

Precision simulations for LHC physics in SHERPA

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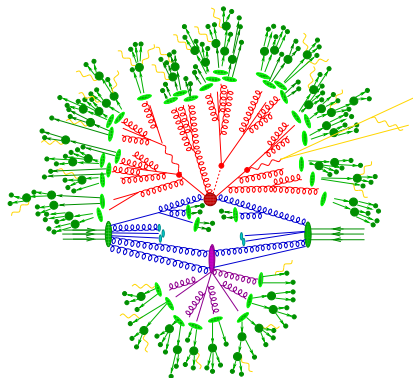
Sheffield University, 22.1.2015



The inner working of event generators ...

simulation: *divide et impera*

- **hard process:**
fixed order perturbation theory
traditionally: Born-approximation
- **bremsstrahlung:**
resummed perturbation theory
- **hadronisation:**
phenomenological models
- **hadron decays:**
effective theories, data
- **"underlying event":**
phenomenological models



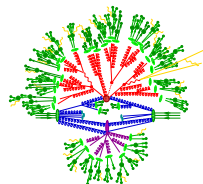
... and possible improvements

possible strategies:

- improving the phenomenological models:
 - “tuning” (fitting parameters to data)
 - replacing by better models, based on more physics
(my hot candidate: “minimum bias” and “underlying event” simulation)
- improving the perturbative description:
 - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

“NLO-Matching” & “Multijet-Merging”

- systematic improvement of the parton shower:
next-to leading (or higher) logs & colours



Reminder: Ingredients of simulations

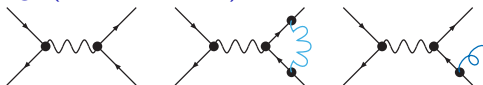
Cross sections at the LHC: Born approximation

$$d\sigma_{ab \rightarrow N} = \int_0^1 dx_a dx_b f_a(x_a, \mu_F) f_b(x_b, \mu_F) \int_{\text{cuts}} d\Phi_N \frac{1}{2\hat{s}} |\mathcal{M}_{p_a p_b \rightarrow N}(\Phi_N; \mu_F, \mu_R)|^2$$

- parton densities $f_a(x, \mu_F)$ (PDFs)
- phase space Φ_N for N -particle final states
- incoming current $1/(2\hat{s})$
- squared matrix element $\mathcal{M}_{p_a p_b \rightarrow N}$
(summed/averaged over polarisations)
- renormalisation and factorisation scales μ_R and μ_F
- complexity demands numerical methods for large N

Higher orders: some general thoughts

- obtained from adding diagrams with additional:
loops (virtual corrections) or
legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F
NLO allows for meaningful estimate of uncertainties
- additional difficulties when going NLO:
ultraviolet divergences in virtual correction
infrared divergences in real and virtual correction

enforce

UV regularisation & renormalisation
IR regularisation & cancellation

(Kinoshita–Lee–Nauenberg–Theorem)

Structure of an NLO calculation

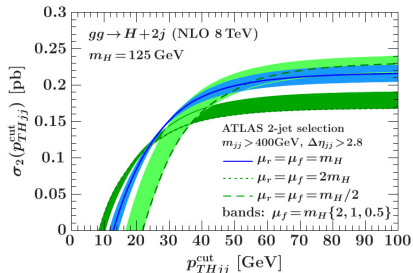
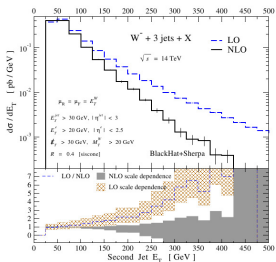
- sketch of cross section calculation

$$\begin{aligned}
 d\sigma_N^{(\text{NLO})} = & \underbrace{d\Phi_N \mathcal{B}_N}_{\substack{\text{Born} \\ \text{approximation}}} + \underbrace{d\Phi_N \mathcal{V}_N}_{\substack{\text{renormalised} \\ \text{virtual correction}}} + \underbrace{d\Phi_{N+1} \mathcal{R}_{N+1}}_{\substack{\text{real correction} \\ \text{IR-divergent}}} \\
 & \text{IR-divergent} \qquad \qquad \text{IR-divergent} \\
 = & d\Phi_N \left[\mathcal{B}_N + \mathcal{V}_N + \mathcal{B}_N \otimes \mathcal{S} \right] + d\Phi_{N+1} \left[\mathcal{R}_{N+1} - \mathcal{B}_N \otimes d\mathcal{S} \right]
 \end{aligned}$$

