

NNLOPS– a comparison

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NNLOs comparison

Two approaches

- POWHEG/MiNLO + reweighting
 $pp \rightarrow h$ [Hamilton, Nason, Re, Zanderighi arXiv:1309.0017](#)
 $pp \rightarrow \ell^+ \ell^-$ [Karlberg, Re, Zanderighi arXiv:1407.2940](#)
- S-Mc@NLO + UNLOPS + q_\perp -slicing
 $pp \rightarrow \ell^+ \ell^-$ [Höche, Li, Prestel arXiv:1405.3607](#)
 $pp \rightarrow h$ [Höche, Li, Prestel arXiv:1407.3773](#)

both only work for singlet production where NNLO contrib only lives at $p_\perp = 0$

\Rightarrow reflects my understanding of these papers, no guarantee that correct in all respects

Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_n B_n(\Phi_n) \left[\Delta_n^{(\mathcal{K})}(t_c, \mu_Q^2) O(\Phi_n) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n(\Phi_1) \Delta_n^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_{n+1}) \right]$$

- splitting kernel $\mathcal{K}_n = \sum \mathcal{K}_i$ and $\mathcal{K}_i(\Phi_1) \propto \frac{\alpha_s}{t} P_i(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots] \quad L = \log \frac{t}{t'}$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t , soft limit at most in $N_c \rightarrow \infty$
 c_1 correct, c_2 at most in $N_c \rightarrow \infty$ approximation
- 1-loop running $\alpha_s(k_\perp)$ catches dominant terms of higher log. order
 \Rightarrow crucial in defining “parton shower accuracy”

Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_n B_n(\Phi_n) \mathcal{F}_n(\mu_Q^2, O)$$

$$\mathcal{F}_n(t, O) = \Delta_n(t_c, t) O(\Phi_n) + \int_{t_c}^t d\hat{\Phi}_1 \mathcal{K}_n(\hat{\Phi}_1) \Delta_n(\hat{t}, t) \mathcal{F}_{n+1}(\hat{t}, O)$$

- generating functional $\mathcal{F}_n(t, O)$ on n -parton state with starting scale t
- unitary higher order approximation with
 - $R_n(\Phi_{n+1}) \approx B_n(\Phi_n) \cdot \mathcal{K}_n(\hat{\Phi}_1) \Theta(\mu_Q^2 - \hat{t}) = D_n(\Phi_{n+1})$
 - $V_n(\Phi_n) \approx -B_n(\Phi_n) \cdot \int d\hat{\Phi}_1 \mathcal{K}_n(\hat{\Phi}_1) \Theta(\mu_Q^2 - \hat{t}) = -\int d\hat{\Phi}_1 D_n(\Phi_n, \hat{\Phi}_1)$
- can improve through (soft limit)
 - matrix element correction $D_n(\Phi_{n+1}) = R_n(\Phi_{n+1}) \Theta(\mu_Q^2 - \hat{t})$
 - $N_c = 3$ splitting functions $D_n(\Phi_{n+1}) = B_n(\Phi_n) \otimes \mathcal{K}_n(\hat{\Phi}_1) \Theta(\mu_Q^2 - \hat{t})$

NLOPS

Two types:

- MC@NLO/POWHEG

$$\langle O \rangle^{\text{NLOPS}} = \int d\Phi_0 \bar{B}_0(\Phi_0) \bar{\mathcal{F}}_0(\mu_Q^2, O) + \int d\Phi_1 \mathbb{H}_0(\Phi_1) \mathcal{F}_1(t_1, O)$$

NLO correction smeared through resummation phase space

- UNLOPS

$$\begin{aligned} \langle O \rangle^{\text{UNLOPS}} &= \int d\Phi_0 B_0(\Phi_0) \bar{\mathcal{F}}_0(\mu_Q^2, O) + \int d\Phi_1 \mathbb{H}_0(\Phi_1) \mathcal{F}_1(t_1, O) \\ &\quad + \int d\Phi_0 [\bar{B}_0 - B_0](\Phi_0) O(\Phi_0) \end{aligned}$$

