



Multijet merging at next-to-leading order

Marek Schönherr

Institute for Particle Physics Phenomenology

Zürich, 12/03/2013



[JHEP09\(2012\)049](#), [Phys.Rev.Lett.110\(2013\)052001](#)

[arXiv:1207.5030](#), [JHEP01\(2013\)144](#)

[Phys.Rev.D86\(2012\)094042](#)*

LHCphenonnet



*in collaboration with T. Gehrmann, S. Höche, F. Krauss, F. Siegert



Contents

① General NLOPS matching

Resummation properties of parton showers

NLOPS matching

Case study

② Multijet merging

MEPS – Multijet merging at leading order

MEPS@NLO – Multijet merging at next-to-leading order

Results

③ Conclusions



Introduction

Importance of matching NLO calculations with parton showers

- exclusive final states
- observable independent combination of fixed order and resummation
- problem double counting: both NLO and PS are approximations to higher order corrections → MC@NLO and POWHEG

Importance of multijet merging

- simultaneous description multijet topologies
→ every jet multiplicity at same fixed order accuracy
- resum hierarchies in emission scales

Uncertainties of NLOs matching/MEPS@NLO merging

- usual μ_R and μ_F variation as in NLO calculations
- also μ_Q -variation as in analytic resummations



Introduction

Importance of matching NLO calculations with parton showers

- exclusive final states
- observable independent combination of fixed order and resummation
- problem double counting: both NLO and PS are approximations to higher order corrections → MC@NLO and POWHEG

Importance of multijet merging

- simultaneous description multijet topologies
→ every jet multiplicity at same fixed order accuracy
- resum hierarchies in emission scales

Uncertainties of NLOs matching/MEPS@NLO merging

- usual μ_R and μ_F variation as in NLO calculations
- also μ_Q -variation as in analytic resummations



Introduction

Importance of matching NLO calculations with parton showers

- exclusive final states
- observable independent combination of fixed order and resummation
- problem double counting: both NLO and PS are approximations to higher order corrections → MC@NLO and POWHEG

Importance of multijet merging

- simultaneous description multijet topologies
→ every jet multiplicity at same fixed order accuracy
- resum hierarchies in emission scales

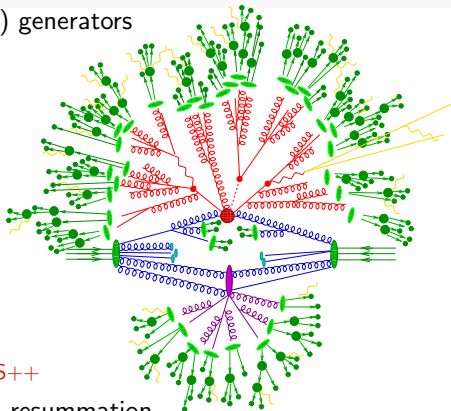
Uncertainties of NLOPS matching/MEPS@NLO merging

- usual μ_R and μ_F variation as in NLO calculations
- also μ_Q -variation as in analytic resummations



The SHERPA event generator framework

- Two multi-purpose Matrix Element (ME) generators
AMEGIC++ JHEP02(2002)044
COMIX JHEP12(2008)039
CS subtraction EPJC53(2008)501
- A Parton Shower (PS) generator
CSSHOWER++ JHEP03(2008)038
- A multiple interaction simulation
à la Pythia **AMISIC++** hep-ph/0601012
- A cluster fragmentation module
AHADIC++ EPJC36(2004)381
- A hadron and τ decay package **HADRONS++**
- A higher order QED generator using YFS-resummation
PHOTONS++ JHEP12(2008)018



Sherpa's traditional strength is the perturbative part of the event

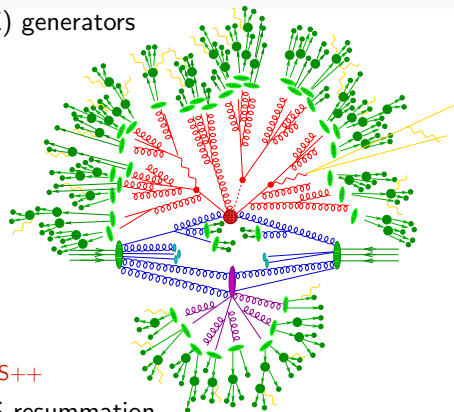
MEPs (CKKW), MC@NLO, MENLOs, MEPS@NLO

→ full analytic control mandatory for consistency/accuracy



The SHERPA event generator framework

- Two multi-purpose Matrix Element (ME) generators
AMEGIC++ JHEP02(2002)044
COMIX JHEP12(2008)039
CS subtraction EPJC53(2008)501
- A Parton Shower (PS) generator
CSSHOWER++ JHEP03(2008)038
- A multiple interaction simulation
à la Pythia **AMISIC++** hep-ph/0601012
- A cluster fragmentation module
AHADIC++ EPJC36(2004)381
- A hadron and τ decay package **HADRONS++**
- A higher order QED generator using YFS-resummation
PHOTONS++ JHEP12(2008)018



Sherpa's traditional strength is the perturbative part of the event

MEPs (CKKW), **Mc@NLO**, MENLOs, **MEPs@NLO**

→ full analytic control mandatory for consistency/accuracy



Contents

1 General NLOPS matching

Resummation properties of parton showers

NLOPS matching

Case study

2 Multijet merging

MEPS – Multijet merging at leading order

MEPS@NLO – Multijet merging at next-to-leading order

Results

3 Conclusions



Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t ,
 c_1 correctly described, c_2 at most in $N_c \rightarrow \infty$ approximation
- $\alpha_s \rightarrow \alpha_s(k_\perp)$ catches dominant terms of higher log. order



Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right]$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t , c_1 correctly described, c_2 at most in $N_c \rightarrow \infty$ approximation
- $\alpha_s \rightarrow \alpha_s(k_\perp)$ catches dominant terms of higher log. order



Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right]$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t ,
 c_1 correctly described, c_2 at most in $N_c \rightarrow \infty$ approximation
- $\alpha_s \rightarrow \alpha_s(k_\perp)$ catches dominant terms of higher log. order



Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right]$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t ,
 c_1 correctly described, c_2 at most in $N_c \rightarrow \infty$ approximation
- $\alpha_s \rightarrow \alpha_s(k_\perp)$ catches dominant terms of higher log. order



Resummation properties of parton showers

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right]$$

- splitting kernel $\mathcal{K}(\Phi_1) \propto \frac{\alpha_s}{t} P(z)$, $\Phi_1 = \{t, z, \phi\}$
- Sudakov form factor

$$\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right] = \exp [c_1 \alpha_s L^2 + c_2 \alpha_s L + \dots]$$

- parton shower starting scale μ_Q plays role of resummation scale, at LO commonly identified with μ_F to recover PDF evolution
- resummation in evolution variable t ,
 c_1 correctly described, c_2 at most in $N_c \rightarrow \infty$ approximation
- $\alpha_s \rightarrow \alpha_s(k_\perp)$ catches dominant terms of higher log. order



General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} = & \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 & + \int d\Phi_R \left[- \sum_i D_i^{(S)}(\Phi_R) \right] O(\Phi_{B_i}) \\
 & + \int d\Phi_R \left[R(\Phi_R) \phantom{- \sum_i D_i^{(S)}(\Phi_R)} \right] O(\Phi_R)
 \end{aligned}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit



General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} &= \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 &+ \int d\Phi_R \left[\sum_i D_i^{(A)}(\Phi_R) O(\Phi_{B_i}) - \sum_i D_i^{(S)}(\Phi_R) O(\Phi_{B_i}) \right] \\
 &+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)} \\
 \langle O \rangle_{\text{corr}}^{(A)} &= \int d\Phi_R \sum_i D_i^{(A)} [O(\Phi_R) - O(\Phi_{B_i})]
 \end{aligned}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit



General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} &= \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 &+ \sum_i \int d\Phi_R \left[D_i^{(A)}(\Phi_R) - D_i^{(S)}(\Phi_R) \right] O(\Phi_{B_i}) \\
 &+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)}
 \end{aligned}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit



General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} &= \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 &+ \sum_i \int d\Phi_{B_i} d\Phi_1^i \left[D_i^{(A)}(\Phi_{B_i}, \Phi_1^i) - D_i^{(S)}(\Phi_{B_i}, \Phi_1^i) \right] O(\Phi_{B_i}) \\
 &+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)}
 \end{aligned}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit



General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} &= \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 &+ \int d\Phi_B \sum_i \int d\Phi_1^i \left[D_i^{(A)}(\Phi_B, \Phi_1^i) - D_i^{(S)}(\Phi_B, \Phi_1^i) \right] O(\Phi_B) \\
 &+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)}
 \end{aligned}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit



General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\langle O \rangle^{\text{NLO}} = \int d\Phi_B \bar{B}^{(A)}(\Phi_B) O(\Phi_B)$$

$$+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)}$$

- introduce second set of subtraction functions $D_i^{(A)}$
- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit



NLOs matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\begin{aligned} \langle O \rangle^{\text{PS}} &= \int d\Phi_B B(\Phi_B) O(\Phi_B) \\ &= \int d\Phi_B B(\Phi_B) O(\Phi_B) + \end{aligned}$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
- $\langle O \rangle_{\text{corr}}^{(\mathcal{K})}$ generated by one parton shower step