- subtraction terms $\mathcal{S}$ (integrated) and $d\mathcal{S}$:
exactly cancel IR divergence in $\mathcal{R}$ – process-independent structures
- result: terms in both brackets **separately infrared finite**

An interesting problem with scales

- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:
unphysical scale choices will yield unphysical results



- so maybe we have to be a bit smarter than just running NLO code

Probabilistic treatment of emissions

- Sudakov form factor

$$\Delta_{ij,k}(t, t_0) = \exp \left[- \int_{t_0}^t d\Gamma_{ij,k}(t) \right]$$

yields probability for **no decay** between scales t_0 and t

- decay width for parton $i(j) \rightarrow ik(j)$ (spectator j)

$$d\Gamma_{ij,k}(t) = \frac{dt}{t} \frac{\alpha_S}{2\pi} \int dz \frac{d\phi}{2\pi} \underbrace{\mathcal{K}_{ij,k}(t, z, \phi)}_{\text{splitting kernel}}$$

- evolution parameter t defined by kinematics

generalised angle (HERWIG++) or transverse momentum (PYTHIA, SHERPA)

- scale choice for strong coupling: $\alpha_S(k_\perp^2)$

resums classes of higher logarithms

- regularisation through cut-off t_0

Emissions off a Born matrix element

- “compound” splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n -particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_S}{2\pi} \sum_{\text{all } \{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}), \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp \left[- \int_{t_0}^t d\Phi_1 \mathcal{K}_n(\Phi_1) \right]$$

- consider first emission only off Born configuration

$$d\sigma_B = d\Phi_N \mathcal{B}_N(\Phi_N)$$

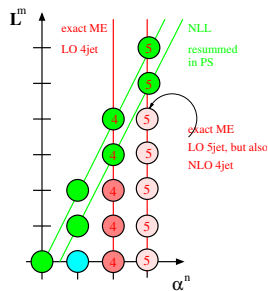
$$\cdot \underbrace{\left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \left[\mathcal{K}_N(\Phi_1) \Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}}_{\text{integrates to unity} \longrightarrow \text{“unitarity” of parton shower}}$$

- further emissions by recursion with $\mu_N^2 \longrightarrow t$ of previous emission

NLO improvements: Matching & Merging

NLO matching: Basic idea

- parton shower resums logarithms
fair description of collinear/soft emissions
jet evolution (where the logs are large)
- matrix elements exact at given order
fair description of hard/large-angle emissions
jet production (where the logs are small)
- adjust (“match”) terms:
 - cross section at NLO accuracy
 - correct hardest emission in PS to exactly reproduce ME at order α_S ($\mathcal{R}$ -part of the NLO calculation)



The POWHEG-trick: modifying the Sudakov form factor

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$d\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \xrightarrow{\text{IR}} d\Phi_1 \frac{\alpha_S}{2\pi} \mathcal{K}_{ij,k}(\Phi_1)$$

- define **modified Sudakov form factor** (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) = \exp \left[- \int_{t_0}^{\mu_N^2} d\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \right],$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_S to parton shower scale

Local K -factors

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- start from Born configuration Φ_N with NLO weight:

("local K -factor")

$$\begin{aligned} d\sigma_N^{(\text{NLO})} &= d\Phi_N \tilde{\mathcal{B}}(\Phi_N) \\ &= d\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N) \otimes \mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)} \right. \\ &\quad \left. + \int d\Phi_1 [\mathcal{R}_N(\Phi_N \otimes \Phi_1) - \mathcal{B}_N(\Phi_N) \otimes dS(\Phi_1)] \right\} \end{aligned}$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes dS$

(relevant for MC@NLO)

NLO accuracy in radiation pattern

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0)$:

$$d\sigma_N^{(\text{NLO})} = d\Phi_N \bar{\mathcal{B}}(\Phi_N) \times \underbrace{\left\{ \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \frac{\mathcal{R}_N(\Phi_N \otimes \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, k_\perp^2(\Phi_1)) \right\}}_{\text{integrating to yield 1 - "unitarity of parton shower"}}$$

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2
- apart from logs: which configurations enhanced by local K -factor

(this is vanilla POWHEG!)