NLO correction localised in Born phase space, not showered

⇒ **same logarithmic accuracy, differ at $\mathcal{O}(\alpha_s^2)$**

⇒ **resum logarithms wrt. Φ_0**

NLOs

Two types:

- MC@NLO/POWHEG

$$\langle O \rangle^{\text{NLOs}} = \int d\Phi_0 \bar{B}_0(\Phi_0) \bar{\mathcal{F}}_0(\mu_Q^2, O) + \int d\Phi_1 \mathbb{H}_0(\Phi_1) \mathcal{F}_1(t_1, O)$$

NLO correction smeared through resummation phase space

- UNLOs

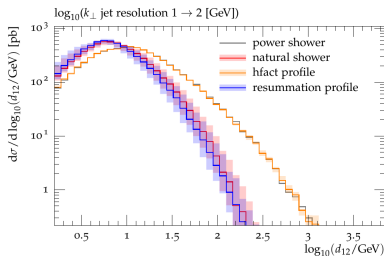
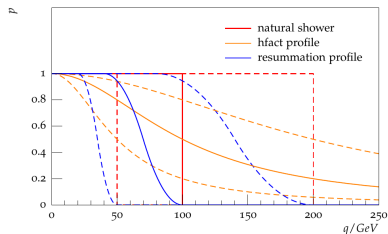
$$\begin{aligned} \langle O \rangle^{\text{UNLOs}} &= \int d\Phi_0 B_0(\Phi_0) \bar{\mathcal{F}}_0(\mu_Q^2, O) + \int d\Phi_1 \mathbb{H}_0(\Phi_1) \mathcal{F}_1(t_1, O) \\ &\quad + \int d\Phi_0 [\bar{B}_0 - B_0](\Phi_0) O(\Phi_0) \end{aligned}$$

NLO correction localised in Born phase space, not showered

- ⇒ same logarithmic accuracy, differ at $\mathcal{O}(\alpha_s^2)$
- ⇒ resum logarithms wrt. Φ_0

$$\begin{aligned} \bar{B} &= B + V + \int d\hat{\Phi}_1 D_0 \Theta(\mu_Q^2 - t) \\ \mathbb{H} &= R - D_0 \Theta(t - \mu_Q^2) \\ \bar{\mathcal{F}}_n(t, O) &= \bar{\Delta}_n(t_c, t) O(\Phi_n) \\ &\quad + \int d\hat{\Phi}_1 \frac{D_n}{B_n} \bar{\Delta}_n(t_c, \hat{t}) \mathcal{F}_{n+1}(\hat{t}, O) \end{aligned}$$

