

Precision observables at the LHC and beyond

Marek Schönherr

IPPP Durham University



Contents

① Introduction

② Analytic resummation

Precision resummation at leading power

Resummation at sub-leading power

③ Resummation in event generators

Precision resummation in parton showers

EW precision resummation

④ Conclusions

Introduction

1 Introduction

2 Analytic resummation

Precision resummation at leading power

Resummation at sub-leading power

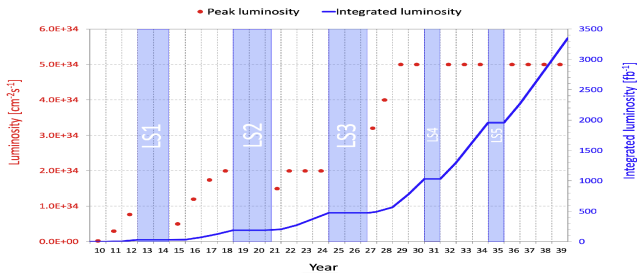
3 Resummation in event generators

Precision resummation in parton showers

EW precision resummation

4 Conclusions

Precision observables at the LHC and beyond



Increasingly **precise data** needs equally **precise theory** predictions to be exploited to its fullest.

The same considerations apply to all future colliders, in particular proposed precision e^+e^- machines (FCC-ee, CEPC, CLIC).

Precision observables at the LHC and beyond

Theory predictions

- **fixed-order calculations**, e.g. $pp \rightarrow tt + X$ @ N³LO
→ differential in parton defined in signature, inclusive in X ,
partons as physical states
- **resummed predictions**, e.g. Drell-Yan q_T spectrum @ N³LL'+N²LO
→ observable specific calculation, often idealised,
many degrees of freedom integrated out (e.g. QCD recoil)
- **Monte-Carlo event generators**, e.g. PYTHIA, HERWIG, SHERPA
→ fully differential, leptons and hadrons as physical states,
but can take into account non-perturbative effects,
often **lowest formal accuracy** due to intricate matching,
generally able to treat on same footing as data

Precision observables at the LHC and beyond

Theory predictions

- **fixed-order calculations**, e.g. $pp \rightarrow tt + X$ @ N³LO
→ differential in parton defined in signature, inclusive in X ,
partons as physical states
- **resummed predictions**, e.g. Drell-Yan q_T spectrum @ N³LL'+N²LO
→ observable specific calculation, often idealised,
many degrees of freedom integrated out (e.g. QCD recoil)
- **Monte-Carlo event generators**, e.g. PYTHIA, HERWIG, SHERPA
→ fully differential, leptons and hadrons as physical states,
but can take into account non-perturbative effects,
often **lowest formal accuracy** due to intricate matching,
generally able to treat on same footing as data

Precision observables at the LHC and beyond

Theory predictions

- **fixed-order calculations**, e.g. $pp \rightarrow tt + X$ @ N³LO
→ differential in parton defined in signature, inclusive in X ,
partons as physical states
- **resummed predictions**, e.g. Drell-Yan q_T spectrum @ N³LL'+N²LO
→ observable specific calculation, often idealised,
many degrees of freedom integrated out (e.g. QCD recoil)
- **Monte-Carlo event generators**, e.g. PYTHIA, HERWIG, SHERPA
→ fully differential, leptons and hadrons as physical states,
but can take into account **non-perturbative effects**,
often **lowest formal accuracy** due to intricate matching,
generally able to treat on same footing as data

Analytic resummation

1 Introduction

2 Analytic resummation

Precision resummation at leading power

Resummation at sub-leading power

3 Resummation in event generators

Precision resummation in parton showers

EW precision resummation

4 Conclusions

Precision resummation at leading power

Analytical resummation

- identify sources of large logarithms spoiling standard perturbative convergence order-by-order
- reorganise perturbative series resumming these logarithms, cross section takes the following form

Example: q_T resummation in Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \underbrace{\sum_{m,n} c_{m,n} \frac{\alpha_s^m \log^n q_T}{q_T}}_{\text{LP}} + \underbrace{\sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T}_{\text{NLP}} + \underbrace{\sum_{m,n} e_{m,n} \alpha_s^m q_T \log^n q_T}_{\text{N}^2\text{LP}} + \dots$$

- LP resummation: CSS formalism, SCET, momentum space

Precision resummation at leading power

Analytical resummation

- identify sources of large logarithms spoiling standard perturbative convergence order-by-order
- reorganise perturbative series resumming these logarithms, cross section takes the following form

Example: q_T resummation in Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \underbrace{\sum_{m,n} c_{m,n} \frac{\alpha_s^m \log^n q_T}{q_T}}_{\text{LP}} + \underbrace{\sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T}_{\text{NLP}} + \underbrace{\sum_{m,n} e_{m,n} \alpha_s^m q_T \log^n q_T}_{\text{N}^2\text{LP}} + \dots$$

- LP resummation: CSS formalism, SCET, momentum space

Precision resummation at leading power

Analytical resummation

- identify sources of large logarithms spoiling standard perturbative convergence order-by-order
- reorganise perturbative series resumming these logarithms, cross section takes the following form

Example: q_T resummation in Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \underbrace{\sum_{m,n} c_{m,n} \frac{\alpha_s^m \log^n q_T}{q_T}}_{\text{LP}} + \underbrace{\sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T}_{\text{NLP}} + \underbrace{\sum_{m,n} e_{m,n} \alpha_s^m q_T \log^n q_T}_{\text{N}^2\text{LP}} + \dots$$

