

A critical appraisal of NLO+PS matching methods

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LHCphenOnet



*in collaboration with S. Höche, F. Krauss, F. Siegert

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Introduction

Importance of matching NLO calculations with parton showers

- exclusive final states
- observable independent combination of fixed order and resummation
- problem double counting: both NLO and PS are approximations to higher order corrections

Two methods appeared in the literature: MC@NLO and POWHEG

- two sides of one medal
- differ in choices of division of resummation and fixed-order part

Uncertainties of NLO+PS matching

- usual μ_R and μ_F variation as in NLO calculations
- also μ_Q -variation as in analytic resummations

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General NLO calculations

- NLO calculation with subtraction methods

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442

Catani, Seymour Nucl.Phys.B485(1997)291-419

$$\begin{aligned}
 \langle O \rangle^{\text{NLO}} &= \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] O(\Phi_B) \\
 &+ \int d\Phi_R \left[- \sum_i D_i^{(S)}(\Phi_R) \right] O(\Phi_{B_i}) \\
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- introduce second set of subtraction functions $D_i^{(A)}$
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 &+ \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(A)}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle_{\text{corr}}^{(A)} \\
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Parton showers and resummation

- parton shower/resummation kernel $\mathcal{K}_i(\Phi_1)$, $\Phi_1 = \{t, z, \phi\}$
 $\rightarrow \mathcal{K}_i$ incorporates divergent propagator and DGLAP splitting kernels

$$\begin{aligned} \langle O \rangle^{\text{PS}} &= \int d\Phi_B B(\Phi_B) O(\Phi_B) \\ &= \int d\Phi_B B(\Phi_B) O(\Phi_B) + \end{aligned}$$

- Sudakov form factor $\Delta^{(\mathcal{K})}(t, t') = \exp \left[- \int_t^{t'} d\Phi_1 \mathcal{K}(\Phi_1) \right]$ contains resummation features
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General NLO+PS matching

$$\begin{aligned}
 \langle O \rangle^{\text{NLO+PS}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \right] \\
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- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale
- POWHEG and MC@NLO now differ in choice of $D_i^{(A)}$ and μ_Q^2

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POWHEG

Special choices:

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $D_i^{(A)} = \rho_i \cdot R$ with $\rho_i = D_i^{(S)} / \sum_i D_i^{(S)}$
 → each $\rho_i \cdot R$ contains only one divergence structure as defined by $D_i^{(S)}$

Consequences:

- no \mathbb{H} -events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2} s_{\text{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $D_i^{(S)}$
- exponentiation of R through matrix element corrected parton shower
 NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2 / (p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
 → $D_i^{(A)} = \rho_i \cdot R \cdot f(p_\perp)$
 → continuous dampening of resummation kernel at large p_\perp

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Mc@NLO – traditional scheme

Special choices:

Frixione, Webber JHEP06(2002)029

- exponentiation kernel $D_i^{(A)} = B \cdot \mathcal{K}_i$ with \mathcal{K}_i parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\max}$ parton shower starting scale
- in general, $D_i^{(A)}$ only leading colour approximation
NLO accuracy depends crucially on correctness of IR-limit

Modifications:

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- introduce soft modification function $f(p_\perp)$ such that

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MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Special choices:

SH, FK, MS, FS arXiv:1111.1220

- exponentiation kernel $D_i^{(A)} = D_i^{(S)}$

Consequences:

- simplification of $\bar{B}^{(A)}$ -integral
- resummation scale $\mu_Q^2 = t_{\max}$ set by phase space limitation of subtraction terms
 - subtraction constrained in parton shower t needed for physical resummation
 - instructive example: use α_{cut} to explore effects Nagy PRD68(2003)094002
- trivially NLO correct independent of the process without arbitrary parameter choices

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MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm

SH, FK, MS, FS arXiv:1111.1220

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{i+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

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$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i) - f(t_i)}{g(t_i) - f(t_i)}$$