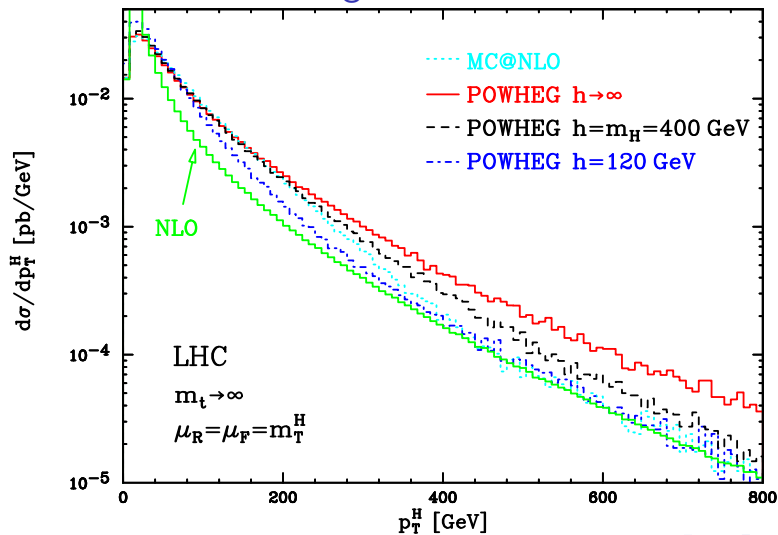
NLOs – resummation region



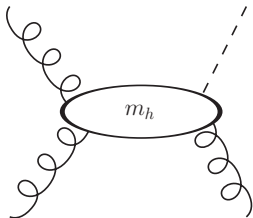
- MC@NLO choice: resummation region $\propto \Theta(\mu_Q^2 - t)$
- POWHEG choice: resummation region $\propto \frac{h^\gamma}{p_{\perp}^\gamma + h^\gamma}$

graphics by S. Plätzer

NLOs – resummation region



CKKW/MiNLO



$$\alpha_s^3(\mu_R) = \alpha_s^3(m_h)$$

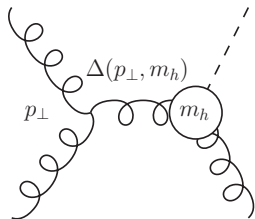
Sudakov resums leading jet p_\perp in $pp \rightarrow hj$ wrt. $pp \rightarrow h$
 $\Delta(p_\perp > m_h, m_h) = 1$
 \rightarrow underlying every multi-jet merging method

standard NLO

$$\langle O \rangle = \int d\Phi_B \alpha_s^2(\mu_R) [B + \alpha_s(\mu_R)V] O(\Phi_B) + \int d\Phi_R \alpha_s^3(\mu_R) R(\Phi_R) O(\Phi_R)$$

- with NLL Sudakov result is finite as $p_\perp \rightarrow 0$
- include proc. dep. B_2 to be NLO also in $pp \rightarrow h$

CKKW/MiNLO



$$\alpha_s^3(\tilde{\mu}_R) = \alpha_s^2(m_h)\alpha_s(p_\perp)$$

Sudakov resums leading jet p_\perp in $pp \rightarrow hj$ wrt. $pp \rightarrow h$

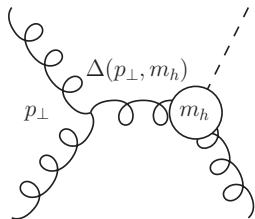
$\Delta(p_\perp > m_h, m_h) = 1$
 \rightarrow underlying every multi-jet merging method

CKKW/MiNLO

$$\begin{aligned} \langle O \rangle = & \int d\Phi_B \alpha_s^2(\tilde{\mu}_R) \Delta(p_\perp, m_h) \\ & \times \left[B + \alpha_s(\tilde{\mu}_R)V + B\Delta^{(1)} \right] O(\Phi_B) \\ & + \int d\Phi_R \alpha_s^3(\tilde{\mu}_R) \Delta(p_\perp, m_h) R(\Phi_R) O(\Phi_R) \end{aligned}$$

- with NLL Sudakov result is finite as $p_\perp \rightarrow 0$
- include proc. dep. B_2 to be NLO also in $pp \rightarrow h$

CKKW/MiNLO



$$\alpha_s^3(\tilde{\mu}_R) = \alpha_s^2(m_h)\alpha_s(p_\perp)$$

Sudakov resums leading jet p_\perp in $pp \rightarrow hj$ wrt. $pp \rightarrow h$

$\Delta(p_\perp > m_h, m_h) = 1$
→ underlying every multi-jet merging method

CKKW/MiNLO

$$\begin{aligned} \langle O \rangle &= \int d\Phi_B \alpha_s^2(\tilde{\mu}_R) \Delta(p_\perp, m_h) \\ &\quad \times \left[B + \alpha_s(\tilde{\mu}_R)V + B\Delta^{(1)} \right] O(\Phi_B) \\ &+ \int d\Phi_R \alpha_s^3(\tilde{\mu}_R) \Delta(p_\perp, m_h) R(\Phi_R) O(\Phi_R) \end{aligned}$$