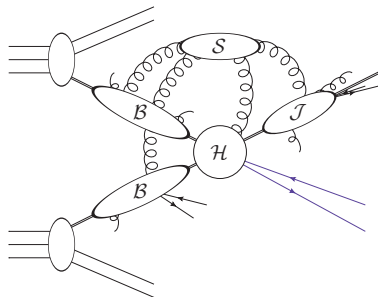
- LP resummation: CSS formalism, SCET, momentum space

Drell-Yan @ $N^3LL' + N^2LO$

Ju, MS '21

LP resummation in SCET

- individual functions to describe dynamics at respective scales
hard - \mathcal{H}
beam - \mathcal{B}
soft - \mathcal{S}
(jet - \mathcal{J})

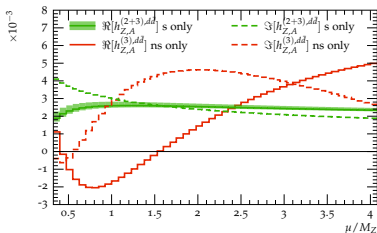
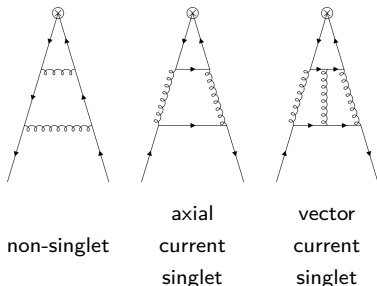


- evolved from their intrinsic scale $\mu_b, \mu_s, (\mu_j)$ to the hard scale μ_h of the event using RGE
- ingredients for N^3LL' :

Logarithmic accuracy	$C_{ns}, C_S^A, C_S^V, C_t, S, \mathcal{B}$	Γ_{cusp}	$\gamma_{t,h,s,b}$
NLL'	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$
N^2LL'	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$
N^3LL'	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^3)$

Drell-Yan @ $N^3LL' + N^2LO$

Ju, MS '21



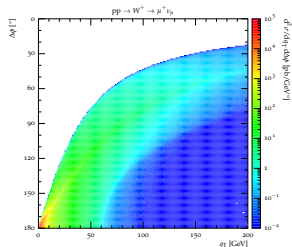
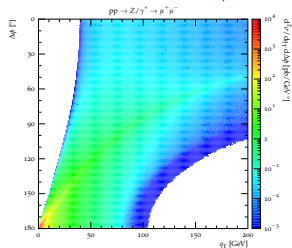
- total singlet contribution about 10% of second order coefficients
- total singlet contribution same size as third order coefficients

⇒ **singlet contribution non-negligible at N^3LL**

Drell-Yan @ $N^3LL' + N^2LO$

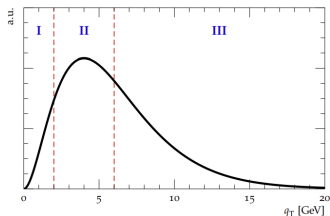
- fiducial regions for W^\pm and Z measurements differ
→ must be accounted for when calculating cross section ratios
- if ratios to be applied to data, it helps to have these ratios as differentially as possible
→ multi-differential distribution
- due to presence of neutrino in W process, only transverse observables useful
→ choose q_T and $\Delta\phi$

Ju, MS '21



Drell-Yan @ $N^3LL' + N^2LO$

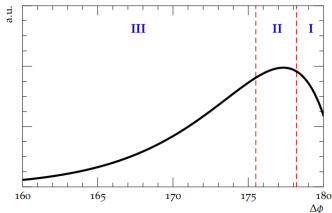
Ju, MS '21



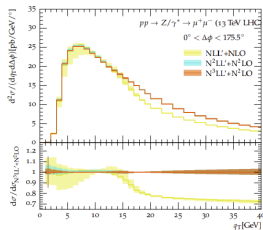
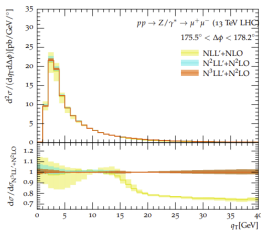
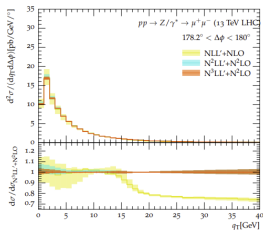
- split q_T and $d\phi$ into three regions: below, on, above Sudakov peak
- e.g., q_T spectra strongly depend on $d\phi$ region

Drell-Yan @ $N^3LL' + N^2LO$

Ju, MS '21

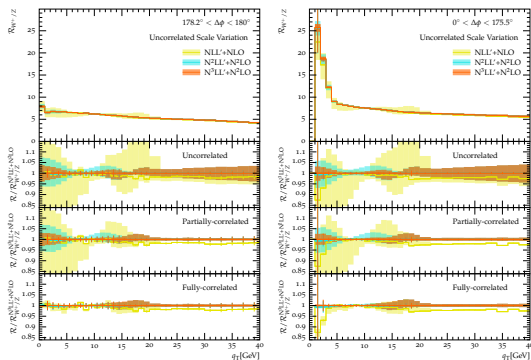


- split q_T and $d\phi$ into three regions: below, on, above Sudakov peak
- e.g., q_T spectra strongly depend on $d\phi$ region