NLOs matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\langle O \rangle^{\text{PS}} = \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right]$$

$$= \int d\Phi_B B(\Phi_B) O(\Phi_B) +$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
- $\langle O \rangle_{\text{corr}}^{(\mathcal{K})}$ generated by one parton shower step



NLOs matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\begin{aligned} \langle O \rangle^{\text{PS}} &= \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right] \\ &= \int d\Phi_B B(\Phi_B) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_R B \cdot \mathcal{K}(\Phi_1) \left[O(\Phi_R) - O(\Phi_B) \right] + \mathcal{O}(\alpha_s^2) \end{aligned}$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
- $\langle O \rangle_{\text{corr}}^{(\mathcal{K})}$ generated by one parton shower step



NLOs matching

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$

$$\begin{aligned} \langle O \rangle^{\text{PS}} &= \int d\Phi_B B(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_Q^2) O(\Phi_B) + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \Delta^{(\mathcal{K})}(t, \mu_Q^2) O(\Phi_R) \right] \\ &= \int d\Phi_B B(\Phi_B) O(\Phi_B) + \langle O \rangle_{\text{corr}}^{(\mathcal{K})} + \mathcal{O}(\alpha_s^2) \end{aligned}$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
- $\langle O \rangle_{\text{corr}}^{(\mathcal{K})}$ generated by one parton shower step



NLOs matching

$$\begin{aligned}
 \langle O \rangle^{\text{NLOs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)}
 \end{aligned}$$

- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale



NLOs matching

$$\begin{aligned}
 \langle O \rangle^{\text{NLOs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R)
 \end{aligned}$$

$$\Delta^{(A)}(t, t') = \exp \left[\int_t^{t'} d\Phi_1 D^{(A)}/B \right]$$

- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale



NLOs matching

$$\begin{aligned}
 \langle O \rangle^{\text{NLOs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R)
 \end{aligned}$$

$$\Delta^{(A)}(t, t') = \exp \left[\int_t^{t'} d\Phi_1 D^{(A)}/B \right]$$

- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\max}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale
- **POWHEG and MC@NLO now differ in choice of $D_i^{(A)}$ and μ_Q^2**



NLOs matching

$$\begin{aligned}
 \langle O \rangle^{\text{NLOs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R)
 \end{aligned}$$

$$\Delta^{(A)}(t, t') = \exp \left[\int_t^{t'} d\Phi_1 D^{(A)}/B \right]$$

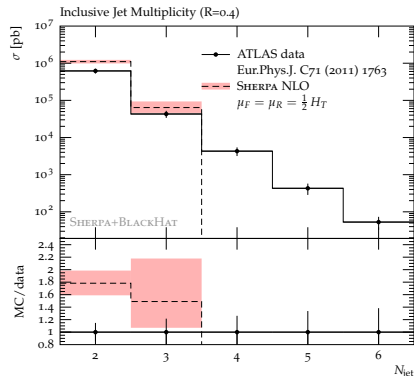
- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\max}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale
- **SHERPA:** $D_i^{(A)} = D_i^{(S)} \Theta(\mu_Q^2 - t)$ ($N_c = 3$ **CS kernels**), μ_Q free



Case study: Inclusive jet & dijet production

NLO:

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- jet- p_{\perp} turn negative in forward region unless y -dependent scale is used (e.g. $H_T^{(y)}$)



no. jets	ATLAS	LO	ME+PS	NLO	NP factor	NLO+NP
≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$559(5)$	$1193(3)^{+130}_{-135}$	$0.95(0.02)$	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$39.7(0.9)$	$54.5(0.5)^{+2.2}_{-19.9}$	$0.92(0.04)$	$50.2(2.1)^{+2.0}_{-18.3}$
≥ 4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$3.97(0.08)$	$5.54(0.12)^{+0.08}_{-2.44}$	$0.92(0.05)$	$5.11(0.29)^{+0.08}_{-2.32}$

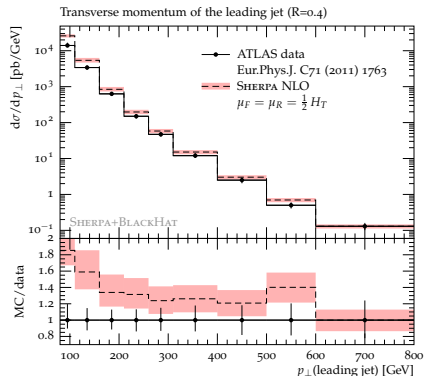
Bern et.al. Phys.Rev.Lett.109(2012)042001



Case study: Inclusive jet & dijet production

NLO:

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- jet- p_{\perp} turn negative in forward region unless y -dependent scale is used (e.g. $H_T^{(y)}$)



no. jets	ATLAS	LO	ME+PS	NLO	NP factor	NLO+NP
≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$559(5)$	$1193(3)^{+130}_{-135}$	$0.95(0.02)$	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$39.7(0.9)$	$54.5(0.5)^{+2.2}_{-19.9}$	$0.92(0.04)$	$50.2(2.1)^{+2.0}_{-18.3}$
≥ 4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$3.97(0.08)$	$5.54(0.12)^{+0.08}_{-2.44}$	$0.92(0.05)$	$5.11(0.29)^{+0.08}_{-2.32}$

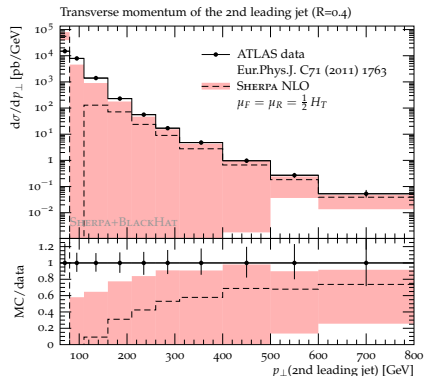
Bern et.al. Phys.Rev.Lett.109(2012)042001



Case study: Inclusive jet & dijet production

NLO:

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- jet- p_{\perp} turn negative in forward region unless y -dependent scale is used (e.g. $H_T^{(y)}$)



no. jets	ATLAS	LO	ME+PS	NLO	NP factor	NLO+NP
≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$559(5)$	$1193(3)^{+130}_{-135}$	$0.95(0.02)$	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$39.7(0.9)$	$54.5(0.5)^{+2.2}_{-19.9}$	$0.92(0.04)$	$50.2(2.1)^{+2.0}_{-18.3}$
≥ 4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$3.97(0.08)$	$5.54(0.12)^{+0.08}_{-2.44}$	$0.92(0.05)$	$5.11(0.29)^{+0.08}_{-2.32}$

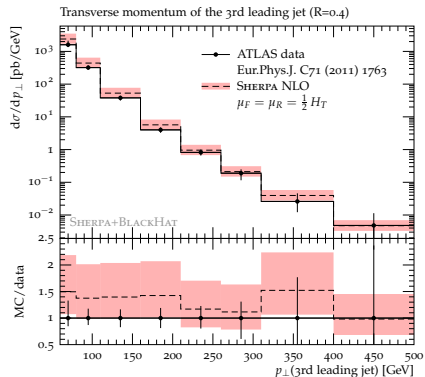
Bern et.al. Phys.Rev.Lett.109(2012)042001



Case study: Inclusive jet & dijet production

NLO:

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- jet- p_{\perp} turn negative in forward region unless y -dependent scale is used (e.g. $H_T^{(y)}$)



no. jets	ATLAS	LO	ME+PS	NLO	NP factor	NLO+NP
≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$559(5)$	$1193(3)^{+130}_{-135}$	$0.95(0.02)$	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$39.7(0.9)$	$54.5(0.5)^{+2.2}_{-19.9}$	$0.92(0.04)$	$50.2(2.1)^{+2.0}_{-18.3}$
≥ 4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$3.97(0.08)$	$5.54(0.12)^{+0.08}_{-2.44}$	$0.92(0.05)$	$5.11(0.29)^{+0.08}_{-2.32}$

Bern et.al. Phys.Rev.Lett.109(2012)042001



Case study: Inclusive jet & dijet production

Describe wealth of experimental data with a single sample (LHC@7TeV)

MC@NLO di-jet production:

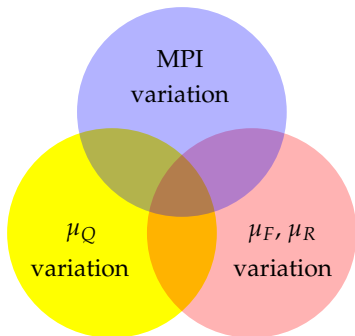
Höche, MS Phys.Rev.D86(2012)094042

- $\mu_{R/F} = \frac{1}{4} H_T$, $\mu_Q = \frac{1}{2} p_{\perp}$
- CT10 PDF ($\alpha_s(m_Z) = 0.118$)
- hadron level calculation, MPI
- virtual MEs from BLACKHAT
Giele, Glover, Kosower
Nucl.Phys.B403(1993)633-670
Bern et.al. arXiv:1112.3940

- $p_{\perp}^{j1} > 20$ GeV, $p_{\perp}^{j2} > 10$ GeV

Uncertainty estimates:

- $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{R/F}^{\text{def}}$
- $\mu_Q \in [\frac{1}{\sqrt{2}}, \sqrt{2}] \mu_Q^{\text{def}}$
- MPI activity in tr. region $\pm 10\%$





Case study: Inclusive jet & dijet production

Describe wealth of experimental data with a single sample (LHC@7TeV)

