Resonant $t\bar{t}$ Production at the LHC

Andrew Papanastasiou*

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*In collaboration with Adrian Signer (IPPP), Paul Mellor (IPPP) and Pietro Falgari (Utrecht)

Outline

Why $t\bar{t}$? / Motivations

LHC: 'top factory'

Top events are significant backgrounds to higgs/new physics events e.g.

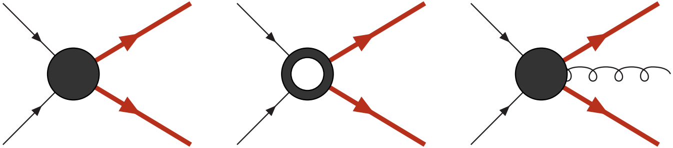
$$H \to W^+W^- \text{ or } H \to \tau^+\tau^-$$

Top mass, M_t: a constraint on SM Higgs mass

TeVatron Forward-Backward asymmetry

Theoretically interesting quarks: decay before hadronizing
$\rightarrow$ for best description should treat as (massive &) unstable.

Stable Top Production, $p_t^2 - M_t^2 = 0$



- top spins summed over

Pro's / Con's

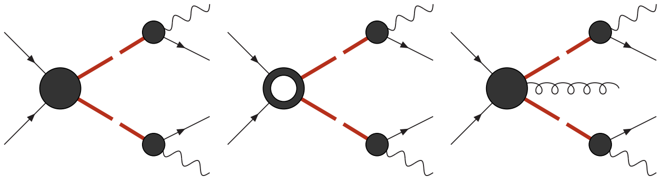
✓ decent approximation for inclusive observables

✗ cuts on final states not possible

✗ no off-shell effects

[P. Nason, S. Dawson, R. K. Ellis '89][W. Beenakker et. al. '91]

On-Shell Top Production and Decay, $p_t^2 - M_t^2 = 0$



- decay of tops included via (improved) Narrrow Width Approximation

$\rightarrow$ Non-Factorizable corrections not included.

Ok, for inclusive observables (σ etc) as effects of non-factorizable corrections are small for these [V. S. Fadin, V. A. Khoze, A. D. Martin '94]

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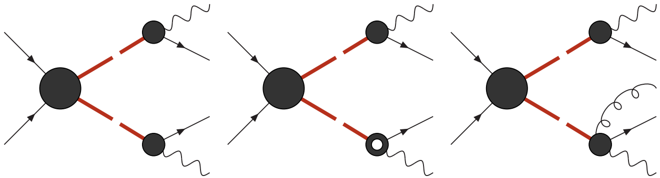

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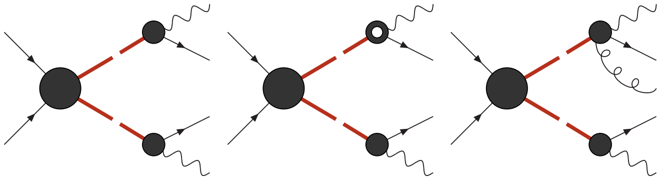

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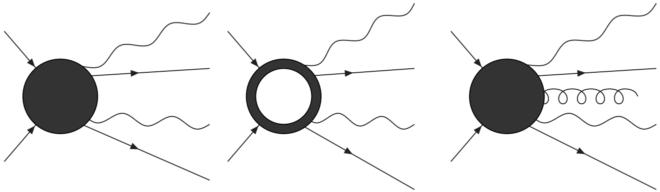


- non-factorizable corrections included
- all background diagrams included

Pro's / Con's

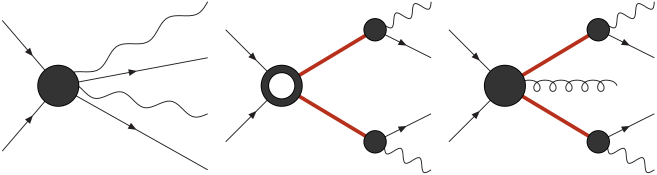
✓ off-shell effects

✓ spin-correlations and cuts on final states

✗ very complicated/difficult calculation

[A. Denner et. al. '10][Bevilacqua at. al. '10]

Resonant Top Production and Decay $p_t^2 - M_t^2 << M_t^2$



- non-factorizable corrections included
- (relevant) background diagrams included

