

Theoretical uncertainties in multileg merged calculations

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LHCphenOnet



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- ① Overview
- ② Perturbative uncertainties
- ③ Non-perturbative uncertainties
- ④ How to combine them

Theoretical predictions at hadron level

Monte-Carlo event generators built upon factorisation assumptions

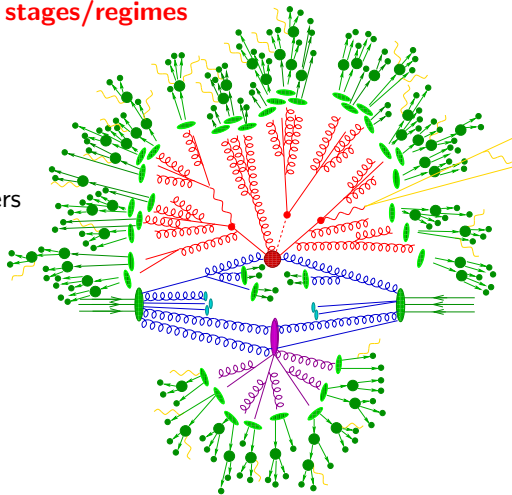
⇒ **divide calculation into different stages/regimes**

Perturbative regimes

- hard interaction
→ fixed-order MEs (LO/NLO)
- parton evolution
→ PDF evolution, parton showers
- QED FSR

Non-perturbative regimes

- PDFs at reference scale
- multiparton interactions
- hadronisation
- hadron decays
→ fit to data



Perturbative uncertainties I

Uncertainties of the hard matrix element:

- arbitrary scales:
 - μ_R from UV regularisation
 - μ_F from factorisation of non-perturbative PDFs
- use that all-orders result independent of these scales \rightarrow variation is an estimate of missing higher-order corrections
- central scale choice arbitrary:
 - choose such that higher order corrections are minimised
- amount of variation arbitrary:
 - customary range $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{R/F}^{\text{def}}$
 - \rightarrow can be done using event weights, keeping the kinematics the same
- $\mu_{R/F}$ independent terms of higher-order corrections cannot be probed
 - \rightarrow (N)NLO might be outside (N)LO uncertainty band
 - \rightarrow all-orders result may be outside uncertainty band

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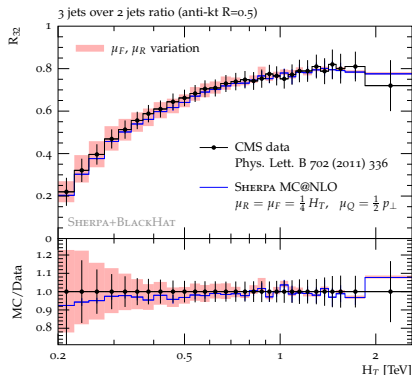
Perturbative uncertainties I

MC@NLO di-jet production

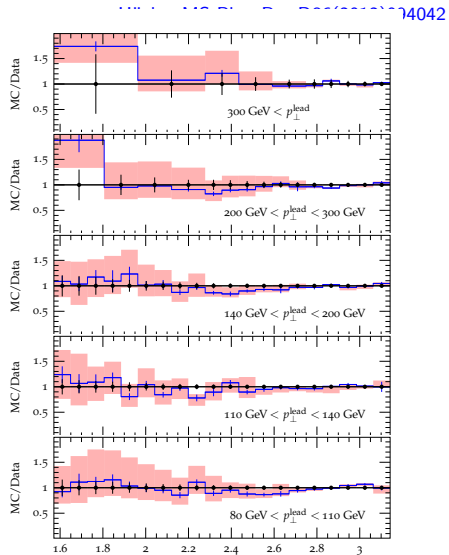
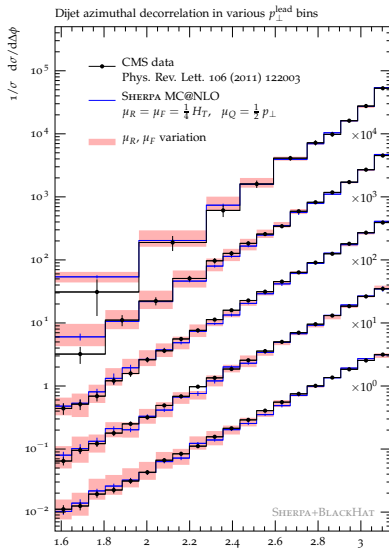
perturbative uncertainty estimate:

