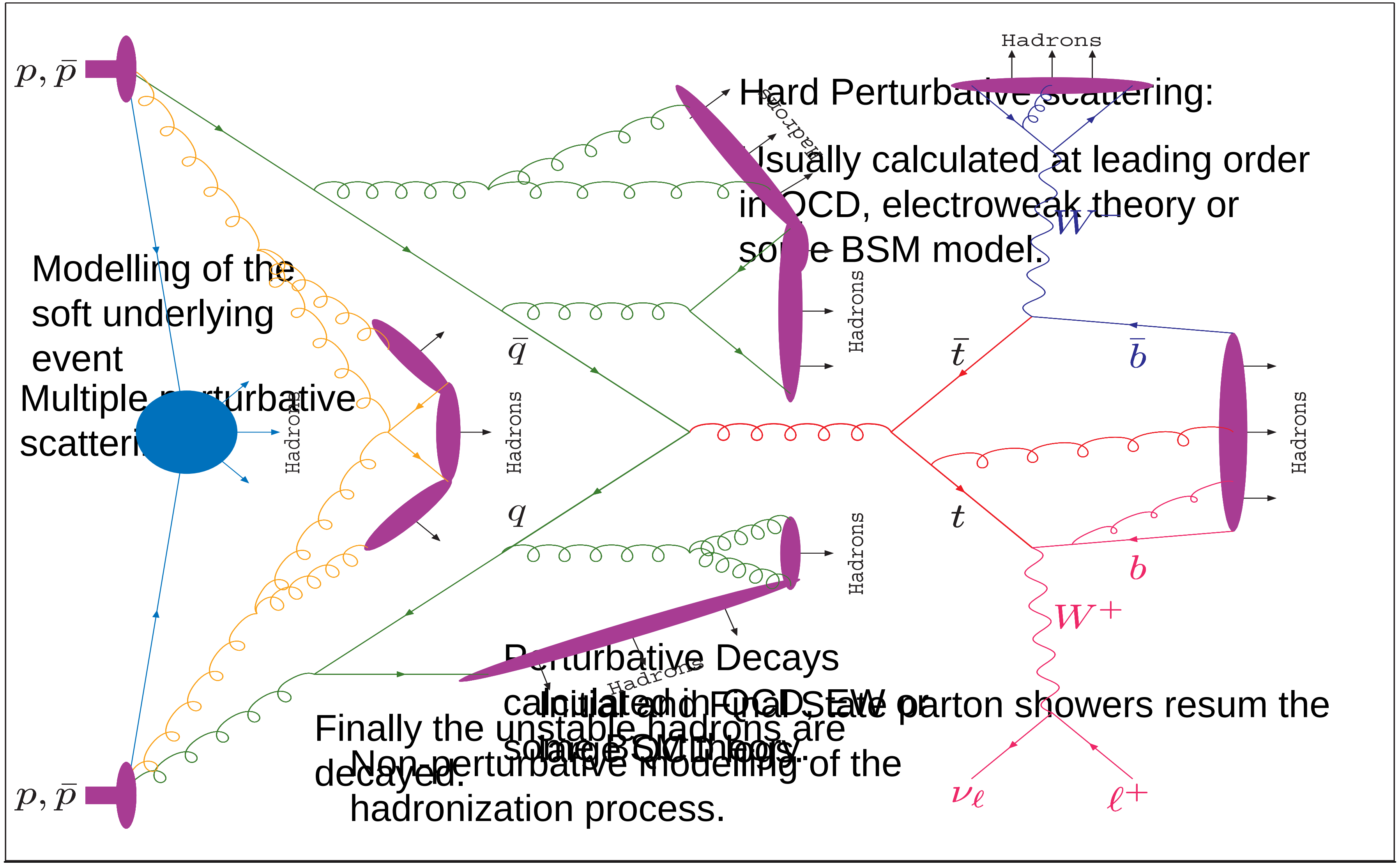
Monte Carlo Event Generators

- At the most basic level a Monte Carlo event generator is a program which simulates particle physics events with the same probability as they occur in nature.
- In essence it performs a large number of integrals and then unweights to give the momenta of the particles which interact with the detector.

Example CDF 2 jet + missing energy event



A Monte Carlo Event



Monte Carlo Event Generators

- All the event generators split the simulation up into the same phases:
  - Hard Process;
  - Parton Shower;
  - Secondary Decays;
  - Hadronization;
  - Multiple Scattering/Soft Underlying Event;
  - Hadron Decays.
- I will discuss the different models and approximations in the different programs as we go along.
- I will try and give a fair and objective comparision, but bear in mind that I’m one of the authors of HERWIG.

Monte Carlo Event Generators

- There are a range of Monte Carlo programs available.
- In general there are two classes of programs

1) General Purpose Event Generators
   - Does everything
2) Specialized Programs
   - Just performs part of the process

- In general need both are needed.

Monte Carlo Event Generators

	General Purpose	Specialized
Hard Processes	HERWIG	Many
Resonance Decays	PYTHIA	HDECAY, SDECAY
Parton Showers	ISAJET	Ariadne/LDC, NLLJet
Underlying Event	SHERPA	DPMJET
Hadronization	+ new C++ versions	None?
Ordinary Decays		TAUOLA/EvtGen

Hard Processes

- Traditionally all the hard processes used were in the event generators.
- These are normally $2\rightarrow2$ scattering processes.
- There are a vast range of processes in both HERWIG and PYTHIA.
- However for the LHC we are often interested in higher multiplicity final states.
- For these need to use specialized matrix element generators.

Processes in HERWIG

IPROC	Process
100	$\ell^+\ell^- \to q\bar q(g)$ (all q flavours)
100+IQ	$\ell^+\ell^- \to q\bar q(g)$ (IQ $=1,2,3,4,5,6$ for $q=d,u,s,c,b,t$)
107	$\ell^+\ell^- \to gg(g)$ (fictitious process)
110	$\ell^+\ell^- \to q\bar q g$ (all flavours)
110+IQ	$\ell^+\ell^- \to q\bar q g$ (IQ as above)
120	$\ell^+\ell^- \to q\bar q$ (all flavours, no hard gluon correction)
120+IQ	$\ell^+\ell^- \to q\bar q$ (IQ as above, no hard gluon correction)
127	$\ell^+\ell^- \to gg$ (fictitious process, no hard gluon correction)
150+IL	$\ell^+\ell^- \to \ell'\bar\ell'$ (IL $=1,2,3$ for $\ell'=e,\mu,\tau$, N.B. $\ell \neq \ell'$)
200	$\ell^+\ell^- \to W^+W^-$ (see sect. 4.3.2 on control of W/Z decays)
250	$\ell^+\ell^- \to Z^0Z^0$ (see sect. 4.3.2 on control of W/Z decays)
300	$\ell^+\ell^- \to Z^0H^0_{SM} \to Z^0q\bar q$ (all flavours)
300+IQ	$\ell^+\ell^- \to Z^0H^0_{SM} \to Z^0q\bar q$ (IQ as above)
306+IL	$\ell^+\ell^- \to Z^0H^0_{SM} \to Z^0\ell\bar\ell$ (IL as above)
310, 311	$\ell^+\ell^- \to Z^0H^0_{SM} \to Z^0W^+W^-,\ Z^0Z^0Z^0$