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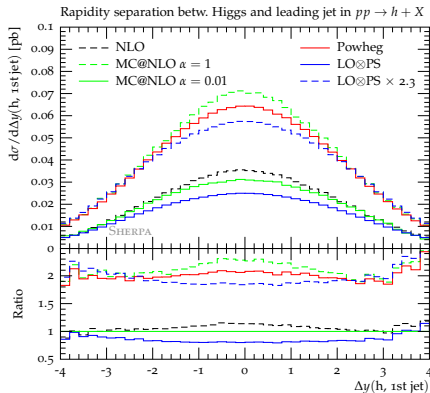
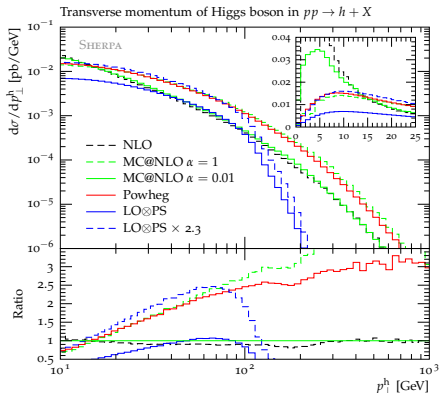
Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow D_i^{(A)}$
- $h(t)$ determined by parton shower $\Rightarrow D_i^{(PS)}$
- $g(t)$ **can be chosen freely** $\Rightarrow \text{const.} \cdot f$
constraints: $\text{sign}(f) = \text{sign}(g)$, $|f| \leq |g|$

Automation in SHERPA framework

- easy to automate and process independent
 - only virtual correction V needs to be supplied
- leading order pieces and phase space generation taken care of by well-tested automated tree-level matrix element generators AMEGIC++ and/or COMIX
 - [Krauss, Kuhn, Soff JHEP02\(2002\)044](#), [Höche, Gleisberg JHEP12\(2008\)039](#)
- based on Catani-Seymour subtraction [Catani, Seymour Nucl.Phys.B485\(1997\)291-419](#)
 - [Nagy PRD68\(2003\)094002](#), [Gleisberg, Krauss EPJC53\(2008\)501-523](#)
- using dipole-like parton shower [Schumann, Krauss JHEP03\(2008\)038](#)
 - ⇒ phase space restriction of dipoles = starting scale for parton shower
- parton showers are easy to correct with matrix elements
 - $D_i^{(A)}/B$ always non-zero and close to pure parton shower result
 - larger analytic weights in soft-gluon regime limited by t_0

Results – $pp \rightarrow h + X$ production in gluon fusion



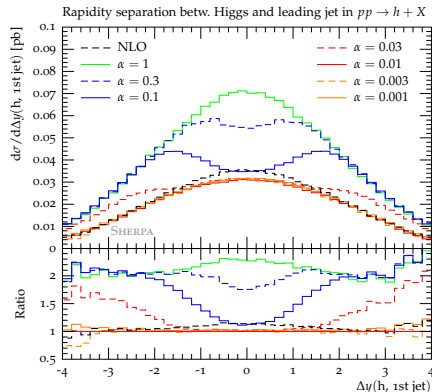
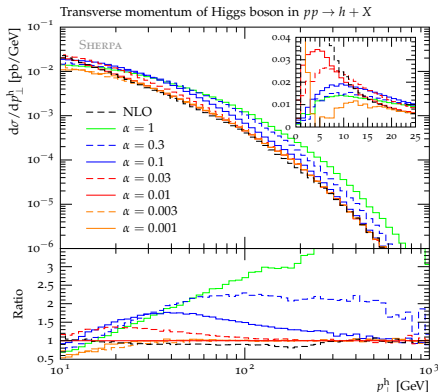
Sanity checks:

- Sudakov shape of LO \otimes PS result for $\alpha_{cut} \rightarrow 1$
- High- p_{\perp} tail of NLO reproduced for $\alpha_{cut} \ll 1$

Ideally both simultaneously, as in traditional MC@NLO,

but currently limited by inappropriate choice of μ_Q^2 through α_{cut}

Results – $pp \rightarrow h + X$ production in gluon fusion

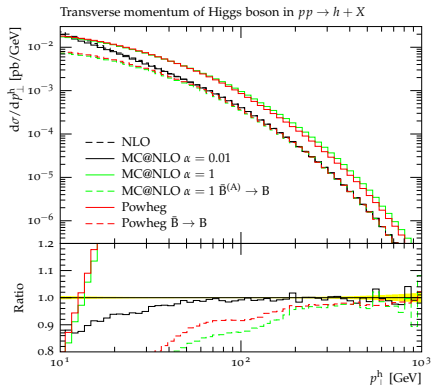


Essential features of [Alioli et.al. JHEP04\(2009\)002](#) reproduced

- hardness of POWHEG/MC@NLO $\alpha_{cut} = 1$
- dip in Δy for intermediate α_{cut}

