An Mc@Nlo EW event generator for lepton colliders

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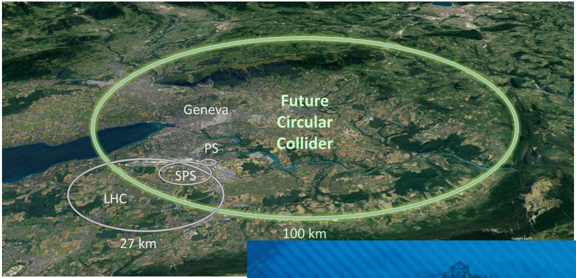
Future
Circular
Collider
Geneva
PS
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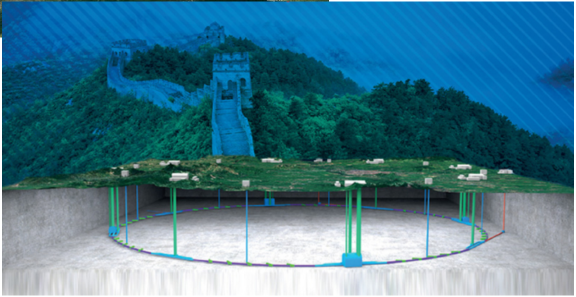
LHC
27 km
100 km



1. Introduction
2. QED dipole shower
3. MC@NLO EW
4. QED shower and resonances
5. MC@NLO QCD+EW
6. Conclusions

Introduction

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Introduction

We need **high-precision event generators** to be able to exploit the projected recorded data of current and future collider experiments to the fullest.

$\rightarrow$ see Emanuele's talk

Disclaimer

Most of the results I am going to show are work in progress and, hence, preliminary and to be taken with a certain grain (pebble? cobble? boulder?) of salt.

QED dipole showers

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Parton showers

Objective

Construct a universal approximation for higher-order corrections

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \sum \mathrm{d}\Phi_1\, \mathcal{K}(\Phi_1) \qquad\qquad \mathrm{d}\Phi_1 = \mathrm{d}t\,\mathrm{d}z\,\mathrm{d}\phi\; J(t,z,\phi)$$

with the splitting functions $\mathcal{K}(t,z,\phi)$ reproducing the soft and collinear limits of the $(n+1)$-particle x-section wrt. the n-particle x-section.
t, z, and ϕ parametrise the one-parton phase space.

Collinear limit: $\mathcal{K}_{\widetilde{ij}}$ **Soft limit:** $\mathcal{K}_{\widetilde{ij},\widetilde{k}}$

DGLAP equation, veto algorithm, ...

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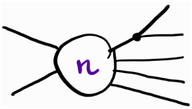
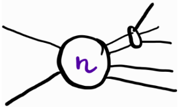
Collinear limit: $\mathcal{K}_{\widetilde{\imath\jmath}}$



Soft limit: $\mathcal{K}_{\widetilde{\imath\jmath},\widetilde{k}}$



DGLAP equation, veto algorithm, ...

QCD parton showers and the large-N_c limit

Full soft-collinear splitting function:

$$\mathcal{K}_{\widetilde{\imath\jmath},\widetilde{k}} = -\mathbf{T}_{\widetilde{k}}\mathbf{T}_{\widetilde{\imath\jmath}} \otimes \mathbf{V}_{\widetilde{\imath\jmath},\widetilde{k}}$$

where the matrix-valued colour correlators have entries of indeterm. sign.

The large-N_c limit.

$$-\mathbf{T}_{\widetilde{k}}\mathbf{T}_{\widetilde{\imath\jmath}} \longrightarrow \begin{cases} C_F \quad + \mathcal{O}(1/N_c^2) & \text{if } \widetilde{\imath\jmath} = q \\ C_A/2 + \mathcal{O}(1/N_c^2) & \text{if } \widetilde{\imath\jmath} = g \\ \qquad\quad \mathcal{O}(1/N_c^2) & \text{otherwise} \end{cases} \text{ and } \widetilde{k} \text{ colour neighbour,}$$

This reduces the matrix-valued colour correlators to Casimirs ($\in \mathbb{R}$).
The splitting functions $\mathcal{K}$ are real-valued positive definite functions in the large-N_c limit.