IPROC	Process
312	$\ell^+\ell^- \to Z^0H^0_{SM} \to Z^0\gamma\gamma$
399	$\ell^+\ell^- \to Z^0H^0_{SM} \to Z^0$ anything
400+ID	$\ell^+\ell^- \to \nu\bar\nu H^0_{SM} + \ell^+\ell^- H^0_{SM}$ (ID as in IPROC $=300+$ ID)
500+ID	$\ell^+\ell^- \to \ell^+\ell^-\gamma\gamma \to \ell^+\ell^- q\bar q/\ell\bar\ell/W^+W^-$ (ID=0–10 as in IPROC $=300+$ ID)
550+ID	$\ell^+\ell^- \to \ell\nu_\ell\gamma W \to \ell\nu_\ell q\bar q'/\ell\bar\ell'$ (ID=0–9 as in IPROC $=300+$ ID)
600	$\ell^+\ell^- \to q\bar q gg, q\bar q q'\bar q'$ (all q flavours)
600+IQ	$\ell^+\ell^- \to q\bar q gg, q\bar q q'\bar q'$ (IQ as above)
	After generation, IHPRO is subprocess (see sect. 4.3.5)
700-99	Minimal Supersymmetric Standard Model (MSSM) processes
700	$\ell^+\ell^- \to$ 2-sparticle processes (sum of 710, 730, 740 and 760)
710	$\ell^+\ell^- \to$ neutralino pairs (all neutralinos)
706+4IN1+IN2	$\ell^+\ell^- \to \tilde\chi^0_{IN1}\tilde\chi^0_{IN2}$ (IN1,2=neutralino mass eigenstate)
730	$\ell^+\ell^- \to$ chargino pairs (all charginos)
728+2IC1+IC2	$\ell^+\ell^- \to \tilde\chi^+_{IC1}\tilde\chi^-_{IC2}$ (IC1,2=chargino mass eigenstate)
740	$\ell^+\ell^- \to$ slepton pairs (all flavours)
736+5IL	$\ell^+\ell^- \to \tilde\ell_{L,R}\tilde\ell^*_{L,R}$ (IL $=1,2,3$ for $\tilde\ell=\tilde e,\tilde\mu,\tilde\tau$)
737+5IL	$\ell^+\ell^- \to \tilde\ell_L\tilde\ell^*_L$ (IL as above)
738+5IL	$\ell^+\ell^- \to \tilde\ell_L\tilde\ell^*_R$ (IL as above)
739+5IL	$\ell^+\ell^- \to \tilde\ell_R\tilde\ell^*_R$ (IL as above)
740+5IL	$\ell^+\ell^- \to \tilde\nu_L\tilde\nu^*_L$ (IL $=1,2,3$ for $\tilde\nu_e,\tilde\nu_\mu,\tilde\nu_\tau$)
760	$\ell^+\ell^- \to$ squark pairs (all flavours)
757+4IQ	$\ell^+\ell^- \to \tilde q_{L,R}\tilde q^*_{L,R}$ (IQ $=1,2,3,4,5,6$ for $\tilde q=\tilde d,\tilde u,\tilde s,\tilde c,\tilde b,\tilde t$)
758+4IQ	$\ell^+\ell^- \to \tilde q_L\tilde q^*_L$ (IQ as above)
759+4IQ	$\ell^+\ell^- \to \tilde q_L\tilde q^*_R$ (IQ as above)
760+4IQ	$\ell^+\ell^- \to \tilde q_R\tilde q^*_R$ (IQ as above)
800-99	R-parity violating supersymmetric processes
800	Single sparticle production, sum of 810–840
810	$\ell^+\ell^- \to \tilde\chi^0\nu_i$, (all neutralinos)
810+IN	$\ell^+\ell^- \to \tilde\chi^0_{IN}\nu_i$, (IN=neutralino mass state)
820	$\ell^+\ell^- \to \tilde\chi^- e_i^+$ (all charginos)
820+IC	$\ell^+\ell^- \to \tilde\chi^-_{IC}e_i^+$, (IC=chargino mass state)
830	$\ell^+\ell^- \to \tilde\nu_i Z^0$ and $\ell^+\ell^- \to \tilde\ell_i^+ W^-$
840	$\ell^+\ell^- \to \tilde\nu_i h^0/H^0/A^0$ and $\ell^+\ell^- \to \tilde\ell_i^+ H^-$
850	$\ell^+\ell^- \to \tilde\nu_i\gamma$
860	Sum of 870 and 880
870	$\ell^+\ell^- \to \ell^+\ell^-$, via LLE only
867+3IL1+IL2	$\ell^+\ell^- \to \ell^+_{IL1}\ell^-_{IL2}$ (IL1,2=1,2,3 for e,μ,τ)

IPROC	Process
880	$\ell^+\ell^- \to d\bar d$, via LLE and LQD
877+3IQ1+IQ2	$\ell^+\ell^- \to d_{IL1}\bar d_{IL2}$ (IQ1,2=1,2,3 for d,s,b)
910	$\ell^+\ell^- \to \nu_e\bar\nu_e h^0 + e^+e^-h^0$
920	$\ell^+\ell^- \to \nu_e\bar\nu_e H^0 + e^+e^-H^0$
960	$\ell^+\ell^- \to Z^0h^0$
970	$\ell^+\ell^- \to Z^0H^0$
955	$\ell^+\ell^- \to H^+H^-$
965	$\ell^+\ell^- \to A^0h^0$
965	$\ell^+\ell^- \to A^0H^0$
1000+ID	$\ell^+\ell^- \to t\,\bar t\,H^0_{SM}$ (ID as in IPROC=300+ID)
1110+IQ	$\ell^+\ell^- \to q\,\bar q\,h^0$ (IQ as in IPROC=100+IQ)
1116+IL	$\ell^+\ell^- \to \ell^+\ell^-h^0$ (IL=1,2,3 for e,μ,τ)
1120+IQ	$\ell^+\ell^- \to q\,\bar q\,H^0$ (IQ as in IPROC=100+IQ)
1126+IL	$\ell^+\ell^- \to \ell^+\ell^-H^0$ (IL=1,2,3 for e,μ,τ)
1130+IQ	$\ell^+\ell^- \to q\,\bar q\,A^0$ (IQ as in IPROC=100+IQ)
1136+IL	$\ell^+\ell^- \to \ell^+\ell^-A^0$ (IL=1,2,3 for e,μ,τ)
1140	$\ell^+\ell^- \to d\,\bar u\,H^+$+ ch. conj.
1141	$\ell^+\ell^- \to s\,\bar c\,H^+$+ ch. conj.
1142	$\ell^+\ell^- \to b\,\bar t\,H^+$+ ch. conj.
1143	$\ell^+\ell^- \to e\,\bar\nu_e H^+$+ ch. conj.
1144	$\ell^+\ell^- \to \mu\,\bar\nu_\mu H^+$+ ch. conj.
1145	$\ell^+\ell^- \to \tau\,\bar\nu_\tau H^+$+ ch. conj.
1200–99	Reserved for other $\ell^+\ell^-$ processes
1300	$q\bar q \to Z^0/\gamma \to q'\bar q'$ (all flavours)
1300+IQ	$q\bar q \to Z^0/\gamma \to q'\bar q'$ (IQ $=1,2,3,4,5,6$ for $q=d,u,s,c,b,t$)
1350	$q\bar q \to Z^0/\gamma \to \ell\bar\ell$ (all lepton species)
1350+IL	$q\bar q \to Z^0/\gamma \to \ell\bar\ell$ (IL $=1-6$ for $\ell=e,\nu_e,\mu,\nu_\mu$, etc.)
1399	$q\bar q \to Z^0/\gamma \to$ anything
1400	$q\bar q \to W^\pm \to q'\bar q''$ (all flavours)
1400+IQ	$q\bar q \to W^\pm \to q'\bar q''$ (q' or q'' as above)
1450	$q\bar q \to W^\pm \to \ell\nu_\ell$ (all lepton species)
1450+IL	$q\bar q \to W^\pm \to \ell\nu_\ell$ (IL $=1,2,3$ for $\ell=e,\mu,\tau$)
1499	$q\bar q \to W^\pm \to$ anything
1500	QCD $2\to2$ hard parton scattering
	After generation, IHPRO is subprocess (see sect. 4.6.2)
1600+ID	$gg/q\bar q \to H^0_{SM}$ (ID as in IPROC $=300+$ ID)
1700+IQ	QCD heavy quark production (IQ as above)
	After generation, IHPRO is subprocess (see sect. 4.6.2)
1800	QCD direct photon + jet production

