Multijet merging at NLO

Marek Schönherr

Institute for Particle Physics Phenomenology

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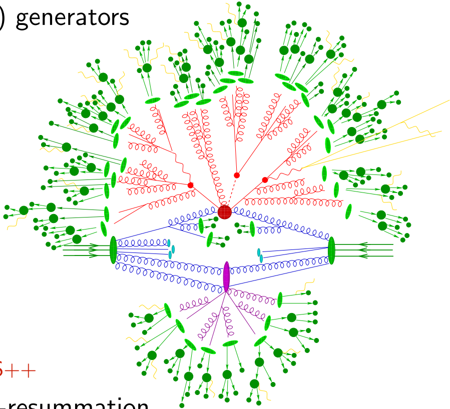
arXiv:1111.1220, arXiv:1201.5882
arXiv:1207.5030, arXiv:1207.5031
arXiv:1208.2815

JHEP02(2009)007

The Sherpa event generator framework



- Two multi-purpose Matrix Element (ME) generators
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  COMIX JHEP12(2008)039
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- A Parton Shower (PS) generator
  CSShower++ JHEP03(2008)038
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  à la Pythia Amisic++ hep-ph/0601012
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- A hadron and τ decay package Hadrons++
- A higher order Qed generator using YFS-resummation
  Photons++ JHEP12(2008)018

Sherpa's traditional strength is the perturbative part of the event
MePs (CKKW), Mc@Nlo, MeNloPs, MePs@Nlo
$\rightarrow$ full analytic control mandatory for consistency/accuracy

JHEP02(2009)007

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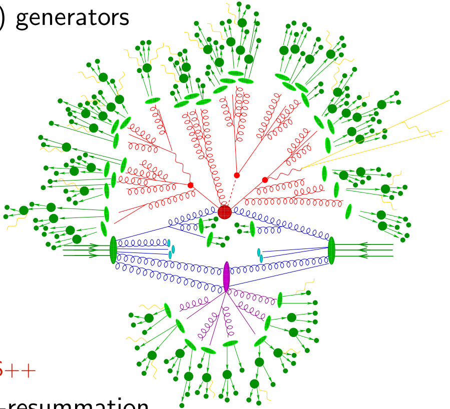

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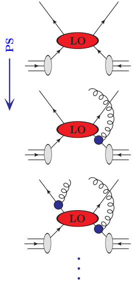

LO $pp \to 2$ with parton showers vs. **LO $pp \to n$ matrix elements**

+ exponentiation of large IR logarithms
− poor hard/wide angle emission pattern

+ dominant terms for hard/wide angle rad.
− breakdown of α_s-expansion in log. region

- MEPS schemes: CKKW-type, MLM-type
  LO+(N)LL accuracy in every jet multiplicity
- scale setting scheme integral to preserve PS-resummation properties

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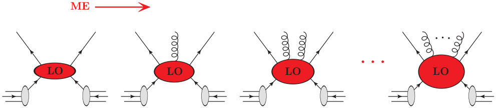

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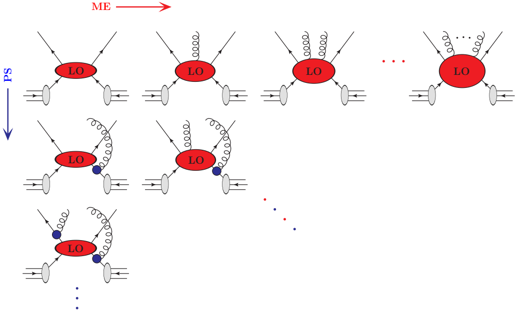

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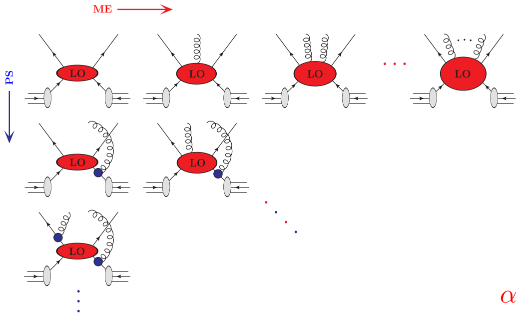