QED parton showers

Well-understood abelian limit of QCD parton showers which suffer from non-existing large-N_c limit. QED is U(1), and $\frac{1}{1}$ is not so small.
$\Rightarrow$ all charge partners of equal importance, but contrib. with diff. signs.

Collinear limit:

coll. emission pattern trivial as only determined by charge of emitter

Soft limit:

sum of eikonals of alternating sign,
neglecting neg. dipoles gets soft limit wrong

alt: multipole rad. Yennie, Frautschi, Suura '61

MS, Krauss '08; Kleiss, Verheyen '17

Additional complication: QED result of a broken gauge group
$SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{QED}}$
in addition to massive fermions
$\Rightarrow$ resonances everywhere

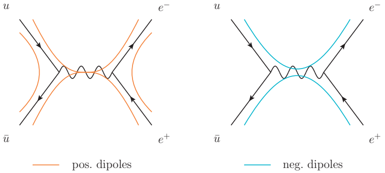
QED dipoles and soft eikonal

Main issue: QED is U(1)

equivalent of large-N_c expansion not very meaningful

need full multipole picture for soft-photon coherence

Example: $u\bar{u} \to e^+e^-$, 6 dipoles (4 opposite sign, 2 same sign)



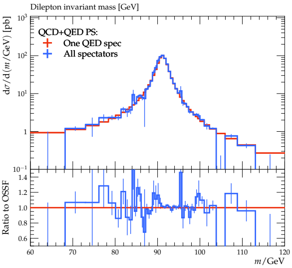
need all dipoles for correct soft-collinear structure,
all dipoles contribute on equal footing
can be handled by weighted shower

QED dipoles

Photon splittings $\gamma \to f\bar{f}$
Pure collinear splitting, free to choose recoil partner to ensure momentum conservation Flower, MS '22

Problem: every $\gamma \to f\bar{f}$ and $g \to q\bar{q}$ introduces $n+1$ new opposite-sign and n new like-sign QED dipoles.

However, in strongly ordered parton shower each such $f\bar{f}$-pair is produced at a well-separated scales and radiation separates into local-dipoles, ie. n opposite-sign/like-sign dipole pairs that cross the intermediate γ/g cancel pairwise.



QED dipoles

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \sum \mathrm{d}\Phi_1 \, \mathcal{K}(\Phi_1) \qquad\qquad \mathrm{d}\Phi_1 = \mathrm{d}t \, \mathrm{d}z \, \mathrm{d}\phi \; J(t, z, \phi)$$

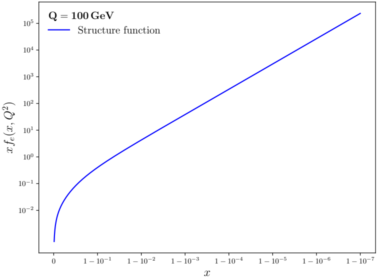
When initial state partons are involved, either as emitter or spectator, they change their momentum fraction x, inducing a change of the structure function associated with it.

$$J_{\mathrm{SF}} \;=\; \frac{f_a\left(\frac{x_{\widetilde{a\imath}}}{z}, t_i\right)}{f_{\widetilde{a\imath}}(x_{\widetilde{a\imath}}, t_i)}$$

While in a proton PDF, all f_a are regular as $x \to 1$, this is not the case for the electron structure function.

The electron structure function

$$f_e(x, Q^2) = \beta \frac{\exp\left(-\gamma_e \beta + \frac{3}{4}\beta_S\right)}{\Gamma(1+\beta)} (1-x)^{\beta-1} + \sum_{n=1}^{\infty} \beta_H^n \mathcal{H}_n(x)$$



$$\beta = \frac{\alpha(Q^2)}{\pi}\left[\log\frac{Q^2}{m_e^2} - 1\right]$$

Lepton structure function
LL solution to DGLAP eq. with initial conditions $f_e(x, m_e^2) = \delta(1-x)$

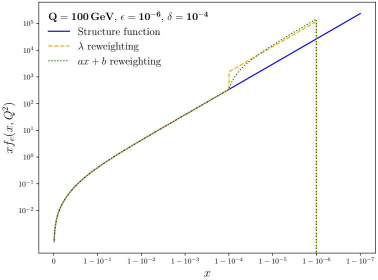
towards higher-orders Frixione '19
Stahlhofen '25; Schnubel, Szafron '25

full PDFs Frixione et.al '19,'23

Integrable singularity at $x = 1$ at any order.