- with NLL Sudakov result is finite as $p_\perp \rightarrow 0$
- include proc. dep. B_2 to be NLO also in $pp \rightarrow h$

NNLOs with POWHEG/MiNLO + reweighting

Hamilton, Nason, Re, Zanderighi arXiv:1309.0017, Karlberg, Re, Zanderighi arXiv:1407.2940

A simulation that is accurate to NLO in both Φ_0 and Φ_1 can be reweighted differentially in Φ_0 by

$$K(\Phi_0) = \frac{d\sigma^{\text{NNLO}}}{d\Phi_0} \bigg/ \frac{d\sigma^{\text{MiNLO}}}{d\Phi_0}$$

to give a NNLO correct result. The resummation accuracy remains untouched.

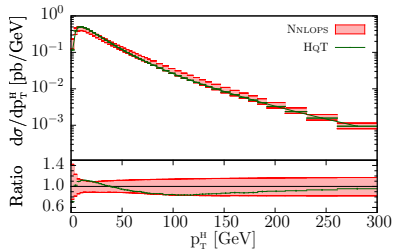
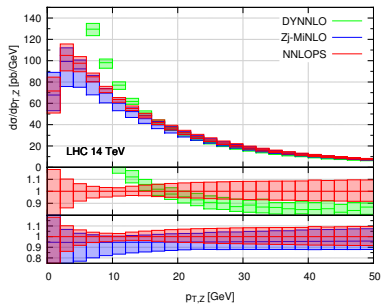
- generate MiNLO $pp \rightarrow hj$, reweight in y_h to HNNLO
- generate MiNLO $pp \rightarrow \ell\ell j$, reweight in $y_{\ell\ell}$, θ_ℓ^* , $a_{m\ell\ell}$ to DYNNLO

problem: cannot use same scales in POWHEG/MiNLO and HNNLO/DYNNLO

- residual effect on observables in Φ_1 beyond accuracy

NNLOs with POWHEG/MiNLO + reweighting

Hamilton, Nason, Re, Zanderighi arXiv:1309.0017, Karlberg, Re, Zanderighi arXiv:1407.2940



NNLOs with S-Mc@NLO + UNLOs + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773

For $q_{\perp} > q_{\perp,\text{cut}}$ ($q_{\perp,\text{cut}} < t_c$) a S-Mc@NLO calculation in Φ_1 can be used to complement an exclusive NNLO calculation at zero q_{\perp} . Same as in NLO multijet merging, overlap of the Sudakov form factor and the NNLO calculation have to be subtracted.

$$\langle \mathcal{O} \rangle_{>q_{\perp,\text{cut}}}^{\text{S-Mc@NLO}} = \int_{q_{\perp,\text{cut}}} d\Phi_1 \bar{B}_1(\Phi_1) \tilde{\mathcal{F}}_1(\mathcal{O}) + \int_{q_{\perp,\text{cut}}} d\Phi_2 \mathbb{H}_1(\Phi_2) \mathcal{F}_2(\mathcal{O})$$

add Δ_0 for resummation wrt. Φ_0 , subtract $\mathcal{O}(\alpha_s)$ expansion

⇒ **match NLO Φ_1 with S-Mc@NLO**
match NNLO Φ_0 UNLOs-style

NNLOPs with S-Mc@NLO + UNLOPs + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773

$$\begin{aligned} \langle O \rangle^{\text{UN}^2\text{LOPs}} &= \int d\Phi_0 \bar{\bar{B}}_0^{q_{\perp}, \text{cut}}(\Phi_0) O(\Phi_0) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 \left[1 - \Delta_0(1 + \Delta_0^{(1)}) \right] B_1(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0(1 + \Delta_0^{(1)}) B_1(\Phi_1) \bar{\mathcal{F}}_1(O) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0] \bar{B}_1^R(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0 \bar{B}_1^R(\Phi_1) \bar{\mathcal{F}}_1(O) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 [1 - \Delta_0] \mathbb{H}_1^R(\Phi_2) O(\Phi_2) + \int_{q_{\perp}, \text{cut}} d\Phi_2 \Delta_0 \mathbb{H}_1^R(\Phi_2) \mathcal{F}_2(O) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 \mathbb{H}_1^E(\Phi_2) \mathcal{F}_2(O) \end{aligned}$$