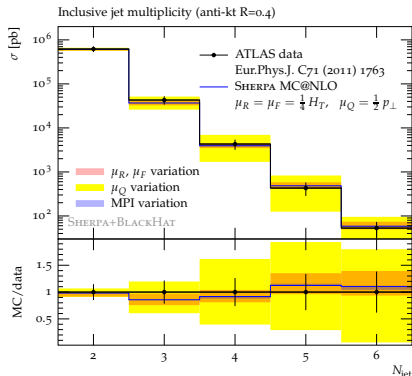
MC@NLO di-jet production:

Höche, *MS Phys.Rev.D86(2012)094042*

- $\mu_{R/F} = \frac{1}{4} H_T$, $\mu_Q = \frac{1}{2} p_{\perp}$
- CT10 PDF ($\alpha_s(m_Z) = 0.118$)
- hadron level calculation, MPI
- virtual MEs from BLACKHAT
Giele, Glover, Kosower
Nucl.Phys.B403(1993)633-670
Bern et.al. *arXiv:1112.3940*
- $p_{\perp}^{j1} > 20$ GeV, $p_{\perp}^{j2} > 10$ GeV

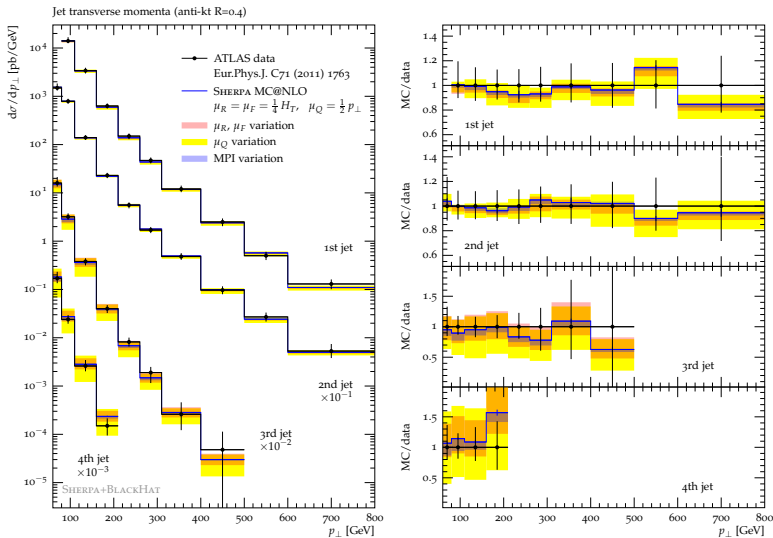
Uncertainty estimates:

- $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{R/F}^{\text{def}}$
- $\mu_Q \in [\frac{1}{\sqrt{2}}, \sqrt{2}] \mu_Q^{\text{def}}$
- MPI activity in tr. region $\pm 10\%$



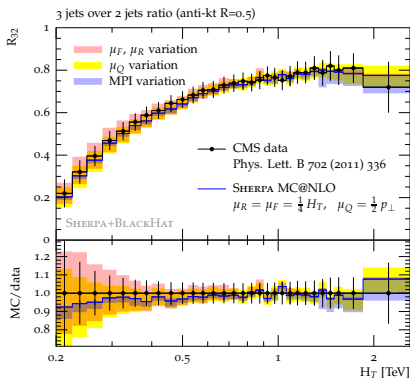


Case study: Inclusive jet & dijet production





Case study: Inclusive jet & dijet production

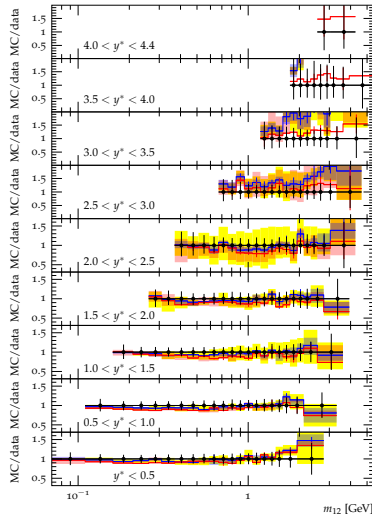
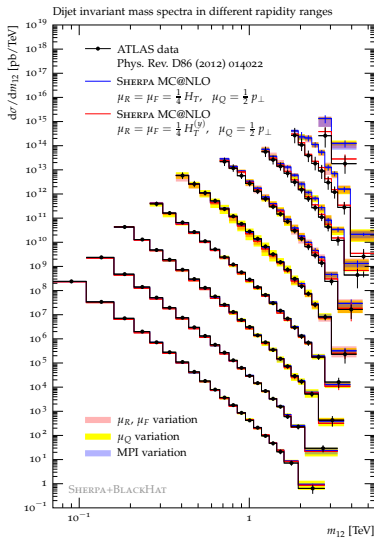


3-jet-over-2-jet ratio

- determined from incl. sample
2-jet rate at NLO+NLL
3-jet rate at LO+LL
- common scale choices
→ varied simultaneously
- at large H_T large MPI
uncertainties
→ better MPI physics needed
(soft QCD)
- similar description of related
ATLAS observables



Case study: Inclusive jet & dijet production





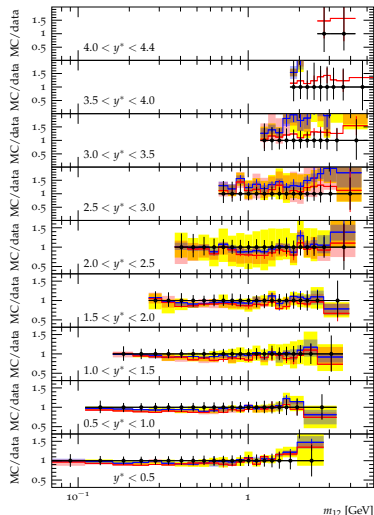
Case study: Inclusive jet & dijet production

Try different scale

- $\mu_{R/F} = \frac{1}{4} H_T^{(y)}$ with $H_T^{(y)} = \sum_{i \in \text{jets}} p_{\perp, i} e^{0.3|y_{\text{boost}} - y_i|}$
with $y_{\text{boost}} = 1/n_{\text{jets}} \sum_{i \in \text{jets}} y_i$
- reduces to $\mu_{R/F} = \frac{1}{2} p_{\perp} e^{0.3y^*}$
with $y^* = \frac{1}{2}|y_1 - y_2|$ for $2 \rightarrow 2$
and captures real emission dynamics
[Ellis, Kunszt, Soper PRD40\(1989\)2188](#)
- better description of data at large rapidities, as expected

description of most other observables worsened

need proper description of forward physics (e.g. (B)FKL)





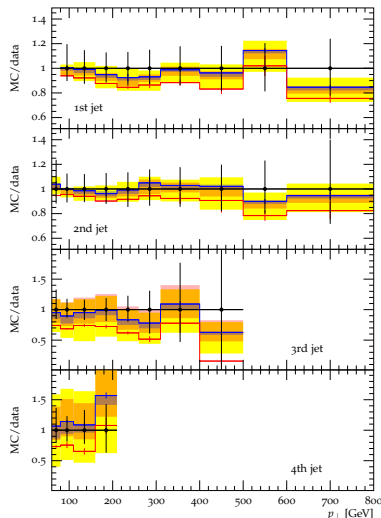
Case study: Inclusive jet & dijet production

Try different scale

- $\mu_{R/F} = \frac{1}{4} H_T^{(y)}$ with $H_T^{(y)} = \sum_{i \in \text{jets}} p_{\perp, i} e^{0.3|y_{\text{boost}} - y_i}$
 with $y_{\text{boost}} = 1/n_{\text{jets}} \sum_{i \in \text{jets}} y_i$
- reduces to $\mu_{R/F} = \frac{1}{2} p_{\perp} e^{0.3y^*}$
 with $y^* = \frac{1}{2}|y_1 - y_2|$ for $2 \rightarrow 2$
 and captures real emission dynamics
 Ellis, Kunszt, Soper PRD40(1989)2188
- better description of data at large rapidities, as expected

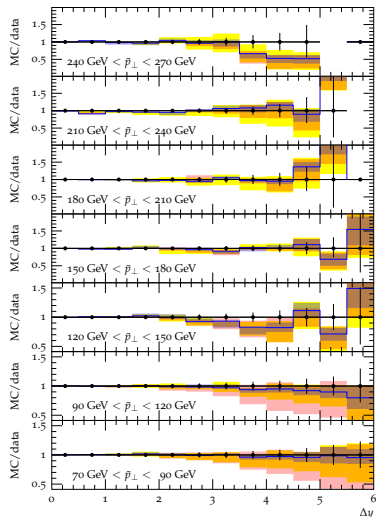
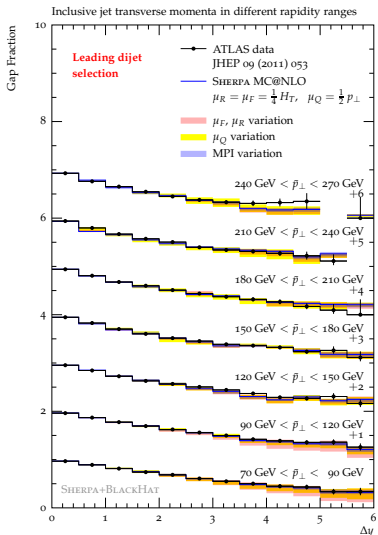
description of most other observables worsened

need proper description of forward physics (e.g. (B)FKL)



