Parton showers and formal accuracy

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Monte-Carlo Event Generators

- Matrix elements
- Parton showers
- Multiple interactions
- Hadronisation
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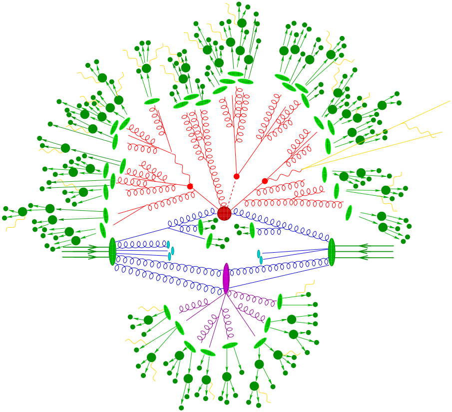

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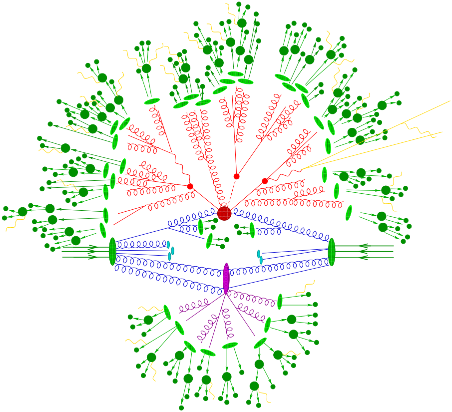

Parton shower development

Parton shower development has been an active field again for the past ten years or so.

- DIRE — Höche, Prestel et.al '15 ff
  (full analytic control over parton shower resummation)
  - work towards fully differential higher-order splitting kernels
- analytic appraisal of parton shower resummation properties — Salam et.al. '18 ff
- full colour evolution (amplitude level exponentiation) — Forshaw, Plätzer et.al. '18 ff
- VINCIA — Skands et.al. '19 ff
- EW showers — Christiansen, Sjöstrand '14; Krauss, Petrov, MS, Spannowsky '14; ...

The DGLAP equation

The DGLAP equation

$$t\frac{\partial}{\partial t}\,f_a(x,t)=\frac{\alpha_s}{2\pi}\sum_b\int\frac{\mathrm{d}z}{z}\,P_{ab}\left(\frac{x}{z}\right)\,f_b(z,t)$$

describes the evolution of parton a.
It is the analogue of the β-function for parton density evolution.
With the introduction of the Sudakov form factor $\Delta_a(t,t_c)$,

$$\Delta_a(t,t_c)=\exp\left[-\sum_b\int\frac{\mathrm{d}t'}{t'}\int\mathrm{d}z\,\frac{\alpha_s(t')}{2\pi}\,\hat{P}_{ab}(z)\right]\,,$$

which describes the probability of parton a not to split between scales t and t_c, the DGLAP equation can be recast as

$$t\frac{\partial}{\partial t}\,f_a(x,t)=\frac{f_a(x,t)}{\Delta_a(t,t_c)}\,t\frac{\partial}{\partial t}\,\Delta_a(t,t_c)+\sum_b\int\frac{\mathrm{d}z}{z}\frac{\alpha_s(t)}{2\pi}\,\hat{P}_{ab}(z)\,f_b\left(\frac{x}{z},t\right)$$

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The DGLAP equation

The DGLAP equation in integral form then reads

$$f_a(x,t) = \Delta_a(t,t_c)\, f_a(x,t_c) + \sum_b \int \frac{\mathrm{d}t'}{t'} \int \frac{\mathrm{d}z'}{z'} \frac{\alpha_s(t')}{2\pi}\, \hat{P}_{ab}(z') \Delta_b(t,t')\, f_b\left(\frac{x}{z'}, t'\right)$$

This is the basis of a parton shower. (Final state showers set $f \equiv 1$.)

The first term describes parton a evolving from t to t_c without emission. The second term has its first emission at scale t', and is inclusive for any further branchings below t'. Iterate by reinsertion.

When integrated over all emissions, the parton shower is unitary.

To generate events, implemented using Monte-Carlo integration using the veto algorithm.

