

Multijet merging at leading and next-to-leading order

Marek Schönherr

Institute for Particle Physics Phenomenology

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LHCphenOnet



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 - NLOPS matching

Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t ,
 c_1 correctly described, c_2 at most in $N_c \rightarrow \infty$ approximation
- $\alpha_s \rightarrow \alpha_s(k_\perp)$ catches dominant terms of higher log. order

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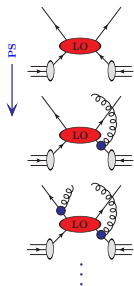
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- ② **Multijet merging**
 - MEPS – Multijet merging at leading order
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- ③ **General NLOPS matching**
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MEPs – Multijet merging at LO



LO $pp \rightarrow 2$ with parton showers

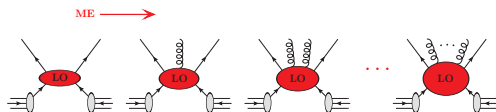
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- poor hard/wide angle emission pattern

vs. **LO** $pp \rightarrow n$ matrix elements

- + dominant terms for hard/wide angle rad.
- breakdown of α_s -expansion in log. region

- MEPS schemes: CKKW-type, MLM-type
- LO+(N)LL accuracy in every jet multiplicity
- scale setting scheme essential to preserve PS-resummation properties

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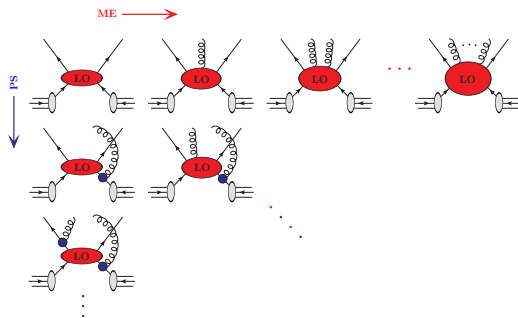
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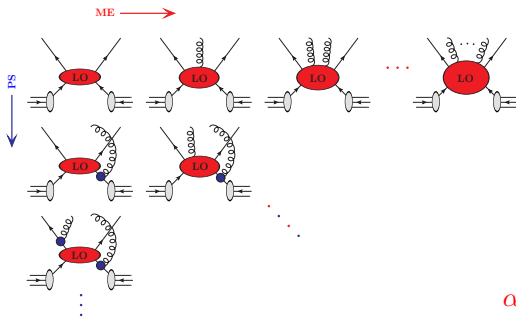
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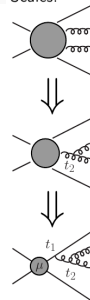
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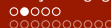
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Multijet merging

LO merging:

- LO accuracy for $n \leq n_{\max}$ -jet processes
- preserve (N)LL accuracy of the parton shower

Catani, Krauss, Kuhn, Webber JHEP11(2001)063

Lönnblad JHEP05(2002)046

Höche, Krauss, Schumann, Siegert JHEP05(2009)053

Hamilton, Richardson, Tully JHEP11(2009)038

Lönnblad, Prestel JHEP03(2012)019

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 \langle O \rangle^{\text{MEPS}} &= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
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 &\quad \times \left[\Delta_{n+1}^{(\mathcal{K})}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \mathcal{K}_{n+1} \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) O_{n+2} \right]
 \end{aligned}$$

- LOPS for n -jet process restricted to region $Q < Q_{\text{cut}}$
- LOPS for $n + 1$ -jet process with LOPS for n -jet process
- truncated showering to account for mismatch of t and Q [Nason JHEP11\(2004\)040](#)

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- α_s scales in $B \cdot \mathcal{K}$ and B_{n+1} must be the same to retain resummation properties of the parton shower
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→ need to use inverse parton shower

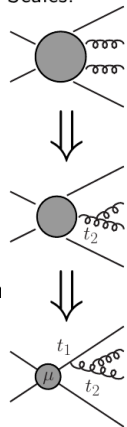
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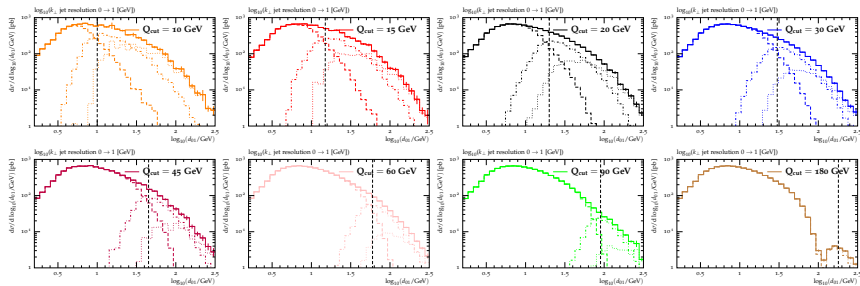
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Merging systematics