Drell-Yan @ $N^3LL' + N^2LO$

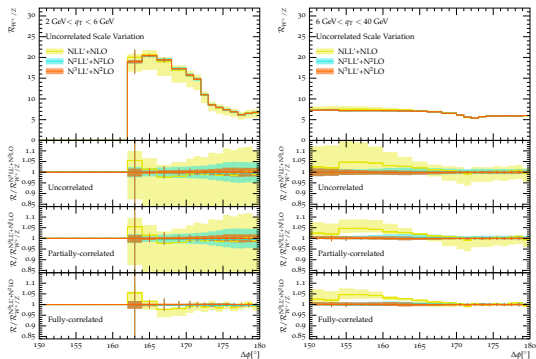
Ju, MS '21



- W^\pm/Z , W^+/W^- ratios depend on phase space region
- strong influence of differing fiducial volumes
- multi-differential predictions help to account for effects

Drell-Yan @ $N^3LL' + N^2LO$

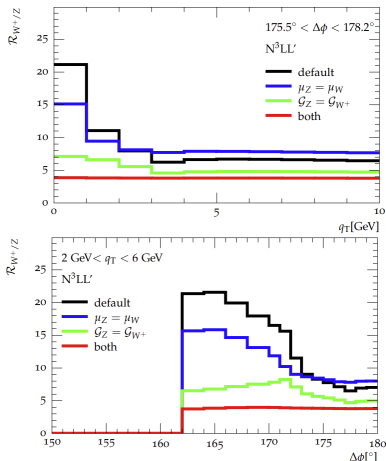
Ju, MS '21



- W^\pm/Z , W^+/W^- ratios depend on phase space region
- strong influence of differing fiducial volumes
- multi-differential predictions help to account for effects

Drell-Yan @ $N^3LL' + N^2LO$

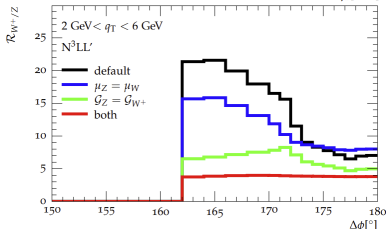
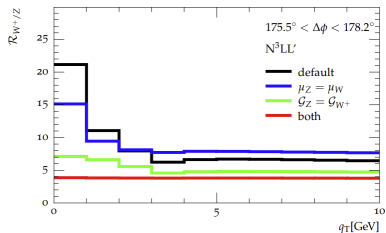
Ju, MS '21



- $\mathcal{R}_{W/Z}$ exhibit a strong shape in the vicinity of the Sudakov peak, induced by
 - different propagator poles, $\mu_W \neq \mu_Z$
 - different fiducial phase space, $G_W \neq G_Z$
- effect induced by different PDF structure for W and Z is comparatively small
- NB: $R_{W^+/W^-} \neq \text{const.}$
in particular $G_{W^+} \neq G_{W^-}$,
 ℓ^+ in $W^+ \Rightarrow \bar{\nu}_\ell$ in W^-

Drell-Yan @ $N^3LL' + N^2LO$

Ju, MS '21



- $R_{W/Z}$ exhibit a strong shape in the vicinity of the Sudakov peak, induced by
 - different propagator poles, $\mu_W \neq \mu_Z$
 - different fiducial phase space, $G_W \neq G_Z$
- effect induced by different PDF structure for W and Z is comparatively small
- NB: $R_{W^+/W^-} \neq \text{const.}$ in particular $G_{W^+} \neq G_{W^-}$, ℓ^+ in $W^+ \Rightarrow \bar{\nu}_\ell$ in W^-

Resummation at subleading power

$$\frac{d\sigma}{dq_T} \sim \underbrace{\sum_{m,n} c_{m,n} \frac{\alpha_s^m \log^n q_T}{q_T}}_{\text{LP}} + \underbrace{\sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T}_{\text{NLP}} + \underbrace{\sum_{m,n} e_{m,n} \alpha_s^m q_T \log^n q_T}_{\text{N}^2\text{LP}} + \dots$$

Resummation at subleading powers

- $\left. \frac{d\sigma}{dq_T} \right|_{\text{exact}} - \left. \frac{d\sigma}{dq_T} \right|_{\text{LP}} \sim \sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T$
 contains integrable divergence for $q_T \rightarrow 0$, problematic numerically
 \rightarrow relevant in particular when LP resummation used to subtract N^kLO infrared divergences (q_T subtraction)
- also relevant at larger q_T where NLP no longer suppressed

Resummation at subleading power

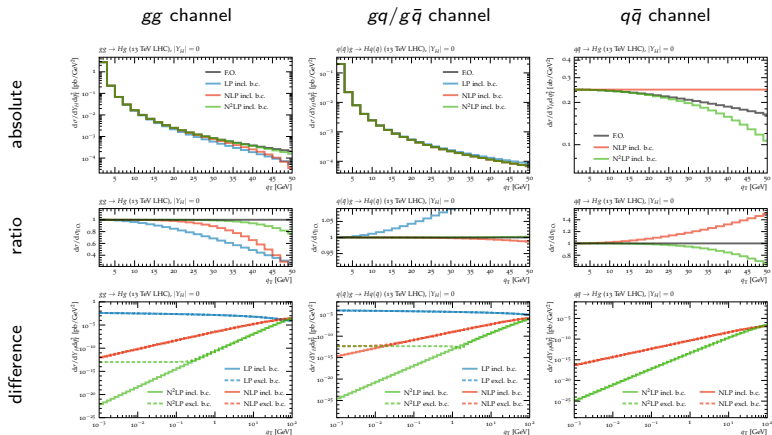
$$\frac{d\sigma}{dq_T} \sim \underbrace{\sum_{m,n} c_{m,n} \frac{\alpha_s^m \log^n q_T}{q_T}}_{\text{LP}} + \underbrace{\sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T}_{\text{NLP}} + \underbrace{\sum_{m,n} e_{m,n} \alpha_s^m q_T \log^n q_T}_{\text{N}^2\text{LP}} + \dots$$