- $\mu_{R/F}^{\text{def}} = \frac{1}{4} H_T$
 $\mu_{R/F} \in \left[\frac{1}{2}, 2\right] \mu_{R/F}^{\text{def}}$

Höche, MS Phys.Rev.D86(2012)094042



Perturbative uncertainties I



Perturbative uncertainties II

- parton showers resum logarithms in their evolution variable t

$$\Delta(t, \mu_Q^2) = \exp \left\{ - \int_t^{\mu_Q^2} dt \int_{z_{\min}}^{z_{\max}} dz \int_0^{2\pi} d\phi \frac{\alpha_s(\mu_R)}{t} \mathcal{K}(t, z, \phi) \right\}$$

$$= \exp \left\{ c_1 \log^2 \frac{t}{\mu_Q^2} + c_2 \log \frac{t}{\mu_Q^2} + \dots \right\}$$

- $t_{\max} = \mu_Q^2$ is the starting scale of the parton shower for this emission
- μ_Q^2 enters the logarithms and plays the role of the resummation scale
- global starting scale μ_Q is a free choice
starting scales for subsequent emissions fixed by scale of previous emission
- initial state shower replaces the inclusive evolution of the PDF (emissions are integrated over) with its own evolution (emissions are sampled = Monte-Carlo integration)
→ choose $\mu_Q \sim \mu_F$ to avoid large logs being left over
- parton shower generates emissions probabilistically, μ_Q is a phase space cut
→ μ_Q **variation cannot be done with event weights, need to rerun**

Perturbative uncertainties II

- parton showers resum logarithms in their evolution variable t

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$$= \exp \left\{ c_1 \log^2 \frac{t}{\mu_Q^2} + c_2 \log \frac{t}{\mu_Q^2} + \dots \right\}$$

- technically, parton showers are only LL accurate (including the correct leading-colour NLL), however
- choice of μ_R in the shower fixed to $\sim k_\perp$ (prefactor tuned)
 - grasp leading terms of higher logarithmic accuracy
 - ⇒ **parton showers resum better than they should**
- but, contrary to higher logarithmic analytic no counter-terms to preserve them when μ_R is varied
 - currently, variation degrades the result
 - discussion in LH2013 on how to address this

Perturbative uncertainties II

Additional (discrete) uncertainties

- evolution variable t (k_{\perp} -like, θ -like, Q^2 -like, etc.)
→ which logarithms are resummed
 - recoil strategy
→ non-logarithmic corrections, but numerically sizeable/important already at moderate t
- ⇒ would need to be assessed as well

Parton shower uncertainty assessment only meaningful if (N)LO+PS combination consistent, i.e. same PDFs and values/evolution of α_s in the full partonic calculation.