.. and numerical integration

$$f_e(x, Q^2) = \beta \, \frac{\exp\left(-\gamma_e \beta + \frac{3}{4}\beta_S\right)}{\Gamma(1+\beta)} \, (1-x)^{\beta-1} + \sum_{n=1}^{\infty} \beta_H^n \mathcal{H}_n(x)$$



Integrable singularity at $x = 1$ is problematic for num. integration

Solution:

$$W_e = \begin{cases} f_e(x) & x \in [0, 1-\delta] \\ w(x) f_e(x) & x \in [1-\delta, 1-\epsilon] \\ 0 & x \in [1-\epsilon, 1] \end{cases}$$

Determine $w(x)$ such that

$$\int_{1-\delta}^{1} \mathrm{d}x\, W_e = \int_{1-\delta}^{1} \mathrm{d}x\, f_e$$

Problems:

- W_e for $w(x) = \text{const.}$ not continuous
- W_e does no longer solves DGLAP eq.

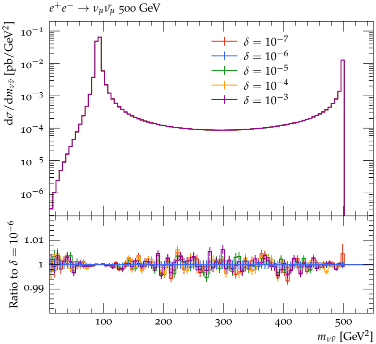
Interplay with initial state QED shower

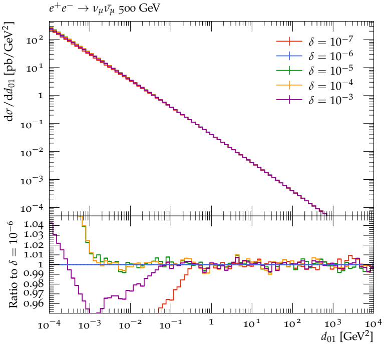
Initial state shower

- backwards evolution from hard scattering to incident beam particles
  $t_{n+1} < t_n$, but $x_{n+1} > x_n$
- contrary to hadron colliders, electron structure function enhances evolution to larger x,
  in particular high probability to evolve from moderate x, eg. $x \approx 0.5$, to large $x \lesssim 1$
- ϵ and δ imply $x_\text{max} < 1$
  choice: $x_\text{max} = 1 - \epsilon$,
  allow one emission into $[1 - \epsilon, 1]$, do not evolve further
- infrared regularisation through cut-off t_c,
  QED IR free, hence $t_c \propto m_e$