IPROC	Process
	After generation, IHPRO is subprocess (see sect. 4.6.5)
1900+ID	$q\bar q \to q'\bar q' W^+W^-/Z^0Z^0 \to q'\bar q' H^0_{SM}$ (ID as in IPROC $=300+$ ID)
2000	t production via $W^\pm$ exchange (sum of 2001–2008)
2001–4	$\bar u b \to \bar d t$, $d\bar b \to u\bar t$, $\bar d\bar b \to \bar u\bar t$, $ub \to dt$
2005–8	$\bar c b \to \bar s t$, $s\bar b \to c\bar t$, $\bar s\bar b \to \bar c\bar t$, $cb \to st$
2100	$W^\pm$ + jet production
2110	$W^\pm$ + jet production (Compton only: $gq \to Wq$)
2120	$W^\pm$ + jet production (annihilation only: $q\bar q \to Wg$)
2150	Z^0 + jet production
2160	Z^0 + jet production (Compton only: $gq \to Zq$)
2170	Z^0 + jet production (annihilation only: $q\bar q \to Zg$)
2200	QCD direct photon pair production
	After generation, IHPRO is subprocess (see sect. 4.6.5)
2300+ID	QCD SM Higgs + jet production (ID as in IPROC=300+ID)
	After generation, IHPRO is subprocess (see sect. 4.6.10)
2400	Mueller-Tang colour singlet exchange
2450	Quark scattering via photon exchange
2500+ID	$gg/q\bar q \to t\bar t H^0_{SM}$ (ID as in IPROC=300+ID)
2600+ID	$q\bar q' \to W^\pm H^0_{SM}$ (ID as in IPROC=300+ID)
2700+ID	$q\bar q \to Z^0H^0_{SM}$ (ID as in IPROC=300+ID)

2800	W^+W^- production in hadron-hadron collisions
2810	Z^0Z^0 production in hadron-hadron collisions (including photon terms)
2815	Z^0Z^0 production in hadron-hadron collisions (Z^0 only)
2820	$W^\pm Z^0$ production in hadron-hadron collisions (including photon terms)
2825	$W^\pm Z^0$ production in hadron-hadron collisions (Z^0 only)
2850	hadron-hadron $\to W^+W^-X$ using MC@NLO
2860	hadron-hadron $\to Z^0Z^0X$ using MC@NLO
2870	hadron-hadron $\to W^+Z^0X$ using MC@NLO
2880	hadron-hadron $\to W^-Z^0X$ using MC@NLO
2900+IQ	$gg+q\bar q \to Q\bar Q Z^0$ for massless Q and $\bar Q$ (IQ=1…6 for $Q=d\ldots t$)
2910+IQ	$gg+q\bar q \to Q\bar Q Z^0$, for massive Q and $\bar Q$ (IQ=1…6 for $Q=d\ldots t$)
3000-3999	Minimal Supersymmetric Standard Model (MSSM) processes
3000	2-parton → 2-sparticle processes (sum of those below)
3010	2-parton → 2-spartons processes
3020	2-parton → 2-gaugino processes
3030	2-parton → 2-slepton processes
3100+ISQ	$gg/q\bar q \to \tilde q\tilde q^* H^\pm$ (ISQ=IPROC−3100 as from table 15)
3200+ISQ	$gg/q\bar q \to \tilde q\tilde q^* h, H, A$ (ISQ=IPROC−3200 as from table 16)

IPROC	Process
3310,3315	$q\bar q' \to W^\pm h^0, H^\pm h^0$ (all q,q' flavours – gauge bosons mediated only)
3320,3325	$q\bar q' \to W^\pm H^0, H^\pm H^0$ (")
3335	$q\bar q' \to H^\pm A^0$ (")
3350	$q\bar q' \to W^\pm H^\mp$ (Higgstrahlung and Higgs mediated)
3355	$q\bar q \to H^\pm H^\mp$ (all q flavours — gauge boson mediated only)
3360,3365	$q\bar q \to Z^0h^0, A^0h^0$ (")
3370,3375	$q\bar q \to Z^0H^0, A^0H^0$ (")
3410	$bg \to b\,h^0$ + ch. conj.
3420	$bg \to b\,H^0$ + ch. conj.
3430	$bg \to b\,A^0$ + ch. conj.
3450	$bg \to t\,H^-$ + ch. conj.
3500	$bq \to bq'H^\pm$ + ch. conj.
3610	$q\bar q/gg \to h^0$ (light scalar Higgs)
3620	$q\bar q/gg \to H^0$ (heavy scalar Higgs)
3630	$q\bar q/gg \to A^0$ (pseudoscalar Higgs)
3710	$q\bar q \to q'\bar q' W^+W^-/Z^0Z^0 \to q'\bar q' h^0$
3720	$q\bar q \to q'\bar q' W^+W^-/Z^0Z^0 \to q'\bar q' H^0$
3810+IQ	$gg+q\bar q \to Q\bar Q h^0$ (all q flavours in s-channel, IQ as usual for Q flavour)
3820+IQ	$gg+q\bar q \to Q\bar Q H^0$ (")
3830+IQ	$gg+q\bar q \to Q\bar Q A^0$ (")
3839	$gg+q\bar q \to b\bar t H^+$ + ch. conjg. (all q flavours in s-channel)
3840+IQ	$gg \to Q\bar Q h^0$ (IQ as above)
3850+IQ	$gg \to Q\bar Q H^0$ (")
3860+IQ	$gg \to Q\bar Q A^0$ (")
3869	$gg \to b\bar t H^+$ + ch. conjg.
3870+IQ	$q\bar q \to Q\bar Q h^0$ (all q flavours in s-channel, IQ as above)
3880+IQ	$q\bar q \to Q\bar Q H^0$ (")
3890+IQ	$q\bar q \to Q\bar Q A^0$ (")
3899	$q\bar q \to b\bar t H^+$ + ch. conjg. (all q flavours in s-channel)
3900–99	Reserved for other hadron-hadron MSSM processes
4000–99	R-parity violating supersymmetric processes via LQD
4000	single sparticle production, sum of 4010–4050
4010	$\bar u_j d_k \to \tilde\chi^0 l_i^-$, $\bar d_j d_k \to \tilde\chi^0\nu_i$ (all neutralinos)
4010+IN	$\bar u_j d_k \to \tilde\chi^0_{IN} l_i^-$, $\bar d_j d_k \to \tilde\chi^0_{IN}\nu_i$ (IN=neutralino mass state)
4020	$\bar u_j d_k \to \tilde\chi^-\nu_i$, $\bar d_j d_k \to \tilde\chi^- e_i^+$ (all charginos)
4020+IC	$\bar u_j d_k \to \tilde\chi^-_{IC}\nu_i$, $\bar d_j d_k \to \tilde\chi^-_{IC}e_i^+$ (IC=chargino mass state)
4040	$u_j\bar d_k \to \tilde\ell_i^+ Z^0$, $u_j\bar d_k \to \tilde\nu_i W^+$ and $d_j\bar d_k \to \tilde\ell_i^+ W^-$
4050	$u_j\bar d_k \to \tilde\ell_i^+ h^0/H^0/A^0$, $u_j\bar d_k \to \tilde\nu_i H^+$ and $d_j\bar d_k \to \tilde\ell_i^+ H^-$