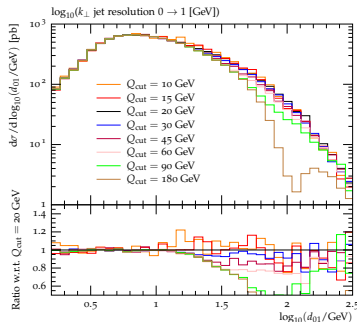
Merging cut Q_{cut} dependence ($pp \rightarrow Z + \text{jets}$ MEPS, up to 2 in ME):



- parton shower is trusted to correctly describe emissions $\lesssim Q_{\text{cut}}$
- changes the region where higher accuracy is used for calculation
 → part of the uncertainty is due to degraded accuracy for large Q_{cut}
- all samples are identical for $Q < Q_{\text{cut}}^{\text{smallest}}$ and $Q > Q_{\text{cut}}^{\text{largest}}$ by construction
- Q_{cut} dependence usually small

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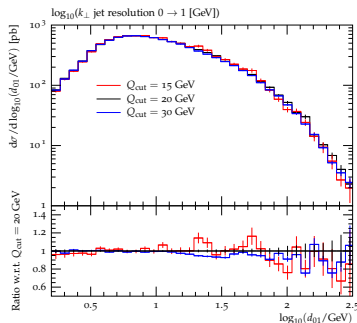
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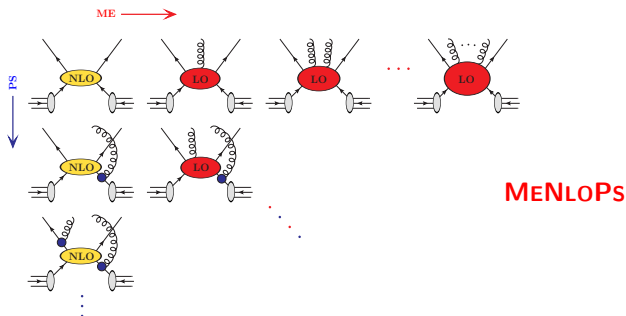
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MENLOPS– Multijet merging with NLO core



- promote core LOPs to NLOs (POWHEG, Mc@NLO)
→ can assess uncertainties (part I)
- combine with LOPs for higher multiplicities into incl. sample (MENLOPS),
preserve NLO+(N)LL accuracy in inclusive observables
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties

MENLOPs for MC@NLO

$$\begin{aligned}
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 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
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- restrict MC@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}

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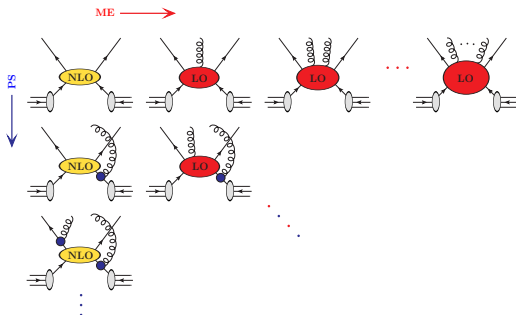
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MENLOPs for MC@NLO

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 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
 & + \int d\Phi_R \left[\frac{\bar{B}^{(A)}(\Phi_B)}{B(\Phi_B)} \left(1 - \frac{\mathbb{H}(\Phi_R)}{R(\Phi_R)} \right) + \frac{\mathbb{H}(\Phi_R)}{R(\Phi_R)} \right] R(\Phi_R) \Delta^{(K)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q - Q_{\text{cut}})
 \end{aligned}$$