The DGLAP equation

The DGLAP equation in integral form then reads

$$1 = \Delta_a(t, t_c)\,\frac{f_a(x, t_c)}{f_a(x, t)} + \sum_b \int \frac{\mathrm{d}t'}{t'} \int \frac{\mathrm{d}z'}{z'}\,\frac{\alpha_s(t')}{2\pi}\,\hat{P}_{ab}(z')\Delta_b(t, t')\,\frac{f_b\left(\frac{x}{z'}, t'\right)}{f_a(x, t)}$$

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Parton shower resummation

Defining characteristics

All FS parton showers implement

$$1 = \Delta_a(t, t_c) + \sum_b \int_{t_c}^{t_0} \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z' \, \mathcal{K}_{ab}(t', z') \Delta_b(t, t')$$

but each makes different choices for t, z, $\mathcal{K}_{ab}$ and momentum maps (recoil scheme).

Nomenclature

- t ordering variable, this is the variable that is resummed in Δ
- z splitting variable, typically a energy or momentum fraction
- $\mathcal{K}$ splitting function, revert to $\hat{P}$ in collinear limit, but generally also include soft limit, possibly non-log terms

Parton shower resummation

Momentum maps and momentum conservation
As we want to interpret phase space points as events, momentum conservation is crucial. This leads to a modification of various integrals, e.g.

$$\int_0^1 \mathrm{d}z \longrightarrow \int_{z_{\mathrm{min}}}^{z_{\mathrm{max}}} \mathrm{d}z$$

Away from soft-collinear limit, $t \to 0$, non-logarithmic terms in splitting functions are sizeable.

Parton masses, m_c and m_b, often play a significant role.

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Parton shower resummation accuracy

How accurate is the parton shower's resummation
DGLAP resums leading single collinear logarithms.
Most parton showers reproduce this by construction.

How well does the parton shower resum related quantities?
Examples: jet rates, thrust, total broadening, ...

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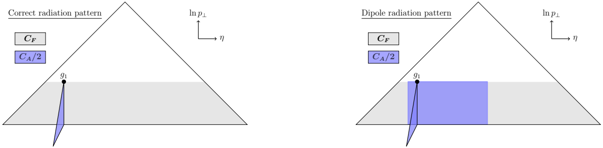
NLL accuracy in parton showers

Splitting functions need to reduce to the correct soft-collinear limits. Collinear limits usually not a problem, the (sub-LC) soft limit sometimes is. Here, Lund diagrams are useful



Dipole showers may assign wrong colour factor for secondary soft emissions. Dasgupta, Dreyer, Hamilton, Monni, Salam '18

Though solutions exist Lönnblad, Gustafson '91

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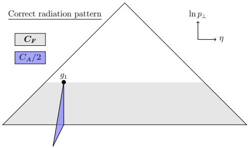

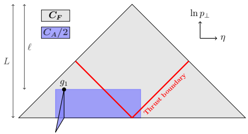

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NLL accuracy in parton showers

Problems may also arise with momentum maps where recoil effects do not vanish quickly enough in soft limit ($\rho \to 0$).

Method of Banfi, Salam, Zanderighi '05

Consider standard (local) dipole momentum map

$$\begin{aligned} p_i &= z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\mathrm{T} \\ p_j &= (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\mathrm{T} \\ p_k &= (1-y)\tilde{p}_k \end{aligned}$$

Consider situation where we first emit $\tilde{p}_{ij}$ from $p_a - p_b$, then $\tilde{p}_{ij} \to p_i p_j$
Transverse momentum of $\tilde{p}_{ij}$ is then $\sim k_\mathrm{T}^{ij} + k_\mathrm{T}^{j}$, thus

$$\Rightarrow \frac{\Delta k_\mathrm{T}^{ij}}{k_\mathrm{T}^{ij}} \to \frac{\rho k_\mathrm{T}^{j}}{\rho k_\mathrm{T}^{ij}} = \mathcal{O}(1)$$

The subsequent emission, through its recoil, has changed the k_T of the former emission too much.

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NLL accuracy in parton showers

How can LL/NLL accuracy be assessed?