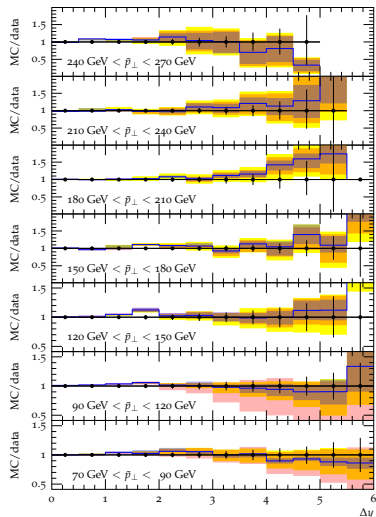
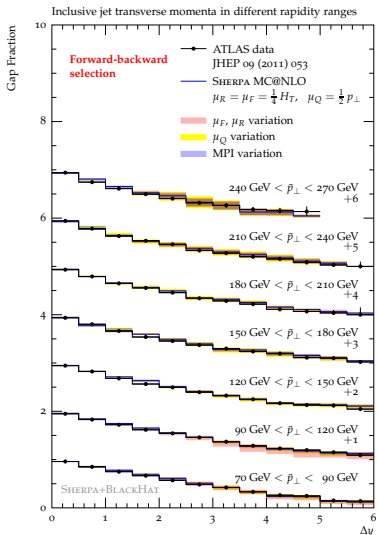


Case study: Inclusive jet & dijet production



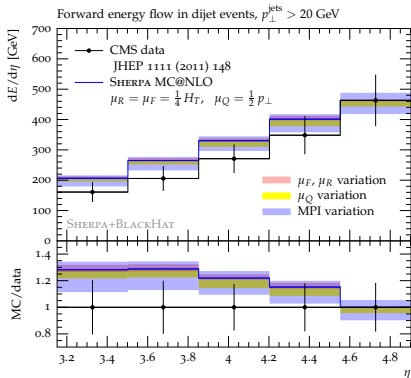


Case study: Inclusive jet & dijet production





Case study: Inclusive jet & dijet production

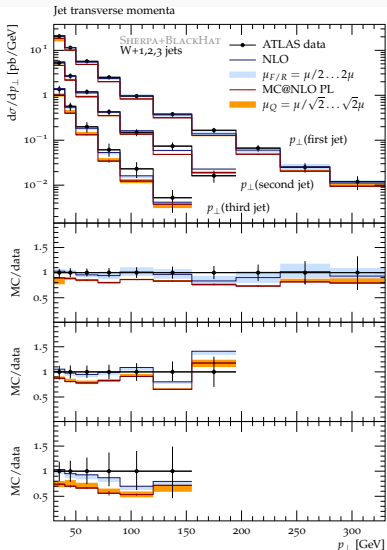


Forward energy flow

- energy flow in rapidity interval per event with a central back-to-back di-jet pair
- normalisation reduces $\mu_{R/F}$ and μ_Q dependence
- dominated by MPI modeling uncertainty



$W + n$ jet production



Höche, Krauss, MS, Siebert

Phys.Rev.Lett.110(2013)052001

$pp \rightarrow W + 1, 2, 3$ jets

- 3 separate samples/calculations
- NLO accuracy for inclusive observables of respective jet multiplicity
- resummation of softest/LO jet, i.e. 4th jet in $pp \rightarrow W + 3$ jets
- no resummation of sample-defining jet multiplicity, i.e. first 3 jets in $pp \rightarrow W + 3$ jets
- scales:

$$\mu_{R/F} = \frac{1}{2} \hat{H}'_T, \mu_Q = p_{\perp}(j_n)$$

Data: ATLAS Phys.Rev.D85(2012)092002



Contents

① General NLOPS matching

Resummation properties of parton showers

NLOPS matching

Case study

② Multijet merging

MEPS – Multijet merging at leading order

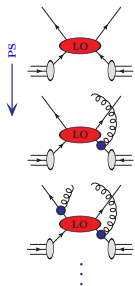
MEPS@NLO – Multijet merging at next-to-leading order

Results

③ Conclusions



MEPs – Multijet merging at LO



LO $pp \rightarrow 2$ with parton showers

- + exponentiation of large IR logarithms
- poor hard/wide angle emission pattern

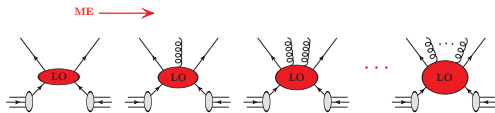
vs. **LO** $pp \rightarrow n$ matrix elements

- + dominant terms for hard/wide angle rad.
- breakdown of α_s -expansion in log. region

- MEPs schemes: CKKW-type, MLM-type
- LO+(N)LL accuracy in every jet multiplicity
- scale setting scheme essential to preserve PS-resummation properties



MEPs – Multijet merging at LO



LO $pp \rightarrow 2$ with parton showers

- + exponentiation of large IR logarithms
- poor hard/wide angle emission pattern

vs.

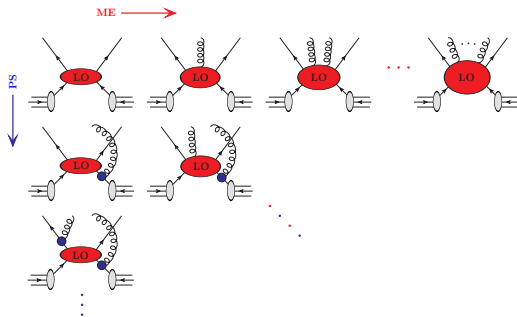
LO $pp \rightarrow n$ matrix elements

- + dominant terms for hard/wide angle rad.
- breakdown of α_s -expansion in log. region

- MEps schemes: CKKW-type, MLM-type
- LO+(N)LL accuracy in every jet multiplicity
- scale setting scheme essential to preserve PS-resummation properties



MEPs – Multijet merging at LO



LO $pp \rightarrow 2$ with parton showers

- + exponentiation of large IR logarithms
- poor hard/wide angle emission pattern

vs. **LO** $pp \rightarrow n$ matrix elements

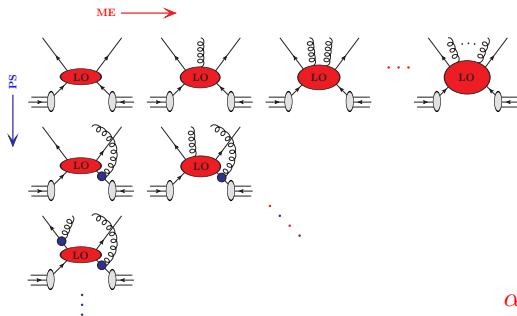
- + dominant terms for hard/wide angle rad.
- breakdown of α_s -expansion in log. region

- MEPs schemes: CKKW-type, MLM-type
LO+(N)LL accuracy in every jet multiplicity

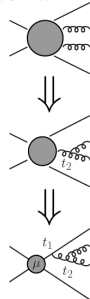
• scale setting scheme essential to preserve PS-resummation properties



MEPs – Multijet merging at LO



Scales:



$$\alpha_s^{k+n}(\mu_{\text{eff}}) = \alpha_s^k(\mu) \alpha_s(t_1) \cdots \alpha_s(t_n)$$

LO $pp \rightarrow 2$ with parton showers vs.

- + exponentiation of large IR logarithms
- poor hard/wide angle emission pattern

LO $pp \rightarrow n$ matrix elements

- + dominant terms for hard/wide angle rad.
- breakdown of α_s -expansion in log. region

- MEPS schemes: CKKW-type, MLM-type
LO+(N)LL accuracy in every jet multiplicity
- scale setting scheme essential to preserve PS-resummation properties



Multijet merging

LO merging:

- LO accuracy for $n \leq n_{\text{max-jet}}$ processes
- preserve (N)LL accuracy of the parton shower

Catani, Krauss, Kuhn, Webber JHEP11(2001)063

Lönnblad JHEP05(2002)046

Höche, Krauss, Schumann, Siegert JHEP05(2009)053

Hamilton, Richardson, Tully JHEP11(2009)038

Lönnblad, Prestel JHEP03(2012)019



MEPs – Multijet merging at LO

$$\begin{aligned}
 \langle O \rangle^{\text{MEPs}} &= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} B_{n+1} \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(\mathcal{K})}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \mathcal{K}_{n+1} \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) O_{n+2} \right]
 \end{aligned}$$

- LOPs for n -jet process restricted to region $Q < Q_{\text{cut}}$
- LOPs for $n + 1$ -jet process
 - implement a correct recombination algorithm for n -jet process
- truncated showering to account for mismatch of t and Q [Nason JHEP11\(2004\)040](#)



MEPs – Multijet merging at LO

$$\begin{aligned}
 \langle O \rangle^{\text{MEPs}} &= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &\quad + \int d\Phi_{n+1} B_{n+1} \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(\mathcal{K})}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \mathcal{K}_{n+1} \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) O_{n+2} \right]
 \end{aligned}$$

- LOPs for n -jet process restricted to region $Q < Q_{\text{cut}}$

- LOPs for $n + 1$ -jet process

- truncated showering to account for mismatch of t and Q [Nason JHEP11\(2004\)040](#)



