Heavy Flavour Physics

and

Effective Field Theories

Epiphany 2015

An inspiring example of *Heavy Flavour*

Alexander Lenz

IPPP Durham

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Contents

1 Introduction

These lectures provide a basic knowledge about flavour physics. To set the notation and conventions the notes start very elementary[1], but they will become more technical later on.

1.1 The standard model in a real nutshell

All currently known elementary particles can be split into up in three groups:

1. **Spin 0 particles:** appear in the process of the **creation of mass**
2. **Spin 1/2 particles: matter** constituents
3. **Spin 1 particles: force** transmitters

These three groups contain altogether 25 (= 1+12+12) fundamental particles, which read explicitly:

1. **Spin 0 particle:** Creating the masses of the fermions and of the weak gauge bosons via the **Higgs mechanism** (Englert and Brout; Higgs; Guralnik, Hagen and Kibble) [2, 3, 4, 5, 6] gives rise to new scalar particles. In the simplest realisation this is a single neutral particle, the so-called **Higgs boson** h, which was predicted in [3, 4, 6][2] and found in 2012 at the **Large Hadron Collider (LHC)** at CERN, Geneva with the experiments ATLAS and CMS [7, 8].

2. **Spin 1/2 particles:** matter is built out of fermions, which are split into two classes: quarks and leptons.

   Quarks: **Leptons**, $\lambda\epsilon\pi\tau o\sigma$ = light, not heavy:

   $$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \qquad\qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

   Quarks take part in the strong interaction, the weak interaction and the electromagnetic interaction. Concerning the latter, the u, c, t quarks have the electric charge $+2/3$ and the d, s, b quarks have charge $-1/3$. Leptons do not take part in the strong interaction, but in the weak interaction. Concerning the electromagnetic interaction, e^-, μ^-, τ^- have charge -1 and thus take part, while neutrinos are electrical neutral and hence they only interact weakly.

[1] For a nice introduction to the standard model see e.g. [1].

[2] All the cited papers can be easily obtained from INSPIRE or arXiv; simply type in Google: "spires" or "arXiv".

3. **Spin 1 particles:** the fundamental interactions are transferred via corresponding interaction quanta, the **gauge bosons**:

   - electro-magnetic interaction: **photon** γ
   - weak interaction: **weak gauge bosons** W^+, W^-, Z^0
   - strong interaction: **gluons** $g_1, ..., g_8$

   The weak bosons $W^\pm$ have the electric charge ± 1, while all other bosons are electrically neutral.

<u>**Remarks:**</u>

- The matter constituents show up in three copies (**generations**), the individual species are called **flavour**, i.e. u, d, c, s, t, b in the case of the quarks. In principle all known matter is made up of the first generation - ordinary matter consists of atoms, which are built of protons, neutrons and electrons and the protons and neutrons itself are built out of up- and down-quarks, at least to a first approximation. Looking more carefully one finds also gluons and different quark-antiquark pairs including a non-negligible portion of strange quarks. Later we will see, what is peculiar about having at least three generations of matter in the standard model.

- Gauge symmetry forces all gauge bosons and fermions to be exactly massless. The weak gauge bosons and fermions will acquire mass via the Higgs mechanism, without violating the gauge principle.

1.2 Masses of the elementary particles

In the theoretical tools used to describe flavour observables the hierarchy between different mass scales will be crucial. Thus we give here a short overview (status: January 2014, PDG [9]) over the masses of the elementary particles.
For comparison: the mass of a proton is $938.272046(21)$ MeV = $1.672621777 \cdot$

10^{-27} kg.

Particle	Physical mass		$\overline{MS}$ – mass	
t	$173.07(89)$	GeV	160	GeV
h	$125.9(4)$	GeV		
Z	$91.1876(21)$	GeV		
W	$80.385(15)$	GeV		
b	$4.78(6)$	GeV	$4.18(3)$	GeV
τ	$1.77682(16)$	GeV		
c	$1.67(7)$	GeV	$1.275(25)$	GeV
μ	$105.6583715(35)$	MeV		
s			$93.5(2.5)$	MeV
d			$4.7(2)$	MeV
u			$2.15(15)$	MeV
e	$510.998928(11)$	keV		GeV
ν	< 1	eV		GeV
$\gamma, g_1, ..., g_8$	0	GeV		GeV

Remarks:

- In principle it is sufficient to remember only rough values of the masses of the elementary particles. Some of the observables we will investigate below, depend however strongly on the masses, e.g. lifetimes of a weakly decaying particle are proportional to the inverse fifth power of the mass of the decaying particle. Hence we provided the precise values of the masses.

- Quarks do not exist as free particles but only within bound states. Thus it is not clear what is actually meant by the mass of a free quark. We give here two commonly used definitions: we identify the pole mass (i.e. the pole of the corresponding quark propagator) with the physical mass. This works well for c, b and t, but not for the light quarks. Another commonly used definition is the $\overline{MS}$-mass [10]. For the three heavy quarks we use $\overline{m}_q(\overline{m}_q)$ and for the three light quarks we quote $\overline{m}_q(2 \text{ GeV})$.

- In order to compare more easily with the literature we will use for the numerical evaluations in this lecture:

$$\overline{m}_b(\overline{m}_b) = 4.248 \text{ GeV} , \qquad m_b^{\text{Pole}} = 4.65 \text{ GeV} , \tag{1}$$
$$\overline{m}_c(\overline{m}_c) = 1.277 \text{ GeV} , \qquad m_c^{\text{Pole}} = 1.471 \text{ GeV} , \tag{2}$$
$$\overline{m}_c(\overline{m}_b) = 0.997 \text{ GeV} . \tag{3}$$

1.3 Outline

Flavour physics is the description of effects related to the change of quark and lepton flavours. In this course we restrict ourselves to quark transitions and since the top quark does not form bound states we will also not discuss it. Mostly we will be treating transitions of bottom and charm quarks.
Many of the theoretical tools used to describe these effects are based on the concept of **effective field theories**, which have also very important applications outside flavour physics.
This lecture course consists of 16 + 6 hours of lectures. *It is split up into the following sections*

1. *General introduction*
2. *Flavour physics and the CKM matrix*
3. *Flavour phenomenology*
4. *Basics of weak decays*
5. *Effective theories, in particular* H_{eff}
6. *Inclusive B-decays*
7. *Lifetimes and lifetime differences - the Heavy Quark Expansion*
8. *Mixing in particle physics*
9. *Mixing of neutral mesons*
10. *Exclusive B-decays*
11. *Search for new physics*
12. *Appendix: collection of useful formulae*

2 Flavour Physics and the CKM matrix

2.1 Heavy hadrons

In this lecture course we are considering hadronic bound states containing a heavy b-quark and/or a heavy c-quark. Mesons consist of a quark and an anti-quark and baryons of three quarks.
The concrete quark content and some basic properties of B-mesons and b-baryons read: (status January 2015, masses from PDG [9] and lifetimes and ratios from HFAG [11] - my own estimates are indicated by $*$):

B-mesons

	$B_d = (\bar{b}d)$	$B^+ = (\bar{b}u)$	$B_s = (\bar{b}s)$	$B_c^+ = (\bar{b}c)$
Mass (GeV)	$5.27955(26)$	$5.27925(26)$	$5.3667(4)$	$6.2745(18)$
Lifetime (ps)	$1.520(4)$	$1.638(4)$	$1.509(4)$	$0.507(9)$
$\tau(X)/\tau(B_d)$	1	1.076 ± 0.004	0.993 ± 0.004	$0.334 \pm 0.006*$

b-baryons

	$\Lambda_b = (udb)$	$\Xi_b^0 = (usb)$	$\Xi_b^- = (dsb)$	$\Omega_b^- = (ssb)$
Mass (GeV)	$5.6194(6)$	$5.7918(5)$	$5.79772(55)$	$6.071(40)$
Lifetime (ps)	$1.467(10)$	$1.465(31)$	$1.559(37)$	$1.57 \left(^{+23}_{-20}\right)$
$\tau(X)/\tau(B_d)$	0.965 ± 0.007	$0.964 \pm 0.020*$	$1.026 \pm 0.024*$	$1.03 \left(^{+15}_{-13}\right) *$

Alternative lifetime averages were, e.g., obtained in [12].
In particular the lifetime ratios provide crucial tests of our calculational tools, since they are not expected to be sizable affected by new physics. If our methods pass these tests we can apply them to quantities which are expected to be sensitive to new physics effects. This will be discussed in detail below.
The quark content and some basic properties of D-mesons and c-baryons read: (status January 2013, masses and lifetimes from PDG [9]:[3]

D-mesons

	$D^0 = (\bar{u}c)$	$D^+ = (\bar{d}c)$	$D_s^+ = (\bar{s}c)$
Mass (GeV)	$1.86486(13)$	$1.86962(15)$	$1.96849(32)$
Lifetime (ps)	$0.4101(15)$	$1.040(7)$	$0.500(7)$
$\tau(X)/\tau(D^0)$	1	2.536 ± 0.017	1.219 ± 0.017

[3] D^0 and D^+ have the same relative precision in the lifetimes.

c-baryons

	$\Lambda_c = (udc)$	$\Xi_c^+ = (usc)$	$\Xi_c^0 = (dsc)$	$\Omega_c = (ssc)$
Mass (GeV)	$2.28646(14)$	$2.4676\left(^{+4}_{-10}\right)$	$2.47109\left(^{+35}_{-100}\right)$	$2.6952\left(^{+18}_{-16}\right)$
Lifetime (ps)	$0.200(6)$	$0.442(26)$	$0.112\left(^{+13}_{-10}\right)$	$0.069(12)$
$\tau(X)/\tau(D^0)$	0.488 ± 0.015	$1.08(6)$	$0.27(3)$	0.17 ± 0.03

The charm sector provides some additional complementary tests of our theoretical tools, since there the expansion parameter is considerably larger.
Later on will also discuss kaons and pions, thus we provide also some of their properties

K-mesons

	$K_S = (\bar{s}d + s\bar{d})$	$K_L = (\bar{s}d - s\bar{d})$	$K^+ = \bar{s}u$
Mass (GeV)	$0.497614(24)$	$0.497614(24)$	$0.493677(16)$
Lifetime (ps)	$89.54(4)$	$51160(210)$	$12380(21)$

Pions

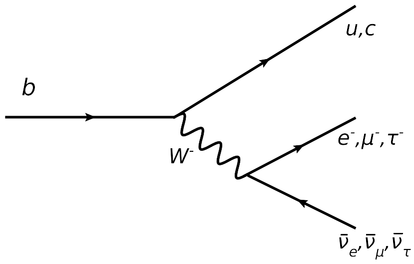
	$\pi^+ = \bar{d}u$	$\pi^0 = (\bar{u}u - d\bar{d})/\sqrt{2}$
Mass (GeV)	$0.13957018(35)$	$0.1349766(6)$
Lifetime (ps)	$26033(5)$	$(8.52 \pm 0.18) \cdot 10^{-5}$

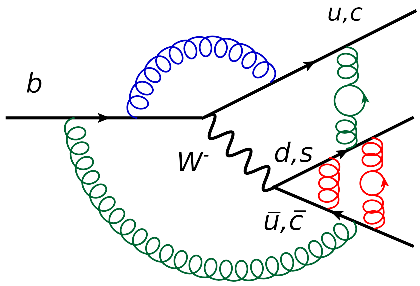
2.2 Weak decays of heavy quarks

All these hadrons decay via the charged weak interaction. The dominant processes are the following tree-level decays:

- free b-quark tree-level decay:

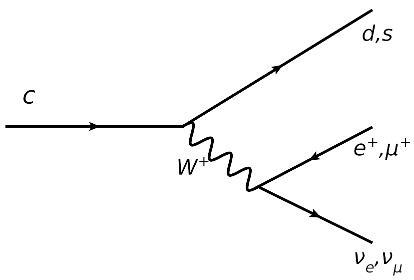
$$b \to \left\{ \begin{array}{c} c \\ u \end{array} \right. + W^- \to \left\{ \begin{array}{c} c \\ u \end{array} \right. + \left\{ \begin{array}{l} \bar{u} + d \\ \bar{c} + s \\ \bar{u} + s \\ \bar{c} + d \\ e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ \tau^- + \bar{\nu}_\tau \end{array} \right.$$

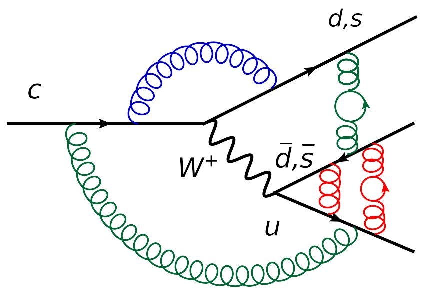




- free c-quark tree-level decay:

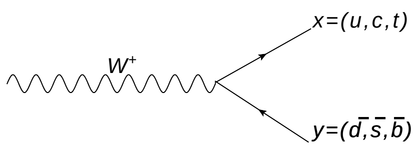
$$c \to \left\{ \begin{array}{l} s \\ d \end{array} \right. + W^+ \to \left\{ \begin{array}{l} s \\ d \end{array} \right. + \left\{ \begin{array}{l} \bar{d} + u \\ \bar{d} + s \\ e^+ + \nu_e \\ \mu^+ + \nu_\mu \end{array} \right.$$



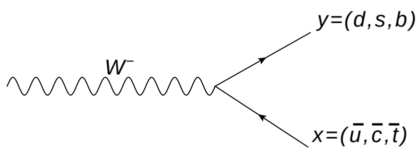


If there are quarks in the final state we have sizable QCD-corrections, which is indicated in the Feynman diagrams by the gluon exchanges. The above transitions are triggered by the charged weak current; they consist of a transition of a x-quark into a y-quark via the exchange of a $W^{\pm}$-boson. The basic vertex reads

$$W^+ : \; i\frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)V_{xy}$$



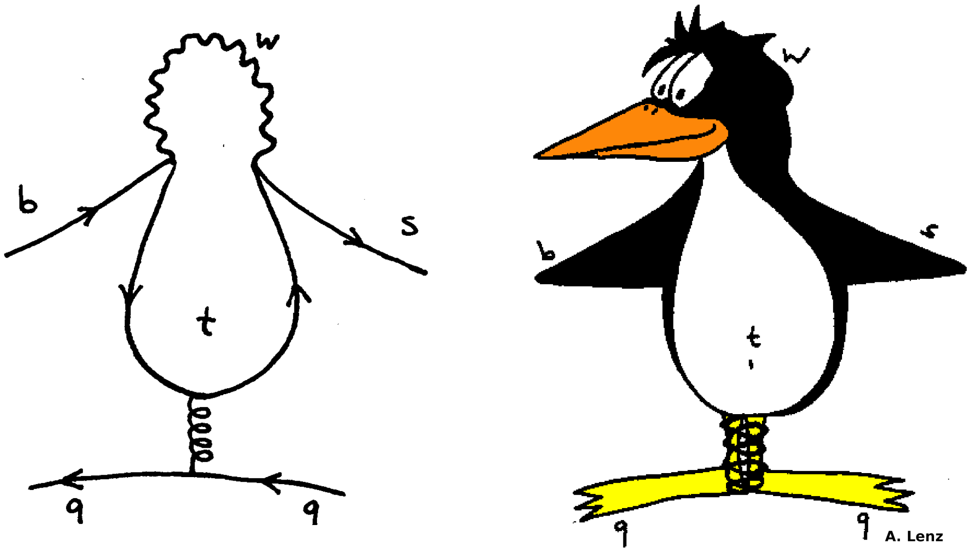
$$W^- : \; i\frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)V_{xy}^*$$



The couplings V_{xy} are the so-called **CKM (Cabibbo-Kobayashi-Maskawa) elements** [13, 14]. The CKM parameters exhibit a pronounced hierarchy. Typically this hierarchy is made explicit by expressing the different CKM-elements in powers of the small **Wolfenstein parameter** [15] $\lambda \approx 0.22551$, see e.g. CKMfitter [16] or UTfit [17]. In the case of inclusive b-decays the following CKM elements appear:

$$\begin{aligned} V_{ud}, V_{cs} &\propto \lambda^0 = 1 \, , \\ V_{us}, V_{cd} &\propto \lambda^1 \, , \\ V_{cb} &\propto \lambda^2 \, , \\ V_{ub} &\propto \lambda^{3.8} \, . \end{aligned}$$

Typically it is stated in the literature that V_{ub} is of order λ^3, but numerically it is much closer to λ^4. At the time, the Wolfenstein parameterisation was proposed (1983), the knowledge about the size of V_{ub} was simply not precise enough to distinguish this difference.
In addition to the above discussed tree-level decays, there are also transitions that appear only on loop-level. In the standard model there is e.g. no tree-level transition of a b-quark into a s-quark. This is the famous absence of **flavour changing neutral currents (FCNC)**. On loop level such a transition is possible within the SM, via so-called **penguin** diagrams (invented in 1975 by Shifman, Vainshtein and Zakharov [18] and baptised by John Ellis in 1977 [19]).

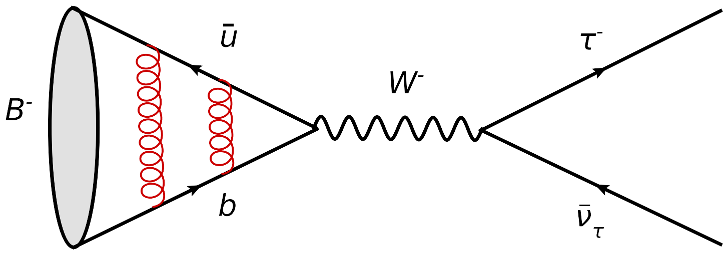


b-decays that proceed only via penguins are $b \to s\bar{s}s, s\bar{s}d, d\bar{d}d, d\bar{d}s, s\gamma, d\gamma, sl^+l^-, dl^+l^-, sg$ and dg. b-decays that proceed via tree-level decays and penguins are $b \to c\bar{c}s, c\bar{c}d, u\bar{u}s$ and $u\bar{u}d$. For $b \to c\bar{c}s$ penguins are a correction of about 9% of the LO decay rate [20], for $b \to u\bar{u}s$ penguins are by far the dominant contribution [21].

2.3 Weak decays of heavy hadrons

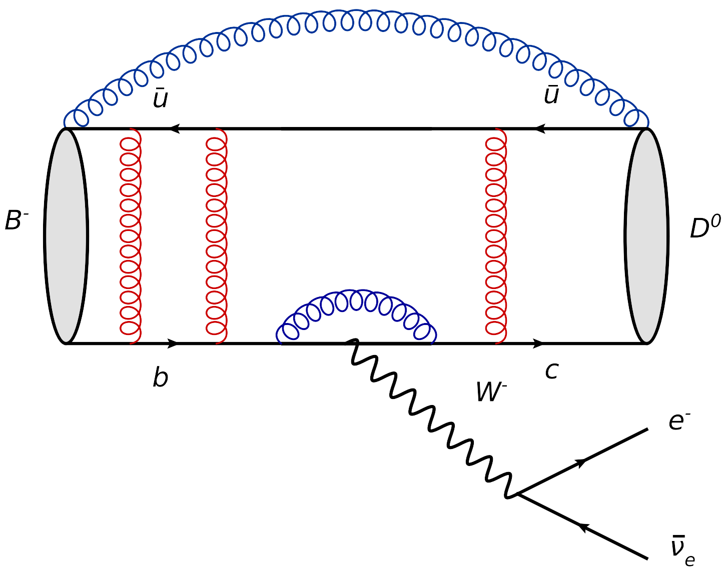
In reality weak decays of mesons are however much more complicated, than the decay of a free quark, because of strong interactions, which is depicted in the following diagrams. In principle the binding of the quarks into a meson is a non-perturbative problem, i.e. the exchange of one gluon is as important as the exchange of numerous ones.
Meson decays can be classified according to their final states:

- **Leptonic decays** have only leptons in the final state, e.g. $B^- \to \tau^- \, \bar{\nu}_\tau$.



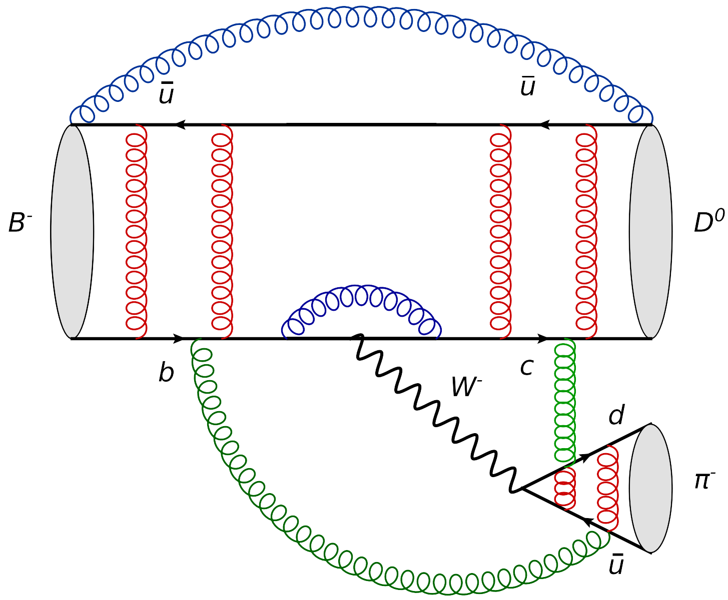
  Such decays have the simplest hadronic structure. Gluons bind the quark of the initial state into a hadron. All non-perturbative effects are described by decay constants.

- **Semi-leptonic decays** have leptons and hadrons in the final state, e.g. $B^- \to D^0 \, e^- \, \bar{\nu}_e$.



Now the hadronic structure is more complicated. We have the binding of hadrons in the initial state and in the final states. Moreover there is the possibility of having strong interactions between the initial and final states. The non-perturbative physics is in this case described by form factors.

- **Non-leptonic decays** have only hadrons in the final state, e.g. $B^- \to D^0\, \pi^-$.



These are the most complicated decays and they can only be treated by making additional assumptions that allow then for a factorisation.

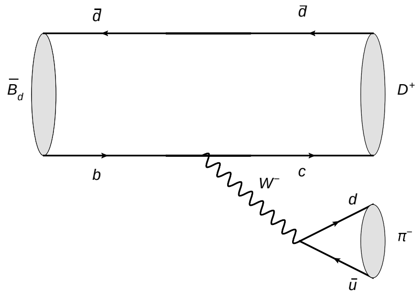
Later on, when investigating these decays in more theoretical detail, we will see, however, that the **free quark decay** is a very good approximation, if the decaying quark is heavy enough. This can be shown within the framework of the **Heavy Quark Expansion (HQE)**, see, e.g., [22] for a review and references therein. For the b-quark this approximation works quite well; it is currently discussed whether it also works for the c-quark.

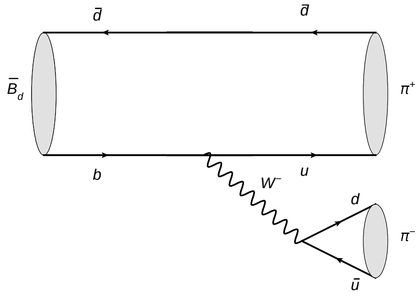
2.4 Exercise 1

1. Draw the Feynman diagrams for the decays $\bar{B}_d \to D^+\pi^-$ and $\bar{B}_d \to \pi^+\pi^-$. What would you expect in theory for the ratio

$$\frac{Br(\bar{B}_d \to D^+\pi^-)}{Br(\bar{B}_d \to \pi^+\pi^-)}?$$

Compare your expectation with the measured values taken from the PDG.
<u>**Solution:**</u>





$$\frac{Br(\bar{B}_d \to D^+\pi^-)}{Br(\bar{B}_d \to \pi^+\pi^-)}^{\text{Theory}} = \frac{|V_{cb}|^2}{|V_{ub}|^2} = \frac{1}{\lambda^4} = 386.667 \,,$$

$$\frac{Br(\bar{B}_d \to D^+\pi^-)}{Br(\bar{B}_d \to \pi^+\pi^-)}^{\text{PDG}} = \frac{(2.68 \pm 0.13) \cdot 10^{-3}}{(5.12 \pm 0.19) \cdot 10^{-6}} = 523.438 \,.$$

The agreement with our naive estimate is quite impressive!

2. What size do you expect for the **semi leptonic branching ratio**

$$B_{sl} = \frac{\Gamma(b \to c\bar{\nu}_e e^-)}{\Gamma_{tot}} \,,$$

if the masses in the final states and sub-dominant $b \to u$-transitions are neglected?

<u>**Solution:**</u>

$$\begin{aligned} B_{sl} &= \frac{\Gamma(b \to c\bar{\nu}_e e^-)}{\Gamma_{tot}} \\ &= \frac{\Gamma(b \to c\bar{\nu}_e e^-)}{\Gamma(b \to c\bar{u}d, s) + \Gamma(b \to c\bar{c}d, s) + 3\Gamma(b \to c\bar{\nu}_e e^-)} \\ &= \frac{1}{3+3+3} = 0.111111 \,. \end{aligned}$$

Accidentally this naive estimate agrees perfectly with the measured value of $B_{sl} = 0.1033 \pm 0.0028$ for B_d mesons [9].

3. Rank all possible inclusive (tree-level) decays according to their branching ratios. Now also the masses of the particles in the final states are taken into

account. The phase space factor for one Charm-quark in the final state is about 0.67, for one tau lepton 0.28, for two charm quarks 0.40 and for one tau and one charm 0.13.

Solution:

Decay		naive	naive	NLO-QCD[20]
$b \to c\bar{u}d$	$\propto$	$\lambda^4 \cdot 3 \cdot PS_1$	$= 41.1999\%$	44.6%
$b \to c\bar{c}s$	$\propto$	$\lambda^4 \cdot 3 \cdot PS_2$	$= 24.597\%$	23.2%
$b \to c\bar{\nu}_e e^-$	$\propto$	$\lambda^4 \cdot 1 \cdot PS_1$	$= 13.7333\%$	11.6%
$b \to c\bar{\nu}_\mu \mu^-$	$\propto$	$\lambda^4 \cdot 1 \cdot PS_1$	$= 13.7333\%$	11.6%
$b \to c\bar{\nu}_\tau \tau^-$	$\propto$	$\lambda^4 \cdot 1 \cdot PS_2$	$= 2.66467\%$	2.7%
$b \to c\bar{u}s$	$\propto$	$\lambda^6 \cdot 3 \cdot PS_1$	$= 2.09521\%$	2.4%
$b \to c\bar{c}d$	$\propto$	$\lambda^6 \cdot 3 \cdot PS_2$	$= 1.25087\%$	1.3%
$b \to u\bar{u}d$	$\propto$	$\lambda^{7.6} \cdot 3 \cdot 1$	$= 0.288548\%$	0.6%
$b \to u\bar{c}s$	$\propto$	$\lambda^{7.6} \cdot 3 \cdot PS_1$	$= 0.193327\%$	0.4%
$!b \to u\bar{u}s$	$\propto$	$\lambda^{9.6} \cdot 3 \cdot 1$	$= 0.0146741\%$	0.2%
$b \to u\bar{\nu}_e e^-$	$\propto$	$\lambda^{7.6} \cdot 1 \cdot 1$	$= 0.0961828\%$	0.2%
$b \to u\bar{\nu}_\mu \mu^-$	$\propto$	$\lambda^{7.6} \cdot 1 \cdot 1$	$= 0.0961829\%$	0.2%
$b \to u\bar{\nu}_\tau \tau^-$	$\propto$	$\lambda^{7.6} \cdot 1 \cdot PS_1$	$= 0.0269312\%$	0.1%
$b \to u\bar{c}d$	$\propto$	$\lambda^{9.6} \cdot 3 \cdot PS_1$	$= 0.00983162\%$	0.00%

For the decay $b \to u\bar{u}s$ the penguin contribution is dominant, so our power counting does not work for this decay.

4. What size do you now expect for the semi leptonic branching ratio?

Solution:

$$B_{sl} = 0.137333\ .$$

Mass corrections turn out to be very sizable. By accident the naive leading estimate reproduced already perfectly the experiment value: $B_{sl} = 0.1033 \pm 0.0028$ for B_d mesons [9]. Later on we will see, that QCD-corrections [20] will bring down again the theoretical value to the experimental one $B_{sl} = 0.116$.

2.5 CKM, FCNC,... within the SM

The Lagrangian of the standard model [23, 24] reads schematically

$$\mathcal{L} \;=\; -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$
\begin{aligned}
&+i\bar{\Psi}\not{D}\Psi \\
&+|D_\mu\Phi|^2 - V(\Phi) \\
&+\bar{\Psi}_i Y_{ij}\Phi\Psi_j + h.c. \quad .
\end{aligned} \tag{4}
$$

The first line of Eq.(4) describes the gauge fields of the strong, weak and electromagnetic interaction, the second line massless fermions and their interaction with the gauge fields. The third line represents the free scalar field, the Higgs potential and the interaction of the scalar field with the gauge fields. The special form of the Higgs potential will result in masses for some of the gauge bosons. The last line describes the interaction between fermions and the scalar field, the so-called Yukawa interaction. When the Higgs field Φ is replaced by its vacuum expectation value $v/\sqrt{2}$ one is left with a fermion mass term of the form $vY_{ij}/\sqrt{2}\cdot\bar{\Psi}_i\Psi_j$, so the mass is given by $m_{ij} = vY_{ij}/\sqrt{2}$.
The full standard model Lagrangian is invariant under Poincare transformations and local $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge transformations - $SU(3)$ describes the strong interaction, c stands for colour, $SU(2)$ describes the weak interaction, L stands for left-handed, $U(1)$ describes the electromagnetic interaction and Y stands for hypercharge. Looking at the $SU(2)_L \times U(1)_Y$-part in more detail one gets in the case of one generation of fermions the following expressions:

$$
\begin{aligned}
\mathcal{L} \;=\;& -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu\,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&+\bar{\Psi}_L\gamma^\mu\left(i\partial_\mu - g_1Y_LB_\mu - g_2q_L\frac{\vec{\sigma}\cdot\vec{W}_\mu}{2}\right)\Psi_L \\
&+\bar{\Psi}_R\gamma^\mu\left(i\partial_\mu - g_1Y_RB_\mu - g_2q_R\frac{\vec{\sigma}\cdot\vec{W}_\mu}{2}\right)\Psi_R \\
&+\left|\left(i\partial_\mu - g_1Y_\Phi B_\mu - g_2q_\Phi\frac{\vec{\sigma}\cdot\vec{W}_\mu}{2}\right)\Phi\right|^2 - V(\Phi^\dagger\Phi) \\
&-\left(\bar{\Psi}_L\Phi^cY_uu_R + \bar{u}_R\Phi^{c\dagger}Y_u\Psi_L\right) - \left(\bar{\Psi}_L\Phi Y_dd_R + \bar{d}_R\Phi^\dagger Y_d\Psi_L\right) \quad .
\end{aligned} \tag{5}
$$

Let us discuss first the notation:

- Ψ_L and Ψ_R denote left- and right-handed spinors describing the fermions

$$
\Psi_{L,R} = \frac{1\pm\gamma_5}{2}\Psi \quad . \tag{6}
$$

  Moreover Ψ_L denotes a $SU(2)_L$ doublet, i.e. one can write

$$
\Psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad . \tag{7}
$$

u_L is the four component Dirac spinor of the up-quark it has weak isospin $+1/2$ and d_L is the four component Dirac spinor of the down quark with weak isospin $-1/2$. Ψ_R denotes $SU(2)_L$ singlets.

- g_1 is the gauge coupling of the $U(1)_Y$ interaction transmitted via the B_μ gauge field, $B_{\mu\nu}$ is the corresponding field strength tensor. $Y_{L,R,\phi}$ are the hyper charges of the left-handed fermions, right-handed fermions and of the Higgs field.

- g_2 is the gauge coupling of the $SU(2)_L$ interaction transmitted via the three $\vec{W}_\mu$ gauge fields, $W^a_{\mu\nu}(a = 1, 2, 3)$ is the corresponding field strength tensor and $\vec{\sigma}$ denotes the Pauli matrices. The fact that only left-handed fermions take part in the weak interaction and right-handed do not, is fulfilled by the following choice of the charges: $q_R = 0$ and $q_L = q_\Phi = 1$. This describes correctly the experimentally found *maximal parity-violation* of the weak interaction.

- Also the Higgs field is a $SU(2)_L$ doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \tag{8}$$

with hypercharge $Y = 1/2$. The complex Higgs doublet has four degrees of freedom and the following quantum numbers.

	ϕ^+	ϕ^0
Q	$+1$	0
T_3	$+1/2$	$-1/2$
Y	$+1/2$	$+1/2$

Using the **unitary gauge** one can expand the Higgs field in the following way

$$\Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} . \tag{9}$$

v is the non-vanishing vacuum expectation value of the Higgs field Φ ($v \approx 246.22$ GeV)[4] and H is the physical Higgs field, which was recently found at the LHC [7, 8].
$Y_{u,d}$ are the Yukawa couplings of the up- and down-quarks. To give both

[4] Originally v is defined as the minimum of the Higgs potential, $v = \sqrt{-\mu^2/\lambda}$. Expressing the gauge boson masses in terms of v one gets $M_W = g_2 v/2$. Comparing this with the definition of the Fermi constant $G_F/\sqrt{2} = g_2^2/(8M_W^2)$ one sees that $v = \sqrt{1/(\sqrt{2}G_F)}$.

the up-quarks and the down-quarks a mass we have to introduce a second Higgs field, which is not independent from the original one (in some extensions of the standard model, it will be independent, e.g. in the Two-Higgs Doublet Model (**2HDM**) or the Minimal Supersymmetric Standard Model (**MSSM**)).

$$\Phi^c = i\sigma_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix} . \tag{10}$$

This field can also be expanded as

$$\Phi^c = \frac{1}{\sqrt{2}} \begin{pmatrix} v+H \\ 0 \end{pmatrix} . \tag{11}$$

After spontaneous symmetry breaking the Yukawa term reads

$$\begin{aligned} \mathcal{L}_{Yukawa} &= -\left(\bar{\Psi}_L \Phi^c Y_u u_R + \bar{u}_R \Phi^{c\dagger} Y_u \Psi_L\right) - \left(\bar{\Psi}_L \Phi Y_d d_R + \bar{d}_R \Phi^\dagger Y_d \Psi_L\right) \\ &= -\frac{vY_u}{\sqrt{2}} \left(\bar{u}_L u_R + \bar{u}_R u_L\right) - \frac{vY_d}{\sqrt{2}} \left(\bar{d}_L d_R + \bar{d}_R d_L\right) . \end{aligned} \tag{12}$$

This is now a simple mass term for the up- and down quarks with the masses $m_{u,d} = vY_{u,d}/\sqrt{2}$.
For three generations of quarks the situation gets still a little more involved. The Yukawa interaction reads now

$$\begin{aligned} &\mathcal{L}_{Yukawa} = \\ &= -\left(\bar{Q}_{1,L}, \bar{Q}_{2,L}, \bar{Q}_{3,L}\right) \Phi^c \hat{Y}_u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + \left(\bar{u}_R, \bar{c}_R, \bar{t}_R\right) \Phi^{c\dagger} \hat{Y}_u \begin{pmatrix} Q_{1,L} \\ Q_{2,L} \\ Q_{3,L} \end{pmatrix} \\ &- \left(\bar{Q}_{1,L}, \bar{Q}_{2,L}, \bar{Q}_{3,L}\right) \Phi \hat{Y}_d \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + \left(\bar{d}_R, \bar{s}_R, \bar{b}_R\right) \Phi^\dagger \hat{Y}_d \begin{pmatrix} Q_{1,L} \\ Q_{2,L} \\ Q_{3,L} \end{pmatrix} , \end{aligned} \tag{13}$$

with the three $SU(2)_L$ doublets

$$Q_{1,L} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_{2,L} = \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \quad Q_{3,L} = \begin{pmatrix} t_L \\ b_L \end{pmatrix}. \tag{14}$$

Note, that now in general the Yukawa coupling matrices $\hat{Y}_{u,d}$ do not have to be diagonal! After spontaneous symmetry breaking one gets the following structure of the fermion mass terms:

$$\bar{\Psi}^u_L \hat{M}_1 \Psi^u_R + \bar{\Psi}^u_R \hat{M}_1 \Psi^u_L + \bar{\Psi}^d_L \hat{M}_2 \Psi^d_R + \bar{\Psi}^d_R \hat{M}_2 \Psi^d_L , \tag{15}$$

with

$$\Psi^u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} , \tag{16}$$

$$\Psi^d = \begin{pmatrix} d \\ s \\ b \end{pmatrix} , \tag{17}$$

$$\hat{M}_1 = \frac{v}{\sqrt{2}} \hat{Y}_u , \tag{18}$$

$$\hat{M}_2 = \frac{v}{\sqrt{2}} \hat{Y}_d . \tag{19}$$

Again, in general the mass matrices $\hat{M}_1$ and $\hat{M}_2$ do not have to be diagonal, but they can be diagonalised with unitary transformations

$$\Psi^u \to U_1 \Psi^u \text{ with } U_1^\dagger U_1 = 1 , \tag{20}$$

$$\Psi^d \to U_2 \Psi^d \text{ with } U_2^\dagger U_2 = 1 . \tag{21}$$

The transformed mass matrices read

$$U_1^\dagger \hat{M}_1 U_1 = \frac{v}{\sqrt{2}} U_1^\dagger \hat{Y}_u U_1 = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} , \tag{22}$$

$$U_2^\dagger \hat{M}_2 U_2 = \frac{v}{\sqrt{2}} U_2^\dagger \hat{Y}_d U_2 = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} . \tag{23}$$

The states that belong to a diagonal mass matrix are called **mass eigenstates** or **physical eigenstates**, the states that couple to the weak gauge bosons are called **weak eigenstates**. In principle the mass matrices could also be diagonal from the beginning on. We will start, however, with the most general possibility and finally experimental data will show what is realised in nature.

The transformation between weak and mass eigenstates does not affect the electromagnetic interaction and also not the neutral weak current. In this cases up-like quarks couple to up-like ones and down-like quarks to down-like ones, so one has always the combinations $U_1^\dagger U_1$ and $U_2^\dagger U_2$ in the interaction terms. By definition this combinations give the unit matrix. Thus all neutral interactions are diagonal, in other words **there are no flavour changing neutral currents (FCNC) in the standard model at tree-level.** The originally diagonal charged current interaction can however become non-diagonal by this transformation

$$\begin{aligned} & (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu \,(1-\gamma_5) \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \\ \to \quad & (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu \,(1-\gamma_5)\, U_1^\dagger U_2 \begin{pmatrix} d \\ s \\ b \end{pmatrix} \\ \to \quad & (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu \,(1-\gamma_5) \begin{pmatrix} .. & .. & .. \\ .. & .. & .. \\ .. & .. & .. \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \end{aligned} \tag{24}$$

This defines the famous **Cabibbo-Kobayashi-Maskawa-Matrix** or **CKM-Matrix**

$$V_{CKM} := U_1^\dagger U_2 \,. \tag{25}$$

From a theory point of view it is not excluded that $U_1^\dagger U_2$ is diagonal (e.g. U_1 and U_2 are unit matrices or $U_1 = U_2$). In the end experimental data will show (and have shown) if the CKM-matrix is non-diagonal and thus allows transitions between different families. Historically this matrix was invented in two steps:

- 1963: 2x2 Quark mixing by Cabibbo [13]
- 1973: 3x3 Quark mixing by Kobayashi and Maskawa [14]; NP 2008

Let us look a little more in the properties of this matrix:
By construction the CKM-Matrix is a unitary matrix, it connects the weak eigenstates q' with the mass eigenstates q. Instead of transforming both the up-type and down-type quark fields one can also solely transform the down-type fields:

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} . \tag{26}$$

One can show, that a general unitary $N \times N$-matrix has $N(N-1)/2$ real parameters and $(N-1)(N-2)/2$ phases, if unphysical phases are discarded (?reference?).

N = 2	1 real parameter	0 phases
N = 3	3 real parameters	1 phase
N = 4	6 real parameters	3 phases

As will be discussed below a complex coupling, e.g. a complex CKM-element, leads to an effect called **CP-violation**. This will have important consequences on the existence of matter in the universe. Kobayashi and Maskawa found in 1973 that one needs at least three families of quarks (i.e. six quarks) to implement CP-violation in the standard model. At that time only three quarks were known, the charm-quark was found in 1974.
As we have seen already, the CKM-Matrix allows non-diagonal couplings of the charged currents, i.e. the u-quark does not only couple to the d-quark via a charged W boson, but it also couples to the s-quark and the b-quark. The entries of the CKM-matrix give the respective coupling strengths

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{27}$$

$$\text{Coupling} \quad \propto \frac{g_2}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) V_{ud} \tag{28}$$

For a unitary 3×3 matrix with 3 real angles and 1 complex phase, different parameterisations are possible. The so-called **standard parameterisation** reads

$$V_{CKM3} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} , \tag{29}$$

with

$$s_{ij} := \sin(\theta_{ij}) \quad \text{and} \quad c_{ij} := \cos(\theta_{ij}) \, . \tag{30}$$

The three angles are θ_{12}, θ_{23} and θ_{13}, the complex phase describing CP-violation is δ_{13}. This parameterisation is exact and it is typically used for numerical calculations. There is also a very ostensive parameterisation, the so-called **Wolfenstein parameterisation** [15]. This parameterisation uses the experimentally found hierarchy $V_{ud} \approx 1 \approx V_{cs}$ and $V_{us} \approx 0.22551 =: \lambda$ to perform a Taylor expansion in λ. Here one also has 3 real parameters λ, A and ρ and one complex coupling denoted by η.

$$V_{CKM} \quad = \quad \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{31}$$

In this form the hierarchies can be read of very nicely. Transitions within a family are strongly favoured, transitions between the first and second family are suppressed by one power of λ, transition between the second and third family are suppressed by two powers of λ and transitions between the first and the third family by at least three powers. The most recent numerical values for the Wolfenstein parameter read (status January 2015 from the CKMfitter page [16])

$$\begin{aligned}
\lambda &= 0.22548^{+0.00068}_{-0.00034}\,, && (32)\\
A &= 0.810^{+0.018}_{-0.024}\,, && (33)\\
\bar{\rho} &= 0.1453^{+0.0133}_{-0.0075}\,, && (34)\\
\bar{\eta} &= 0.343^{+0.011}_{-0.012}\,. && (35)
\end{aligned}$$

Remarks:

- The non-vanishing value of η describes CP-violation within the standard model.
- Numerically one gets $|V_{ub}| = 0.00355 = \lambda^{3.78699}$, so V_{ub} is more of the order λ^4 than λ^3 as historically assumed.

For the values of all CKM elements one gets (status January from the CKMfitter page [16])

$$V_{CKM} = \begin{pmatrix} 0.974242^{+0.000079}_{-0.000158} & 0.22548^{+0.00068}_{-0.00034} & 0.00355^{+0.00017}_{-0.00015} \\ 0.22534^{+0.00068}_{-0.00034} & 0.97341^{+0.00011}_{-0.00018} & 0.04117^{+0.00090}_{-0.00114} \\ 0.00855^{+0.00021}_{-0.00027} & 0.04043^{+0.00088}_{-0.00112} & 0.999146^{+0.000046}_{-0.000038} \end{pmatrix}. \tag{36}$$

Remarks:

- From this experimental numbers we clearly can see, that the CKM-matrix is non-diagonal. So our initial ansatz with non-diagonal Yukawa interactions was necessary!
- One also clearly sees the hierarchy of the CKM-matrix. Transitions within a family are clearly favoured, while changes of the family are disfavoured. In the lepton sector there is a very different hierarchy.
- The above given numbers have very small uncertainties. This relies crucially on the assumption of having a unitary 3×3 CKM matrix. Giving up this assumption, e.g. in models with four fermion generations the uncertainties will be considerably larger.

2.6 Exercise II:

Derive the Wolfenstein parameterisation!

1. Experimentally it was known that $1 \approx V_{ud} > V_{us} \gg V_{ub}$. Start by defining the expansion parameter $\lambda := V_{us}$ and derive $c_{13} \approx 1$.

2. Write down the standard parameterisation of the CKM-matrix, where s_{12} and c_{12} are expressed in terms of λ. Include corrections up to order λ^3.

3. Looking closer at experimental data one finds $1 \approx V_{ud} > V_{us} > V_{cb} > V_{ub}$. Make the ansatz $V_{cb} =: A\lambda^2$ and $V_{ub} =: A\lambda^3(\rho - i\eta)$ and express the whole CKM matrix in terms of λ.