IPROC	Process
4060	Sum of 4070 and 4080
4070	$\bar u_j d_k \to \bar u_l d_m$ and $\bar d_j d_k \to \bar d_l d_m$, via LQD only
4080	$\bar u_j d_k \to \nu_j l_k^-$ and $\bar d_j d_k \to l_j^+ l_k^-$, via LQD and LLE
4100-99	R-parity violating supersymmetric processes via UDD
4100	single sparticle production, sum of 4110–4150
4110	$u_i d_j \to \tilde\chi^0\bar d_k$, $d_j d_k \to \tilde\chi^0\bar u_i$ (all neutralinos)
4110 +IN	$u_i d_j \to \tilde\chi^0_{IN}\bar d_k$, $d_j d_k \to \tilde\chi^0_{IN}\bar u_i$(IN as above)
4120	$u_i d_j \to \tilde\chi^+\bar u_k$, $d_j d_k \to \tilde\chi^-\bar d_i$ (all charginos)
4120 +IC	$u_i d_j \to \tilde\chi^+_{IC}\bar u_k$, $d_j d_k \to \tilde\chi^-_{IC}\bar d_i$ (IC as above)
4130	$u_i d_j \to \tilde g\bar d_k$, $d_j d_k \to \tilde g\bar u_i$
4140	$u_i d_j \to \tilde b_1^* Z^0$, $d_j d_k \to \tilde t_1^* Z^0$, $u_i d_j \to \tilde t_i^* W^+$ and $d_j d_k \to \tilde b_i^* W^-$
4150	$u_i d_j \to \tilde d_{k1}^* h^0/H^0/A^0$, $d_j d_k \to \tilde u_{i1}^* h^0/H^0/A^0$, $u_i d_j \to \tilde u_{k\alpha}^* H^+$, $d_j d_k \to \tilde d_{i\alpha}^* H^-$
4160	$u_i d_j \to u_l d_m$, $d_j d_k \to d_l d_m$ via UDD.
4200-99	Graviton resonance production
4200	Sum of 4210, 4250 and 4270
4210	$gg/q\bar q \to G \to gg/q\bar q$ (all partons)
4210+IQ	$gg/q\bar q \to G \to q\bar q$ (IQ as above)
4220	$gg/q\bar q \to G \to gg$
4250	$gg/q\bar q \to G \to \ell\bar\ell$ (all leptons)
4250+IL	$gg/q\bar q \to G \to \ell\bar\ell$ (IL $=1-6$ for $\ell=e,\nu_e,\mu,\nu_\mu$, etc.)
4260	$gg/q\bar q \to G \to \gamma\gamma$
4270	$gg/q\bar q \to G \to W^+W^-/Z^0Z^0/H^0_{SM}H^0_{SM}$
4271	$gg/q\bar q \to G \to W^+W^-$
4272	$gg/q\bar q \to G \to Z^0Z^0$
4273	$gg/q\bar q \to G \to H^0_{SM}H^0_{SM}$
5000	Pointlike photon-hadron jet production (all flavours)
5100+IQ	Pointlike photon heavy flavour pair production (IQ as above)
5200+IQ	Pointlike photon heavy flavour single excitation (IQ as above)
	After generation, IHPRO is subprocess (see sect. 4.6.5)
5300	Quark-photon Compton scattering
5500	Pointlike photon production of light (u,d,s) L=0 mesons
5510,20	S=0 mesons only, S=1 mesons only
	After generation, IHPRO is subprocess (see sect. 4.6.5)
6000	$\gamma\gamma \to q\bar q$ (all flavours)
6000+IQ	$\gamma\gamma \to q\bar q$ (IQ as above)
6006+IL	$\gamma\gamma \to \ell\bar\ell$ (IL $=1,2,3$ for $\ell=e,\mu,\tau$)
6010	$\gamma\gamma \to W^+W^-$
7000 –	Baryon-number violating and other multi-$W^\pm$ processes

IPROC	Process
7999	generated by HERBVI package
8000	Minimum bias soft hadron-hadron event
9000	Deep inelastic lepton scattering (all neutral current)
9000+IQ	Deep inelastic lepton scattering (NC on flavour IQ)
9010	Deep inelastic lepton scattering (all charged current)
9010+IQ	Deep inelastic lepton scattering (CC on flavour IQ)
9100	Boson-gluon fusion in neutral current DIS (all flavours)
9100+IQ	Boson-gluon fusion in neutral current DIS (IQ as above)
9107	J/ψ + gluon production by boson-gluon fusion
9110	QCD Compton process in neutral current DIS (all flavours)
9110+IP	QCD Compton process in NC DIS (IP=1–12 for $d-t,\bar d-\bar t$)
9130	All $\mathcal{O}(\alpha_S)$ NC processes (i.e. 9100+9110)
9140+IP	Heavy quark production by charged-current boson-gluon fusion
	IP: $1=s\bar c, 2=b\bar c, 3=s\bar t, 4=b\bar t$ (+ ch. conj.)
9500+ID	$W^+W^-/Z^0Z^0 \to H^0_{SM}$ in DIS (ID as in IPROC $=300+$ ID)
10000+IP	as IPROC = IP but with soft underlying event
	(soft remnant fragmentation in lepton-hadron) suppressed

Processes in PYTHIA

In	No.	Subprocess
+	1	$f_i\bar{f}_i \rightarrow \gamma^*/Z^0$
+	2	$f_i\bar{f}_j \rightarrow W^+$
+	3	$f_i\bar{f}_i \rightarrow h^0$
	4	$\gamma W^+ \rightarrow W^+$
+	5	$Z^0Z^0 \rightarrow h^0$
	6	$Z^0W^+ \rightarrow W^+$
	7	$W^+W^- \rightarrow Z^0$
+	8	$W^+W^- \rightarrow h^0$
+	10	$f_if_j \rightarrow f_kf_l$ (QFD)
+	11	$f_if_j \rightarrow f_if_j$ (QCD)
+	12	$f_i\bar{f}_i \rightarrow f_k\bar{f}_k$
+	13	$f_i\bar{f}_i \rightarrow gg$
+	14	$f_i\bar{f}_i \rightarrow g\gamma$
+	15	$f_i\bar{f}_i \rightarrow gZ^0$
+	16	$f_i\bar{f}_j \rightarrow gW^+$
	17	$f_i\bar{f}_i \rightarrow gh^0$
+	18	$f_i\bar{f}_i \rightarrow \gamma\gamma$
+	19	$f_i\bar{f}_i \rightarrow \gamma Z^0$
+	20	$f_i\bar{f}_j \rightarrow \gamma W^+$
	21	$f_i\bar{f}_i \rightarrow \gamma h^0$
+	22	$f_i\bar{f}_i \rightarrow Z^0Z^0$
+	23	$f_i\bar{f}_j \rightarrow Z^0W^+$
+	24	$f_i\bar{f}_i \rightarrow Z^0h^0$
+	25	$f_i\bar{f}_i \rightarrow W^+W^-$
+	26	$f_i\bar{f}_j \rightarrow W^+h^0$
	27	$f_i\bar{f}_i \rightarrow h^0h^0$
+	28	$f_ig \rightarrow f_ig$
+	29	$f_ig \rightarrow f_i\gamma$
+	30	$f_ig \rightarrow f_iZ^0$
+	31	$f_ig \rightarrow f_kW^+$
	32	$f_ig \rightarrow f_ih^0$
+	33	$f_i\gamma \rightarrow f_ig$
+	34	$f_i\gamma \rightarrow f_i\gamma$
+	35	$f_i\gamma \rightarrow f_iZ^0$
+	36	$f_i\gamma \rightarrow f_kW^+$
	37	$f_i\gamma \rightarrow f_ih^0$
	38	$f_iZ^0 \rightarrow f_ig$
	39	$f_iZ^0 \rightarrow f_i\gamma$

In	No.	Subprocess
	40	$f_iZ^0 \rightarrow f_iZ^0$
	41	$f_iZ^0 \rightarrow f_kW^+$
	42	$f_iZ^0 \rightarrow f_ih^0$
	43	$f_iW^+ \rightarrow f_kg$
	44	$f_iW^+ \rightarrow f_k\gamma$
	45	$f_iW^+ \rightarrow f_kZ^0$
	46	$f_iW^+ \rightarrow f_kW^+$
	47	$f_iW^+ \rightarrow f_kh^0$
	48	$f_ih^0 \rightarrow f_ig$
	49	$f_ih^0 \rightarrow f_i\gamma$
	50	$f_ih^0 \rightarrow f_iZ^0$
	51	$f_ih^0 \rightarrow f_kW^+$
	52	$f_ih^0 \rightarrow f_ih^0$
+	53	$gg \rightarrow f_k\bar{f}_k$
+	54	$g\gamma \rightarrow f_k\bar{f}_k$
	55	$gZ^0 \rightarrow f_k\bar{f}_k$
	56	$gW^+ \rightarrow f_k\bar{f}_l$
	57	$gh^0 \rightarrow f_k\bar{f}_k$
+	58	$\gamma\gamma \rightarrow f_k\bar{f}_k$
	59	$\gamma Z^0 \rightarrow f_k\bar{f}_k$
	60	$\gamma W^+ \rightarrow f_k\bar{f}_l$
	61	$\gamma h^0 \rightarrow f_k\bar{f}_k$
	62	$Z^0Z^0 \rightarrow f_k\bar{f}_k$
	63	$Z^0W^+ \rightarrow f_k\bar{f}_l$
	64	$Z^0h^0 \rightarrow f_k\bar{f}_k$
	65	$W^+W^- \rightarrow f_k\bar{f}_k$
	66	$W^+h^0 \rightarrow f_k\bar{f}_l$
	67	$h^0h^0 \rightarrow f_k\bar{f}_k$
+	68	$gg \rightarrow gg$
+	69	$\gamma\gamma \rightarrow W^+W^-$
+	70	$\gamma W^+ \rightarrow Z^0W^+$
+	71	$Z^0Z^0 \rightarrow Z^0Z^0$ (longitudinal)
+	72	$Z^0Z^0 \rightarrow W^+W^-$ (longitudinal)
+	73	$Z^0W^+ \rightarrow Z^0W^+$ (longitudinal)
	74	$Z^0h^0 \rightarrow Z^0h^0$
	75	$W^+W^- \rightarrow \gamma\gamma$
+	76	$W^+W^- \rightarrow Z^0Z^0$ (longitudinal)
+	77	$W^+W^\pm \rightarrow W^+W^\pm$ (longitudinal)
	78	$W^+h^0 \rightarrow W^+h^0$
	79	$h^0h^0 \rightarrow h^0h^0$
+	80	$q_i\gamma \rightarrow q_k\pi^\pm$

