NLO+PS matching and multijet merging

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arXiv:1207.5030, arXiv:1207.5031
arXiv:1208.2815

Introduction

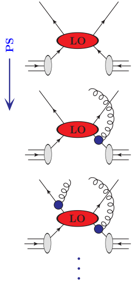
- Monte-Carlo event generators heavily used tools for fully exclusive SM/BSM predictions on hadron level
  $\rightarrow$ directly comparable to experimental data
- standard LO+PS calculations have limited accuracy in regions probed by current colliders
  $\rightarrow$ multiple hard objects in final state
- standard NLO calculations have limited accuracy in strongly hierarchical configurations

$\Rightarrow$ Tevatron and LHC allow for both at the same time

- a number of solutions to combine the observable independent resummation of PS with fixed-order accuracy in hard multiparton final states exist

$\Rightarrow$ **imperative to know about implicit choices/assumptions and how to assess uncertainties for phenomenological studies**

Motivation



LO $pp \to 2$ with parton showers
+ exponentiation of large IR logarithms
– poor hard/wide angle emission pattern

vs.

LO $pp \to n$ matrix elements
+ dominant terms for hard/wide angle rad.
– breakdown of α_s-expansion in log. region

- MEPs schemes: CKKW-type, MLM-type
  LO+(N)LL accuracy in every jet multiplicity
- scale setting scheme integral to preserve PS-resummation properties

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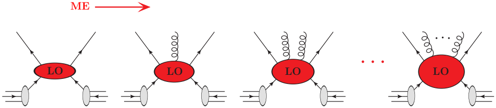

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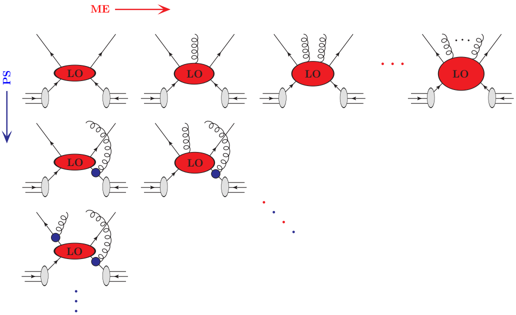

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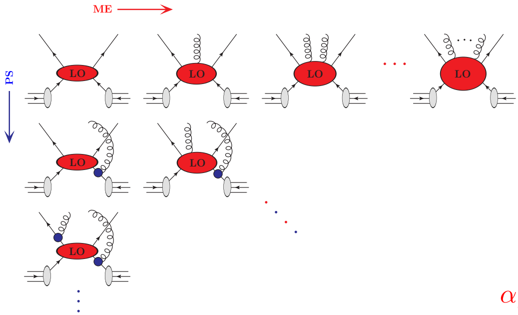

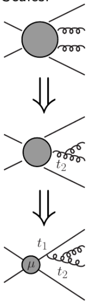

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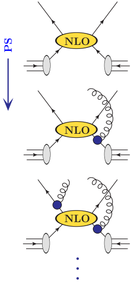

- promote LoPs to NloPs (Powheg, Mc@Nlo)
  → can assess uncertainties (part I)
- combine NloPs for successive multiplicities into incl. sample (MePs@Nlo),
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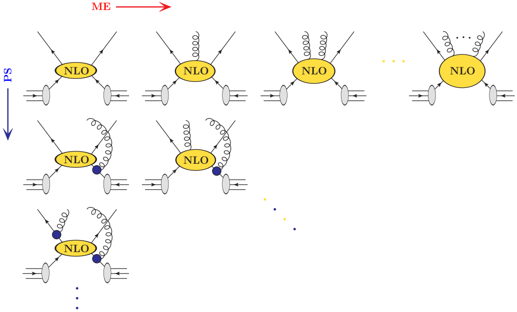

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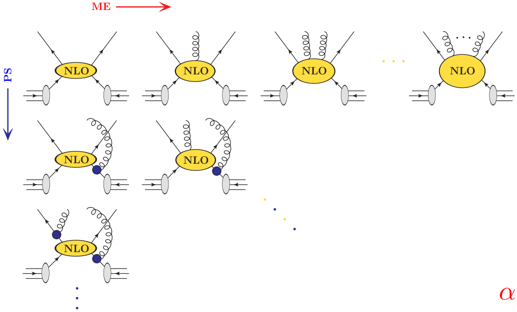
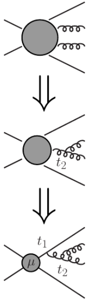


Scales:



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JHEP02(2009)007

The Sherpa event generator framework

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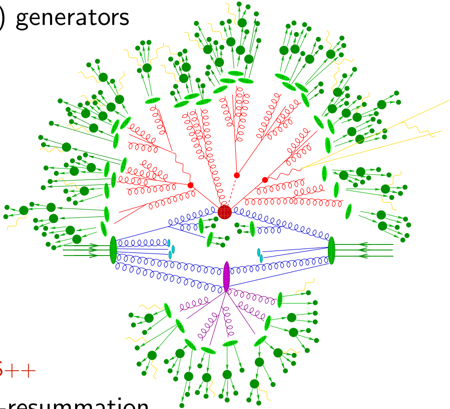

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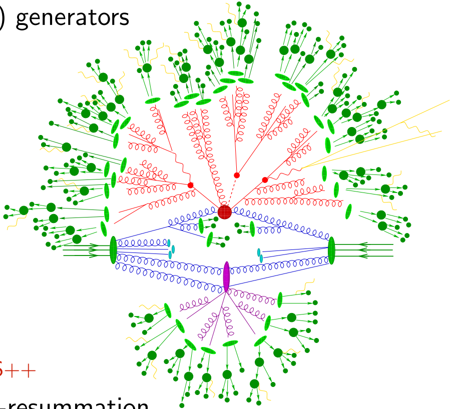

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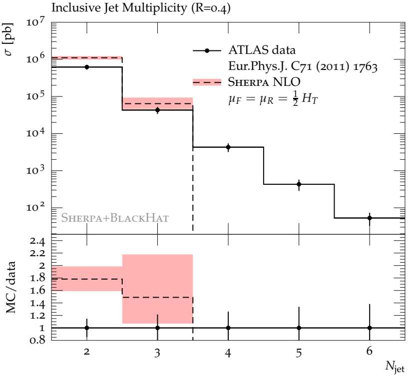
Short-comings of fixed-order QCD

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- very pronounced in inclusive & dijet production
- jet-$p_\perp$ turn negative in forward region unless y-dependent scale is used (e.g. $H_T^{(y)}$)



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≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$559(5)$	$1193(3)^{+130}_{-135}$	$0.95(0.02)$	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$39.7(0.9)$	$54.5(0.5)^{+2.2}_{-19.9}$	$0.92(0.04)$	$50.2(2.1)^{+2.0}_{-18.3}$
≥ 4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$3.97(0.08)$	$5.54(0.12)^{+0.08}_{-2.44}$	$0.92(0.05)$	$5.11(0.29)^{+0.08}_{-2.32}$