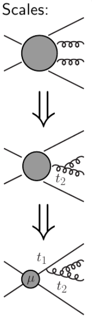

$$\alpha_s^{k+n}(\mu_{\text{eff}}) = \alpha_s^k(\mu)\,\alpha_s(t_1)\cdots\alpha_s(t_n)$$

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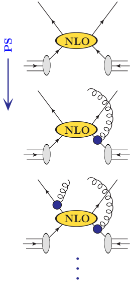

- promote LoPs to NloPs (Powheg, Mc@Nlo)
  $\rightarrow$ (part I)

Mc@Nlo

Frixione, Webber JHEP06(2002)029

$$\langle O\rangle^{\text{NLO+PS}} = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathrm{A})}(\Phi_B)\left[\Delta^{(\mathrm{A})}(t_0,\mu_Q^2)\, O(\Phi_B) + \sum_i \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1^i\, \frac{\mathrm{D}_i^{(\mathrm{A})}(\Phi_B,\Phi_1)}{\mathrm{B}(\Phi_B)}\, \Delta^{(\mathrm{A})}(t,\mu_Q^2)\, O(\Phi_R)\right] + \int \mathrm{d}\Phi_R\left[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\mathrm{A})}(\Phi_R)\right] O(\Phi_R)$$

Höche, Krauss, MS, Siegert arXiv:1111.1220

- NLO weighted Born configuration $\bar{\mathrm{B}}^{(\mathrm{A})} = \mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I} + \int \mathrm{d}\Phi_1 [\mathrm{D}^{(\mathrm{A})} - \mathrm{D}^{(\mathrm{S})}]$
- use $\mathrm{D}_i^{(\mathrm{A})}$ as resummation kernels: choice $\mathrm{D}_i^{(\mathrm{A})} = \mathrm{D}_i^{(\mathrm{S})}\Theta(\mu_Q^2 - t)$

  $\Delta^{(\mathrm{A})}(t,t') = \exp\left[-\int_t^{t'} \mathrm{d}\Phi_1\, \mathrm{D}^{(\mathrm{A})}/\mathrm{B}\right] = \exp\left[-\alpha_s \log^2(t/t') \cdot f + \ldots\right]$
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$

  → starting scale of parton shower evolution (exponent vanishes)

  → should be of the order of the hard resummation scale

  ⇒ first implementation to allow to study μ_Q uncertainty

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Case study: Inclusive jet & dijet production

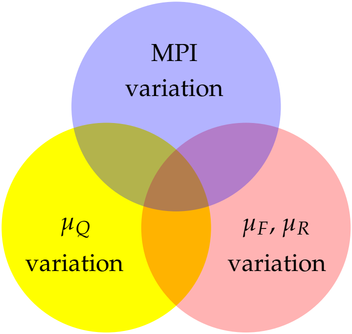
Describe wealth of experimental data with a single sample (LHC@7TeV)

MC@NLO di-jet production: Höche, MS arXiv:1208.2815

- $\mu_{R/F} = \frac{1}{4}\,H_T$, $\mu_Q = \frac{1}{2}\,p_\perp$
- CT10 PDF $(\alpha_s(m_Z) = 0.118)$
- hadron level calculation
  fully hadronised including MPI
- virtual MEs from BLACKHAT
  Giele, Glover, Kosower Nucl.Phys.B403(1993)633-670
  Bern et.al. arXiv:1112.3940
- $p_\perp^{j_1} > 20$ GeV, $p_\perp^{j_2} > 10$ GeV

Uncertainty estimates:

- $\mu_{R/F} \in [\frac{1}{2}, 2]\,\mu_{R/F}^{\text{def}}$
- $\mu_Q \in [\frac{1}{\sqrt{2}}, \sqrt{2}]\,\mu_Q^{\text{def}}$
- MPI activity in tr. region $\pm$ 10%