Purely academic, rather define $D^{(A)}$ properly \Rightarrow **phase space separation in t**

Results – $pp \rightarrow h + X$ production in gluon fusion



replace weight of \mathbb{S} -events
by LO weight

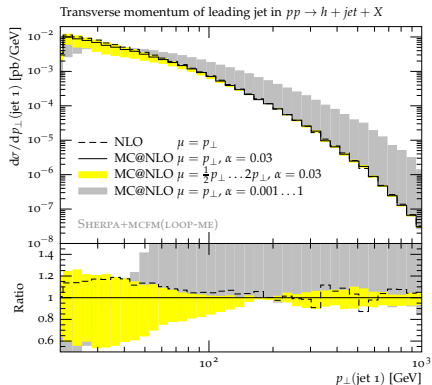
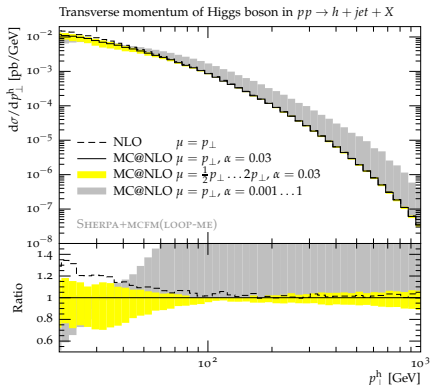
- for $t \rightarrow s_{\text{had}}$ approaches NLO result, but

$$\Delta^{(A)}(m_h^2, s_{\text{had}}) = \exp \left[- \int_{m_h^2}^{s_{\text{had}}} dt \frac{R(t)}{B} \right] < 1$$

- non-negligible Sudakov suppression at $p_{\perp} \sim m_h \ll s_{\text{had}}$, induced by erroneous phase space boundaries for resummation
- spuriously large subleading logarithms

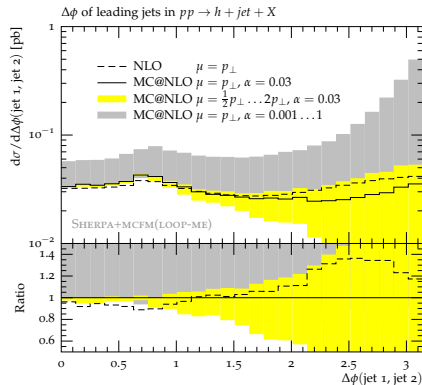
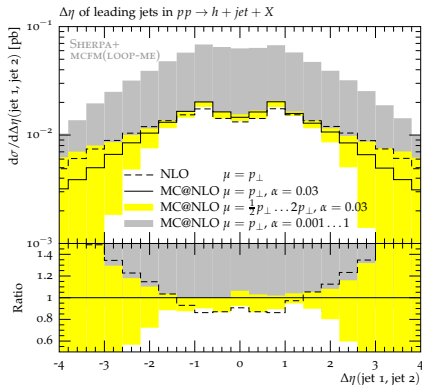
Banfi, Salam, Zanderighi JHEP08(2004)062

Results – $pp \rightarrow h + \text{jet} + X$ production in gl. fusion



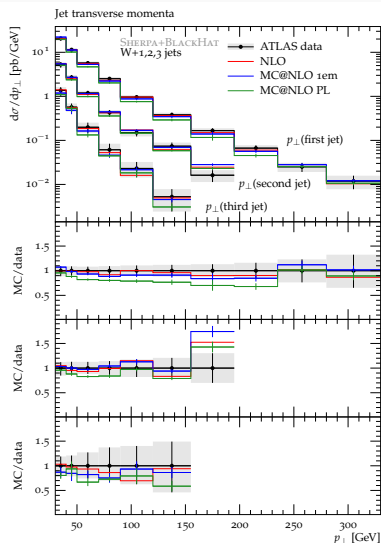
- increased parton multiplicity worsens problems
- large dependence on exponentiated phase space
- unphysical results for $\mu_Q^2 \rightarrow \frac{1}{2} s_{\text{had}}$