Bern et.al. arXiv:1112.3940

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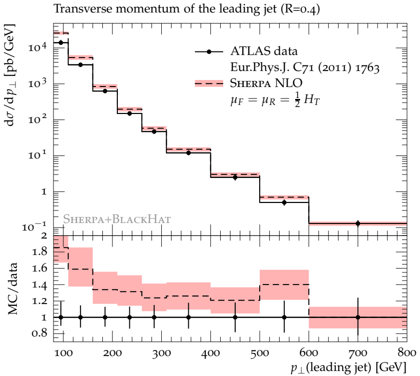

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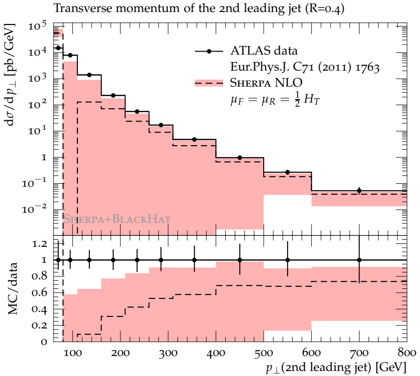

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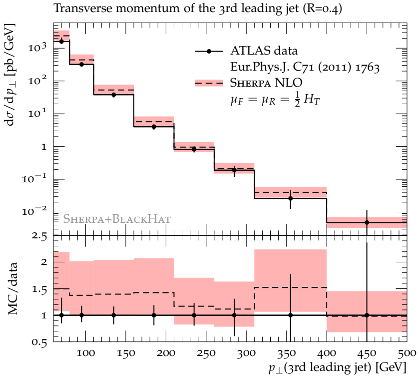

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Case study: Inclusive jet & dijet production

Describe wealth of experimental data with a single sample (LHC@7TeV)

Mc@Nlo di-jet production: Höche, MS arXiv:1208.2815

- $\mu_{R/F} = \frac{1}{4}\,H_T$, $\mu_Q = \frac{1}{2}\,p_\perp$
- CT10 PDF $(\alpha_s(m_Z) = 0.118)$
- hadron level calculation
  fully hadronised including MPI
- virtual MEs from BlackHat
  Giele, Glover, Kosower Nucl.Phys.B403(1993)633-670
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- $p_\perp^{j_1} > 20$ GeV, $p_\perp^{j_2} > 10$ GeV

Uncertainty estimates:

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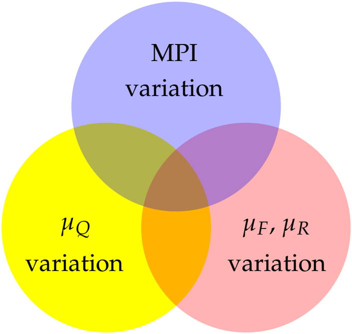

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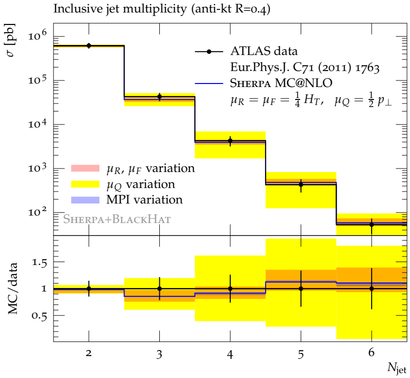

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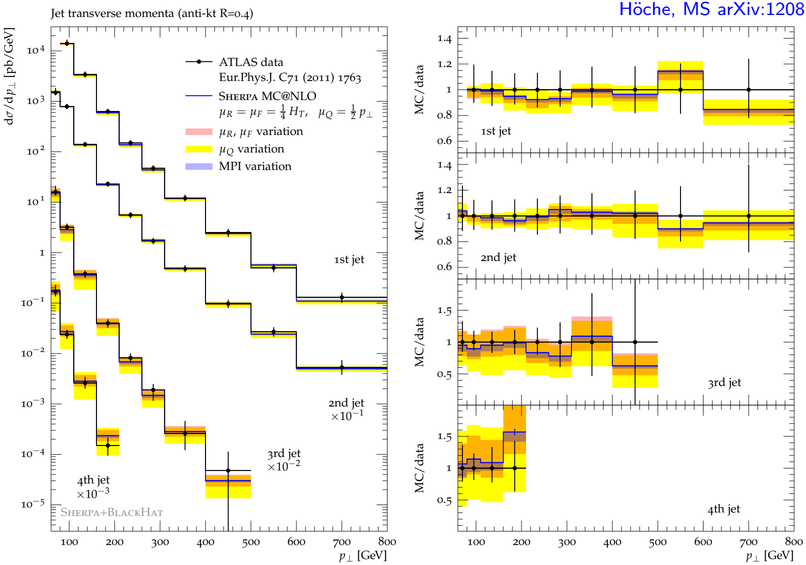

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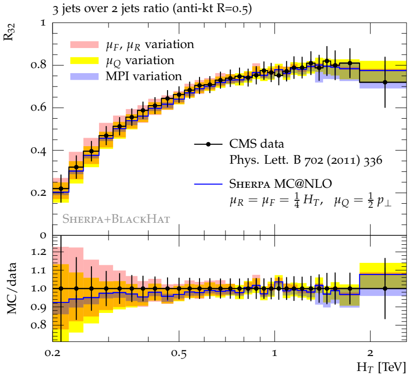

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3-jet-over-2-jet ratio

- determined from incl. sample
  2-jet rate at NLO+NLL
  3-jet rate at LO+LL
- common scale choices
  → varied simultaneously
- at large H_T large MPI uncertainties
  → better MPI physics needed (soft QCD)
- similar description of related ATLAS observables

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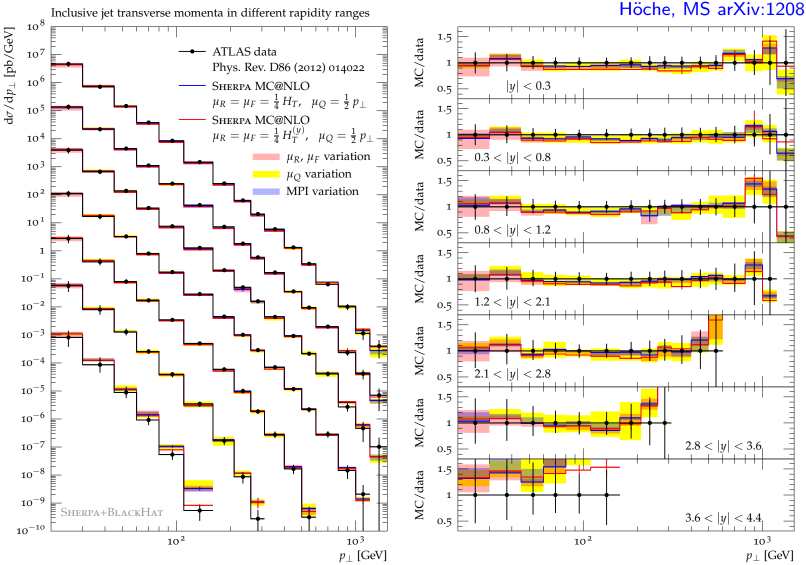