(K -factor for inclusive production of X adequate for X + jet at large $p_\perp$?)

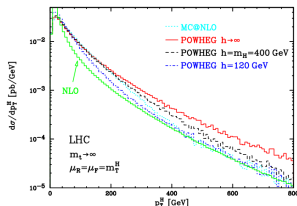
Improved POWHEG

(S. Alioli, P. Nason, C. Oleari, & E. Re, JHEP 0904 (2009) 002)

- split real-emission ME as

$$\mathcal{R} = \mathcal{R} \left(\underbrace{\frac{h^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(F)}} \right)$$

- can “tune” h to mimick NNLO - or maybe resummation result
- differential event rate up to first emission



$$d\sigma = d\Phi_B \bar{\mathcal{B}}^{(R^{(S)})} \left[\Delta^{(R^{(S)}/B)}(s, t_0) + \int_{t_0}^s d\Phi_1 \frac{\mathcal{R}^{(S)}}{B} \Delta^{(R^{(S)}/B)}(s, k_\perp^2) \right] + d\Phi_R \mathcal{R}^{(F)}(\Phi_R)$$

Resummation in MC@NLO

(S. Fixione & B. Webber, JHEP 0602 (2002) 029)

(S. Hoeche, F. Krauss, M. Schoenherr, & F. Siegert, JHEP 1209 (2012) 049)

- divide $\mathcal{R}_N$ in **soft** (“S”) and **hard** (“H”) part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes d\mathcal{S}_1 + \mathcal{H}_N$$

- identify subtraction terms and shower kernels $d\mathcal{S}_1 \equiv \sum_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1st emission to account for colour)

$$d\sigma_N = d\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \mathcal{K}_{ij,k}(\Phi_1) \Delta_N^{(\mathcal{K})}(\mu_N^2, k_{\perp}^2) \right] + d\Phi_{N+1} \mathcal{H}_N$$

- effect: only resummed parts modified with local K -factor

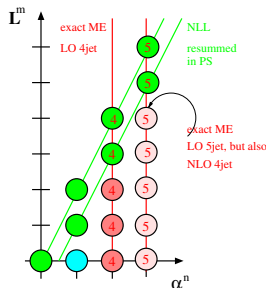
Multijet merging @ leading order

Multijet merging: basic idea

(S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111 (2001) 063,

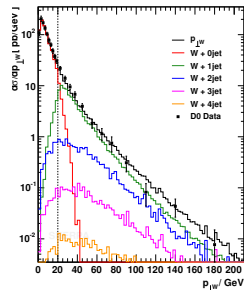
L. Lonnblad, JHEP 0205 (2002) 046, & F. Krauss, JHEP 0208 (2002) 015)

- parton shower resums logarithms
fair description of collinear/soft emissions
jet evolution (where the logs are large)
- matrix elements exact at given order
fair description of hard/large-angle emissions
jet production (where the logs are small)
- combine (“merge”) both:
result: “towers” of MEs with increasing
number of jets evolved with PS
 - multijet cross sections at **Born accuracy**
 - maintain **(N)LL accuracy** of parton shower



Separating jet evolution and jet production

- separate regions of jet production and jet evolution with jet measure Q_J
(“truncated showering” if not identical with evolution parameter)
- matrix elements populate hard regime
- parton showers populate soft domain



First emission(s), again

(S. Hoeche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905 (2009) 053)

$$d\sigma = d\Phi_N \mathcal{B}_N \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \\ + d\Phi_{N+1} \mathcal{B}_{N+1} \Delta_N^{(\mathcal{K})}(\mu_{N+1}^2, t_{N+1}) \Theta(Q_{N+1} - Q_J)$$

- note: $N + 1$ -contribution includes also $N + 2$, $N + 3$, ...