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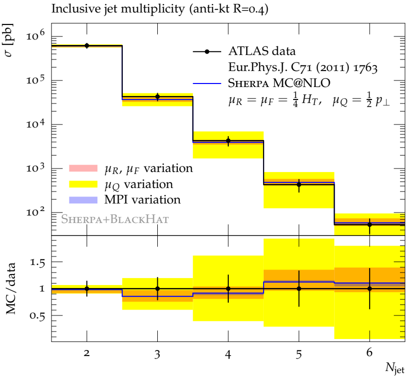

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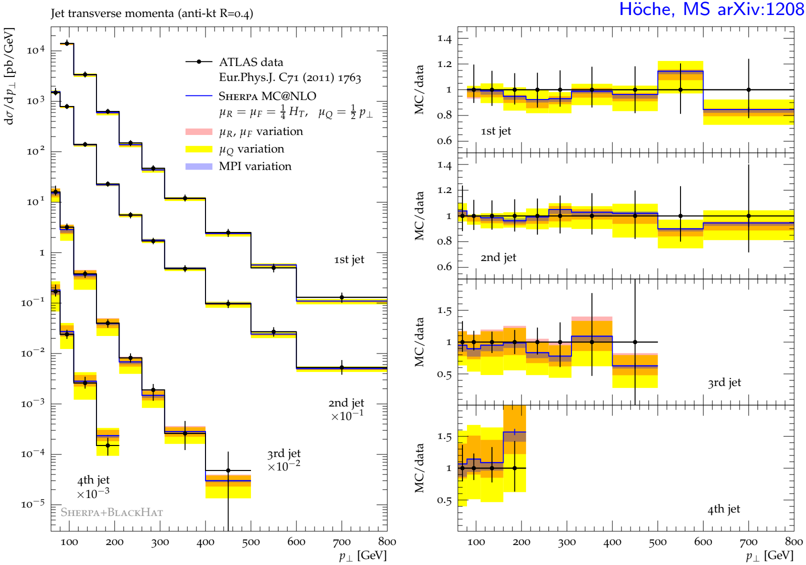

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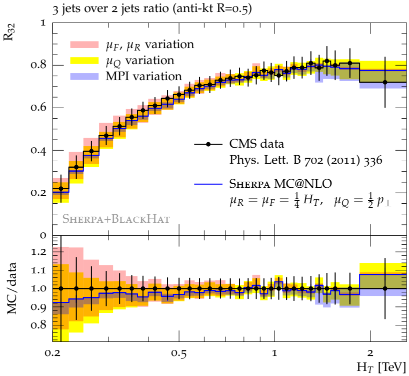

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3-jet-over-2-jet ratio

- determined from incl. sample
  2-jet rate at NLO+NLL
  3-jet rate at LO+LL
- common scale choices
  → varied simultaneously
- at large H_T large MPI uncertainties
  → better MPI physics needed (soft QCD)
- similar description of related ATLAS observables

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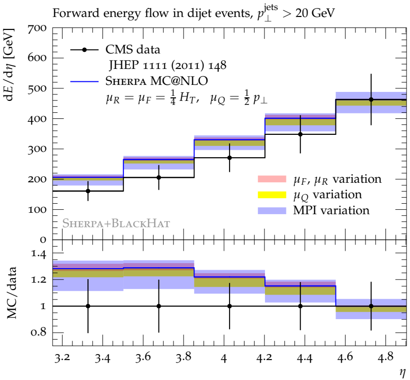
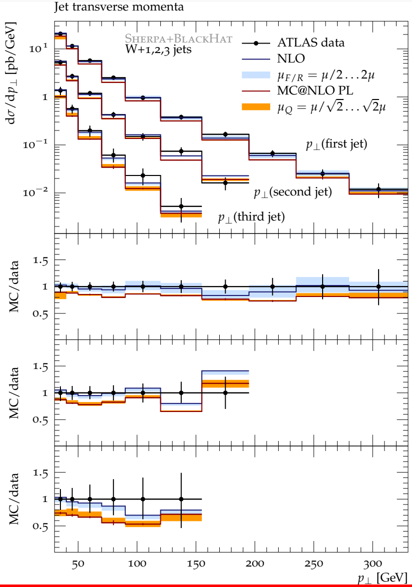