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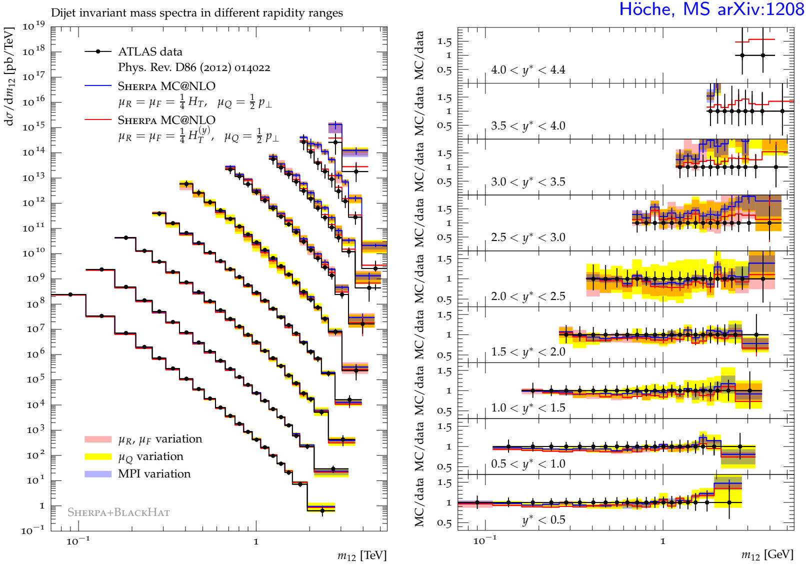

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Try different scale

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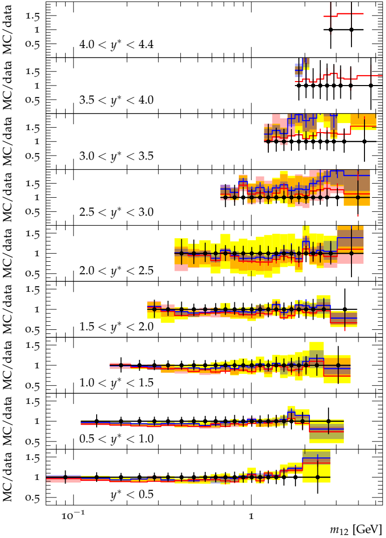
description of most other observables worsened

need proper description of forward physics (e.g. (B)FKL)



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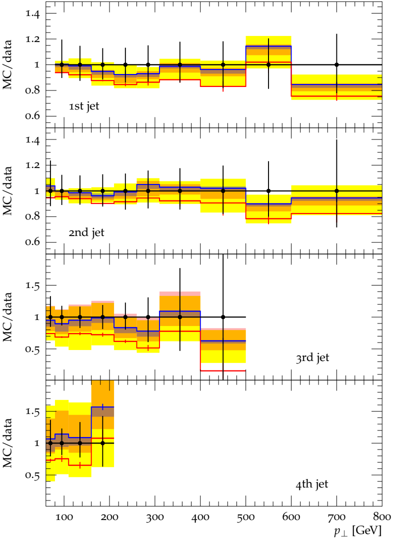

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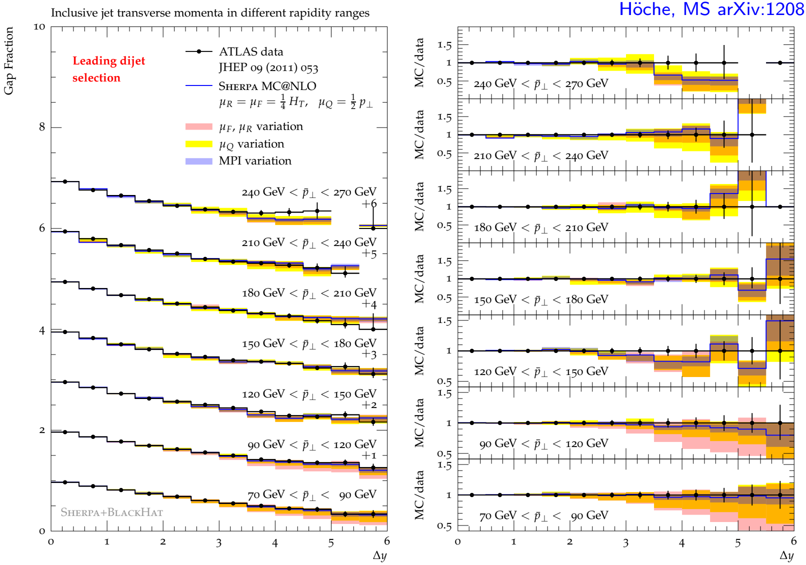

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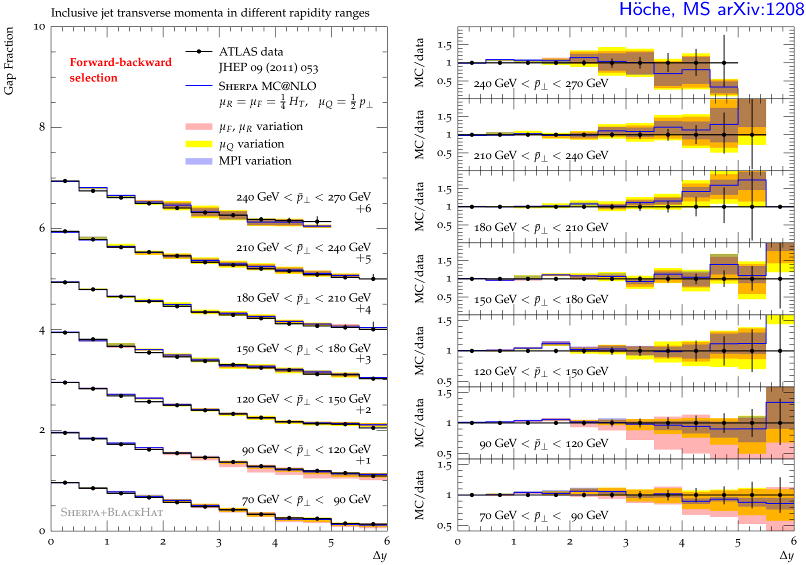