2.7 A clue to explain existence

In this section we will motivate the huge interest in effects related to the violation of a mathematical symmetry called CP (Charge Parity). This is an extremely fundamental issue and it is related to the origin of matter in the universe.
There is an observed asymmetry between matter and antimatter in the universe, which can be parameterised by the baryon to photon ratio η_B, which was measured by PLANCK [25] to be

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx (6.05 \pm 0.07) \cdot 10^{-10} \tag{37}$$

n_B is the number of baryons in the universe, $n_{\bar{B}}$ the number of anti-baryons and n_γ the number of photons. The tiny[5] matter excess is responsible for the whole visible universe! In the very early universe the relative excess of matter over antimatter was much smaller, compared to now

$$\eta_B(t \approx 0) = \frac{10000000001 - 10000000000}{n_\gamma} \tag{38}$$

$$\eta_B(today) = \frac{1 - 0}{n_\gamma} \tag{39}$$

Now we have two possibilities for the initial conditions:

- $\eta_B(t = 0) = 0$: this seems to be *natural*, but how can then $\eta_B(t > 0) \neq 0$ be produced?
  Starting from symmetric initial conditions in the big bang everything should

[5]The numerical value is obtained by investigating primordial nucleosynthesis and the cosmic microwave background, see e.g. the PLANCK homepage.

have annihilated itself, which we do not observe (because we exist!) or there are regions in the universe, which consist of antimatter, so that there is in total an exactly equal amount of matter and antimatter. This we also do not observe.[6]

- $\eta_B(t = 0) \neq 0$: is not excluded, even if it might seem *unnatural*. **But** during an inflationary phase (and everything points towards this scenario) every finite value of η_B will be almost perfectly thinned out to zero.

Sakharov has shown in 1967 [26] how one can solve this puzzle. If the basic laws of nature have certain properties, then one can create a baryon asymmetry dynamically (**Baryogenesis**). In order not to be wiped out by inflation one expects that the asymmetry has to be produced somewhere between the time of inflation ($T \geq 10^{16}$ GeV) and the electroweak phase transition ($T \approx 100$ GeV). The basics properties Sakharov found are:

a) **C and CP-Violation:** C is the charge parity, it changes the sign of the charges of the elementary particles; P is the usual parity, a space reflection. The violation of parity in the weak interaction was theoretically proposed in 1956 by Lee and Yang (NP 1957) [27] and almost immediately verified by the experiment of Wu [28]. In 1964 a tiny CP violation effect was found in the neutral K-system - in an observable denoted by ϵ_K - by Christenson, Cronin, Fitch, Turlay [29] (NP 1980).
We know three ways of implementing CP violation in our models:

   1. via complex Yukawa-couplings, as in the CKM matrix.
   2. via complex parameters in the Higgs potential, see e.g. 2 Higgs doublett models in the end of the lectures
   3. a la strong CP - this we will not be discussed in this lecture notes

b) **B Violation:** The necessity to violate the baryon number is obvious.
Examples for baryon number violating processes are:

   1. Sphalerons in the SM
   2. Decay of heavy X, Y Bosons in GUTs - triggers proton decay
   3. SUSY without R-Parity - triggers proton decay

c) **Phase out of thermal equilibrium:** In order to decide whether one is in thermal equilibrium or not one has to compare the expansion rate of the universe with the reaction rate of processes that can create a matter-antimatter

[6]See e.g. the homepage of the **A**lpha **M**agnetic **S**pectrometer experiment; the current bound for the anti-He to He ratio will be improved from 10^{-6} to 10^{-9}.

asymmetry. In principle the universe is after inflation almost always in thermal equilibrium. Deviations of it are possible via

- Out-of-equilibrium decay of heavy particles, e.g.
  - Nucleo synthesis
  - Decoupling of Neutrinos
  - Decoupling of Photons
- First order phase transitions, e.g.
  - Inflation
  - Electroweak phase transition?

Remarks:

- Sakharov's paper was sent to the journal on 23.9.1966 and published on 1.1.1967; it was cited for the first time in 1976 by Okun and Zeldovich; beginning of 2015 it has 2080 citations $\Rightarrow$ be patient with your papers!
- The paper is quite cryptic; it discusses the decays of maximons ($m \approx M_{Planck}$)...
- All three ingredients have to be part of the fundamental theory, not only in principal, but also to a sufficient extent.

For lecture notes on baryogenesis see e.g. [30, 31, 32, 33]. Currently there a three main types of models discussed, which could create a baryon asymmetry.

2.7.1 Electroweak Baryogenesis

Here one assumes that the baryon asymmetry will be created during the electroweak phase transition at an energy/temperature of about $T \approx 100$ GeV. The first candidate for this scenario is clearly the standard model. So let us see, whether the Sakharov criteria might be fulfilled within the standard model.

a) In the standard model C and CP violation are implemented. For a measure of the magnitude of CP violation one typically uses the Jarlskog invariant J[34], which reads in the standard model

$$J = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \cdot A \,. \tag{40}$$

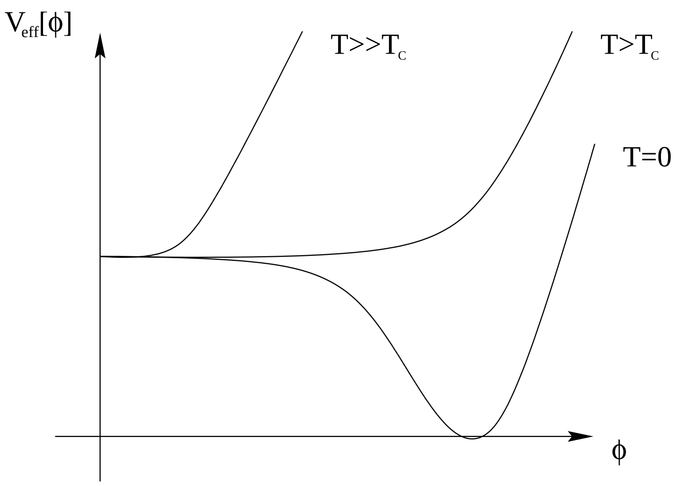
m_q denotes the mass of the quark q and A the area of the unitarity triangle, which will be discussed below. A is large, if the CKM-elements have also large imaginary, i.e. CP violating contributions. Normalising J to the scale of the electroweak phase transition one gets a very small number:

$$\frac{J}{(100\,\mathrm{GeV})^{12}} \approx 10^{-20} \ll 6 \times 10^{-10} \approx \eta_B \tag{41}$$

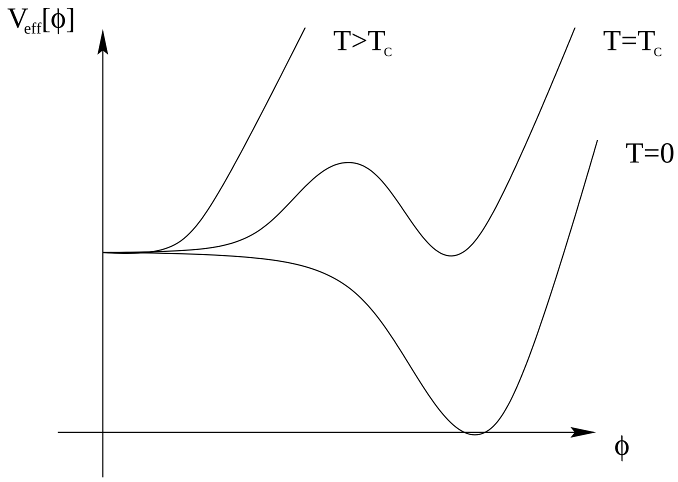
see e.g. [35]. So it seems that the amount of CP violation in the standard model is not sufficient to explain the baryon asymmetry.

b) In the standard model baryon number (B) and lepton number (L) are conserved to leading order in perturbation theory. Including quantum effects (in particular the Adler-Bell-Jackiew anomaly) one finds that B and L are no longer conserved separately, but $B - L$ is still conserved. Considering also non-perturbative effects (there exist no Feynman diagrams!), in particular thermal effects one can create the needed violation of B. These effects are called *sphalerons* (greek: weak, dangerous) [36, 37]. At temperatures T$<$ 100 GeV this effect is exponentially suppressed, while it grows very rapidly above 100 GeV.

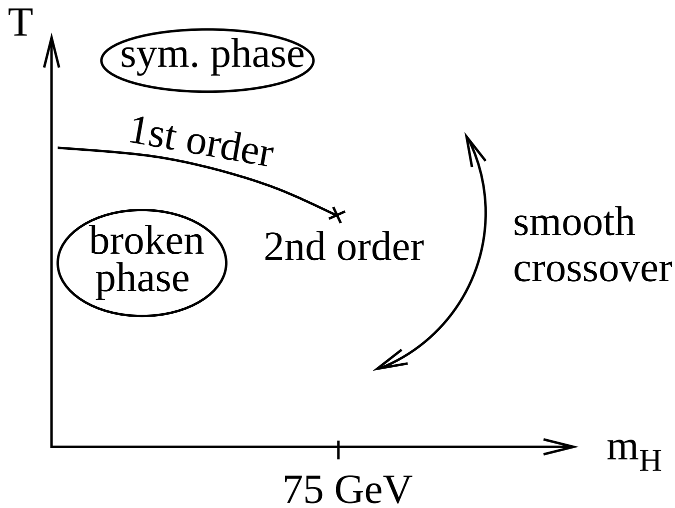
c) Finally one needs to be out of thermal equilibrium at 100 GeV. During a second order phase transition the parameters change in a continuous way and one stays always in thermal equilibrium:



In order to leave thermal equilibrium a first order transition is needed:



To answer the question of the nature of the electroweak phase transition one has to calculate the effective Higgs potential (classical potential plus quantum effects) in dependence of the Higgs mass at finite temperature. One finds for masses $m_H < 72$ GeV a first order transition, while the transition is continuous for higher masses, see e.g. [38, 39, 40, 41].



Thus the experimental value of the Higgs mass of 126 GeV clearly points towards a continuous transition within the standard model.

To summarise: in the standard model we have C and CP violation, we have B number violation and we have a possibility to have a phase out of thermal equilibrium. Looking closer one finds however that the amount of CP violation is not sufficient and that the experimental measured value of the Higgs mass is too high

to give a first order phase transition.
Thus we have to extend the standard model in order to create the observed baryon asymmetry. **This is a very strong indication for physics beyond the standard model.**
Staying with baryogenesis at the electroweak scale there are several possibilities to extend the standard model in such a way that the Sakharov criterias can be fulfilled, e.g.:

- Extended fermion sector, e.g. fourth generation models
  - The Jarlskog measure can easily be increased by 10 orders of magnitude [42]
  - Non perturbative effects due to large Yukawa couplings might modify the effective potential

  The most simple fourth generations are, however, excluded by the measured properties of the Higgs boson [43, 44, 45]

- Extended Higgs sectors, e.g. 2 Higgs-Doublet model:
  - New CP violating effects can appear in the Higgs sector, see e.g. [46]
  - Now a first order phase transition is possible, see e.g. [47, 48, 49, 50, 51, 52, 53].

- SUSY without R-parity:
  - New CP-violating effects possible
  - First order phase transition possible for certain mass spectra, see e.g.[54]

- ?Out-of-Equilibrium decay of new unknown particles with masses $m \approx 100$ GeV???

- ...

2.7.2 GUT-Baryo genesis

Here one assumes that the baryon asymmetry will be created during the electroweak phase transition at an energy/temperature of about $T \approx 10^{15}$ GeV, the unification scale of the strong, weak and electromagnetic interaction.

a) Due to the extended Higgs sector there is a lot of room for new CP violating effects.

b) Baryon number violating processes are now already induced pertubatively by the decays of the heavy X and Y bosons.

c) When the temperature falls below the GUT-scale ($\approx m_{X,Y}$), the production and the decay of the X,Y leave the equilibrium.

If a baryon asymmetry is produced at a very high scale, like the GUT scale, then there is always the danger that this asymmetry will be washed out later by sphaleron processes. Wash-out Processes (i.e. B asymmetry $\to$ B symmetry via inverse decay and rescattering) were investigated e.g. by Rocky Kolb (Post-Doc) and Stephen Wolfram (Grad. Student, founder of MATHEMATICA) [55, 56].

2.7.3 Lepto genesis

For a review of lepto genesis see e.g. [57].
There is no experimental bound on

$$\eta_L = \frac{n_L - n_{\bar{L}}}{n_\gamma} \,, \tag{42}$$

since charged leptons can always transform in more or less invisible neutrinos.
The basic idea of lepto genesis (1986, Fukugita and Yanagida [58]) consists of two steps:

1. Produce first a lepton asymmetry via neutrino processes (in order not to violate charge conservation). For this a violation of CP is mandatory.
   One possibility would be the decay of super-heavy right-handed Majorana neutrinos ($\Delta L = 2$). Such neutrinos could also explain the origin of the small neutrino masses.

2. Transform the lepton asymmetry into a baryon asymmetry via Sphalerons ($B + L$ is violated, while $B - L$ is conserved).

2.8 CP violation

A violation of the CP symmetry corresponds to the appearance of a complex coupling in the theory. In the standard model this happens in the Yukawa sector, in particular the CKM elements can be complex if there are at least three generations of fermions.

3 Flavour phenomenology

3.1 Overview

Decays of hadrons containing a beauty-quark or a charm-quark are perfectly suited for:

a) **Precise determination of the standard model parameter**, like CKM elements and quark masses.

b) **Indirect search for new physics** - heavy new particle might might give virtual contributions that are of comparable size as the standard model contributions.

c) Understanding of the origin and the mechanism of **CP violation**.

Having a closer look at these decays, one finds:

a) The CKM parameter appearing in decays of b-hadrons are among the least well known: V_{cb}, V_{ub}, V_{tb}, V_{ts} and V_{td}.

b) Certain decay modes, e.g. $b \to s\bar{s}s$ can not happen at tree-level in the standard model since we do not have FCNCs. But such decays can proceed via loop effects in the standard model. The probability for penguin decays is typically much smaller than for tree-level decays since penguins are an higher order effect in the weak interaction. Virtual corrections due to heavy new physics particles might have a similar size as penguin decays. Although being only a tiny correction to tree-level decays, new physics effects might be a large effect in loop induced b-decays and therefore these decay modes are especially well suited for the search for new physics.

c) CP violating effects are expected to be large in the b-system. In the K-system these effects are of the order of 10^{-3} (the analogue of ϵ_K in the B-system are the semi leptonic asymmetries which are also very small, see the discussion below). Bigi and Sanda pointed out in 1981 that the size of CP violation in exclusive decays of B-mesons might be large, i.e. of order one [59].[7]

[7] This was based on a work by Carter and Sanda [60]. Ashton Baldwin ”Ash” Carter (born on September 24, 1954) was nominated by President Barack Obama on December 5, 2014 to become the United States Secretary of Defense.

3.2 The unitarity triangle

This programme is closely related to the determination of the CKM-matrix and in particular to the determination of the so-called unitarity triangle. By construction we have

$$V_{CKM} V_{CKM}^{\dagger} = 1 \; . \tag{43}$$

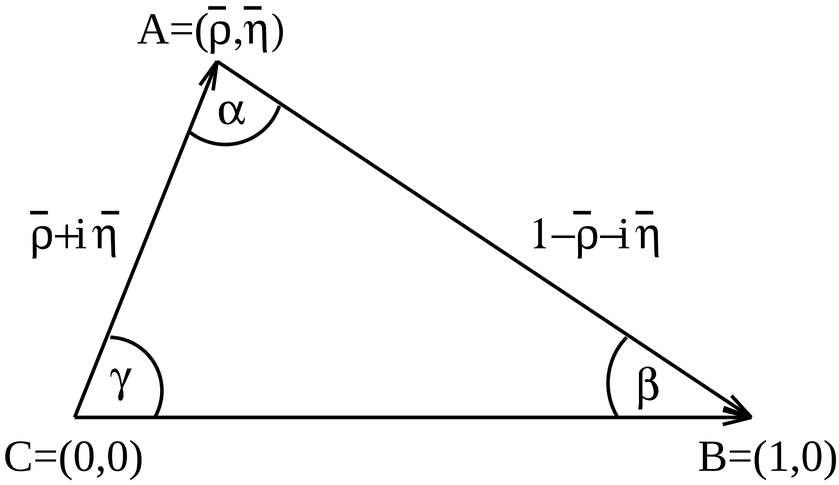
In the case of three generations this gives us nine conditions. Three combinations of CKM elements, whose sum is equal to one and six combinations whose sum is equal to zero, in particular

$$V_{ud} V_{ub}^{*} + V_{cd} V_{cb}^{*} + V_{td} V_{tb}^{*} = 0 \; . \tag{44}$$

Using the Wolfenstein parameterisation we get for this sum

$$A\lambda^3 \left[(\overline{\rho} + i\overline{\eta}) - 1 + (1 - (\overline{\rho} + i\overline{\eta})) \right] = 0 \, . \tag{45}$$

Since A and λ are already quite well known one concentrates on the determination of ρ and η. The above sum of three complex numbers can be represented graphically as a triangle, the so-called *unitarity triangle*, in the complex $\rho - \eta$ plane.



The determination of the unitarity triangle is in particular interesting since a non-vanishing η describes CP-violation in the standard model.
In principle the following strategy is used (for a review see e.g.[61]):
Compare the experimental value of some flavour observable with the corresponding theory expression, where ρ and η are left as free parameters and plot the constraint on these two parameters in the complex $\rho - \eta$ plane e.g.:

- The amplitude of a beauty-quark decaying into an up-quark is proportional to V_{ub}. Therefore the branching fraction of B-mesons decaying semi leptonically into mesons that contain the up-quark from the beauty decay is proportional to $|V_{ub}|^2$:

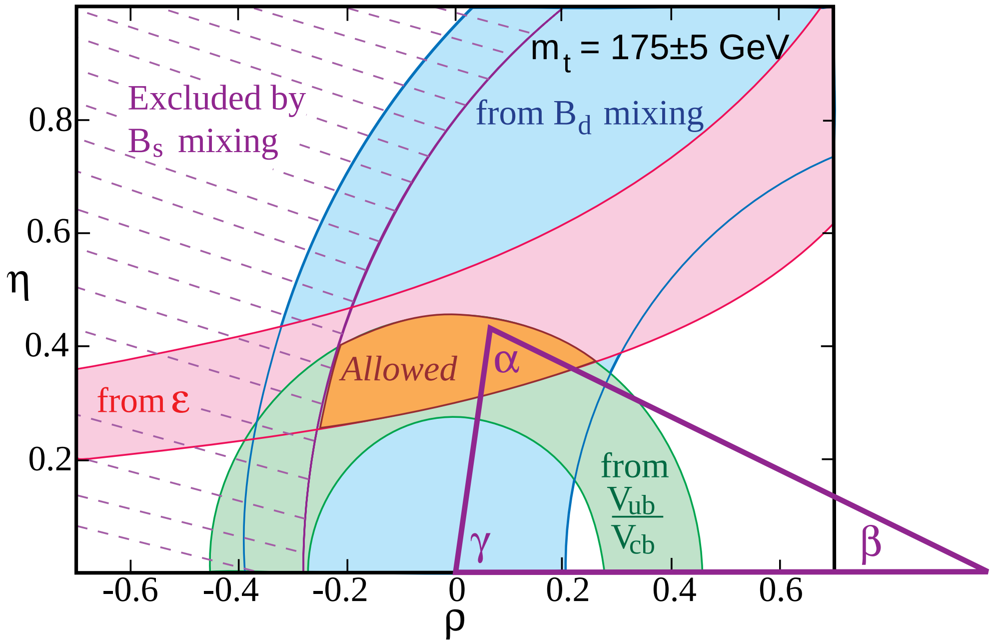
$$\begin{aligned} B(B \to X_u e \overline{\nu}) &= \tilde{a}_{\text{theory}} \cdot |V_{ub}|^2 = a_{\text{theory}} \cdot \left(\rho^2 + \eta^2\right) , \\ \Rightarrow \rho^2 + \eta^2 &= \frac{B^{\text{Exp.}}(B \to X_u e \overline{\nu})}{a_{\text{theory}}} , \end{aligned} \tag{46}$$

  where a contains the result of the theoretical calculation. By comparing experiment and theory for this decay and leaving ρ and η as free parameters we get a constraint in the $\rho - \eta$-plane in the form of a circle around $(0,0)$ with the radius $B^{\text{Exp.}}(B \to X_u e \overline{\nu})/a_{\text{theory}}$.

- Investigating the system of neutral B-mesons one finds that the physical eigenstates are a mixture of the flavour eigenstates. This effect will be discussed in more detail below. As a result of this mixing the two physical eigenstates have different masses, the difference of the two masses is denoted by ΔM_{B_d}. Theoretically one finds $\Delta M_{B_d} \propto |V_{td}|^2 \propto (\rho - 1)^2 + \eta^2$. Comparing experiment and theory we obtain a circle around $(1,0)$.

- Comparing theory and experiment for the CP-violation effect in the neutral K-system, denoted by the quantity ϵ_K, we get an hyperbola in the $\rho - \eta$-plane.

The overlap of all these regions gives finally the values for ρ and η. In the following figure all the above discussed quantities are included schematically. The constraint from the semi leptonic decay is shown in green, the constraint from B-mixing is shown in blue and the hyperbolic constraint from ϵ_k is displayed in pink. This figure is just meant to visualise the method in principle, later on we show a plot with the latest experimental numbers.



Remarks:

- The above programme was performed in the last years with great success! As a result of these efforts Kobayashi and Maskawa were awarded with the Nobel Prize in 2008.

- The presented method to determine ρ and η is equivalent to an indirect determination of the angle β. Bigi and Sanda have shown that this angle can be extracted directly, with almost no theoretical uncertainty from the following CP-asymmetry in exclusive B-decays [59].

$$a_{CP} \quad := \quad \frac{\Gamma(B \to J/\Psi + K_S) - \Gamma(\bar{B} \to J/\Psi + K_S)}{\Gamma(B \to J/\Psi + K_S) + \Gamma(\bar{B} \to J/\Psi + K_S)} \propto \sin 2\beta \quad (47)$$

  Because of its theoretical cleanness this decay mode is called the *gold-plated mode*.

3.3 Flavour experiments

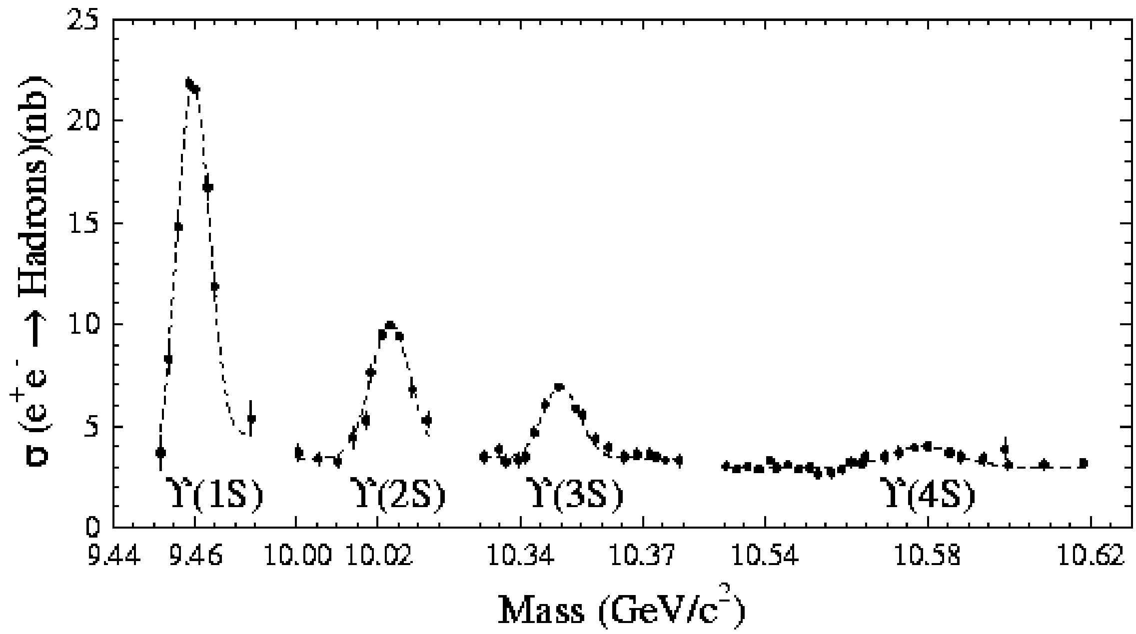
In order to be able to measure flavour quantities as precisely as possible one needs a huge number of B-mesons. So the obvious aim was to build accelerators that create as many B-mesons as possible. Currently there are two classes of accelerators that can fulfil this task:

- So-called *B factories*. This are $e^+ - e^-$-colliders, that seem to be in particular advantageous to perform precision measurements because of their low background. This was done since 1999 in SLAC, Stanford, USA with the PEP accelerator and the BaBar detector and in KEK, Japan with the Belle detector. Currently the machine and the detector in KEK are upgraded to a *Super B factory*.

  B-mesons will then be produced according to the following reaction

$$e^+ - e^- \rightarrow \Upsilon(= b\bar{b} - \text{resonances}) \rightarrow B + \bar{B} \,. \tag{48}$$

  There are several excitations of the Υ-resonance, with different masses and different production-cross-sections.



  Now one has to check, whether the production of B-mesons is kinematically allowed. The lightest mesons have a mass of $2m_{B^{\pm,0}} \approx 10559$ MeV, therefore our machine has to run on the $\Upsilon(4s)$-resonance, to have the highest possible production cross section. The price to pay is, that we can not produce any B_s, B_c or Λ_b in such a machine. To produce also B_s mesons one has to switch to the $\Upsilon(5s)$-resonance, with the prize of a lower cross section - this was only done at KEK.

  Another problem we have is the short lifetime of the b-hadrons, $\tau_b \approx 10^{-12}$ s. In order to be able to measure the tracks of the b-hadrons, it was decided to build asymmetric accelerators, where the produced B-mesons have a large boost and therefore due to time dilatation a lifetime long enough to be measured.

- Hadron colliders like the TeVatron at Fermilab ($p-\bar{p}$-collisions) and the LHC at CERN ($p-p$-collisions) were not primarily built for flavour physics, but they have huge $b-$ and charm production cross sections

$$
\begin{aligned}
\sigma(pp \to \bar{b}b + X) &= 284\mu b \text{ (7 TeV)} && (49)\\
\sigma(pp \to \bar{c}c + X) &= 6100\mu b \text{ (7 TeV)} && (50)\\
\sigma(e^+e^- \to \bar{b}b) &\approx 1nb \text{ (BaBar, Belle)} && (51)
\end{aligned}
$$

This means that an integrated luminosity of 1fb^{-1} [8] at LHC corresponds to 6×10^{12} $c\bar{c}$ pairs, approximately 1/6 of them is detected by LHCb. Thus the currently achieved 3 fb^{-1} correspond to about 10^{11} detected $b\bar{b}$ pairs, which has to be compared with about 10^9 $b\bar{b}$ pairs at Belle. Moreover in addition to B_d and B^+ the heavier hadrons like B_s, B_c and Λ_b are accessible in hadron machines. This led to the fact that recently the detector LHCb (for certain decay modes also ATLAS and CMS) started to dominate the field of experimental heavy flavour physics.

In the following table we give a brief list of some accelerators producing b-hadrons. Besides the kind of accelerated particles and their energy the *luminosity* $\mathcal{L}$ is one of the most important key numbers of an accelerator.

[8]The number of events of a certain kind is related to the cross section of this event and the luminosity in the following way

$$\# \text{ of events} = \int \mathcal{L} dt \cdot \sigma \tag{52}$$

$\int \mathcal{L} dt$ is also called the integrated luminosity and it is measured in units of e.g. fb$^{-}1$.

machine	location	particles energy	Exp.	circum-ference	luminosity $[\text{cm}^{-2}s^{-1}]$
LEP '89 - '01	CERN Geneva, CH	$e^+ + e^-$ 105 GeV	ALEPH, DELPHI OPAL, ZEUS	27 km	10^{32}
TEVATRON '87 - 30.9.2011	FERMILAB Chicago, USA	$p + \bar{p}$ 1 TeV	CDF D 0	6.3 km	$4 \cdot 10^{32}$ $10 fb^{-1}$
CESR '79 -	CLEO Cornell, USA	$e^+ + e^-$ 6 GeV		0.8 km	$1.3 \cdot 10^{33}$
PEP-II '99 - '08	SLAC Stanford USA	$e^+ + e^-$ e^+ : 3.1GeV e^- : 9 GeV	BaBar	2.2 km	$1.2069 \cdot 10^{34}$ (Peak) $557/433(4s) fb^{-1}$
KEKB '99 - '10	KEK Japan	$e^+ + e^-$ e^+ : 3.5 GeV e^- : 8 GeV	Belle	0.8 km	$2.1083 \cdot 10^{34}$ (Peak) $1040 fb^{-1}$
LHC '10 - '13 '15 - ...	CERN Geneva, CH	$p + p$ p : 8 TeV	LHCb	27 km	$8 \cdot 10^{33}$ (Peak) $3 fb^{-1}$ (LHCb)
SuperKEKB '17/'18 -	KEK Japan	$e^+ + e^-$ e^+ : 4 GeV e^- : 7 GeV	Belle	0.8 km	$8 \cdot 10^{35}$ (Peak) $50 ab^{-1}$ in '22/'23

3.4 Current status of flavour phenomenology

These experiments have achieved unprecedentedly high precision in flavour physics, for a recent review see, e.g. [62] and references therein; some highlights are:

- Precise determination of the CKM matrix:
  - Assuming the validity of the standard model, fits of the CKM matrix give very precise values, see Eq.(36).
  - In particular the previously quite unknown elements V_{cb} and V_{ub} are now strongly constrained.
  - One of the basic motivations of the B-factories was a direct determination of the angle β in the unitarity triangle via investigation of the decay $B_d \to J/\Psi + K_s$. A combination of the results from BaBar and Belle gives
    $$\sin 2\beta = 0.679 \pm 0.020 \,. \tag{53}$$
    This result is in very good agreement with the indirect determination of β via fits of the CKM matrix.
  - There is also a precise determination of the angle α as well as the first direct measurement of the angle γ available. The size of γ is directly

proportional to the size of CP violation in the CKM matrix. The most precise value for this value stems from LHCb

$$\gamma = \left(73^{+9}_{-10}\right)^\circ . \tag{54}$$

All these achievements were awarded in 2008 with the Nobel Prize for Kobayashi and Maskawa.

- CP violation:
  - Large CPV in B decays, in particular *indirect CP-violation* (= the physical eigenstate is not a pure CP-eigenstate) was found in the decay $B_d \to J/\Psi + K_s$. This was the first discovery of CP-violation outside the K-system.
  - *Direct CP-violation* (= the decay itself violates CP) was discovered, e.g. in the decays

$$\begin{aligned} B_d &\to \pi^+\pi^- , K^+ + \pi^- , && (55) \\ B^+ &\to DK^+ , && (56) \\ B_s &\to K^-\pi^+ . && (57) \end{aligned}$$

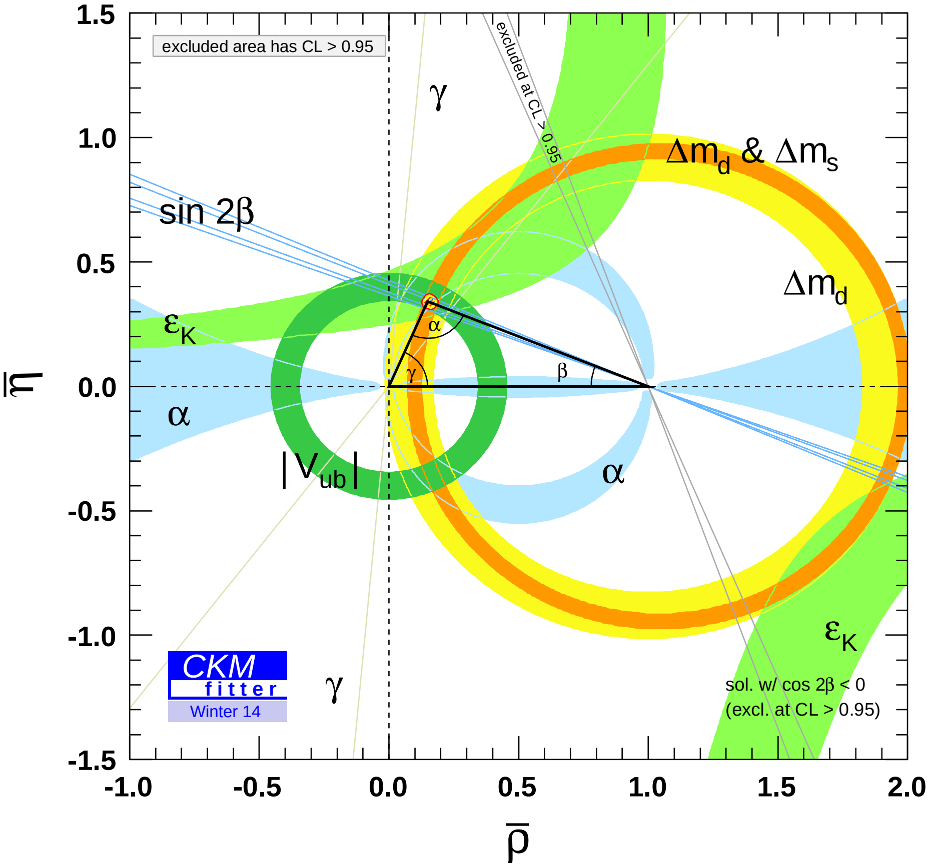
  - There are hints for direct CPV in the charm sector at the several per mille level.
  - CPV effects in mixing of neutral B-mesons are searched for intensively; these effects are tiny in the SM - so they present a nice nulltest.

- The lifetimes of the B-mesons and b-baryons were measured with a high precision. Since the lifetime is one of the fundamental properties of a particle, it is very desirable to understand this quantities theoretically. Moreover, lifetimes are expected to be only marginally affected by new physics contributions, thus they present a very clean test of our theoretical tools to describe heavy hadron decays, in particular the expansion in inverse powers of the heavy quark mass. We will present below the state of the art in calculating lifetimes of heavy hadrons, see [22] for a review.
  Of particular interest was the measurement of the decay rate difference in the neutral B_s system, $\Delta\Gamma_s$ by the LHCb collaboration from 2012 on, as well as newer results from ATLAS and CMS. This measurement provided a strong confirmation of the validity of the Heavy Quark Expansion.

- Numerous rare decays like $B_s \to \mu^+\mu^-, B \to X_s\gamma, B_d \to K^*\mu^+\mu^-, \ldots$ were measured with branching fractions as low as $3 \cdot 10^{-9}$. These modes are ideal for the search for new physics contributions, as well as the results from B-mixing.

- Numerous hadronic decays like $B \to \pi\pi, K\pi, KK, \ldots$ were measured, with branching fractions of the order of 10^{-5} and below. These decay modes are an interesting testing ground for attempts to describe the strong interaction effects in hadronic decays, which is more complicated than lifetimes.

In the following figure the current status (January 2015) of the determination of the unitarity triangle is shown.

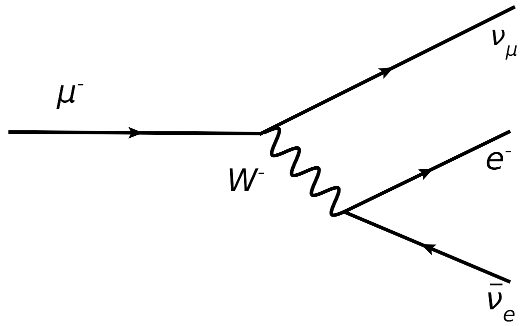


All measurements agree very well with the CKM mechanism, nevertheless there is still some sizeable room for new physics effects and we have about 10 deviations of experiment and standard model at the three sigma level.

4 Weak decays I - Basics

4.1 The myon decay

The muon decay $\mu^- \to \nu_\mu \; e^- \; \bar{\nu}_e$ represents the most simple weak decay, because there are no QCD effects involved.[9] This process is given by the following Feynman diagram.



Hence the total decay rate of the muon reads (see, e.g., [63] for an early reference)

$$\Gamma_{\mu \to \nu_\mu + e + \bar{\nu}_e} = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e}{m_\mu}\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} c_{3,\mu} \, . \tag{58}$$

$G_F = g_2^2/(4\sqrt{2}M_W^2)$ denotes the Fermi constant and f the phase space factor for one massive particle in the final state. It is given by

$$f(x) \;\; = \;\; 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln(x) \, . \tag{59}$$

The coefficient $c_{3,\mu}$ is introduced here to be consistent with our later notation. The result in Eq.(58) is already very instructive, since we get now for the measurable lifetime of the muon

$$\tau = \frac{1}{\Gamma} = \frac{192\pi^3}{G_F^2 m_\mu^5 f\left(\frac{m_e}{m_\mu}\right)} \, . \tag{60}$$

Thus the lifetime of a weakly decaying particle is proportional to the inverse of the fifth power of the mass of the decaying particle. Using the measured values [9] for $G_F = 1.1663787(6) \cdot 10^{-5}$ GeV^{-2} , $m_e = 0.510998928(11)$ MeV and $m_\mu = 0.1056583715(35)$ GeV we predict[10] the lifetime of the muon to be

$$\tau_\mu^{Theo.} = 2.18776 \cdot 10^{-6}\,\mathrm{s} \, , \tag{61}$$

[9] This statements holds to a high accuracy. QCD effects arise for the first time at the two loop order.

[10] This is of course not really correct, because the measured muon lifetime was used to determine the Fermi constant, but for pedagogical reasons we assume that the Fermi constant is known from somewhere else.

which is in excellent agreement with the measured value [9] of

$$\tau_\mu^{Exp.} = 2.1969811(22) \cdot 10^{-6}\,\mathrm{s}\ . \tag{62}$$

The remaining tiny difference (the prediction is about 0.4% smaller than the experimental value) is due to higher order electro-weak corrections. These corrections are crucial for a high precision determination of the Fermi constant. The dominant contribution is given by the 1-loop QED correction, calculated already in the 1950s [64, 65]:

$$c_{3,\mu} = f\left(\frac{m_e}{m_\mu}\right)\left[1+\frac{\alpha}{4\pi}2\left(\frac{25}{4}-\pi^2\right)\right]\ . \tag{63}$$

Taking this effect into account ($\alpha = 1/137.035999074(44)$ [9]) we predict

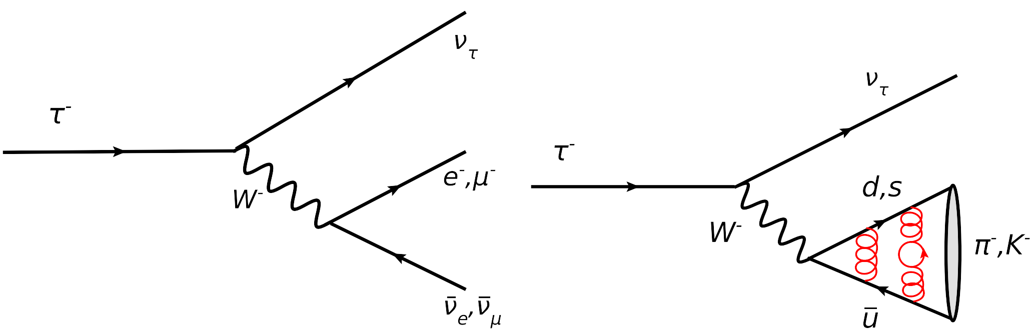
$$\tau_\mu^{Theo.} = 2.19699 \cdot 10^{-6}\,\mathrm{s}\ , \tag{64}$$

which is almost identical to the measured value given in Eq.(62). The complete 2-loop QED corrections have been determined in [66], a review of loop-corrections to the muon decay is given in [67] and two very recent higher order calculations can be found in, e.g., [68, 69].
The phase space factor is almost negligible for the muon decay - it reads $f(m_e/m_\mu) = 0.999813 = 1 - 0.000187051$ - but it will turn out to be quite sizable for a decay of a b-quark into a charm quark.

4.2 The tau decay

Moving to the tau lepton, we have now two leptonic decay channels as well as decays into quarks:

$$\tau \quad\to\quad \nu_\tau + \begin{cases} e^- + \bar\nu_e \\ \mu^- + \bar\nu_\mu \\ d + \bar u \\ s + \bar u \end{cases} \ .$$



Heavier quarks, like charm- or bottom-quarks cannot be created, because the lightest meson containing such quarks ($D^0 = c\bar{u}; M_{D^0} \approx 1.86\,\mathrm{GeV}$) is heavier than the tau lepton ($m_\tau = 1.77682(16)$ GeV). Thus the total decay rate of the tau lepton reads

$$\begin{aligned}\Gamma_\tau &= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[f\left(\frac{m_e}{m_\tau}\right) + f\left(\frac{m_\mu}{m_\tau}\right) + N_c\,|V_{ud}|^2\, g\left(\frac{m_u}{m_\tau}, \frac{m_d}{m_\tau}\right) + N_c\,|V_{us}|^2\, g\left(\frac{m_u}{m_\tau}, \frac{m_s}{m_\tau}\right)\right] \\ &=: \frac{G_F^2 m_\tau^5}{192\pi^3} c_{3,\tau}\,. \end{aligned} \tag{65}$$

The factor $N_c = 3$ is a colour factor and g denotes a new phase space function, when there are two massive particles in the final state. If we neglect the phase space factors ($f(m_e/m_\tau) = 1 - 7 \cdot 10^{-7}; f(m_\mu/m_\tau) = 1 - 0.027; ...$) and if we use $V_{ud}^2 + V_{us}^2 \approx 1$, then we get $c_{3,\tau} = 5$ and thus the simple approximate relation

$$\frac{\tau_\tau}{\tau_\mu} = \left(\frac{m_\mu}{m_\tau}\right)^5 \frac{1}{5}\,. \tag{66}$$

Using the experimental values for τ_μ, m_μ and m_τ we predict

$$\tau_\tau^{Theo.} = 3.26707 \cdot 10^{-13}\ \mathrm{s}\,, \tag{67}$$

which is quite close to the experimental value of

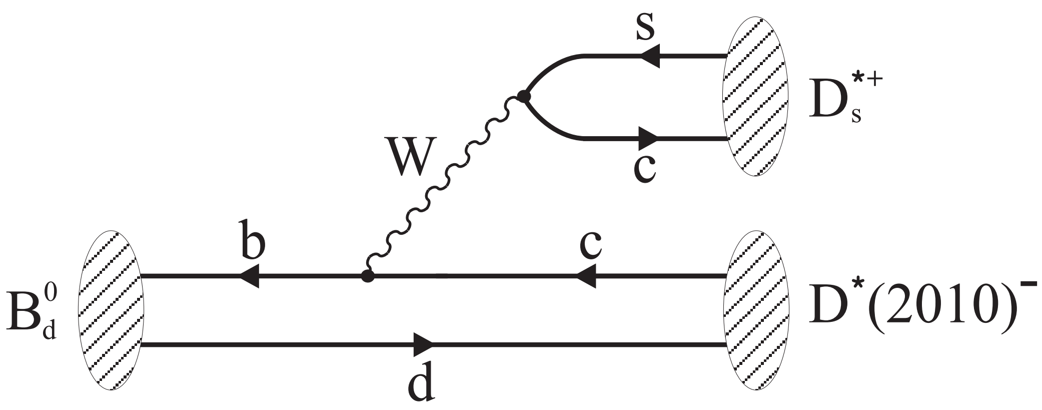
$$\tau_\tau^{Exp.} = 2.906(1) \cdot 10^{-13}\ \mathrm{s}\,. \tag{68}$$

Now the theory prediction is about 12% larger than the measured value. This is mostly due to sizable QCD corrections, when there are quarks in the final state - which was not possible in the muon decay. These QCD corrections are currently calculated up to five loop accuracy [70], a review of higher order corrections can be found in [71].
Because of the pronounced and clean dependence on the strong coupling, tau decays can also be used for precision determinations of α_s, see, e.g., the review [72]. This example shows already, that a proper treatment of QCD effects is mandatory for precision investigations of lifetimes. In the case of meson decays this will even be more important.

4.3 Meson decays - Definitions

As a starting point of the discussion of weak decays of mesons, we introduce two classes of decays - *inclusive* and *exclusive* decays.