Höche, MS arXiv:1208.2815

Forward energy flow

- energy flow in rapidity interval per event with a central back-to-back di-jet pair
- normalisation reduces $\mu_{R/F}$ and μ_Q dependence
- dominated by MPI modeling uncertainty

$W + n$ jet production



Höche, Krauss, MS, Siegert arXiv:1201.5882

$pp \to W + 1, 2, 3$ jets

- 3 separate samples/calculations
- NLO accuracy for inclusive observables of respective jet multiplicity
- resummation of softest/LO jet, i.e. 4th jet in $pp \to W + 3$ jets
- no resummation of sample-defining jet multiplicity, i.e. first 3 jets in $pp \to W + 3$ jets
- scales: $\mu_{R/F} = \frac{1}{2}\,\hat{H}'_T$, $\mu_Q = p_\perp(j_n)$

Data: ATLAS Phys.Rev.D85(2012)092002

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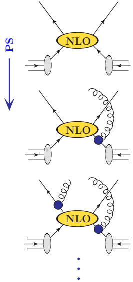

- promote LOPS to NLOPS (POWHEG, MC@NLO)
  → (part I)
- combine NLOPS for successive multiplicities into incl. sample (MEPS@NLO), preserve NLO+(N)LL accuracy in every jet multiplicity
  restore resummation wrt. to inclusive sample (part II)
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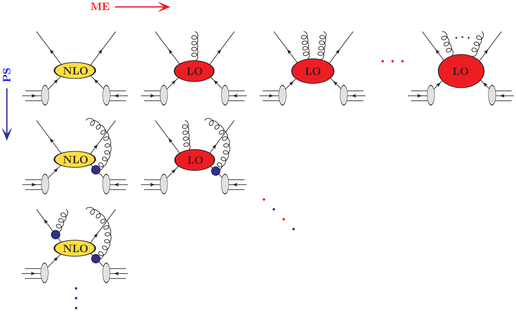

- promote LoPs to NloPs (Powheg, Mc@Nlo)
  → (part I)
- promote lowest multiplicity to NLO:
  merge one NloPs with MePs ⇒ MeNloPs
- combine NloPs for successive multiplicities into incl. sample (MePs@Nlo),
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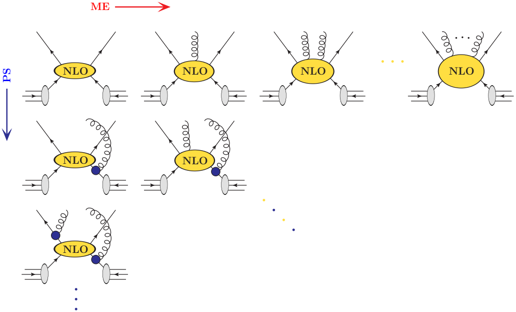

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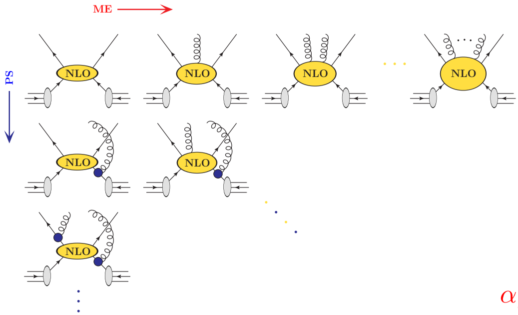

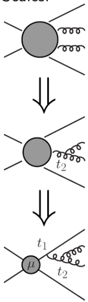

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NLO merging

LO merging:

- LO accuracy for $n \leq n_{\text{max}}$-jet processes
- preserve LL accuracy of the parton shower