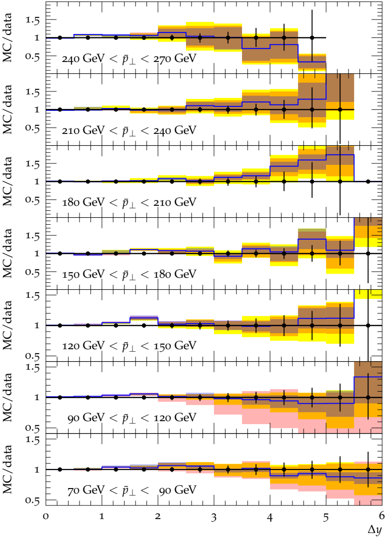
Case study: Inclusive jet & dijet production

Höche, MS arXiv:1208.2815

- small-Δy region
  $\Rightarrow$ small uncertainty on additional jet production
- large-Δy region
  $\Rightarrow$ all uncertainties sizable

- small-$\bar{p}_\perp$ region
  $\Rightarrow$ dominated by perturbative uncertainties
- large-$\bar{p}_\perp$ region
  $\Rightarrow$ non-perturbative uncertainties as large as perturbative uncertainties

Reduction of theoretical uncertainty necessitates better understanding of soft QCD and non-factorisable contributions



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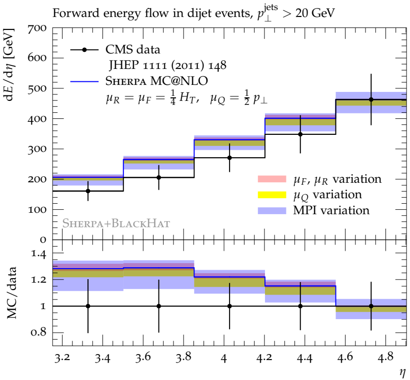
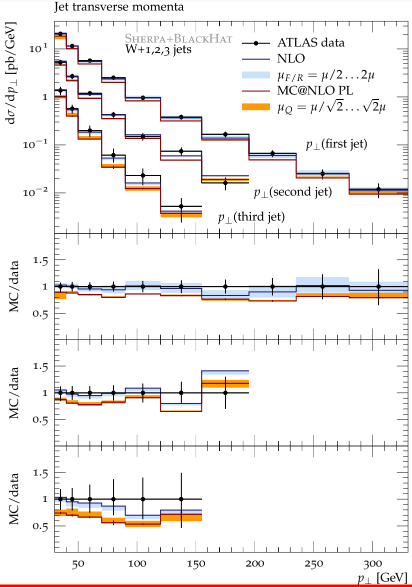


Höche, MS arXiv:1208.2815

Forward energy flow

- energy flow in rapidity interval per event with a central back-to-back di-jet pair
- normalisation reduces $\mu_{R/F}$ and μ_Q dependence
- dominated by MPI modeling uncertainty

$W + n$ jet production



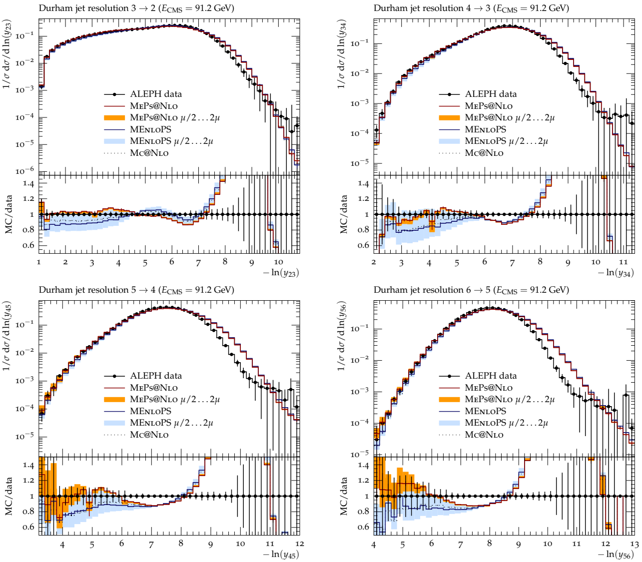
Höche, Krauss, MS, Siegert arXiv:1201.5882

$pp \to W + 1, 2, 3$ jets

- 3 separate samples/calculations
- NLO accuracy for inclusive observables of respective jet multiplicity
- resummation of softest/LO jet, i.e. 4th jet in $pp \to W + 3$ jets
- no resummation of sample-defining jet multiplicity, i.e. first 3 jets in $pp \to W + 3$ jets
- scales: $\mu_{R/F} = \frac{1}{2}\,\hat{H}'_T,\ \mu_Q = p_\perp(j_n)$

Data: ATLAS Phys.Rev.D85(2012)092002

Results: $e^+e^- \to$ hadrons



$ee \to$ hadrons (2,3,4 @ NLO; 5,6 @ LO)

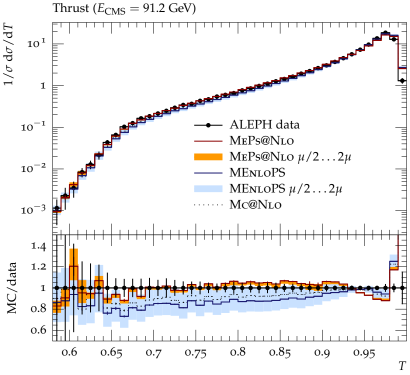
Jet resolutions (Durham measure)

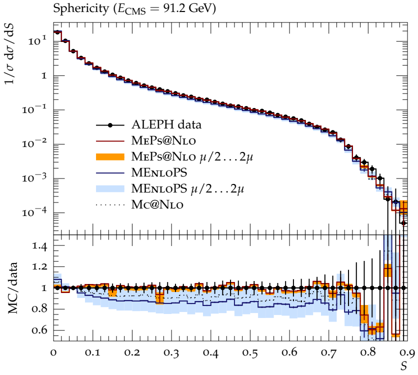
- MEPS@NLO vs MENLOPS
- at $y \ll 1$ dominated by hadr. effects $\to$ needs retuning
- much improved ren. scale dependence

ALEPH data EPJC35(2004)457-486

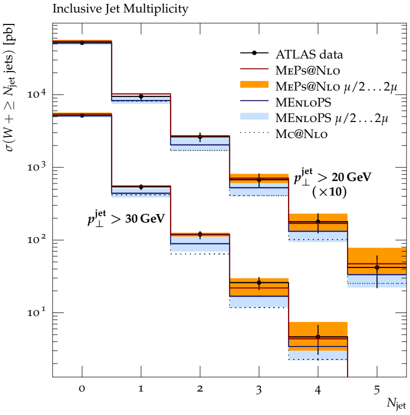
Results: $e^+e^- \to$ hadrons

ALEPH data EPJC35(2004)457-486





Results: pp → W+jets

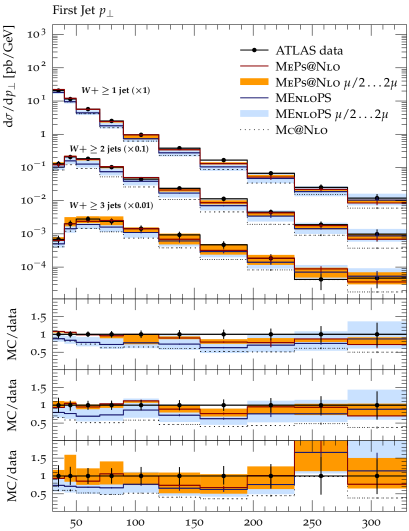


$pp \to W$+jets (0,1,2 @ NLO; 3,4 @ LO)