MEPs – Multijet merging at LO

$$\begin{aligned}
 \langle O \rangle^{\text{MEPs}} &= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} B_{n+1} \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(\mathcal{K})}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \mathcal{K}_{n+1} \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) O_{n+2} \right]
 \end{aligned}$$

- LOPs for n -jet process restricted to region $Q < Q_{\text{cut}}$
- LOPs for $n + 1$ -jet process with additional Sudakov wrt. n -jet process
 → implements correct resummation behaviour wrt. incl. sample
- truncated showering to account for mismatch of t and Q [Nason JHEP11\(2004\)040](#)



MEPs – Multijet merging at LO

$$\begin{aligned}
 \langle O \rangle^{\text{MEPs}} &= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} B_{n+1} \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(\mathcal{K})}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \mathcal{K}_{n+1} \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) O_{n+2} \right]
 \end{aligned}$$

- LOPs for n -jet process restricted to region $Q < Q_{\text{cut}}$
- LOPs for $n + 1$ -jet process with additional Sudakov wrt. n -jet process
 → implements correct resummation behaviour wrt. incl. sample
- truncated showering to account for mismatch of t and Q [Nason JHEP11\(2004\)040](#)



MEPs – Multijet merging at LO

$$\begin{aligned}
 \langle O \rangle^{\text{MEPs}} &= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} B_{n+1} \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(\mathcal{K})}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \mathcal{K}_{n+1} \Delta_{n+1}^{(\mathcal{K})}(t_{n+2}, t_{n+1}) O_{n+2} \right]
 \end{aligned}$$

- LOPs for n -jet process restricted to region $Q < Q_{\text{cut}}$
- LOPs for $n + 1$ -jet process with additional Sudakov wrt. n -jet process
→ implements correct resummation behaviour wrt. incl. sample
- truncated showering to account for mismatch of t and Q [Nason JHEP11\(2004\)040](#)



MEPs – Multijet merging at LO

 $\langle O \rangle^{\text{MEPs}}$

$$= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\ \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \left(\mathcal{K}_n \Theta(Q_{\text{cut}} - Q) + \frac{B_{n+1}}{B_n} \Theta(Q - Q_{\text{cut}}) \right) \right. \\ \left. \times \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) O_{n+2} \right]$$

- α_s scales in $B \cdot \mathcal{K}$ and B_{n+1} must be the same to retain resummation properties of the parton shower
- interpret B_{n+1} as PS splitting on top of B
 → need to use inverse parton shower



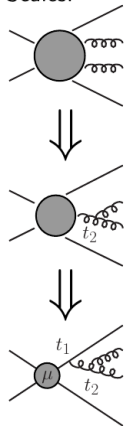
MEPs – Multijet merging at LO

 $\langle O \rangle^{\text{MEPs}}$

$$= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n + \int_{t_0}^{\mu_Q^2} d\Phi_1 \left(\mathcal{K}_n \Theta(Q_{\text{cut}} - Q) + \frac{B_{n+1}}{B_n} \Theta(Q - \right. \right. \\ \left. \left. \times \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) O_{n+2} \right) \right]$$

- α_s scales in $B \cdot \mathcal{K}$ and B_{n+1} must be the same to retain resun properties of the parton shower
- interpret B_{n+1} as PS splitting on top of B
→ need to use inverse parton shower

Scales:



$$\alpha_s^{k+n}(\mu_{\text{eff}}) = \alpha_s^k(\mu) \alpha_s(t_1) \cdots \alpha_s(t_n)$$



MEPs – Multijet merging at LO

 $\langle O \rangle^{\text{MEPs}}$

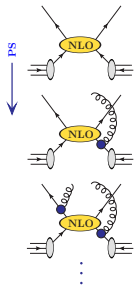
$$= \int d\Phi_n B_n \left[\Delta_n^{(\mathcal{K})}(t_0, \mu_Q^2) O_n \right. \\ \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \left(\mathcal{K}_n \Theta(Q_{\text{cut}} - Q) + \frac{B_{n+1}}{B_n} \Theta(Q - Q_{\text{cut}}) \right) \right. \\ \left. \times \Delta_n^{(\mathcal{K})}(t_{n+1}, \mu_Q^2) O_{n+2} \right]$$

mismatch of $\mathcal{O}(\frac{1}{N_c} \alpha_s L)$

- α_s scales in $B \cdot \mathcal{K}$ and B_{n+1} must be the same to retain resummation properties of the parton shower
- interpret B_{n+1} as PS splitting on top of B
→ need to use inverse parton shower



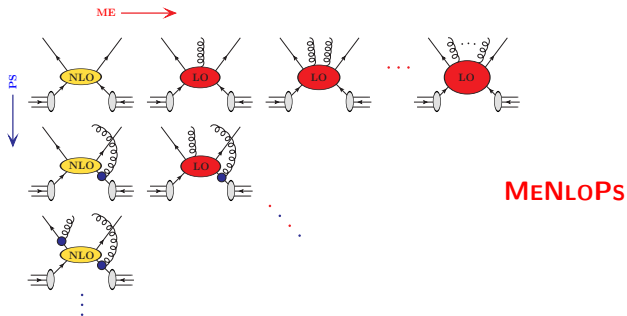
MEPs@NLO – Multijet merging at NLO



- promote LOPs to NLOs (POWHEG, Mc@NLO)
→ can assess uncertainties (part I)
- combine NLOs for successive multiplicities into incl. sample (MEPs@NLO),
preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties



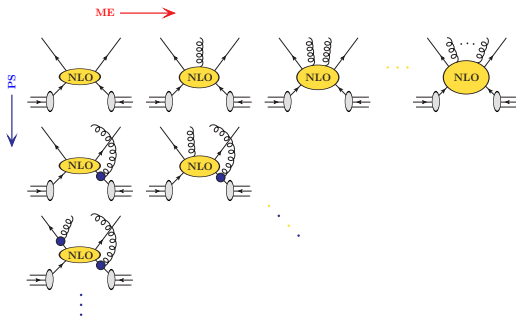
MEPs@NLO – Multijet merging at NLO



- promote LOPs to NLOs (POWHEG, Mc@NLO)
→ can assess uncertainties (part I)
- combine NLOs for successive multiplicities into incl. sample (MEPs@NLO),
preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties



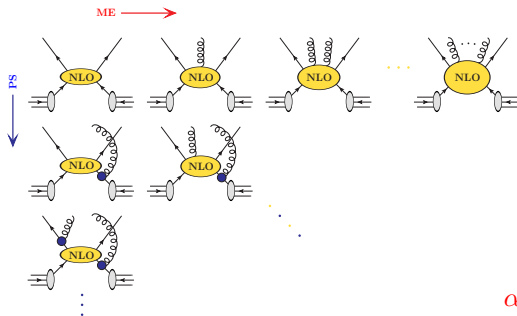
MEPs@NLO – Multijet merging at NLO



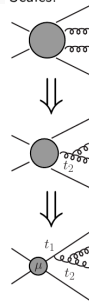
- promote LOPs to NLOs (POWHEG, MC@NLO)
→ can assess uncertainties (part I)
- combine NLOs for successive multiplicities into incl. sample (MEPs@NLO),
preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties



MEPs@NLO – Multijet merging at NLO



Scales:



$$\alpha_s^{k+n}(\mu_{\text{eff}}) = \alpha_s^k(\mu) \alpha_s(t_1) \cdots \alpha_s(t_n)$$

- promote LOPs to NLOs (POWHEG, MC@NLO)
→ can assess uncertainties (part I)
- combine NLOs for successive multiplicities into incl. sample (MEPs@NLO),
preserve NLO+(N)LL accuracy in every jet multiplicity
restore resummation wrt. to inclusive sample (part II)
- scale setting scheme essential to preserve PS-resummation properties



MEPs@NLO – Multijet merging at NLO

LO merging:

- LO accuracy for $n \leq n_{\max}$ -jet processes
- preserve (N)LL accuracy of the parton shower

Catani, Krauss, Kuhn, Webber JHEP11(2001)063

Lönnblad JHEP05(2002)046

Höche, Krauss, Schumann, Siegert JHEP05(2009)053

Hamilton, Richardson, Tully JHEP11(2009)038

Lönnblad, Prestel JHEP03(2012)019

- NLO accuracy for $n \leq n_{\max}$ -jet processes
- preserve (N)LL accuracy of the parton shower

Lavesson, Lönnblad JHEP12(2008)070

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031



MEPs@NLO – Multijet merging at NLO

 $\langle O \rangle_{\text{MEPs@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\begin{aligned}
 &= \int d\Phi_n \bar{B}_n^{(A)} \left[\Delta_n^{(A)}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
 &+ \int d\Phi_{n+2} \left[R_{n+1} - D_{n+1}^{(A)} \right] \Theta(Q - Q_{\text{cut}}) O_{n+2}
 \end{aligned}$$



MEPs@NLO – Multijet merging at NLO

$\langle O \rangle_{\text{MEPs@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\begin{aligned}
 &= \int d\Phi_n \bar{B}_n^{(A)} \left[\Delta_n^{(A)}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
 &+ \int d\Phi_{n+2} \left[R_{n+1} - D_{n+1}^{(A)} \right] \Theta(Q - Q_{\text{cut}}) O_{n+2}
 \end{aligned}$$



MEPs@NLO – Multijet merging at NLO

 $\langle O \rangle_{\text{MEPs@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\begin{aligned}
 &= \int d\Phi_n \bar{B}_n^{(A)} \left[\Delta_n^{(A)}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
 &+ \int d\Phi_{n+2} \left[R_{n+1} - D_{n+1}^{(A)} \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) O_{n+2}
 \end{aligned}$$