Catani, Krauss, Kuhn, Webber JHEP11(2001)063

Lönnblad JHEP05(2002)046

Höche, Krauss, Schumann, Siegert JHEP05(2009)053

Hamilton, Richardson, Tully JHEP11(2009)038

Lönnblad, Prestel JHEP03(2012)019

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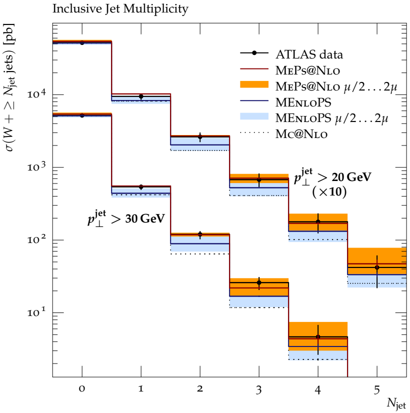
Lavesson, Lönnblad JHEP12(2008)070

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

Results: $pp \to W$+jets



$pp \to W$+jets (0,1,2 @ NLO; 3,4 @ LO)

- $\mu_{R/F} \in [\frac{1}{2}, 2]\,\mu_{\text{CKKW}}$
  scale uncertainty much reduced
- NLO dependece for
  $pp \to W$+0,1,2 jets
  LO dependence for
  $pp \to W$+3,4 jets
- $Q_{\text{cut}} = 30$ GeV
- good data description

ATLAS data Phys.Rev.D85(2012)092002

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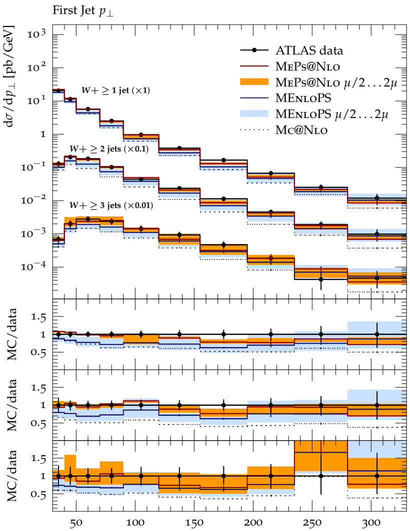

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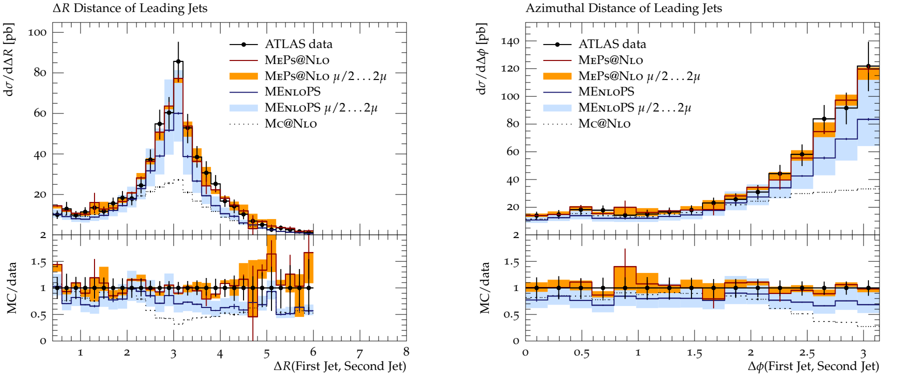
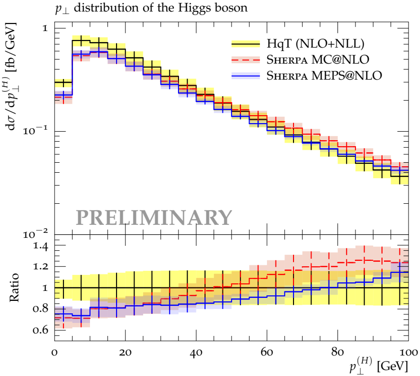


Results: pp → h+jets



$pp \to h$+jets (0,1 @ NLO; 2 @ LO)