- $\mu_{R/F} \in [\frac{1}{2}, 2]\,\mu_{\text{def}}$ scale uncertainty much reduced
- NLO dependence for $pp \to W$+0,1,2 jets LO dependence for $pp \to W$+3,4 jets
- $Q_{\text{cut}} = 30$ GeV
- good data description

ATLAS data Phys.Rev.D85(2012)092002

Results: $pp \to W$+jets



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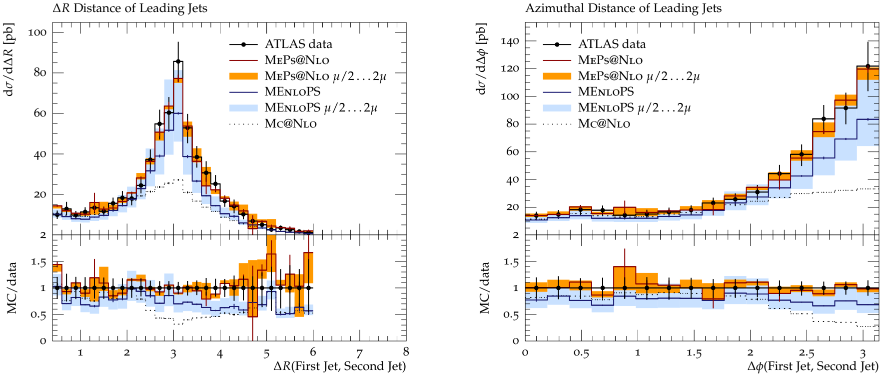
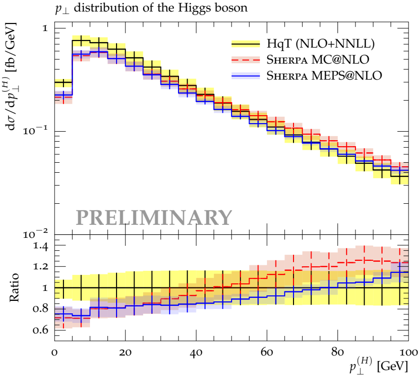
ATLAS data Phys.Rev.D85(2012)092002

Results: pp $\to$ W+jets

ATLAS data Phys.Rev.D85(2012)092002



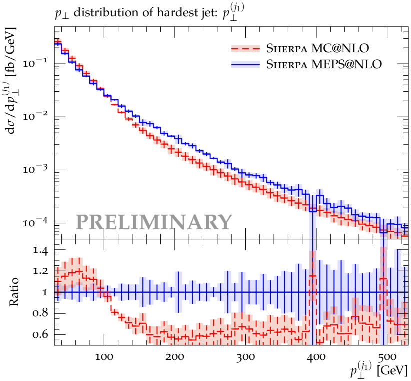
Results: $pp \to h$+jets



$pp \to h$+jets (0,1 @ NLO; 2 @ LO)

- $\mu_{R/F} \in [\frac{1}{2}, 2]\,\mu_{\text{CKKW}}$
  scale uncertainty much reduced
- NLO dependence for $pp \to h$+0,1 jets
  LO dependence for $pp \to h$+2 jets
- $Q_{\text{cut}} = 30$ GeV
- HqT: $\mu_{R/F} \in [\frac{1}{2}, 2] \cdot \frac{1}{2}\, m_h$
  $\mu_R^{\text{exp}} \propto p_\perp^h$

Results: pp → h+jets



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Conclusions

- SHERPA's MC@NLO formulation allows full evaluation of perturbative uncertainties (μ_F, μ_R, μ_Q)
- MC@NLO can be easily combined with MEPS $\rightarrow$ MENLOPS
- MC@NLO is a necessary input for NLO merging $\rightarrow$ MEPS@NLO
- MEPS@NLO gives full benefits of NLO calculations (scale dependences, normalisations) while also retaining full (N)LL accuracy of parton shower

$\Rightarrow$ will be included in SHERPA-2.0.α (upcoming)

Current release: SHERPA-1.4.2

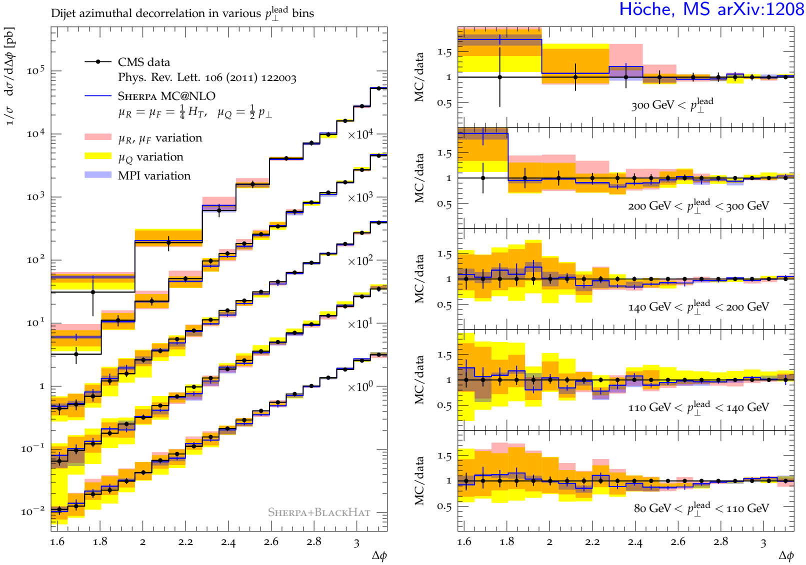
http://sherpa.hepforge.org

- better description of perturbative QCD is only part of the story to achieve higher precission for (hard) collider observables

Thank you for your attention!