- In exclusive modes every final state hadron is identified.



This is in principle what experiments can do well, while theory has the problem to describe the hadronic binding in the final states. For the quark-level decay $\bar{b} \to \bar{c}\; c\; \bar{s}$ we have among many more the following options:

$$\begin{aligned} B_d^0 &\to D^*(2010)^- D_s^{*+} , \\ &\to D^- D_s^{*+} , \\ &\to D^*(2010)^- D_s^+ , \\ &\to D^- D_s^+ . \end{aligned}$$

- In inclusive modes we only care about the quarks in the final states:

$$\bar{b} \to \bar{c}\; c\; \bar{s} .$$

  This is clearly theoretically easier, while experiments have the problem of summing up all decays that belong to a certain inclusive decay mode.

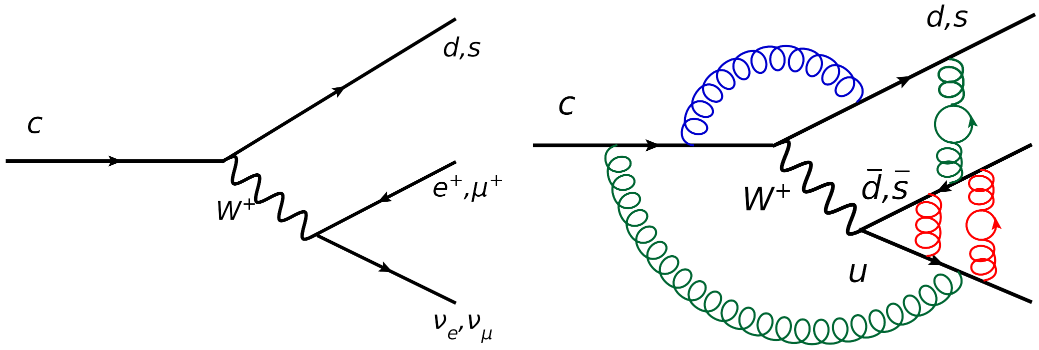
To get a feeling for the arising branching fractions we list the theory value [20] for $b \to c\; \bar{c}\; s$, with some measured [9] exclusive branching ratios.

$$\mathrm{Br}(b \to c\bar{c}s) = (23 \pm 2)\% , \tag{69}$$
$$\mathrm{Br}(D^{*-}\; D_s^{*+}) = (1.77 \pm 0.14)\% , \tag{70}$$
$$\mathrm{Br}(D^{*-}\; D_s^{+}) = (8.0 \pm 1.1) \cdot 10^{-3} , \tag{71}$$
$$\mathrm{Br}(D^{-}\; D_s^{*+}) = (7.4 \pm 1.6) \cdot 10^{-3} , \tag{72}$$
$$\mathrm{Br}(D^{-}\; D_s^{+}) = (7.2 \pm 0.8) \cdot 10^{-3} , \tag{73}$$
$$\mathrm{Br}(J/\Psi\; K_S) = (8.73 \pm 0.32) \cdot 10^{-4} . \tag{74}$$

Here one can already guess that quite some number of exclusive decay channels has to be summed up in order to obtain the inclusive branching ratio.

4.4 Charm-quark decay

Before trying to investigate the complicated meson decays, let us look at the decay of free c- and b-quarks. Later on we will show that the free quark decay is the leading term in a systematic expansion in the inverse of the heavy (decaying) quark mass - the Heavy Quark Expansion (HQE).
A charm quark can decay weakly into a strange- or a down-quark and a W^+-boson, which then further decays either into leptons (semi-leptonic decay) or into quarks (non-leptonic decay).



Calculating the total inclusive decay rate of a charm-quark we get

$$\Gamma_c = \frac{G_F^2 m_c^5}{192\pi^3} |V_{cs}|^2 c_{3,c} \, , \tag{75}$$

with

$$\begin{aligned} c_{3,c} = \quad & g\left(\frac{m_s}{m_c}, \frac{m_e}{m_c}\right) + g\left(\frac{m_s}{m_c}, \frac{m_\mu}{m_c}\right) + N_c |V_{ud}|^2 h\left(\frac{m_s}{m_c}, \frac{m_u}{m_c}, \frac{m_d}{m_c}\right) \\ & + N_c |V_{us}|^2 h\left(\frac{m_s}{m_c}, \frac{m_u}{m_c}, \frac{m_s}{m_c}\right) \\ & + \left|\frac{V_{cd}}{V_{cs}}\right|^2 \left\{ g\left(\frac{m_d}{m_c}, \frac{m_e}{m_c}\right) + g\left(\frac{m_d}{m_c}, \frac{m_\mu}{m_c}\right) + N_c |V_{ud}|^2 h\left(\frac{m_d}{m_c}, \frac{m_u}{m_c}, \frac{m_d}{m_c}\right) \right. \\ & \left. + N_c |V_{us}|^2 h\left(\frac{m_d}{m_c}, \frac{m_u}{m_c}, \frac{m_s}{m_c}\right) \right\} \end{aligned} \tag{76}$$

h denotes a new phase space function, when there are three massive particles in the final state. If we set all phase space factors to one ($f(m_s/m_c) = f(0.0935/1.471) = 1 - 0.03, \ldots$ with $m_s = 93.5(2.5)$ MeV [9]) and use $|V_{ud}|^2 + |V_{us}|^2 \approx 1 \approx |V_{cd}|^2 + |V_{cs}|^2$, then we get $|V_{cs}|^2 c_{3,c} = 5$, similar to the τ decay. In that case we predict a charm lifetime of

$$\tau_c \quad = \left\{ \begin{array}{l} 0.84 \text{ ps} \\ 1.70 \text{ ps} \end{array} \right. \qquad \text{for } m_c = \left\{ \begin{array}{lll} 1.471 & \text{GeV} & (\text{Pole-scheme}) \\ 1.277(26) & \text{GeV} & (\overline{MS} - \text{scheme}) \end{array} \right. \tag{77}$$

These predictions lie roughly in the ball-bark of the experimental numbers for D-meson lifetimes, but at this stage some comments are appropriate:

- Predictions of the lifetimes of free quarks have a huge parametric dependence on the definition of the quark mass ($\propto\ m_q^5$). This is the reason, why typically only lifetime ratios (the dominant m_q^5 dependence as well as CKM factors and some sub-leading non-perturbative corrections cancel) are determined theoretically. We show in this introduction for pedagogical reasons the numerical results of the theory predictions of lifetimes and not only ratios. In our case the value obtained with the $\overline{MS}-$ scheme for the charm quark mass is about a factor of 2 larger than the one obtained with the pole-scheme. In LO-QCD the definition of the quark mass is completely arbitrary and we have these huge uncertainties. If we calculate everything consistently in NLO-QCD, the treatment of the quark masses has to be defined within the calculation, leading to a considerably weaker dependence of the final result on the quark mass definition.
  Bigi, Shifman, Uraltsev and Vainshtein have shown in 1994 [73] that the pole mass scheme is always affected by infra-red renormalons, see also the paper of Beneke and Braun [74] that appeared on the same day on the arXiv and the review in this issue [75]. Thus short-distance definitions of the quark mass, like the $\overline{\mathrm{MS}}$-mass [10] seem to be better suited than the pole mass. More recent suggestions for quark mass concepts are the kinetic mass from Bigi, Shifman, Uraltsev and Vainshtein [76, 77] introduced in 1994, the potential subtracted mass from Beneke [78] and the $\Upsilon(1s)$-scheme from Hoang, Ligeti and Manohar [79, 80], both introduced in 1998. In [20] we compared the above quark mass schemes for inclusive non-leptonic decay rates and found similar numerical results for the different short distance masses. Thus we rely in this review - for simplicity - on predictions based on the $\overline{\mathrm{MS}}$-mass scheme and we discard the pole mass, even if we give several times predictions based on this mass scheme for comparison.
  Concerning the concrete numerical values for the quark masses we also take the same numbers as in [20]. In that work relations between different quark mass schemes were strictly used at NLO-QCD accuracy (higher terms were discarded), therefore the numbers differ slightly from the PDG [9]-values, which would result in

$$\tau_c = \begin{cases} 0.44 \text{ ps} \\ 1.71 \text{ ps} \end{cases} \quad \text{for } m_c = \begin{cases} 1.67(7) & \text{GeV (Pole-scheme)} \\ 1.275(25) & \text{GeV } (\overline{MS}-\text{scheme}) \end{cases} \tag{78}$$

  Since our final lifetime predictions are only known up to NLO accuracy and we expand every expression consistently up to order α_s, we will stay with the parameters used in [20].

- Taking only the decay of the c-quark into account, one obtains the same lifetimes for all charm-mesons, which is clearly a very bad approximation, taking the large spread of lifetimes of different D-mesons into account. Below we will see that in the case of charmed mesons a very sizable contribution comes from non-spectator effects where also the valence quark of the D-meson is involved in the decay.

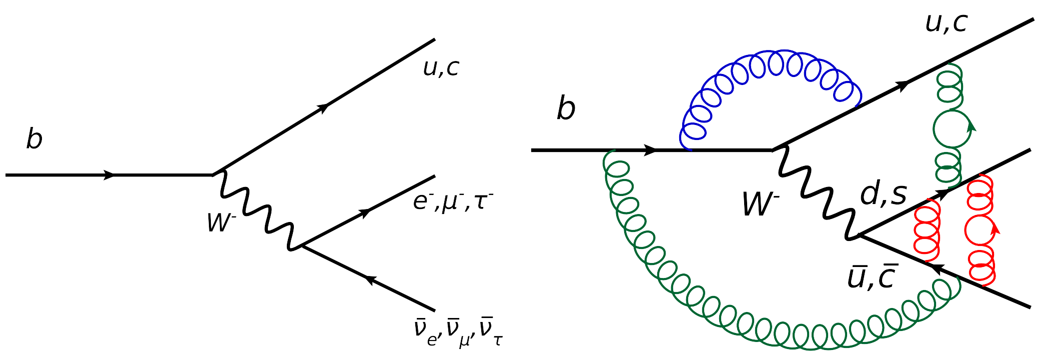
- Perturbative QCD corrections will turn out to be very important, because $\alpha_s(m_c)$ is quite large.

- In the above expressions we neglected, e.g., annihilation decays like $D^+ \to l^+ \nu_l$, which have very small branching ratios [9] (the corresponding Feynman diagrams have the same topology as the decay $B^- \to \tau^- \bar{\nu}_\tau$, that was mentioned earlier). In the case of D_s^+ meson the branching ratio into $\tau^+ \nu_\tau$ will, however, be sizable [9] and has to be taken into account.

$$\mathrm{Br}(D_s^+ \to \tau^+ \nu_\tau) = (5.43 \pm 0.31)\% \,. \tag{79}$$

In the framework of the HQE the non-spectator effects will turn out to be suppressed by $1/m_c$ and since m_c is not very large, the suppression is also not expected to be very pronounced. This will change in the case of B-mesons. Because of the larger value of the b-quark mass, one expects a better description of the meson decay in terms of the simple b-quark decay.

4.5 Bottom-quark decay



Calculating the total inclusive decay rate of a b-quark we get

$$\Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 c_{3,b} \,, \tag{80}$$

with

$$
\begin{aligned}
c_{3,b} = \quad & \left\{ g\left(\frac{m_c}{m_b},\frac{m_e}{m_b}\right) + g\left(\frac{m_c}{m_b},\frac{m_\mu}{m_b}\right) + g\left(\frac{m_c}{m_b},\frac{m_\tau}{m_b}\right) \right. \\
& \quad + N_c|V_{ud}|^2 h\left(\frac{m_c}{m_b},\frac{m_u}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{us}|^2 h\left(\frac{m_c}{m_b},\frac{m_u}{m_b},\frac{m_s}{m_b}\right) \\
& \quad \left. + N_c|V_{cd}|^2 h\left(\frac{m_c}{m_b},\frac{m_c}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{cs}|^2 h\left(\frac{m_c}{m_b},\frac{m_c}{m_b},\frac{m_s}{m_b}\right) \right\} \\
& + \left|\frac{V_{ub}}{V_{cb}}\right|^2 \left\{ g\left(\frac{m_u}{m_b},\frac{m_e}{m_b}\right) + g\left(\frac{m_u}{m_b},\frac{m_\mu}{m_b}\right) + g\left(\frac{m_u}{m_b},\frac{m_\tau}{m_b}\right) \right. \\
& \quad + N_c|V_{ud}|^2 h\left(\frac{m_u}{m_b},\frac{m_u}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{us}|^2 h\left(\frac{m_u}{m_b},\frac{m_u}{m_b},\frac{m_s}{m_b}\right) \\
& \quad \left. + N_c|V_{cd}|^2 h\left(\frac{m_u}{m_b},\frac{m_c}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{cs}|^2 h\left(\frac{m_u}{m_b},\frac{m_c}{m_b},\frac{m_s}{m_b}\right) \right\} .
\end{aligned}
\tag{81}
$$

In this formula penguin induced decays have been neglected, they will enhance the decay rate by several per cent, see [20]. More important will, however, be the QCD corrections. To proceed further we can neglect the masses of all final state particles, except for the charm-quark and for the tau lepton. In addition we can neglect the contributions proportional to $|V_{ub}|^2$ since $|V_{ub}/V_{cb}|^2 \approx 0.01$. Using further $|V_{ud}|^2 + |V_{us}|^2 \approx 1 \approx |V_{cd}|^2 + |V_{cs}|^2$, we get the following simplified formula

$$
c_{3,b} = \left[(N_c+2) f\left(\frac{m_c}{m_b}\right) + g\left(\frac{m_c}{m_b},\frac{m_\tau}{m_b}\right) + N_c g\left(\frac{m_c}{m_b},\frac{m_c}{m_b}\right) \right] . \tag{82}
$$

If we have charm quarks in the final states, then the phase space functions show a huge dependence on the numerical value of the charm quark mass (values taken from [20])

$$
f\left(\frac{m_c}{m_b}\right) = \left\{ \begin{array}{l} 0.484 \\ 0.518 \\ 0.666 \end{array} \right. \quad \text{for} \left\{ \begin{array}{llll} m_c^{\rm Pole} & = 1.471 \ \text{GeV}, & m_b^{\rm Pole} & = 4.650 \ \text{GeV} \\ \bar{m}_c(\bar{m}_c) & = 1.277 \ \text{GeV}, & \bar{m}_b(\bar{m}_b) & = 4.248 \ \text{GeV} \\ \bar{m}_c(\bar{m}_b) & = 0.997 \ \text{GeV}, & \bar{m}_b(\bar{m}_b) & = 4.248 \ \text{GeV} \end{array} \right. . \tag{83}
$$

The big spread in the values for the space functions clearly shows again that the definition of the quark mass is a critical issue for a precise determination of lifetimes. The value for the pole quark mass is only shown to visualise the strong mass dependence. As discussed above short-distance masses like the $\overline{\rm MS}$-mass are theoretically better suited. Later on we will argue further for using $\bar{m}_c(\bar{m}_b)$ and $\bar{m}_b(\bar{m}_b)$ - so both masses at the scale m_b -, which was suggested in [81], in order to sum up large logarithms of the form $\alpha_s^n (m_c/m_b)^2 \log^n (m_c/m_b)^2$ to all orders. Thus only the result using $\bar{m}_c(\bar{m}_b)$ and $\bar{m}_b(\bar{m}_b)$ should be considered as the theory prediction, while the additional numbers are just given for completeness. The phase space function for two identical particles in the final states reads [82, 83, 84, 85] (see [86] for the general case of three different masses)

$$g(x) = \sqrt{1-4x^2}\left(1 - 14x^2 - 2x^4 - 12x^6\right) + 24x^4\left(1-x^4\right)\log\frac{1+\sqrt{1-4x^2}}{1-\sqrt{1-4x^2}}\,, \tag{84}$$

with $x = m_c/m_b$. Thus we get in total for all the phase space contributions

$$c_{3,b} = \left\{ \begin{array}{l} 9 \\ 2.97 \\ 3.25 \\ 4.66 \end{array} \right. \quad \text{for} \quad \left\{ \begin{array}{l} m_c = 0, \\ m_c^{\rm Pole}, m_b^{\rm Pole} \\ \bar{m}_c(\bar{m}_c), \bar{m}_b(\bar{m}_b) \\ \bar{m}_c(\bar{m}_b), \bar{m}_b(\bar{m}_b) \end{array} \right. . \tag{85}$$

The phase space effects are now quite dramatic. For the total b-quark lifetime we predict (with $V_{cb} = 0.04151^{+0.00056}_{-0.00115}$ from [16], for similar results see [17].)

$$\tau_b \;=\; 2.60\ \text{ps} \qquad \text{for}\ \ \bar{m}_c(\bar{m}_b), \quad \bar{m}_b(\bar{m}_b) \quad . \tag{86}$$

This number is about 70% larger than the experimental number for the B-meson lifetimes. There are in principle two sources for that discrepancy: first we neglected several CKM-suppressed decays, which are however not phase space suppressed as well as penguin decays. An inclusion of these decays will enhance the total decay rate roughly by about 10% and thus reduce the lifetime prediction by about 10%. Second, there are large QCD effects, that will be discussed in the next subsection; including them will bring our theory prediction very close to the experimental number. For completeness we show also the lifetime predictions, for different (theoretically less motivated) values of the quark masses.

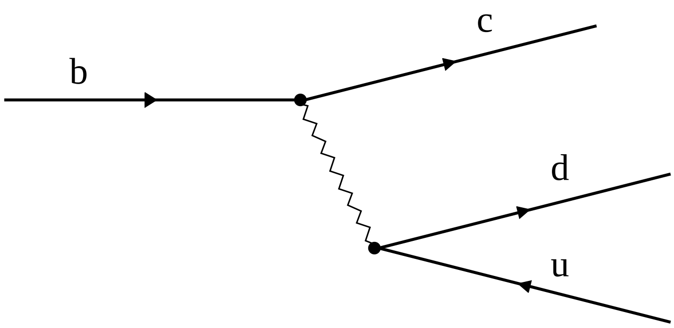
$$\tau_b \;=\; \left\{ \begin{array}{l} 0.90\ \text{ps} \\ 1.42\ \text{ps} \\ 2.59\ \text{ps} \\ 3.72\ \text{ps} \end{array} \right. \quad \text{for} \left\{ \begin{array}{ll} m_c = 0, & m_b^{\rm Pole} \\ m_c = 0, & \bar{m}_b(\bar{m}_b) \\ m_c^{\rm Pole}, & m_b^{\rm Pole} \\ \bar{m}_c(\bar{m}_c), & \bar{m}_b(\bar{m}_b) \end{array} \right. . \tag{87}$$

By accident a neglect of the charm quark mass can lead to predictions that are very close to experiment. As argued above, only the value in Eq.(86) should be considered as the theory prediction for the b-quark lifetime and not the ones in Eq.(87). Next we introduce the missing, but necessary concepts for making reliable predictions for the lifetimes of heavy hadrons.

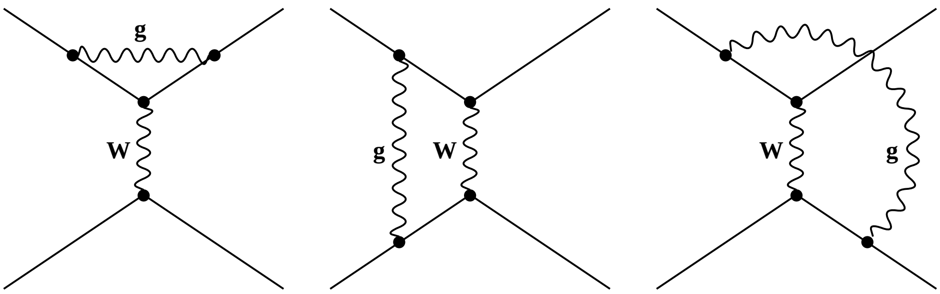
5 Weak decays II - The effective Hamiltonian

5.1 Motivation

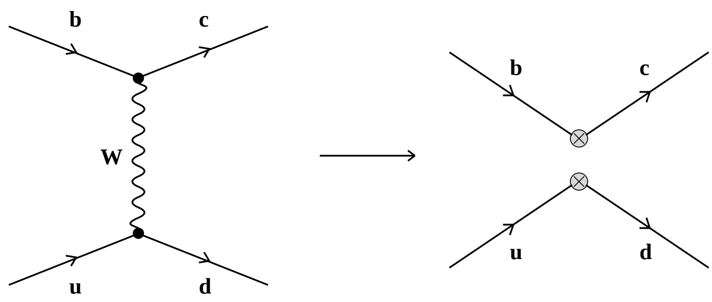
Weak decays are dominantly triggered by the exchange of heavy W-bosons. The decay $b \to c + W^- \to c + \bar{u} + d$ is described by the following Feynman diagram.



In this problem two scales arises, the mass of the W-boson ($\approx$ 80 GeV) and the mass of the b-quark ($\approx$ 5 GeV). If one includes now perturbative QCD corrections



one finds that in the calculation big logarithms arise. As a net result we do not get a Taylor expansion in α_s but an expansion in $\alpha_s \ln\left(\frac{m_b^2}{M_W^2}\right) \approx 6\alpha_s$ which clearly spoils our perturbative approach.
Using the fact that the particles triggering the weak decay are much heavier than the b-quark ($m_W \gg m_b$) one can integrate them out by performing an operator product expansion (OPE I), see, e.g., [87] for a nice introduction, as well as [88, 89, 90]. Schematically one contracts the W-propagator to a point



$$\frac{ig_2 V_{cb}^*}{2\sqrt{2}} \frac{1}{k^2 - M_W^2} \frac{ig_2 V_{ud}}{2\sqrt{2}} \approx \left(\frac{g_2}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} V_{CKM} =: \frac{G_F}{\sqrt{2}} V_{CKM} \tag{88}$$

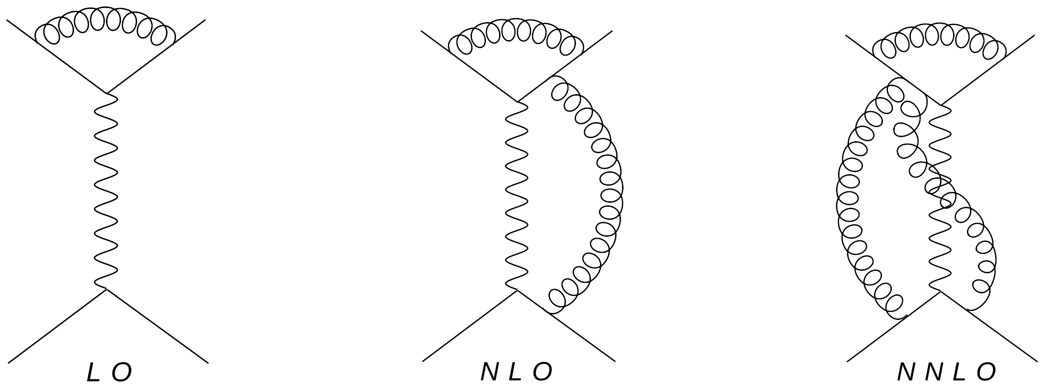
One is then left with the **Fermi-theory** of the weak interaction. Corrections are of the order of $k^2/M_W^2 \approx m_b^2/M_W^2 \approx 3.6 \cdot 10^{-3}$. Including also QCD-effects we will arrive at the **effective weak Hamiltonian.**

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3}^{6} C_j Q_j \right] . \tag{89}$$

Without QCD corrections only the operator Q_2 arises and the *Wilson coefficient* $C_2 = 1$. The operator Q_2 has a current-current structure:

$$\begin{aligned} Q_2 &= (\bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha) \times \left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) u_\beta\right) , \\ &=: (\bar{c}_\alpha b_\alpha)_{V-A} \times \left(\bar{d}_\beta u_\beta\right)_{V-A} , \end{aligned} \tag{90}$$

where α and β denote colour indices. The Vs describe different combinations of CKM elements. With the inclusion of QCD one gets additional operators. Q_1 has the same quark structure as Q_2, but it has a different colour structure, $Q_3, .., Q_6$ arise from penguin decays. Due to renormalisation all Wilson coefficients become scale dependent functions. Numerically C_2 is of order one, C_1 of order 20$\%$ and the penguin coefficients are below 5%, with the exception of C_8, the coefficient of the chromomagnetic operator.
The effective Hamiltonian in Eq.(89) was already obtained in 1974 in LO-QCD [91], a nice review of the NLO-results is given in [88]. Currently also NNLO results are available [92]. For the LO Wilson coefficients 1-loop diagrams have to be calculated, for NLO 2-loop diagrams and for NNLO 3-loop diagrams:



Before introducing the concept of the effective Hamiltonian in detail, one might ask: Why do we not simply calculate in the full standard model?
There are several reasons for that:

1. In the standard model large logarithms arise, when one includes virtual corrections due to the strong interaction, which are not negligible. In the end one will not have an expansion in the strong coupling α_s ($\alpha_s(m_b) \approx 0.2$) but an expansion in $\ln(m_b/M_W)^2\alpha_s \approx 1$. So the convergence of the expansion is not ensured. The general structure of the perturbative expansion reads

$$\begin{array}{llll} 1 & - & - & - \\ \alpha_s \ln & \alpha_s & - & - \\ \alpha_s^2 \ln^2 & \alpha_s^2 \ln & \alpha_s^2 & - \\ \alpha_s^3 \ln^3 & \alpha_s^3 \ln^2 & \alpha_s^3 \ln & \alpha_s^3 \\ \ldots & \ldots & \ldots & \ldots \end{array} \tag{91}$$

   Calculating within the standard model corresponds to calculate line by line. Calculating within the framework of the effective Hamiltonian corresponds to calculate row by row and summing up the large logarithms to all orders. An example for such a summation is given by the solution of the renormalisation group equations for the strong coupling, which is discussed in detail in the appendix.

2. In the decay of a meson besides perturbatively calculable short-distance QCD effects (e.g. the scale M_W) also long-distance strong interaction effect arise (e.g. the scale Λ_{QCD}), these are of non-perturbative origin. The effective Hamiltonian allows a well-defined separation of scales. The high energy physics is described by the Wilson coefficients, they can be calculated in perturbation theory. The low energy physics is described by the matrix elements of the operators $Q_1, .., Q_6$. Here one needs non-perturbative methods like lattice QCD or sum rules.

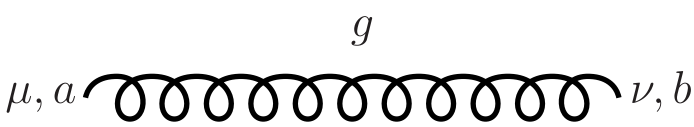
3. Calculations within the framework of the effective Hamiltonian are technically simpler, because fewer propagators appear in the formulae.

5.2 The effective Hamiltonian in LO-QCD

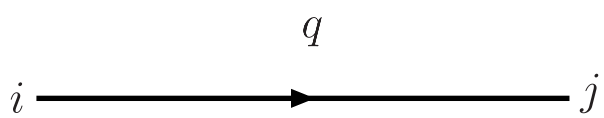
In the following we describe the derivation of the LO effective Hamiltonian. We closely follow the Les Houches Lectures of Andrzej Buras [87].

5.2.1 Basics - Feynman rules

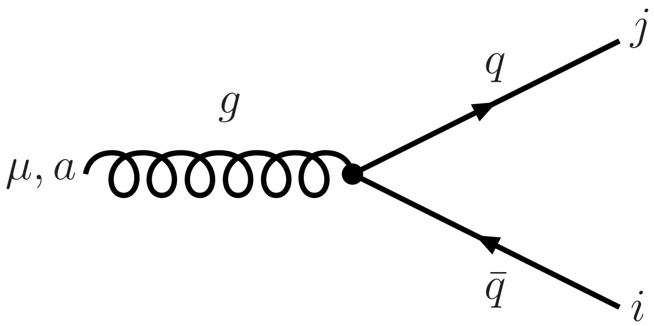
We are using the following set of Feynman rules (corresponding to Buras and Itzykson-Zuber; but different from e.g. Muta).



$-i\delta^{ab}\frac{g_{\mu\nu}}{p^2}$



$i\delta^{ij}\frac{\not{p}+m}{p^2-m^2}$



$ig\gamma^{\mu}\left(T^a\right)_{ij}$

$$\gamma \to f\bar{f} \qquad -ie\gamma_\mu Q_f \tag{92}$$

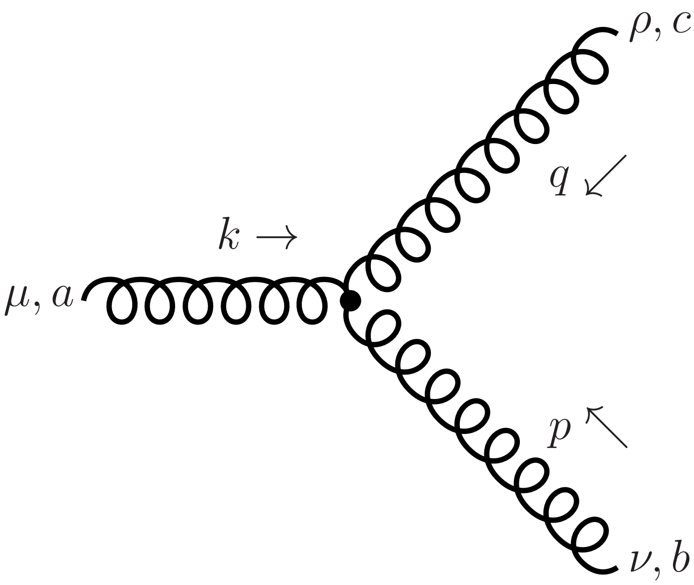
$$Z \to f\bar{f} \qquad i\frac{g_2}{2\cos\theta_W}\gamma_\mu(v_f - a_f\gamma_\mu) \tag{93}$$

$$W \to t\bar{d} \qquad i\frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)V_{td} \tag{94}$$

p denotes the momentum of the propagating particle, its direction is from the left to the right. The indices i, j denote colour ($i, j = 1, 2, 3$), the indices a, b, c denote the different gluons ($a, b, c = 1, \ldots, 8$) and μ, ν and ρ are the usual Dirac indices. g is the strong coupling and the T^as in the quark gluon vertex are the SU(3) matrices.

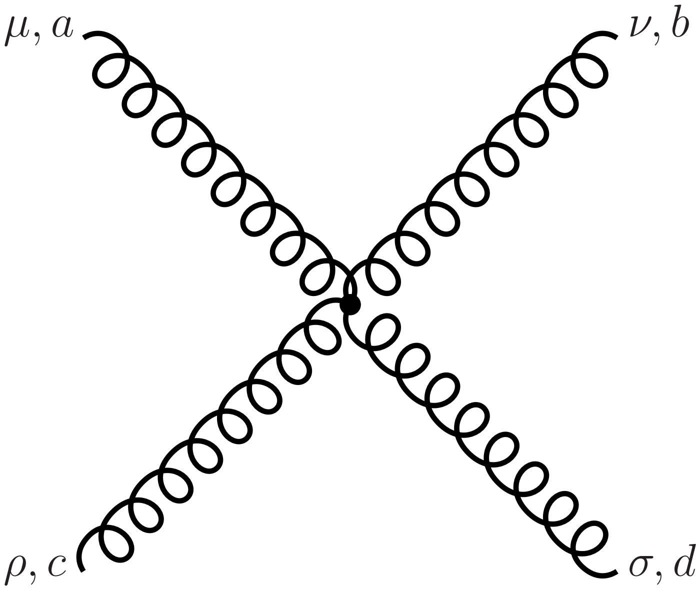
Compared to QED we have some completely new contributions. Because of SU(3) being a non-abelian group we get new contributions in the field strength tensor when constructing a SU(3) gauge theory. This new contribution results in a self-interaction of the gluon; we get the following new fundamental vertices:

3-gluon-vertex



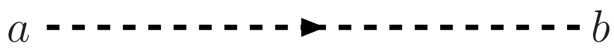
$$g f^{abc} \left[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu} \right] \tag{95}$$

4-gluon vertex

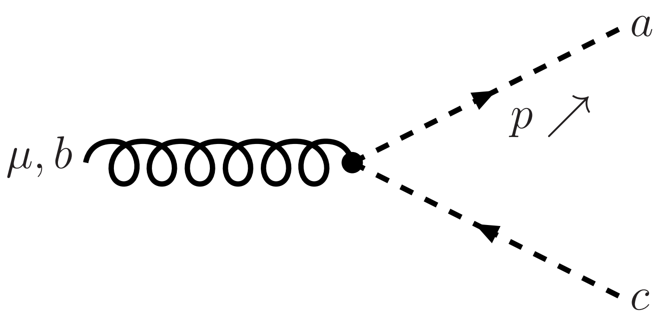


$$-ig^2 \left[f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} \right) \right] \tag{96}$$

with the antisymmetric SU(3) structure constants f^{abc}.
When trying to quantise a non-abelian gauge field theory the freedom of choosing arbitrary gauges results in problems which can be circumvented by choosing a particular gauge. Part of the term in the Lagrangian, which fixes the gauge can be rewritten in a form that corresponds to virtual particles, the so-called *Faddeev-Popov-ghosts* [93]. These particles have no physical meaning, it is just a calculational trick to fix the gauge. Although being spin-0 particles, their properties are governed by the Fermi-Dirac statistics. The following Feynman rules hold for the ghost fields:



$$-i\delta^{ab}\frac{1}{p^2}$$



$$-gf^{abc}p^{\mu}$$

Note that we used a convention where vertices have upper Dirac indices and the gluon propagator has lower Dirac indices. Finally we have Feynman rules for virtual particle loops.

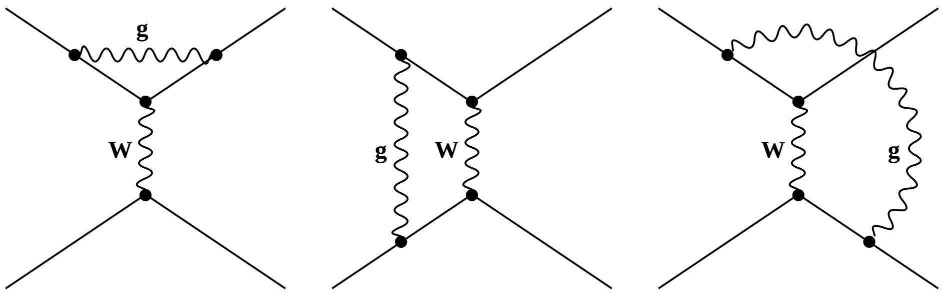
$$\begin{aligned} \text{for each loop} &: \int \frac{d^4k}{(2\pi)^4} && (97) \\ \text{fermion loop} &: -1 \ \text{ and Dirac-trace} && (98) \\ \text{ghost loop} &: -1 && (99) \end{aligned}$$

An additional rule for pure gauge loops is the symmetry factor 1/2.
Now we have all Feynman rules at hand which we need to perform perturbative calculations.

5.2.2 The initial conditions

In order to determine the Wilson coefficients C_1 and C_2 at the scale M_W (initial condition), we calculate the tree level decay $b \to c\bar{u}d$ both in the SM and in the effective theory, where $C_{1,2}(M_W)$ appear as unknown parameter. Equating the two results will give an expression for the Wilson coefficients.

- SM amplitude:
  At 1 loop the following diagrams (and their symmetric counterparts). are contributing



Calculating them (under the assumption $m_i = 0; p^2 < 0$), we get the full amplitude

$$A^{(0)}_{full} = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} \qquad \left[\left(1 + 2C_f \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) \right) \langle Q_2 \rangle_{tree} \right.$$
$$+ \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} \langle Q_2 \rangle_{tree}$$
$$\left. -3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} \langle Q_1 \rangle_{tree} \right] . \qquad (100)$$

Remarks:

- $^{(0)}$ denotes the unrenormalised amplitude. The singularities could be removed by quark field renormalisation; but they will cancel anyway in the determination of the Wilson coefficients.
- μ is an unphysical renormalisaton scale, which had to be introduced because of dimensional reasons when doing dimensional regularisation. In principle it can be chosen arbitrarily, in practice it will be chosen in such a way to not produce artificially large logarithms.

– Now two operators appear

$$Q_2 = (\bar{c}_\alpha b_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A} \, , \tag{101}$$
$$Q_1 = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{d}_\beta u_\alpha)_{V-A} \, . \tag{102}$$

Without QCD only the operator Q_2 arises. Taking colour effects into accounts and using in particular

$$T^a_{\alpha\beta} T^a_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta} \, , \tag{103}$$

the second operator Q_1 arises. Finally we have the colour factor $C_f = \frac{N^2-1}{2N} = \frac{4}{3}$.

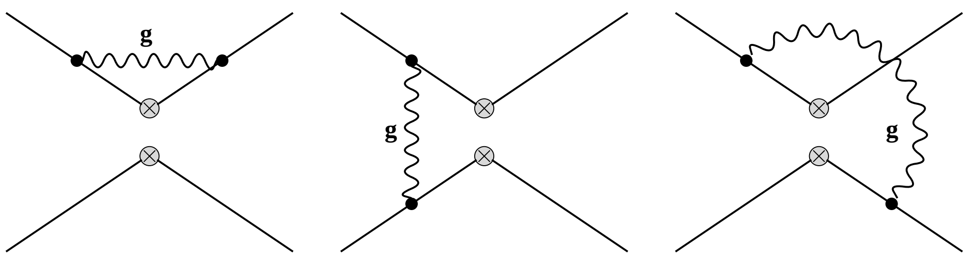
– Choosing all external momenta to be equal and all quark masses to be zero, does not change the final result for the Wilson coefficients, but it considerably simplifies the calculation.

– Constant terms of $\mathcal{O}(\alpha_s)$ have been discarded, while logarithmic terms have been kept; this corresponds to the leading log approximation.

– "Amplitude" in the above sense is an amputated Greens function (i.e. multiplied by i). Gluonic self energy corrections are not included.

Exercise: Calculate A_{full} in LO-QCD

- Effective theory contribution:
In the effective theory we study the 1-loop corrections to the insertions of the operators Q_1 and Q_2 in the following Feynman diagrams.



Calculating all these diagrams (and the symmetric ones) we get the QCD corrections to Q_1 and Q_2 in the effective theory:

$$\begin{aligned}
\langle Q_1\rangle^{(0)} = \quad & \left(1 + 2C_f \frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)\right)\langle Q_1\rangle_{tree} \\
& + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)\langle Q_1\rangle_{tree} \\
& - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)\langle Q_2\rangle_{tree} \ , \qquad (104)
\end{aligned}$$

$$\begin{aligned}
\langle Q_2\rangle^{(0)} = \quad & \left(1 + 2C_f \frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)\right)\langle Q_2\rangle_{tree} \\
& + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)\langle Q_2\rangle_{tree} \\
& - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\right)\langle Q_1\rangle_{tree} \ . \qquad (105)
\end{aligned}$$

<u>Remarks</u>:

- Now we have additional divergencies; our effective theory is actually non-renormalisable. Working to finite order in perturbation theory we can, however, renormalise it with additional renormalisation constants, which we are introducing now.
  The first divergencies in the above expressions cancel in the matching; alternatively one could do a field renormalisation. The new divergencies appearing in the second and third line require an additional renormalisation, the operator renormalisation:

  $$Q_i^{(0)} = \hat{Z}_{ij} Q_j \ . \qquad (106)$$

  $\hat{Z}_{ij}$ is a 2×2 matrix.
  For the amputated Greens functions we get

  $$\langle Q_i\rangle^{(0)} = Z_q^{-2}\hat{Z}_{ij}\langle Q_j\rangle \ . \qquad (107)$$

  The quark field renormalisation Z_q removes the first divergence and the operator renormalisation $\hat{Z}_{ij}$ removes the second divergencies. We directly can read off the operator renormalisation matrix

  $$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\begin{pmatrix} \frac{3}{N} & -3 \\ -3 & \frac{3}{N} \end{pmatrix} \ . \qquad (108)$$

Thus we get for the renormalised operators

$$\begin{aligned}
\langle Q_1\rangle = \quad & \left(1 + 2C_f \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right) \langle Q_1\rangle_{tree} && (109)\\
& + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \langle Q_1\rangle_{tree} - 3\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \langle Q_2\rangle_{tree} \, , \\
\langle Q_2\rangle = \quad & \left(1 + 2C_f \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right) \langle Q_2\rangle_{tree} && (110)\\
& + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \langle Q_2\rangle_{tree} - 3\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \langle Q_1\rangle_{tree} \, .
\end{aligned}$$

Exercise: Calculate A_{full} in LO-QCD

5.2.3 Matching:

Finally we do the **matching** of our calculations with the standard model and the effective theory

$$A_{full} = A_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[C_1 \langle Q_1\rangle + C_2 \langle Q_2\rangle\right] \, . \tag{111}$$

Comparing our results for A_{full} with the ones for $\langle Q_{1,2}\rangle$ - be aware to treat the divergencies in the same manner in the full and the effective theory! - we obtain the Wilson coefficients

$$C_1 = 0 - 3\frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2} \, , \tag{112}$$

$$C_2 = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2} \, . \tag{113}$$

Remarks:

- Switching off QCD, i.e. setting the strong coupling to zero, we get $C_2 = 1$ and $C_1 = 0$, as expected.
- A different look to the renormalisation:
  Renormalisation can also be done with the usual counter term method. We start with the effective Hamiltonian and consider the Wilson coefficients to be coupling constants. Fields and couplings are renormalised according to

$$q^{(0)} = Z_q^{1/2} q, \qquad C_i^{(0)} = Z_{ij}^c C_j \, . \tag{114}$$

Inserting this in the effective Hamiltonian we get

$$\begin{aligned}\mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}} V_{CKM} C_i^{(0)} Q_i[q^{(0)}] \\ &= \frac{G_F}{\sqrt{2}} V_{CKM} Z_{ij}^c C_j Z_q^2 Q_i[q] \\ &= \frac{G_F}{\sqrt{2}} V_{CKM} \left\{ C_i Q_i[q] + \left(Z_{ij}^c Z_q^2 - \delta_{ij} \right) C_j Q_i[q] \right\} \ . \end{aligned} \tag{115}$$

In the first term of the last expression everything is expressed in terms of renormalised couplings and renormalised quark fields, the second term is the counter term.
Using this we get for the renormalised effective amplitude

$$\begin{aligned} A_{eff} &= \frac{G_F}{\sqrt{2}} V_{CKM} Z_{ij}^c Z_q^2 C_j \langle Q_i[q] \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{CKM} Z_{ij}^c Z_q^2 C_j \langle Q_i \rangle^{(0)} \ . \end{aligned} \tag{116}$$

On the other hand we can use also our operator renormalisation to get

$$\begin{aligned} A_{eff} &= \frac{G_F}{\sqrt{2}} V_{CKM} C_j \langle Q_j \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{CKM} C_j Z_q^2 Z_j i^{-1} \langle Q_i \rangle^{(0)} \ . \end{aligned} \tag{117}$$

Comparing the two expressions we get

$$Z_{ij}^c = Z_{ji}^{-1} \ . \tag{118}$$

- Operator Mixing:
  We have seen that the operators Q_1 and Q_2 mix under renormalisation, i.e. $\hat{Z}$ is a non-diagonal matrix. That means the renormalisation of Q_2 requires a counter term proportional to Q_2 and one proportional to Q_1.
  We can diagonalise the $Q_1 - Q_2$ system via

$$Q_\pm = \frac{Q_2 \pm Q_1}{2} \ , \tag{119}$$

$$C_\pm = C_2 \pm C_1 \ . \tag{120}$$

  The we get for the renormalisation

$$Q_\pm^{(0)} = Z_\pm Q_\pm \ , \tag{121}$$

with

$$Z_{\pm} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\left(\mp 3\frac{N \mp 1}{N}\right) . \qquad (122)$$

Now the amplitude reads

$$A = \frac{G_f}{\sqrt{2}} V_{cb} V_{ud}^* \left(C_+(\mu)\langle Q_+(\mu)\rangle + C_-(\mu)\langle Q_-(\mu)\rangle\right) , \qquad (123)$$

with

$$\begin{aligned} \langle Q_{\pm}(\mu)\rangle &= \left(1 + 2C_F\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2}\right) Q_{\pm,tree} \\ &\quad + \left(\frac{3}{N} \mp 3\right)\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2} Q_{\pm,tree} , \qquad (124) \\ C_{\pm}(\mu) &= 1 + \left(\frac{3}{N} \mp 3\right)\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2} . \qquad (125) \end{aligned}$$

Now both Wilson coefficients have the value 1 without QCD.