- $\mu_{R/F} \in [\frac{1}{2}, 2]\, \mu_{\text{CKKW}}$
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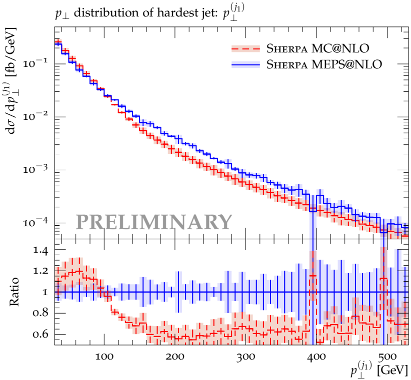
- NLO dependece for $pp \to h$+0,1 jets
  LO dependence for $pp \to h$+2 jets
- $Q_{\text{cut}} = 30$ GeV
- HqT: $\mu_{R/F} \in [\frac{1}{2}, 2] \cdot \frac{1}{2}\, m_h$

Results: pp → h+jets



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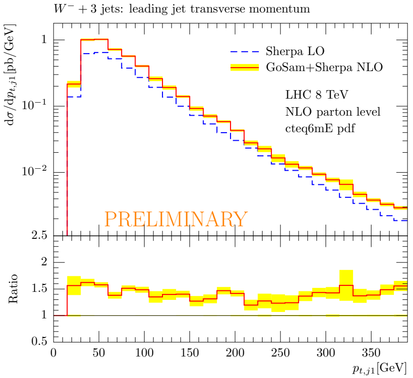
Public availability

NLO merging automated in SHERPA except for virtual

- GOSAM (public)
  automated one-loop code
  also convenience libraries
- NJETS (public)
  one-loop code for $pp \to$ jets
- OPENLOOPS (to be public)
  automated one-loop code
- BLACKHAT (to be public)
  one-loop library for
  $pp \to (V+)$jets

either dedicated interfaces or BLHA interface



scales: $\mu_{R/F} = H_T$

Merging with massive quarks

MEPS (CKKW) with massive quarks

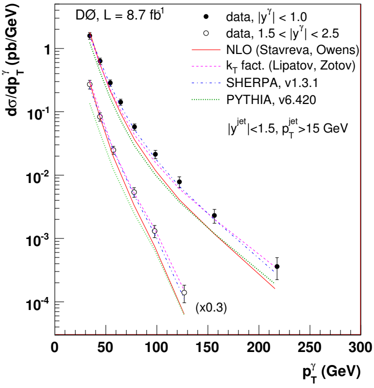
- e.g. $m_b, m_c \neq 0$
- no conceptual problem
- take massive quarks also into initial state (5 flavour PDFs)
- factorisation of massive initial state à la ACOT
- consistent ME and PS
- merging still only one parameter procedure (Q_{cut})

same as MEPS (CKKW) with massless quarks

no additional heavy flavour overlap removal or similar

available since SHERPA-1.3.0

$pp \to \gamma + b$-jet production



DØ data arXiv:1203.5865

Conclusions

- SHERPA's MC@NLO formulation allows full evaluation of perturbative uncertainties (μ_F, μ_R, μ_Q)
  non-perturbative uncertainties can be consistently estimated
- MC@NLO can be easily combined with MEPS $\rightarrow$ MENLOPS
- MC@NLO is a necessary input for NLO merging $\rightarrow$ MEPS@NLO
- MEPS@NLO gives full benefits of NLO calculations (scale dependences, normalisations) while also retaining full (N)LL accuracy of parton shower

$\Rightarrow$ will be included in next major release

Current release: SHERPA-1.4.2

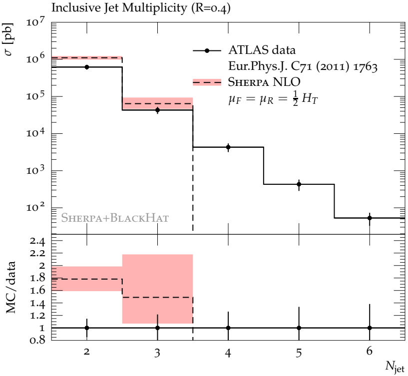
http://sherpa.hepforge.org

- precise theoretical calculations need to be confronted with data as differentially as possible over as large a phase space as possible

Thank you for your attention!