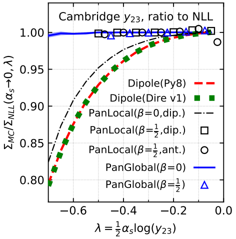
Evaluate in the limit $\alpha_s \to 0$, $\lambda = \alpha_s L = \text{const}$

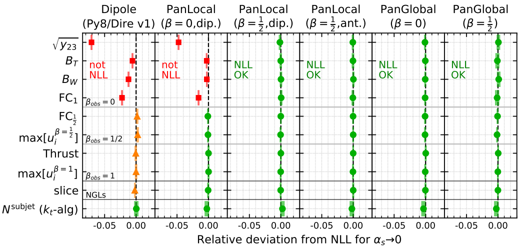
$$
\begin{aligned}
\frac{\Sigma^{\text{shower}}}{\Sigma^{\text{NLL}}} \propto \quad & \exp\left(f_{\text{shower}}^{\text{LL}} - L g_1(\alpha_s L)\right) \\
& \times \exp\left(f_{\text{shower}}^{\text{NLL}} - g_2(\alpha_s L)\right) \\
& \times \exp\left(\mathcal{O}(\alpha_s^{n+1} L^n)\right) \\
\to 1 &
\end{aligned}
$$

if shower reproduces LL and NLL.

NLL accuracy in parton showers

Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez '20





- the standard PYTHIA and DIRE showers are not NLL accurate
- newly formulated accademic PANGLOBAL and PANLOCAL family of parton showers designed for NLL accuracy

ALARIC – A Logarithmically Accurate Resummation

ALARIC – A Logarithmically Accurate Resummation

Herre, Höche, Krauss, Reichelt, MS '22

Aim: Implement a NLL-accurate parton shower for e^+e^- collisions that trivially generalises to *pp* and *ep* colliders, and can be extended for full use in the matching and merging formalisms of a practical event generator.

ALARIC – A Logarithmically Accurate Resummation

A NLL-accurate parton shower must describe both the collinear and soft limits accurately, ie. soft and coll. limits must be appropriately matched.

Starting point: Eikonal

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} \equiv \frac{W_{ik,j}}{E_j^2}$$

Separate according to collinear regions.

Solution 1: Marchesini, Webber '88

$$W_{ik,j} = \widetilde{W}_{ik,j}^{i} + \widetilde{W}_{ki,j}^{k}$$

with

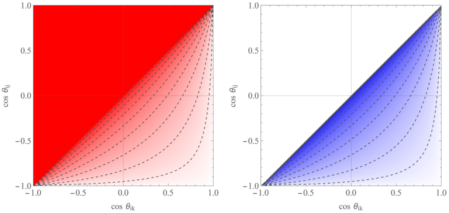
$$\widetilde{W}_{ik,j} = \tfrac{1}{2}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

ALARIC – A Logarithmically Accurate Resummation

Marchesini, Webber '88

Additive radiator function

$$\widetilde{W}_{ik,j} = \tfrac{1}{2}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right)$$



$$\int \mathrm{d}\Phi\, \widetilde{W}^{i}_{ik,j} = \begin{cases} 1 & \text{if } \theta_{j^i} < \theta^i_k \\ 0 & \text{else} \end{cases}$$

angular ordering,
coherent branching formalism

→ problems with non-global logs (observables not insensitive to the distribution of wide-angle radiation)

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Separate according to collinear regions.
Solution 2: Catani, Seymour '97

$$W_{ik,j} = \overline{W}^{i}_{ik,j} + \overline{W}^{k}_{ki,j}$$

with

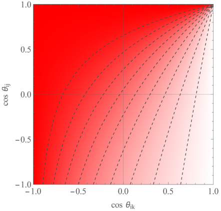
$$\overline{W}_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{jk})}$$

ALARIC – A Logarithmically Accurate Resummation

Catani, Seymour '97

Multiplicative radiator function:

$$\overline{W}_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{jk})}$$



$$\int \mathrm{d}\Phi \overline{W}^{i}_{ik,j} \neq 0 \text{ everywhere}$$

approaches the correct limits in all soft and collinear regions

→ captures all angular correlations of spin-summed soft eikonal, and describes all wide-angle soft emissions correctly