Resummation at subleading powers

- $\left. \frac{d\sigma}{dq_T} \right|_{\text{exact}} - \left. \frac{d\sigma}{dq_T} \right|_{\text{LP}} \sim \sum_{m,n} d_{m,n} \alpha_s^m \log^n q_T$
 contains integrable divergence for $q_T \rightarrow 0$, problematic numerically
 \rightarrow relevant in particular when LP resummation used to subtract N^kLO infrared divergences (q_T subtraction)
- also relevant at larger q_T where NLP no longer suppressed

Sub-leading power corrections of q_T in ggh

Ferrera, Ju, MS '23



- crucial for $q_T \rightarrow 0$ (eg. in N^kLO q_T -subtraction)
- improves control over medium q_T region and matching to fixed-order

Overview

1 Introduction

2 Analytic resummation

Precision resummation at leading power

Resummation at sub-leading power

3 Resummation in event generators

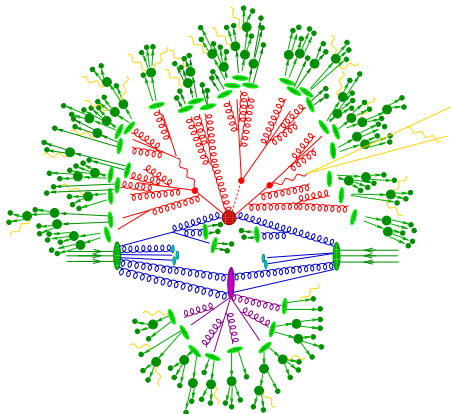
Precision resummation in parton showers

EW precision resummation

4 Conclusions

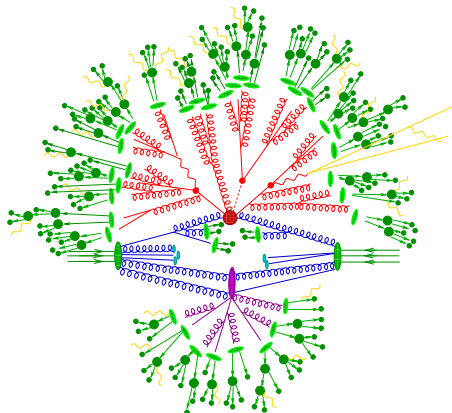
Monte-Carlo Event Generators

- Matrix elements
- Parton showers
- Multiple interactions
- Hadronisation
- Hadron decays
- QED radiation



Monte-Carlo Event Generators

- Matrix elements
- Parton showers
- Multiple interactions
- Hadronisation
- Hadron decays
- QED radiation



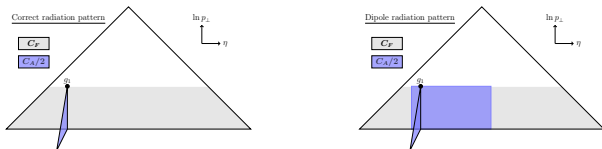
Parton shower development

Parton shower development has been an active field again for the past ten years or so.

- DIRE Höche, Prestel et.al '15 ff
(full analytic control over parton shower resummation)
 - work towards fully differential higher-order splitting kernels
- analytic appraisal of parton shower resummation properties Salam et.al. '18 ff
- full colour evolution (amplitude level exponentiation) Forshaw, Plätzer et.al. '18 ff
- VINCIA Skands et.al. '19 ff
- EW showers Christiansen, Sjöstrand '14; Krauss, Petrov, MS, Spannowsky '14; ...

NLL accuracy in parton showers

Splitting functions need to reduce to the correct soft-collinear limits. Collinear limits usually not a problem, the (sub-LC) soft limit sometimes is (at LC $C_A = 2C_F$). Here, Lund diagrams are useful



Dipole showers may assign wrong colour factor for secondary soft emissions.

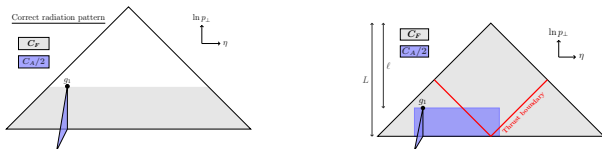
Though solutions exist.

Dasgupta, Dreyer, Hamilton, Monni, Salam '18

Lönnblad, Gustafson '91

NLL accuracy in parton showers

Splitting functions need to reduce to the correct soft-collinear limits. Collinear limits usually not a problem, the (sub-LC) soft limit sometimes is (at LC $C_A = 2C_F$). Here, Lund diagrams are useful



Dipole showers may assign wrong colour factor for secondary soft emissions.

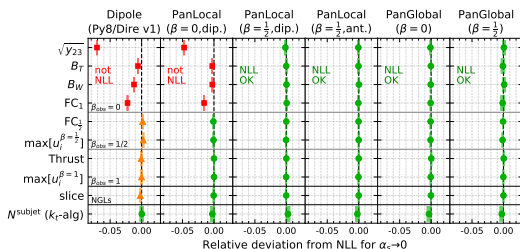
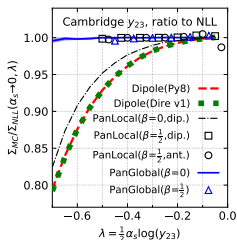
Though solutions exist.