Case study: Inclusive jet & dijet production

Höche, MS arXiv:1208.2815



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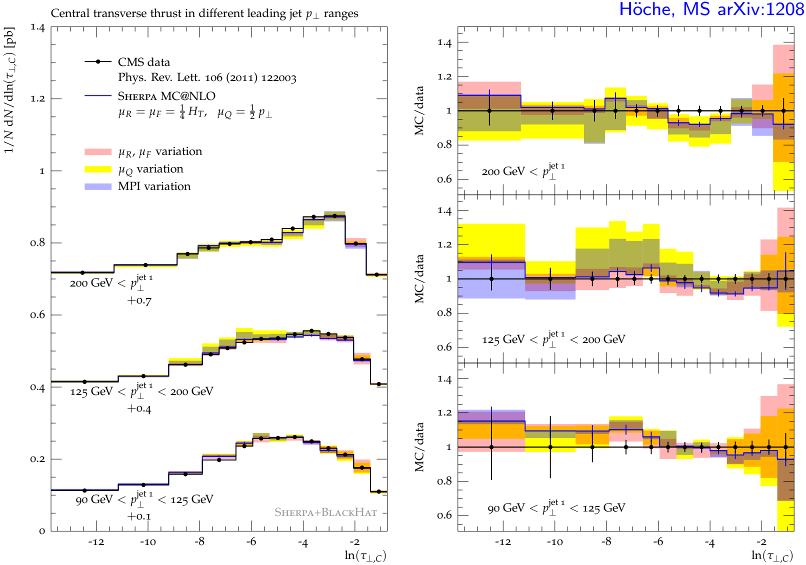

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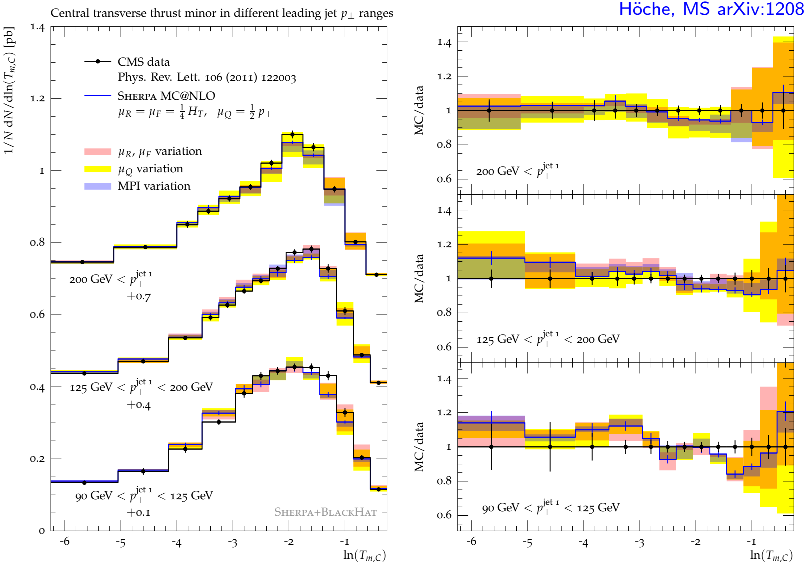

General NLO calculations

- NLO calculation with subtraction methods

$$\langle O\rangle^{\text{NLO}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}(\Phi_B) + \mathrm{V}(\Phi_B) + \mathrm{I}(\Phi_B)\Big]\, O(\Phi_B)$$

$$+ \int \mathrm{d}\Phi_R \Big[- \sum_i \mathrm{D}_i^{(\mathrm{S})}(\Phi_R)\, O(\Phi_{B_i})\Big]$$

$$+ \int \mathrm{d}\Phi_R \Big[\mathrm{R}(\Phi_R) \Big]\, O(\Phi_R)$$

- introduce second set of subtraction functions $\mathrm{D}_i^{(\mathrm{A})}$
- $\mathrm{D}_i^{(\mathrm{A})}$ and $\mathrm{D}_i^{(\mathrm{S})}$ need to have same momentum maps and IR limit
  - full spin-correlations in collinear limit
  - full colour-correlations in soft limit

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$$+ \int \mathrm{d}\Phi_R \Big[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\mathsf{A})}(\Phi_R)\Big]\, O(\Phi_R) + \langle O\rangle^{(\mathsf{A})}_{\text{corr}}$$

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$$
\begin{aligned}
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&+ \sum_i \int \mathrm{d}\Phi_R \quad \Big[\mathrm{D}_i^{(\text{A})}(\Phi_R) \quad - \mathrm{D}_i^{(\text{S})}(\Phi_R) \quad \Big]\, O(\Phi_{B_i}) \\
&+ \int \mathrm{d}\Phi_R \Big[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\text{A})}(\Phi_R)\Big]\, O(\Phi_R) + \langle O\rangle^{(\text{A})}_{\text{corr}}
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$$+ \sum_i \int \mathrm{d}\Phi_{B_i} \mathrm{d}\Phi_1^i \Big[\mathrm{D}_i^{(\text{A})}(\Phi_{B_i}, \Phi_1^i) - \mathrm{D}_i^{(\text{S})}(\Phi_{B_i}, \Phi_1^i)\Big]\, O(\Phi_{B_i})$$
$$+ \int \mathrm{d}\Phi_R \Big[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\text{A})}(\Phi_R)\Big]\, O(\Phi_R) + \langle O\rangle_{\text{corr}}^{(\text{A})}$$

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Parton showers and resummation

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$
  $\to \mathcal{K}_i$ incorporates divergent propagator and DGLAP splitting kernels

$$\langle O\rangle^{\mathsf{PS}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

$$= \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B) +$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t,t') = \exp\left[-\int_t^{t'} \mathrm{d}\Phi_1\, \mathcal{K}(\Phi_1)\right]$ contains resummation features
- $\langle O\rangle^{(\mathcal{K})}_{\mathsf{corr}}$ generated by one parton shower step

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$$\langle O\rangle^{\text{PS}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[\Delta^{(\mathcal{K})}(t_0, \mu_F^2)\, O(\Phi_B) + \int\limits_{t_0}^{\mu_F^2} \mathrm{d}\Phi_1 \mathcal{K}(\Phi_1)\, \Delta^{(\mathcal{K})}(t, \mu_F^2)\, O(\Phi_R) \right]$$

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$$= \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B) + \int\limits_{t_0}^{\mu_F^2}\mathrm{d}\Phi_R\,\mathrm{B}\cdot\mathcal{K}(\Phi_1)\Big[O(\Phi_R) - O(\Phi_B)\Big] + \mathcal{O}(\alpha_s^2)$$

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General NLO+PS matching

$$\langle O\rangle^{\text{NLO+PS}} = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathsf{A})}(\Phi_B) \left[\Delta^{(\mathsf{A})}(t_0,\mu_Q^2)\, O(\Phi_B) + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}^{(\mathsf{A})}(\Phi_B,\Phi_1)}{\mathrm{B}(\Phi_B)}\, \Delta^{(\mathsf{A})}(t,\mu_Q^2)\, O(\Phi_R) \right]$$

$$+ \int \mathrm{d}\Phi_R \left[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\mathsf{A})}(\Phi_R) \right] O(\Phi_R) + \langle O\rangle^{(\mathsf{A})}_{\text{corr}}$$