- Factorisation of SD and LD:
  We just have seen one of the most important features of the OPE, the separation of SD and LD contributions:

$$\left(1 + \alpha_s G \ln\frac{M_W^2}{-p^2}\right) = \left(1 + \alpha_s G \ln\frac{M_W^2}{\mu^2}\right)\left(1 + \alpha_s G \ln\frac{\mu^2}{-p^2}\right) . \qquad (126)$$

  The large logarithm at the l.h.s. arises in the full theory, the first term on the r.h.s. corresponds to the Wilson coefficient and the second term to the matrix element of the operator.
  The splitting of the logarithm

$$\ln\frac{M_W^2}{-p^2} = \ln\frac{M_W^2}{\mu^2} + \ln\frac{\mu^2}{-p^2} \qquad (127)$$

  corresponds to a splitting of the momentum integration in the following form

$$\int_{-p^2}^{M_W^2}\frac{dk^2}{k^2} = \int_{-p^2}^{\mu^2}\frac{dk^2}{k^2} + \int_{\mu^2}^{M_W^2}\frac{dk^2}{k^2} . \qquad (128)$$

  This means the matrix elements contains the low scale physics ($[-p^2, \mu^2]$) and the Wilson coefficients contains the high scale physics ($[\mu^2, M_W^2]$). The renormalisation scale μ acts as a separation scale between SD and LD.

- IR divergencies cancel in the matching, if they are properly renormalised.

5.2.4 The renormalisation group evolution

We just obtained the following result

$$C_+(\mu) \quad = \quad 1 + \left(\frac{3}{N} + 3 \right) \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2} \ . \tag{129}$$

At a scale of $\mu = 4.8$ this gives $C_+(4.8 \text{ GeV}) = 1 + 0.36$, which is already a very sizable correction and at lower scales these corrections will exceed one and the perturbative approach breaks down.
Within the framework of the renormalisation group we will be able to sum up these logarithms to all orders.
Remember (or have a look in the appendix): For the running coupling we obtained

$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{1 - \beta_0 \frac{\alpha_s(M_Z)}{2\pi} \ln \left(\frac{M_Z}{\mu} \right)} \ , \tag{130}$$

with $\beta_0 = (11N - 2f)/3$**. Expanding the above formula we get**

$$\alpha_s(\mu) = \alpha_s(M_Z) \left[1 - \sum_{n=1}^{\infty} \left(\beta_0 \frac{\alpha_s(M_Z)}{2\pi} \ln \left(\frac{M_Z}{\mu} \right) \right)^n \right] \ , \tag{131}$$

which shows that the renormalisation group sums up the large logs automatically to all orders.
Now we apply the same framework to the effective Hamiltonian. The starting point is the fact that the unrenormalised quantities do not depend on the renormalisation scale and the relation between renormalised and unrenormalised quantities:

$$Q_\pm^{(0)} = Z_\pm Q_\pm \Rightarrow C_\pm = Z_\pm C_\pm^{(0)} \ . \tag{132}$$

From that we get

$$\begin{aligned} \frac{dC_\pm(\mu)}{d \ln \mu} &= \frac{dZ_\pm(\mu)}{d \ln \mu} C_\pm^{(0)} \\ &= \frac{1}{Z_\pm} \frac{dZ_\pm(\mu)}{d \ln \mu} Z_\pm C_\pm^{(0)} \\ &=: \gamma_\pm C_\pm \ , \end{aligned} \tag{133}$$

with the anomalous dimension

$$\begin{aligned} \gamma_\pm &= \frac{1}{Z_\pm} \frac{dZ_\pm(\mu)}{d \ln \mu} \\ &= \frac{1}{Z_\pm} \frac{dZ_\pm(\mu)}{dg} \frac{dg}{d \ln \mu} \ . \end{aligned} \tag{134}$$

$dg/d\ln\mu$ is simply the running of the strong coupling. There the following formula holds

$$\frac{dg}{d\ln\mu} = -\epsilon g - \beta_0 \frac{g^3}{(4\pi)^2} + \mathcal{O}(g^5)\ , \tag{135}$$

with $\beta_0 = (11N - 2f)/3$. Using in addition

$$\begin{aligned} Z_\pm &= 1 + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\left(\mp 3\frac{N \mp 1}{N}\right) \\ &= 1 - \frac{1}{2}\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\gamma_\pm^{(0)}\ , \end{aligned} \tag{136}$$

with

$$\gamma_\pm^{(0)} = \pm 6\frac{N \mp 1}{N} \tag{137}$$

we get for the anomalous dimension

$$\gamma_\pm = \frac{1}{Z_\pm}\frac{dZ_\pm(\mu)}{dg}\frac{dg}{d\ln\mu} \tag{138}$$

$$= \left(1 + \frac{1}{2}\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\gamma_\pm^{(0)}\right)\cdot\left(-\frac{1}{2}\frac{2g}{(4\pi)^2}\frac{1}{\epsilon}\gamma_\pm^{(0)}\right)\cdot\left(-\epsilon g - \beta_0\frac{g^3}{(4\pi)^2}\right) \tag{139}$$

$$= \frac{g^2}{(4\pi)^2}\gamma_\pm^{(0)} = \frac{\alpha_s}{4\pi}\gamma_\pm^{(0)}\ . \tag{140}$$

Now we can solve the differential equation for the μ-evolution of the Wilson coefficients

$$\begin{aligned} \frac{dC_\pm(\mu)}{d\ln\mu} &=: \gamma_\pm(g) C_\pm(\mu)\ , \\ \frac{dC_\pm(\mu)}{dg}\frac{dg}{d\ln\mu} &=: \gamma_\pm(g) C_\pm(\mu)\ , \\ \frac{dC_\pm(\mu)}{dg}\beta(g) &=: \gamma_\pm(g) C_\pm(\mu)\ , \\ \frac{dC_\pm(\mu)}{C_\pm(\mu)} &=: \frac{\gamma_\pm(g)}{\beta(g)}dg\ , \\ \int\limits_{\mu_0}^{\mu}\frac{dC_\pm(\mu)}{C_\pm(\mu)} &=: \int\limits_{g(\mu_0)}^{g(\mu)}\frac{\gamma_\pm(g)}{\beta(g)}dg\ , \\ \ln\frac{C_\pm(\mu)}{C_\pm(\mu_0)} &=: \int\limits_{g(\mu_0)}^{g(\mu)}\frac{\gamma_\pm(g)}{\beta(g)}dg\ , \\ C_\pm(\mu) &= C_\pm(\mu_0) e^{\int\limits_{g(\mu_0)}^{g(\mu)}\frac{\gamma_\pm(g)}{\beta(g)}dg}\ . \end{aligned} \tag{141}$$

This is the formal solution of the renormalisation group evolution of the Wilson coefficient. Now we insert our LO results for β and γ and write down an analytical formula the Wilson coefficients.

1. Step 1:
$$\frac{\gamma_\pm(g)}{\beta(g)} = \frac{\frac{\alpha_s}{4\pi}\gamma_\pm^{(0)}}{-\beta_0 \frac{g^3}{(4\pi)^2}} = -\frac{1}{g}\frac{\gamma_\pm^{(0)}}{\beta_0} .$$

2. Step 2:
$$\int\limits_{g(\mu_0)}^{g(\mu)} \frac{\gamma_\pm(g)}{\beta(g)} dg = -\frac{\gamma_\pm^{(0)}}{\beta_0} \int\limits_{g(\mu_0)}^{g(\mu)} \frac{1}{g} dg = -\frac{\gamma_\pm^{(0)}}{\beta_0} \ln \frac{g(\mu)}{g(\mu_0)} .$$

3. Step 3:
$$e^{\int\limits_{g(\mu_0)}^{g(\mu)} \frac{\gamma_\pm(g)}{\beta(g)} dg} = \left[\frac{g(\mu)}{g(\mu_0)}\right]^{-\frac{\gamma_\pm^{(0)}}{\beta_0}} = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\frac{\gamma_\pm^{(0)}}{2\beta_0}} .$$

So now we arrived at our final formula for scale dependence of C_1 and C_2:

$$\begin{aligned} C_\pm(\mu) &= \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\frac{\gamma_\pm^{(0)}}{2\beta_0}} C_\pm(\mu_0) \\ &= \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\frac{\gamma_\pm^{(0)}}{2\beta_0}} \left[1 + \left(\frac{3}{N} \mp 3\right) \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu_0^2}\right] . \end{aligned} \tag{142}$$

This is the general result for the Wilson coefficients $C_\pm$.
Remarks:

- The first terms sums up potentially large logarithms due to the different scales μ and μ_0; the second term gives the fixed order perturbation theory calculation for the initial condition.

- Expanding in powers of $\alpha_s(\mu_0)$ one gets
$$C_\pm(\mu) = 1 + \frac{\alpha_s(\mu_0)}{4\pi} \begin{pmatrix} -2 \\ 4 \end{pmatrix} \ln \frac{M_W^2}{\mu^2} . \tag{143}$$

  This almost looks like the initial condition alone, except that we now have the strong coupling at the scale μ_0 instead of μ.
  By expanding explicitly in powers of α_s we ”destroy” the summing of the logarithms.

- To make full use of the RGE we take $\mu_0 = M_W$, then the have a small logarithm (here: exactly zero) in the initial condition and the large logarithms $\ln M_W^2 / \mu^2$ are summed up within the RGE:

$$C_\pm(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right]^{-\frac{\gamma_\pm^{(0)}}{2\beta_0}} \cdot 1 \,. \tag{144}$$

For $f = 5$ we get finally at the scale μ_b

$$C_+(\mu_b) = \left[\frac{\alpha_s(\mu_b)}{\alpha_s(M_W)}\right]^{-\frac{6}{23}} , \qquad C_-(\mu_b) = \left[\frac{\alpha_s(\mu_b)}{\alpha_s(M_W)}\right]^{+\frac{12}{23}} \tag{145}$$

or

$$C_1 = \frac{C_+ - C_-}{2} = \frac{1}{2}\left(\left[\frac{\alpha_s(\mu_b)}{\alpha_s(M_W)}\right]^{\frac{6}{23}} - \left[\frac{\alpha_s(\mu_b)}{\alpha_s(M_W)}\right]^{-\frac{12}{23}}\right) , \tag{146}$$

$$C_2 = \frac{C_+ + C_-}{2} = \frac{1}{2}\left(\left[\frac{\alpha_s(\mu_b)}{\alpha_s(M_W)}\right]^{\frac{6}{23}} + \left[\frac{\alpha_s(\mu_b)}{\alpha_s(M_W)}\right]^{-\frac{12}{23}}\right) . \tag{147}$$

Remarks:

- Programming the above formulae and using $\alpha_s = 0.1184$ one obtains for $f = 5$ the following numerical values:

$$\alpha_s(5) = 0.20395 \,, \tag{148}$$

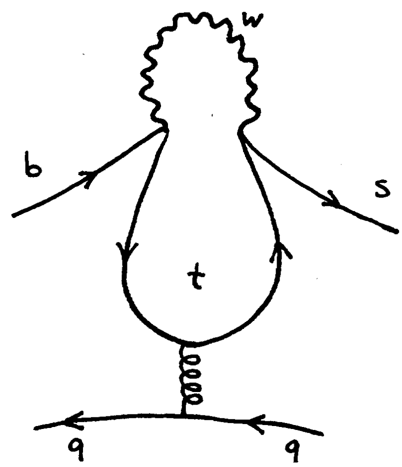
$$C_+(5) = 0.871912 \,, \tag{149}$$

$$C_-(5) = 1.31539 \,, \tag{150}$$

$$C_1(5) = -0.22174 \,, \tag{151}$$

$$C_2(5) = 1.09365 \,. \tag{152}$$

- Besides the current-current operators Q_1 and Q_2 also so-called Penguin operators arise. The QCD-penguin operators $Q_{3,\ldots,6}$ stem from the following diagrams:



Electro-weak penguins (here the gluon is exchange with a photon or a Z-boson) are denoted by $Q_{7,...,10}$; penguins with only an on-shell photon are denoted by $Q_{7\gamma}$ and penguins with an on-shell gluon are denoted by Q_{8g}.

- Final theory remarks:
  - The unphysical renormalisation dependence (μ) cancels up to the calculated order in perturbation theory between the Wilson coefficients and matrix elements of the 4-quark operators.
  - The theoretical error due to missing higher order corrections is thus estimated via a variation of the renormalisation scale; it became convention to use the following range:

$$\frac{m_b}{2} < \mu < 2m_b \; .$$

  - Threshold effects have to be taken into account, when we pass with the renormalisation group evolution the b-quark or c-quark mass scale.

5.3 The effective Hamiltonian in NLO and NNLO-QCD

Why should we bother about calculating higher orders? (2-loops or even 3-loops)

- Renormalisation scale is often the dominant uncertainty - this can be reduced by including higher order corrections.
- For some decays, NLO-effect can be the dominant effect.

6 Weak decays III - Inclusive B-decays

6.1 Inclusive B-decays at LO-QCD

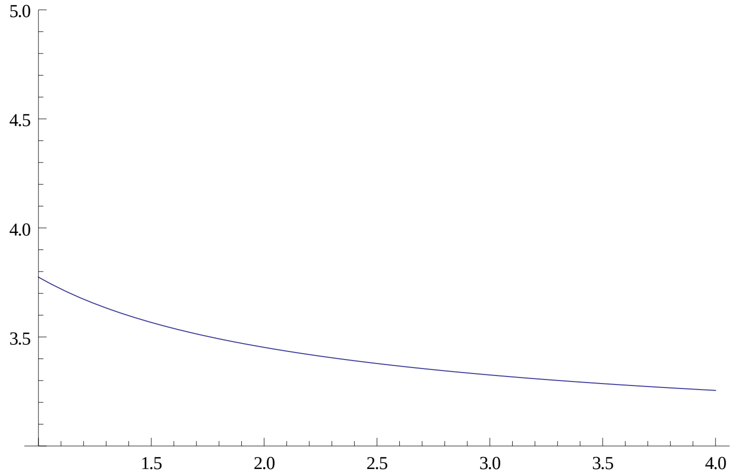
Now we can calculate the free quark decay starting from the effective Hamiltonian instead of the full standard model. If we again neglect penguins, we get in leading logarithmic approximation the same structure as in Eq.(81) and the coefficient c_3 reads now:

$$\begin{aligned} c_{3,b}^{\mathrm{LO-QCD}} &= c_{3,b}^{ce\bar{\nu}_e} + c_{3,b}^{c\mu\bar{\nu}_\mu} + c_{3,b}^{c\bar{u}d} + c_{3,b}^{c\bar{u}s} + c_{3,b}^{c\tau\bar{\nu}_\tau} + c_{3,b}^{c\bar{c}s} + c_{3,b}^{c\bar{c}d} \ldots \\ &= \left[(2+\mathcal{N}_a(\mu))\, f\left(\frac{m_c}{m_b}\right) + g\left(\frac{m_c}{m_b}, \frac{m_\tau}{m_b}\right) + \mathcal{N}_a(\mu) g\left(\frac{m_c}{m_b}, \frac{m_c}{m_b}\right)\right] . \end{aligned} \tag{153}$$

Thus the inclusion of the effective Hamiltonian is equivalent with changing the colour factor $N_c = 3$ - stemming from QCD - into

$$\mathcal{N}_a(\mu) = 3C_1^2(\mu) + 3C_2^2(\mu) + 2C_1(\mu)C_2(\mu) \approx 3.3 \ \ (\mathrm{LO},\ \mu = 4.248 \ \mathrm{GeV}) \ . \tag{154}$$

The dependence of $\mathcal{N}_a(\mu)$ on the renormalisation scale μ is shown in the following graph:



This effect enhances the total decay rate by about 10% and thus brings down (if also the sub-leading decays are included) the prediction for the lifetime of the b-quark to about

$$\tau_b \approx 2.10 \ \mathrm{ps} \qquad \text{for } \ \bar{m}_c(\bar{m}_b), \quad \bar{m}_b(\bar{m}_b) \quad . \tag{155}$$

Exercise: B-decay in LO-QCD with the effective Hamiltonian

6.2 B_{sl} and n_c at NLO-QCD

Going to next-to-leading logarithmic accuracy we have to use the Wilson coefficients of the effective Hamiltonian to NLO accuracy and we have to determine one-loop QCD corrections within the effective theory. These NLO-QCD corrections turned out to be very important for the inclusive b-quark decays. For massless final state quarks the calculation was done in 1991 [94]:

$$c_{3,b} = c_{3,b}^{\mathrm{LO-QCD}} + 8\frac{\alpha_s}{4\pi}\left[\left(\frac{25}{4} - \pi^2\right) + 2\left(C_1^2 + C_2^2\right)\left(\frac{31}{4} - \pi^2\right) - \frac{4}{3}C_1 C_2\left(\frac{7}{4} + \pi^2\right)\right] . \tag{156}$$

The first QCD corrections in Eq.(156) stems from semi-leptonic decays. It can be guessed from the correction to the muon decay in Eq.(63) by decomposing the factor 8 in Eq.(156) as $8 = 3 \cdot C_F \cdot 2$: 3 comes from the three leptons e^-, μ^-, τ^-, C_F is a QCD colour factor and 2 belongs to the correction in Eq.(63). The second and the third term in Eq.(156) stem from non-leptonic decays.
It turned out, however, that effects of the charm quark mass are crucial, see, e.g., the estimate in [95]. NLO-QCD corrections with full mass dependence were determined for $b \to c l^- \bar{\nu}$ already in 1983 [96], for $b \to c \bar{u} d$ in 1994 [97], for $b \to c \bar{c} s$ in 1995 [98], for $b \to$ no charm in 1997 [21] and for $b \to sg$ in 2000 [99, 100]. Since there were several misprints in [98]- leading to IR divergent expressions -, the corresponding calculation was redone in [20] and the numerical result was updated.[11] With the results in [20] we predict (using $\bar{m}_c(\bar{m}_b)$ and $\bar{m}_b(\bar{m}_b)$)

$$c_{3,b} = \begin{cases} 9 & (m_c = 0 = \alpha_s) \\ 5.29 \pm 0.35 & (\mathrm{LO-QCD}) \\ 6.88 \pm 0.74 & (\mathrm{NLO-QCD}) \end{cases} . \tag{157}$$

Comparing this result with Eq.(85) one finds a huge phase space suppression, which reduces the value of $C_{3,b}$ from 9 in the mass less case to about 4.7 when including charm quark mass effect. Switching on in addition QCD effects $c_{3,b}$ is enhanced back to a value of about 6.9. The LO $b \to c$ transitions contribute about 70% to this value, the full NLO-QCD corrections about 24% and the $b \to u$ and penguin contributions about 6% [20].
For the total lifetime we predict thus

$$\tau_b = (1.65 \pm 0.24) \text{ ps} , \tag{158}$$

which is our final number for the lifetime of a free b-quark. This number is now very close to the experimental numbers in Eq.(2.1), unfortunately the uncertainty

[11] The authors of [98] left particle physics and it was not possible to obtain the correct analytic expressions. The numerical results in [98] were, however, correct.

is still quite large. To reduce this, a calculation at the NNL order would be necessary. Such an endeavour seems to be doable nowadays. The dominant Wilson coefficients C_1 and C_2 are known at NNLO accuracy [92] and the two loop corrections in the effective theory have been determined e.g. in [101, 102, 103, 104, 105, 106] for semi-leptonic decays and partly in [107] for non-leptonic decays.
It is amusing to note, that a naive treatment with vanishing charm quark masses and neglecting the sizable QCD-effects, see Eq.(86), yields by accident a similar result as in Eq.(158). The same holds also for the semi leptonic branching ratio, where a naive treatment ($m_c = 0 = \alpha_s$) gives

$$B_{sl} = \frac{\Gamma(b \to c e^- \bar{\nu}_e)}{\Gamma_{\rm tot}} = \frac{1}{9} = 11.1\% \;, \tag{159}$$

while the full treatment (following [20]) gives

$$B_{sl} = (11.6 \pm 0.8)\% \;. \tag{160}$$

This number agrees well with recent measurements [9, 108]

$$\begin{aligned} B_{sl}(B_d) &= (10.33 \pm 0.28)\% \;, \\ B_{sl}(B^+) &= (10.99 \pm 0.28)\% \;, \\ B_{sl}(B_s) &= (10.61 \pm 0.89)\% \;. \end{aligned} \tag{161}$$

7 The Heavy Quark Expansion

7.1 Calculation of inclusive decay rates

Now we are ready to derive the heavy quark expansion for inclusive decays. The decay rate of the transition of a B-meson to an inclusive final state X can be expressed as a phase space integral over the square of the matrix element of the effective Hamiltonian sandwiched between the initial B-meson[12] state and the final state X. Summing over all final states X with the same quark quantum numbers we obtain

$$\Gamma(B \to X) = \frac{1}{2m_B} \sum_X \int_{\text{PS}} (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X | \mathcal{H}_{eff} | B \rangle|^2 \,. \tag{162}$$

If we consider, e.g., a decay into three particles, i.e. $B \to 1+2+3$, then the phase space integral reads

$$\int_{\text{PS}} = \prod_{i=1}^{3} \left[\frac{d^3 p_i}{(2\pi)^3 2E_i} \right] \tag{163}$$

and $p_X = p_1 + p_2 + p_3$. With the help of the optical theorem the total decay rate in Eq.(162) can be rewritten as

$$\Gamma(B \to X) = \frac{1}{2m_B} \langle B | \mathcal{T} | B \rangle \,, \tag{164}$$

with the transition operator

$$\mathcal{T} = \text{Im}\, i \int d^4x T \left[\mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) \right] \,, \tag{165}$$

consisting of a non-local double insertion of the effective Hamiltonian.
This can be visualised via
vspace4cm

7.2 The expansion in inverse masses

A second operator-product-expansion, exploiting the large value of the b-quark mass m_b, yields for $\mathcal{T}$

$$\mathcal{T} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[c_{3,b} \bar{b} b + \frac{c_{5,b}}{m_b^2} \bar{b} g_s \sigma_{\mu\nu} G^{\mu\nu} b + 2 \frac{c_{6,b}}{m_b^3} (\bar{b} q)_\Gamma (\bar{q} b)_\Gamma + \ldots \right] \tag{166}$$

[12] The replacements one has to do when considering a D-meson decay are either trivial or we explicitly comment on them.

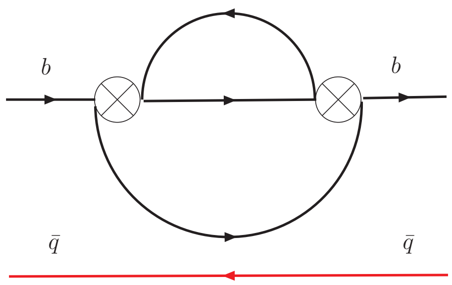
and thus for the decay rate

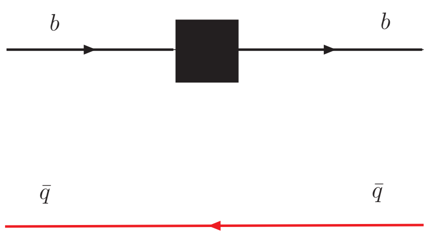
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[c_{3,b}\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2}\frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + ...\right] . \tag{167}$$

The individual contributions in Eq.(167) will be discussed in detail now.

7.3 Leading term in the HQE

To get the first term of Eq.(167) we contracted all quark lines, except the beauty-quark lines, in the product of the two effective Hamiltonians. This leads to the following two-loop diagram on the l.h.s., where the circles with the crosses denote the $\Delta B = 1$-operators from the effective Hamiltonian.





Performing the loop integrations in this diagram we get the Wilson coefficient $c_{3,b}$ that contains all the loop functions and the dimension-three operator $\bar{b}b$, which is denoted by the black square in the diagram on the r.h.s. . This has been done already in Eq.(153), Eq.(156) and Eq.(157).
A crucial finding for the HQE was the fact, that the matrix element of the dimension-three operator $\bar{b}b$ can also be expanded in the inverse of the b-quark mass. According to the Heavy Quark Effective Theory (HQET) we get[13]

$$\frac{\langle B|\bar{b}b|B\rangle}{2M_B} = 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) , \tag{168}$$

with the matrix element of the kinetic operator μ_π^2 and the matrix element of the

[13]We use here the conventional relativistic normalisation $\langle B|B\rangle = 2EV$, where E denotes the energy of the meson and V the space volume. In the original literature sometimes different normalisations have been used, which can lead to confusion.

chromo-magnetic operator μ_G^2, defined in the B-rest frame as[14]

$$\mu_\pi^2 = \frac{\langle B|\bar{b}(i\vec{D})^2 b|B\rangle}{2M_B} + \mathcal{O}\left(\frac{1}{m_b}\right), \quad (169)$$

$$\mu_G^2 = \frac{\langle B|\bar{b}\frac{g_s}{2}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \mathcal{O}\left(\frac{1}{m_b}\right). \quad (170)$$

With the above definitions for the non-perturbative matrix-elements the expression for the total decay rate in Eq.(167) becomes

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left\{ c_{3,b}\left[1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right] + 2c_{5,b}\left[\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right] + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots \right\} \quad (171)$$

The leading term in Eq.(171) describes simply the decay of a free quark. Since here the spectator-quark (red) is not involved in the decay process at all, this contribution will be the same for all different b-hadrons, thus predicting the same lifetime for all b-hadrons.
The first corrections are already suppressed by two powers of the heavy b-quark mass - we have no corrections of order $1/m_b$! This non-trivial result explains, why our description in terms of the free b-quark decay was so close to the experimental values of the lifetimes of B-mesons.
In the case of D-mesons the expansion parameter $1/m_c$ is not small and the higher order terms of the HQE will lead to sizable corrections. The leading term $c_{3,c}$ for charm-quark decays gives at the scale $\mu = M_W$ for vanishing quark mass $c_{3,c} = 5$. At the scale $\mu = \bar{m}_c(\bar{m}_c)$ and realistic values of final states masses we get

$$c_{3,c} = \left\{ \begin{array}{ll} 5 & (m_s = 0 = \alpha_s) \\ 6.29 \pm 0.72 & (\mathrm{LO-QCD}) \\ 11.61 \pm 1.55 & (\mathrm{NLO-QCD}) \end{array} \right. . \quad (172)$$

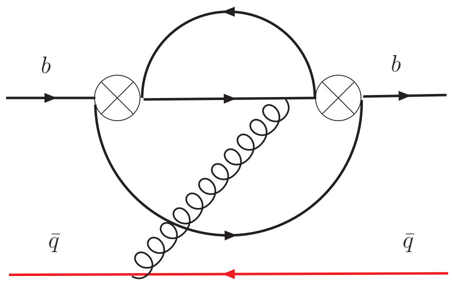
Here we have a large QCD enhancement of more than a factor of two, while phase space effects seem to be negligible.
The $1/m_b^2$-corrections in Eq.(171) have two sources: first the expansion in Eq.(168) and the second one - denoted by the term proportional to $c_{5,b}$ - will be discussed below.
Concerning the different $1/m_b^3$-corrections, indicated in Eq.(171), we will see that the first two terms of the expansion in Eq.(167) are triggered by a two-loop diagram, while the third term is given by a one-loop diagram. This will motivate,

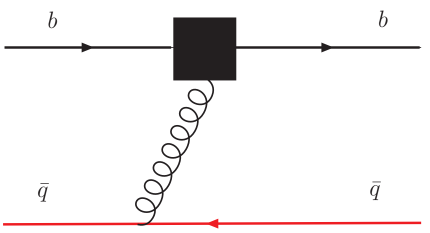
[14]We use here $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. In the original literature sometimes the notation $i\sigma G := i\gamma_\mu\gamma_\nu G^{\mu\nu}$ was used, which differs by a factor of i from our definition of σ.

why the $1/m_b^3$-corrections proportional to $c_{3,b}$ and $c_{5,b}$ can be neglected in comparison to the $1/m_b^3$-corrections proportional to $c_{6,b}$; the former ones will, however, be important for precision determination of semi-leptonic decay rates.

7.4 Second term of the HQE

To get the second term in Eq.(167) we couple in addition a gluon to the vacuum. This is denoted by the diagram below, where a gluon is emitted from one of the internal quarks of the two-loop diagram. Doing so, we obtain the so-called chromo-magnetic operator $\bar{b} g_s \sigma_{\mu\nu} G^{\mu\nu} b$, which already appeared in the expansion in Eq.(168).





Since this operator is of dimension five, the corresponding contribution is - as seen before - suppressed by two powers of the heavy quark mass, compared to the leading term. The corresponding Wilson coefficient $c_{5,b}$ reads, e.g., for the semi-leptonic decay $b \to c e^- \bar{\nu}_e$[15] and the non-leptonic decays $b \to c\bar{u}d$ and $b \to c\bar{c}s$

$$c_{5,b}^{ce\bar{\nu}_e} = -(1-z)^4 \left[1 + \frac{\alpha_s}{4\pi} \ldots\right] , \tag{173}$$

$$c_{5,b}^{c\bar{u}d} = -|V_{ud}|^2 (1-z)^3 \left[\mathcal{N}_a(\mu)(1-z) + 8C_1C_2 + \frac{\alpha_s}{4\pi} \ldots\right] , \tag{174}$$

$$\begin{aligned} c_{5,b}^{c\bar{c}s} = & -|V_{cs}|^2 \left\{ \mathcal{N}_a(\mu) \left[\sqrt{1-4z}(1-2z)(1-4z-6z^2) + 24z^4 \log\left(\frac{1+\sqrt{1-4z}}{1-\sqrt{1-4z}}\right)\right] \right. \\ & \left. +8C_1C_2 \left[\sqrt{1-4z}\left(1+\frac{z}{2}+3z^2\right) - 3z(1-2z^2)\log\left(\frac{1+\sqrt{1-4z}}{1-\sqrt{1-4z}}\right)\right] + \frac{\alpha_s}{4\pi}\ldots \right\} , \end{aligned} \tag{175}$$

with the quark mass ratio $z = (m_c/m_b)^2$. For vanishing charm-quark masses and $V_{ud} \approx 1$ we get $c_{5,b}^{c\bar{u}d} = -3$ at the scale $\mu = M_W$, which reduces in LO-QCD to about -1.2 at the scale $\mu = m_b$.

[15]The result in Eq.(94) of the review [109] has an additional factor 6 in $c_5^{ce\bar{\nu}_e}$.

For the total decay rate we have to sum up all possible quark level-decays

$$c_{5,b} = c_{5,b}^{ce\bar{\nu}_e} + c_{5,b}^{c\mu\bar{\nu}_\mu} + c_{5,b}^{c\tau\bar{\nu}_\tau} + c_{5,b}^{c\bar{u}d} + c_{5,b}^{c\bar{c}s} + \ldots \; . \tag{176}$$

Neglecting penguin contributions we get numerically

$$c_{5,b} = \begin{cases} \approx -9 & (m_c = 0 = \alpha_s) \\ -3.8 \pm 0.3 & (\bar{m}_c(\bar{m}_c)\, , \alpha_s(m_b)) \end{cases} \; , \tag{177}$$

For $c_{5,b}$ both QCD effects as well as phase space effects are quite pronounced. The overall coefficient of the matrix element of the chromo-magnetic operator μ_G^2 normalised to $2m_b^2$ in Eq.(171) is given by $c_{3,b}+4c_{5,b}$, which is sometimes denoted as $c_{G,b}$. For semi-leptonic decays like $b \to ce^-\bar{\nu}_e$, it reads[16]

$$c_{G,b}^{ce\bar{\nu}_e} = c_{3,b}^{ce\bar{\nu}_e} + 4c_{5,b}^{ce\bar{\nu}_e} \;=\; (-3)\left[1 - \frac{8}{3}z + 8z^2 - 8z^3 + \frac{5}{3}z^4 + 4z^2\ln(z)\right] \tag{178}$$

For the sum of all inclusive decays we get

$$c_{G,b} = \begin{cases} -27 = -3c_3 & (m_c = 0 = \alpha_s) \\ -7.9 \approx -1.1c_3 & (\bar{m}_c(\bar{m}_c)\, , \alpha_s(m_b)) \end{cases} \; , \tag{179}$$

leading to the following form of the total decay rate

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left[c_{3,b} - c_{3,b}\frac{\mu_\pi^2}{2m_b^2} + c_{G,b}\frac{\mu_G^2}{2m_b^2} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + ...\right] . \tag{180}$$

Both $1/m_b^2$-corrections are reducing the decay rate and their overall coefficients are of similar size as $c_{3,b}$. To estimate more precisely the numerical effect of the $1/m_b^2$ corrections, we still need the values of μ_π^2 and μ_G^2. Current values [111, 112] of these parameters read for the case of B_d and B^+-mesons

$$\mu_\pi^2(B) \;=\; (0.414 \pm 0.078)\ \mathrm{GeV}^2\ , \tag{181}$$

$$\mu_G^2(B) \;\approx\; \frac{3}{4}\left(M_{B^*}^2 - M_B^2\right) \approx (0.35 \pm 0.07)\ \mathrm{GeV}^2\ . \tag{182}$$

For B_s-mesons only small differences compared to B_d and B^+-mesons are predicted [113]

$$\mu_\pi^2(B_s) - \mu_\pi^2(B_d) \;\approx\; (0.08\ldots0.10)\ \mathrm{GeV}^2\ , \tag{183}$$

$$\frac{\mu_G^2(B_s)}{\mu_G^2(B_d)} \;\approx\; 1.07 \pm 0.03\ , \tag{184}$$

[16]We differ here slightly from Eq.(7) of [110], who have a different sign in the coefficients of z^2 and z^3. We agree, however, with the corresponding result in [86].

while sizable differences are expected [113] for Λ_b-baryons.

$$\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d) \approx (0.1 \pm 0.1)\ \mathrm{GeV}^2\ , \tag{185}$$

$$\mu_G^2(\Lambda_b) = 0\ . \tag{186}$$

Inserting these values in Eq.(180) we find that the $1/m_b^2$-corrections are decreasing the decay rate slightly ($m_b = \bar{m}_b(\bar{m}_b) = 4.248$ GeV):

	B_d	B^+	B_s	Λ_d
$-\frac{\mu_\pi^2}{2m_b^2}$	-0.011	-0.011	-0.014	-0.014
$\frac{c_{G,b}}{c_{3,b}} \frac{\mu_\pi^2}{2m_b^2}$	-0.011	-0.011	-0.011	0.00

(187)

The kinetic and the chromo-magnetic operator each reduce the decay rate by about 1%, except for the case of the Λ_b-baryon, where the chromo-magnetic operator vanishes. The $1/m_b^2$-corrections exhibit now also a small sensitivity to the spectator-quark. Different values for the lifetimes of b-hadrons can arise due to different values of the non-perturbative parameters μ_G^2 and μ_π^2, the corresponding numerical effect will, however, be small.

$X:$	B^+	B_s	Λ_d
$\frac{\mu_\pi^2(X)-\mu_\pi^2(B_d)}{2m_b^2}$	0.000 ± 0.000	0.002 ± 0.000	0.003 ± 0.003
$\frac{c_{G,b}}{c_{3,b}} \frac{\mu_G^2(X)-\mu_G^2(B_d)}{2m_b^2}$	0.000 ± 0.000	$0.000...0.001$	-0.011 ± 0.003

(188)

Thus we find that the $1/m_b^2$-corrections give no difference in the lifetimes of B^+- and B_d-mesons, they enhance the B_s-lifetime by about 3 per mille, compared to the B_d-lifetime and they reduce the Λ_b-lifetime by about 1% compared to the B_d-lifetime.
To get an idea of the size of these corrections in the charm-system, we first investigate the Wilson coefficient c_5.

$$c_{5,c} = \begin{cases} \approx -5 & (m_c = 0 = \alpha_s) \\ -1.7 \pm 0.3 & (\bar{m}_c(\bar{m}_c)\ , \alpha_s(m_b)) \end{cases} \ , \tag{189}$$

At the scale $\mu = m_c$ the non-leptonic contribution to c_5 is getting smaller than in the bottom case and it even changes sign. For the coefficient c_G we find

$$c_{G,c} = \begin{cases} \approx -15 = -3\, c_{3,c} & (m_c = 0 = \alpha_s) \\ 4.15 \pm 1.48 = (0.37 \pm 0.13)\, c_{3,c} & (\bar{m}_c(\bar{m}_c)\ , \alpha_s(m_b)) \end{cases} \ . \tag{190}$$

We see for that for the charm case the overall coefficient of the chromo-magnetic operator has now a positive sign and the relative size is less than in the bottom case. For D^0- and D^+-mesons the value of the chromo-magnetic operator reads

$$\mu_G^2(D) \approx \frac{3}{4}\left(M_{D^*}^2 - M_D^2\right) \approx 0.41\ \mathrm{GeV}^2\ , \tag{191}$$

which is of similar size as in the B-system. Normalising this value to the charm quark mass $m_c = \bar{m}_c(\bar{m}_c) = 1.277$ GeV, we get however a bigger contribution compared to the bottom case and also a different sign.

$$c_{G,c}\frac{\mu_G^2(D)}{2m_c^2} \approx +0.05\, c_{3,c} \; . \tag{192}$$

Now the second order corrections are non-negligible, with a typical size of about + 5% of the total decay rate. Concerning lifetime differences of D-mesons, we find no visible effect due to the chromo-magnetic operator [114]

$$\frac{\mu_G^2(D^+)}{\mu_G^2(D^0)} \approx 0.993 \; , \tag{193}$$

$$\frac{\mu_G^2(D_s^+)}{\mu_G^2(D^0)} \approx 1.012 \pm 0.003 \; . \tag{194}$$

For the kinetic operator a sizable SU(3) flavour breaking was found by Bigi, Mannel and Uraltsev [113]

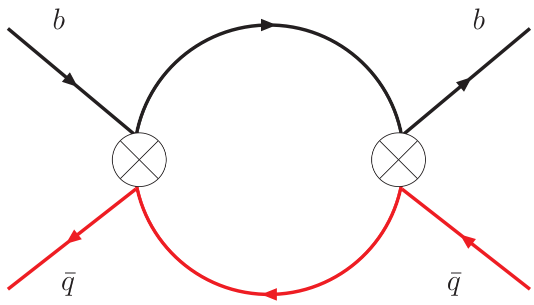
$$\mu_\pi^2(D_s^+) - \mu_\pi^2(D^0) \approx 0.1 \text{ GeV}^2 \; , \tag{195}$$

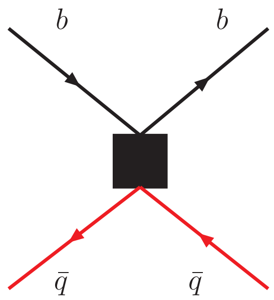
leading to an reduction of the D_s^+-lifetime of the order of 3% compared to the D^0-lifetime

$$\frac{\mu_\pi^2(D_s^+) - \mu_\pi^2(D^0)}{2m_c^2} \approx 0.03 \; . \tag{196}$$

7.5 Third term of the HQE

The next term in Eq.(167) is obtained by only contracting two quark lines in the product of the two effective Hamiltonian in Eq.(165). The b-quark and the spectator quark of the considered hadron are not contracted. For B_d-mesons ($q = d$) and B_s-mesons ($q = s$) we get the following so-called *weak annihilation* diagram.

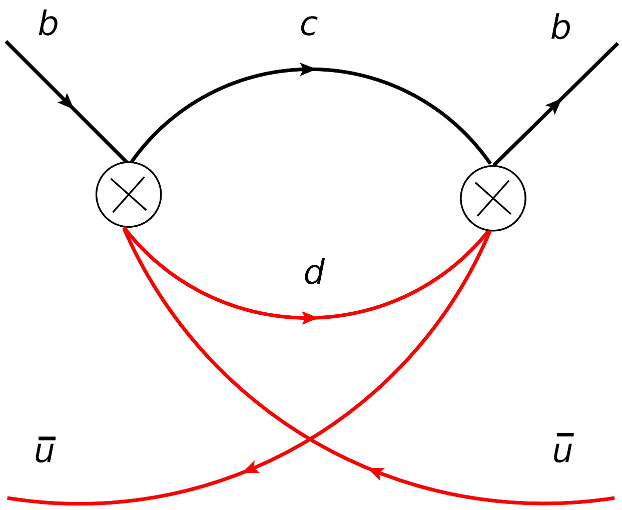




Performing the loop integration on the diagram on the l.h.s. we get the Wilson coefficient c_6 and dimension six four-quark operators $(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma$, with Dirac structures Γ. The corresponding matrix elements of these $\Delta B = 0$ operators are typically written as

$$\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle = c_\Gamma f_B^2 M_B B_\Gamma \; , \tag{197}$$

with the bag parameter B_Γ, the decay constant f_B and a numerical factor c_Γ that contains some colour factors and sometimes also ratios of masses.
For the case of the B^+-meson we get a similar diagram, with the only difference that now the external spectator-quark lines are crossed, this is the so-called *Pauli interference* diagram.