MEPs@NLO – Multijet merging at NLO

 $\langle O \rangle_{\text{MEPs@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\begin{aligned}
 &= \int d\Phi_n \bar{B}_n^{(A)} \left[\Delta_n^{(A)}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
 &+ \int d\Phi_{n+2} \left[R_{n+1} - D_{n+1}^{(A)} \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) O_{n+2}
 \end{aligned}$$



MEPs@NLO – Multijet merging at NLO

 $\langle O \rangle_{\text{MEPs@NLO}}$

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

$$\begin{aligned}
 &= \int d\Phi_n \bar{B}_n^{(A)} \left[\Delta_n^{(A)}(t_0, \mu_Q^2) O_n \right. \\
 &\quad \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q) O_{n+1} \right] \\
 &+ \int d\Phi_{n+1} \left[R_n - D_n^{(A)} \right] \Theta(Q_{\text{cut}} - Q) O_{n+1} \\
 &+ \int d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) \\
 &\quad \times \left[\Delta_{n+1}^{(A)}(t_0, t_{n+1}) O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) O_{n+2} \right] \\
 &+ \int d\Phi_{n+2} \left[R_{n+1} - D_{n+1}^{(A)} \right] \Delta_n^{(K)}(t_{n+1}, \mu_Q^2) \Theta(Q - Q_{\text{cut}}) O_{n+2}
 \end{aligned}$$



NLO merging – Generation of MC counterterm

$$\left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right]$$

- same form as exponent of Sudakov form factor $\Delta_n^{(\text{PS})}(t_{n+1}, \mu_Q^2)$
- truncated parton shower on n -parton configuration underlying $n + 1$ -parton event
 - 1 no emission \rightarrow retain $n + 1$ -parton event as is
 - 2 first emission at t' with $Q > Q_{\text{cut}}$, multiply event weight with $B_{n+1}/\bar{B}_{n+1}^{(A)}$, restart evolution at t' , do not apply emission kinematics
 - 3 treat every subsequent emission as in standard truncated vetoed shower
- generates

$$\left[1 + \frac{B_{n+1}}{\bar{B}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 \mathcal{K}_n \right] \Delta_n^{(\text{PS})}(t_{n+1}, \mu_Q^2)$$

\Rightarrow **identify $\mathcal{O}(\alpha_s)$ counterterm with the omitted emission**



NLO merging

Renormalisation scales:

- determined by clustering using PS probabilities and taking the respective nodal values t_i

$$\alpha_s(\mu_R^2)^k = \prod_{i=1}^k \alpha_s(t_i)$$

- change of scales $\mu_R \rightarrow \tilde{\mu}_R$ in MEs necessitates one-loop counter term

$$\alpha_s(\tilde{\mu}_R^2)^k \left(1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi} \beta_0 \sum_{i=1}^k \ln \frac{t_i}{\tilde{\mu}_R^2} \right)$$

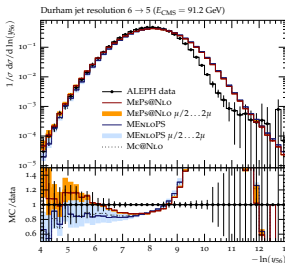
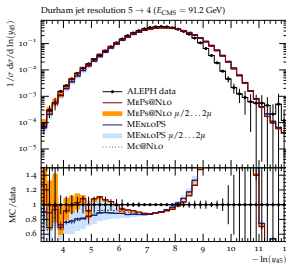
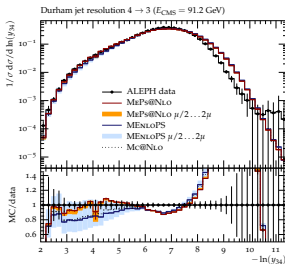
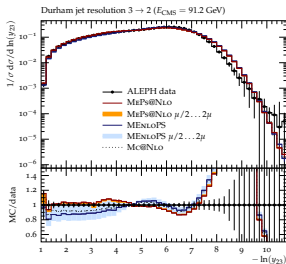
Factorisation scale:

- μ_F determined from core n -jet process
- change of scales $\mu_F \rightarrow \tilde{\mu}_F$ in MEs necessitates one-loop counter term

$$B_n(\Phi_n) \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi} \log \frac{\mu_F^2}{\tilde{\mu}_F^2} \left(\sum_{c=q,g}^n \int_{x_a}^1 \frac{dz}{z} P_{ac}(z) f_c(x_a/z, \tilde{\mu}_F^2) + \dots \right)$$



Results: $e^+e^- \rightarrow$ hadrons



$ee \rightarrow$ hadrons
(2,3,4 @ NLO;
5,6 @ LO)

Jet resolutions
(Durham measure)

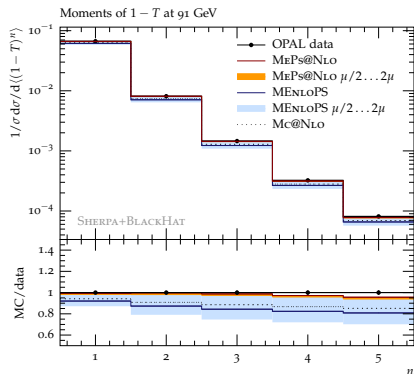
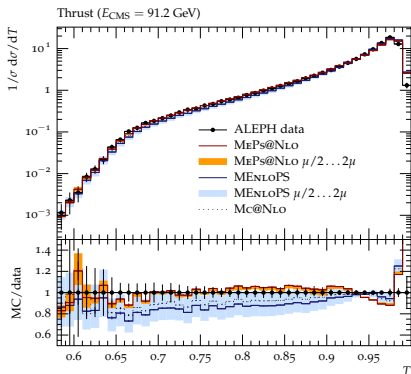
- MEPS@NLO vs MENLOPS
- at $y \ll 1$ dominated by hadr. effects \rightarrow needs retuning
- much improved ren. scale dependence

ALEPH data
EPJC35(2004)457-486



Results: $e^+e^- \rightarrow \text{hadrons}$

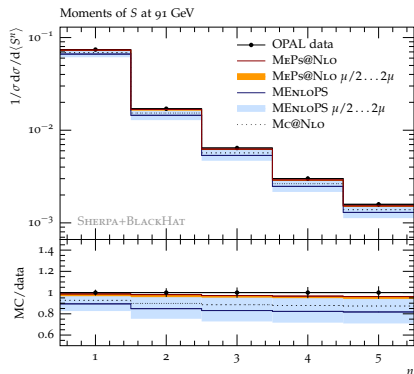
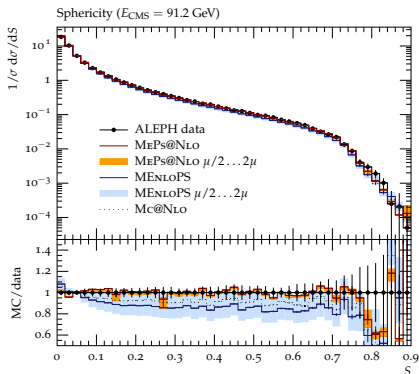
ALEPH data EPJC35(2004)457-486, OPAL data EPJC40(2005)287-316





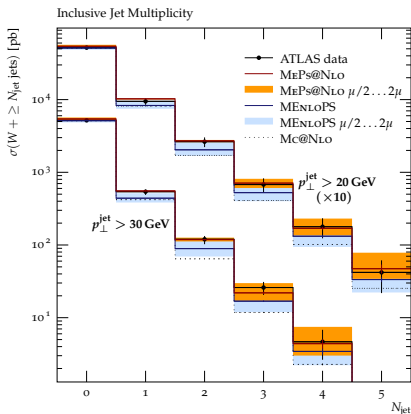
Results: $e^+e^- \rightarrow \text{hadrons}$

ALEPH data EPJC35(2004)457-486, OPAL data EPJC40(2005)287-316





Results – $pp \rightarrow W + \text{jets}$



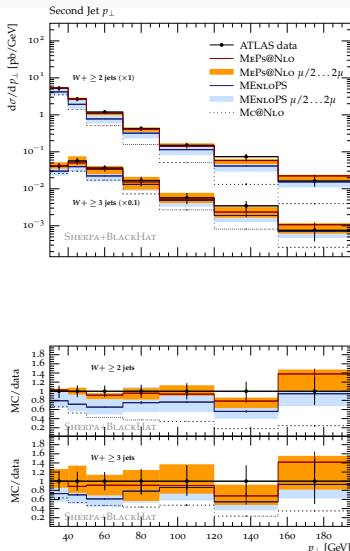
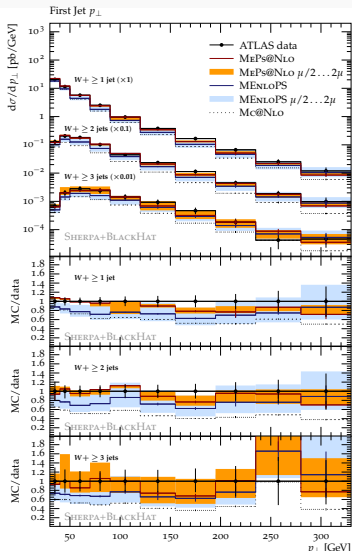
$pp \rightarrow W + \text{jets}$ (0,1,2 @ NLO; 3,4 @ LO)