Alaric – A Logarithmically Accurate Resummation

Matching soft and collinear splitting functions

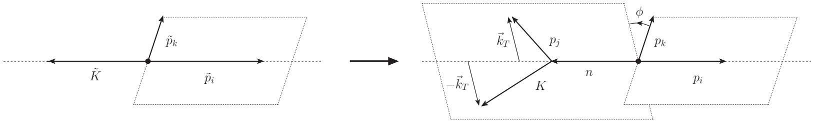
$$\mathcal{K}_{\{(ij),k\}\to\{ij,k\}} = \frac{1}{2p_ip_j}\,P_{(ij)i}(z) + \delta_{(ij)i}\,\mathbf{T}_i^2\,\frac{\overline{W}_{ik,j}^{i} - \overline{W}_{ik,j}^{i,(\mathrm{coll})}}{E_j^2}$$

Colour spectator k introduces azimuthal dependence in spin-averaged parton evolution kernel.

ALARIC – A Logarithmically Accurate Resummation

As argued above, local recoil schemes invite the possibility of non-vanishing recoil effects in soft limit. Dasgupta, Dreyer, Hamilton, Monni, Salam '18

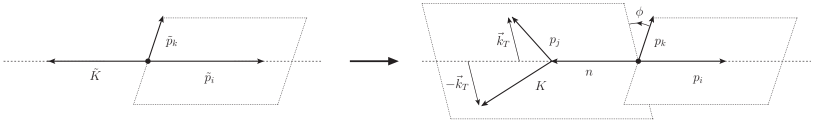
Choice: global recoil scheme (emission-by-emission)
Inspired by multipole radiation in soft-photon resummation. MS, Krauss '08



With splitter $\tilde{p}_i$, recoil momentum $\tilde{K}$, and colour spectator $\tilde{p}_k$.
The recoil momentum $\tilde{K}$ is a hard momentum, typically a (subset of) the hard radiator.

Alaric – A Logarithmically Accurate Resummation

$$\{\tilde{p}_i, \tilde{p}_k; \tilde{K}\} \quad \longrightarrow \quad \{p_i, p_j, p_k; K\}$$



construct p_j from evolution variable $\mathbf{k}_\mathrm{T}^2$ and z

momentum map

$$p_i^\mu = z\tilde{p}_i^\mu$$ absorbs collinear recoil

$$p_k^\mu = \tilde{p}_k^\mu$$ defines azimuthal angle ϕ

$$\sum_l k_l^\mu = K^\mu = \Lambda^\mu_{\ \nu}\tilde{K}^\nu = \sum_l \Lambda^\mu_{\ \nu}\tilde{k}_l^\nu \qquad K^2 = \tilde{K}^2$$

absorbs transverse recoil

with $\Lambda^\mu_{\ \nu} = g^\mu_{\ \nu} - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})_\nu}{(K+\tilde{K})^2} + 2\frac{K^\mu\tilde{K}_\nu}{\tilde{K}^2}$ Catani, Seymour '97

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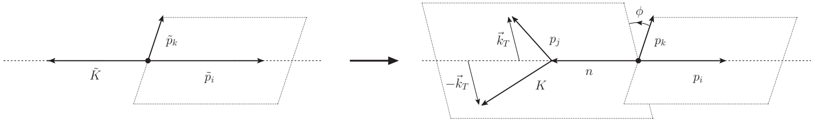

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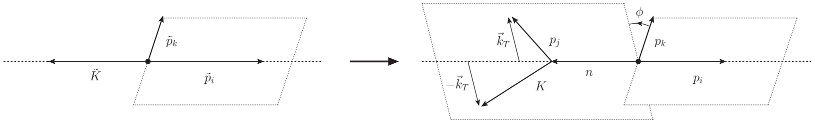

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ALARIC – NLL accuracy

Analytic proof of NLL accuracy

With the proper soft-collinear splitting functions (in large N_c), need to show

- colour assignment is correct for multiple emissions
- recoil effects vanish in soft limit, $\rho \to 0$

Colour assigments

This is trivially realised by disentangling colour and recoil spectators.