Results – $pp \rightarrow h + \text{jet} + X$ production in gl. fusion



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Results – $pp \rightarrow W + n \text{ jet} + X$ production



$W + 1, 2, 3$ jets at LHC (ATLAS data)

SH, FK, MS, FS arXiv:1201.5882

- complexity not a problem
- speed limited by the virtual amplitude in $W + 3$ jet
- scales:

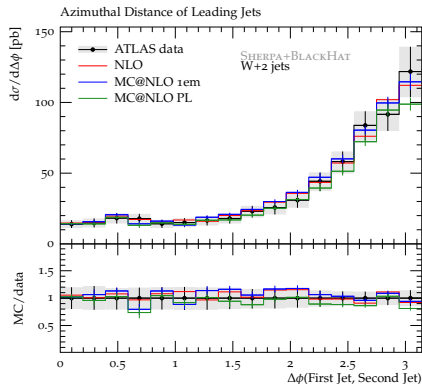
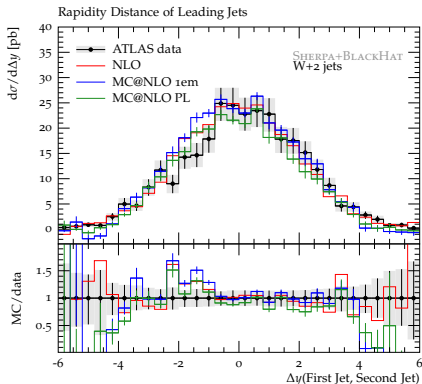
$$\mu_R = \mu_F = \frac{1}{2} \hat{H}'_T$$

$$\mu_{\text{exp}} = \frac{1}{(1/p_{\perp}^2 + 1/\mu_R^2)^{\frac{1}{2}}}$$

- fixed order behaviour at high p_{\perp}
→ smoother transition to $\overline{\text{MS}}$ -events



Results – $pp \rightarrow W + n \text{ jet} + X$ production



data: [ATLAS arXiv:1201.1276](https://arxiv.org/abs/1201.1276)

MENLOPs for Mc@NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MENLOPs}} = & \int d\Phi_B \bar{B}^{(A)}(\Phi_B) \left[\Delta^{(A)}(t_0, \mu_Q^2) O(\Phi_B) \right. \\
 & \left. + \int_{t_0}^{\mu_Q^2} d\Phi_1 \frac{D^{(A)}(\Phi_B, \Phi_1)}{B(\Phi_B)} \Delta^{(A)}(t, \mu_Q^2) O(\Phi_R) \Theta(Q_{\text{cut}} - Q) \right] \\
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- restrict Mc@NLO expression to region $Q < Q_{\text{cut}}$
- add in real radiation explicitly, as in ME+PS
- restore logarithmic behaviour by explicit Sudakov
- local K-factor for continuity at Q_{cut}

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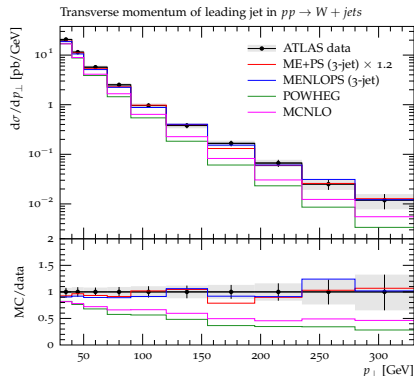
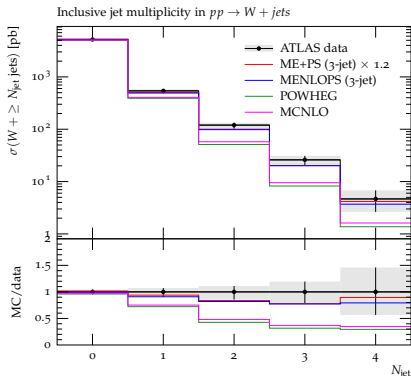
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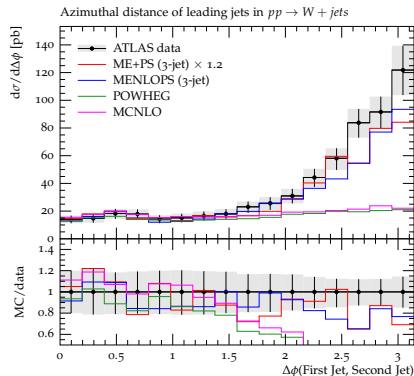
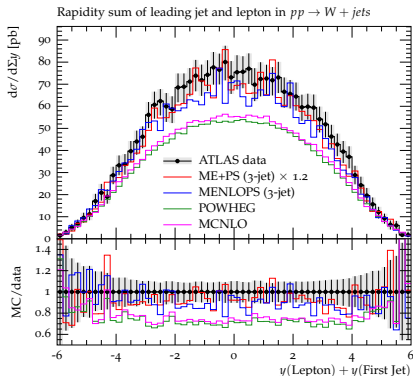
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Choice of exponentiated phase space