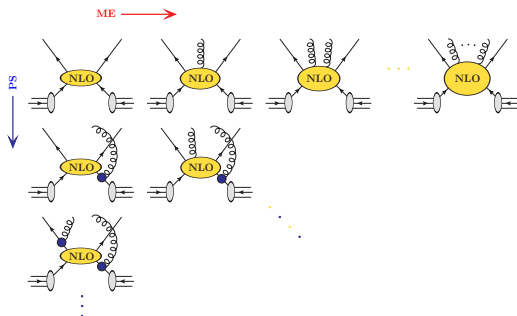
- restrict MC@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}

MEPs@NLO – Multijet merging at NLO



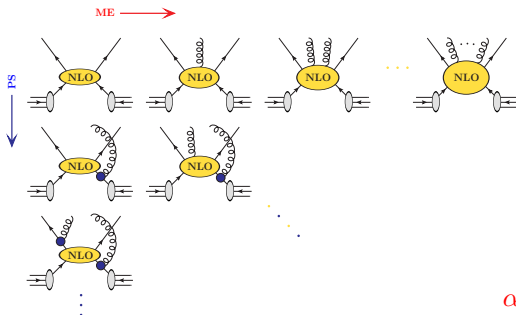
- promote LOPs to NLOs (POWHEG, Mc@NLO)
→ can assess uncertainties (part I)
- combine NLOs for successive multiplicities into incl. sample (MEPs@NLO),
preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties

MEPs@NLO – Multijet merging at NLO

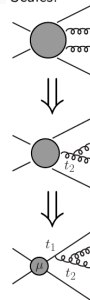


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preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties

MEPs@NLO – Multijet merging at NLO



Scales:



$$\alpha_s^{k+n}(\mu_{\text{eff}}) = \alpha_s^k(\mu) \alpha_s(t_1) \cdots \alpha_s(t_n)$$

- promote LOPs to NLOs (POWHEG, Mc@NLO)
→ can assess uncertainties (part I)
- combine NLOs for successive multiplicities into incl. sample (MEPs@NLO),
preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties

MEPs@NLO – Multijet merging at NLO

LO merging:

- LO accuracy for $n \leq n_{\max}$ -jet processes
- preserve (N)LL accuracy of the parton shower

Catani, Krauss, Kuhn, Webber JHEP11(2001)063

Lönnblad JHEP05(2002)046

Höche, Krauss, Schumann, Siegert JHEP05(2009)053

Hamilton, Richardson, Tully JHEP11(2009)038

Lönnblad, Prestel JHEP03(2012)019

- NLO accuracy for $n \leq n_{\max}$ -jet processes
- preserve (N)LL accuracy of the parton shower

Lavesson, Lönnblad JHEP12(2008)070

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

MEPs@NLO – Multijet merging at NLO

 $\langle O \rangle_{\text{MEPs@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\begin{aligned}
 &= \int d\Phi_n \bar{B}_n^{(A)} \left[\Delta_n^{(A)}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
 &+ \int d\Phi_{n+2} \left[R_{n+1} - D_{n+1}^{(A)} \right] \Theta(Q - Q_{\text{cut}}) O_{n+2}
 \end{aligned}$$

MEPS@NLO – Multijet merging at NLO

 $\langle O \rangle^{\text{MEPS@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

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 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
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MEPS@NLO – Multijet merging at NLO

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 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \left[1 + \frac{B_{n+1}}{B_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
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MEPs@NLO – Multijet merging at NLO

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Höche, Krauss, MS, Siegert arXiv:1207.5030

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Höche, Krauss, MS, Siebert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siebert JHEP01(2013)144

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 \end{aligned}$$

NLO merging – Generation of MC counterterm

$$\left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right]$$

- same form as exponent of Sudakov form factor $\Delta_n^{(PS)}(t_{n+1}, \mu_Q^2)$
- truncated parton shower on n -parton configuration underlying $n + 1$ -parton event
 - ① no emission \rightarrow retain $n + 1$ -parton event as is
 - ② first emission at t' with $Q > Q_{\text{cut}}$, multiply event weight with $B_{n+1}/\bar{B}_{n+1}^{(A)}$, restart evolution at t' , do not apply emission kinematics
 - ③ treat every subsequent emission as in standard truncated vetoed shower
- generates

$$\left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right] \Delta_n^{(PS)}(t_{n+1}, \mu_Q^2)$$

\Rightarrow **identify $\mathcal{O}(\alpha_s)$ counterterm with the omitted emission**

NLO merging

Renormalisation scales:

- determined by clustering using PS probabilities and taking the respective nodal values t_i

$$\alpha_s(\mu_R^2)^{n+k} = \alpha_s(\mu_{\text{core}})^n \prod_{i=1}^k \alpha_s(t_i)$$

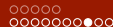
- change of scales $\mu_R \rightarrow \tilde{\mu}_R$ in MEs necessitates one-loop counter term

$$\alpha_s(\tilde{\mu}_R^2)^k \left(1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi} \beta_0 \sum_{i=1}^k \ln \frac{t_i}{\tilde{\mu}_R^2} \right)$$

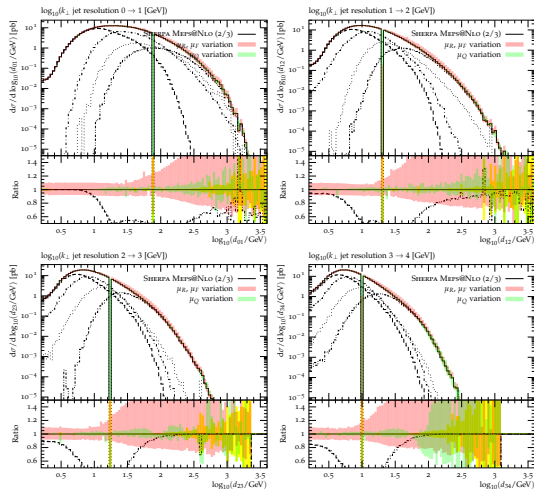
Factorisation scale:

- μ_F determined from core n -jet process
- change of scales $\mu_F \rightarrow \tilde{\mu}_F$ in MEs necessitates one-loop counter term

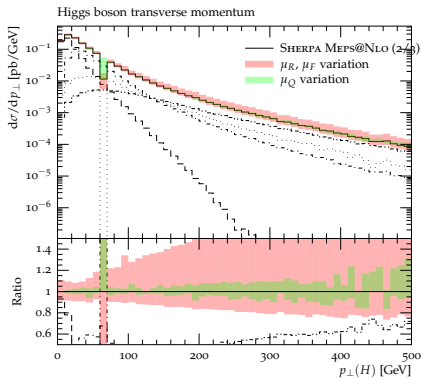
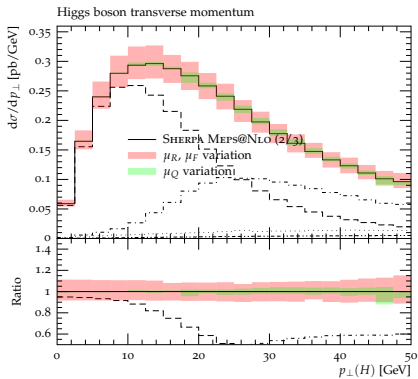
$$B_n(\Phi_n) \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi} \log \frac{\mu_F^2}{\tilde{\mu}_F^2} \left(\sum_{c=q,g}^n \int_{x_a}^1 \frac{dz}{z} P_{ac}(z) f_c(x_a/z, \tilde{\mu}_F^2) + \dots \right)$$



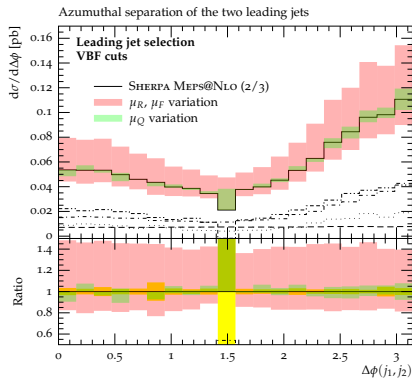
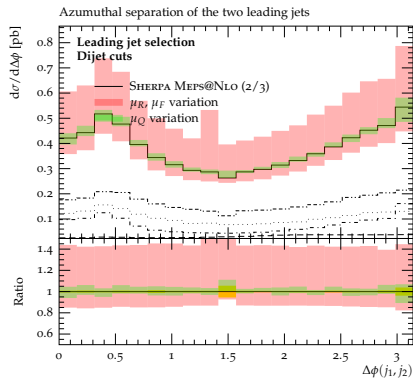
Merging systematics – $pp \rightarrow h + \text{jets}$ prelim