Dasgupta, Dreyer, Hamilton, Monni, Salam '18

Lönblad, Gustafson '91

NLL accuracy in parton showers

Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez '20



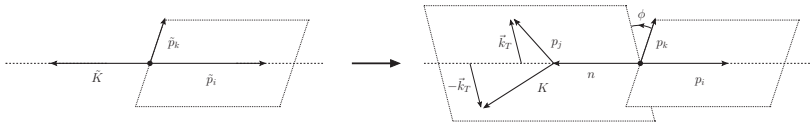
- the standard PYTHIA and DIRE showers are not NLL accurate for investigated e^+e^- event shapes (SHERPA's CSSHOWER similar)
- PANGLOBAL and PANLOCAL family of parton showers designed for NLL accuracy

ALARIC – A Logarithmically Accurate Resummation

A NLL-accurate parton shower must describe both the collinear and soft limits accurately, ie. soft and coll. limits must be appropriately matched.

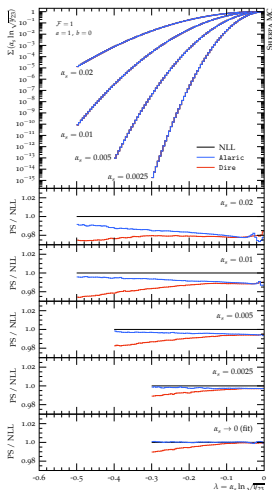
Choice: global recoil scheme (emission-by-emission)

Inspired by multipole radiation in soft-photon resummation. [MS, Krauss '08](#)



With splitter \tilde{p}_i , recoil momentum \tilde{K} , and colour spectator \tilde{p}_k .
The recoil momentum \tilde{K} is a hard momentum, typically a (subset of) the hard radiator.

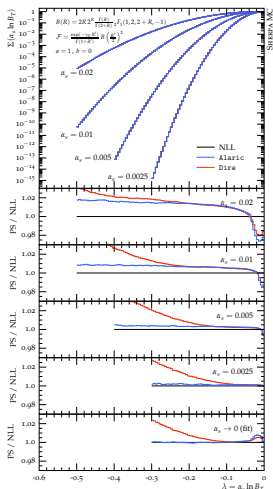
ALARIC – NLL accuracy



limit: $\alpha_s \rightarrow 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

- Durham jet rate y_{23} $\beta = 0$
- Total jet broadening B_T $\beta = 0$
- Durham jet rate $FC_{1/2}$ $\beta = \frac{1}{2}$
- Thrust $1 - T$ $\beta = 1$
- Hemisphere mass M_H $\beta = 1$

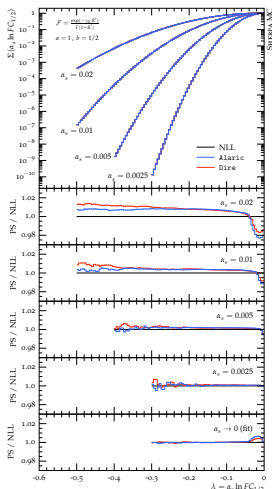
ALARIC – NLL accuracy



limit: $\alpha_s \rightarrow 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

- Durham jet rate y_{23} $\beta = 0$
- Total jet broadening B_T $\beta = 0$
- Durham jet rate $FC_{1/2}$ $\beta = \frac{1}{2}$
- Thrust $1 - T$ $\beta = 1$
- Hemisphere mass M_H $\beta = 1$

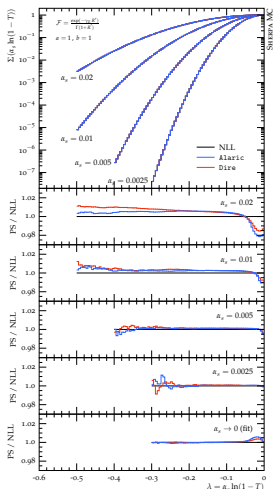
ALARIC – NLL accuracy



limit: $\alpha_s \rightarrow 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

- Durham jet rate y_{23} $\beta = 0$
- Total jet broadening B_T $\beta = 0$
- Durham jet rate $FC_{1/2}$ $\beta = \frac{1}{2}$
- Thrust $1 - T$ $\beta = 1$
- Hemisphere mass M_H $\beta = 1$

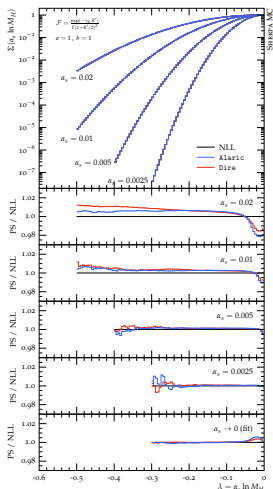
ALARIC – NLL accuracy



limit: $\alpha_s \rightarrow 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

- Durham jet rate y_{23} $\beta = 0$
- Total jet broadening B_T $\beta = 0$
- Durham jet rate $FC_{1/2}$ $\beta = \frac{1}{2}$
- Thrust $1 - T$ $\beta = 1$
- Hemisphere mass M_H $\beta = 1$