(no Sudakov suppression below t_{n+1} , see further slides for iterated expression)

- potential occurrence of different shower start scales: $\mu_{N,N+1}, \dots$
- “unitarity violation” in square bracket: $\mathcal{B}_N \mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

(cured with UMEPs formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &

S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])

Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \rightarrow \text{hadrons}$
Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor α_S and up to $\log^2 \frac{E_{\text{c.m.}}}{Q_J}$ per jet
- use **Sudakov form factor** for resummation &
replace **approximate fixed order** by exact expression:



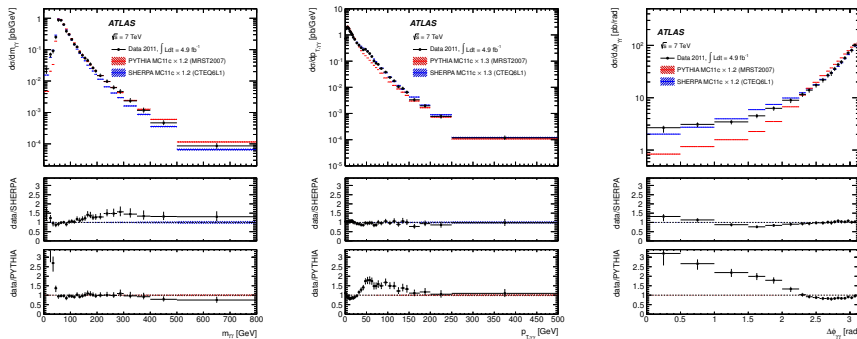
$$\mathcal{R}_2(Q_J) = [\Delta_q(E_{\text{c.m.}}^2, Q_J^2)]^2$$



$$\begin{aligned} \mathcal{R}_3(Q_J) = & 2\Delta_q(E_{\text{c.m.}}^2, Q_J^2) \int_{Q_J^2}^{E_{\text{c.m.}}^2} \frac{dk_\perp^2}{k_\perp^2} \left[\frac{\alpha_S(k_\perp^2)}{2\pi} dz \mathcal{K}_q(k_\perp^2, z) \right. \\ & \times \Delta_q(E_{\text{c.m.}}^2, k_\perp^2) \Delta_q(k_\perp^2, Q_J^2) \Delta_g(k_\perp^2, Q_J^2) \left. \right] \end{aligned}$$

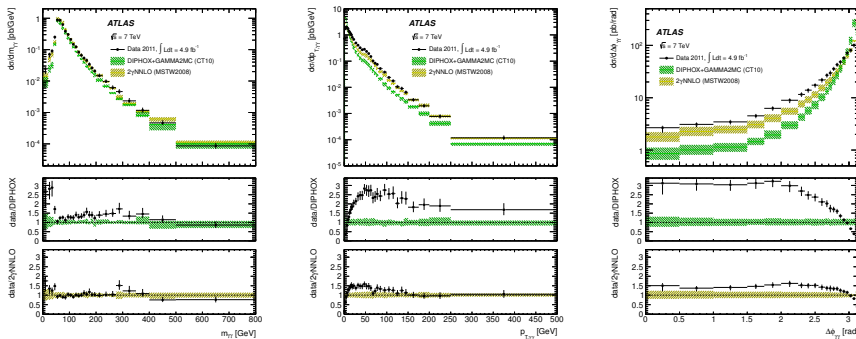
Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])



Aside: Comparison with higher order calculations

(arXiv:1211.1913 [hep-ex])



Multijet merging @ next-to leading order

Multijet-merging at NLO: MEPs@NLO

(arXiv: 1207.5030, 1207.5031 [hep-ph])