- $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{\text{def}}$
scale uncertainty much reduced
- NLO dependence for $pp \rightarrow W + 0,1,2$ jets
LO dependence for $pp \rightarrow W + 3,4$ jets
- $Q_{\text{cut}} = 30 \text{ GeV}$
- good data description

ATLAS data Phys.Rev.D85(2012)092002



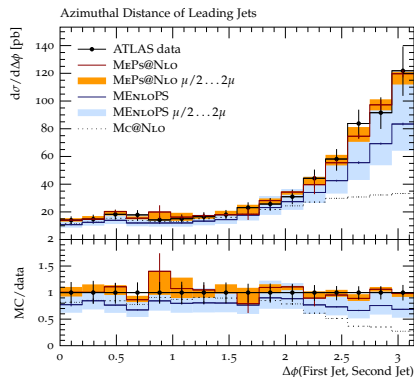
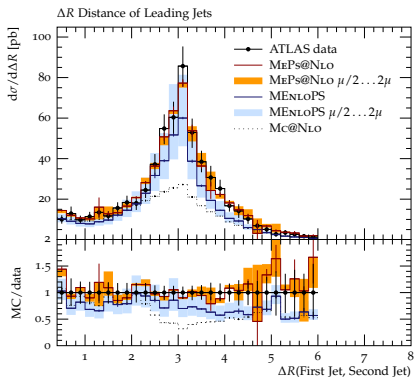
Results – $pp \rightarrow W + \text{jets}$





Results – $pp \rightarrow W + \text{jets}$

ATLAS data Phys.Rev.D85(2012)092002

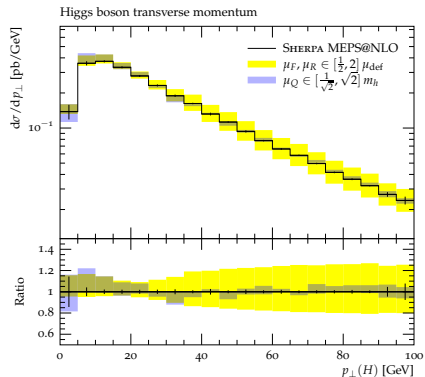




Results – $pp \rightarrow h + \text{jets}$

Setup: $pp \rightarrow H + \text{jets}$ (ggF)

- purely perturbative calculation (no hadronisation, MPI, etc.)
- 0,1 jets @ NLO
2,3 jets @ LO
 $Q_{\text{cut}} = 20 \text{ GeV}$, $N_{\text{max}} = 3$
- perturbative scale variations
 $\mu_{R/F} \in [\frac{1}{2}, 2] \mu_{\text{def}}$
 $\mu_Q \in [\frac{1}{\sqrt{2}}, \sqrt{2}] m_h$
- variation of merging parameter
 $Q_{\text{cut}} \in \{15, 20, 30\} \text{ GeV}$ and
max. ME multi $N_{\text{max}} \in \{1, 2, 3\}$
- inclusive calculation
→ scales of each jet-multiplicity
subsample not independent



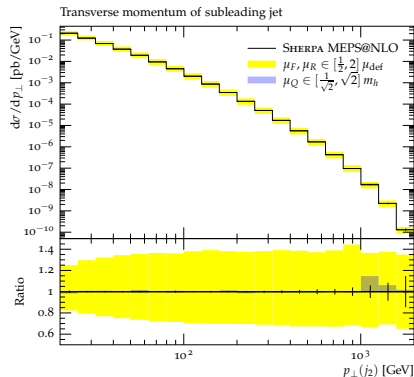
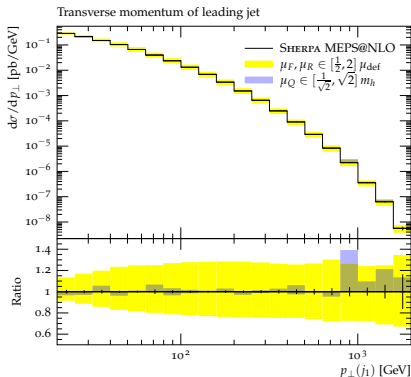
- inclusive cross section

$$\sigma = 15.2^{+2.5}_{-1.5} {}^{+1.0}_{-0.4} {}^{+0.3}_{-0.0} {}^{+0.2}_{-1.0} \text{ pb}$$

$$\mu_{R/F}^-, \mu_Q^-, Q_{\text{cut}}^-, N_{\text{max}} \text{ var.}$$



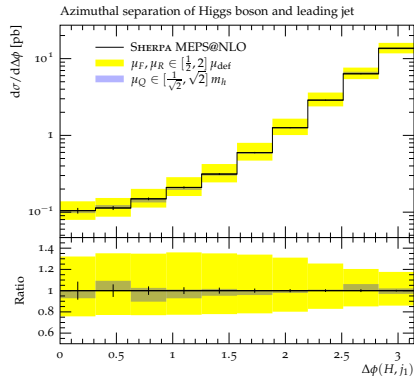
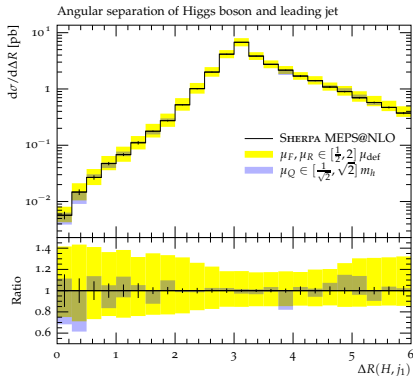
Results – pp → h + jets



- transverse momenta of leading jets
- only small resummation scale dependence
- bulk of scale dependence comes from μ_R -dependence of effective vertex



Results – pp → h + jets

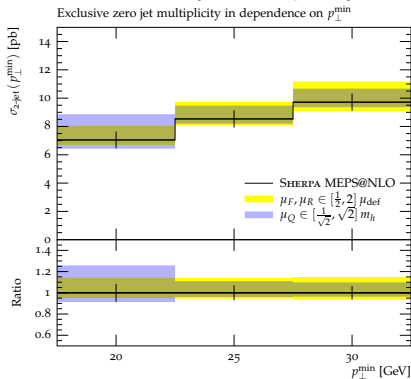


- angular correlations of leading jet and Higgs
- shape rather stable against variation



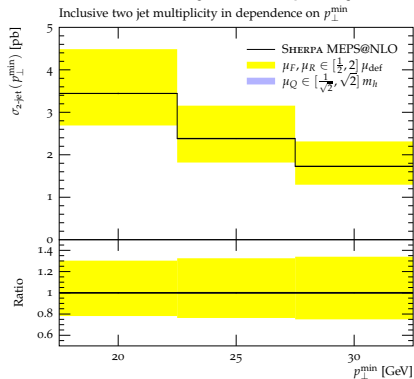
Results – pp → h + jets

exclusive 0-jet multiplicity



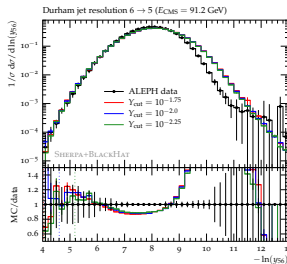
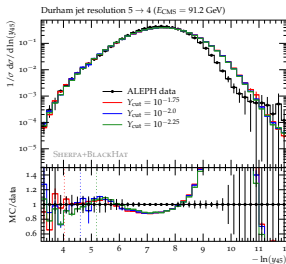
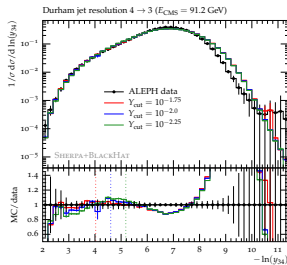
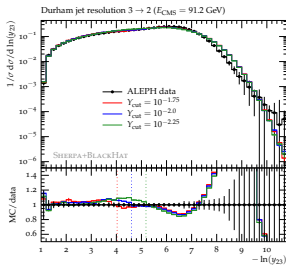
- Determined from inclusive sample
 - common scale choice
 - correlated variations
 - needs improved resummation

inclusive 2-jet multiplicity





Q_{cut} dependence

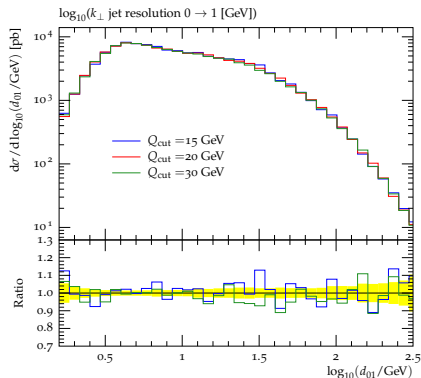


$$y_{\text{cut}} = \left(\frac{Q_{\text{cut}}}{E_{\text{CMS}}} \right)^2 = \begin{cases} 10^{-1.75} \\ 10^{-2.0} \\ 10^{-2.25} \end{cases}$$

- variation up to 10% in merging region
- very stable above and below
- incl. cross section $< 1\%$



Q_{cut} dependence



$pp \rightarrow W + \text{jets}$

- 0,1 @ NLO, 2 @ LO
- $Q_{\text{cut}} = 15, 20, 30 \text{ GeV}$
- variation $\approx 5\%$



Conclusions

- NLOPS matching methods allow for observable independent matching of NLO calculations and parton shower resummation
→ allow for continuation into non-perturbative regime (hadronisation, multiple parton interactions)
- NLOPS is LO+(N)LL matching
- multijet merging at NLO proceeds schematically as at LO
→ introduce MC-counterterm to retain NLO accuracy
- scale setting essential for recovering PS resummation
- can be improved by adding higher order calculations
 - (N)NLL resummation
 - NNLO corrections

to be released SHERPA-2.0. β_2 , when fully tuned SHERPA-2.0.0

<http://sherpa.hepforge.org>

○
○○○
○○○○○○○○○

○○○
○○○○
○○○○○○○○○○○

Thank you for your attention!