Assessment of uncertainties

- limit discussion to $gg \rightarrow h$ because effects are largest and cleanest here (large NLO k-factor, very simple colour/dipole structure)
 - large rate difference for \mathbb{S} and \mathbb{H} events
 - setup: k_{\perp} -ordered parton shower based on Catani-Seymour dipoles
 - highlights what happens at resummation scale $\mathbf{k}_{\perp}^{\max}$
 - renormalisation scale $\mu_R = m_h$, $\mu_R^{\exp} = \sqrt{\frac{1}{1/p_{\perp}^2 + 1/\mu_R^2}} \xrightarrow{p_{\perp} \rightarrow \infty} \mu_R$
 - vary resummation scale $\mathbf{k}_{\perp}^{\max}$, i.e. starting conditions of MC@NLO-shower
 - starting conditions of POWHEG-shower fixed at $\mathbf{k}_{\perp}^{\max} = \frac{1}{2} \sqrt{s_{\text{had}}}$
 - effect of suppression function not investigated
 - introduces arbitrary free parameter (not fixed to be of order of m_h)
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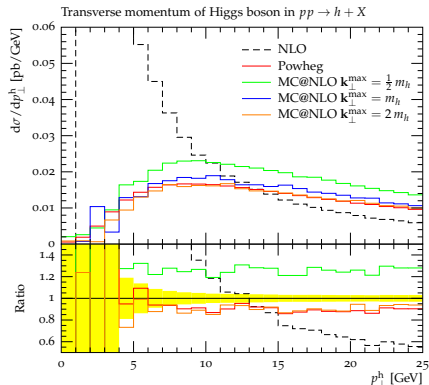
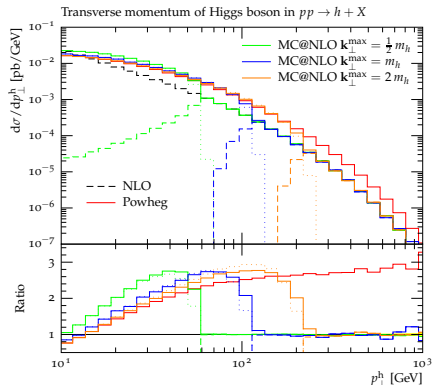
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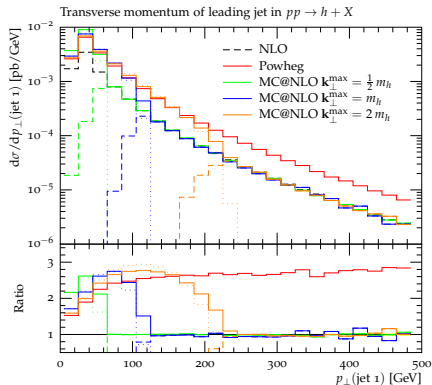
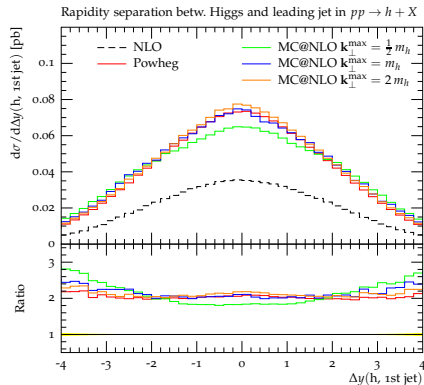
traditional MC@NLO and POWHEG choices of splitting kernels