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

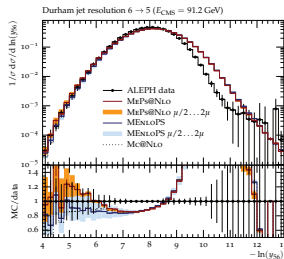
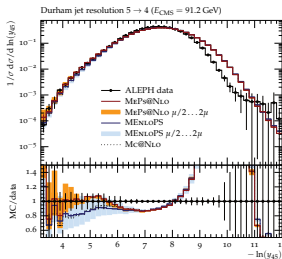
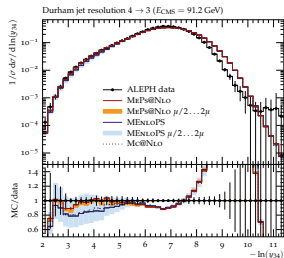
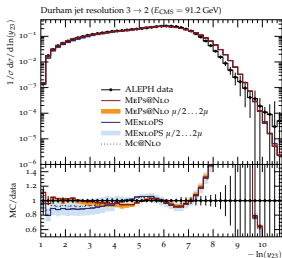
maintain NLO and LL accuracy of ME and PS

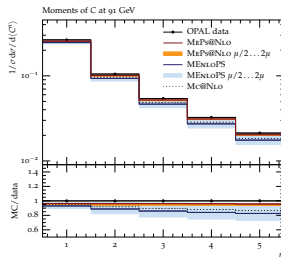
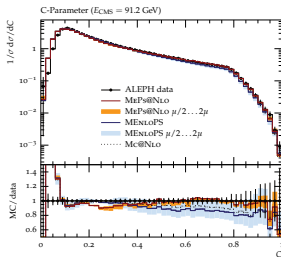
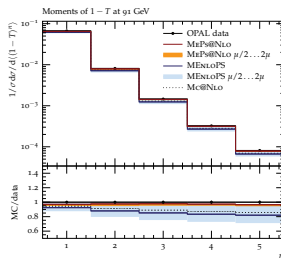
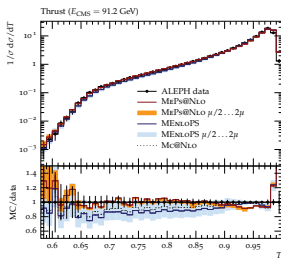
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities

First emission(s), once more

$$\begin{aligned}
 d\sigma = & d\Phi_N \tilde{\mathcal{B}}_N \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \\
 & + d\Phi_{N+1} \mathcal{H}_N \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \\
 & + d\Phi_{N+1} \tilde{\mathcal{B}}_{N+1} \left(1 + \frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}} \int_{t_{N+1}}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \right) \Theta(Q_{N+1} - Q_J) \\
 & \cdot \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \cdot \left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_0) + \int_{t_0}^{t_{N+1}} d\Phi_1 \mathcal{K}_{N+1} \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2}) \right] \\
 & + d\Phi_{N+2} \mathcal{H}_{N+1} \Delta_N^{(\mathcal{K})}(\mu_N^2, t_{N+1}) \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2}) \Theta(Q_{N+1} - Q_J) + \dots
 \end{aligned}$$

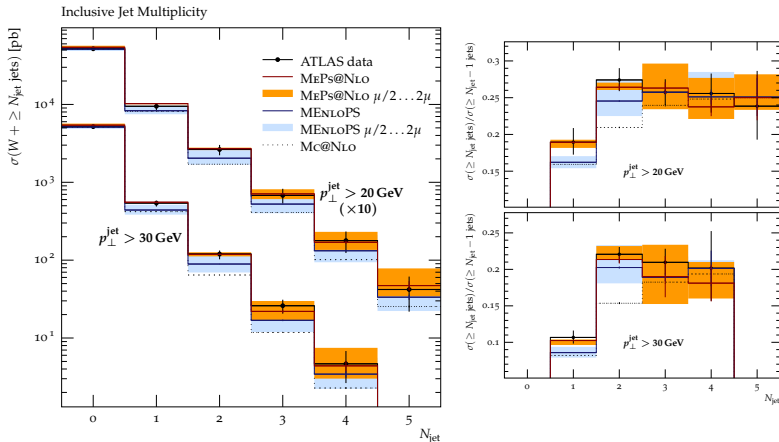
MEPs@NLO: validation in $e^-e^+ \rightarrow \text{hadrons}$