- use $\mathrm{D}_i^{(\mathsf{A})}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$
  → starting scale of parton shower evolution
  → should be of the order of the scale of the hard interaction
- **POWHEG and MC@NLO now differ in choice of $\mathrm{D}_i^{(\mathsf{A})}$ and μ_Q^2**

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$$\Delta^{(\mathsf{A})}(t,t') = \exp\left[- \int_t^{t'} \mathrm{d}\Phi_1\, \mathrm{D}^{(\mathsf{A})}/\mathrm{B} \right] = \exp\left[-\alpha_s \log^2 \tfrac{t}{t'} + \ldots \right]$$

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POWHEG

Special choices: Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $\mathrm{D}_i^{(\mathsf{A})} = \rho_i \cdot \mathrm{R}$ with $\rho_i = \mathrm{D}_i^{(\mathsf{S})} / \sum_i \mathrm{D}_i^{(\mathsf{S})}$
  $\rightarrow$ each $\rho_i \cdot \mathrm{R}$ contains only one divergence structure as defined by $\mathrm{D}_i^{(\mathsf{S})}$

Consequences:

- no $\mathbb{H}$-events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2}\, s_{\mathsf{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $\mathrm{D}_i^{(\mathsf{S})}$
- exponentiation of R through matrix element corrected parton shower
  NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2/(p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
  $\rightarrow \mathrm{D}_i^{(\mathsf{A})} = \rho_i \cdot \mathrm{R} \cdot f(p_\perp)$
  $\rightarrow$ continuous dampening of resummation kernel at large $p_\perp$

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Mc@Nlo – traditional scheme

Special choices: Frixione, Webber JHEP06(2002)029

- exponentiation kernel $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{B}\cdot\mathcal{K}_i$ with $\mathcal{K}_i$ parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\mathsf{max}}$ parton shower starting scale
- in general, $\mathrm{D}_i^{(\mathsf{A})}$ only leading colour approximation and spin-avaraged
  NLO accuracy depends crucially on correctness of IR-limit

Modifications: Frixione, Nason, Webber JHEP08(2003)007

- introduce soft modification function $f(p_\perp)$ such that

$$\sum \mathrm{B}\cdot\mathcal{K}_i\cdot f(p_\perp) \xrightarrow{p_\perp\to 0} \sum \mathrm{D}_i^{(\mathsf{S})}$$

- $f(p_\perp)$ process dependent in general

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MC@NLO – $\mathrm{D}_i^{(A)} = \mathrm{D}_i^{(S)}$ scheme

Special choices: SH, FK, MS, FS arXiv:1111.1220

- exponentiation kernel $\mathrm{D}_i^{(A)} = \mathrm{D}_i^{(S)}\,\Theta(\mu_Q^2 - t)$

Consequences:

- simplification of $\bar{\mathrm{B}}^{(A)}$-integral
- resummation scale $\mu_Q^2 = t_{\mathrm{max}}$ set by phase space limitation of subtraction terms
  $\rightarrow$ subtraction constrained in parton shower t needed for physical resummation
- trivially NLO correct independent of the process without arbitrary parameter choices

MC@NLO – $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{D}_i^{(\mathsf{S})}$ scheme

Special choices: SH, FK, MS, FS arXiv:1111.1220

- exponentiation kernel $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{D}_i^{(\mathsf{S})}\,\Theta(\mu_Q^2 - t)$

Consequences:

- simplification of $\bar{\mathrm{B}}^{(\mathsf{A})}$-integral
- resummation scale $\mu_Q^2 = t_{\mathsf{max}}$ set by phase space limitation of subtraction terms
  $\rightarrow$ subtraction constrained in parton shower t needed for physical resummation
- trivially NLO correct independent of the process without arbitrary parameter choices

MC@NLO – $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{D}_i^{(\mathsf{S})}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $\mathrm{D}_i^{(\mathsf{A})} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm SH, FK, MS, FS arXiv:1111.1220

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
  Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)}\,g(t)\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t_{n+1}}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)g(t_i)\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)}\,h(t)\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,h(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t_{n+1}}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)h(t_i)\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,h(\bar{t})\right\}\right]$$

$$w(t,t_1,\ldots,t_n) = \frac{g(t)}{h(t)}\prod_{i=1}^{n}\frac{g(t_i)}{h(t_i)}\,\frac{h(t_i)-f(t_i)}{g(t_i)-f(t_i)}$$

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$$w(t,t_1,\ldots,t_n) = \frac{g(t)}{h(t)}\prod_{i=1}^{n}\frac{g(t_i)}{h(t_i)}\,\frac{h(t_i)-f(t_i)}{g(t_i)-f(t_i)}$$

Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow \mathrm{D}_i^{(\mathsf{A})}$
- $h(t)$ determined by parton shower $\Rightarrow \mathrm{D}_i^{(\mathsf{PS})}$
- $g(t)$ **can be chosen freely** $\Rightarrow$ const. $\cdot\, f$
  constraints: sign(f) = sign(g), $|f| \leq |g|$

NLO merging

LO merging:

- LO accuracy for $n \leq n_{\mathrm{max}}$-jet processes
- preserve LL accuracy of the parton shower

Catani, Krauss, Kuhn, Webber JHEP11(2001)063
Lönnblad JHEP05(2002)046
Höche, Krauss, Schumann, Siegert JHEP05(2009)053
Hamilton, Richardson, Tully JHEP11(2009)038
Lönnblad, Prestel JHEP03(2012)019

NLO merging:

- NLO accuracy for $n \leq n_{\mathrm{max}}$-jet processes
- preserve LL accuracy of the parton shower

Lavesson, Lönnblad JHEP12(2008)070
Höche, Krauss, MS, Siegert arXiv:1207.5030
Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

NLO merging

Höche, Krauss, MS, Siegert arXiv:1207.5030
Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$= \int \mathrm{d}\Phi_n\; \bar{\mathrm{B}}_n^{(\mathsf{A})} \left[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\, O_n + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\; O_{n+1} \right]$$

$$+ \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})} \right] \; O_{n+1}$$

NLO merging

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$= \int d\Phi_n\, \bar{\mathrm{B}}_n^{\text{(A)}} \left[\Delta_n^{\text{(A)}}(t_0,\mu_Q^2)\, O_n + \int_{t_0}^{\mu_Q^2} d\Phi_1\, \frac{\mathrm{D}_n^{\text{(A)}}}{\mathrm{B}_n}\, \Delta_n^{\text{(A)}}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\text{cut}}-Q)\, O_{n+1}\right]$$

$$+ \int d\Phi_{n+1}\left[\mathrm{R}_n - \mathrm{D}_n^{\text{(A)}}\right] \Theta(Q_{\text{cut}}-Q)\, \Delta_n^{\text{(PS)}}(t_{n+1},\mu_Q^2)\, O_{n+1}$$

$$+ \int d\Phi_{n+1}\, \bar{\mathrm{B}}_{n+1}^{\text{(A)}}\, \Theta(Q-Q_{\text{cut}})$$

$$\times \left[\Delta_{n+1}^{\text{(A)}}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} d\Phi_1\, \frac{\mathrm{D}_{n+1}^{\text{(A)}}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{\text{(A)}}(t_{n+2},t_{n+1})\, O_{n+2}\right]$$

$$+ \int d\Phi_{n+2}\left[\mathrm{R}_{n+1} - \mathrm{D}_{n+1}^{\text{(A)}}\right] \Theta(Q-Q_{\text{cut}})\, O_{n+2}$$