Merging systematics – $pp \rightarrow h + \text{jets}$ prelim



Merging systematics – $pp \rightarrow h + \text{jets}$ prelim



Contents

- ① Resummation properties of parton showers
- ② Multijet merging
 - MEPS – Multijet merging at leading order
 - MEPS@NLO – Multijet merging at next-to-leading order
- ③ **General NLOPS matching**
 - NLOPS matching

General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} = & \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 & + \int d\Phi_R \left[- \sum_i D_i^{(S)}(\Phi_R) O(\Phi_{B_i}) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) \right] O(\Phi_R)
 \end{aligned}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit

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 &+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)} \\
 \langle O \rangle_{\text{corr}}^{(A)} &= \int d\Phi_R \sum_i D_i^{(A)} [O(\Phi_R) - O(\Phi_{B_i})]
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NLOs matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\begin{aligned} \langle O \rangle^{\text{PS}} &= \int d\Phi_B B(\Phi_B) O(\Phi_B) \\ &= \int d\Phi_B B(\Phi_B) O(\Phi_B) + \end{aligned}$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
- $\langle O \rangle_{\text{corr}}^{(\mathcal{K})}$ generated by one parton shower step

NLOs matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right]$$

$$= \int d\Phi_B B(\Phi_B) O(\Phi_B) +$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
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NLOPS matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\begin{aligned} \langle O \rangle^{\text{PS}} &= \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right] \\ &= \int d\Phi_B B(\Phi_B) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_R B \cdot \mathcal{K}(\Phi_1) \left[O(\Phi_R) - O(\Phi_B) \right] + \mathcal{O}(\alpha_s^2) \end{aligned}$$

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- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
- $\langle O \rangle_{\text{corr}}^{(\mathcal{K})}$ generated by one parton shower step

NLOPS matching

$$\begin{aligned}
 \langle O \rangle^{\text{NLOPS}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)}
 \end{aligned}$$

- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale

NLOPS matching

$$\begin{aligned} \langle O \rangle^{\text{NLOPS}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\ & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\ & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \end{aligned}$$

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- **POWHEG and MC@NLO now differ in choice of $D_i^{(A)}$ and μ_Q^2**

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- **SHERPA:** $D_i^{(A)} = D_i^{(S)} \Theta(\mu_Q^2 - t)$ ($N_c = 3$ CS kernels), μ_Q free

POWHEG

Special choices:

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $D_i^{(A)} = \rho_i \cdot R$ with $\rho_i = D_i^{(S)} / \sum_i D_i^{(S)}$
 → each $\rho_i \cdot R$ contains only one divergence structure as defined by $D_i^{(S)}$

Consequences:

- no H-events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2} s_{\text{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $D_i^{(S)}$
- exponentiation of R through matrix element corrected parton shower
 NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2 / (p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
 → $D_i^{(A)} = \rho_i \cdot R \cdot f(p_\perp)$
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Mc@NLO – traditional scheme

Special choices:

Frixione, Webber JHEP06(2002)029

- exponentiation kernel $D_i^{(A)} = B \cdot \mathcal{K}_i$ with \mathcal{K}_i parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\max}$ parton shower starting scale
- in general, $D_i^{(A)}$ only leading colour approximation
NLO accuracy depends crucially on correctness of IR-limit

Modifications:

Frixione, Nason, Webber JHEP08(2003)007

- introduce soft modification function $f(p_\perp)$ such that

$$\sum B \cdot \mathcal{K}_i \cdot f(p_\perp) \xrightarrow{p_\perp \rightarrow 0} \sum D_i^{(S)}$$

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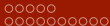
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MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Special choices:

Höche, Krauss, MS, Siegert JHEP09(2012)049

- exponentiation kernel $D_i^{(A)} = D_i^{(S)}$

Consequences:

- simplification of $\bar{B}^{(A)}$ -integral
- resummation scale $\mu_Q^2 = t_{\max}$ set by phase space limitation of subtraction terms
 - subtraction constrained in parton shower t needed for physical resummation
 - instructive example: use α_{cut} to explore effects Nagy PRD68(2003)094002
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MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm Höche, Krauss, MS, Siebert JHEP09(2012)049

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

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Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow D_i^{(A)}$
- $h(t)$ determined by parton shower $\Rightarrow D_i^{(PS)}$
- $g(t)$ **can be chosen freely** $\Rightarrow \text{const.} \cdot f$
constraints: $\text{sign}(f) = \text{sign}(g)$, $|f| \leq |g|$