In	No.	Subprocess
+	81	$f_i\bar{f}_i \rightarrow Q_k\bar{Q}_k$
+	82	$gg \rightarrow Q_k\bar{Q}_k$
+	83	$q_if_j \rightarrow Q_kf_l$
+	84	$g\gamma \rightarrow Q_k\bar{Q}_k$
+	85	$\gamma\gamma \rightarrow F_k\bar{F}_k$
+	86	$gg \rightarrow J/\psi g$
+	87	$gg \rightarrow \chi_{0c}g$
+	88	$gg \rightarrow \chi_{1c}g$
+	89	$gg \rightarrow \chi_{2c}g$
+	91	elastic scattering
+	92	single diffraction ($AB \rightarrow XB$)
+	93	single diffraction ($AB \rightarrow AX$)
+	94	double diffraction
+	95	low-$p_\perp$ production
+	96	semihard QCD $2 \rightarrow 2$
+	99	$\gamma^*q \rightarrow q$
	101	$gg \rightarrow Z^0$
+	102	$gg \rightarrow h^0$
+	103	$\gamma\gamma \rightarrow h^0$
+	104	$gg \rightarrow \chi_{0c}$
+	105	$gg \rightarrow \chi_{2c}$
+	106	$gg \rightarrow J/\psi\gamma$
+	107	$g\gamma \rightarrow J/\psi g$
+	108	$\gamma\gamma \rightarrow J/\psi\gamma$
+	110	$f_i\bar{f}_i \rightarrow \gamma h^0$
+	111	$f_i\bar{f}_i \rightarrow gh^0$
+	112	$f_ig \rightarrow f_ih^0$
+	113	$gg \rightarrow gh^0$
+	114	$gg \rightarrow \gamma\gamma$
+	115	$gg \rightarrow g\gamma$
	116	$gg \rightarrow \gamma Z^0$
	117	$gg \rightarrow Z^0Z^0$
	118	$gg \rightarrow W^+W^-$
	119	$\gamma\gamma \rightarrow gg$
+	121	$gg \rightarrow Q_k\bar{Q}_kh^0$
+	122	$q_i\bar{q}_i \rightarrow Q_k\bar{Q}_kh^0$
+	123	$f_if_j \rightarrow f_if_jh^0$ (ZZ fusion)
+	124	$f_if_j \rightarrow f_kf_lh^0$ (W^+W^- fusion)
+	131	$f_i\gamma^*_T \rightarrow f_ig$
+	132	$f_i\gamma^*_L \rightarrow f_ig$
+	133	$f_i\gamma^*_T \rightarrow f_i\gamma$

In	No.	Subprocess
+	134	$f_i\gamma^*_L \rightarrow f_i\gamma$
+	135	$g\gamma^*_T \rightarrow f_i\bar{f}_i$
+	136	$g\gamma^*_L \rightarrow f_i\bar{f}_i$
+	137	$\gamma^*_T\gamma^*_T \rightarrow f_i\bar{f}_i$
+	138	$\gamma^*_T\gamma^*_L \rightarrow f_i\bar{f}_i$
+	139	$\gamma^*_L\gamma^*_T \rightarrow f_i\bar{f}_i$
+	140	$\gamma^*_L\gamma^*_L \rightarrow f_i\bar{f}_i$
+	141	$f_i\bar{f}_i \rightarrow \gamma/Z^0/Z'^0$
+	142	$f_i\bar{f}_j \rightarrow W'^+$
+	143	$f_i\bar{f}_j \rightarrow H^+$
+	144	$f_i\bar{f}_j \rightarrow R$
+	145	$q_i\ell_j \rightarrow L_Q$
+	146	$e\gamma \rightarrow e^*$
+	147	$dg \rightarrow d^*$
+	148	$ug \rightarrow u^*$
+	149	$gg \rightarrow \eta_{tc}$
+	151	$f_i\bar{f}_i \rightarrow H^0$
+	152	$gg \rightarrow H^0$
+	153	$\gamma\gamma \rightarrow H^0$
+	156	$f_i\bar{f}_i \rightarrow A^0$
+	157	$gg \rightarrow A^0$
+	158	$\gamma\gamma \rightarrow A^0$
+	161	$f_ig \rightarrow f_kH^+$
+	162	$q_ig \rightarrow \ell_kL_Q$
+	163	$gg \rightarrow L_Q\bar{L}_Q$
+	164	$q_i\bar{q}_i \rightarrow L_Q\bar{L}_Q$
+	165	$f_i\bar{f}_i \rightarrow f_k\bar{f}_k$ (via γ^*/Z^0)
+	166	$f_i\bar{f}_j \rightarrow f_k\bar{f}_l$ (via $W^\pm$)
+	167	$q_iq_j \rightarrow q_kd^*$
+	168	$q_iq_j \rightarrow q_ku^*$
+	169	$q_i\bar{q}_i \rightarrow e^\pm e^{*\mp}$
+	171	$f_i\bar{f}_i \rightarrow Z^0H^0$
+	172	$f_i\bar{f}_j \rightarrow W^+H^0$
+	173	$f_if_j \rightarrow f_if_jH^0$ (ZZ fusion)
+	174	$f_if_j \rightarrow f_kf_lH^0$ (W^+W^- fusion)
+	176	$f_i\bar{f}_i \rightarrow Z^0A^0$
+	177	$f_i\bar{f}_j \rightarrow W^+A^0$
+	178	$f_if_j \rightarrow f_if_jA^0$ (ZZ fusion)
+	179	$f_if_j \rightarrow f_kf_lA^0$ (W^+W^- fusion)
+	181	$gg \rightarrow Q_k\bar{Q}_kH^0$
+	182	$q_i\bar{q}_i \rightarrow Q_k\bar{Q}_kH^0$