There are two very interesting things to note. First this is now a one-loop diagram. Although being suppressed by three powers of the b-quark mass it is enhanced by a phase space factor of $16\pi^2$ compared to the leading two-loop diagrams. Second, now we are really sensitive to the flavour of the spectator-quark, because in principle, each different spectator quark gives a different contribution[17]. These observations are responsible for the fact that lifetime differences in the system of heavy hadrons are almost entirely due to the contribution of weak annihilation and Pauli interference diagrams.
In the case of the B_d meson four different four-quark operators arise

$$\begin{aligned} Q^q &= \bar{b}\gamma_\mu(1-\gamma_5)q \times \bar{q}\gamma^\mu(1-\gamma_5)b, \\ Q^q_S &= \bar{b}(1-\gamma_5)q \times \bar{q}(1-\gamma_5)b, \\ T^q &= \bar{b}\gamma_\mu(1-\gamma_5)T^a q \times \bar{q}\gamma^\mu(1-\gamma_5)T^a b, \\ T^q_S &= \bar{b}(1-\gamma_5)T^a q \times \bar{q}(1-\gamma_5)T^a b, \end{aligned} \tag{198}$$

[17] This difference is, however, negligible, if one considers, e.g., B_s vs. B_d.

with $q = d$ for the case of B_d-mesons. Q denotes colour singlet operators and T colour octet operators. For historic reasons the matrix elements of these operator are typically expressed as

$$\frac{\langle B_d|Q^d|B_d\rangle}{M_{B_d}} = f_B^2 B_1 M_{B_d} \, , \qquad \frac{\langle B_d|Q_S^d|B_d\rangle}{M_{B_d}} = f_B^2 B_2 M_{B_d} \, , \tag{199}$$

$$\frac{\langle B_d|T^d|B_d\rangle}{M_{B_d}} = f_B^2 \epsilon_1 M_{B_d} \, , \qquad \frac{\langle B_d|T_S^d|B_d\rangle}{M_{B_d}} = f_B^2 \epsilon_2 M_{B_d} \, . \tag{200}$$

The bag parameters $B_{1,2}$ are expected to be of order one in vacuum insertion approximation, while the $\epsilon_{1,2}$ vanish in that limit. We will discuss below several estimates of B_i and ϵ_i. Decay constants can be determined with lattice-QCD, see, e.g., the reviews of FLAG [115] or with QCD sum rules, see, e.g., the recent determination in [116]. Later on, we will see, however, that the Wilson coefficients of B_1 and B_2 are affected by sizable numerical cancellations, enhancing hence the relative contribution of the colour suppressed ϵ_1 and ϵ_2. The corresponding Wilson coefficients of the four operators can be written as

$$\begin{aligned} c_6^{Q^d} &= 16\pi^2 \left[|V_{ud}|^2 F^u + |V_{cd}|^2 F^c\right] , \\ c_6^{Q_S^d} &= 16\pi^2 \left[|V_{ud}|^2 F_S^u + |V_{cd}|^2 F_S^c\right] , \\ c_6^{T^d} &= 16\pi^2 \left[|V_{ud}|^2 G^u + |V_{cd}|^2 G^c\right] , \\ c_6^{T_S^d} &= 16\pi^2 \left[|V_{ud}|^2 G_S^u + |V_{cd}|^2 G_S^c\right] . \end{aligned} \tag{201}$$

F^q describes an internal $c\bar{q}$ loop in the above weak annihilation diagram. The functions F and G are typically split up in contributions proportional to C_2^2, $C_1 C_2$ and C_1^2.

$$F^u = C_1^2 F_{11}^u + C_1 C_2 F_{12}^u + C_2^2 F_{22}^u \, , \tag{202}$$

$$F_S^u = \ldots \, . \tag{203}$$

Next, each of the F_{ij}^q can be expanded in the strong coupling

$$F_{ij}^u = F_{ij}^{u,(0)} + \frac{\alpha_s}{4\pi} F_{ij}^{u,(1)} + \ldots \, , \tag{204}$$

$$F_{S,ij}^u = \ldots \, . \tag{205}$$

As an example we give the following LO results

$$F_{11}^{u,(0)} = -3(1-z)^2 \left(1 + \frac{z}{2}\right) , \qquad F_{S,11}^{u,(0)} = 3(1-z)^2 (1+2z) \, , \tag{206}$$

$$F_{12}^{u,(0)} = -2(1-z)^2 \left(1 + \frac{z}{2}\right) , \qquad F_{S,12}^{u,(0)} = 2(1-z)^2 (1+2z) \, , \tag{207}$$

$$F_{22}^{u,(0)} = -\frac{1}{3}(1-z)^2 \left(1 + \frac{z}{2}\right) , \qquad F_{S,22}^{u,(0)} = \frac{1}{3}(1-z)^2 (1+2z) \, , \tag{208}$$

$$G_{22}^{u,(0)} = -2(1-z)^2 \left(1 + \frac{z}{2}\right) , \qquad G_{S,22}^{u,(0)} = 2(1-z)^2 (1+2z) \, , \tag{209}$$

with $z = m_c^2/m_b^2$.
Putting everything together we arrive at the following expression for the decay rate of a B_d-meson

$$\begin{aligned}\Gamma_{B_d} &= \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left[c_3 - c_3 \frac{\mu_\pi^2}{2m_b^2} + c_G \frac{\mu_G^2}{2m_b^2} + \frac{16\pi^2 f_B^2 M_{B_d}}{m_b^3} \tilde{c}_6^{B_d} + \mathcal{O}\left(\frac{1}{m_b^3}, \frac{16\pi^2}{m_b^4}\right)\right] \\ &\approx \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left[c_3 - 0.01 c_3 - 0.01 c_3 + \frac{16\pi^2 f_B^2 M_{B_d}}{m_b^3} \tilde{c}_6^{B_d} + \mathcal{O}\left(\frac{1}{m_b^3}, \frac{16\pi^2}{m_b^4}\right)\right], \end{aligned} \tag{210}$$

with

$$\begin{aligned}\tilde{c}_6^{B_d} &= |V_{ud}|^2 \left(F^u B_1 + F_S^u B_2 + G^u \epsilon_1 + G_S^u \epsilon_2\right) \\ &+ |V_{cd}|^2 \left(F^c B_1 + F_S^c B_2 + G^c \epsilon_1 + G_S^c \epsilon_2\right) \ . \end{aligned} \tag{211}$$

The size of the third contribution in Eq.(210) is governed by size of $\tilde{c}_6$ and its pre-factor. The pre-factor gives

$$\frac{16\pi^2 f_{B_d}^2 M_{B_d}}{m_b^3} \approx 0.395 \approx 0.05\, c_3 \ , \tag{212}$$

where we used $f_{B_d} = (190.5 \pm 4.2)$ MeV [115] for the decay constant. If $\tilde{c}_6$ is of order 1, we would expect corrections of the order of 5% to the total decay rate, which are larger than the formally leading $1/m_b^2$-corrections. The LO-QCD expression for $\tilde{c}_6^{B_d}$ can be written as

$$\begin{aligned}\tilde{c}_6^{B_d} &= |V_{ud}|^2 (1-z)^2 \left\{ \left(3C_1^2 + 2C_1 C_2 + \frac{1}{3} C_2^2\right) \left[(B_2 - B_1) + \frac{z}{2}(4B_2 - B_1)\right] \right. \\ & \left. \qquad + 2C_2^2 \left[(\epsilon_2 - \epsilon_1) + \frac{z}{2}(4\epsilon_2 - \epsilon_1)\right] \right\} \ . \end{aligned} \tag{213}$$

However, in Eq.(213) several cancellations are arising. In the first line there is a strong cancellation among the bag parameters B_1 and B_2. In vacuum insertion approximation $B_1 - B_2$ is zero and the next term proportional to $4B_2 - B_1$ is suppressed by $z \approx 0.055$. Using the latest lattice determination of these parameters [117] - dating back already to 2001! -

$$B_1 = 1.10 \pm 0.20 \ , \quad B_2 = 0.79 \pm 0.10 \ , \quad \epsilon_1 = -0.02 \pm 0.02 \ , \quad \epsilon_2 = 0.03 \pm 0.01 \tag{214}$$

one finds $B_1 - B_2 \in [0.01, 0.61]$ and $(4B_2 - B_1)z/2 \in [0.07, 0.12]$, so the second contribution is slightly suppressed compared to the first one. Moreover there is an additional cancellation among the $\Delta B = 1$ Wilson coefficients. Without QCD the

combination $3C_1^2 + 2C_1C_2 + \frac{1}{3}C_2^2$ is equal to $1/3$, in LO-QCD this combination is reduced to about 0.05 ± 0.05 at the scale of m_b (varying the renormalisation scale between $m_b/2$ and $2m_b$). Hence B_1 and B_2 give a contribution between 0 and 0.07 to $\tilde{c}_6^{B_d}$, leading thus at most to a correction of about 4 per mille to the total decay rate. This statement depends, however, crucially on the numerical values of the bag parameters, where we are lacking a state-of-the-art determination.
There is no corresponding cancellation in the coefficients related to the colour-suppressed bag parameters $\epsilon_{1,2}$. According to [117] $\epsilon_2 - \epsilon_1 \in [0.02, 0.08]$, leading to a correction of at most 1.0% to the decay rate. Relying on the lattice determination in [117] we find that the colour-suppressed operators can be numerical more important than the colour allowed operators and the total decay rate of the B_d-meson can be enhanced by the weak annihilation at most by about 1.4%. The status at NLO-QCD will be discussed below.
The Pauli interference contribution to the B^+-decay rate gives

$$\tilde{c}_6^{B^+} = (1-z)^2 \left[\left(C_1^2 + 6C_1C_2 + C_2^2 \right) B_1 + 6 \left(C_1^2 + C_2^2 \right) \epsilon_1 \right] . \quad (215)$$

The contribution of the colour-allowed operator is slightly suppressed by the $\Delta B = 1$ Wilson coefficients. Without QCD the bag parameter B_1 has a pre-factor of one, which changes in LO-QCD to about -0.3. Taking again the lattice values for the bag parameter from [117], we expect Pauli interference contributions proportional to B_1 to be of the order of about -1.8% of the total decay rate. In the coefficient of ϵ_1 no cancellation is arising and we expect (using again [117]) this contribution to be between 0 and -1.5% of the total decay rate. All in all Pauli interference seems to reduce the total B^+-decay rate by about 1.8% to 3.3%. The status at NLO-QCD will again be discussed below.
In the charm system the pre-factor of the coefficient c_6 reads

$$\frac{16\pi^2 f_D^2 M_D}{m_c^3} \approx \begin{cases} 6.2 \approx 0.6\, c_3 & \text{for} \quad D^0, D^+ \\ 9.2 \approx 0.8\, c_3 & \text{for} \quad D_s^+ \end{cases} , \quad (216)$$

where we used $f_{D^0} = (209.2 \pm 3.3)$ MeV and $f_{D_s^+} = (248.3 \pm 2.7)$ MeV [115] for the decay constants. Depending on the strength of the cancellation among the $\Delta C = 1$ Wilson coefficients and the bag parameters, large corrections seem to be possible now: In the case of the weak annihilation the cancellation of the $\Delta C = 1$ Wilson coefficients seems to be even more pronounced than at the scale m_b. Thus a knowledge of the colour-suppressed operators is inalienable. In the case of Pauli interference no cancellation occurs and we get values for the coefficient of B_1, that are smaller than -1 and we get a sizable, but smaller contribution from the colour-suppressed operators. Unfortunately there is no lattice determination of the $\Delta C = 0$ matrix elements available, so we cannot make any final, profound statements about the status in the charm system. Numerical results for the NLO-QCD case will also be discussed below.

7.6 Fourth term of the HQE

If one takes in the calculation of the weak annihilation and Pauli interference diagrams also small momenta and masses of the spectator quark into account, one gets corrections that are suppressed by four powers of m_b compared to the free-quark decay. These dimension seven terms are either given by four-quark operators times the small mass of the spectator quark or by a four quark operator with an additional derivative. Examples are the following $\Delta B = 0$ operators

$$P_1 = \frac{m_{d,s}}{m_b} \bar{b}_i (1-\gamma_5) d_i \times \bar{d}_j (1-\gamma_5) b_j \, , \tag{217}$$

$$P_2 = \frac{m_{d,s}}{m_b} \bar{b}_i (1+\gamma_5) d_i \times \bar{d}_j (1+\gamma_5) b_j \, , \tag{218}$$

$$P_3 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\rho \gamma_\mu (1-\gamma_5) D^\rho d_i \times \bar{d}_j \gamma^\mu (1-\gamma_5) b_j \, , \tag{219}$$

$$P_4 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho d_i \times \bar{d}_j (1+\gamma_5) b_j \, . \tag{220}$$

These operators have currently only been estimated within vacuum insertion approximation. However, for the corresponding operators appearing in the decay rate difference of neutral B-meson first studies with QCD sum rules have been performed [118, 119].
Putting everything together we arrive at the *Heavy-Quark Expansion* of decay rates of heavy hadrons

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots \, , \tag{221}$$

where the expansion parameter is denoted by Λ/m_b. From the above explanations it is clear that Λ is not simply given by Λ_{QCD} - the pole of the strong coupling constant - as stated often in the literature. Very naively one expects Λ to be of the order of Λ_{QCD}, because both denote non-perturbative effects. The actual value of Λ, has, however, to be determined by an explicit calculation for each order of the expansion separately. At order $1/m_b^2$ one finds that Λ is of the order of μ_π or μ_G, so roughly below 1 GeV. For the third order Λ^3 is given by $16\pi^2 f_B^2 M_B$ times a numerical suppression factor, leading to values of Λ larger than 1 GeV. Moreover, each of the coefficients Γ_j, which is a product of a perturbatively calculable Wilson coefficient and a non-perturbative matrix element, can be expanded in the strong coupling

$$\Gamma_j = \Gamma_j^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_j^{(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\Gamma_j^{(2)} + \ldots \, . \tag{222}$$

Before we apply this framework to experimental observables, we would like to make some comments of caution.

7.7 Violation of quark-hadron duality

A possible drawback of this approach might be that the expansion in the inverse heavy quark mass does not converge well enough — advocated under the labelling *violation of quark hadron duality*. There is a considerable amount of literature about theoretical attempts to prove or to disprove duality, but all of these attempts have to rely on strong model assumptions.

Uraltsev published some general investigations of quark hadron duality violation in [120, 121] and some investigations within the two dimensional 't Hooft model [122, 123], that indicated the validity of quark hadron duality. Other investigations in that direction were e.g. performed by Grinstein and Lebed in 1997 [124] and 1998 [125] and by Grinstein in 2001 [126, 127]. In our opinion the best way of tackling this question is to confront precise HQE-based predictions with precise experimental data. An especially well suited candidate for this problem is the decay $b \to c\bar{c}s$, which is CKM dominant, but phase space suppressed. The actual expansion parameter of the HQE is in this case not $1/m_b$ but $1/(m_b\sqrt{1-4z})$; so violations of duality should be more pronounced. Thus a perfect observable for testing the HQE is the decay rate difference $\Delta\Gamma_s$ of the neutral B_s mesons, which is governed by the $b \to c\bar{c}s$ transition. The first measurement of this quantity in 2012 and several follow-up measurements are in perfect agreement with the HQE prediction and exclude thus huge violations of quark hadron duality, see [128] and the discussion below.

7.8 Status of lifetime predictions

In this final section we update several of the lifetime predictions and compare them with the most recent data, obtained many times at the LHC experiments.

7.8.1 B-meson lifetimes

The most recent theory expressions for $\tau(B^+)/\tau(B_s)$ and $\tau(B_s)/\tau(B_d)$ are given in [175] (based on the calculations in [81, 129, 130, 117]). For the charged B-meson we get the updated relation (including α_s-corrections and $1/m_b$-corrections)

$$\begin{aligned}\frac{\tau(B^+)}{\tau(B_d)}^{\mathrm{HQE\,2014}} &= 1 + 0.03\left(\frac{f_{B_d}}{190.5\,\mathrm{MeV}}\right)^2\left[(1.0\pm0.2)B_1 + (0.1\pm0.1)B_2\right.\\ &\qquad \left. -(17.8\pm0.9)\epsilon_1 + (3.9\pm0.2)\epsilon_2 - 0.26\right]\\ &= 1.04^{+0.05}_{-0.01}\pm0.02\pm0.01\,. \end{aligned} \tag{223}$$

Here we have used the lattice values for the bag parameters from [117]. Using all the available values for the bag parameters in the literature, see [22], the central

value of our prediction for $\tau(B^+)/\tau(B_d)$ varies between 1.03 and 1.09. This is indicated by the first asymmetric error and clearly shows the urgent need for more profound calculations of these non-perturbative parameters. The second error in Eq.(223) stems from varying the matrix elements of [117] in their allowed range and the third error comes from the renormalisation scale dependence as well as the dependence on m_b.
Next we update also the prediction for the B_s-lifetime given in [175], by including also $1/m_b^2$-corrections discussed in Eq.(188).

$$\begin{aligned}\frac{\tau(B_s)}{\tau(B_d)}^{\rm HQE\,2014} &= 1.003 + 0.001 \left(\frac{f_{B_s}}{231\,\rm MeV}\right)^2 [(0.77 \pm 0.10)B_1 + (1.0 \pm 0.13)B_2 \\ &\qquad +(36 \pm 5)\epsilon_1 + (51 \pm 7)\epsilon_2] \\ &= 1.001 \pm 0.002\,. \end{aligned} \tag{224}$$

The values in Eq.(223) and Eq.(224) differ slightly from the ones in [175], because we have used updated lattice values for the decay constants[18] and we included the SU(3)-breaking of the $1/m_b^2$-correction - see Eq.(188) - for the B_s-lifetime, which was previously neglected. Comparing these predictions with the measurements given in Eq.(2.1), we find a perfect agreement for the B_s-lifetime, leaving thus only a little space for, e.g., hidden new B_s-decay channels, following, e.g., [131, 132]. There is a slight tension in $\tau(B^+)/\tau(B_d)$, which, however, could solely be due to the unknown values of the hadronic matrix elements. A value of, e.g., $\epsilon_1 = -0.092$ - and leaving everything else at the values given in Eq.(214) - would perfectly match the current experimental average from Eq.(2.1).
The most recent experimental numbers for these lifetime ratios have been updated by the LHCb Collaboration in 2014 [133].

7.8.2 b-baryon lifetimes

There was a long standing puzzle related to the lifetime of Λ_b-baryon. Old measurements hinted towards a value that was considerably smaller than the B_d lifetime. Recent measurements, in particular from the experiments at Tevatron and the LHC, haven proven, however, that the Λ_b-lifetime is comparable to the one of the B_d-meson. The current HFAG average given in Eq.(2.1) clearly rules out now the old small values of the Λ_b-lifetime. Updating the NLO-calculation from the Rome group [134] and including $1/m_b$-corrections from [130] we get for the current HQE prediction

$$\begin{aligned}\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\rm HQE\,2014} &= 1 - (0.8 \pm 0.5)\%_{\frac{1}{m_b^2}} - (4.2 \pm 3.3)\%^{\Lambda_b}_{\frac{1}{m_b^3}} - (0.0 \pm 0.5)\%^{B_d}_{\frac{1}{m_b^3}} - (1.6 \pm 1.2)\%_{\frac{1}{m_b^4}} \\ &= 0.935 \pm 0.054\,, \end{aligned} \tag{225}$$

[18] We have used $f_{B_s} = 227.7$ MeV [115].

where we have split up the corrections coming from the $1/m_b^2$-corrections discussed in Eq.(188), the $1/m_b^3$-corrections coming from the Λ_b-matrix elements, the $1/m_b^3$-corrections coming from the B_d-matrix elements and finally $1/m_b^4$-corrections studied in [130]. The origin of these numerical values is discussed in detail in [22]. All in all, now the new measurements of the Λ_b-lifetime are in nice agreement with the HQE result. This is now a very strong confirmation of the validity of the HQE and this makes also the motivation of many of the studies trying to explain the Λ_b-lifetime puzzle, e.g., [135, 136, 137], invalid.
In [81] it was shown that the lifetime ratio of the Ξ_b-baryons can be in principle be determined quite precisely, because here the above mentioned problems with penguin contractions do not arise. Unfortunately there exists no non-perturbative determination of the matrix elements for Ξ_b-baryons. So, we are left with the possibility of assuming that the matrix elements for Ξ_b are equal to the ones of Λ_b. In that case we can give a rough estimate for the expected lifetime ratio. In order to get rid of unwanted $s \to u$-transitions we define (following [81])

$$\frac{1}{\bar{\tau}(\Xi_b)} = \bar{\Gamma}(\Xi_b) = \Gamma(\Xi_b) - \Gamma(\Xi_b \to \Lambda_b + X) \,. \tag{226}$$

For a numerical estimate we again scan over all the results for the Λ_b-matrix elements. Using also recent values for the remaining input parameters we obtain

$$\frac{\bar{\tau}(\Xi_b^0)}{\bar{\tau}(\Xi_b^+)}^{\mathrm{HQE}\,2014} = 0.95 \pm 0.04 \pm 0.01 \pm ??? \,, \tag{227}$$

where the first error comes from the range of the values used for r, the second denotes the remaining parametric uncertainty and $???$ stands for some unknown systematic errors, which comes from the approximation of the Ξ_b-matrix elements by the Λ_b-matrix elements. We expect the size of these unknown systematic uncertainties not to exceed the error stemming from r, thus leading to an estimated overall error of about ± 0.06. As soon as Ξ_b-matrix elements are available the ratio in Eq.(227) can be determine more precisely than $\tau(\Lambda_b)/\tau(B_d)$.
If we further approximate $\bar{\tau}(\Xi_b^0) = \tau(\Lambda_b)$ - here similar cancellations are expected to arise as in τ_{B_s}/τ_{B_d} - , then we arrive at the following prediction

$$\frac{\tau(\Lambda_b)}{\bar{\tau}(\Xi_b^+)}^{\mathrm{HQE}\,2014} = 0.95 \pm 0.06 \,. \tag{228}$$

From the new measurements of the LHCb Collaboration [138, 139] (see also the CDF update [140]), we deduce

$$\frac{\tau(\Xi_b^0)}{\tau(\Xi_b^+)}^{\mathrm{LHCb}\,2014} = 0.92 \pm 0.03 \,, \tag{229}$$

$$\frac{\tau(\Xi_b^0)}{\tau(\Lambda_b)}^{\mathrm{LHCb\,2014}} = 1.006 \pm 0.021 \,, \tag{230}$$

$$\frac{\tau(\Lambda_b)}{\tau(\Xi_b^+)}^{\mathrm{LHCb\,2014}} = 0.918 \pm 0.028 \,, \tag{231}$$

which is in perfect agreement with the predictions above in Eq.(227) and Eq.(228), within the current uncertainties.

7.8.3 D-meson lifetimes

In [114] the NLO-QCD corrections for the D-meson lifetimes were completed. Including $1/m_c$-corrections as well as some assumptions about the hadronic matrix elements one obtains

$$\frac{\tau(D^+)}{\tau(D^0)}^{\mathrm{HQE\,2013}} = 2.2 \pm 0.4^{\mathrm{(hadronic)}}{}^{+0.03\mathrm{(scale)}}_{-0.07} \,, \tag{232}$$

$$\frac{\tau(D_s^+)}{\tau(D^0)}^{\mathrm{HQE\,2013}} = 1.19 \pm 0.12^{\mathrm{(hadronic)}}{}^{+0.04\mathrm{(scale)}}_{-0.04} \,, \tag{233}$$

being very close to the experimental values shown in the beginning of this lecture. Therefore this result seems to indicate that one might apply the HQE also to lifetimes of D-mesons, but definite conclusions cannot not be drawn without a reliable non-perturbative determination of the hadronic matrix elements, which is currently missing.

8 Mixing in Particle Physics

8.1 Overview

Mixing occurs at several stages within the standard model of particle physics. One example we discussed already in the derivation of the CKM matrix. Mixing simply describes the fact that states of particles, which have fixed quantum numbers are in general not the mass eigenstates. Some examples for mixing are:

1. **Quarks:**
   Creating quark masses with the Yukawa interaction one observes the possibility that in general the mass matrices might not be diagonal, i.e. the flavour eigenstates - defined by their interactions - differ from the mass eigenstates. Diagonalising the mass matrix one finds the Cabibbo-Kobayashi-Maskawa (CKM) matrix [13, 14] in the weak charged interaction. If in the beginning the mass eigenstates are not identical to the flavour eigenstates, then the CKM matrix might have non-diagonal entries. This possibility has now been firmly established by experiment and Kobayashi and Maskawa received 2008 for their findings the Nobel Prize of physics.

2. **Leptons:**
   In analogy to the quark sector one can introduce a lepton mixing matrix, the so-called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [141], which connects the flavour and mass eigenstates.

3. **Elektroweak gauge bosons:**
   Starting with the eigenstates W_1, W_2, W_3 and B of the $SU(2)_L \times U(1)_Y$ gauge symmetry one finds that these states differ from the corresponding mass eigenstates W^+, W^-, Z^0 and A [24]. Glashow, Salam and Weinberg received 1979 the Nobel Prize of physics for the construction of the standard model.

4. **Neutrino oscillations:**
   Since neutrinos exists as free particles - in contrast to quarks - their oscillations can be observed as a kind of macroscopic quantum effect. The first hint for oscillations was found in solar neutrinos:
   For many years considerably less neutrinos were observed [142] from the sun than expected [143]. As one solution it was suggested that the weak eigenstates of the neutrinos, which are produced in the sun differ from the mass eigenstates that propagate on their way to the earth (Neutrino oscillations were suggested by Bruno Pontecorvo [144, 145, 146]). Davis and Koshiba received 2002 the Nobel Prize of physics for the verification of neutrino oscillations.

5. **Neutral Mesons:**
   Mixing was observed as a macroscopic quantum effect in the study of neutral mesons, in particular

1955 K^0-system: Mixing in the neutral K-system was theoretically developed in 1955 by Gell-Mann and Pais [147]. Based on that framework the phenomenon of *regeneration* was predicted in the same year by Pais and Piccioni [148]. Experimentally regeneration was confirmed in 1960 [149]. A huge lifetime difference between the two neutral K-mesons was established already in 1956 [150].

1986 B_d-system: Mixing in the B_d-system was found 1986 by UA1 at CERN [151] (UA1 attributed the result however to B_s mixing) and 1987 by ARGUS at DESY[152]. The large result for the mass difference ΔM_d can be seen as the first clear hint for an (at that time) unexpected large value of the top quark mass[153] [19]. For the decay rate difference currently only upper bounds are available, see [11] for the most recent and most precise bound.

2006/12 B_s-system: The large mass difference in the B_s-system was established by the CDF collaboration at TeVatron [155]. In 2012 the LHCb Collaboration presented at Moriond for the first time a non-vanishing value of the decay rate difference in the B_s-system [156]. In the meantime this quantity is quite precisely known from measurements of LHCb, ATLAS, CMS, D0 and CDF.

2007/12 D^0-system: Here we had several experimental evidences (BaBar, Belle, Cleo, CDF, E791, E831) for values of $\Delta\Gamma/\Gamma$ and $\Delta M/\Gamma$ at the per cent level, but the first single measurement with a statistical significance of more than five standard deviations was done only in 2012 by the LHCb Collaboration [157].

Here we do not consider the neutral pion, which is its own anti particle and we also do not consider excited states of these mesons, because they decay too fast (due to the strong interaction) for mixing to occur.
The mesons denoted by K^0, D^0, B_d^0 and B_s^0 are defined by their quark content, therefore they are called flavour eigenstates. Due to the weak interaction transitions between the flavour eigenstates of the neutral mesons and their antiparticles are possible. Now again the mass eigenstates differ

[19] To avoid a very large value of the top quark mass, also different new physics scenarios were investigated, in particular a scenario with a heavy fourth generation of fermions and a top quark mass of the order of 50 GeV, see e.g. [154].

from the flavour eigenstates. Mixing leads to mass differences of the neutral mesons with macroscopic oscillations lengths, so we have here a real macroscopic quantum effect.

Below we discuss the latter three examples a little more in detail.

8.2 Weak gauge bosons

The relation between the interaction eigenstates W_1, W_2, W_3 - from SU(2)$_L$ and B from U(1)$_Y$ and the mass eigenstates W^+, W^-, Z^0 (intermediate vector bosons) and A (photon) is given by

$$\begin{pmatrix} W^+ \\ W^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} W^1 \\ W^2 \end{pmatrix} , \tag{234}$$

$$\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\,\mu} \end{pmatrix} , \tag{235}$$

with the Weinberg angle θ_W, which was introduced by Glashow in 1961 (*w* = weak). The numerical value of the Weinberg angle is an important observable of the standard model. It can be measured very precisely and also be calculated very precisely, thus providing a stringent consistency check of the model. The actual value of the weinberg angle depends on the concrete renormalisation procedure used. In the $\overline{MS}$ scheme one finds [9]:

$$\sin^2(\theta_W) = 0.2312 \pm 0.0001 \Rightarrow \sin(\theta_W) \approx 0.48 \Rightarrow \theta_W \approx 0.50 \approx 28.7^\circ \tag{236}$$

8.3 Neutrino oscillations

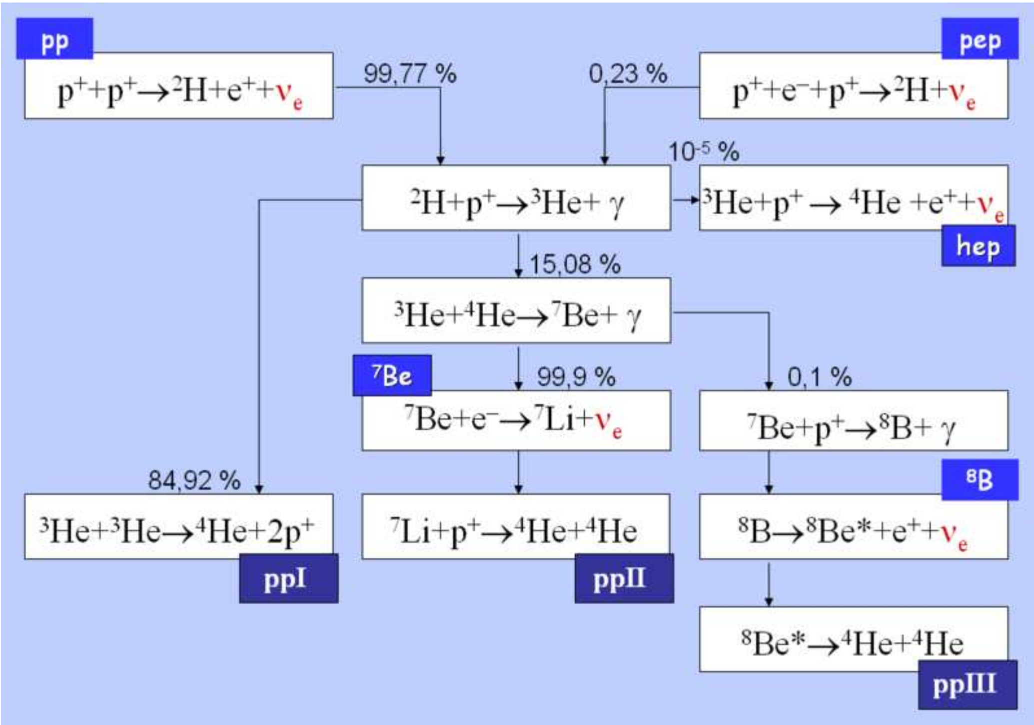
We explain the concept of neutrino oscillations with the example of solar neutrinos:

<u>Production in the sun:</u>

Neutrinos are produced in the sun by the weak interaction.

$$4p \to^4 \text{He} + 2e^+ + 2\nu_e \tag{237}$$

In more detail the production mechanism looks like



Thus the fundamental production process is an inverse β-decay: $p \to n + e^+ + \nu_e$ or on quark level $u \to d + e^+ + \nu_e$. The corresponding Feynman diagram reads

Feynman diagram

The produced neutrino, we denote it by ν_e is defined by its coupling (together with the positron) to the force carrier of the weak interaction, the W^+ boson. Hence we call ν_e the weak (interaction) eigenstate. Naively we would expect that ν_e has also a definite mass, but quantum mechanics allows that the basis of weak eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ differs from the basis of mass eigenstates, which we we denote by ν_1, ν_2 and ν_3. Such a difference results in an interesting effect, that we will derive below.

Propagation:

For simplicity we explain only the mixing of two neutrino flavours. The general relation (quantum mechanical basis transformation) between weak and mass eigenstates reads

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} . \tag{238}$$

The electron neutrino which was produced as a weak eigenstate is a linear combination of the mass eigenstates ν_1 and ν_2.

$$\nu_e = \cos\theta \cdot \nu_1 + \sin\theta \cdot \nu_2 \ . \tag{239}$$

In the vacuum these two eigenstates will propagate with the corresponding masses m_1 and m_2.

$$\begin{aligned} \nu_1(t) &= \nu_1(0) \cdot e^{im_1 t} \ , && (240) \\ \nu_2(t) &= \nu_2(0) \cdot e^{im_2 t} \ . && (241) \end{aligned}$$

Due to their different wavelength the relative composition of the original electron neutrino in terms of ν_1 and ν_2 will change over time.

$$\begin{aligned} \nu_e(t) &= \cos\theta \cdot \nu_1(t) + \sin\theta \cdot \nu_2(t) && (242) \\ &= \cos\theta \cdot \nu_1(0) \cdot e^{iE(m_1)t} + \sin\theta \cdot \nu_2(0) \cdot e^{iE(m_2)t} \ . && (243) \end{aligned}$$

This can again be expressed in terms of ν_e and ν_μ.

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{244}$$

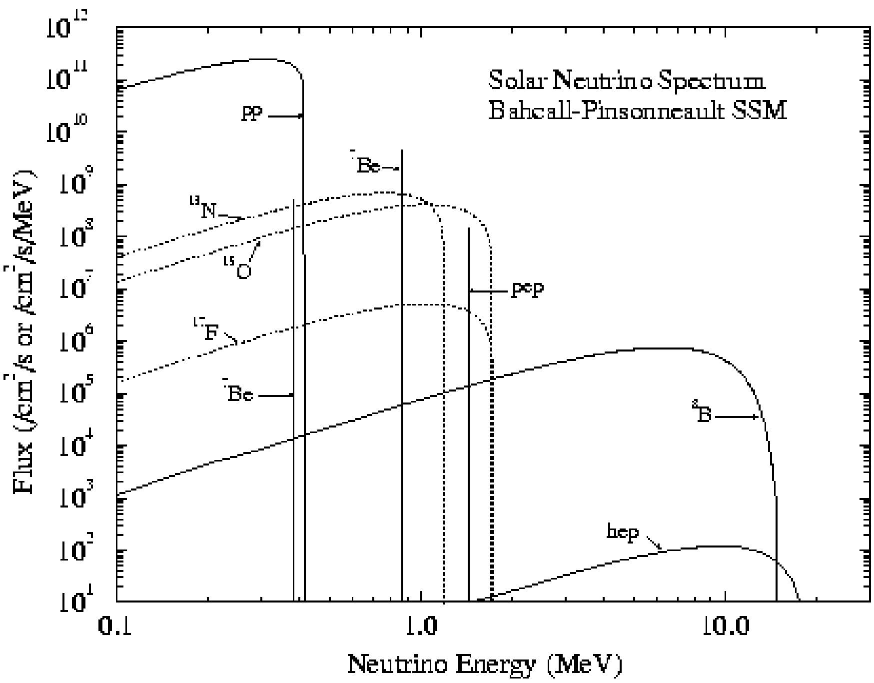
and one obtains

$$\begin{aligned} \nu_e(t) &= \cos\theta \left(\cos\theta \cdot \nu_e(0) - \sin\theta \cdot \nu_\mu(0)\right) e^{iE(m_1)t} \\ &\quad + \sin\theta \left(\sin\theta \cdot \nu_e(0) + \cos\theta \cdot \nu_\mu(0)\right) e^{iE(m_2)t} && (245) \\ &= \left(\cos^2\theta \cdot e^{iE(m_1)t} + \sin^2\theta \cdot e^{iE(m_2)t}\right) \nu_e(0) \\ &\quad + \cos\theta \sin\theta \left(e^{iE(m_2)t} - e^{iE(m_1)t}\right) \nu_\mu(0) \ . && (246) \end{aligned}$$

From this formula one can read off, that the electron neutrino can oscillate in a muon neutrino, **if** $m_1 \neq m_2$ **and** $\theta \neq 0$.
The probability for the change of a flavour a to a flavour b is given by

$$\begin{aligned} P(\nu_a \to \nu_b) &= \left|\langle \nu_a(t) | \nu_b(0) \rangle\right|^2 \\ &= \left|\cos\theta \sin\theta \left(e^{iE(m_2)t} - e^{iE(m_1)t}\right)\right|^2 \\ &= \frac{1}{2} \sin^2 2\theta \left\{1 - \cos\left[E(m_1) - E(m_2)\right] t\right\} \\ &= \ldots \\ &= \sin^2(2\theta) \cdot \sin^2\left(\frac{m_2^2 - m_1^2}{4} \frac{L}{E}\right) \ . && (247) \end{aligned}$$

The energy E of the neutrinos depends on the creation process.



L corresponds to the distance between creation and detection, which is more or less the distance of the sun and the earth. The remaining two parameters of the mixing formulae are

- **Mixing angle** θ
  In the lepton sector we have an analogue of the CKM matrix - the Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS). Its entries are determined by the results of different neutrino oscillation experiments.

- **Difference of squared masses** $m_2^2 - m_1^2$
  The neutrino mass is a fundamental parameter of nature, it also can have cosmological consequences.

Detection on the earth:

The detection of the neutrino also proceeds via the weak interaction, i.e. the detection is only sensitive to the weak eigenstate. Any charges reaction that involves a electron can only detect a solar electron neutrino, but not a muon neutrino (compare tagging). Such experiments were e.g.

$$\begin{aligned} \mathrm{Cl}^{37} + \nu_e &\rightarrow \mathrm{Ar}^{37} + e^- \quad \text{Davies, Homestoke} \\ n + \nu_e &\rightarrow p + e^- \end{aligned}$$

$$\begin{aligned}\mathrm{Ga}^{71} + \nu_e &\to \mathrm{Ge}^{71} + e^- \quad \text{Gallex, Sage, GNO} \\ n + \nu_e &\to p + e^-\end{aligned}$$

The result was that always too few electron neutrinos were found. This was the so-called solar neutrino problem.
The SNO experiment had different detection channels: one channel that was also only sensitive to electron neutrino - here they found too few event, but also channels that were sensitive to all three neutrino flavours (neutral current) - here they found the expected number of neutrinos.
Proof of neutrino oscillations!
Current data
Our current (PDG 2014) knowledge about neutrino mixing can be summarised as [9]

$$\Delta m^2_{sun} \approx 7.54 \pm 0.24 \cdot 10^{-5}\mathrm{eV}^2 \tag{248}$$
$$\Delta m^2_{atm} \approx 2.43 \pm 0.06 \cdot 10^{-3}\mathrm{eV}^2 \tag{249}$$
$$\sin^2\theta_{12} \approx 0.308 \pm 0.017 \Rightarrow \theta_{12} = 33.7^\circ \tag{250}$$
$$\sin^2\theta_{23} \approx 0.455 \pm 0.035 \Rightarrow \theta_{23} = 42.4^\circ \tag{251}$$
$$\sin^2\theta_{13} \approx 0.0234 \pm 0.0020 \Rightarrow \theta_{13} = 8.8^\circ \tag{252}$$

9 Mixing of neutral mesons

9.1 General Introduction

Neutral mesons like B_d^0 and their anti particles $\bar{B}_d^0$ form a two state system, which can be described with a Schrödinger like equation

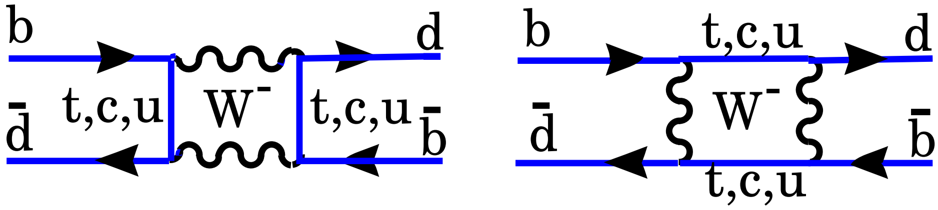
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} B_d^0 \\ \bar{B}_d^0 \end{pmatrix} = \hat{H}\begin{pmatrix} B_d^0 \\ \bar{B}_d^0 \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & 0 \\ 0 & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} B_d^0 \\ \bar{B}_d^0 \end{pmatrix} \tag{253}$$

This is equivalent to the following time evolution of B mesons:

$$\Rightarrow B_i(t) = e^{\frac{1}{i\hbar}\left(M_{ii} - \frac{i}{2}\Gamma_{ii}\right)t} = e^{\frac{1}{i\,\hbar}M_{ii}t}e^{-\frac{1}{2\,\hbar}\Gamma_{ii}t} \tag{254}$$

- $M_{11,22}$ is the mass of the particle
- $\Gamma_{11,22}$ is the decay rate of the particle
- CPT invariance implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

Due to the weak interaction, however, transitions of a B_d^0-meson to a $\bar{B}_d^0$ are possible via the so-called **box diagrams**.



The box diagrams lead to off-diagonal terms in the Hamiltonian

$$\hat{H} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Γ_{12} corresponds to intermediate on-shell states, like $(c\bar{c})$, while M_{12} corresponds to virtual intermediate, i.e. off-Shell states. Therefore the top quark as well as other hypothetical new physics particles contribute only to M_{12}. Thus we are are left with non-diagonal mass matrices and decay rate matrices. A non-diagonal mass matrix means simply that the flavour eigenstates of the mesons are not mass eigenstates.
CPT invariance implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$ and hermiticity gives $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{12}^*$.
In order to obtain meson states we simply have to diagonalise H, we get then new eigenstates, which we denote by the index H=Heavy and L=Light.

$$\begin{aligned} B_H &= pB_d^0 - q\bar{B}_d^0 \\ B_L &= pB_d^0 + q\bar{B}_d^0 \end{aligned} \tag{255}$$

with $p = p(M_{12}, \Gamma_{12})$ and $q = q(M_{12}, \Gamma_{12})$. The new eigenstates B_H and B_L have now definite masses M_H, M_L and definite decay rates Γ_H and Γ_L. By diagonalisation one gets the following observables

$$\begin{aligned} \Delta\Gamma &= \Gamma_L - \Gamma_H = \Delta\Gamma(M_{12}, \Gamma_{12}) \\ \Delta M &= M_H - M_L = \Delta M(M_{12}, \Gamma_{12}) \,, \end{aligned} \tag{256}$$

where the following relations hold

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4\,|M_{12}|^2 - |\Gamma_{12}|^2 \,, \tag{257}$$

$$\Delta M \cdot \Delta\Gamma = -4\text{Re}\,(M_{12}\Gamma_{12}^*) \,, \tag{258}$$

$$\frac{p}{q} = -\frac{\Delta M + \frac{i}{2}\Delta\Gamma}{2M_{12} - i\Gamma_{12}} \,. \tag{259}$$

Later on we will discuss the solutions of these euqations for different systems. Now we can derive in the same way as we did for the neutrino the time evolution of the B mesons. For the mass eigenstates the time evolution is trivial

$$|B_{H,L}(t)\rangle = e^{-\left(iM_{H,L}+\Gamma_{H,L}/2\right)t}|B_{H,L}(0)\rangle \ . \tag{260}$$

For the flavour eigenstates it reads

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle \ , \tag{261}$$

$$|\bar{B}^0(t)\rangle = \frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle \ , \tag{262}$$

with the coefficients

$$g_+(t) = e^{-imt}e^{-\Gamma/2t}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} - i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] , \tag{263}$$

$$g_-(t) = e^{-imt}e^{-\Gamma/2t}\left[-\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] \tag{264}$$

Here we used the averaged masses m and decay rates Γ:

$$m = \frac{M_H + M_L}{2}, \qquad \Gamma = \frac{\Gamma_H + \Gamma_L}{2} \ . \tag{265}$$

$g_+(t)$ and $g_-(t)$ give directly the probability for mixing and non-mixing:

$$\left|\langle B^0|B^0(t)\rangle\right|^2 = |g_+(t)|^2 = \left|\langle \bar{B}^0|\bar{B}^0(t)\rangle\right|^2 \ , \tag{266}$$

$$\left|\langle \bar{B}^0|B^0(t)\rangle\right|^2 = \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \ . \tag{267}$$

The arguments of the trigonometric and hyperbolic functions can be rewritten as

$$\frac{\Delta M t}{2} = \frac{1}{2}x\frac{t}{\tau} \text{ with } x := \frac{\Delta M}{\Gamma} \tag{268}$$

$$\frac{\Delta\Gamma t}{4} = \frac{1}{2}y\frac{t}{\tau} \text{ with } y := \frac{\Delta\Gamma}{2\Gamma} \ , \tag{269}$$

where the lifetime τ is related to the total decay rate Γ via $\tau = 1/\Gamma$. The oscillation length of the trigonometric functions can be determined via

$$\frac{\Delta M t}{2} = \pi \Rightarrow t = \frac{2\pi}{\Delta M} \tag{270}$$

$$\Rightarrow x = vt' = \beta\gamma ct = \beta\gamma\frac{2\pi c}{\Delta M} \ . \tag{271}$$

9.2 Experimental results for the different mixing systems:

After huge experiemental efforts, that are stil going on, the following values for the mixing parameters were obtained

	K^0	D^0	B_d	B_s
ΔM in ps^{-1}	$5.293 \pm 0.009 \cdot 10^{-3^b}$	0.00999756^c	0.510 ± 0.003^a	17.757 ± 0.021^a
ΔM in eV	$3.484 \pm 0.006 \cdot 10^{-6^b}$	$6.58051 \cdot 10^{-6^c}$	$3.35688 \cdot 10^{-4^c}$	0.0116879^c
$x = \frac{\Delta M}{\Gamma}$	0.946^c	$0.0041^{+0.0014^a}_{-0.0015}$	0.7752^c	26.7953^c
$\frac{2\pi c}{\Delta M}$ in mm	$356^c\beta\gamma$	$188.411^c\beta\gamma$	$3.69343^c\beta\gamma$	$0.106079^c\beta\gamma$
$\Delta\Gamma$ in ps^{-1}	0.0111^c	0.0307242^c	$0.000657895 \pm 0.00657895^c$	0.081 ± 0.006^a
$\frac{\Delta\Gamma}{\Gamma}$	1.99^c	0.0126^c	0.002 ± 0.020^a	0.122229^c
$y = \frac{\Delta\Gamma}{2\Gamma}$	1.00^c	0.0063 ± 0.0007^a	0.001 ± 0.0010^c	0.0611145^c
$\frac{2\pi c}{\Delta\Gamma}$ in mm	$169^c\beta\gamma$	$61.3084^c\beta\gamma$	$> 260.286\beta\gamma$	$23.255^c\beta\gamma$
$\Delta\Gamma/\Delta M$	2.1063^c	3.07317^c	$0.00128999 \pm 0.0128999^c$	0.00456158^c

^{a}: HFAG: March 2015; ^{b}: PDG: June 2013; ^{c}: derived by myself, no error estimate.