Short-comings of fixed-order QCD

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- very pronounced in inclusive & dijet production
- jet-$p_\perp$ turn negative in forward region unless y-dependent scale is used (e.g. $H_T^{(y)}$)



no. jets	ATLAS	LO	ME+PS	NLO	NP factor	NLO+NP
≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$559(5)$	$1193(3)^{+130}_{-135}$	$0.95(0.02)$	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$39.7(0.9)$	$54.5(0.5)^{+2.2}_{-19.9}$	$0.92(0.04)$	$50.2(2.1)^{+2.0}_{-18.3}$
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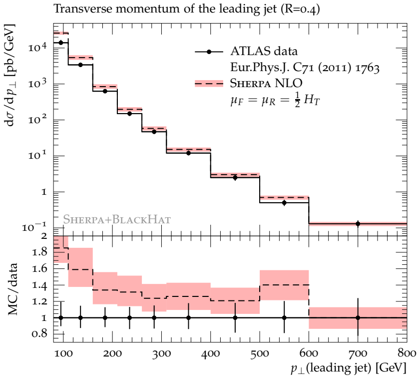
taken from Bern et.al. arXiv:1112.3940

Short-comings of fixed-order QCD

- poor description in phase space regions with strongly hierarchical scales
- poor perturbative jet-modeling (at most two constituents)
- no hadronisation, MPI effects
- very pronounced in inclusive & dijet production
- jet-$p_\perp$ turn negative in forward region unless y-dependent scale is used (e.g. $H_T^{(y)}$)



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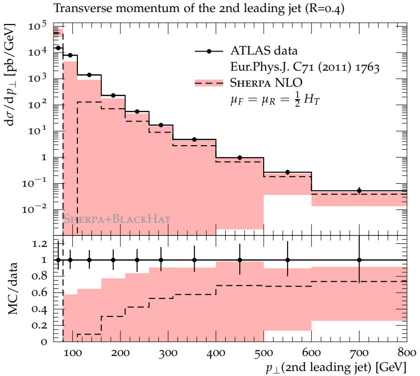

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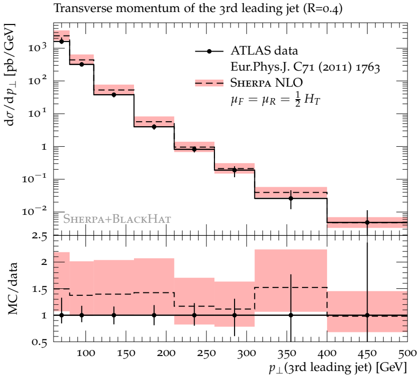

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NLO merging

Höche, Krauss, MS, Siegert arXiv:1207.5030

Gehrmann, Höche, Krauss, MS, Siegert arXiv:1207.5031

$$
\begin{aligned}
\langle O\rangle^{\text{MEPS@NLO}}
&= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathrm{A})} \left[\Delta_n^{(\mathrm{A})}(t_0,\mu_Q^2)\, O_n \right.\\
&\qquad \left. + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathrm{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathrm{A})}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\text{cut}}-Q)\, O_{n+1}\right]\\
&\quad + \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathrm{A})}\right] \Theta(Q_{\text{cut}}-Q)\, \Delta_n^{(\mathrm{PS})}(t_{n+1},\mu_Q^2)\, O_{n+1}\\
&\quad + \int \mathrm{d}\Phi_{n+1}\, \bar{\mathrm{B}}_{n+1}^{(\mathrm{A})}\, \Theta(Q-Q_{\text{cut}})\\
&\qquad \times \left[\Delta_{n+1}^{(\mathrm{A})}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_{n+1}^{(\mathrm{A})}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{(\mathrm{A})}(t_{n+2},t_{n+1})\, O_{n+2}\right]\\
&\quad + \int \mathrm{d}\Phi_{n+2} \left[\mathrm{R}_{n+1} - \mathrm{D}_{n+1}^{(\mathrm{A})}\right] \Theta(Q-Q_{\text{cut}})\, O_{n+2}
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$$\langle O\rangle^{\text{MEPS@NLO}}$$