In practice – LO+PS QED $e^+e^- \to \nu_\mu\bar{\nu}_\mu$ @ 500 GeV

Dependence on δ

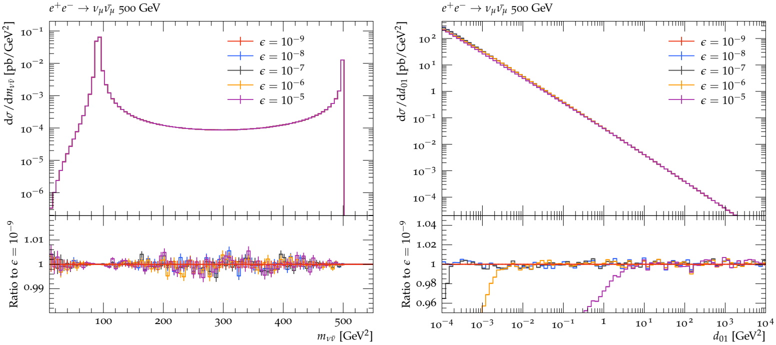




- dependence below accuracy if δ small enough, default $\delta = 10^{-4}$

In practice – LO+PS QED $e^+e^- \to \nu_\mu\bar{\nu}_\mu$ @ 500 GeV

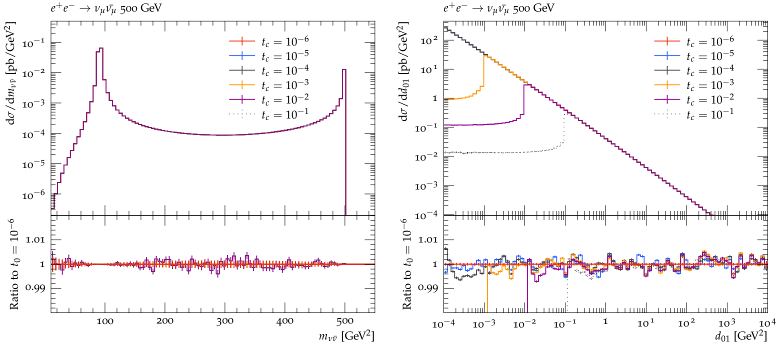
Dependence on ϵ



- dependence below accuracy if ϵ small enough, default $\epsilon = 10^{-8}$

In practice – LO+PS QED $e^+e^- \to \nu_\mu\bar{\nu}_\mu$ @ 500 GeV

Dependence on t_c



- dependence below accuracy if t_c small enough, default $t_c = m_e^2$

MC@NLO EW for lepton colliders

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MC@NLO EW for lepton colliders

MC@NLO matching in SHERPA

$$\langle\mathcal{O}\rangle = \int \mathrm{d}\Phi_B\, \overline{\mathrm{B}}(\Phi_B)\, \mathcal{F}^A(t_0, O) + \int \mathrm{d}\Phi_R\, \mathrm{H}(\Phi_R)\, \mathcal{F}(t_1, O)$$

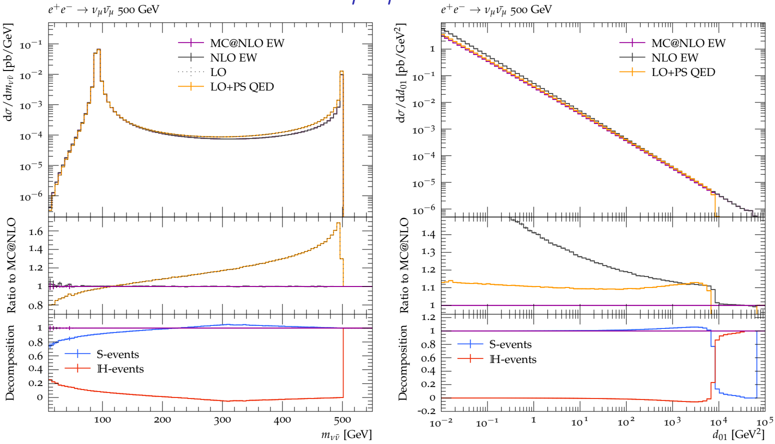
MC@NLO EW follows the established NLO QCD matching methods

Höche, Krauss, MS, Siegert '11

$$\begin{aligned}\overline{\mathrm{B}}(\Phi_B) &= \mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B) + \mathrm{I}^S(\Phi_B)\\ &\qquad + \int \mathrm{d}\Phi_1 \sum_i \left[\mathrm{D}_i^A(\Phi_B, \Phi_1) - \mathrm{D}_i^S(\Phi_B, \Phi_1)\right]\\ \mathrm{H}(\Phi_R) &= \mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^A(\Phi_B^i, \Phi_1^i)\end{aligned}$$

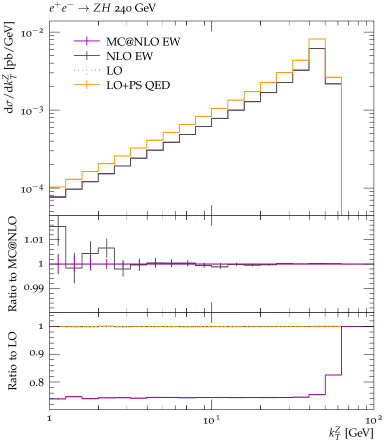
with the spin-correlated splitting functions D_i^A
and IR subtraction $\mathrm{I}^S = \sum_i \int \mathrm{d}\Phi_1\, \mathrm{D}_i^S$.