$$\bar{B}^R = \bar{B} - B$$

$$\bar{\mathbb{H}}^R = \mathbb{H} \Theta(t_2 - t_1) \Theta(t_1 - t_c)$$

$$\bar{\mathbb{H}}^E = \bar{\mathbb{H}} - \bar{\mathbb{H}}^R$$



NNLOPs with S-Mc@NLO + UNLOPs + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773

$\langle O \rangle^{\text{UN}^2\text{LOPs}}$

$$\begin{aligned}
 &= \int d\Phi_0 \bar{\bar{B}}_0^{q_{\perp}, \text{cut}}(\Phi_0) O(\Phi_0) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0(1 + \Delta_0^{(1)})] B_1(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0(1 + \Delta_0^{(1)}) B_1(\Phi_1) \bar{\mathcal{F}}_1(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0] \bar{B}_1^R(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0 \bar{B}_1^R(\Phi_1) \bar{\mathcal{F}}_1(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 [1 - \Delta_0] \mathbb{H}_1^R(\Phi_2) O(\Phi_2) + \int_{q_{\perp}, \text{cut}} d\Phi_2 \Delta_0 \mathbb{H}_1^R(\Phi_2) \mathcal{F}_2(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 \mathbb{H}_1^E(\Phi_2) \mathcal{F}_2(O)
 \end{aligned}$$

$$\bar{B}^R = \bar{B} - B$$

$$\bar{\mathbb{H}}^R = \mathbb{H} \Theta(t_2 - t_1) \Theta(t_1 - t_c)$$

$$\bar{\mathbb{H}}^E = \bar{\mathbb{H}} - \bar{\mathbb{H}}^R$$

excl. NNLO contribution



NNLOPs with S-MC@NLO + UNLOPs + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773

$$\begin{aligned}
 \langle O \rangle^{\text{UN}^2\text{LOPs}} &= \int d\Phi_0 \bar{\bar{B}}_0^{q_{\perp}, \text{cut}}(\Phi_0) O(\Phi_0) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0(1 + \Delta_0^{(1)})] B_1(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0(1 + \Delta_0^{(1)}) B_1(\Phi_1) \bar{\mathcal{F}}_1(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0] \bar{B}_1^R(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0 \bar{B}_1^R(\Phi_1) \bar{\mathcal{F}}_1(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 [1 - \Delta_0] \mathbb{H}_1^R(\Phi_2) O(\Phi_2) + \int_{q_{\perp}, \text{cut}} d\Phi_2 \Delta_0 \mathbb{H}_1^R(\Phi_2) \mathcal{F}_2(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 \mathbb{H}_1^E(\Phi_2) \mathcal{F}_2(O)
 \end{aligned}$$

$$\bar{B}^R = \bar{B} - B$$

$$\bar{\mathbb{H}}^R = \mathbb{H} \Theta(t_2 - t_1) \Theta(t_1 - t_c)$$

$$\bar{\mathbb{H}}^E = \bar{\mathbb{H}} - \bar{\mathbb{H}}^R$$



S-MC@NLO in Φ_1 , with Δ_0 resummation

NNLOPS with S-Mc@NLO + UNLOPS + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773

$$\begin{aligned} \langle O \rangle^{\text{UN}^2\text{LOPS}} &= \int d\Phi_0 \bar{B}_0^{q_{\perp}, \text{cut}}(\Phi_0) O(\Phi_0) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0(1 + \Delta_0^{(1)})] B_1(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0(1 + \Delta_0^{(1)}) B_1(\Phi_1) \bar{\mathcal{F}}_1(O) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 [1 - \Delta_0] \bar{B}_1^R(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0 \bar{B}_1^R(\Phi_1) \bar{\mathcal{F}}_1(O) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 [1 - \Delta_0] \mathbb{H}_1^R(\Phi_2) O(\Phi_2) + \int_{q_{\perp}, \text{cut}} d\Phi_2 \Delta_0 \mathbb{H}_1^R(\Phi_2) \mathcal{F}_2(O) \\ &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 \mathbb{H}_1^E(\Phi_2) \mathcal{F}_2(O) \end{aligned}$$