Exercise:

Update the above table with the following new inputs from HFAG 2014

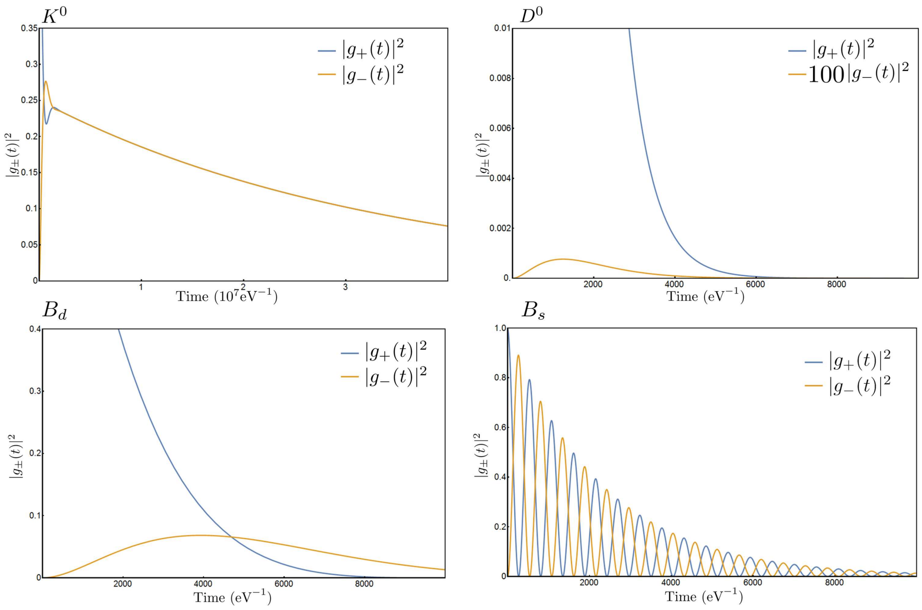
$$\Delta M_d = 0.510 \pm 0.003 ps^{-1} \quad (272)$$
$$\Delta M_s = 17.757 \pm 0.021 ps^{-1} \quad (273)$$
$$x_D = 0.41^{+0.14}_{-0.15} \quad (274)$$
$$y_D = 0.63 \pm 0.07 \quad (275)$$

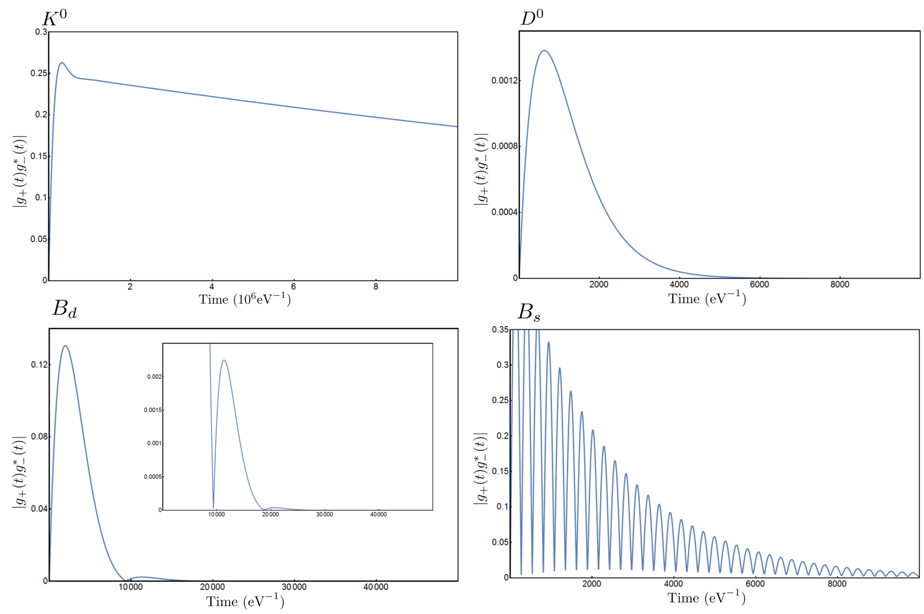
At this stage some comments are in order:

1. The kaon system is special, because kaons can decay hadronically only into 2 pions or 3 pions and there is a huge phase space difference for these final states. The physical kaon states are almost CP eigenstates and the 2 pion and the the 3 pion final state differs in the CP quantum number. Therefore K_L has only a very small phase space - and therefore lives much longer - compared to K_S.

2. For all other neutral mesons there is plenty of phase space for final states with different CP quantum numbers. Nevertheless we have e.g. in x a large range a values. Where does the ratio $x_{B_s^0}/x_{D^0} \approx 26.63/0.0063 \approx 42 \cdot 10^2$ come from?

3. Having this numerical values at hand, we can now compare time evolution for the different neutral mesons by ploting $|g_+(t)|^2$, $|g_-(t)|^2$ and $|g_+(t)g_-(t)|^2$





4. Comparison of the absolute values of the mass differences

	ΔM in ps^{-1}	ΔM in eV	$2\pi c/\Delta M$ in mm$\beta\gamma$
B_s	17.7	0.01	0.1
B_d	0.5	0.0003	3.7
D^0	0.02	0.00001	123
K^0	0.005	0.000003	356

This clearly shows that mixing is a macroscopic quantum effect, the oscil-

lation lengths vary between 0.1 and 356 mm $\beta\gamma$.
Where do the large differences in the size of the mixing parameters - a factor of more than 3500 - come from?

5. Comparison of the absolute values of the decay rate differences

	$\Delta\Gamma$in ps^{-1}	$2\pi c/\Delta\Gamma$in mm$\beta\gamma$
B_s	0.089	21
D^0	0.037	52
K^0	0.011	169
B_d	< 0.010	> 190

This again shows that mixing is a macroscopic quantum effect, the oscillation lengths vary between 21 and more than 190 mm $\beta\gamma$.
The differences in the absolute values are now less pronounced, a factor of more than 8.9.

6. Comparison of the relative values of the mass differences

	$\Delta M/\Gamma$
B_s	26.6
K^0	0.95
B_d	0.77
D^0	0.0063

Where do the large differences in the size of the mixing parameters - a factor of more than 4200 - come from?

7. Comparison of the relative values of the decay rate differences

	$\Delta\Gamma/\Gamma$
K^0	2
B_s	0.13
D^0	0.015
B_d	< 0.015

Where do the large differences in the size of the mixing parameters - a factor of more than 100 - come from?

8. Comparison of decay rate difference vs. mass difference

	$\Delta\Gamma/\Delta M$
D^0	2.4
K^0	2.1
B_d	< 0.02
B_s	0.0050

Exercise:
Produce nice plots for the different systems

9.3 Standard model predictions for mixing of neutral mesons

9.3.1 Observables

In the B_q^0-system $\Gamma_{12}^q \ll M_{12}^q$ holds, therefore one can simplify the expressions for $\Delta\Gamma_q$ and ΔM_q. We get

$$\Delta M_q = 2|M_{12}^q| \left(1 - \frac{1}{8}\frac{|\Gamma_{12}^q|^2}{|M_{12}^q|^2}\sin^2\phi_q + ...\right) \tag{276}$$

$$\approx 2|M_{12}^q| \,, \tag{277}$$

$$\Delta\Gamma_q = 2|\Gamma_{12}^q|\cos\phi_q \left(1 + \frac{1}{8}\frac{|\Gamma_{12}^q|^2}{|M_{12}^q|^2}\sin^2\phi_q + ...\right) \tag{278}$$

$$\approx 2|\Gamma_{12}^q|\cos\phi_q \,, \tag{279}$$

with the weak mixing phase $\phi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$. There was actually a lot of confusion related to the definition of this phase, see [158]. The weak mixing phase appears also in the flavour-specific or semi leptonic CP asymmetries. A flavour specific decay $B_q^0 \to f$ is defined by

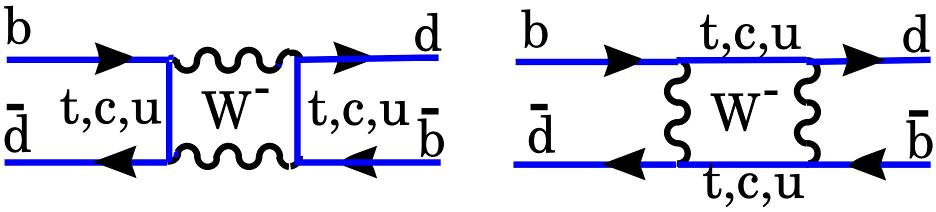
- $\bar{B}_q^0 \to f$ and $B_q^0 \to \bar{f}$ are forbidden.
- No direct CP violation arises, i.e. $|\langle f|B_q^0\rangle| = |\langle\bar{f}|\bar{B}_q^0\rangle|$

Example for flavour-specific decays are e.g. $B_s^0 \to D_s^-\pi^+$ or $B_q^0 \to Xl\nu$ - therefore the second name. The asymmetry reads

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\overline{B}_q(t)\to f) - \Gamma(B_q(t)\to\overline{f})}{\Gamma(\overline{B}_q(t)\to f) + \Gamma(B_q(t)\to\overline{f})} = -2\left(\left|\frac{q}{p}\right| - 1\right)$$

$$= \left|\frac{\Gamma_{12}}{M_{12}}\right|\sin\phi \left(= \text{Im}\frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M}\tan\phi\right) \tag{280}$$

Reminder: the mixing stems from the box diagrams:



M_{12} is the dispersive part (sensitive to heavy internal particles) and Γ_{12} is the absorptive part (sensitive to light internal particles) of these box diagrams.
Below we discuss the calculation of M_{12} and Γ_{12}

Exercise:

Plot of Box diagrams for Γ_{12} and M_{12} for all four systems = 12 diagrams
Search for all the different conventions for ϕ_q

9.3.2 First estimates

In the following we will explain our theoretical tools to calculate Γ_{12}^q and M_{12}^q. One big goal of flavor physics is the search for new physics. We have currently some hints for deviations of measurements from the SM predictions. Therefore we really have to make sure that we control the SM predictions, in particular the hadronic effects.
First we look at all the diagrams contributing to M_{12}^d. For each topology we get nine contributions

$$\begin{aligned} M_{12}^d &= \lambda_u^2 F(u,u) + \lambda_u\lambda_c F(u,c) + \lambda_u\lambda_t F(u,t) + \\ &\quad \lambda_c\lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c\lambda_t F(c,t) + \\ &\quad \lambda_t\lambda_u F(t,u) + \lambda_t\lambda_c F(t,c) + \lambda_t^2 F(t,t) \end{aligned} \tag{281}$$

with the CKM structures $\lambda_q = V_{qd}V_{qb}^*$.
Next we can use unitarity of the CKM matrix ($\lambda_u + \lambda_c + \lambda_t = 0$) to eliminate λ_u.

$$\begin{aligned} M_{12}^d &= \lambda_c^2 \left(F(c,c) - 2F(u,c) + F(u,u)\right) \\ &\quad + 2\lambda_c\lambda_t \left(F(c,t) - F(u,t) - F(u,c) + F(u,u)\right) \\ &\quad + \lambda_t^2 \left(F(t,t) - 2F(u,t) + F(u,u)\right) \end{aligned} \tag{282}$$

Doing the loop calculation one finds

$$F(p,q) = f_0 + f(m_q, m_p)\,, \tag{283}$$

with a large constant value f_0 and a mass dependent term $f(m_q, m_p)$ that grows with the mass. Thus one finds that f_0 cancels in M_{12} due to GIM cancellation. If all internal masses would be equal (or zero), M_{12} would vanish. Looking at the

CKM hierarchy we find

$$\lambda_c^2 \propto \lambda^6 , \tag{284}$$
$$\lambda_c \lambda_t \propto \lambda^6 , \tag{285}$$
$$\lambda_t^2 \propto \lambda^6 , \tag{286}$$

so all three contribution have a similar size of the CKM factors, but the first two terms are strongly GIM suppressed [159]. To a good approximation we can write

$$M_{12}^d = \lambda_t^2 \left(F(t,t) - 2F(u,t) + F(u,u) \right) = \lambda_t^2 S(m_t) , \tag{287}$$

with the Inami-Lim function $S(m_t)$ [160].
In the B_s-system we have

$$\lambda_c^2 \propto \lambda^4 , \tag{288}$$
$$\lambda_c \lambda_t \propto \lambda^4 , \tag{289}$$
$$\lambda_t^2 \propto \lambda^4 , \tag{290}$$

and we can thus again approximate

$$M_{12}^s = \lambda_t^2 \left(F(t,t) - 2F(u,t) + F(u,u) \right) = \lambda_t^2 S(m_t) , \tag{291}$$

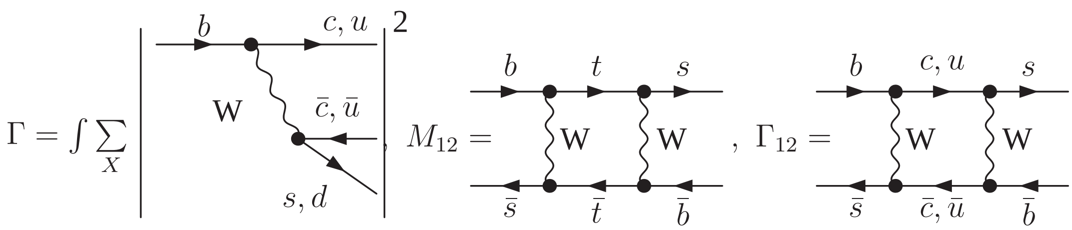
Hence we expects

$$\frac{\Delta M_s}{\Delta M_d} = \frac{1}{\lambda^2} = 25 \tag{292}$$

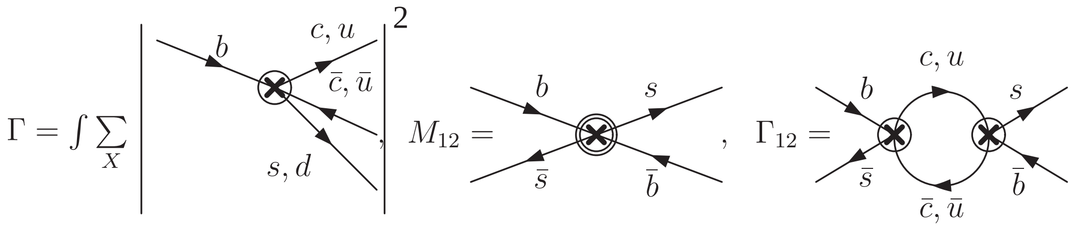
which fits already quite well with the experimental findings.

9.3.3 The SM predictions for mixing quantities

In practice, the calculation of the mixing quantities is still a little more involved. When calculating QCD corrections we will find large logarithms that can be summed up to all orders if we integrate out all heavy particles, i.e. the top-quark and the W boson.
For an illustration we compare now the determination of the total lifetime $\tau_s = 1/\Gamma_s$, M_{12}^s and Γ_{12}^s. These quantities are given by the following diagrams



Integrating out the heavy particles we find



The vertices in the diagrams for Γ and Γ_{12} are effective four-quark operators with $\Delta B = 1$, like

$$Q_2 = \bar{c}\gamma_\mu(1-\gamma_5)b \times \bar{s}\gamma^\mu(1-\gamma_5)c \tag{293}$$

$$=: (\bar{c}b)_{V-A}(\bar{s}c)_{V-A} \; , \tag{294}$$

while the vertex in the diagram for M_{12} is an effective four-quark operator with $\Delta B = 2$

$$Q = (\bar{s}b)_{V-A}(\bar{s}b)_{V-A} \; . \tag{295}$$

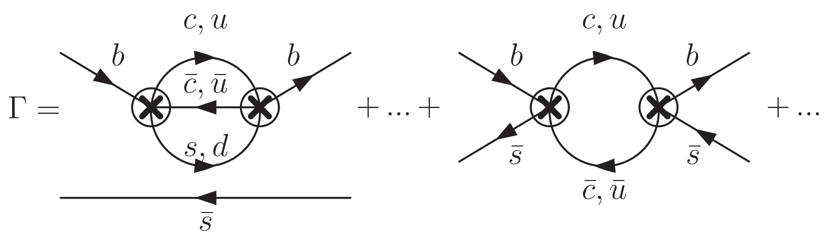
For M_{12} we have now already the final local operator, whose matrix element has to be determined with some non-perturbative QCD-method.
Calculating the box diagram with internal top quarks one obtains

$$M_{12,q} = \frac{G_F^2}{12\pi^2}(V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B \; . \tag{296}$$

The Inami-Lim function $S_0(x_t = \bar{m}_t^2/M_W^2)$ was discussed above. It results from the box diagram without any gluon corrections. The NLO QCD correction is parameterised by $\hat{\eta}_B \approx 0.84$ [161]. The non-perturbative matrix element is parameterised by the bag parameter B and the decay constant f_B

$$\langle \bar{B}_q | Q | B_q \rangle = \frac{8}{3} f_{B_q}^2 B_{B_q}^2 M_{B_q} \; . \tag{297}$$

As a next step we rewrite the expression for Γ in a form that is almost identical to the one of Γ_{12}. With the help of the optical theorem Γ can be rewritten (diagramatically: a mirror reflection on the right end of the decay diagram followed by all possible Wick contractions of the quark lines) in

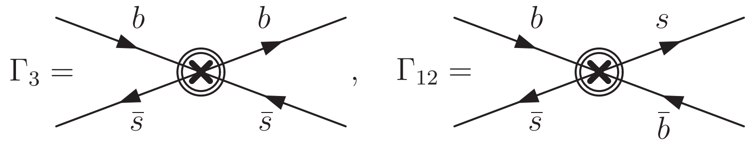


Γ_0 Γ_3

The first term ($=: \Gamma_0$) corresponds to the decay of a free b-quark. This term gives the same contribution to all b-hadrons. The lifetime differences we are interested in will only appear in subleading terms of this expansion like the second diagram ($=: \Gamma_3$), which looks very similar to the diagram for Γ_{12}. Counting the mass dimensions of the external lines one can write formally an expansion of the total decay rate in inverse powers of the heavy quark mass m_b:

$$\Gamma = \Gamma_0 + \frac{\Lambda}{m_b}\Gamma_1 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \dots . \tag{298}$$

The parameter Λ is expected to be of the order of Λ_{QCD}, its actual size can however only be determined by explicit calculation. The expressions for Γ_i and Γ_{12} are however still non-local, so we perform a second OPE (OPE II) using the fact that the b-quark mass is heavier than the QCD scale ($m_b \gg \Lambda_{QCD}$). The OPE II is called the heavy quark expansion (HQE) and was discussed in Section The resulting diagrams for Γ_3 and Γ_{12} look like the final diagram for M_{12}:



Now we are left with local four-quark operators ($\Delta B = 0$ for τ and $\Delta B = 2$ for Γ_{12}). The non-perturbative matrix elements of these operators are expressed in terms of decay constants f_B and bag parameters B. In the standard model one gets one operator for M_{12} (Q) and three operators for Γ_{12} - including Q

$$Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta , \tag{299}$$

$$Q_S = \bar{s}^\alpha (1+\gamma_5) b^\alpha \times \bar{s}^\beta (1+\gamma_5) b^\beta , \tag{300}$$

$$\tilde{Q}_S = \bar{s}^\alpha (1+\gamma_5) b^\beta \times \bar{s}^\beta (1+\gamma_5) b^\alpha , \tag{301}$$

and e.g. four operators for $\tau(B^+)/\tau(B_d)$ [20] - in extensions of the standard model more operators can arise. It turns out that Q, Q_S and $\tilde{Q}_S$ are not independent, so one of them can be eliminated. Historically $\tilde{Q}_S$ was eliminated; later on it turned out that this was a bad choice.

Before discussing the full standard model result let us have a short look at the differences between Γ_{12}^s and Γ_{12}^d. Both quantities have three different CKM contributions

$$\Gamma_{12}^s = -\left[(\lambda_c^s)^2 \Gamma_{12}^{cc,s} + 2\lambda_c^s \lambda_u^s \Gamma_{12}^{uc,s} + (\lambda_u^s)^2 \Gamma_{12}^{uu,s}\right] \tag{302}$$

$$\Gamma_{12}^d = -\left[\left(\lambda_c^d\right)^2 \Gamma_{12}^{cc,d} + 2\lambda_c^d \lambda_u^d \Gamma_{12}^{uc,d} + \left(\lambda_u^d\right)^2 \Gamma_{12}^{uu,d}\right] \tag{303}$$

One sees that in Γ_{12}^s there is the CKM leading contribution $\lambda_c^s \propto \lambda^2$ and thus the expression is dominated by the first term - this will, however, not hold for the imaginary part. On the other hand Γ_{12}^d is CKM subleading ($\lambda_c^d \propto \lambda^3$) and all three contributions seem to be of similar size.

Another way of looking at the mixing systems is the investigation of the ratio Γ_{12}^q/M_{12}^q. In this ratio many of the leading uncertainties cancel, e.g. the factor $(f_{B_q} M_{B_q})^2$, thus one expects - up to different CKM structures - similar results for the B_d and B_s mesons. The three physical mixing observables ΔM_q, $\Delta\Gamma_q$ and a_{sl}^q can be expressed in terms of this clean ratio:

$$a_{sl}^q = \text{Im}\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right), \tag{304}$$

$$\frac{\Delta\Gamma_q}{\Delta M_q} = -\text{Re}\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right). \tag{305}$$

Moreover, the ratio Γ_{12}^q/M_{12}^q can be simplified considerably if the unitarity of the CKM matrix is used, i.e. $\lambda_u + \lambda_c + \lambda_t = 0$

$$-\frac{\Gamma_{12}^q}{M_{12}^q} = \frac{\lambda_c^2 \Gamma_{12}^{cc,q} + 2\lambda_c\lambda_u \Gamma_{12}^{uc,q} + \lambda_u^2 \Gamma_{12}^{uu,q}}{\lambda_t^2 \tilde{M}_{12,q}} \tag{306}$$

$$= \frac{\Gamma_{12}^{cc,q}}{\tilde{M}_{12,q}} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{cc,q} - \Gamma_{12}^{uc,q}}{\tilde{M}_{12,q}} + \left(\frac{\lambda_u}{\lambda_t}\right)^2 \frac{\Gamma_{12}^{cc,q} - 2\Gamma_{12}^{uc,q} + \Gamma_{12}^{uu,q}}{\tilde{M}_{12,q}} \tag{307}$$

$$\approx 10^{-4}\left[(51 \pm 10) - \frac{\lambda_u}{\lambda_t}(10 \pm 2) - \left(\frac{\lambda_u}{\lambda_t}\right)^2 (0.16 \pm 0.03)\right] \tag{308}$$

The three numerical coefficients in (308) are almost identical for the B_d and B_s system. The CKM factor reads $\lambda_u/\lambda_t = -0.008 + 0.021\,i$ in the B_s system and $\lambda_u/\lambda_t = -0.033 - 0.439\,i$ in the B_d system. Hence the real part of Γ_{12}^q/M_{12}^q and

[20] This statements hold only at order $1/m_b^3$.

thus $\Delta\Gamma_q/\Delta M_q$ is dominated by the first coefficient. It is interesting to note, that a knowledge of $\Gamma_{12}^{cc,d}$ is sufficient to get a precise SM value of $\Delta\Gamma_d$ via the relation

$$\Delta\Gamma_d = -\mathrm{Re}\left(\frac{\Gamma_{12}^d}{M_{12}^d}\right)\Delta M_d^{\mathrm{Exp.}} \ , \tag{309}$$

while one needs all three diagrams $\Gamma_{12}^{cc,d}$, $\Gamma_{12}^{uu,d}$ and $\Gamma_{12}^{uc,d}$, if one determines $\Delta\Gamma_d$ via the relation

$$\Delta\Gamma_d = 2\left|\Gamma_{12}^d\right|\cos(\phi_d) \ . \tag{310}$$

Moreover, an imaginary part can only appear in (308) in the second and third contribution, which therefore describes the semi-leptonic CP asymmetries, whose final sizes are given by the values of the CKM elements. In the B_s system the CKM factor has a small imaginary value and a_{sl}^s gets therefore a small numerical value. The third term in (308) is negligible in the B_s system. In the B_d system the CKM ratio is larger and it has a sizable imaginary part – it is about a factor of 20 larger than in the B_s-system – giving rise to a semi-leptonic CP asymmetry in the B_d sector that is also about 20 times larger than the one in the B_s system.
The full expression for Γ_{12} can be expanded as

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + ...\right) + \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + ...\right) + ... \ . \tag{311}$$

Each of the $\Gamma_i^{(0)}$ is a product of perturbative Wilson coefficients and non-perturbative matrix elements. In Γ_3 these matrix elements arise from dimension 6 four-quark operators, in Γ_4 from dimesion 7 operators and so on.
The leading term $\Gamma_3^{(0)}$ was calculated already quite long ago [19, 162, 163, 164, 165, 166] The $1/m_b$-corrections ($\Gamma_4^{(0)}$) were determined in [167] and they turned out to be quite sizeable. NLO QCD-corrections were done for the first time in [168], they also were quite large. Five years later the QCD-corrections were confirmed and also subleading CKM structures were included [169, 134]. Unfortunately it turned out that $\Delta\Gamma$ is not well-behaved [170]. All corrections are unexpectedly large and they go in the same direction. This problem could be solved by using Q and $\tilde{Q}_S$ as the two independent operators instead of Q and Q_S, so just a change of the operator basis [171]. As an illustration of the improvement we show the expressions for Γ_{12}/M_{12} in the old and the new basis:

$$\frac{\Delta\Gamma_s}{\Delta M_s}^{\text{Old}} = 10^{-4}\cdot\left[0.9 + 40.9\frac{B'_S}{B} - 25.0\frac{B_R}{B}\right] \ , \tag{312}$$

$$\frac{\Delta\Gamma_s}{\Delta M_s}^{\text{New}} = 10^{-4}\cdot\left[46.2 + 10.6\frac{\tilde{B}'_S}{B} - 11.9\frac{B_R}{B}\right] \ , \tag{313}$$

where B'_S is the bag parameter of Q_S, $\tilde{B}'_S$ is the bag parameter of $\tilde{Q}_S$ and B_R denotes the bag parameters of the dimension 7 operators. Now the term that is completely free of any non-perturbative uncertainties is numerical dominant. Moreover the $1/m_b$-corrections became smaller and undesired cancellations are less pronounced. For more details we refer the reader to [171]. Currently also $1/m_b$-corrections for the subleading CKM structures in Γ_{12} [172] and $1/m_b^2$-corrections for $\Delta\Gamma_s$ [173] are available - they are relatively small.

9.3.4 Numerical Results

Mixing:
The mixing quantities have been re-investigated recently [175] (update of [171]). Numerically we obtain for the mass differences

$$\Delta M_d^{\rm SM} = 0.54 \pm 0.09\,\text{ps}^{-1}\,, \qquad \Delta M_s^{\rm SM} = 17.3 \pm 2.6\,\text{ps}^{-1}\,. \tag{314}$$

The mass differences have been measured with great precision at LEP, TeVatron and the B factories[**?**, **?**, **?**, **?**]

$$\Delta M_d = 0.507 \pm 0.005\,\text{ps}^{-1}\,, \qquad \Delta M_s = 17.77 \pm 0.10 \pm 0.07\,\text{ps}^{-1}\,. \tag{315}$$

The numbers agree well, but the theory error is still more than an order of magnitude larger than the experimental error.
For the decay rate differences we obtain the following predictions

$$\Delta\Gamma_d^{\rm SM} = (2.89 \pm 0.72)\cdot 10^{-3}\,\text{ps}^{-1}\,, \qquad \Delta\Gamma_s^{\rm SM} = 0.087 \pm 0.021\,\text{ps}^{-1}\,, \tag{316}$$

$$\left(\frac{\Delta\Gamma_d}{\Gamma_d}\right)^{\rm SM} = (4.11 \pm 0.78)\cdot 10^{-3}\,, \qquad \left(\frac{\Delta\Gamma_s}{\Gamma_s}\right)^{\rm SM} = 0.137 \pm 0.027\,, \tag{317}$$

$$\left(\frac{\Delta\Gamma_d}{\Delta M_d}\right)^{\rm SM} = (53.2 \pm 10.1)\cdot 10^{-4}\,, \qquad \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm SM} = (50.4 \pm 10.1)\cdot 10^{-4}\,. \tag{318}$$

The predictions for $\Delta\Gamma_s/\Gamma_s$ and $\Delta\Gamma_d/\Gamma_d$ are obtained under the assumption that there are no new physics contributions in ΔM_d and ΔM_s. The decay rate differences have not been measured yet, but we have already interesting bounds

$$\left(\frac{\Delta\Gamma_d}{\Gamma_d}\right) = (10 \pm 37)\cdot 10^{-3}\,, \qquad \left(\frac{\Delta\Gamma_s}{\Gamma_s}\right) = 0.092^{+0.052}_{-0.054}\,. \tag{319}$$

Here we are eagerly waiting for more precise results from TeVatron and from LHC!

Finally we present the numerical updates for the mixing phases and the flavor-specific asymmetries

$$\phi_d^{\rm SM} = -0.085 \pm 0.025\,, \qquad \phi_s^{\rm SM} = (4.2 \pm 1.3) \cdot 10^{-3}\,, \tag{320}$$
$$a_{fs}^{d,\rm SM} = (-4.5 \pm 0.8) \cdot 10^{-4}\,, \qquad a_{fs}^{s,\rm SM} = (2.11 \pm 0.36) \cdot 10^{-5}\,. \tag{321}$$

From this list one sees the strong suppression of ϕ and a_{sl} in the standard model. In addition we give also the updated prediction for the dimuon asymmetry and the difference between the two semileptonic CP-asymmetries that will be measured at LHCb

$$A_{SL}^{\rm SM} = -\,(0.22 \pm 0.04) \cdot 10^{-3}\,, \tag{322}$$
$$a_{\rm fs}^{s,\rm SM} - a_{\rm fs}^{d,\rm SM} = (0.47 \pm 0.08) \cdot 10^{-3}\,. \tag{323}$$

We will compare these numbers with experimental data in the new physics section. At that stage it is instructive to look also at the detailed list of the different sources of the theoretical error for observables in the B_s mixing system. We compare this numbers with the corresponding ones from Reference [171] (the table and numerical values are from [175]). For the mass difference we have

ΔM_s	This work	hep-ph/0612167
Central Value	$17.3\,\mathrm{ps}^{-1}$	$19.3\,\mathrm{ps}^{-1}$
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(B)$	2.9%	7.1%
$\delta(m_t)$	1.1%	1.8%
$\delta(\alpha_s)$	0.4%	2.0%
$\delta(\gamma)$	0.3%	1.0%
$\delta(V_{ub}/V_{cb})$	0.2%	0.5%
$\delta(m_b)$	0.1%	$---$
$\sum \delta$	14.0%	34.6%

For the mass difference we observe a considerable reduction of the overall error from 34.6% in 2006 to 14%. This is mainly driven by the progress in the lattice determination of the decay constant and the bag parameter B. To further improve the accuracy we need more precise values of the decay constant.

For the decay rate difference we get

$\Delta\Gamma_s$	This work	hep-ph/0612167
Central Value	$0.087\,\text{ps}^{-1}$	$0.096\,\text{ps}^{-1}$
$\delta(B_{R_2})$	17.2%	15.7%
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(\mu)$	7.8%	13.7%
$\delta(B_3)$	4.8%	3.1%
$\delta(B_{R_0})$	3.4%	3.0%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(B)$	2.7%	6.6%
$\delta(B_{\tilde{R}_1})$	1.9%	$---$
$\delta(\bar{z})$	1.5%	1.9%
$\delta(m_s)$	1.0%	1.0%
$\delta(B_{R_1})$	0.8%	$---$
$\delta(B_{\tilde{R}_3})$	0.5%	$----$
$\delta(\alpha_s)$	0.4%	0.1%
$\delta(\gamma)$	0.3%	1.0%
$\delta(B_{R_3})$	0.2%	$---$
$\delta(V_{ub}/V_{cb})$	0.2%	0.5%
$\delta(m_b)$	0.1%	1.0%
$\sum\delta$	24.5%	40.5%

For the decay rate difference we also find a strong reduction of the overall error from 40.5% in 2006 to 24.5%. This is again due to our more precise knowledge about the decay constant and the bag parameter B, but also from our change to the $\overline{MS}$-scheme for the quark masses, which leads to a sizeable reduction of the renormalisation scale dependence. In [171] we were using in addition the pole scheme, and our numbers and errors were averages of these two quark mass schemes. It is very interesting to note, that now the dominant uncertainty stems from the value of the matrix element of the power suppressed operator $\tilde{R}_2$.
To further improve the accuracy a non-perturbative determination of $B_{\tilde{R}_2}$ and B_2 as well as a more precise value of f_{B_s} is mandatory. In addition the calculation of the α_s/m_b and the α_s^2-corrections will reduce the μ-dependence.

For the ratio of $\Delta\Gamma$ ΔM the decay constant cancels.

$\Delta\Gamma_s/\Delta M$	This work	hep-ph/0612167
Central Value	$50.4 \cdot 10^{-4}$	$49.7 \cdot 10^{-4}$
$\delta(B_{R_2})$	17.2%	15.7%
$\delta(\mu)$	7.8%	9.1%
$\delta(B_3)$	4.8%	3.1%
$\delta(B_{R_0})$	3.4%	3.0%
$\delta(B_{\tilde{R}_1})$	1.9%	$---$
$\delta(\bar{z})$	1.5%	1.9%
$\delta(m_b)$	1.4%	1.0%
$\delta(m_t)$	1.1%	1.8%
$\delta(m_s)$	1.0%	0.1%
$\delta(\alpha_s)$	0.8%	0.1%
$\delta(B_{R_1})$	0.8%	$---$
$\delta(B_{\tilde{R}_3})$	0.5%	$----$
$\delta(B_{R_3})$	0.2%	$---$
$\delta(B)$	0.1%	0.5%
$\delta(\gamma)$	0.0%	0.1%
$\delta(V_{ub}/V_{cb})$	0.0%	0.1%
$\delta(V_{cb})$	0.0%	0.0%
$\sum \delta$	20.1%	18.9%

For the ratio of $\Delta\Gamma/\Delta M$ we do not have any improvement. The decay constant cancels out in that ratio and therefore we did not profit from the progress in lattice simulations. Also the CKM dependence cancels to a large extent. The improvement in the renormalisation scale dependence is less pronounced than in $\Delta\Gamma$ alone.

To improve the precision, we have to improve the precision on $\Delta\Gamma$ as described above (except for the decay constant).

For the semileptonic CP-asymmetries we get

a_{fs}^s	This work	hep-ph/0612167
Central Value	$2.11 \cdot 10^{-5}$	$2.06 \cdot 10^{-5}$
$\delta(V_{ub}/V_{cb})$	11.6%	19.5%
$\delta(\mu)$	8.9%	12.7%
$\delta(\bar{z})$	7.9%	9.3%
$\delta(\gamma)$	3.1%	11.3%
$\delta(B_{\tilde{R}_3})$	2.8%	2.5%
$\delta(m_s)$	2.0%	3.7%
$\delta(\alpha_s)$	1.8%	0.7%
$\delta(B_{R_3})$	1.2%	1.1%
$\delta(m_t)$	1.1%	1.8%
$\delta(B_3)$	0.6%	0.4%
$\delta(B_{R_0})$	0.3%	$---$
$\delta(B_{\tilde{R}_1})$	0.2%	$---$
$\delta(B)$	0.2%	0.6%
$\delta(m_s)$	0.1%	0.1%
$\delta(B_{R_2})$	0.1%	$---$
$\delta(B_{R_1})$	0.0%	$---$
$\delta(V_{cb})$	0.0%	0.0%
$\sum \delta$	17.3%	27.9%

Finally we also have a large improvement for the flavor specific asymmetries. The overall error went down from 27.9% to 17.3%. In a_{fs} also the decay constant cancels, but in contrast to $\Delta\Gamma/\Delta M$ we now have a strong dependence on the CKM elements. Here we benefited from more precise values of the CKM values and also from a more sizeable reduction of the renormalisation scale dependence. Here a further improvement in the CKM values of V_{ub} and the charm quark mass will help, as well as the reduction of the μ-dependence via the calculation of higher order terms.

Lifetimes:

While the theoretical framework for the determination of the mass differences is very solid, the applicability of the HQE for Γ_{12} was questioned sometimes in the literature. We will test the HQE with the lifetime ratio of mesons, which are practically free of hadronic uncertainties.

The theoretically best investigated lifetime ratio is τ_{B_s}/τ_{B_d}. Here large cancellations occur so the ratio is expected to be very close to one [175]

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.00^{0}_{-0.004} \,. \tag{324}$$

The theoretically next best lifetime ratio is τ_{B^+}/τ_{B_d}. One obtains [175]

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.044 \pm 0.024\,. \tag{325}$$

NLO-QCD corrections turned out to be important, subleading $1/m_b$-corrections are small. Care has to be taken with the arising matrix elements of the four-quark operators: it turned out that the Wilson coefficients of the colour-suppressed operators are numerically enhanced, see [**?**]. But the matrix elements of these operators are only known with large relative errors. Currently two determinations on the lattice are available [**?**, **?**].
Experimentally we have

$$\begin{aligned}
\frac{\tau(B_s)}{\tau(B_d)} &= 1 - 0.027 \pm 0.015\,, & (326)\\
\frac{\tau(B^+)}{\tau(B_d)} &= 1.081 \pm 0.006\,. & (327)
\end{aligned}$$

we find that the HQE gives numbers that are close to experiment, but to perform a precise comparison of experiment and theory an updated of the bag parameters is mandatory. The state of the art is here already 10 years old! For the B_s life we are waiting for much more precise experimental values.
In Fig. (1) we visualise the theory predictions for these lifetimes ratios. Predictions for the Λ_b have to be taken with more care. In that case the NLO-QCD corrections are not complete and only preliminary lattice values [**?**] are available. A typical value quoted in the literature [**?**] is

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.88 \pm 0.05\,. \tag{328}$$

The lifetime of the doubly heavy meson B_c has been investigated e.g. in [**?**], but only in LO QCD.

$$\tau(B_c)_{\rm LO} = 0.52^{+0.18}_{-0.12}\,\text{ps}\,.$$

In addition to the b-quark now also the c-quark can decay, giving rise to the biggest contribution to the total decay rate.
An interesting quantity is the lifetime ratio of the Ξ_b-baryons, which was investigated in NLO-QCD in [**?**]. This quantity can in principle be determined as precise as τ_{B^+}/τ_{B_d} ($\pm 3\%$). However, up to now the matrix elements for the Ξ_b baryons are not available. Assuming that the matrix elements for Ξ_b are equal to the ones of Λ_b we can give a rough estimate for the expected lifetime ratio. In order to get rid of unwanted $s \to u$-transitions we define (following [**?**])

$$\frac{1}{\bar\tau(\Xi_b)} = \bar\Gamma(\Xi_b) = \Gamma(\Xi_b) - \Gamma(\Xi_b \to \Lambda_b + X)\,. \tag{329}$$

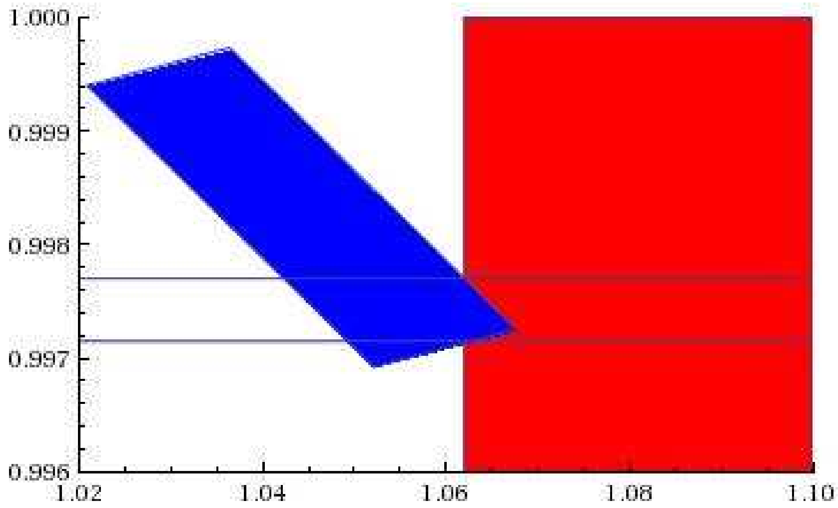


Figure 1: *Allowed range for τ_{B_s}/τ_{B_d} and τ_{B^+}/τ_{B_d}. The prediction for τ_{B^+}/τ_{B_d} derived with the decade-old hadronic parameters of Ref. [**?**] barely overlaps with the experimental 3σ region. This shows the importance of a modern lattice calculation of these parameters.*

Using the preliminary lattice values [**?**] for the matrix elements of Λ_b we obtain

$$\frac{\bar{\tau}(\Xi_b^0)}{\bar{\tau}(\Xi_b^+)} = 1 - 0.12 \pm 0.02 \pm ???\,, \tag{330}$$

where $???$ stands for some unknown systematic errors. As a further approximation we equate $\bar{\tau}(\Xi_b^0)$ to $\tau(\Lambda_b)$ - here similar cancellations arise as in τ_{B_s}/τ_{B_d} - , so we arrive at the following prediction

$$\frac{\tau(\Lambda_b)}{\bar{\tau}(\Xi_b^+)} = 0.88 \pm 0.02 \pm ???\,. \tag{331}$$

The PDG quotes [**?**] the following numbers

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.99 \pm 0.10\,, \qquad \tau(B_c) = 0.453 \pm 0.041\,\text{ps}\,. \tag{332}$$

The situation for the Λ_b-baryon is not settled yet. First several theoretical improvements have to be included, second there are two different experimental numbers on the market [**?**, **?**]. For B_c the number lies in the right ball park, but here also a full NLO-QCD calculation would be desirable to make the comparison more quantitative. Finally we are waiting for a first result for the lifetimes of the Ξ_b-baryons.

9.4 Mixing of D mesons

9.4.1 What is so different compared to the B system?

Let us start with a very naive estimate of contributions to the Box-diagrams ($\lambda \approx 0.2; x_q = \frac{m_q^2}{M_W^2}$)

$$
\begin{array}{ll}
K^0 & : \left\{ \begin{array}{lll} (V_{us}V_{ud}^*)^2 \propto \lambda^2 & x_u \approx 1.5 \cdot 10^{-9} & V_{CKM}S(x_q) \approx 7 \cdot 10^{-11} \\ (V_{cs}V_{cd}^*)^2 \propto \lambda^2 & x_c \approx 0.00035 & V_{CKM}S(x_q) \approx 2 \cdot 10^{-5} \\ (V_{ts}V_{td}^*)^2 \propto \lambda^{10} & x_t \approx 4.8 & V_{CKM}S(x_q) \approx 7 \cdot 10^{-7} \end{array} \right. \\
B_d^0 & : \left\{ \begin{array}{lll} (V_{ub}V_{ud}^*)^2 \propto \lambda^6 & x_u \approx 1.5 \cdot 10^{-9} & V_{CKM}S(x_q) \approx 2 \cdot 10^{-13} \\ (V_{cb}V_{cd}^*)^2 \propto \lambda^6 & x_c \approx 0.00035 & V_{CKM}S(x_q) \approx 4 \cdot 10^{-8} \\ (V_{tb}V_{td}^*)^2 \propto \lambda^6 & x_t \approx 4.8 & V_{CKM}S(x_q) \approx 3 \cdot 10^{-4} \end{array} \right. \\
B_s^0 & : \left\{ \begin{array}{lll} (V_{ub}V_{us}^*)^2 \propto \lambda^8 & x_u \approx 1.5 \cdot 10^{-9} & V_{CKM}S(x_q) \approx 8 \cdot 10^{-15} \\ (V_{cb}V_{cs}^*)^2 \propto \lambda^4 & x_c \approx 0.00035 & V_{CKM}S(x_q) \approx 8 \cdot 10^{-7} \\ (V_{tb}V_{ts}^*)^2 \propto \lambda^4 & x_t \approx 4.8 & V_{CKM}S(x_q) \approx 6 \cdot 10^{-3} \end{array} \right. \\
D^0 & : \left\{ \begin{array}{lll} (V_{cd}V_{ud}^*)^2 \propto \lambda^2 & x_d \approx 6 \cdot 10^{-9} & V_{CKM}S(x_q) \approx 3 \cdot 10^{-10} \\ (V_{cs}V_{us}^*)^2 \propto \lambda^2 & x_s \approx 1 \cdot 10^{-6} & V_{CKM}S(x_q) \approx 4 \cdot 10^{-8} \\ (V_{cb}V_{ub}^*)^2 \propto \lambda^{10} & x_b \approx 0.003 & V_{CKM}S(x_q) \approx 8 \cdot 10^{-10} \end{array} \right.
\end{array}
$$

All contributions are small and of similar size in D-mixing, but It comes worse!!!
Why naive?
In the derivation of the Inami-Lim functions already the unitarity of the CKM-matrix has been used

$$
\begin{aligned}
M_{12} \propto \quad & \lambda_d\lambda_d f(d,d) + \lambda_d\lambda_s f(d,s) + \lambda_d\lambda_b f(d,b) \\
+ \quad & \lambda_s\lambda_d f(s,d) + \lambda_s\lambda_s f(s,s) + \lambda_s\lambda_b f(s,b) \\
+ \quad & \lambda_b\lambda_d f(b,d) + \lambda_b\lambda_s f(b,s) + \lambda_b\lambda_b f(b,b) \qquad (333) \\
= \quad & \lambda_s^2 \left[f(s,s) - 2f(s,d) + f(d,d) \right] \\
+ \quad & 2\lambda_s\lambda_b \left[f(b,s) - f(b,d) - f(s,d) + f(d,d) \right] \\
+ \quad & \lambda_b^2 \left[f(b,b) - 2f(b,d) + f(d,d) \right] \qquad (334) \\
=: \quad & \lambda_s^2 S(x_s) + 2\lambda_s\lambda_b S(x_s,x_b) + \lambda_b^2 S(x_b) \qquad (335)
\end{aligned}
$$

What problems do arise in the charm system?