ALARIC – NLL accuracy

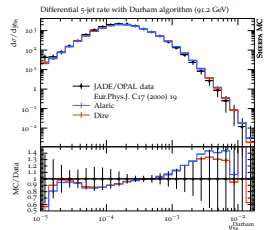
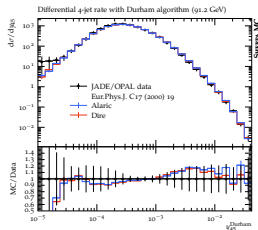
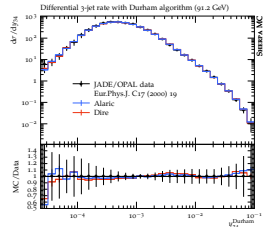
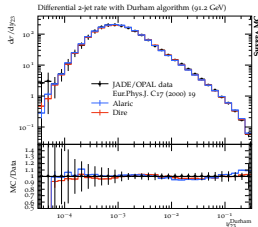


limit: $\alpha_s \rightarrow 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

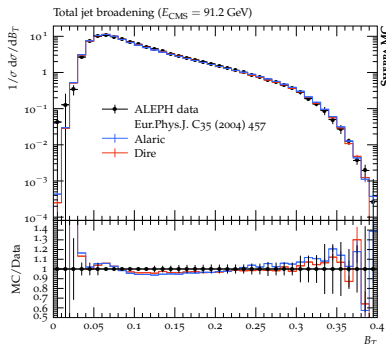
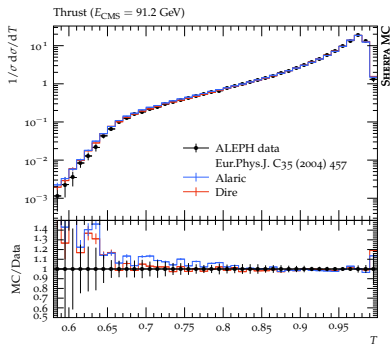
- Durham jet rate y_{23} $\beta = 0$
- Total jet broadening B_T $\beta = 0$
- Durham jet rate $FC_{1/2}$ $\beta = \frac{1}{2}$
- Thrust $1 - T$ $\beta = 1$
- Hemisphere mass M_H $\beta = 1$

ALARIC – LEP phenomenology

- ALARIC+PYTHIA string had.
- hadronisation models are not infrared safe and depend on distribution of soft gluons
- tunes are shower specific
- no matching or multijet merging

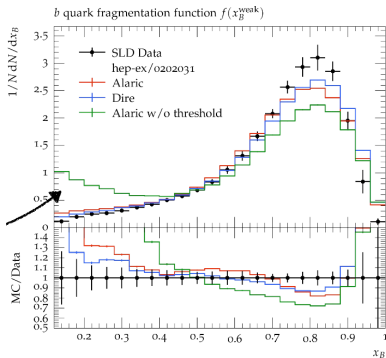


ALARIC – LEP phenomenology



- no matching to fixed-order $3j$, $4j$, etc.

ALARIC – LEP phenomenology



- ALARIC is constructed with massless quarks so far
- quark masses are phenomenologically relevant
- quick fix: flavour thresholds for $g \rightarrow c\bar{c}$ and $g \rightarrow b\bar{b}$, properly implemented in

Assi, Höche '23

EW precision resummation

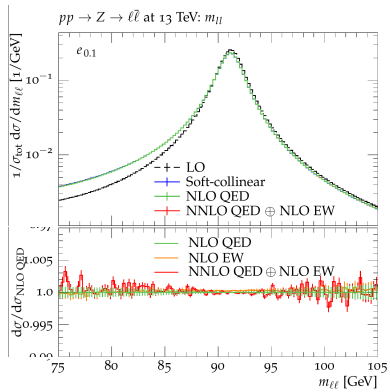
Yennie-Frautschi-Suura soft-photon resummation

Yennie, Frautschi, Suura '61

- method of choice for highest precision QED, e.g.
 - KKMC (YFS+NNLO) for $e^+e^- \rightarrow \mu^+\mu^-$ Jadach et.al '00
- has been implemented for generic QED FSR in SHERPA with up to YFS+NNLO QED+NLO EW for $Z \rightarrow \ell\ell$ MS, Krauss '08
Krauss, Lindert, Linten, MS '18
- extended to include collinear $\gamma \rightarrow \bar{f}f$ splittings Flower, MS '22
- QED ISR for e^+e^- colliders in SHERPA Krauss, Price, MS '22

EW precision resummation

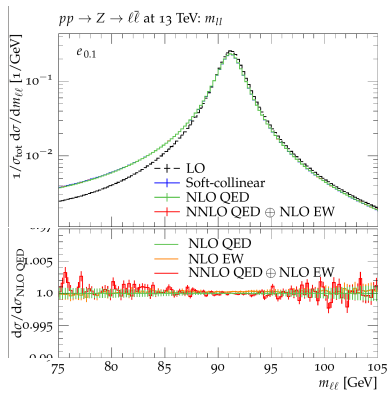
Krauss, Lindert, Linten, MS '18



- soft photon resummation matched to NNLO QED + NLO EW
- double-hard-photon emission corrections negligible
- permille accuracy in the description of the $Z \rightarrow \ell\bar{\ell}$ kinematics ?

EW precision resummation

Krauss, Lindert, Linten, MS '18



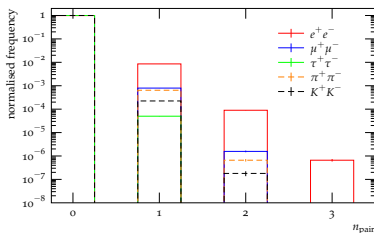
- soft photon resummation matched to NNLO QED + NLO EW
- double-hard-photon emission corrections negligible
- permille accuracy in the description of the $Z \rightarrow \ell\bar{\ell}$ kinematics ?