$$\bar{B}^R = \bar{B} - B$$

$$\bar{\mathbb{H}}^R = \mathbb{H} \Theta(t_2 - t_1) \Theta(t_1 - t_c)$$

$$\bar{\mathbb{H}}^E = \bar{\mathbb{H}} - \bar{\mathbb{H}}^R$$

UNLOPS subtraction



NNLOs with S-Mc@NLO + UNLOs + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773

$$\begin{aligned}
 \langle O \rangle^{\text{UN}^2\text{LOs}} &= \int d\Phi_0 \bar{\bar{B}}_0^{q_{\perp}, \text{cut}}(\Phi_0) O(\Phi_0) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 \left[1 - \Delta_0(1 + \Delta_0^{(1)}) \right] B_1(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0(1 + \Delta_0^{(1)}) B_1(\Phi_1) \bar{\mathcal{F}}_1(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_1 \left[1 - \Delta_0(1 + \Delta_0^{(1)}) \right] \tilde{B}_1^R(\Phi_1) O(\Phi_1) + \int_{q_{\perp}, \text{cut}} d\Phi_1 \Delta_0(1 + \Delta_0^{(1)}) \tilde{B}_1^R(\Phi_1) \bar{\mathcal{F}}_1(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 \left[1 - \Delta_0(1 + \Delta_0^{(1)}) \right] \mathbb{H}_1^R(\Phi_2) O(\Phi_2) + \int_{q_{\perp}, \text{cut}} d\Phi_2 \Delta_0(1 + \Delta_0^{(1)}) \mathbb{H}_1^R(\Phi_2) \mathcal{F}_2(O) \\
 &+ \int_{q_{\perp}, \text{cut}} d\Phi_2 \mathbb{H}_1^E(\Phi_2) \mathcal{F}_2(O)
 \end{aligned}$$

$$\bar{B}^R = \bar{B} - B$$

$$\bar{\mathbb{H}}^R = \mathbb{H} \Theta(t_2 - t_1) \Theta(t_1 - t_c)$$

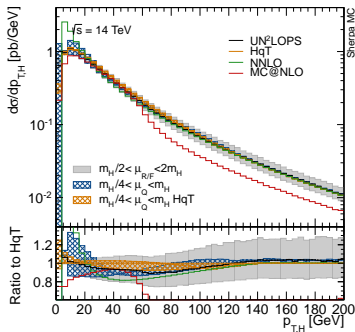
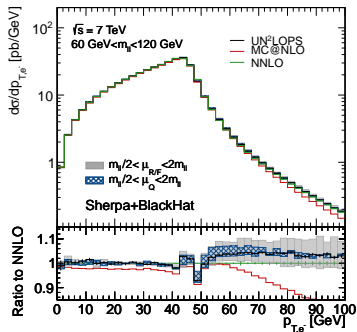
$$\bar{\mathbb{H}}^E = \bar{\mathbb{H}} - \bar{\mathbb{H}}^R$$

differs in higher order terms
possibly smoother for large NNLO corr.



NNLOs with S-Mc@NLO + UNLOs + q_{\perp} -slicing

Höche, Li, Prestel arXiv:1405.3607, arXiv:1407.3773



Conclusions

- both ansatzes differ by terms beyond NNLOPS accuracy
→ different matching conditions lead to different uncontrolled terms of relative $\mathcal{O}(\alpha_s^3)$, no impact on resummation accuracy
- if NNLO corrections are large, difference between schemes is large

Thank you for your attention!

Backup