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ALARIC – NLL accuracy

Recoil effects

Decompose boost vector Λ into large and small components

$$\Lambda^\mu_{\ \nu} = g^\mu_{\ \nu} + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

in the limit $\rho \to 0$

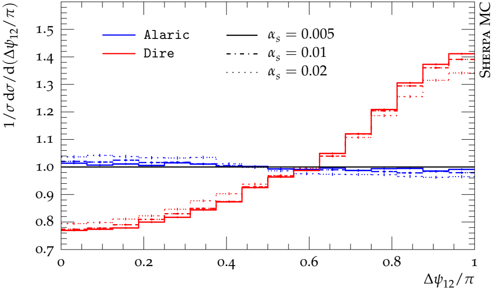
$$A^\nu \to 2\frac{\tilde{K}X}{\tilde{K}^2}\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2} \qquad \text{and} \qquad B^\nu \to \frac{\tilde{K}^\nu}{\tilde{K}^2}$$

where X^ν vanishes fast enough as $\rho \to 0$. Thus, $\Lambda^\mu_{\ \nu} \to g^\mu_{\ \nu}$.
When applied to soft momentum p_l, this gives

$$\frac{\Delta k^l_{\mathrm{T}}}{k^l_{\mathrm{T}}} \to 0$$

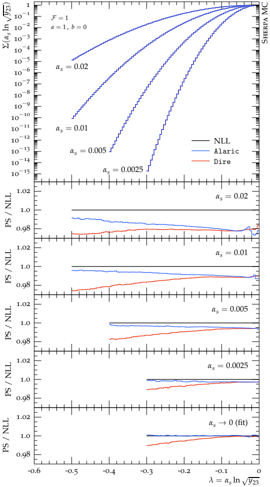
Full details in Herren, Höche, Krauss, Reichelt, MS '22

Alaric – NLL accuracy



$\rightarrow$ subsequent emissions are uncorrelated to NLL accuracy

ALARIC – NLL accuracy



limit: $\alpha_s \to 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

- Durham jet rate y_{23} $\beta = 0$
- Total jet broadening B_T $\beta = 0$
- Durham jet rate $FC_{1/2}$ $\beta = \frac{1}{2}$
- Thrust $1 - T$ $\beta = 1$

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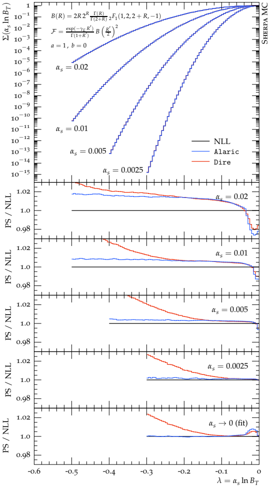

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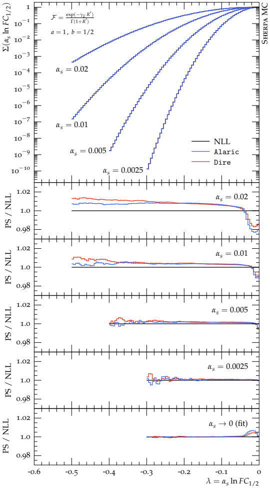

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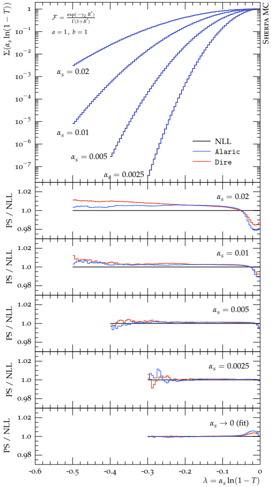