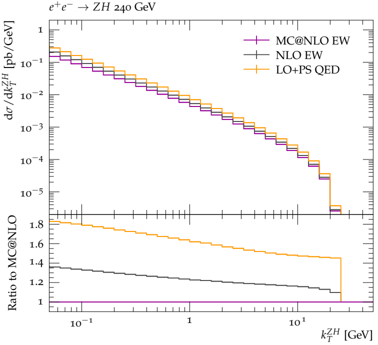
MC@NLO QED $e^+e^- \to \nu_\mu \bar{\nu}_\mu$ @ 500 GeV



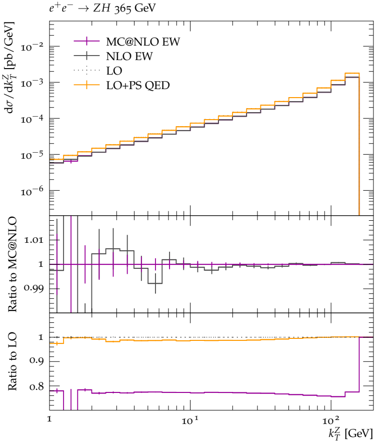
- $m_{\nu\nu}$ identical between NLO EW and MC@NLO EW (should be)
- Sudakov factor regularises

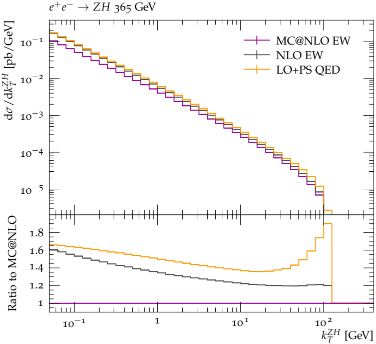
MC@NLO QED $e^+e^- \to ZH$ @ 240 GeV





Mc@Nlo QED $e^+e^- \to ZH$ @ 365 GeV





QED showers and resonances

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QED showers and resonances

QCD parton shower can large avoid resonances as the top quark is the only colour-charged resonance in the SM.

Resonances are ubiquitous in the the EW SM.
All process contain resonances at the LHC and future colliders contain resonances, if not at LO, then at higher orders.

Soft-collinear limit of splitting functions has no information of transverse momentum balance (recoil), in principle arbitrary.

Recoil assignment (momentum conservation) must not alter virtuality of resonance.

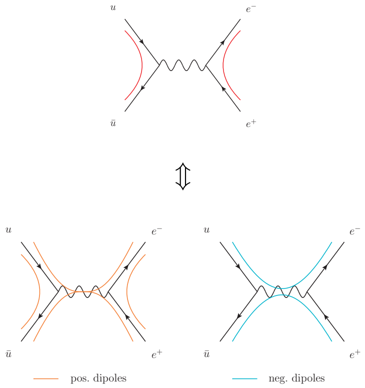
Infrared subtraction and parton shower splitting kernels follow the same physics principles. for FKS Jezo, Nason '15, Frederix et.al '16

Neutral resonances

Physical picture

$t \gtrsim \Gamma^2$ photon emission **can** resolve the resonance, production and decay emit separately.

$t \lesssim \Gamma^2$ photon emission **cannot** resolve the resonance, production and decay emit coherently.



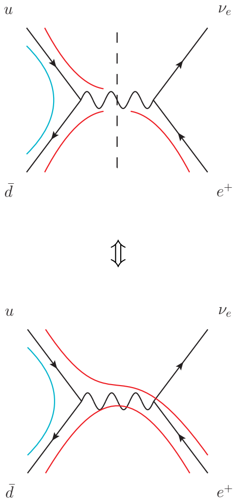
Charged resonances

Physical picture same as for charged resonance, but need dipoles with resonance as emitter and spectator

Resonance production
$\mathrm{IF}_{\mathrm{res}}$ and $\mathrm{FF}_{\mathrm{res}}$ dipoles