In	No.	Subprocess
+	183	$f_i\bar{f}_i \rightarrow gH^0$
+	184	$f_ig \rightarrow f_iH^0$
+	185	$gg \rightarrow gH^0$
+	186	$gg \rightarrow Q_k\bar{Q}_kA^0$
+	187	$q_i\bar{q}_i \rightarrow Q_k\bar{Q}_kA^0$
+	188	$f_i\bar{f}_i \rightarrow gA^0$
+	189	$f_ig \rightarrow f_iA^0$
+	190	$gg \rightarrow gA^0$
+	191	$f_i\bar{f}_i \rightarrow \rho^0_{tc}$
+	192	$f_i\bar{f}_j \rightarrow \rho^\pm_{tc}$
+	193	$f_i\bar{f}_i \rightarrow \omega^0_{tc}$
+	194	$f_i\bar{f}_i \rightarrow f_k\bar{f}_k$
+	195	$f_i\bar{f}_j \rightarrow f_k\bar{f}_l$
+	201	$f_i\bar{f}_i \rightarrow \tilde{e}_L\tilde{e}^*_L$
+	202	$f_i\bar{f}_i \rightarrow \tilde{e}_R\tilde{e}^*_R$
+	203	$f_i\bar{f}_i \rightarrow \tilde{e}_L\tilde{e}^*_R + \tilde{e}^*_L\tilde{e}_R$
+	204	$f_i\bar{f}_i \rightarrow \tilde{\mu}_L\tilde{\mu}^*_L$
+	205	$f_i\bar{f}_i \rightarrow \tilde{\mu}_R\tilde{\mu}^*_R$
+	206	$f_i\bar{f}_i \rightarrow \tilde{\mu}_L\tilde{\mu}^*_R + \tilde{\mu}^*_L\tilde{\mu}_R$
+	207	$f_i\bar{f}_i \rightarrow \tilde{\tau}_1\tilde{\tau}^*_1$
+	208	$f_i\bar{f}_i \rightarrow \tilde{\tau}_2\tilde{\tau}^*_2$
+	209	$f_i\bar{f}_i \rightarrow \tilde{\tau}_1\tilde{\tau}^*_2 + \tilde{\tau}^*_1\tilde{\tau}_2$
+	210	$f_i\bar{f}_j \rightarrow \tilde{\ell}_L\tilde{\nu}^*_\ell + \tilde{\ell}^*_L\tilde{\nu}_\ell$
+	211	$f_i\bar{f}_j \rightarrow \tilde{\tau}_1\tilde{\nu}^*_\tau + \tilde{\tau}^*_1\tilde{\nu}_\tau$
+	212	$f_i\bar{f}_j \rightarrow \tilde{\tau}_2\tilde{\nu}_\tau{}^* + \tilde{\tau}^*_2\tilde{\nu}_\tau$
+	213	$f_i\bar{f}_i \rightarrow \tilde{\nu}_\ell\tilde{\nu}^*_\ell$
+	214	$f_i\bar{f}_i \rightarrow \tilde{\nu}_\tau\tilde{\nu}^*_\tau$
+	216	$f_i\bar{f}_i \rightarrow \tilde{\chi}_1\tilde{\chi}_1$
+	217	$f_i\bar{f}_i \rightarrow \tilde{\chi}_2\tilde{\chi}_2$
+	218	$f_i\bar{f}_i \rightarrow \tilde{\chi}_3\tilde{\chi}_3$
+	219	$f_i\bar{f}_i \rightarrow \tilde{\chi}_4\tilde{\chi}_4$
+	220	$f_i\bar{f}_i \rightarrow \tilde{\chi}_1\tilde{\chi}_2$
+	221	$f_i\bar{f}_i \rightarrow \tilde{\chi}_1\tilde{\chi}_3$
+	222	$f_i\bar{f}_i \rightarrow \tilde{\chi}_1\tilde{\chi}_4$
+	223	$f_i\bar{f}_i \rightarrow \tilde{\chi}_2\tilde{\chi}_3$
+	224	$f_i\bar{f}_i \rightarrow \tilde{\chi}_2\tilde{\chi}_4$
+	225	$f_i\bar{f}_i \rightarrow \tilde{\chi}_3\tilde{\chi}_4$
+	226	$f_i\bar{f}_i \rightarrow \tilde{\chi}^\pm_1\tilde{\chi}^\mp_1$
+	227	$f_i\bar{f}_i \rightarrow \tilde{\chi}^\pm_2\tilde{\chi}^\mp_2$
+	228	$f_i\bar{f}_i \rightarrow \tilde{\chi}^\pm_1\tilde{\chi}^\mp_2$

In	No.	Subprocess
+	229	$f_i\bar{f}_j \rightarrow \tilde{\chi}_1\tilde{\chi}^\pm_1$
+	230	$f_i\bar{f}_j \rightarrow \tilde{\chi}_2\tilde{\chi}^\pm_1$
+	231	$f_i\bar{f}_j \rightarrow \tilde{\chi}_3\tilde{\chi}^\pm_1$
+	232	$f_i\bar{f}_j \rightarrow \tilde{\chi}_4\tilde{\chi}^\pm_1$
+	233	$f_i\bar{f}_j \rightarrow \tilde{\chi}_1\tilde{\chi}^\pm_2$
+	234	$f_i\bar{f}_j \rightarrow \tilde{\chi}_2\tilde{\chi}^\pm_2$
+	235	$f_i\bar{f}_j \rightarrow \tilde{\chi}_3\tilde{\chi}^\pm_2$
+	236	$f_i\bar{f}_j \rightarrow \tilde{\chi}_4\tilde{\chi}^\pm_2$
+	237	$f_i\bar{f}_i \rightarrow \tilde{g}\tilde{\chi}_1$
+	238	$f_i\bar{f}_i \rightarrow \tilde{g}\tilde{\chi}_2$
+	239	$f_i\bar{f}_i \rightarrow \tilde{g}\tilde{\chi}_3$
+	240	$f_i\bar{f}_i \rightarrow \tilde{g}\tilde{\chi}_4$
+	241	$f_i\bar{f}_j \rightarrow \tilde{g}\tilde{\chi}^\pm_1$
+	242	$f_i\bar{f}_j \rightarrow \tilde{g}\tilde{\chi}^\pm_2$
+	243	$f_i\bar{f}_i \rightarrow \tilde{g}\tilde{g}$
+	244	$gg \rightarrow \tilde{g}\tilde{g}$
+	246	$f_ig \rightarrow \tilde{q}_{iL}\tilde{\chi}_1$
+	247	$f_ig \rightarrow \tilde{q}_{iR}\tilde{\chi}_1$
+	248	$f_ig \rightarrow \tilde{q}_{iL}\tilde{\chi}_2$
+	249	$f_ig \rightarrow \tilde{q}_{iR}\tilde{\chi}_2$
+	250	$f_ig \rightarrow \tilde{q}_{iL}\tilde{\chi}_3$
+	251	$f_ig \rightarrow \tilde{q}_{iR}\tilde{\chi}_3$
+	252	$f_ig \rightarrow \tilde{q}_{iL}\tilde{\chi}_4$
+	253	$f_ig \rightarrow \tilde{q}_{iR}\tilde{\chi}_4$
+	254	$f_ig \rightarrow \tilde{q}_{jL}\tilde{\chi}^\pm_1$
+	256	$f_ig \rightarrow \tilde{q}_{jL}\tilde{\chi}^\pm_2$
+	258	$f_ig \rightarrow \tilde{q}_{iL}\tilde{g}$
+	259	$f_ig \rightarrow \tilde{q}_{iR}\tilde{g}$
+	261	$f_i\bar{f}_i \rightarrow \tilde{t}_1\tilde{t}^*_1$
+	262	$f_i\bar{f}_i \rightarrow \tilde{t}_2\tilde{t}^*_2$
+	263	$f_i\bar{f}_i \rightarrow \tilde{t}_1\tilde{t}^*_2 + \tilde{t}^*_1\tilde{t}_2$
+	264	$gg \rightarrow \tilde{t}_1\tilde{t}^*_1$
+	265	$gg \rightarrow \tilde{t}_2\tilde{t}^*_2$
+	271	$f_if_j \rightarrow \tilde{q}_{iL}\tilde{q}_{jL}$
+	272	$f_if_j \rightarrow \tilde{q}_{iR}\tilde{q}_{jR}$
+	273	$f_if_j \rightarrow \tilde{q}_{iL}\tilde{q}_{jR} + \tilde{q}_{iR}\tilde{q}_{jL}$
+	274	$f_i\bar{f}_j \rightarrow \tilde{q}_{iL}\tilde{q}^*_{jL}$
+	275	$f_i\bar{f}_j \rightarrow \tilde{q}_{iR}\tilde{q}^*_{jR}$
+	276	$f_i\bar{f}_j \rightarrow \tilde{q}_{iL}\tilde{q}^*_{jR} + \tilde{q}_{iR}\tilde{q}^*_{jL}$
+	277	$f_i\bar{f}_i \rightarrow \tilde{q}_{jL}\tilde{q}^*_{jL}$