$$= \int \mathrm{d}\Phi_n\, \bar{\mathrm{B}}_n^{(\mathsf{A})} \left[\Delta_n^{(\mathsf{A})}(t_0,\mu_Q^2)\, O_n + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_n^{(\mathsf{A})}}{\mathrm{B}_n}\, \Delta_n^{(\mathsf{A})}(t_{n+1},\mu_Q^2)\, \Theta(Q_{\mathsf{cut}}-Q)\, O_{n+1}\right]$$

$$+ \int \mathrm{d}\Phi_{n+1} \left[\mathrm{R}_n - \mathrm{D}_n^{(\mathsf{A})}\right] \Theta(Q_{\mathsf{cut}}-Q)\, O_{n+1}$$

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$$+ \int \mathrm{d}\Phi_{n+1}\, \bar{\mathrm{B}}_{n+1}^{(\mathsf{A})} \quad \Theta(Q-Q_{\mathsf{cut}})$$

$$\times \bigg[\Delta_{n+1}^{(\mathsf{A})}(t_0,t_{n+1})\, O_{n+1} + \int_{t_0}^{t_{n+1}} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_{n+1}^{(\mathsf{A})}}{\mathrm{B}_{n+1}}\, \Delta_{n+1}^{(\mathsf{A})}(t_{n+2},t_{n+1})\, O_{n+2} \bigg]$$

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NLO merging

Renormalisation scales:

- determined by clustering using PS probabilities and taking the respective nodal values t_i

$$\alpha_s(\mu_R^2)^k = \prod_{i=1}^{k} \alpha_s(t_i)$$

- change of scales $\mu_R \to \tilde{\mu}_R$ in MEs necessitates one-loop counter term

$$\alpha_s(\tilde{\mu}_R^2)^k \left(1 - \frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\,\beta_0 \sum_{i=1}^{k} \ln \frac{t_i}{\tilde{\mu}_R^2}\right)$$

Factorisation scale:

- μ_F determined from core n-jet process
- change of scales $\mu_F \to \tilde{\mu}_F$ in MEs necessitates one-loop counter term

$$\mathrm{B}_n(\Phi_n)\,\frac{\alpha_s(\tilde{\mu}_R^2)}{2\pi}\,\log\frac{\mu_F^2}{\tilde{\mu}_F^2}\left(\sum_{c=q,g}^{n}\int_{x_a}^{1}\frac{\mathrm{d}z}{z}\,P_{ac}(z)\,f_c(x_a/z,\tilde{\mu}_F^2) + \ldots\right)$$

NLO merging – Generation of MC counterterm

$$\left[1 + \frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} \mathrm{d}\Phi_1 \, \mathrm{K}_n \right]$$

- same form as exponent of Sudakov form factor $\Delta_n^{(\mathsf{PS})}(t_{n+1}, \mu_Q^2)$
- truncated parton shower on n-parton configuration underlying $n+1$-parton event
  1. no emission $\rightarrow$ retain $n+1$-parton event as is
  2. first emission at t' with $Q > Q_{\mathsf{cut}}$, multiply event weight with $\mathrm{B}_{n+1}/\bar{\mathrm{B}}_{n+1}^{(\mathsf{A})}$, restart evolution at t', do not apply emission kinematics
  3. treat every subsequent emission as in standard truncated vetoed shower
- generates

$$\left[1 + \frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}} \int_{t_{n+1}}^{\mu_Q^2} \mathrm{d}\Phi_1 \, \mathrm{K}_n \right] \Delta_n^{(\mathsf{PS})}(t_{n+1}, \mu_Q^2)$$

$\Rightarrow$ **identify $\mathcal{O}(\alpha_s)$ counterterm with the emitted emission**