Pro's / Con's

- ✓ off-shell effects included
- ✓ spin-correlations and cuts on final states
- ✓ simpler calculation
- ✗ not valid outside resonant region $p_t^2 \sim M_t^2$

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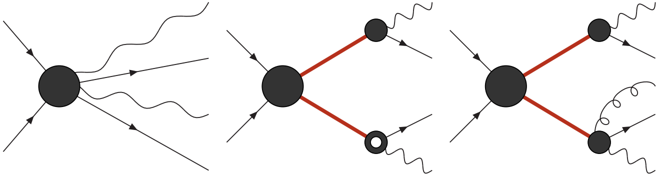

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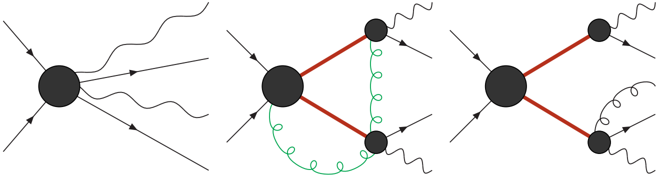

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Calculation Simplified

Look at di-lepton channel

$$P(q,g)\,P(\bar{q},g) \to W^+\, b\, W^-\, \bar{b} \to l^+\, \nu_l\, b\, l^-\, \bar{\nu}_l\, \bar{b}$$

(decay of W $\to$ leptons included via (improved narrow width approximation))

Conditions: $p_t^2 = (p_{W^+} + p_b)^2 \sim M_t^2$ and $p_{\bar{t}}^2 = (p_{W^-} + p_{\bar{b}})^2 \sim M_t^2$

Exploit widely separated scales: $p_t^2 - M_t^2 \sim p_{\bar{t}}^2 - M_t^2 \sim M_t\Gamma_t << M_t^2$

$\to$ Effective Theory inspired approach

[A.P. Chapovsky, V.A. Khoze, A. Signer, W.J. Stirling '01]

[M. Beneke, A.P. Chapovsky, A. Signer, G. Zanderighi '04]

[P. Falgari, P. Mellor, A. Signer '10], [P. Falgari, F. Giannuzzi, P. Mellor, A. Signer '11]

$\to$ Expand full amplitudes in the small parameters α_s and $\frac{(p_W+p_b)^2 - M_t^2}{M_t^2}$

Tree-Level Expansion

Full tree-level amplitude takes the form:

$$\mathcal{A}^{\text{tree}} = \frac{\mathcal{K}^D(p_i)}{(p_t^2 - M_t^2)(p_{\bar{t}}^2 - M_t^2)} + \frac{\mathcal{K}^{S,t}(p_i)}{(p_t^2 - M_t^2)} + \frac{\mathcal{K}^{S,\bar{t}}(p_i)}{(p_{\bar{t}}^2 - M_t^2)} + \mathcal{J}(p_i).$$

(Double) Pole Expansion: [A. Aeppli, G. J. v Oldenborgh, D. Wyler '94]

$$\mathcal{A}^{\text{tree}} = \frac{\mathcal{K}^D(p_i, p_t^2 = p_{\bar{t}}^2 = \mu_t^2)}{\Delta_t \Delta_{\bar{t}}}(1 + \delta\mathcal{R}_D)$$

$$+ \frac{1}{\Delta_t}\left[\mathcal{K}^{S,t}(p_i, p_t^2 = p_{\bar{t}}^2 = \mu_t^2)(1 + \delta\mathcal{R}_{S,t}) + \frac{\partial \mathcal{K}^D}{\partial p_{\bar{t}}^2}(p_i, p_t^2 = p_{\bar{t}}^2 = \mu_t^2)\right] + \frac{1}{\Delta_{\bar{t}}}[t \leftrightarrow \bar{t}]$$

$$+ \ldots\ldots$$

with $\Delta_t = p_t^2 - \mu_t^2, \quad \Delta_{\bar{t}} = p_{\bar{t}}^2 - \mu_t^2, \quad \mu_t^2 = M_t^2 - iM_t\Gamma_t$

$(1 + \delta\mathcal{R}_D)$, $(1 + \delta\mathcal{R}_{S,t})$ are the residues of the double / single poles at $p_t^2 = \mu_t^2$.

Power Counting

Want to combine expansion in Δ_t and $\Delta_{\bar{t}}$ with expansion in α_s and α_{ew}.