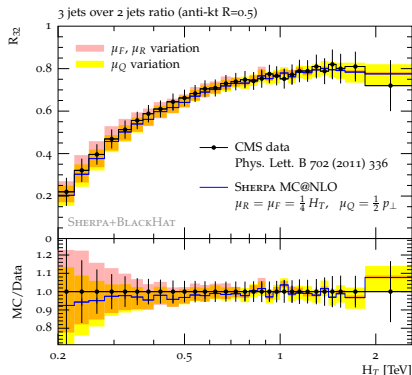
Perturbative uncertainties II

MC@NLO di-jet production

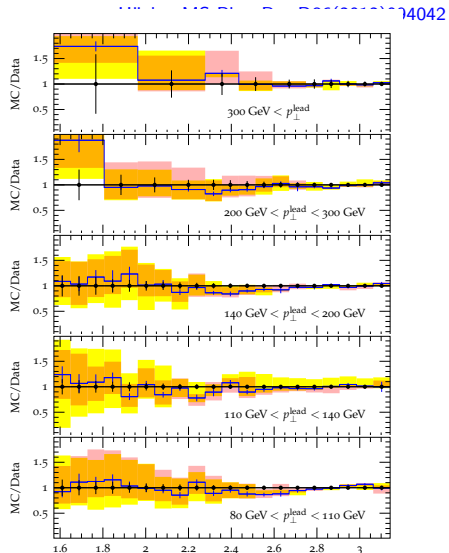
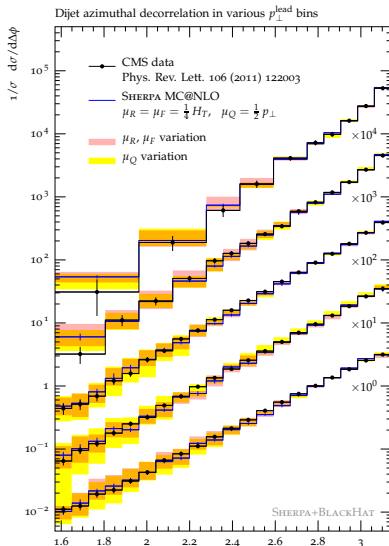
perturbative uncertainty estimate:

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 $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{R/F}^{\text{def}}$
- $\mu_Q^{\text{def}} = \frac{1}{2} p_{\perp}$
 $\mu_Q \in [\frac{1}{\sqrt{2}}, \sqrt{2}] \mu_Q^{\text{def}}$

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Perturbative uncertainties II



Perturbative uncertainties in merged samples

- consistent combination of fixed-order and parton shower
- scale choice

$$\alpha_s(\mu_R^{\text{def}})^{n+k} = \alpha_s^k(\mu_R^{\text{core}}) \prod_{i=0}^n \alpha_s(t_i)$$

to (N)LO accuracy in (N)LO merging necessary to preserve resummation behaviour

- can vary μ_R in NLO merging, necessitates counterterm
- similar considerations for μ_F
- resummation of parton shower unchanged

→ same considerations for μ_Q apply

→ one global μ_Q

parton shower of n-jet configuration fixed to start at t_n

- merging scale Q_{cut} :

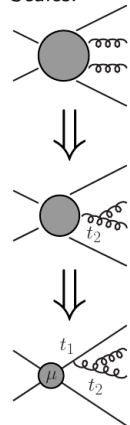
residual dependence beyond formal accuracy → uncertainty

→ vary within region such that parton shower approximation still sensible

→ changes the range of (N)LO acc.,

cannot be done via event weights

Scales:



Perturbative uncertainties in merged samples

MEPs@NLO Higgs+jets production

perturbative uncertainty estimate:

- $\mu_{core}^{def} = m_h$
 $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{R/F}^{def}$

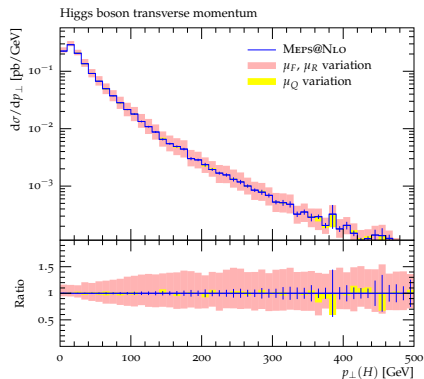
- $\mu_Q^{def} = m_h$
 $\mu_Q \in [\frac{1}{\sqrt{2}}, \sqrt{2}] \mu_Q^{def}$

- $Q_{cut} = 20 \text{ GeV}$

- $Q_{cut} \in [15, 30] \text{ GeV}$

be aware:

different curves have different accuracies in the region 15-30 GeV



Perturbative uncertainties in merged samples

MEPS@NLO Higgs+jets production

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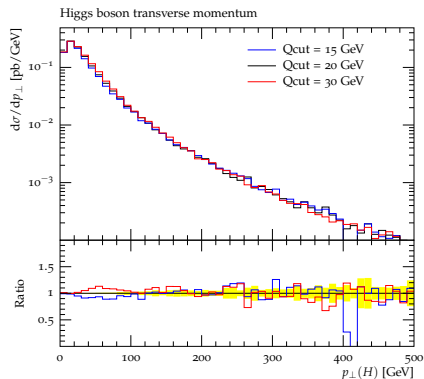
- $\mu_{core}^{def} = m_h$
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- $\mu_Q^{def} = m_h$
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Non-perturbative uncertainties