NLO merging

Höche, Krauss, MS, Siegert arXiv:1207.5030
Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathsf{A})} \Bigg[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\, O_n + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\mathsf{cut}}-Q)\, O_{n+1}\Bigg]$$

$$+ \int \mathrm{d}\Phi_{n+1}\Big[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})}\Big]\, \Theta(Q_{\mathsf{cut}}-Q)\, \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, O_{n+1}$$

NLO merging

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$\langle O\rangle^{\text{MEPS@NLO}}$

$$
\begin{aligned}
&= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathsf{A})} \bigg[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\, O_n \\
&\qquad\qquad + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\text{cut}}-Q)\, O_{n+1} \bigg] \\
&\quad + \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})}\right] \Theta(Q_{\text{cut}}-Q)\, \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, O_{n+1} \\
&\quad + \int \mathrm{d}\Phi_{n+1}\, \bar{\mathrm{B}}_{n+1}^{(\mathsf{A})} \left[1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_n\right] \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}}) \\
&\qquad\qquad \times \left[\Delta_{n+1}^{(\mathsf{A})}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_{n+1}^{(A)}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{(\mathsf{A})}(t_{n+2},t_{n+1})\, O_{n+2} \right] \\
&\quad + \int \mathrm{d}\Phi_{n+2} \left[\mathrm{R}_{n+1} - \mathrm{D}_{n+1}^{(\mathsf{A})}\right] \Delta_{n+1}^{(\mathsf{PS})}(t_{n+2},t_{n+1})\, \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}})\, O_{n+2}
\end{aligned}
$$

NLO merging

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$\begin{aligned}
&= \int \mathrm{d}\Phi_n\; \bar{\mathrm{B}}_n^{(\mathsf{A})} \Bigg[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\, O_n \\
&\qquad\qquad + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\text{cut}}-Q)\, O_{n+1} \Bigg] \\
&\quad + \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})}\right] \Theta(Q_{\text{cut}}-Q)\, \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, O_{n+1} \\
&\quad + \int \mathrm{d}\Phi_{n+1}\; \bar{\mathrm{B}}_{n+1}^{(\mathsf{A})}\; \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}}) \\
&\qquad\qquad \times \left[\Delta_{n+1}^{(\mathsf{A})}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_{n+1}^{(A)}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{(\mathsf{A})}(t_{n+2},t_{n+1})\, O_{n+2} \right] \\
&\quad + \int \mathrm{d}\Phi_{n+2} \left[\mathrm{R}_{n+1} - \mathrm{D}_{n+1}^{(\mathsf{A})}\right] \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\text{cut}})\, O_{n+2}
\end{aligned}$$

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$$\begin{aligned}
\langle O\rangle^{\text{MePs@Nlo}}
&= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathrm{A})} \Bigg[\Delta_n^{(\mathrm{A})}(t_0,\mu_Q^2)\, O_n \\
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&\quad + \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathrm{A})}\right] \Theta(Q_{\mathrm{cut}}-Q)\, \Delta_n^{(\mathrm{PS})}(t_{n+1},\mu_Q^2)\, O_{n+1} \\
&\quad + \int \mathrm{d}\Phi_{n+1}\, \bar{\mathrm{B}}_{n+1}^{(\mathrm{A})} \left[1 + \frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_n \right] \Delta_n^{(\mathrm{PS})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\mathrm{cut}}) \\
&\qquad \times \left[\Delta_{n+1}^{(\mathrm{A})}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_{n+1}^{(A)}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{(\mathrm{A})}(t_{n+2},t_{n+1})\, O_{n+2} \right] \\
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\end{aligned}$$

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$\langle O \rangle^{\text{MEPS@NLO}}$

$$
\begin{aligned}
&= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathsf{A})} \Bigg[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\, O_n \\
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&\quad + \int \mathrm{d}\Phi_{n+2} \left[\mathrm{R}_{n+1} - \mathrm{D}_{n+1}^{(\mathsf{A})}\right] \Delta_{n+1}^{(\mathsf{PS})}(t_{n+2},t_{n+1})\, \Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)\, \Theta(Q-Q_{\mathsf{cut}})\, O_{n+2}
\end{aligned}
$$

NLO merging – Generation of MC counterterm

$$\left[1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}}\int_{t_{n+1}}^{\mu_Q^2}\mathrm{d}\Phi_1\,\mathrm{K}_n\right]$$

- same form as exponent of Sudakov form factor $\Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)$
- truncated parton shower on n-parton configuration underlying $n+1$-parton event
  1. no emission $\rightarrow$ retain $n+1$-parton event as is
  2. first emission at t' with $Q > Q_{\mathsf{cut}}$, multiply event weight with $\mathrm{B}_{n+1}/\bar{\mathrm{B}}^{(\mathsf{A})}_{n+1}$, restart evolution at t', do not apply emission kinematics
  3. treat every subsequent emission as in standard truncated vetoed shower
- generates

$$\left[1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}}\int_{t_{n+1}}^{\mu_Q^2}\mathrm{d}\Phi_1\,\mathrm{K}_n\right]\Delta_n^{(\mathsf{PS})}(t_{n+1},\mu_Q^2)$$

$\Rightarrow$ **identify $\mathcal{O}(\alpha_s)$ counterterm with the emitted emission**

NLO merging

Renormalisation scales:

- determined by clustering using PS probabilities and taking the respective nodal values t_i

$$\alpha_s(\mu_R^2)^k = \prod_{i=1}^{k} \alpha_s(t_i)$$

- change of scales $\mu_R \to \tilde{\mu}_R$ in MEs necessitates one-loop counter term

$$\alpha_s(\tilde{\mu}_R^2)^k \left(1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\,\beta_0 \sum_{i=1}^{k} \ln \frac{t_i}{\tilde{\mu}_R^2}\right)$$

Factorisation scale:

- μ_F determined from core n-jet process
- change of scales $\mu_F \to \tilde{\mu}_F$ in MEs necessitates one-loop counter term

$$\mathrm{B}_n(\Phi_n)\,\frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\,\log\frac{\mu_F^2}{\tilde{\mu}_F^2}\left(\sum_{c=q,g}^{n}\int_{x_a}^{1}\frac{\mathrm{d}z}{z}\,P_{ac}(z)\,f_c(x_a/z,\tilde{\mu}_F^2) + \ldots\right)$$