Resonance decay
$\mathrm{FI}_{\mathrm{res}}$ dipoles

Basso, Dittmaier, Huss, Oggero '15



Mc@Nlo QCD+EW

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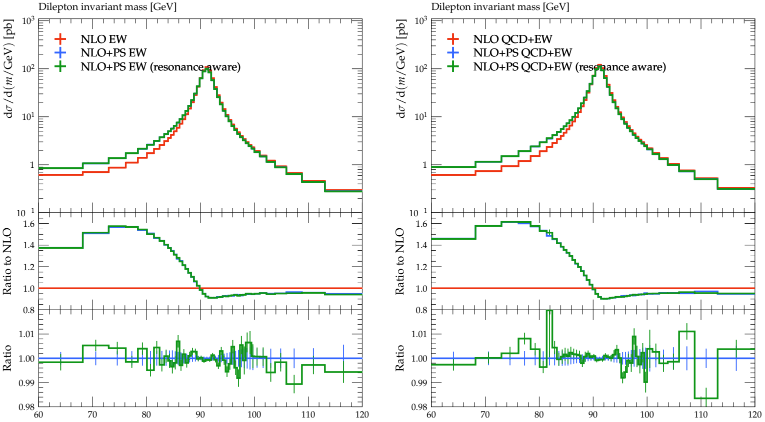
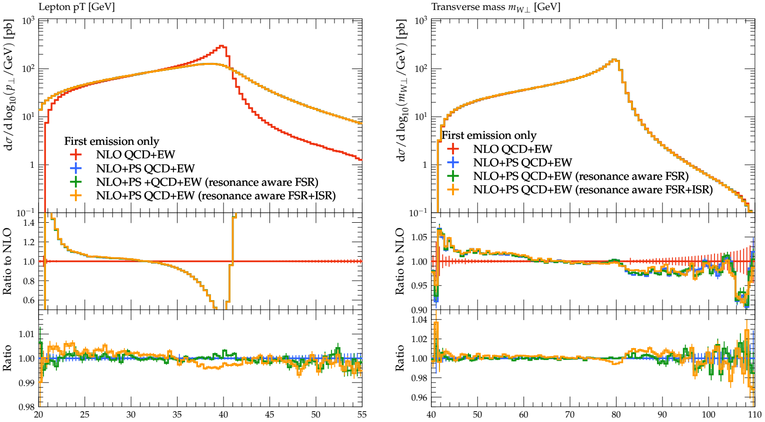
MC@NLO QCD+EW $pp \to \ell\ell$



- resonance awareness not very relevant
  (physics IF+FI dipoles cancel almost exactly)

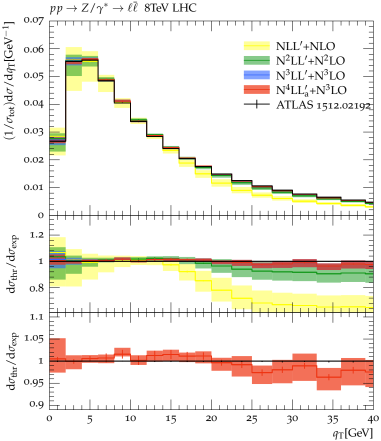
Mc@Nlo QCD+EW $pp \to \ell\nu$



- resonance awareness very relevant — see also Mück, Oymanns '16
- size of corrections impacted by $\overline{\mathrm{B}}_{\mathrm{QCD+EW}}/\mathrm{B}$ K-factor

$N^4LL_a+N^3LO$ $pp \to e^+e^-$

Resummation framework of Ju, MS '21
Assi, Campbell, Höche, Ju, MS to appear



Conclusions

e^+e^- colliders offer great physics opportunities, but we need the tools to seize them!

- QED dipole showers implement natively the soft and collinear limits of QED radiation, and can be straight forwardly interleaved with QCD dipole showers
- QED initial state showers for lepton initial states at lepton colliders need some care regarding integrable singularity at $x = 1$ in lepton structure function
- can produce negative weights when $n_{\text{ch}} > 2$, but physics rescues us
- MC@NLO EW proceeds along the same lines as MC@NLO QCD
- resonance awareness necessary for many processes

http://sherpa.hepforge.org

Thank you!

Backup