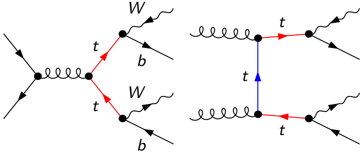
Introduce scalings: $\delta \sim \alpha_s^2 \sim \alpha_{ew} \sim \frac{\Delta_t}{M_t^2} \sim \frac{\Delta_{\bar{t}}}{M_t^2}$

$\rightarrow$ assign a power of δ to each diagram

(Note: this will correspond to the leading scaling in δ for each diagram.)

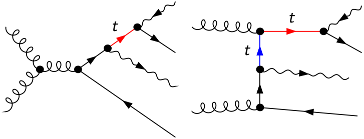
Power Counting

Double Resonant



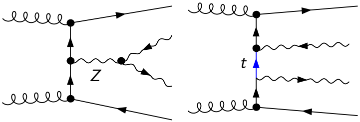
$$\sim \alpha_s \alpha_{ew} \frac{1}{\Delta_t \Delta_{\bar{t}}} \sim \frac{\delta^{\frac{1}{2}} \delta}{\delta^2} \sim \delta^{-\frac{1}{2}}$$

Single Resonant



$$\sim \alpha_s \alpha_{ew} \frac{1}{\Delta_t} \sim \frac{\delta^{\frac{1}{2}} \delta}{\delta} \sim \delta^{\frac{1}{2}}$$

Non-Resonant



$$\sim \alpha_s \alpha_{ew} \sim \delta^{\frac{1}{2}} \delta \sim \delta^{\frac{3}{2}}$$

Write tree-level amplitude (gg-inital state)

$$\mathcal{A}^{\text{tree}} = \alpha_{ew}\alpha_s A_{-2}^{(1,1)} + \alpha_{ew}\alpha_s A_{-1}^{(1,1)} + \alpha_{ew}\alpha_s A_{0}^{(1,1)}$$

$$\sim \delta^{-\frac{1}{2}} \qquad \sim \delta^{\frac{1}{2}} \qquad \sim \delta^{\frac{3}{2}}$$

Squaring:

$$\mathcal{M}^{\text{tree}} = \alpha_{ew}^2\alpha_s^2 \left(|A_{-2}^{(1,1)}|^2 + 2\text{Re}\left(A_{-2}^{(1,1)} A_{-1}^{(1,1)*}\right) + |A_{-1}^{(1,1)}|^2 + 2\text{Re}\left(A_{-1}^{(1,1)} A_{0}^{(1,1)*}\right) + \ldots \right)$$

$$\sim \delta^{-1} \qquad \sim \delta^{0} \qquad \sim \delta^{1} \qquad \sim \delta^{1} \qquad \sim \delta^{>1}$$

Define: 'LO' $\sim \delta^{-1}$

Going beyond 'LO' consistently requires us to include loop and real emision diagrams.

Aim: compute Matrix Element to $\mathcal{O}(\delta^{-\frac{1}{2}})$ in counting

$\rightarrow \mathcal{O}(\alpha_s)$-corrections to double resonant contributions

$$\mathcal{M}^{\text{NLO}} \sim \alpha_{ew}^2\alpha_s^3 \, 2\text{Re}\left(A_{-2}^{(1,1)} A_{-2}^{(2,1)*}\right) \sim \delta^{-\frac{1}{2}}$$

Define: 'NLO' $\sim \delta^{-\frac{1}{2}}$

Important: Expansion of Amplitude / Matrix Element is strictly gauge invariant at every order in δ.

Loop diagrams: Method of Regions

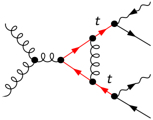
Wish to compute the contributions from virtual diagrams which scale as $\sim \delta^0$.