MENLOPs for Mc@NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q - Q_{\text{cut}})
 \end{aligned}$$

- restrict Mc@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}



MENLOPs for Mc@NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
 & + \int d\Phi_R R(\Phi_R) O(\Phi_R) \Theta(Q - Q_{\text{cut}})
 \end{aligned}$$

- restrict Mc@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}



MENLOPs for MC@NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
 & + \int d\Phi_R \left[\frac{\bar{B}^{(A)}(\Phi_B)}{B(\Phi_B)} \left(1 - \frac{H(\Phi_R)}{R(\Phi_R)} \right) + \frac{H(\Phi_R)}{R(\Phi_R)} \right] R(\Phi_R) \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q - Q_{\text{cut}})
 \end{aligned}$$

- restrict MC@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}



MENLOPs for MC@NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
 & + \int d\Phi_R \left[\frac{\bar{B}^{(A)}(\Phi_B)}{B(\Phi_B)} \left(1 - \frac{H(\Phi_R)}{R(\Phi_R)} \right) + \frac{H(\Phi_R)}{R(\Phi_R)} \right] R(\Phi_R) \Delta^{(K)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q - Q_{\text{cut}})
 \end{aligned}$$

- restrict MC@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}



MENLOPs for MC@NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
 & + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \\
 & + \int d\Phi_R \left[\frac{\bar{B}^{(A)}(\Phi_B)}{B(\Phi_B)} \left(1 - \frac{\mathbb{H}(\Phi_R)}{R(\Phi_R)} \right) + \frac{\mathbb{H}(\Phi_R)}{R(\Phi_R)} \right] R(\Phi_R) \Delta^{(K)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q - Q_{\text{cut}})
 \end{aligned}$$

- restrict MC@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in MEPS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm Höche, Krauss, MS, Siebert JHEP09(2012)049

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)} h(t) \exp \left\{ - \int_t^{t_1} d\bar{t} h(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) h(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} h(\bar{t}) \right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm [Höche, Krauss, MS, Siegert JHEP09\(2012\)049](#)

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)} h(t) \exp \left\{ - \int_t^{t_1} d\bar{t} h(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) h(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} h(\bar{t}) \right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm Höche, Krauss, MS, Siegert JHEP09(2012)049

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)} h(t) \exp \left\{ - \int_t^{t_1} d\bar{t} h(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) h(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} h(\bar{t}) \right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$

Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow D_i^{(A)}$
- $h(t)$ determined by parton shower $\Rightarrow D_i^{(PS)}$
- $g(t)$ **can be chosen freely** $\Rightarrow \text{const.} \cdot f$
constraints: $\text{sign}(f) = \text{sign}(g)$, $|f| \leq |g|$



POWHEG

Special choices:

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $D_i^{(A)} = \rho_i \cdot R$ with $\rho_i = D_i^{(S)} / \sum_i D_i^{(S)}$
 → each $\rho_i \cdot R$ contains only one divergence structure as defined by $D_i^{(S)}$

Consequences:

- no \mathbb{H} -events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2} s_{\text{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $D_i^{(S)}$
- exponentiation of R through matrix element corrected parton shower
 NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2 / (p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
 → $D_i^{(A)} = \rho_i \cdot R \cdot f(p_\perp)$
 → continuous dampening of resummation kernel at large p_\perp



POWHEG

Special choices:

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $D_i^{(A)} = \rho_i \cdot R$ with $\rho_i = D_i^{(S)} / \sum_i D_i^{(S)}$
 \rightarrow each $\rho_i \cdot R$ contains only one divergence structure as defined by $D_i^{(S)}$

Consequences:

- no \mathbb{H} -events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2} s_{\text{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $D_i^{(S)}$
- exponentiation of R through matrix element corrected parton shower
 NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2 / (p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
 $\rightarrow D_i^{(A)} = \rho_i \cdot R \cdot f(p_\perp)$
 \rightarrow continuous dampening of resummation kernel at large p_\perp



POWHEG

Special choices:

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $D_i^{(A)} = \rho_i \cdot R$ with $\rho_i = D_i^{(S)} / \sum_i D_i^{(S)}$
 → each $\rho_i \cdot R$ contains only one divergence structure as defined by $D_i^{(S)}$

Consequences:

- no \mathbb{H} -events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2} s_{\text{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $D_i^{(S)}$
- exponentiation of R through matrix element corrected parton shower
 NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2 / (p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
 → $D_i^{(A)} = \rho_i \cdot R \cdot f(p_\perp)$
 → continuous dampening of resummation kernel at large p_\perp



Mc@NLO – traditional scheme

Special choices:

Frixione, Webber JHEP06(2002)029

- exponentiation kernel $D_i^{(A)} = B \cdot \mathcal{K}_i$ with \mathcal{K}_i parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\max}$ parton shower starting scale
- in general, $D_i^{(A)}$ only leading colour approximation
NLO accuracy depends crucially on correctness of IR-limit

Modifications:

Frixione, Nason, Webber JHEP08(2003)007

- introduce soft modification function $f(p_\perp)$ such that

$$\sum B \cdot \mathcal{K}_i \cdot f(p_\perp) \xrightarrow{p_\perp \rightarrow 0} \sum D_i^{(S)}$$

- $f(p_\perp)$ process dependent in general



Mc@NLO – traditional scheme

Special choices:

Frixione, Webber JHEP06(2002)029

- exponentiation kernel $D_i^{(A)} = B \cdot \mathcal{K}_i$ with \mathcal{K}_i parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\max}$ parton shower starting scale
- in general, $D_i^{(A)}$ only leading colour approximation
NLO accuracy depends crucially on correctness of IR-limit

Modifications:

Frixione, Nason, Webber JHEP08(2003)007

- introduce soft modification function $f(p_\perp)$ such that

$$\sum B \cdot \mathcal{K}_i \cdot f(p_\perp) \xrightarrow{p_\perp \rightarrow 0} \sum D_i^{(S)}$$

- $f(p_\perp)$ process dependent in general



Mc@NLO – traditional scheme

Special choices:

Frixione, Webber JHEP06(2002)029

- exponentiation kernel $D_i^{(A)} = B \cdot \mathcal{K}_i$ with \mathcal{K}_i parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\max}$ parton shower starting scale
- in general, $D_i^{(A)}$ only leading colour approximation
NLO accuracy depends crucially on correctness of IR-limit

Modifications:

Frixione, Nason, Webber JHEP08(2003)007

- introduce soft modification function $f(p_\perp)$ such that

$$\sum B \cdot \mathcal{K}_i \cdot f(p_\perp) \xrightarrow{p_\perp \rightarrow 0} \sum D_i^{(S)}$$

- $f(p_\perp)$ process dependent in general



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Special choices:

Höche, Krauss, MS, Siebert JHEP09(2012)049

- exponentiation kernel $D_i^{(A)} = D_i^{(S)}$

Consequences:

- simplification of $\bar{B}^{(A)}$ -integral
- resummation scale $\mu_Q^2 = t_{\max}$ set by phase space limitation of subtraction terms
 - subtraction constrained in parton shower t needed for physical resummation
 - instructive example: use α_{cut} to explore effects Nagy PRD68(2003)094002
- trivially NLO correct independent of the process without arbitrary parameter choices



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Special choices:

Höche, Krauss, MS, Siebert JHEP09(2012)049

- exponentiation kernel $D_i^{(A)} = D_i^{(S)}$

Consequences:

- simplification of $\bar{B}^{(A)}$ -integral
- resummation scale $\mu_Q^2 = t_{\max}$ set by phase space limitation of subtraction terms
 - subtraction constrained in parton shower t needed for physical resummation
 - instructive example: use α_{cut} to explore effects [Nagy PRD68\(2003\)094002](#)
- trivially NLO correct independent of the process without arbitrary parameter choices



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm Höche, Krauss, MS, Siebert JHEP09(2012)049

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)} h(t) \exp \left\{ - \int_t^{t_1} d\bar{t} h(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) h(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} h(\bar{t}) \right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm [Höhe, Krauss, MS, Siegert JHEP09\(2012\)049](#)

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)} h(t) \exp \left\{ - \int_t^{t_1} d\bar{t} h(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) h(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} h(\bar{t}) \right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$



MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm [Höche, Krauss, MS, Siegert JHEP09\(2012\)049](#)

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)} h(t) \exp \left\{ - \int_t^{t_1} d\bar{t} h(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) h(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} h(\bar{t}) \right\} \right]$$

$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$

Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow D_i^{(A)}$
- $h(t)$ determined by parton shower $\Rightarrow D_i^{(PS)}$
- $g(t)$ **can be chosen freely** $\Rightarrow \text{const.} \cdot f$
constraints: $\text{sign}(f) = \text{sign}(g)$, $|f| \leq |g|$