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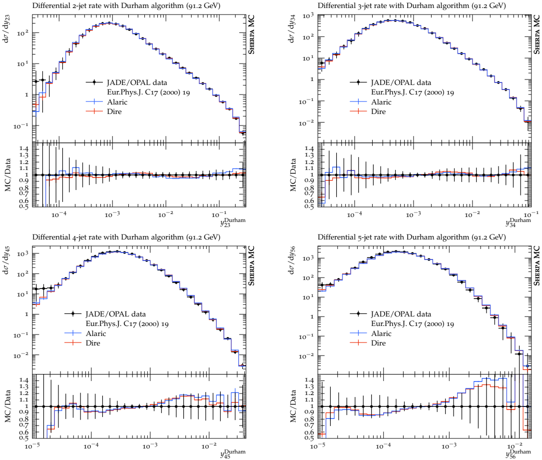
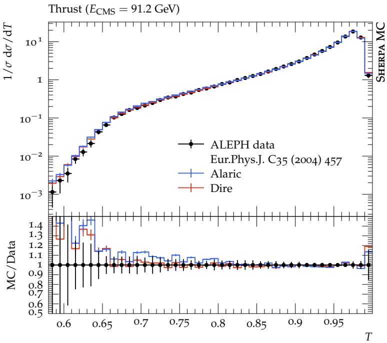
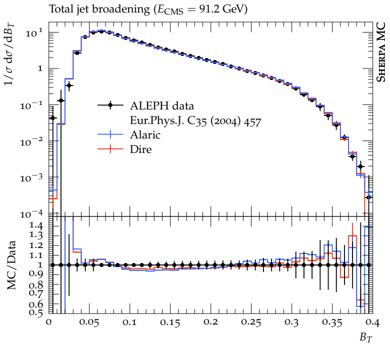
Alaric – LEP phenomenology

- Alaric +Pythia string had.
- hadronisation models are not infrared safe and depend on distribution of soft gluons
- tunes are shower specific
- no matching or multijet merging



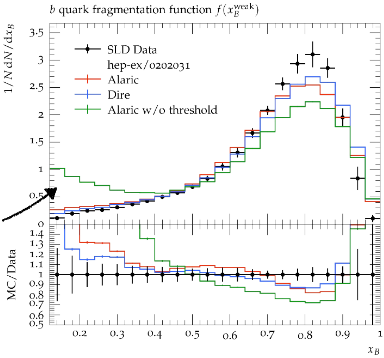
Alaric – LEP phenomenology





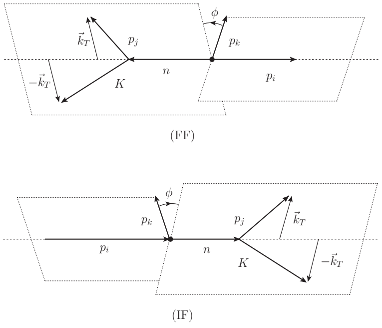
- no matching to fixed-order $3j$, $4j$, etc.

ALARIC – LEP phenomenology



- ALARIC is constructed with massless quarks so far
- quark masses are phenomenologically relevant
- quick fix: flavour thresholds for $g \to c\bar{c}$ and $g \to b\bar{b}$

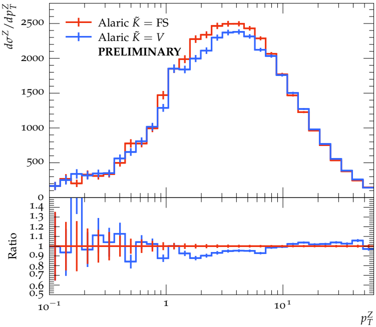
Alaric – LHC outlook



- formalism of Herren, Höche, Krauss, Reichelt, MS '22 completely general and applicable to initial state evolution
- more freedom in choice of
  - evolution variable, hard radiator $\tilde{K}$

ALARIC – LHC outlook

- formalism of Herre, Höche, Krauss, Reichelt, MS '22 completely general and applicable to initial state evolution
- more freedom in choice of
  - evolution variable
  - hard radiator $\tilde{K}$ (in DY either EW boson or full final state)
  - ...



Conclusions

- parton showers are a numerical observable-independent and fully exclusive resummation tool for precision QCD predictions
- formal NLL accuracy crucial in parton shower development
  - meaningful uncertainty assignment
  - subleading colour evolution
  - NLO splitting kernels
  - N^2LO matching
  - ...
- formal NLL accuracy not necessarily phenomenologically relevant, other effects usually dominant
- phenomenological impact of higher-accuracy resummation cancelled by poorly understood hadronisation process

http://sherpa.hepforge.org

Thank you!

Backup