In	No.	Subprocess
+	278	$f_i\bar{f}_i \rightarrow \tilde{q}_{jR}\tilde{q}^*_{jR}$
+	279	$gg \rightarrow \tilde{q}_{iL}\tilde{q}^*_{iL}$
+	280	$gg \rightarrow \tilde{q}_{iR}\tilde{q}^*_{iR}$
+	281	$bq \rightarrow \tilde{b}_1\tilde{q}_L$ (q not b)
+	282	$bq \rightarrow \tilde{b}_2\tilde{q}_R$
+	283	$bq \rightarrow \tilde{b}_1\tilde{q}_R + \tilde{b}2\tilde{q}_L$
+	284	$b\bar{q} \rightarrow \tilde{b}_1\tilde{q}^*_L$
+	285	$b\bar{q} \rightarrow \tilde{b}_2\tilde{q}^*_R$
+	286	$b\bar{q} \rightarrow \tilde{b}_1\tilde{q}^*_R + \tilde{b}_2\tilde{q}^*_L$
+	287	$q\bar{q} \rightarrow \tilde{b}_1\tilde{b}^*_1$
+	288	$q\bar{q} \rightarrow \tilde{b}_2\tilde{b}^*_2$
+	289	$gg \rightarrow \tilde{b}_1\tilde{b}^*_1$
+	290	$gg \rightarrow \tilde{b}_2\tilde{b}^*_2$
+	291	$bb \rightarrow \tilde{b}_1\tilde{b}_1$
+	292	$bb \rightarrow \tilde{b}_2\tilde{b}_2$
+	293	$bb \rightarrow \tilde{b}_1\tilde{b}_2$
+	294	$bg \rightarrow \tilde{b}_1\tilde{g}$
+	295	$bg \rightarrow \tilde{b}_2\tilde{g}$
+	296	$b\bar{b} \rightarrow \tilde{b}_1\tilde{b}^*_2 + \tilde{b}^*_1\tilde{b}_2$
+	297	$f_i\bar{f}_j \rightarrow H^\pm h^0$
+	298	$f_i\bar{f}_j \rightarrow H^\pm H^0$
+	299	$f_i\bar{f}_i \rightarrow Ah^0$
+	300	$f_i\bar{f}_i \rightarrow AH^0$
+	301	$f_i\bar{f}_i \rightarrow H^+H^-$
+	341	$\ell_i\ell_j \rightarrow H^{\pm\pm}_L$
+	342	$\ell_i\ell_j \rightarrow H^{\pm\pm}_R$
+	343	$\ell_i\gamma \rightarrow H^{\pm\pm}_L e^\mp$
+	344	$\ell_i\gamma \rightarrow H^{\pm\pm}_R e^\mp$
+	345	$\ell_i\gamma \rightarrow H^{\pm\pm}_L \mu^\mp$
+	346	$\ell_i\gamma \rightarrow H^{\pm\pm}_R \mu^\mp$
+	347	$\ell_i\gamma \rightarrow H^{\pm\pm}_L \tau^\mp$
+	348	$\ell_i\gamma \rightarrow H^{\pm\pm}_R \tau^\mp$
+	349	$f_i\bar{f}_i \rightarrow H^{++}_L H^{--}_L$
+	350	$f_i\bar{f}_i \rightarrow H^{++}_R H^{--}_R$
+	351	$f_if_j \rightarrow f_kf_lH^{\pm\pm}_L$ (WW) fusion)
+	352	$f_if_j \rightarrow f_kf_lH^{\pm\pm}_R$ (WW) fusion)
+	353	$f_i\bar{f}_i \rightarrow Z^0_R$
+	354	$f_i\bar{f}_j \rightarrow W^+_R$
+	361	$f_i\bar{f}_i \rightarrow W^+_L W^-_L$
+	362	$f_i\bar{f}_i \rightarrow W^\pm_L \pi^\mp_{tc}$

In	No.	Subprocess
+	363	$f_i\bar{f}_i \rightarrow \pi^+_{tc}\pi^-_{tc}$
+	364	$f_i\bar{f}_i \rightarrow \gamma\pi^0_{tc}$
+	365	$f_i\bar{f}_i \rightarrow \gamma\pi'^0_{tc}$
+	366	$f_i\bar{f}_i \rightarrow Z^0\pi^0_{tc}$
+	367	$f_i\bar{f}_i \rightarrow Z^0\pi'^0_{tc}$
+	368	$f_i\bar{f}_i \rightarrow W^\pm\pi^\mp_{tc}$
+	370	$f_i\bar{f}_j \rightarrow W^\pm_L Z^0_L$
+	371	$f_i\bar{f}_j \rightarrow W^\pm_L \pi^0_{tc}$
+	372	$f_i\bar{f}_j \rightarrow \pi^\pm_{tc} Z^0_L$
+	373	$f_i\bar{f}_j \rightarrow \pi^\pm_{tc}\pi^0_{tc}$
+	374	$f_i\bar{f}_j \rightarrow \gamma\pi^\pm_{tc}$
+	375	$f_i\bar{f}_j \rightarrow Z^0\pi^\pm_{tc}$
+	376	$f_i\bar{f}_j \rightarrow W^\pm\pi^0_{tc}$
+	377	$f_i\bar{f}_j \rightarrow W^\pm\pi'^0_{tc}$
+	391	$f\bar{f} \rightarrow G^*$
+	392	$gg \rightarrow G^*$
+	393	$q\bar{q} \rightarrow gG^*$
+	394	$qg \rightarrow qG^*$
+	395	$gg \rightarrow gG^*$

Matrix Element Calculations

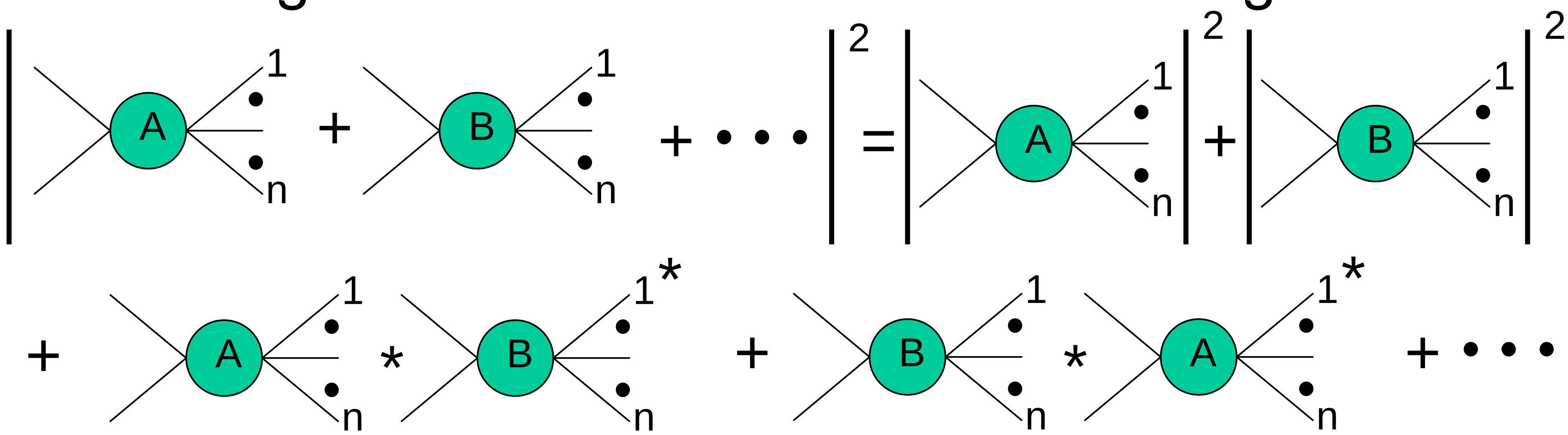
- There are two parts to calculating the cross section.
  1) Calculating the matrix element for a given point in phase space.
  2) Integrating it over the phase space with whatever cuts are required.
- Often the matrix elements have peaks and singularities and integrating them is hard particularly for the sort of high multiplicity final states we are interested in for the Tevatron and LHC.