1. Exact treatment of V_{CKM}
   $\Rightarrow$ Huge GIM cancellation between the 3 contributions

2. $\alpha_s(m_c) \approx \mathcal{O}(50\%)$
   $\Rightarrow$ convergence of QCD perturbative Expansion?

3. Λ/m_c not so small
   $\Rightarrow$ convergence of Heavy Quark Expansions?

4. Exp. $\Gamma_{12} \approx M_{12}$
   Use exact formulae for diagonalisation

How to solve these problems

1. Because of huge cancellations: be very careful with approximation that might seem justified on first sight.

2. Simply try by explicit calculation!

3. Test with charm lifetimes and simply try by explicit calculation.

4. The most easy part: the exact relations read

$$\begin{aligned} (\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2, \\ \Delta M \Delta\Gamma &= 4|M_{12}||\Gamma_{12}|\cos(\phi)\,. \end{aligned} \tag{336}$$

   If $|\Gamma_{12}/M_{12}| \ll 1$, as in the case of the B_s system ($\approx 5 \cdot 10^{-3}$) or if $\phi \ll 1$, one gets the famous approximate formulae

$$\Delta M = 2|M_{12}|\,, \qquad \Delta\Gamma = 2|\Gamma_{12}|\cos\phi\,.$$

   In the D-system $|\Gamma_{12}/M_{12}| \approx 1$ possible — Solve Eigenvalue equation exactly
   A numerical estimate shows: $\Delta\Gamma \leq 2|\Gamma_{12}|$

9.4.2 SM predictions

Theoretical Tools:
There are two approaches to describe the SM contribution to D-mixing. They are state of the art, but they are more an estimate than a calculation

- Exclusive Approach
  Falk, Grossman, Ligeti, Petrov PRD65 (2002)
  Falk, Grossman, Ligeti, Nir, Petrov PRD69 (2004)

- Inclusive Approach
  Georgi, PLB 297 (1992), Ohl, Ricciardi, Simmons, NPB 403 (1993)
  Bigi, Uraltsev, NPB 592 (2001)

$\Rightarrow$ x, y up to 1% not excluded

$\Rightarrow$ Essential no CPV in mixing — unambiguous signal for NP!!!

Comments on the exclusive approach:
y due to final states common to D and $\overline{D}$

$$y = \frac{1}{\Gamma} \sum_n \rho_n \langle \overline{D}^0 | \mathcal{H}_W^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_W^{\Delta C=1} | D^0 \rangle$$

This is much too complicated to calculate exclusive decay rates exactly!

- Estimate only SU(3) violating phase space effects (mild assumptions about $\vec{p}$-dependence of matrix elements) = calculable source of SU(3) breaking
- Assume hadronic matrix elements are SU(3) invariant
- Assume CP invariance of D decays
- Assume no cancellations with other sources of SU(3) breaking
- Assume no cancellations between different SU(3) multipletts

$\Rightarrow$ individual effects of 1% possible: $y^{Exp} \approx 1\% \not\Rightarrow$ NP

- ”our analysis does not amount to a SM calculation of y”

We try to push the inclusive approach to its limit.
Charm lifetimes:
The following is just a naive estimate - a quantitative analysis has to be done!
Experimentally we get relatively large differences in the lifetimes of D mesons:

$$\text{Exp.:}\ \frac{\tau(D^+)}{\tau(D^0)} = \frac{1040\,\text{fs}}{410\,\text{fs}} \approx 2.5 \qquad \frac{\tau(D_s^+)}{\tau(D^0)} = \frac{500\,\text{fs}}{410\,\text{fs}} \approx 1.2$$

We assume now that the HQE can also be applied to charm system and we investigate how large the HQE would have to be in order to reproduce the experimental findings.

- Applying the HQE for D-system we get the following diagramatic contributions
  - D^0: weak annihilation (=WA)
  - D^+, D_s^+: Pauli interference (=PI); PI (D_s^+) $= (V_{us}/V_{ud})^2$ PI (D^+)
- This can be compared with the HQE contributions for the B-system

- B_d, B_s: WA, similar CKM structure, differences due to phase space
- B^+: PI (larger than WA)

According to the HQE the total decay rate can be written as a leading term that describes the decay of a free charm quark and some corrections that depend also on the flavor of the spectator quark.

$$\Gamma(D_x) = \Gamma(c) + \delta\Gamma(D_x)$$

With our above assumptions we easily see that the experimental constraints are full-filled for

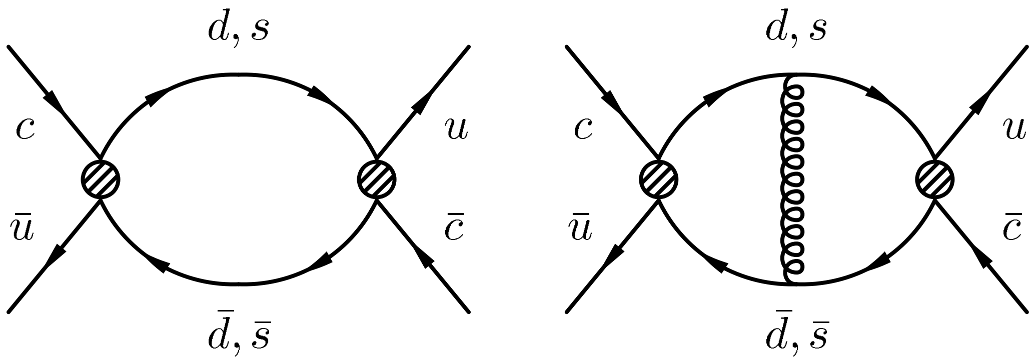
$$\frac{\delta\Gamma(D^+)}{\Gamma(c)} \approx -53\%\,, \qquad \frac{\delta\Gamma(D^0)}{\Gamma(c)} \approx +19\%$$

First of all, the size of the correction is in the expected range, since $(m_b/m_c)^3 \approx 20...30$. Next the expected corrections are large, but not so large that an application of the HQE is a priori meaningless.
Here it would be very valuable to have a real HQE calculation of the lifetime ratios of charm mesons.

9.4.3 HQE for decay rate difference

The problem:

$$\Gamma_{12} = -(\lambda_s^2\Gamma_{ss} + 2\lambda_s\lambda_d\Gamma_{sd} + \lambda_d^2\Gamma_{dd})$$



$$\begin{aligned}
\lambda_d &= V_{cd}V_{ud}^* = -c_{12}c_{23}c_{13}s_{12} - c_{12}^2 c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}\left(\lambda^1 + i\lambda^5\right),\\
\lambda_s &= V_{cs}V_{us}^* = +c_{12}c_{23}c_{13}s_{12} - s_{12}^2 c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}\left(\lambda^1 + i\lambda^7\right),\\
\lambda_b &= V_{cb}V_{ub}^* = c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}\left(\lambda^5 + i\lambda^5\right),
\end{aligned}$$

For $m_s = m_d$ we have an exact cancellation! <u>Approximations are dangerous:</u>
Common folklore $\lambda_b \approx 0$ (looks reasonable!)

$$\text{Unitarity: } \lambda_d + \lambda_s = 0 \Rightarrow \Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}\right)$$

- Γ_{12} **vanishes in the SU(3)$_F$ limit**
  Use the results for B_s-mixing from Beneke, Buchalla, (Greub), A.L., Nierste 1998; 2003; Ciuchini, Franco, Lubicz, Mescia, Tarantino 2003, A.L., Nierste 2006

$$\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd} \approx 1.2 \frac{m_s^4}{m_c^4} - 59 \frac{m_s^6}{m_c^6}$$

Golowich, Petrov 2005, Bobrowski, A.L., Riedl, Rohrwild 2009

- Γ_{12} **is real to a very high accuracy**

$$\lambda_s^2 = \mathcal{O}\left(\lambda^2 + i\lambda^8\right) \Rightarrow \text{Arg}\left(\lambda_s^2\right) \approx \frac{1}{\lambda^6} \approx 10^{-4} \Rightarrow 10^{-3} = \text{NP}$$

- Overall result much too small

$$y \approx \mathcal{O}(10^{-6})$$

Huge cancellations $\Rightarrow$ be careful with approximations !!!
D= 6,7 without folklore!!!! Bobrowski, A.L., Riedl, Rohrwild 2009, 2010
Unitarity: $\lambda_d + \lambda_s + \lambda_b = 0$

$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}\right) + 2\lambda_s\lambda_b \left(\Gamma_{sd} - \Gamma_{dd}\right) - \lambda_b^2 \Gamma_{dd}$$

$$\begin{aligned} \Gamma_{sd}^{D=6,7} &= 1.8696 - 2.7616 \frac{m_s^2}{m_c^2} - 7.4906 \frac{m_s^4}{m_c^4} + \dots . \\ \Gamma_{dd}^{D=6,7} &= 1.8696 \end{aligned}$$

$$\Gamma_{12} \propto \lambda_s^2 \frac{m_s^6}{m_c^6} + 2\lambda_s\lambda_b \frac{m_s^2}{m_c^2} - \lambda_b^2 \, 1$$

$$\begin{aligned} 10^7 \Gamma_{12}^{D=6,7} &= -14.6 + 0.0009i(\text{1st term}) - 6.7 - 16i(\text{2nd term}) + 0.3 - 0.3i(\text{3rd term}) \\ &= -21.1 - 16.0i = (11...39)\, e^{-i(0.5...2.6)} . \end{aligned}$$

- not zero in SU(3)$_F$ limit

- large phase ($\mathcal{O}(1)$) possible!!!
- $y_D \in [0.5, 1.9] \cdot 10^{-6} \Rightarrow$ still much smaller than experiment ($8 \cdot 10^{-3}$)

What does this mean?

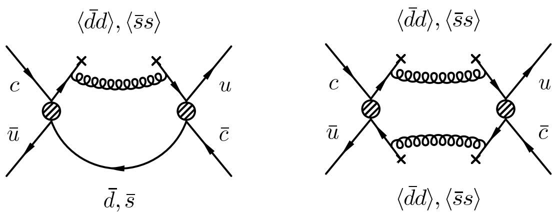
1. Standard argument for "arg Γ_{12} is negligible" is wrong

2. Can there be a sizeable phase in D-mixing?
   - Phase of Γ_{12} is unphysical
   - Phase of M_{12}/Γ_{12} is physical $\Rightarrow$ determine also M_{12}

3. $\Gamma_{12}^{D=6,7}$ has a large phase, but $y^{D=6,7} \ll y^{Exp.}$
   - Georgi 1992; Ohl, Ricciardi, Simmons 1993; Bigi, Uraltsev 2001
     Higher orders in the HQE might be dominant: $y^{D\geq 9} = y^{Exp.}$ not excluded
   - Bobrowski, A.L., Riedl, Rohrwild 2009, 2010
     If estimate of Bigi/Uraltsev is correct + our findings for D=6:
     $y^{Theory} = y^{Exp.}$ and 5 per mille CP-violation not excluded
   - Bobrowski, A.L. 2010; Bobrowski, Braun, A.L., Nierste, Prill in progress
     Do the real calculation for $D \geq 9$

Try by explicit calculation if HQE works:

Idea: higher orders in HQE might be dominant if GIM is less pronounced
Georgi; Ohl, Ricciardi, Simmons; Bigi, Uraltsev



naive expectation for a single diagram:

y_D	no GIM	with GIM	
$D = 6, 7$	$2 \cdot 10^{-2}$	$1 \cdot 10^{-6}$	Calculation
$D = 9$	$2 \cdot 10^{-2} ... 5 \cdot 10^{-4}$	$???$	Dimensional Estimate
$D = 12$	$2 \cdot 10^{-2} ... 1 \cdot 10^{-5}$	$???$	Dimensional Estimate

? Can one obtain $y_D^{Exp.}$? ?How big can ϕ be?

Our dimensional estimates

- Determine Γ_{12}: Imaginary part of 1-loop
- Estimate D = 9:
  - Quark condensate: $\langle \bar{s}s \rangle / m_c^3$
  - $4\pi\alpha_s$ relative to LO diagram
  - GIM : $(m_s/m_c)^3$ and m_s/m_c

Suppressed by about $2 \cdot 10^{-5}, 3 \cdot 10^{-3}$ compared to D=6 diagram
D=6 GIM suppressed by about $5 \cdot 10^{-5}$ $\Rightarrow$! IMPORTANT !

Dimensional estimate in Bigi, Uraltsev 2001

- Determine M_{12}: 0-loop
- Estimate D = 9: Quark condensate: $\mu^3_{hadron.}/m_c^3$ soft GIM : $m_s/\mu_{hadr.}$
- Estimate Γ_{12} via dispersion integral over M_{12}

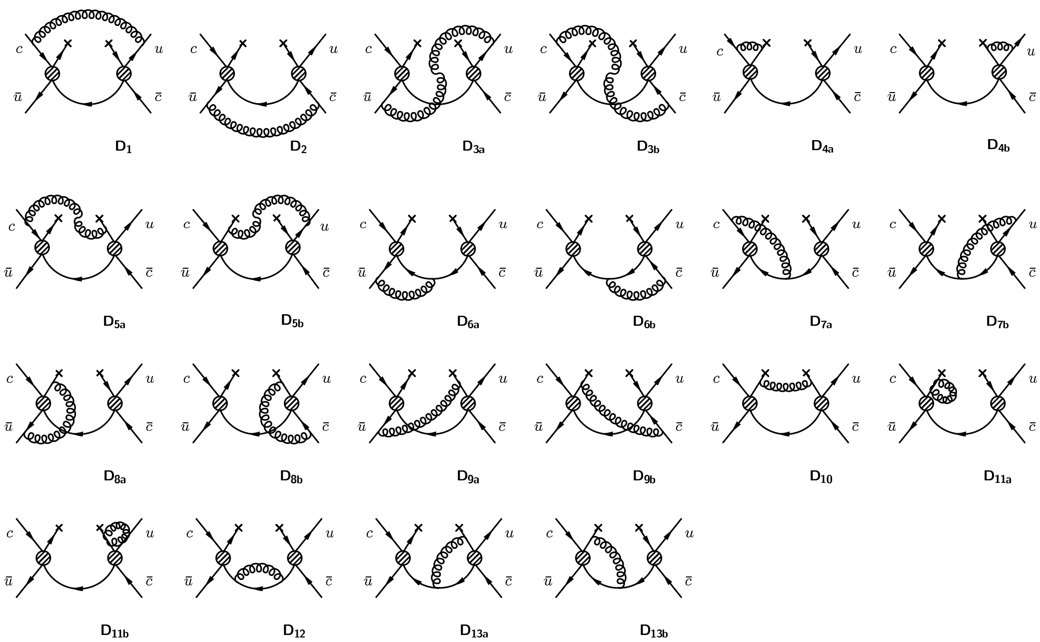
Difference: $\frac{\langle \bar{s}s \rangle m_s}{m_c^4}$ vs. $\frac{m_s \mu^2_{hadron.}}{m_c^3}$ or better $\langle \bar{q}q \rangle \approx (0.24\text{GeV})^3$ vs. $\mu_{hadr.} \approx 1$ GeV

$\Rightarrow$ BU/BBLNP ≈ 80 $\Rightarrow$ Calculation has to decide!

Our Research Program

1. Redo D=6 without any approximations
   Bobrowski, A.L, Riedl, Rohrwild, JHEP 2010
2. Calculate D$\geq$9
   Bobrowski, A.L. 2010; Bobrowski, Braun, A.L., Nierste, Prill unpublished
3. Calculate D$\geq$12
4. Calculate M_{12}
5. Calculate lifetimes of D mesons
6. Give a much more relieable range for the SM values of the possible size of CP violation in D mixing

Determination of D= 9,10,... in factorisation approximation



- Factorisation approximation, expected to hold up to $1/N_c$

- Enhancement of $\mathcal{O}(15)$ compared to leading term
  Large effect, but not as large as estimated by Bigi, Uraltsev

- GIM cancellation reduced to: $\propto m_s^3$

$$
\begin{aligned}
\Gamma_{12} &\propto \lambda_s^2 \cdot \frac{m_s^6}{m_c^6} + 2\lambda_s\lambda_b \cdot \frac{m_s^2}{m_c^2} + \lambda_b^2 \cdot 1 \\
\to \Gamma_{12} &\propto \lambda_s^2 \cdot \frac{m_s^3}{m_c^3} + 2\lambda_s\lambda_b \cdot \frac{m_s^2}{m_c^2} + \lambda_b^2 \cdot 1
\end{aligned}
$$

y_D	no GIM	with GIM	CP violation	
$D = 6,7$	$2 \cdot 10^{-2}$	$1 \cdot 10^{-6}$	$\mathcal{O}(1)$	Calculation
$D = 9$	$2 \cdot 10^{-2}$...$(3.5 \cdot 10^{-3})$...$5 \cdot 10^{-4}$	$1.5 \cdot 10^{-5}$	$\mathcal{O}(5\%)$	Calculation
$D = 12$	$2 \cdot 10^{-2}$...$1 \cdot 10^{-5}$	???		Dim. Estimate

next Dim 12!

9.5 Search for new physics

9.5.1 Model independent analyses in B-mixing

In [171] a model independent way to determine new physics effects in the mixing sector was presented. We assume that new physics does not alter Γ_{12} - at least not

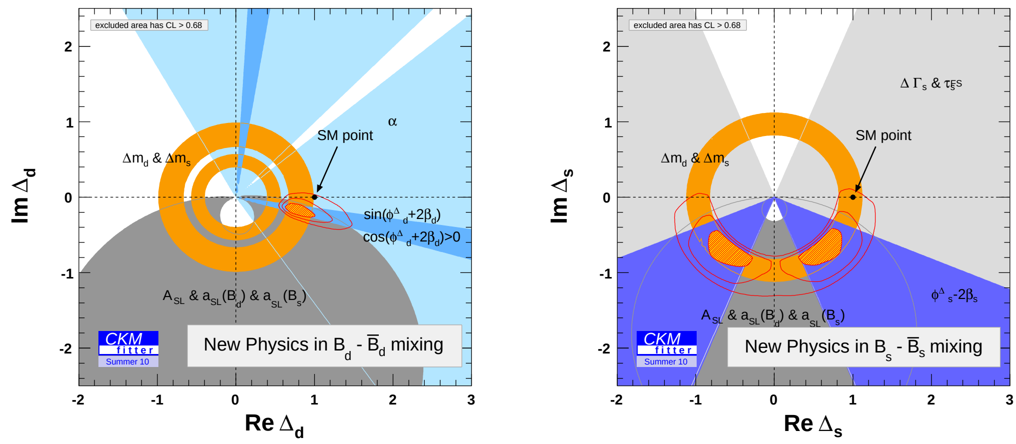


Figure 2: *Allowed regions for Δ_d and Δ_s [**?**].*

more than the intrinsic QCD uncertainties, but it might have a considerable effect on M_{12}. Therefore we write

$$\Gamma_{12}^q = \Gamma_{12}^{q,\text{SM}} \qquad M_{12}^q = M_{12}^{q,\text{SM}} \cdot \Delta_q \tag{337}$$

By comparing experiment and theory for the different mixing observables we get bounds in the complex Δ-plane, see [171]. In [**?**] we performed a fit of the complex parameters Δ_d and Δ_s. The result is shown in Fig. 2 We found that the SM is excluded by 3.6 standard deviations.

9.5.2 Search for new physics in D mixing

Contrary to expectation: Γ_{12} is sensitive to new physics!!!

$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}\right) + 2\lambda_s\lambda_b \left(\Gamma_{sd} - \Gamma_{dd}\right) - \lambda_b^2 \Gamma_{dd}$$

Γ_{12} is small, because

1. $\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}$ is small
2. λ_b is small

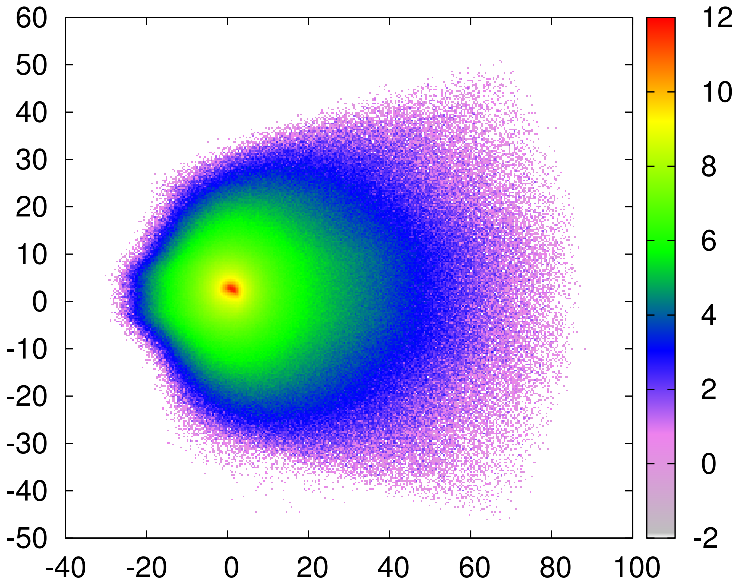
$\Rightarrow$ 2 possibilities for enhancements

1. Enhance $\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}$
   e.g. Golowich et al.: small corrections to the individual decay rates that do not cancel via GIM

2. "Enhance λ_b"
   The resurrection of the SM 4

   $$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}\right) + 2\lambda_s(\lambda_b + \lambda_{b'}) \left(\Gamma_{sd} - \Gamma_{dd}\right) - (\lambda_b + \lambda_{b'})^2 \Gamma_{dd}$$

   $\lambda_b \propto \lambda^{5...6}$ - still possible $\lambda_{b'} \propto \lambda^3$ (arXiv:0902.4883) see also Melic et al, Kou et al., Soni et. al, Hou et al. ...



9.6 Open Questions

- How large is the SM contribution to D-mixing?

$$\left.\frac{x_{B_s^0}}{x_{D^0}}\right|_{\mathrm{Exp}} = \mathbf{42} \cdot 10^2 \tag{338}$$

  Continue the full calculation of D-mixing within the HQE approach and look for other ideas.

- How large is weak mixing phase in the B_s-system

$$\Phi_s^{\mathrm{SM}} = \mathbf{42} \cdot 10^{-4} \tag{339}$$

  LHCb will show!

- Is the Dimuon result from D0 real or only a statistical fluctuation?

$$\frac{A_{SL}^{\mathrm{D0}}}{A_{SL}^{\mathrm{SM}}} = \mathbf{42} \tag{340}$$

  More results from TeVatron and LHC...

9.7 Comments

Exercise: Calculate M_{12}
Exercise: Calculate $\Delta\Gamma_s$
Lecture: Discuss NLO-QCD and lattice
The final success: $\Delta\Gamma_s$ vs. Quark-hadron duality

10 Exclusive B-decays

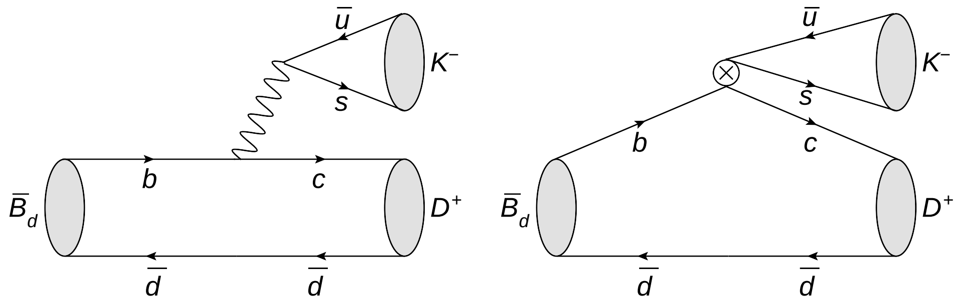
10.1 Decay topologies and QCD factorisation

(following Chapter 3 of the lecture notes from Thorsten Feldmann)
As an example for different decay topologies we consider several $B \to DK$ decays:

a) $\bar{B}_d \to D^+K^-$: The branching ratio of this decay is measured [9] to be

$$Br\left(\bar{B}_d \to D^+K^-\right) = (1.97 \pm 0.21) \cdot 10^{-4} , \tag{341}$$

the decay proceeds via the following tree-level diagrams (in the SM and in the effective theory)

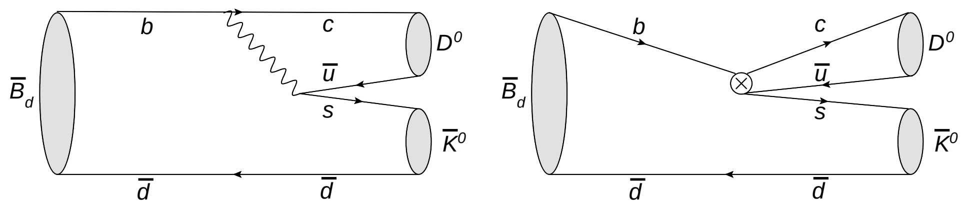


This topology is called **tree-level topology (class I)**. Naive colour counting gives two colour loops and thus a numerical factor N_c^2.

b) $\bar{B}_d \to D^0\bar{K}^0$: The branching ratio of this decay is measured [9] to be

$$Br\left(\bar{B}_d \to D^0\bar{K}^0\right) = (5.2 \pm 0.7) \cdot 10^{-5} , \tag{342}$$

the decay proceeds now via a different tree-level topology,

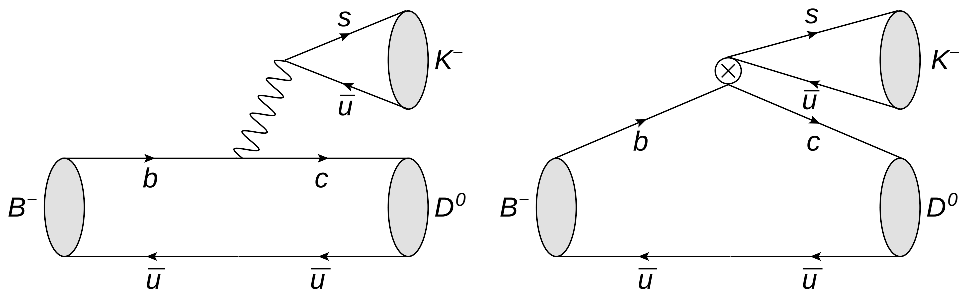


which is called **tree-level topology (class II)**. Naive colour counting gives only one colour loop and thus a numerical factor N_c.

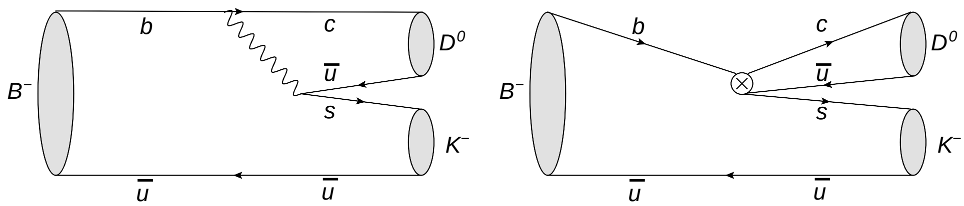
c) $B^- \to D^0 K^-$: The branching ratio of this decay is measured [9] to be

$$Br\left(B^- \to D^0 K^-\right) = (3.70 \pm 0.17) \cdot 10^{-4}\ . \tag{343}$$

Here we have both class I topology



and class II topology

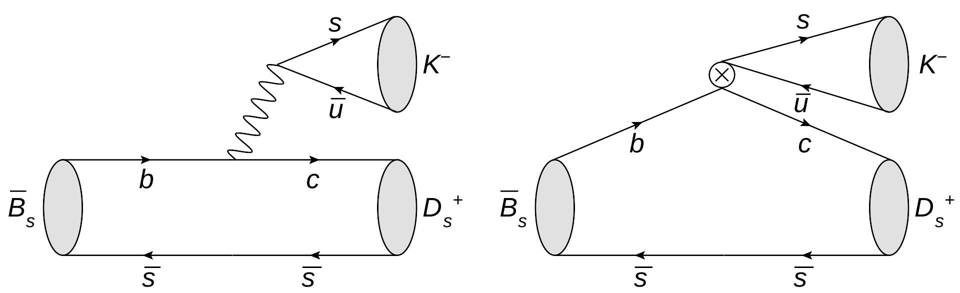


numerically class I is dominant.

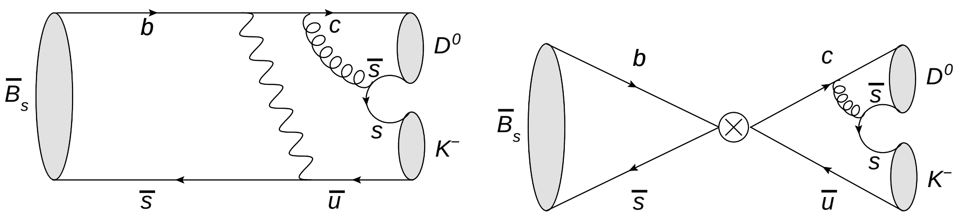
d) $\bar{B}_s \to D_s^+ K^-$: The branching ratio of this decay is measured [9] to be

$$Br\left(\bar{B}_s \to D_s^+ K^-\right) = (2.03 \pm 0.28) \cdot 10^{-4} \tag{344}$$

This decay proceeds via class I tree-level topology (in the SM and in the effective theory)



Besides the class I topology we have a new one that is called **annihilation topology**.



Naive colour counting gives for the annihilation one colour loop and thus a numerical factor N_c.

Now we would like to investigate the above decays a little more quantitatively. In the above naive colour estimates we implicitly assumed the insertion of the colour singlett operator Q_2, now we will do the general case of the effective Hamiltonian.

a) Tree-level topology (Class I):
The amplitude for the $\bar{B}_d \to D^+ K^-$ decay reads

$$\langle D^+ K^- | \mathcal{H}_{eff} | \bar{B}_d \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \sum_{i=1}^{2} C_i(\mu) \langle D^+ K^- | Q_i | \bar{B}_d \rangle \quad (345)$$

In principle we have to determine the matrix elements of the operators Q_1 and Q_2 non-perturbatively; in practice we cannot do this yet. Thus we have to rely on some additional assumptions.
The naive factorisation approximation states

$$\begin{aligned} \langle D^+ K^- | Q_2 | \bar{B}_d \rangle &\approx \langle D^+ | j^{(b\to c)} | \bar{B}_d \rangle \langle K^- s j^{(u \to c)} | \bar{B}_d \rangle && (346) \\ &= F^{B\to D}(q^2 = M_K^2) \cdot f_K && (347) \end{aligned}$$

The first object is called a form factor and the second one is a decay constant. In order to get the contribution of the operator Q_1 we have to express this operator in terms of colour singlett operator and colour octett operator. Using $\tilde{1} = \frac{1}{3} \cdot 1 +$ octett (see appendix) and keeping in mind that only the colour singlett part contributes, we find thus that C_1 appears with an additions factor $1/3$. Hence we get for the amplitude in naive factorisation approximation:

$$\langle D^+ K^- | \mathcal{H}_{eff} | \bar{B}_d \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \left[\frac{1}{3} C_1(\mu) + C_2(\mu) \right] F^{B\to D}(q^2 = M_K^2) \cdot f_K \quad (348)$$

The numerical value of the combination of the Wilson coefficients reads

$$C_2[5] + \frac{1}{3} C_1[5] = 1.01974 \; . \quad (349)$$

Since the scale dependence of the Wilson coefficients cannot be compensated by the form factors and the decay constants (the have no scale dependence) it is clear that naive factorisation is just a naive approximation and theoretically not consistent.
This problem is solved by the QCD (improved) factorisation approach [178, 179, 180, 181], which gives an expression of the following form,

$$\sum_{i=1}^{2} C_i(\mu)\langle D^+K^-|Q_i|\bar{B}_d\rangle = F^{B\to D}(q^2 = M_K^2)\cdot f_K$$
$$\int_0^1 du \left[\frac{1}{3}C_1(\mu) + C_2(\mu) + \frac{\alpha_s(\mu)}{4\pi}t(u,\mu)\phi_k(u,\mu)\right] + \mathcal{O}\left(\frac{\Lambda}{m_b}\right) \quad (350)$$

where t is a function that can be calculated in perturbation theory and ϕ_K is the so-called distribution amplitude of the kaon.

b) Tree-level topology (Class II)

$$C_1[5] + \frac{1}{3}C_2[5] \quad = \quad 0.142811 \quad (351)$$

Experimentally we get for

$$\frac{Br\left(\bar{B}_d \to D^+K^-\right)}{Br\left(\bar{B}_d \to D^0\bar{K}^0\right)} = 3.788 \quad (352)$$

Naive factorisation predicts a ratio of

$$\frac{C_2[5] + \frac{1}{3}C_1[5]}{C_1[5] + \frac{1}{3}C_2[5]} = 50.9865. \quad (353)$$

Thus the theory is off by a factor of $13.4584 = (3.66857)^2$. Naive colour counting would work here better: QCD factorisation predicts ....

c) Annihilation topology

10.2 Heavy Quark Effective Theory

a method to calculate form factors
Chapter 5 of Thorsten Lecture Notes

10.3 Different Methods

LCSR
BBNS
Example $\Lambda_b \to p\nu l$?

11 Search for new physics

11.1 Model independent analyses

This was done above for B-mixing, similar results have been obtains for other observables....

11.2 SM4

11.3 2HDM

- project of Matthew

11.4 Vector-like quarks

Heiko

11.5 MSSM

12 Acknowledgements

I would like to thank my PhD student Gilberto Tetlamatzi-Xolocotzi for creating most of the Feynman diagrams in these notes; my summer students Lucy Budge, Jonathan Cullen and Liu for updating the mixing observables, doing the plots for the time evolution and comparing different definitions for the mixing phases.

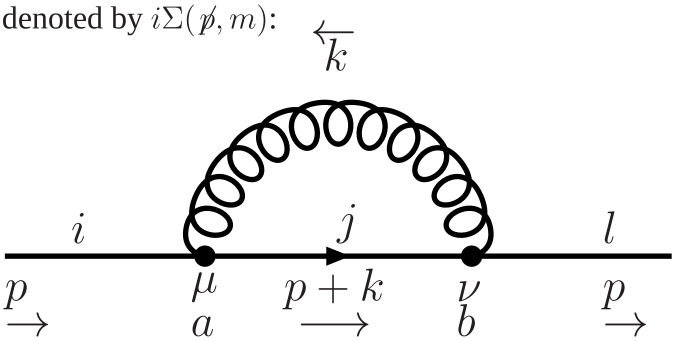
13 Appendix A: Basic QCD calculations

13.1 One-Loop Corrections

In this section we derive in detail the one-loop corrections to all elementary objects in QCD: the quark propagator, the gluon propagator and the quark gluon vertex. In section 13.1.4 we have collected a list of useful formulae which we are using extensively in the following.

13.1.1 Quark Self Energy

The one-loop correction for the quark self energy is given by the following Feynman diagram, denoted by $i\Sigma(\not p, m)$:



p and k denote the momenta, i, j and l denote the colour of the quark, μ and ν are the usual Dirac indices and a and b denote different gluons. The Feynman rules give the following expression:

$$\begin{aligned}
\Sigma(\not p, m) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{i} \left(ig\gamma_\nu (T^b)_{jl} \right) \left(-i\delta^{ab} \frac{g^{\mu\nu}}{k^2} \right) \left(i \frac{\not p + \not k + m}{(p+k)^2 - m^2} \right) \left(ig\gamma_\mu (T^a)_{ij} \right) \\
&= -g^2 (T^a T^a)_{il} \int \frac{d^4k}{(2\pi)^4 i} \frac{\gamma^\mu \left(\not p + \not k + m \right) \gamma_\mu}{\left[(p+k)^2 - m^2 \right] k^2} \,. \qquad (354)
\end{aligned}$$

We use dimensional regularisation ($D = 4 - 2\epsilon$, $g \to g\mu^\epsilon$) to evaluate this integral. With $(T^a T^a)_{il} = C_F \delta_{il}$, $C_F = 4/3$ one gets

$$\Sigma(\not p, m) = -g^2 C_F \delta_{il} \mu^{2\epsilon} \int \frac{d^Dk}{(2\pi)^D i} \frac{(2-D)(\not p + \not k) + Dm}{\left[(p+k)^2 - m^2 \right] k^2} \,. \qquad (355)$$

According to the so-called Feynman-trick (see section 13.1.4) the two propagators can be rewritten in the following way

$$\frac{1}{\left[(p+k)^2 - m^2 \right] k^2} = \int_0^1 \frac{dx}{\left[\tilde{k}^2 - M^2 \right]^2} \,, \qquad (356)$$

$$\text{with} \qquad \tilde{k} = k + px, M^2 = x \left(m^2 - p^2(1-x) \right) \,.$$

Performing the shift $k \to \tilde{k}$ and using the fact, that terms which are linear in the momentum k vanish after integration we get

$$\begin{aligned}\Sigma(\not{p}, m) &= -g^2 C_F \delta_{il} \mu^{2\epsilon} \int_0^1 dx \left[(2-D)\ \not{p}(1-x) + Dm\right] \\ &\quad \cdot \int \frac{d^D k}{(2\pi)^D i} \frac{1}{\left[k^2 - M^2\right]^2} \,. \end{aligned} \tag{357}$$

The momentum integral is already in standard form, so we can apply our formulae from section 13.1.4. With

$$\int \frac{d^D k}{(2\pi)^D i} \frac{1}{\left[k^2 - M^2\right]^2} = \frac{1}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma\left(2 - \frac{D}{2}\right)}{(M^2)^{2-\frac{D}{2}}} \,, \tag{358}$$

we get for the quark self energy

$$\Sigma(\not{p}, m) = -\frac{g^2 C_F \delta_{il}}{(4\pi)^2} \mu^{2\epsilon} \Gamma(\epsilon) (4\pi)^\epsilon \int_0^1 dx \frac{(2-D)\ \not{p}(1-x) + Dm}{(M^2)^\epsilon} . \tag{359}$$

For massless quarks we immediately get the final result

$$\begin{aligned}\Sigma(\not{p}, 0) &= -\frac{\alpha_s}{4\pi} C_F \delta_{il}\ \not{p} \frac{\mu^{2\epsilon}(4\pi)^\epsilon \Gamma(\epsilon)}{(-p^2)^\epsilon} (2-D) \int_0^1 dx (1-x)^{1-\epsilon} x^{-\epsilon} \\ &= \frac{\alpha_s}{2\pi} C_F \delta_{il}\ \not{p} \left(\frac{4\pi\mu^2}{-p^2}\right)^\epsilon \frac{\Gamma(\epsilon)\Gamma^2(2-\epsilon)}{\Gamma(3-2\epsilon)} \\ &\approx \frac{\alpha_s}{3\pi} \delta_{il}\ \not{p} \left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + \ln \frac{\mu^2}{-p^2} + 1 + \mathcal{O}(\epsilon)\right] \,, \end{aligned} \tag{360}$$

where we have performed a Taylor expansion around $\epsilon = 0$ in the last step.
For the case of non-vanishing quark masses it seems to be easier to perform the Taylor expansion around $\epsilon = 0$ in Eq. (359) before the x-integration.

$$\begin{aligned}\Sigma(\not{p}, m) &= 2\frac{\alpha_s}{3\pi} \delta_{il} \int_0^1 dx \Gamma(\epsilon) \left(\frac{\mu^2 4\pi}{x\left(m^2 - p^2(1-x)\right)}\right)^\epsilon \left[(1-\epsilon)\ \not{p}(1-x) - (2-\epsilon)m\right] \\ &=: m\Sigma_1(p^2, m) + \not{p}\Sigma_2(p^2, m) \,. \end{aligned} \tag{361}$$

Performing the Taylor expansion and the subsequent x-integration we obtain the final result for Σ_1 and Σ_2:

$$\Sigma_1(p^2, m) \approx -4\frac{\alpha_s}{3\pi} \delta_{il} \int_0^1 dx \left[\frac{1}{\epsilon} - \gamma_e + \ln\left(\mu^2 4\pi\right) - \ln x - \ln\left(m^2 - p^2(1-x)\right) - \frac{1}{2}\right]$$

$$
\begin{aligned}
&= -4\frac{\alpha_s}{3\pi}\delta_{il}\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)+\frac{3}{2}-\ln\frac{m^2-p^2}{\mu^2}+\frac{m^2}{p^2}\ln\frac{m^2-p^2}{m^2}\right] . \\
\Sigma_2(p^2,m) &\approx 2\frac{\alpha_s}{3\pi}\delta_{il}\int\limits_0^1 dx(1-x)\left[\frac{1}{\epsilon}-\gamma_e+\ln\left(\mu^2 4\pi\right)-\ln x-\ln\left(m^2-p^2(1-x)\right)-1\right] \\
&= \frac{\alpha_s}{3\pi}\delta_{il}\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)+1+\frac{m^2}{p^2}-\ln\frac{m^2-p^2}{\mu^2}+\frac{m^4}{p^4}\ln\frac{m^2-p^2}{m^2}\right] .
\end{aligned}
\tag{362}
$$

For the determination of the so-called *pole mass* of the quark we need the self energy evaluated at $p^2 = m^2$.

$$
\begin{aligned}
\Sigma_1(m^2,m) &\approx -4\frac{\alpha_s}{3\pi}\delta_{il}\int\limits_0^1 dx\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)-\frac{1}{2}+\ln\frac{\mu^2}{m^2}-2\ln x\right] \\
&= -4\frac{\alpha_s}{3\pi}\delta_{il}\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)+\frac{3}{2}+\ln\frac{\mu^2}{m^2}\right] . \\
\Sigma_2(m^2,m) &\approx 2\frac{\alpha_s}{3\pi}\delta_{il}\int\limits_0^1 dx(1-x)\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)-1+\ln\frac{\mu^2}{m^2}-2\ln x\right] \\
&= \frac{\alpha_s}{3\pi}\delta_{il}\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)+2+\ln\frac{\mu^2}{m^2}\right] . \\
\Sigma(\not{p}=m,m) &= -3\frac{\alpha_s}{3\pi}\delta_{il}m\left[\frac{1}{\epsilon}-\gamma_e+\ln(4\pi)+\frac{4}{3}+\ln\frac{\mu^2}{m^2}\right] .
\end{aligned}
\tag{363}
$$

The same diagram can be easily calculated with the use of an cut-off instead of dimensional regularisation. Therefore we start with the form given in Eq. (357)

$$
\Sigma(\not{p},m) = 2g^2C_F\delta_{il}\int\limits_0^1 dx\left[\not{p}(1-x)-2m\right]\int\frac{d^4k}{(2\pi)^4 i}\frac{1}{\left[k^2-M^2\right]^2} . \tag{364}
$$

Applying the *Wick rotation* we can write

$$
\int\frac{d^4k}{(2\pi)^4 i}f(k^2) = \frac{1}{(4\pi)^2}\int\limits_0^\infty dk_E^2 k_E^2 f(-k_E^2) . \tag{365}
$$

Inserting this in the quark self energy we get

$$\Sigma(\not{p}, m) = 2\frac{g^2}{(4\pi)^2} C_F \delta_{il} \int\limits_0^1 dx \, [\not{p}(1-x) - 2m] \int\limits_0^\infty dk^2 k^2 \frac{1}{[k^2 + M^2]^2} . \tag{366}$$

The momentum integral is solved by introducing a cut-off Λ.

$$\begin{aligned} \int\limits_0^\Lambda dk^2 k^2 \frac{1}{[k^2 + M^2]^2} &= \ln\left(1 + \frac{\Lambda}{M^2}\right) + \frac{M^2}{M^2 + \Lambda} - 1 \\ &\approx \ln\frac{\Lambda}{M^2} - 1 . \end{aligned} \tag{367}$$

Finally we get the following result, that can be compared with the result obtained by dimensional regularisation.

$$\begin{aligned} \Sigma^\Lambda(\not{p}, m) &= \frac{\alpha_s}{2\pi} C_F \delta_{il} \int\limits_0^1 dx \Big\{ [\not{p}(1-x) - 2m] \\ & \left(\ln\frac{\Lambda}{\tilde{M}^2} - \ln\frac{x(m^2 - p^2(1-x))}{\tilde{M}^2} - 1\right)\Big\} . \end{aligned} \tag{368}$$

$$\begin{aligned} \Sigma^\epsilon(\not{p}, m) &= \frac{\alpha_s}{2\pi} C_F \delta_{il} \int\limits_0^1 dx \Big\{ [\not{p}(1-x) - 2m] \\ & \left(\frac{1}{\epsilon} - \gamma_e + \ln 4\pi - \ln\frac{x(m^2 - p^2(1-x))}{\mu^2} - 1\right) - m\Big\} . \end{aligned} \tag{369}$$

Comparing this two very similar looking expressions, we find the following correspondence.