Technically this is achieved by computing the intergrals via the Method of Regions [M. Beneke, V. A. Smirnov '98]

This involves splitting the integrals into 'hard' and 'soft' parts:

- expand the integrands in the 2 cases of the loop momentum scaling as $q^\mu \sim 1$ and $q^\mu \sim \delta << 1$
- only keep contributions which scale as δ^0

Method of Regions: Hard-Soft separation: Example



$$\sim \alpha_s^2 \alpha_{ew} \langle p_b - | \ldots \frac{1}{\Delta_t} \int d^4 q \; \frac{1}{q^2} \; \frac{\not{p}_t + \not{q} + M_t}{(p_t+q)^2 - M_t^2} \; \gamma^\nu \; \frac{\not{p}_{\bar{t}} - \not{q} - M_t}{(p_{\bar{t}}+q)^2 - M_t^2} \; \frac{1}{\Delta_{\bar{t}}} \ldots | p_{\bar{b}} - \rangle$$

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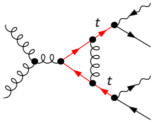


$$\sim \alpha_s^2 \alpha_{ew} \langle p_b - | \ldots \frac{1}{\Delta_t} \int d^4 q \, \frac{1}{q^2} \, \frac{\not{p}_t + \not{q} + M_t}{(p_t + q)^2 - M_t^2} \, \gamma^\nu \, \frac{\not{p}_{\bar{t}} - \not{q} - M_t}{(p_{\bar{t}} + q)^2 - M_t^2} \, \frac{1}{\Delta_{\bar{t}}} \ldots | p_{\bar{b}} - \rangle$$

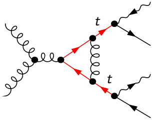
Hard: $q^\mu \sim 1, \quad d^4 q \sim 1, \quad \Delta_t \sim \Delta_{\bar{t}} \sim \delta$

E.g. $\quad (q + p_t)^2 - M_t^2 \; = \; q^2 + 2q.p_t + \Delta_t \; \rightarrow \; q^2 + 2q.p_t$

$$\rightarrow \alpha_s^2 \alpha_{ew} \langle p_b - | \ldots \frac{1}{\Delta_t} \int d^4 q \, \frac{1}{q^2} \, \frac{\not{p}_t + \not{q} + M_t}{q^2 + 2q.p_t} \, \gamma^\nu \, \frac{\not{p}_{\bar{t}} - \not{q} - M_t}{q^2 - 2q.p_{\bar{t}}} \, \frac{1}{\Delta_{\bar{t}}} \ldots | p_{\bar{b}} - \rangle$$

$\delta \quad \delta \quad \frac{1}{\delta} \quad 1 \quad 1 \quad 1 \quad 1 \quad \frac{1}{\delta} \quad \sim \delta^0$ keep!

Method of Regions: Hard-Soft separation: Example



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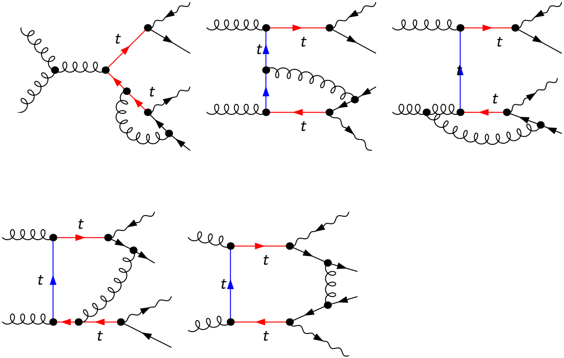
Soft: $q^\mu \sim \delta, \quad d^4q \sim \delta^4, \quad \Delta_t \sim \Delta_{\bar{t}} \sim \delta$

E.g. $\quad (q + p_t)^2 - M_t^2 \; = \; q^2 + 2q.p_t + \Delta_t \; \rightarrow \; 2q.p_t + \Delta_t$

$$\rightarrow \alpha_s^2 \alpha_{ew} \langle p_b - | \ldots \frac{1}{\Delta_t} \int d^4q \, \frac{1}{q^2} \, \frac{\not{p}_t + M_t}{2q.p_t + \Delta_t} \, \gamma^\nu \, \frac{\not{p}_{\bar{t}} - M_t}{-2q.p_{\bar{t}} + \Delta_{\bar{t}}} \, \frac{1}{\Delta_{\bar{t}}} \ldots | p_{\bar{b}} - \rangle$$

$\delta \quad \delta \qquad \frac{1}{\delta} \qquad \delta^4 \quad \frac{1}{\delta^2} \qquad \frac{1}{\delta} \qquad \frac{1}{\delta} \qquad \frac{1}{\delta} \qquad \sim \delta^0$ keep!