- ▶ large uncertainties when varying k_{\perp}^{\max}
- ▶ driven by diff. in normalisation of \mathbb{S} - and \mathbb{H} -events and size of $\ln^2(k_{\perp}^2/\mu_Q^2)$
- ▶ shape difference driven by unitarity constraint

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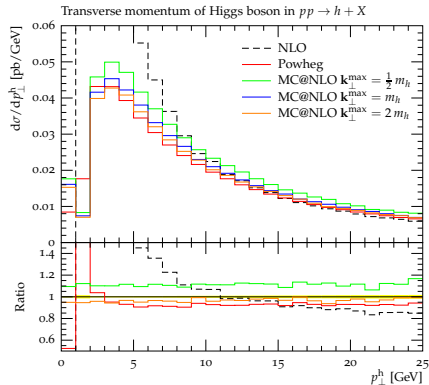
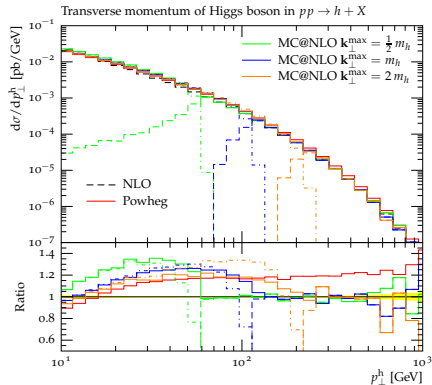
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- uncertainty on jet rates with $p_{\perp} \sim 100\text{GeV}$: 2.5
- no dip in $\Delta y \rightarrow$ originates in HERWIG's radiation pattern

Choice of splitting kernel – $D^{(A)}/\bar{B}^{(A)}$

here: change splitting kernels $K \rightarrow (1 + \alpha_s \cdot \text{const.}) K$

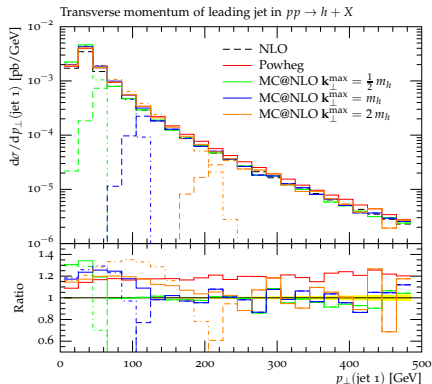
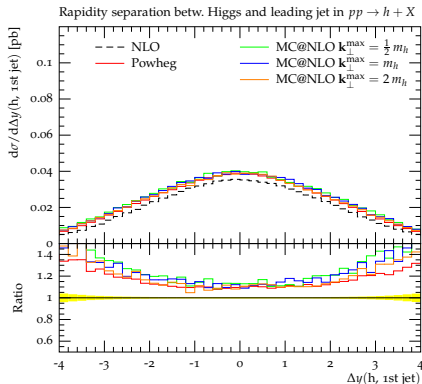


- ▶ uncertainties much lower, smooth transition at μ_Q^2
- ▶ much closer to NLO fixed order result for “hard” emissions
- ▶ price of spuriously large LL prefactor \rightarrow Sudakov peak differs



Choice of splitting kernel – $D^{(A)}/\bar{B}^{(A)}$

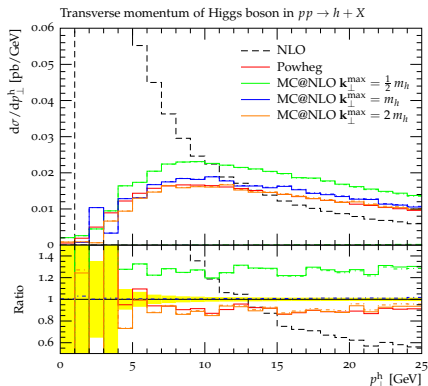
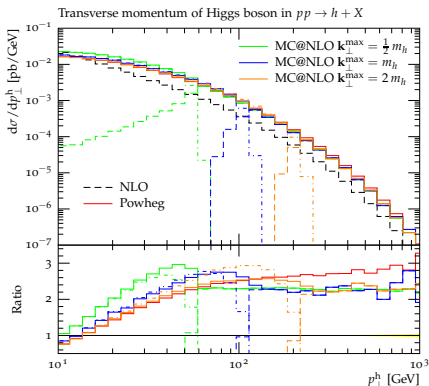
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Choice of higher order correction – $\bar{B}^{(A)}/B \cdot \mathbb{H}$