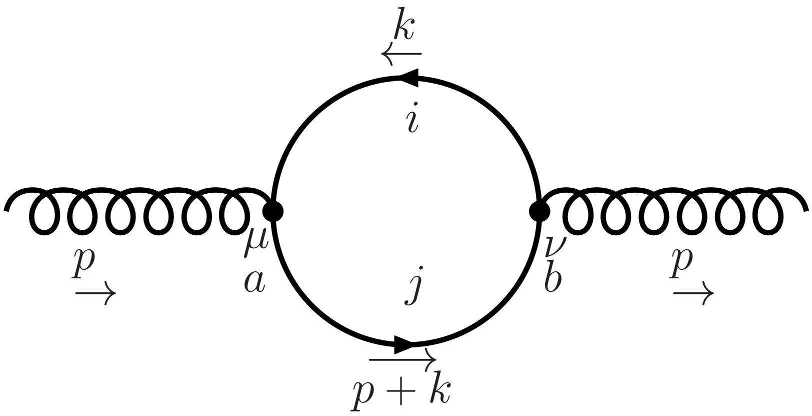
$$\frac{1}{\epsilon} - \gamma_e + \ln 4\pi \Leftrightarrow \ln\frac{\Lambda}{\tilde{M}} . \tag{370}$$

$$\mu \Leftrightarrow \tilde{M} . \tag{371}$$

The remaining difference is the finite term $-m$ in the dimensional regularisation.

13.1.2 Gluon Self Energy

There are several one-loop corrections for the gluon self energy. We start our calculation with the contribution of virtual quarks which is given by the following Feynman diagram, denoted by $i\Pi^{q,ab}_{\mu\nu}(p, m)$.



p and k denote the momenta, i and j denote the colour of the quark, μ and ν are the usual Dirac indices and a and b denote different gluons. The Feynman rules give the following expression.

$$\begin{aligned}\Pi^{q,ab}_{\mu\nu}(p,m) &= (-1)\int \frac{d^Dk}{(2\pi)^D i}\mathrm{Tr}\left[\left(ig\mu^\epsilon\gamma_\nu(T^b)_{ji}\right)\left(i\frac{\not p+\not k+m}{(p+k)^2-m^2}\right)\right.\\ &\qquad\left.\left(ig\mu^\epsilon\gamma_\mu(T^a)_{ij}\right)\left(i\frac{\not k+m}{k^2-m^2}\right)\right]\\ &= -g^2\mu^{2\epsilon}(T^aT^b)_{ii}\int\frac{d^Dk}{(2\pi)^D i}\frac{\mathrm{Tr}\left[\gamma_\nu\left(\not p+\not k+m\right)\gamma_\mu\left(\not k+m\right)\right]}{\left[(p+k)^2-m^2\right]\left[k^2-m^2\right]}. \quad (372)\end{aligned}$$

Using the fact that the trace of three γ-matrices vanishes, $(T^aT^b)_{ii} = \delta^{ab}/2$ and the Feynman-trick

$$\begin{aligned}\frac{1}{\left[(p+k)^2-m^2\right]\left[k^2-m^2\right]} &= \int_0^1 \frac{dx}{\left[\tilde k^2-M^2\right]^2}, \quad (373)\\ \text{with} &\quad \tilde k = k+px, M^2 = m^2-p^2x(1-x)\end{aligned}$$

we obtain after a shift $k\to\tilde k$ the following expression (linear terms in k vanish)

$$\Pi^{q,ab}_{\mu\nu}(p,m) = -\frac{g^2\mu^{2\epsilon}}{2}\delta^{ab}\int_0^1 dx\int\frac{d^Dk}{(2\pi)^D i}\frac{\mathrm{Tr}\left[\gamma_\nu\not k\gamma_\mu\not k - x(1-x)\gamma_\nu\not p\gamma_\mu\not p+m^2\gamma_\nu\gamma_\mu\right]}{\left[k^2-M^2\right]^2}. \quad (374)$$

In that problem two kinds of k-integrals appear which can be solved using the formulae in section 13.1.4.

$$B = \int\frac{d^Dk}{(2\pi)^D i}\frac{1}{\left[k^2-M^2\right]^2} = \frac{1}{(4\pi)^2}\frac{(4\pi)^\epsilon\Gamma(\epsilon)}{(M^2)^\epsilon}. \quad (375)$$

$$B^{\mu\nu} = \int \frac{d^Dk}{(2\pi)^D i} \frac{k^\mu k^\nu}{\left[k^2 - M^2\right]^2} = \frac{1}{(4\pi)^2} \frac{(4\pi)^\epsilon \Gamma(\epsilon)}{(M^2)^\epsilon} \frac{g^{\mu\nu}}{2} \frac{M^2}{1-\epsilon}\,. \qquad (376)$$

Therefore we get for the gluon self energy

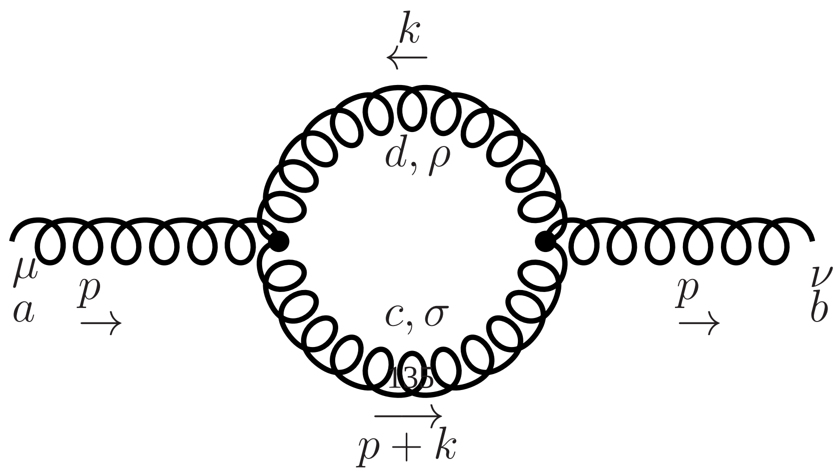
$$\begin{aligned}
\Pi^{q,ab}_{\mu\nu}(p,m) &= -\frac{g^2}{(4\pi)^2}\frac{\delta^{ab}}{2}(4\pi\mu^2)^\epsilon\Gamma(\epsilon)\int\limits_0^1 dx \frac{1}{(M^2)^\epsilon} \\
&\quad \mathrm{Tr}\left[\frac{\gamma_\nu\gamma_\alpha\gamma_\mu\gamma^\alpha}{2(1-\epsilon)}M^2 - x(1-x)\gamma_\nu \not{p}\gamma_\mu \not{p} + m^2\gamma_\nu\gamma_\mu\right] \\
&= -\frac{\alpha_s}{4\pi}\frac{\delta^{ab}}{2}(4\pi\mu^2)^\epsilon\Gamma(\epsilon)\int\limits_0^1 dx \frac{x(1-x)}{(m^2 - p^2x(1-x))^\epsilon}\mathrm{Tr}\left[\gamma_\nu\gamma_\mu p^2 - \gamma_\nu \not{p}\gamma_\mu \not{p}\right] \\
&= -\frac{\alpha_s}{\pi}\delta^{ab}\left(p^2 g^{\mu\nu} - p^\mu p^\nu\right)(4\pi\mu^2)^\epsilon\Gamma(\epsilon)\int\limits_0^1 dx \frac{x(1-x)}{(m^2 - p^2x(1-x))^\epsilon}. \qquad (377)
\end{aligned}$$

Performing a Taylor expansion we get

$$\begin{aligned}
\Pi^{q,ab}_{\mu\nu}(p,m) &= -\frac{\alpha_s}{6\pi}\delta^{ab}\left(p^2 g^{\mu\nu} - p^\mu p^\nu\right) \\
&\quad \left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 6\int\limits_0^1 dx x(1-x)\ln\frac{m^2 - p^2x(1-x)}{\mu^2}\right]. \qquad (378)
\end{aligned}$$

The last integral can of course be solved analytically, but we think it is more elegant to express the result in the given form.

The next one-loop correction we are considering is due to virtual gluons:



p and k denote the momenta, μ, ρ, σ and ν are usual Dirac indices and a, b, c and d denote different gluons. The Feynman rules give the following expression.

$$\Pi^{3g,ab}_{\mu\nu}(p,m) = \int \frac{d^Dk}{(2\pi)^D i}\left(-i\frac{1}{(p+k)^2}\right)\left(-i\frac{1}{k^2}\right)X^{ab}_{\mu\nu}\,. \tag{379}$$

In $X^{ab}_{\mu\nu}$ we have encoded the Feynman rules for the three gluon vertex and the metric tensors from the gluon propagators. The denominators of the two propagators can be combined in the same way as in the previous example, with a simpler form of M^2.

$$M^2 = -p^2x(1-x)\,. \tag{380}$$

For $X^{ab}_{\mu\nu}$ we get

$$\begin{aligned} X^{ab}_{\mu\nu} &= g\mu^\epsilon f^{acd}\left[g^{\mu\sigma}(2p+k)^\rho - g^{\sigma\rho}(2k+p)^\mu + g^{\rho\mu}(k-p)^\sigma\right]g_{\rho\rho'}g_{\sigma\sigma'} \\ &\quad g\mu^\epsilon f^{dcb}\left[-g^{\rho'\sigma'}(2k+p)^\nu + g^{\sigma'\nu}(2p+k)^{\rho'} + g^{\nu\rho'}(k-p)^{\sigma'}\right] \\ &= -g^2\mu^{2\epsilon}f^{acd}f^{bcd}\left[g^{\mu\nu}(5p^2+2pk+2k^2) + (D-1)(2k+p)^\mu(2k+p)^\nu\right. \\ &\qquad \left.-(2p+k)^\mu(2p+k)^\nu - (p-k)^\mu(p-k)^\nu\right]\,. \end{aligned} \tag{381}$$

When performing the momentum integration we make a shift in the integration variable k, in practice this means that we exchange k by $k - xp$.

$$\begin{aligned} X^{ab}_{\mu\nu} &= -3g^2\mu^{2\epsilon}\delta^{ab}\left[g^{\mu\nu}\left(p^2(5-2x+2x^2)+2k^2\right)\right. \\ &\qquad +(4D-6)k^\mu k^\nu \\ &\qquad \left.+\left(D(1-2x)^2 - 6(1-x+x^2)\right)p^\mu p^\nu\right]\,. \end{aligned} \tag{382}$$

Now we can insert everything in Eq. (379).

$$\begin{aligned} \Pi^{3g,ab}_{\mu\nu}(p,m) &= 3g^2\mu^{2\epsilon}\delta^{ab}\int_0^1 dx \int \frac{d^Dk}{(2\pi)^D i}\frac{1}{[k^2-M^2]^2}\left[g^{\mu\nu}\left(p^2(5-2x+2x^2)+2k^\alpha k^\beta g_{\alpha\beta}\right)\right. \\ &\qquad +(4D-6)k^\mu k^\nu \\ &\qquad \left.+\left(D(1-2x)^2 - 6(1-x+x^2)\right)p^\mu p^\nu\right]\,. \end{aligned} \tag{383}$$

Performing all the momentum integrals we get

$$\begin{aligned} \Pi^{3g,ab}_{\mu\nu}(p,m) &= 3\frac{g^2}{(4\pi)^2}\delta^{ab}\int_0^1 dx \frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2x(1-x)]^\epsilon}\left[g^{\mu\nu}p^2(5-2x+2x^2)\right. \\ &\qquad +Dg^{\mu\nu}\frac{-p^2x(1-x)}{1-\epsilon} + (2D-3)g^{\mu\nu}\frac{-p^2x(1-x)}{1-\epsilon} \end{aligned}$$

$$+\left(D(1-2x)^2-6(1-x+x^2)\right)p^\mu p^\nu\Big]$$

$$= 3\frac{g^2}{(4\pi)^2}\delta^{ab}\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2]^\epsilon}\int_0^1\frac{dx}{[x(1-x)]^\epsilon}\Big[-3(3-2\epsilon)g^{\mu\nu}p^2\frac{x(1-x)}{1-\epsilon}$$
$$+g^{\mu\nu}p^2(5-2x+2x^2)-p^\mu p^\nu\left(2+2\epsilon+(10-8\epsilon)x-(10-8\epsilon)x^2\right)\Big]\,. \tag{384}$$

Before we perform all x-integrations — only β-functions appear — we will use a simple trick to simplify our expressions. In the above integral we can exchange the integration variable x with $1-x$. The denominator is symmetric in x and $1-x$ and in the denominator we have constants, linear and quadratic terms in x. We can split up x in $1/2x+1/2x$ and exchange in one term x with $1-x$, i.e $x\to 1/2x+1/2(1-x)=1/2$, this means we can replace every linear term in the numerator with 1/2.

$$\Pi^{3g,ab}_{\mu\nu}(p,m) = 3\frac{g^2}{(4\pi)^2}\delta^{ab}\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2]^\epsilon}\int_0^1 dx x^{-\epsilon}(1-x)^{-\epsilon}\left[\frac{g^{\mu\nu}p^2}{1-\epsilon}\left((11-8\epsilon)x^2-(\frac{1}{2}+\epsilon)\right)\right.$$
$$\left.+p^\mu p^\nu\left(2(5-4\epsilon)x^2-(7-2\epsilon)\right)\right]\,. \tag{385}$$

Now we perform the x-integration. With

$$\int_0^1 dx x^{-\epsilon}(1-x)^{-\epsilon} = \beta(1-\epsilon,1-\epsilon)=\frac{\beta(2-\epsilon,2-\epsilon)}{1-\epsilon}2(3-2\epsilon) \tag{386}$$

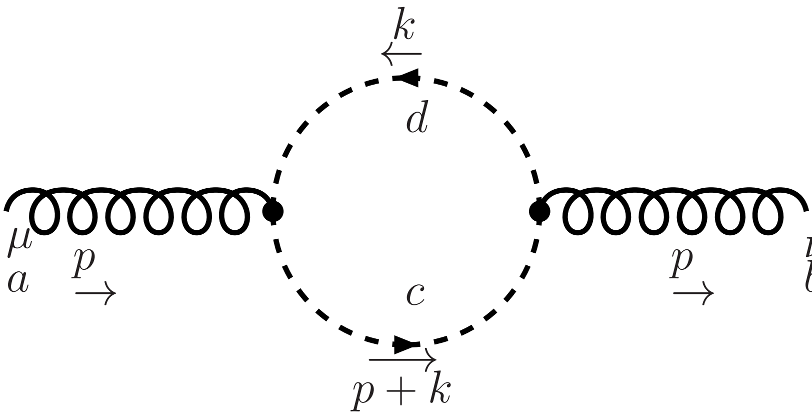
$$\int_0^1 dx x^{2-\epsilon}(1-x)^{-\epsilon} = \beta(3-\epsilon,1-\epsilon)=\frac{\beta(2-\epsilon,2-\epsilon)}{1-\epsilon}(2-\epsilon) \tag{387}$$

one gets

$$\Pi^{3g,ab}_{\mu\nu}(p,m) = 3\frac{\alpha_s}{4\pi}\delta^{ab}\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2]^\epsilon}\frac{\beta(2-\epsilon,2-\epsilon)}{1-\epsilon}\left[g^{\mu\nu}p^2\left(19-12\epsilon\right)-2p^\mu p^\nu\left(11-7\epsilon\right)\right]\,. \tag{388}$$

We will perform the Taylor expansion only after all gauge contributions have been summed up.

Next we consider the contribution of virtual Faddeev-Popov-ghosts:



p and k denote the momenta, i and j denote the colour of the quark, μ and ν are the usual Dirac indices and a and b denote different gluons. The Feynman rules give the following expression.

$$
\begin{aligned}
\Pi^{FP,ab}_{\mu\nu}(p,m) &= (-1)\int \frac{d^Dk}{(2\pi)^D i}\left(-i\frac{1}{(p+k)^2}\right)\left(-i\frac{1}{k^2}\right)\left(-g\mu^\epsilon f^{cad}(p+k)^\mu\right)\left(-g\mu^\epsilon f^{dbc}k^v\right) \\
&= -3g^2\mu^{2\epsilon}\delta^{ab}\int \frac{d^Dk}{(2\pi)^D i}\frac{(p+k)^\mu k^v}{(p+k)^2k^2} \\
&= -3g^2\mu^{2\epsilon}\delta^{ab}\int_0^1 dx \int \frac{d^Dk}{(2\pi)^D i}\frac{k^\mu k^v - x(1-x)p^\mu p^v}{\left[k^2-p^2x(1-x)\right]^2} \\
&= -3\frac{g^2}{(4\pi)^2}\delta^{ab}\int_0^1 dx\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2x(1-x)]^\epsilon}\frac{g^{\mu\nu}}{2}\frac{-p^2x(1-x)}{1-\epsilon} - x(1-x)p^\mu p^v \\
&= 3\frac{\alpha_s}{4\pi}\delta^{ab}\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2]^\epsilon}\int_0^1 dx x^{1-\epsilon}(1-x)^{1-\epsilon}\left[\frac{g^{\mu\nu}}{2}\frac{p^2}{1-\epsilon}+p^\mu p^\nu\right] \\
&= 3\frac{\alpha_s}{4\pi}\delta^{ab}\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2]^\epsilon}\frac{\Gamma^2(2-\epsilon)}{\Gamma(4-2\epsilon)}\left[\frac{g^{\mu\nu}}{2}\frac{p^2}{1-\epsilon}+p^\mu p^\nu\right] .
\end{aligned} \tag{389}
$$

Summing up the final results for the virtual gluon (including the symmetry factor 1/2) and the virtual ghost we get

$$
\Pi^{g,ab}_{\mu\nu}(p,m) = 3\frac{\alpha_s}{2\pi}\delta^{ab}\frac{(4\pi\mu^2)^\epsilon\Gamma(\epsilon)}{[-p^2]^\epsilon}\frac{\Gamma^2(2-\epsilon)}{\Gamma(4-2\epsilon)}\frac{5-3\epsilon}{1-\epsilon}\left[g^{\mu\nu}p^2-p^\mu p^\nu\right] . \tag{390}
$$

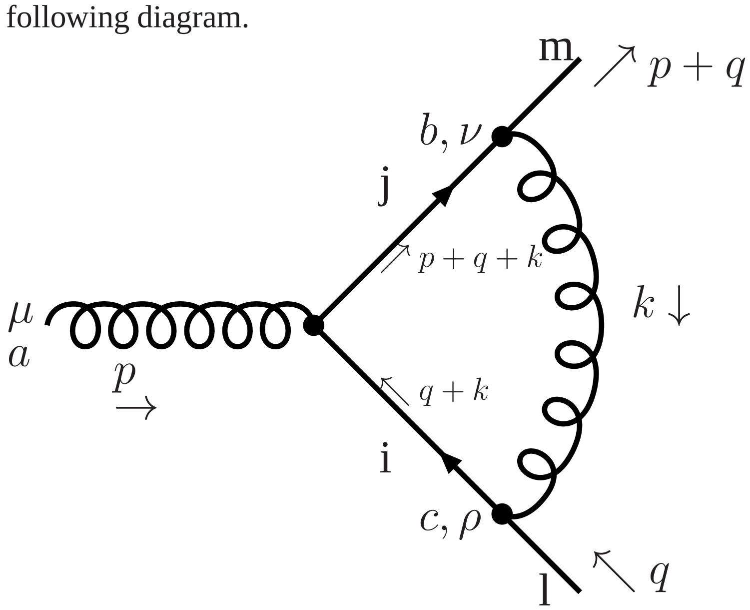
Performing a Taylor expansion in ϵ we arrive at

$$
\Pi^{g,ab}_{\mu\nu}(p,m) = 5\frac{\alpha_s}{4\pi}\delta^{ab}\left[g^{\mu\nu}p^2-p^\mu p^\nu\right]\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi+\frac{31}{15}-\ln\frac{-p^2}{\mu^2}\right) .
$$

(391)

13.1.3 Vertex Correction

Now we come to the last class of corrections, to virtual vertex corrections, which are given by the following diagram.



p, q and k denote the momenta, i,j, l and m denote the colour of the quark, μ, ν and σ are the usual Dirac indices and a, b and c denote different gluons. The Feynman rules give the following expression.

$$\begin{aligned}\Gamma^\mu(p,q,m) &= \int \frac{d^Dk}{(2\pi)^D}\left(ig\gamma_\nu(T^b)_{jm}\right)\left(i\frac{\not p+\not q+\not k+m}{(p+q+k)^2-m^2}\right)\left(ig\gamma_\mu(T^a)_{ij}\right)\\ &\quad \left(i\frac{\not q+\not k+m}{(q+k)^2-m^2}\right)\left(ig\gamma_\rho(T^c)_{li}\right)\left(-i\delta^{bc}\frac{g^{\rho\nu}}{k^2}\right)\\ &= g^3(T^bT^aT^b)_{lm}\int\frac{d^Dk}{(2\pi)^D}\frac{\gamma_\nu\left(\not p+\not q+\not k+m\right)\gamma_\mu\left(\not q+\not k+m\right)\gamma^\nu}{\left[(p+q+k)^2-m^2\right]\left[(q+k)^2-m^2\right]k^2}\\ &= -\frac{g^3}{6}T^a_{lm}\int\frac{d^Dk}{(2\pi)^D}\frac{\gamma_\nu\left(\not k+\not p+m\right)\gamma_\mu\left(\not k+m\right)\gamma^\nu}{\left[(p+k)^2-m^2\right]\left[k^2-m^2\right](k-q)^2}\,,\end{aligned} \tag{392}$$

where we made a shift in the integration momentum k. The loop integral is only logarithmically divergent, therefore we can extract the ultra-violet divergence simply by setting p, q and m equal to zero.

$$\Gamma^\mu(0,0,0) = -\frac{g^3}{6}T^a_{lm}\int\frac{d^Dk}{(2\pi)^D}\frac{\gamma_\nu\not k\gamma_\mu\not k\gamma^\nu}{(k^2)^3}$$

$$= \quad -(2-D)\frac{g^3}{6}T^a_{lm}\int\frac{d^Dk}{(2\pi)^D}\frac{\not k\gamma_\mu\not k}{(k^2)^3}\,. \tag{393}$$

From symmetry reasons we can replace $k^\alpha k^\beta$ with $k^2 g^{\alpha\beta}/D$.

$$\Gamma^\mu(0,0,0) \quad = \quad -\frac{(2-D)^2}{6D}g^3\gamma_\mu T^a_{lm}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2)^2}\,. \tag{394}$$

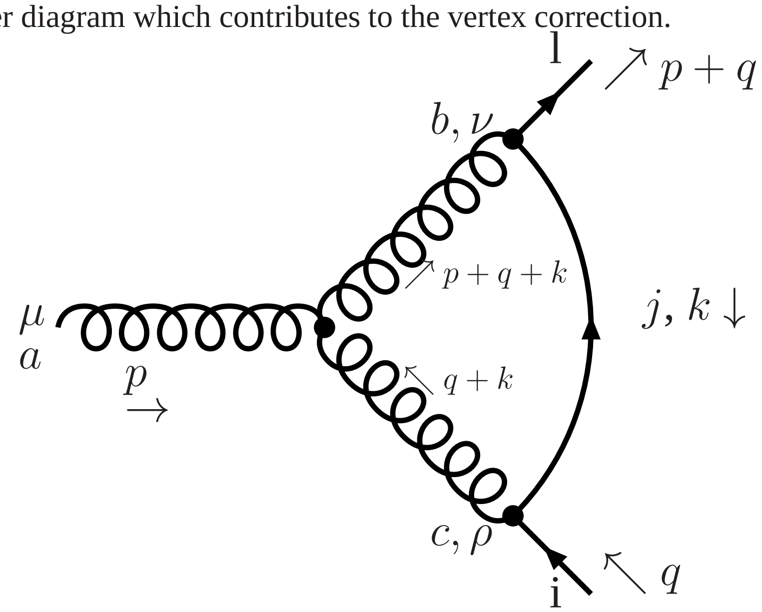
The appearing integral can be treated as follows.

$$\begin{aligned}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2)^2} &= \lim_{M\to 0}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-M^2)^2}\\ &= \lim_{M\to 0}\frac{i}{(4\pi)^2}\frac{(4\pi)^\epsilon\Gamma(\epsilon)}{(M^2)^\epsilon}\\ &= \lim_{M\to 0}\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+\ldots\right]\,. \end{aligned}\tag{395}$$

We get for the divergent part of the vertex correction

$$\Gamma^\mu_{UV}(0,0,0) \quad = \quad -\frac{1}{6}ig\gamma_\mu T^a_{lm}\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\,. \tag{396}$$

In order to compute the finite parts of this integral we have to keep the external momenta and the masses in the calculation.
There is another diagram which contributes to the vertex correction.



p, q and k denote the momenta, i,j and l denote the colour of the quark, μ, ν and σ are the usual Dirac indices and a, b and c denote different gluons. The Feynman rules give the following expression.

$$
\begin{aligned}
\Gamma^\mu(p,q,m) &= \int \frac{d^Dk}{(2\pi)^D} \left(ig\gamma_\nu (T^b)_{jl}\right) \left(i\frac{-\not k + m}{k^2-m^2}\right) \left(ig\gamma_\rho (T^c)_{ij}\right) \\
& \left(-i\frac{1}{(q+k)^2-m^2}\right)\left(-i\frac{1}{(p+q+k)^2-m^2}\right) \\
& \left(gf^{acb}\left[g^{\mu\rho}(p-q-k)^\nu + g^{\nu\rho}(p+2q+2k)^\mu - g^{\nu\mu}(2p+q+k)^\rho\right]\right) \\
= ig^3 f^{acb}(T^cT^b)_{il} & \int \frac{d^Dk}{(2\pi)^D} \frac{(\not p - \not q - \not k)\,(m-\not k)\,\gamma_\mu + (p+2q+2k)^\mu \gamma_\nu\,(m-\not k)\,\gamma_\nu}{\left[(p+q+k)^2-m^2\right]\left[(q+k)^2-m^2\right]k^2} \\
& - \frac{\gamma_\mu\,(m-\not k)\,(2\not p + \not q + \not k)}{\left[(p+q+k)^2-m^2\right]\left[(q+k)^2-m^2\right]k^2}\,. \qquad (397)
\end{aligned}
$$

Setting the external momenta and the masses to zero, we get

$$
\begin{aligned}
\Gamma^\mu(p,q,m) &= \frac{3}{2} g^3 T^a_{il} \int \frac{d^Dk}{(2\pi)^D} \frac{\not k\,\not k \gamma_\mu - 2(2-D)k^\mu\,\not k + \gamma_\mu\,\not k\,\not k}{\left[k^2\right]^3}\,. \\
&= 6g^3 T^a_{il}\gamma_\mu \frac{D-1}{D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{\left[k^2\right]^2} \qquad (398)
\end{aligned}
$$

The UV-divergent part reads

$$
\Gamma^\mu_{UV}(0,0,0) = \frac{9}{2} i g T^a_{il} \gamma^\mu \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\,. \qquad (399)
$$

Now we have determined the divergencies of all basic ingredients of QCD - the fermion propagator, the gluon propagator and the quark-gluon vertex. Before we proceed to renormalise our theory we list up a useful formulae for performing perturbative calculations. Part of these formulae have been copied from a previous QCD-course of Prof. Vladimir Braun.

13.1.4 Useful Formulae

SU(3)-Algebra

The SU(N)-algebra is defined by the following commutation relation for the *generators* T^a with $a = 1, ..., N^2-1$

$$
\left[T^a, T^b\right] = i f^{abc} T^c\,. \qquad (400)
$$

The generators can be *represented* as matrices. Commonly used representations are the *fundamental representation* in N dimensions and the *adjoint representation* in N^2-1 dimensions. For the fundamental representation we demand the following normalisation

$$
\mathrm{Tr}\left[T^aT^b\right] = \frac{1}{2}\delta^{ab}\,. \qquad (401)
$$

Then the following relations hold.

$$\begin{aligned}
\mathrm{Tr}\,[T^a] &= 0 && (402)\\
T^a_{ij}T^a_{kl} &= \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\right) . && (403)\\
(T^aT^a)_{ij} &= \frac{N^2-1}{2N}\delta_{ij} && (404)\\
\left(T^aT^bT^a\right)_{ij} &= -\frac{1}{2N}T^b_{ij} && (405)\\
if^{abc}T^bT^c &= \frac{N}{2}T^a && (406)\\
f^{acd}f^{bcd} &= N\delta^{ab} && (407)
\end{aligned}$$

Dirac-Algebra in 4 Dimensions

Traces with even number of γ-matrices:

$$\begin{aligned}
\mathrm{Tr}\{\mathbf{1}\} &= 4 && (408)\\
\mathrm{Tr}\{\gamma_\mu\gamma_\nu\} &= 4g_{\mu\nu} && (409)\\
\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\} &= 4[g_{\mu\nu}g_{\alpha\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}] && (410)
\end{aligned}$$

Traces with odd number of γ-matrices:

$$\mathrm{Tr}\{\gamma_{\mu_1}\ldots\gamma_{\mu_{2k+1}}\} = 0\,, \qquad k = 0,1,2,\ldots \qquad (411)$$

Traces including a γ_5-matrix:

$$\begin{aligned}
\mathrm{Tr}\{\gamma_5\} &= 0 && (412)\\
\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_5\} &= 0 && (413)\\
\mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\gamma_5\} &= 4i\epsilon_{\mu\nu\alpha\beta} && (414)\\
\mathrm{Tr}\{\gamma_{\mu_1}\ldots\gamma_{\mu_{2k+1}}\gamma_5\} &= 0\,, \qquad k = 0,1,2,\ldots && (415)
\end{aligned}$$

Useful identities for products of γ-matrices:

$$\begin{aligned}
\gamma_\mu\gamma^\mu &= 4 && (416)\\
\gamma_\mu\gamma_\alpha\gamma^\mu &= -2\gamma_\alpha && (417)\\
\gamma_\mu\gamma_\alpha\gamma_\beta\gamma^\mu &= 4g_{\alpha\beta} && (418)\\
\gamma_\mu\gamma_\alpha\gamma_\beta\gamma_\rho\gamma^\mu &= -2\gamma_\rho\gamma_\beta\gamma_\alpha && (419)\\
\gamma_\mu\gamma_\alpha\gamma_\nu &= g_{\alpha\mu}\gamma_\nu + g_{\alpha\nu}\gamma_\mu - g_{\mu\nu}\gamma_\alpha + i\epsilon_{\mu\alpha\nu\beta}\gamma_5\gamma_\beta && (420)
\end{aligned}$$

Useful identities for products of ϵ-tensors:

$$\epsilon_{\alpha\beta\mu\nu}\epsilon^{\alpha\beta\mu\nu} = -24 \tag{421}$$
$$\epsilon_{\alpha\beta\mu\nu}\epsilon^{\rho\beta\mu\nu} = -6g_\alpha^\rho \tag{422}$$
$$\epsilon_{\alpha\beta\mu\nu}\epsilon^{\rho\sigma\mu\nu} = -2[g_\alpha^\rho g_\beta^\sigma - g_\alpha^\sigma g_\beta^\rho] \tag{423}$$
$$\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\epsilon^{\beta_1\beta_2\beta_3\beta_4} = -\det\left(g_{\alpha_i}^{\beta_k}\right) \tag{424}$$
$$\frac{1}{2}\epsilon_{\alpha\beta\mu\nu}\sigma^{\mu\nu} = i\sigma_{\alpha\beta}\gamma_5 \tag{425}$$

We use definitions from Bjorken and Drell:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \epsilon_{0123} = +1 \tag{426}$$

Some other (equally famous) books use different definitions:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \epsilon^{0123} = -\epsilon_{0123} = +1 \qquad \text{Itzykson, Zuber} \tag{427}$$
$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = -i\gamma^0\gamma^1\gamma^2\gamma^3\,, \qquad \epsilon^{0123} = -\epsilon_{0123} = +1 \qquad \text{Okun} \tag{428}$$

This ambiguity is a standard source of sign errors!

Integration in the 4 Dimensional Euclidian Space

Definitions:

$$k_o \rightarrow ik_4 \tag{429}$$
$$d^4k = dk_o d^3\vec{k} = id^4k_E \tag{430}$$
$$k^2 = k_0^2 - \vec{k}^2 = -(k_1^2 + k_2^2 + k_3^2 + k_4^2) = -k_E^2 \tag{431}$$

Integration:

$$\int d^Dk_E f(k_E^2) = \int d\Omega_D \int\limits_0^\infty dk_E k_E^{D-1} f(k_E^2) \tag{432}$$
$$= \frac{\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int\limits_0^\infty dk_E^2 \left(k_E^2\right)^{\frac{D}{2}-1} f(k_E^2) \tag{433}$$

Dimensional Regularisation ($D = 4 - 2\epsilon$)

Definitions:

$$\int d^4k \rightarrow \int d^Dk \tag{434}$$
$$e_0 \rightarrow e_0\mu^{2-\frac{D}{2}} \tag{435}$$

Dirac-Algebra in D Dimensions

Since there are some subtleties in defining the ϵ-tensor and γ_5 in D dimensions, we will leave out here the corresponding formulae.

$$\gamma_\mu \gamma^\mu = D \tag{436}$$
$$\gamma_\mu \gamma_\alpha \gamma^\mu = (2-D)\gamma_\alpha \tag{437}$$
$$\gamma_\mu \gamma_\alpha \gamma_\beta \gamma^\mu = 4g_{\alpha\beta} + (D-4)\gamma_\alpha\gamma_\beta \tag{438}$$
$$\gamma_\mu \gamma_\alpha \gamma_\beta \gamma_\rho \gamma^\mu = -2\gamma_\rho\gamma_\beta\gamma_\alpha + (4-D)\gamma_\alpha\gamma_\beta\gamma_\rho \tag{439}$$

Feynman-Trick

Feynman parameter integrals for products of propagators:

$$\frac{1}{A\cdot B} = \int_0^1 dx \frac{1}{[xA+(1-x)B]^2} \tag{440}$$
$$\frac{\Gamma(a)\Gamma(b)}{A^a\cdot B^b} = \int_0^1 dx x^{a-1}(1-x)^{b-1}\frac{\Gamma(a+b)}{[xA+(1-x)B]^{a+b}} \tag{441}$$
$$\frac{1}{A_1A_2\ldots A_n} = \int_0^1 dx_1\ldots dx_n \frac{\delta\left(\sum x_i - 1\right)(n-1)!}{[x_1A_1+\ldots+x_nA_n]^n} \tag{442}$$
$$\frac{1}{A_1^{m_1}\ldots A_n^{m_n}} = \int_0^1 dx_1\ldots dx_n \frac{\delta\left(\sum x_i - 1\right)\Pi x_i^{m_i-1}}{[x_1A_1+\ldots+x_nA_n]^{\sum m_i}}\frac{\Gamma(m_1+\ldots+m_n)}{\Gamma(m_1)\cdot\ldots\cdot\Gamma(m_n)} \tag{443}$$

Loop Integrals in D Dimensions

$$\int d^Dk\,\frac{\Gamma(a)}{[-k^2-A-i\epsilon]^a} = i\pi^{\frac{D}{2}}\frac{\Gamma\left(a-\frac{D}{2}\right)}{[-A]^{a-\frac{D}{2}}} \tag{444}$$
$$\int d^Dk\,\frac{\Gamma(a)}{[-k^2-A-i\epsilon]^a}\,k_\mu k_\nu = i\pi^{\frac{D}{2}}\left(-\frac{g_{\mu\nu}}{2}\right)\frac{\Gamma\left(a-1-\frac{D}{2}\right)}{[-A]^{a-1-\frac{D}{2}}} \tag{445}$$

Taylor Expansion in ϵ

$$\Gamma(x+1) = x\Gamma(x) \tag{446}$$
$$\Gamma(\epsilon) \approx \frac{1}{\epsilon} - \gamma_E \tag{447}$$
$$x^\epsilon = \exp(\ln x) \approx 1 + \epsilon x + \dots \tag{448}$$

with the Euler constant $\gamma_E = 0.57721....$

Feynman Parameter Integrals

$$\beta(p,q) := \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)},$$
$$\beta(p,q) = \int_0^\infty \frac{t^{p-1}}{(1+t)^{p+q}}dt,$$
$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx.$$

13.2 Renormalisation

We summarise here the results for the divergent parts of the quark self energy, the gluon self energy and the vertex correction.

$$i\Sigma^{UV}(\not p, m) = \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{4}{3}\cdot i\delta_{il}\left(\not p - 4m\right) \tag{449}$$
$$i\Pi^{q,ab}_{\mu\nu}(p,m) = \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{-2}{3}\cdot i\delta^{ab}\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right) \tag{450}$$
$$i\Pi^{g,ab}_{\mu\nu}(p,m) = \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot 5\cdot i\delta^{ab}\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right) \tag{451}$$
$$\Gamma^\mu(p,q,m) = \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{13}{3}\cdot ig\gamma^\mu T^a \tag{452}$$

The renormalisation process starts with a redefinition of the fields, the masses and the couplings.

$$\Psi^0 = Z_2^{\frac{1}{2}}\Psi_R \tag{453}$$
$$A^{\mu,0} = Z_3^{\frac{1}{2}}A^\mu_R \tag{454}$$
$$m^0 = Z_m m_R \tag{455}$$
$$g^0 = Z_g g_R \tag{456}$$

Z_1 describes the renormalisation of the vertex.
Inserting this relations in the Lagrangian of QCD expressed in terms of the naked

quantities, we can split up the QCD Lagrangian in a part that contains only renormalised quantities and in a part that contains renormalised quantities and the renormalisation constants. The latter part is called the *counterterm Lagrangian*. For the counterterms we obtain the following Feynman rules.

$$\begin{aligned} \text{gluon propagator} &: \quad i\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right)(Z_3 - 1) && (457)\\ \text{quark propagator} &: \quad -i\left(\not{p}(Z_2 - 1) - m(Z_2 Z_M - 1)\right) && (458)\\ \text{vertex} &: \quad -ig\gamma^\mu T^a (Z_1 - 1) && (459)\end{aligned}$$

Summing up our results for the one-loop diagrams with the counterterms and demanding that the ϵ-pole cancels, we obtain

$$\begin{aligned} Z_m &= 1 + \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot 4, && (460)\\ Z_1 &= 1 + \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{13}{3}, && (461)\\ Z_2 &= 1 + \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{4}{3}, && (462)\\ Z_3 &= 1 - \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\left(5 - \frac{2}{3}n_f\right). && (463)\end{aligned}$$

The renormalisation constant of the coupling can be obtained from Z_1, Z_2 and Z_3

$$\begin{aligned} Z_g &= Z_1^{-1} Z_2 Z_3^{\frac{1}{2}} && (464)\\ &= 1 - \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{1}{2}\left(11 - \frac{2}{3}n_f\right). && (465)\end{aligned}$$

13.3 The Running Coupling

Between the naked g_0 and the renormalised coupling g_R the following relation holds

$$g_0 = Z_g g \mu^\epsilon. \qquad (466)$$

The naked coupling clearly does not depend on the renormalisation scale, therefore we obtain

$$\begin{aligned} 0 &= \frac{d}{d\mu}g_0 = \frac{dZ_g}{d\mu}g\mu^\epsilon + Z_g\frac{dg}{d\mu}\mu^\epsilon + \epsilon Z_g g\mu^{\epsilon-1} && (467)\\ \Rightarrow \quad & \frac{dg}{d\mu} = \frac{dg}{d\ln\mu}\frac{d\ln\mu}{d\mu} = -\epsilon g\mu^{-1} - \frac{dZ_g}{d\mu}\frac{g}{Z_g} && (468)\\ \Rightarrow \quad & \frac{dg}{d\ln\mu} = -\epsilon g - \frac{dZ_g}{d\ln\mu}\frac{g}{Z_g}. && (469)\end{aligned}$$

The renormalisation constant Z_g can be expanded in the following form

$$Z_g =: 1 + \frac{g^2}{(4\pi)^2} z_g + \mathcal{O}(g^4), \tag{470}$$

with

$$z_g = -\frac{1}{\epsilon}\frac{1}{2}\left(11 - \frac{2}{3}n_F\right) \ . \tag{471}$$

Now we can insert again in Eq.(469).

$$\begin{aligned}
\beta(g,\epsilon) := \frac{dg}{d\ln\mu} &= -\epsilon g - 2\frac{g^2}{(4\pi)^2} z_g \frac{dg}{d\ln\mu}\frac{1}{Z_g} && (472)\\
&\approx -\epsilon g + \epsilon 2\frac{g^3}{(4\pi)^2} z_g && (473)\\
&= -\epsilon g - \frac{g^3}{(4\pi)^2}\left(11 - \frac{2}{3}n_F\right) \ . && (474)
\end{aligned}$$

z_g contains a pole in ϵ. In the limit $\epsilon \to 0$ only the second term survives:

$$\begin{aligned}
\beta(g) &= -\beta_0 \frac{g^3}{(4\pi)^2} + \mathcal{O}(g^5), && (475)\\
\text{with } \beta_0 &= -2\epsilon z_g. && (476)
\end{aligned}$$

With the results from the previous section we have

$$z_g = -\frac{1}{\epsilon}\frac{1}{2}\left(11 - \frac{2}{3}n_F\right) \tag{477}$$

and therefore

$$\beta_0 = \left(11 - \frac{2}{3}n_F\right). \tag{478}$$

Now we can easily derive a solution for $\alpha(\mu)$:

$$\begin{aligned}
\frac{dg}{d\ln\mu} &= -\beta_0 \frac{g^3}{(4\pi)^2} && (479)\\
\Rightarrow \frac{dg}{g^3} &= -\frac{\beta_0}{(4\pi)^2} d\ln\mu && (480)\\
\int_{g_0}^{g_1} \frac{dg}{g^3} &= -\frac{\beta_0}{(4\pi)^2}\int_{\mu_0}^{\mu_1} d\ln\mu && (481)
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow \left[\frac{1}{-2g^2}\right]_{g_0}^{g_1} &= -\frac{\beta_0}{(4\pi)^2}\left[\ln \mu_1 - \ln \mu_0\right] && (482)\\
\frac{1}{g_1^2} - \frac{1}{g_0^2} &= \frac{2\beta_0}{(4\pi)^2} \ln \frac{\mu_1}{\mu_0} && (483)\\
\frac{1}{g_1^2} &= \frac{1}{g_0^2} + \frac{2\beta_0}{(4\pi)^2} \ln \frac{\mu_1}{\mu_0} && (484)\\
g_1^2 &= \frac{1}{\frac{1}{g_0^2} + \frac{2\beta_0}{(4\pi)^2} \ln \frac{\mu_1}{\mu_0}} && (485)\\
\frac{g_1^2}{4\pi} &= \frac{\frac{g_0^2}{4\pi}}{1 + \frac{2\beta_0}{4\pi}\frac{g_0^2}{4\pi} \ln \frac{\mu_1}{\mu_0}} && (486)\\
\Rightarrow \alpha(\mu_1) &= \frac{\alpha(\mu_0)}{1 + \frac{2\beta_0}{4\pi}\frac{g_0^2}{4\pi} \ln \frac{\mu_1}{\mu_0}} && (487)
\end{aligned}
$$

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