- only phenomenological models for non-perturbative physics (Had/UE), or parametrisation (PDF at reference scale)
 - unclear in how far they describe all aspects of the physics process
 - **different models/parametrisations should be taken for unc.**
 - models fitted to data
 - has statistical interpretation within a given (parameter) model
 - use PDF eigenvectors or model eigentunes to estimate uncertainties
- [Richardson, Winn EPJC72\(2012\)2178](#)
- PDF uncertainty on hard ME can be assessed using event weights
 - Had/UE uncertainties cannot be assessed using weights as the event structure changes (UE scatters and primordial Hadrons are probabilistic)
 - **need to rerun**
 - cumbersome for >15 parameters
 - simplified idea discussed in LH2011
 - vary MPI activity in transverse region $\pm 10\%$ (consistent with data)
changes the overall MPI activity

Perturbative uncertainties II

MC@NLO di-jet production

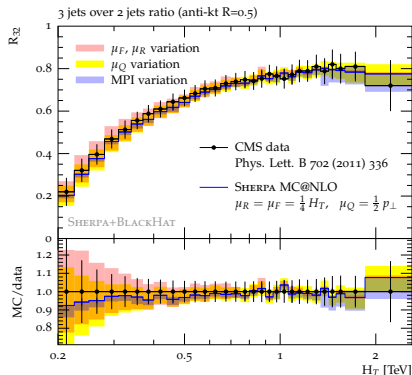
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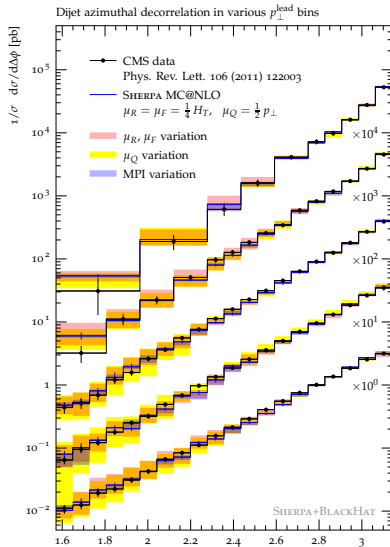
non-pert. uncertainty estimate:

- MPI activity in tr. region $\pm 10\%$

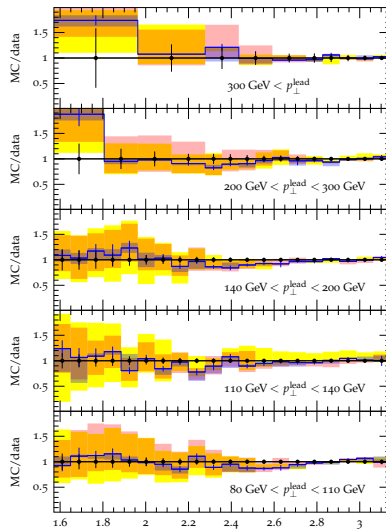
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Non-perturbative uncertainties



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How to combine them (?)

- no statistical interpretation of uncertainty estimate of a perturbative calculation, scale independent terms cannot be included
- uncertainties on fits to data can be interpreted statistically, however have to consider that model is incomplete (no first principles)
→ vary the model/parametrisation
- total uncertainty – how to combine them:
 - envelope?
→ minimal solution (underestimation very likely)
 - add in quadrature?
→ seems like good solution in between extremes
 - add linearly?
→ maximal solution (very conservative, overestimation likely)
- not all theoretical uncertainties can be assessed using event weights
→ need to rerun (at least) for parton shower, hadronisation, multiparton interaction uncertainties

Thank you for your attention!