Method of Regions: Hard - Soft separation



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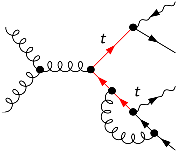
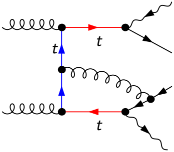
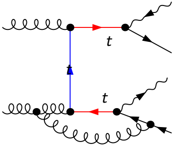


H $\sim \delta^0$ ✓
S $\sim \delta^0$ ✓



H $\sim \delta$ ✗
S $\sim \delta$ ✗



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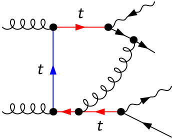

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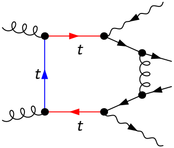


H $\sim \delta$ ✗
S $\sim \delta^0$ ✓

Method of Regions: Hard - Soft separation

Note: Expansion provides a gauge invariant separation of:

- 'Hard corrections' $\leftrightarrow$ Factorizable Corrections, i.e. corrections to on-shell production/decay
- 'Soft corrections' $\leftrightarrow$ Non-Factorizable Corrections, i.e. corrections connecting production and decay

[A.P. Chaposvsky, V. Khoze, A. Signer, W. J. Stirling '01]

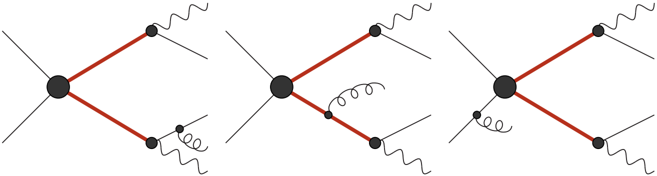
Our Calculation:

Hard Corrections: [S. Badger, R. Sattler, V. Yundin '11]

Soft Corrections: with Pietro Falgari - all but one integral completed.

Real

Work in progress.



Harder than virtual corrections: don't know what to expand in:

$$(p_W + p_b)^2 - M_t^2 \sim \delta \text{ or } (p_W + p_b + p_g)^2 - M_t^2 \sim \delta.$$

Effective Theory: gluon momentum can introduce a new scale to the problem, which can spoil the power-counting/expansion.

Subtraction

A solution:

Usually write:

$$\int d\Phi_{n+1}\, |M_{n+1}^{\text{Real}}|^2 = \int d\Phi_{n+1}\, \left(|M_{n+1}^{\text{Real}}|^2 - |M_{n+1}^{\text{c.t.}}|^2\right) + \int d\Phi_{n+1}\, |M_{n+1}^{\text{c.t.}}|^2$$

where $\int d\Phi_1\, |M_{n+1}^{\text{c.t.}}|^2$ cancels the epsilon poles in $|M_n^{\text{Virt.}}|^2$.

Now write:

$$\int d\Phi_{n+1}\, |M_{n+1}^{\text{Real}}|^2 = \int d\Phi_{n+1}\, \left(|M_{n+1}^{\text{Real}}|^2 - |M_{n+1}^{\text{c.t.}}|^2\right) + \int d\Phi_{n+1}\, |M_{n+1}^{\text{c.t., exp}}|^2$$

where $\int d\Phi_1\, |M_{n+1}^{\text{c.t., exp}}|^2$ cancels the epsilon poles in $|M_n^{\text{Virt., exp}}|^2$.

Drawbacks to this method:

Ideally, would like 'hard/soft' virtual singularities to be cancelled by 'hard/soft' real singularities

$\rightarrow$ could then consistently use a hard ($\mu_h = M_t$) and a soft scale ($\mu_s = \Gamma_t$) for $\alpha_s(\mu)$ for the 'hard' and 'soft' contributions
(Now use common (hard) scale for both sets of contributions)

$\rightarrow$ would lead to an enhancement of the soft/non-factorizable corrections.

Conclusions

- Various approaches to unstable particle production and decay
- E.T. inspired approach $\rightarrow$ gauge invariant expansion of amplitude in resonant region

Method is systematically improvable: know what we have to compute if we want to go to the next order in δ.

- Hard/Soft separation of virtual corrections $\leftrightarrow$ gauge invariant separation of factorizable / non-factorizable corrections

Outlook

- Finish $t\bar{t}$ calculation; study off-shell effects in distributions, mass determination
- Fully understand hard/soft separation of real corrections and cancellation of associated singularities with hard/soft virtual corrections
- Resummation of $\log(\frac{\mu_s}{\mu_h}) \sim \log(\frac{\Gamma_t}{M_t})$
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- Straight forward to include anomalous decays of tops

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Thanks for listening!