Matrix Element Calculations

There are a number of ways of calculating the matrix element

1) Trace Techniques

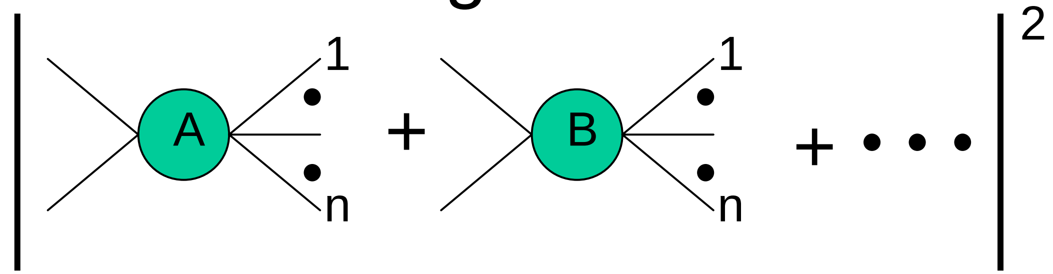
– We all learnt this as students, but spend goes like N^2 with the number of diagrams



Matrix Element Calculations

2) Helicity Amplitudes

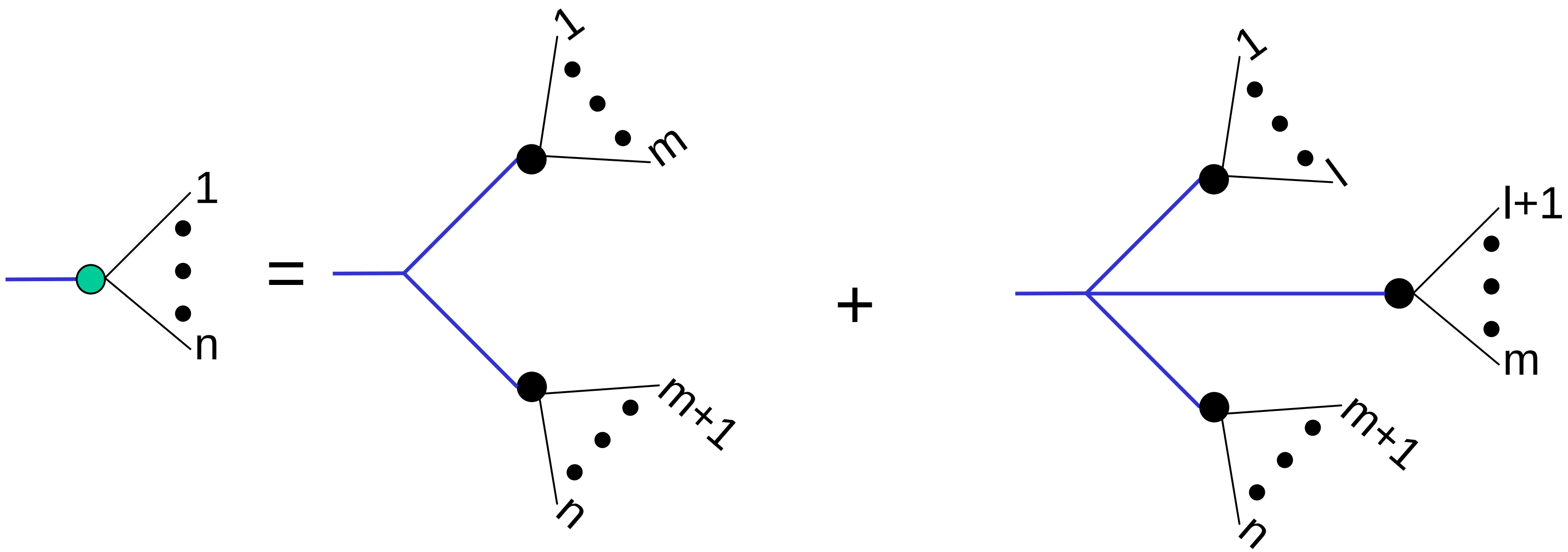
- Work out the matrix element as a complex number, sum up the diagrams and then square.
- Numerically, grows like N with the number of diagrams.



Matrix Element Calculations

3) Off-Shell recursion relations (Berends-Giele, ALPHA, Schwinger-Dyson.)
   - Don’t draw diagrams at all use a recursion relation to get final states with more particles from those with fewer particles.

Matrix Element Calculations

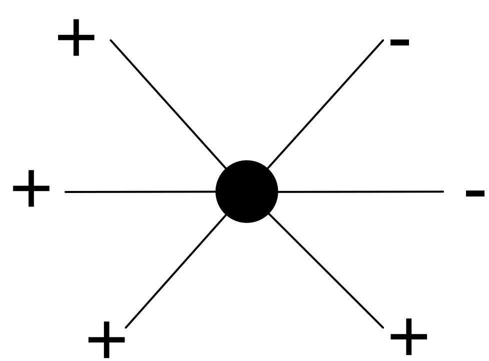


- Blue lines are off-shell gluons, other lines are on-shell.
- Full amplitudes are built up from simpler amplitudes with fewer particles.
- This is a recursion built from off-shell currents.

MHV/BCFW Recursion

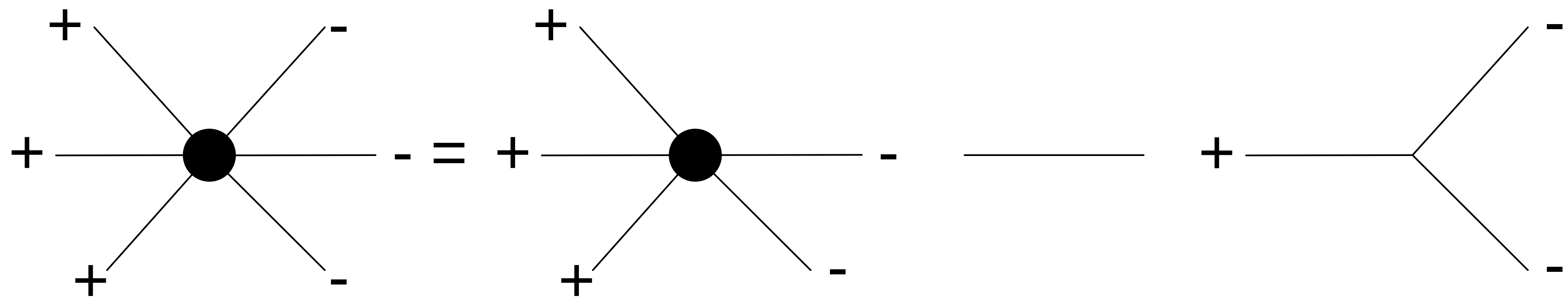
4) All the new techniques are essential on-shell recursion relations.

- The first results Cachazo, Svrcek and Witten were for the combination of maximum helicity violating (MHV) amplitudes.



- Has two negative helicity gluons (all others positive)

MHV Recursion



- Here all the particles on-shell, with rules for combining them.

BCFW Recursion

- Initially only for gluons.
- Many developments since then culminating (for the moment) in the BCFW recursion relations.
- This has been generalised to massive particles.

Integration

- All of this is wonderful for evaluating the matrix element at one phase space point.
- However it has to be integrated.
- The problem is that the matrix element has a lot of peaks.
- Makes numerical integration tricky.

Integration

- As we have seen there are essentially two techniques.

1) Adaptive integration VEGAS and variants
   - Automatically smooth out peaks to reduce error
   - Often not good if peaks not in one of the integration variables
2) Multi-Channel integration
   - Analytically smooth out the peaks in many different channels
   - Automatically optimise the different channels.

Integration

- In practice the best programs use a combination of the two.
  - Multichannel to get rid of the worst behaviour.
  - Then adaptive to make it more efficient.

Programs

	Trace	Helicity Amplitude	Off-shell recursion	On-Shell Recursion
Adaptive	COMPHEP CALCHEP Slowest	None	ALPGEN Faster	None
Multi Channel	None	MADEVENT SHERPA Fastest	None	None

Faster

Faster

Integration

- The integration of the matrix elements is the problem not the calculation of the matrix element.
- The best programs have the best integration NOT the best calculation of the matrix element.
- In Monte Carlo event generators most of the work is always involved with the phase space.

Summary

- In this morning's lecture we have looked at
  - The Monte Carlo integration technique
  - Basics of phase space integration
  - The different parts of the Monte Carlo event generator.
  - Calculation of the hard matrix element
- This afternoon we will go on and consider the simulation of perturbative QCD in Monte Carlo simulations.