Resolving the photon cloud

Flower, MS '22

What happens to the photons after they were radiated?

How does this impact the physical dressed lepton?



Photons **split into other flavours** (leptons, hadrons) and thereby remove themselves from a naïvely defined dressed lepton.

This exposes the measurement to **logarithms** of the lightest splitting product's mass, m_e .

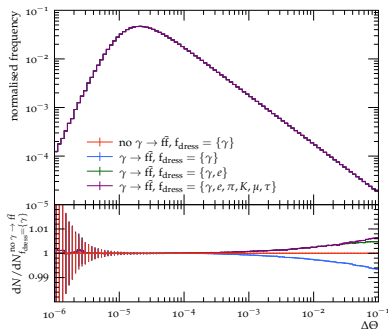
A more inclusive dressed lepton definition will result in a stabler result.

Resolving the photon cloud

Flower, MS '22

On-shell $Z \rightarrow \ell\ell$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- impact for small dressing cones moderate
- larger dressing cones mandate expanded dress. prescription

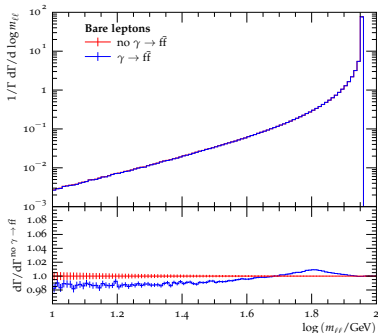


Resolving the photon cloud

Flower, MS '22

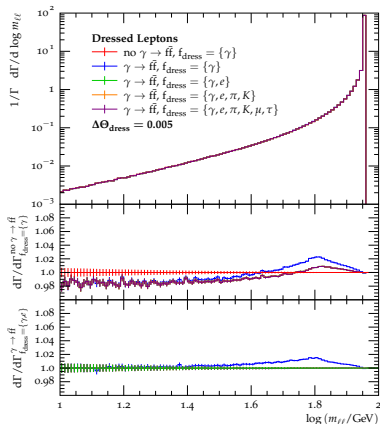
On-shell $Z \rightarrow \ell\ell$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- impact for small dressing cones moderate
- larger dressing cones mandate expanded dress. prescription



Resolving the photon cloud

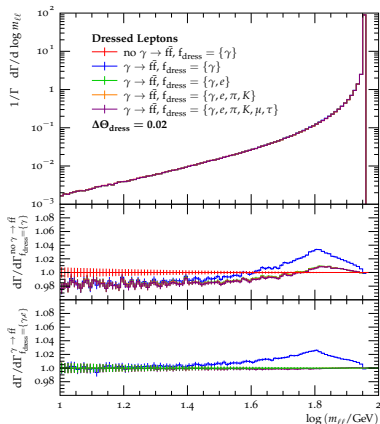
Flower, MS '22

On-shell $Z \rightarrow \ell\ell$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- impact for small dressing cones moderate
- larger dressing cones mandate expanded dress. prescription

Resolving the photon cloud

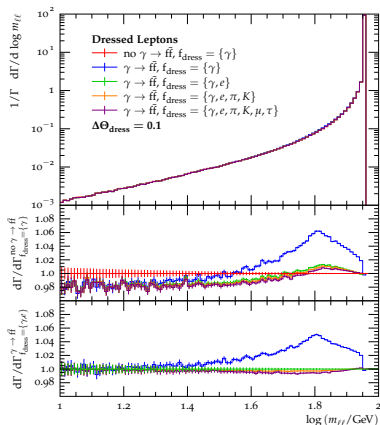
Flower, MS '22

On-shell $Z \rightarrow \ell\ell$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- impact for small dressing cones moderate
- larger dressing cones mandate expanded dress. prescription

Resolving the photon cloud

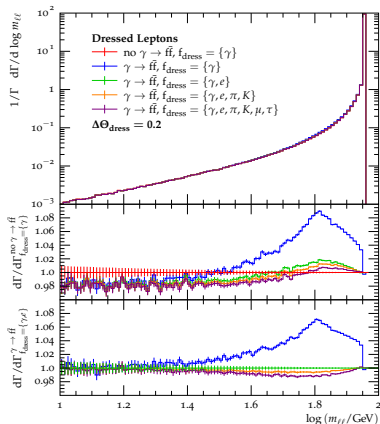
Flower, MS '22

On-shell $Z \rightarrow \ell\ell$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- impact for small dressing cones moderate
- larger dressing cones mandate expanded dress. prescription

Resolving the photon cloud

Flower, MS '22

On-shell $Z \rightarrow \ell\ell$ decays

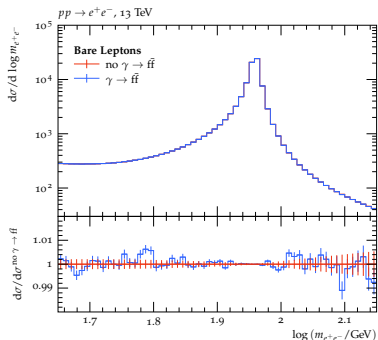
- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- impact for small dressing cones moderate
- larger dressing cones mandate expanded dress. prescription