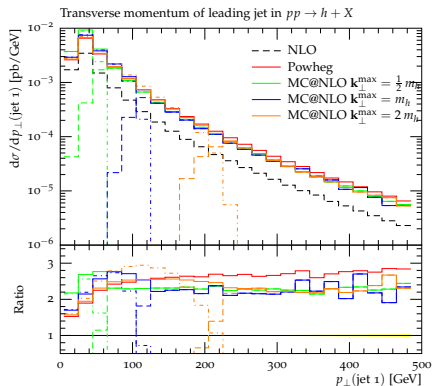
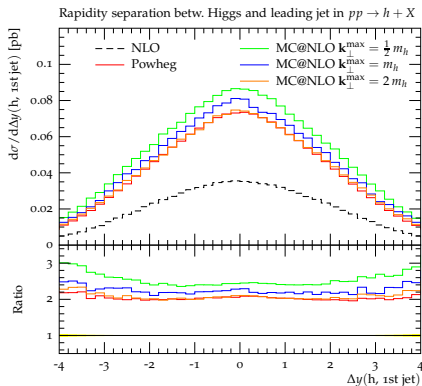
here: modify \mathbb{H} -term with arbitrary higher order corrections $\mathbb{H} \rightarrow \frac{\bar{B}^{(A)}}{B} \mathbb{H}$



- ▶ PS resummation left void of higher order terms
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Conclusions

- care must be taken to preserve the correct resummation properties as not to introduce large spurious subleading logarithms

Banfi, Salam, Zanderighi JHEP08(2004)062

- MC@NLO $D^{(A)} = D^{(S)}$ scheme facilitates implementation of process independent correct soft-gluon limit needed for NLO accuracy
- no conceptual problems for high-multiplicity processes
- independent check of POWHEG \leftrightarrow MC@NLO differences
- both MC@NLO and POWHEG can be trivially combined with ME+PS merging \Rightarrow MENLOPS

SHERPA-1.4.0

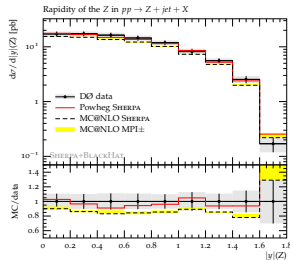
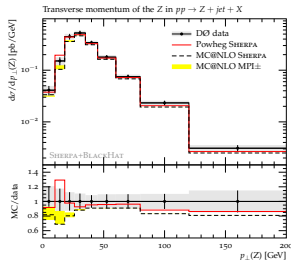
<http://sherpa.hepforge.org/trac/wiki/SherpaDownloads/Sherpa-1.4.0>

Conclusions

- NLO+PS is LO+(N)LL matching
- uncertainties studied occur in every process and are inherent to methods
→ $gg \rightarrow h$ just presents a clean environment
- exploit freedom left at the respective level of accuracy
→ each with merits and drawbacks
- choices constrained by adding higher order calculations
 - (N)NLL resummation
 - NNLO corrections
 - NLO \otimes NLO merging with $Q_{\text{cut}} < \mu_Q^2$

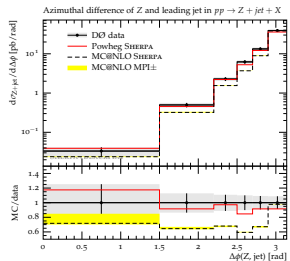
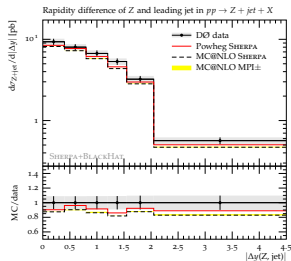
Thank you for your attention!

POWHEG and MC@NLO for $pp \rightarrow Z + \text{jet} + X$



POWHEG

- LC-NLO correct only
- too large emission probability cancelled by too soft MPI



MC@NLO

- $\alpha_{\text{cut}} = 0.03$
- rather large non-perturbative effects
- MPI too soft