Resolving the photon cloud

Flower, MS '22

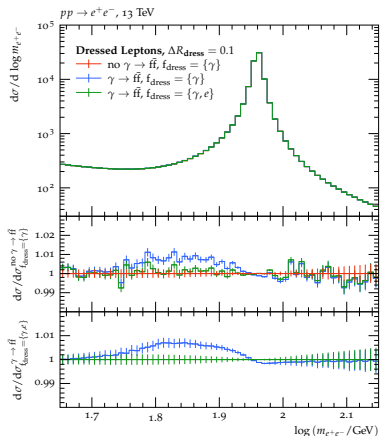
Off-shell $pp \rightarrow e^+e^-$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- can be compensated by more inclusive dressing procedure (ideally: flavoured EW jet)



Resolving the photon cloud

Flower, MS '22

Off-shell $pp \rightarrow e^+e^-$ decays

- hard wide-angle photons have higher probability to split
- $\mathcal{O}(\alpha^2)$ effect, but impact much larger than NNLO γ -radiation corrections as leading logarithms not already in resummation
- can be compensated by more inclusive dressing procedure (ideally: flavoured EW jet)

Conclusions

Analytic resummation

- highest logarithmic accuracy is process and observable dependent
 - structure for q_T differs from 0-jettiness differs from ...
 - singlet contribs in DY are non-negligible
- is inclusive for certain final state components, e.g. QCD recoil in N^k LL QCD resummation
- non-logarithmic or sub-leading power effects (e.g. phase space boundaries, momentum conservation, ...) can be sizeable

Höche, Reichelt, Siebert '17

and are often needed

Conclusions

Resummation on Monte-Carlo event generators

- parton showers are a numerical observable-independent and fully exclusive resummation tool for precision QCD predictions
- formal NLL accuracy crucial in parton shower development
 - meaningful uncertainty assignment
 - subleading colour evolution
 - NLO splitting kernels
 - N²LO matching

but not necessarily phenomenologically relevant, other effects often dominant

- phenomenological impact of higher-accuracy resummation cancelled by poorly understood hadronisation process

<http://sherpa.hepforge.org>

Thank you!

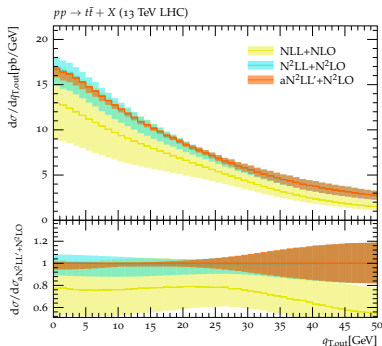
Backup

Top-pair production at $aN^2LL'+N^2LO$

Ju, MS '22

$pp \rightarrow t\bar{t}$ production

- projected transverse momentum $q_{T,out}$ and $\Delta\phi$ resummed at $aN^2LL'+N^2LO$
- removes azimuthally asymmetric contributions otherwise present for general $d^2\sigma/d\vec{q}_T$

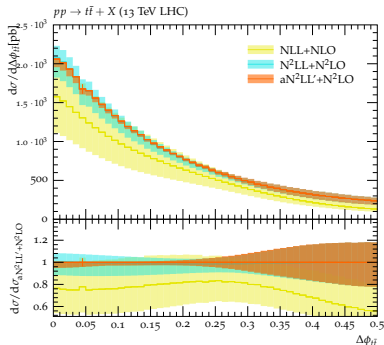


Top-pair production at $aN^2LL'+N^2LO$

Ju, MS '22

$pp \rightarrow t\bar{t}$ production

- projected transverse momentum $q_{T,out}$ and $\Delta\phi$ resummed at $aN^2LL'+N^2LO$
- removes azimuthally asymmetric contributions otherwise present for general $d^2\sigma/d\vec{q}_T$



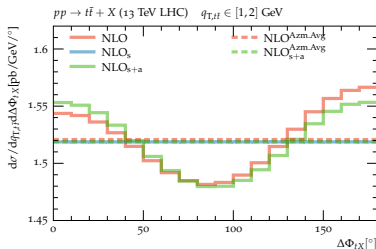
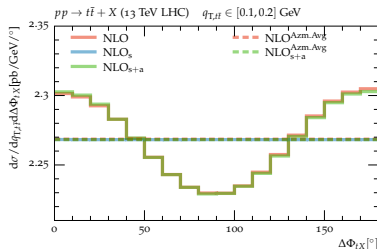
Top-pair production at aN²LL'+N²LO

Ju, MS '22

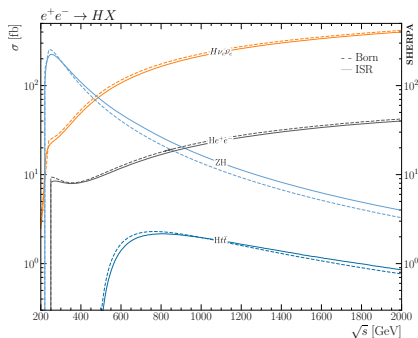
- azimuthally asymmetric coefficients in general present, but neglected

$$\frac{d^2\sigma}{d\vec{q}_T} \propto \sum_{m,n} \alpha_s^m L^n [s_m(q_T) + a_m(\vec{q}_T)]$$

- $a_m(\vec{q}_T)$ needed to reproduce full IR limit



Precision calculations for future e^+e^- colliders



Krauss, Price, MS '22

Soft-photon resummation

Yennie, Frautschi, Suura '61

- supplement collinear logarithms to $\mathcal{O}(\alpha^3 L^3)$
- account for coherent radiation off initial state electrons
- detailed prediction of photon momenta and transverse recoil