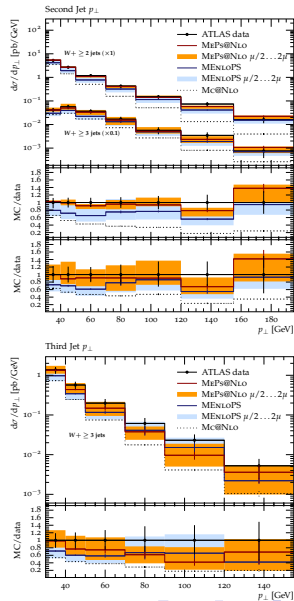
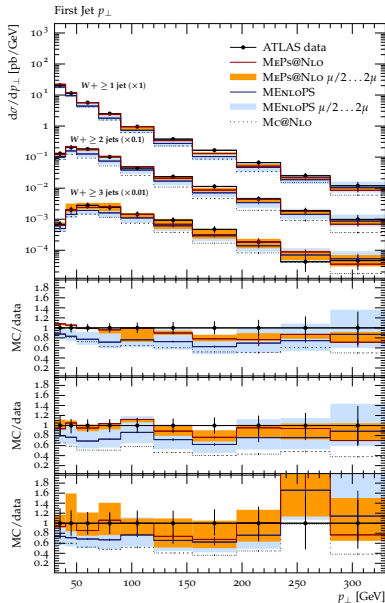


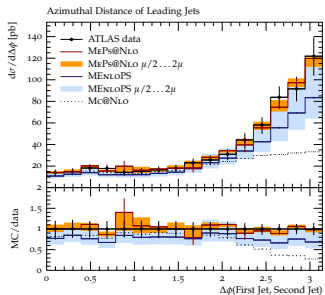
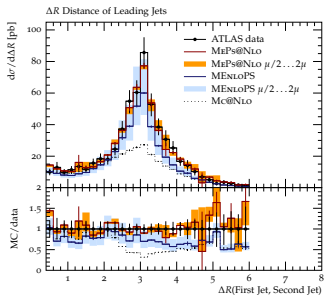
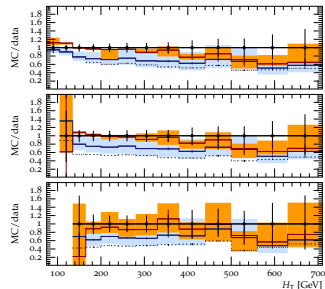
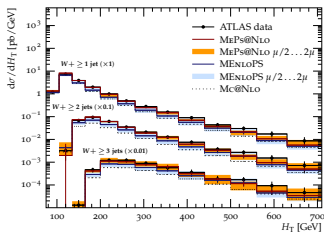


MEPS@NLO: validation in W +jets

(S. Hoeche, F. Krauss, M. Schoenherr & F. Siegert, JHEP 1304 (2013) 027)





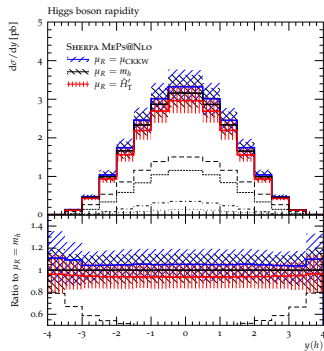
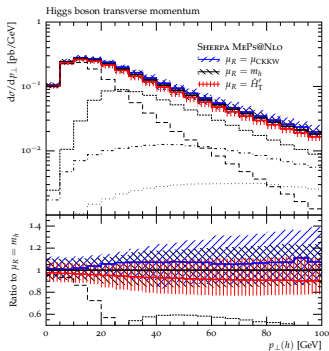


Multijet merging @ next-to leading order: $gg \rightarrow H$

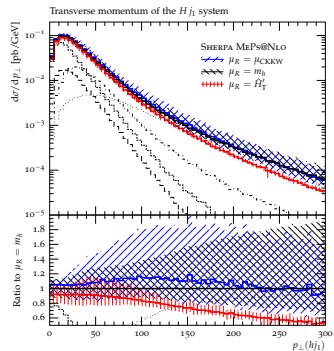
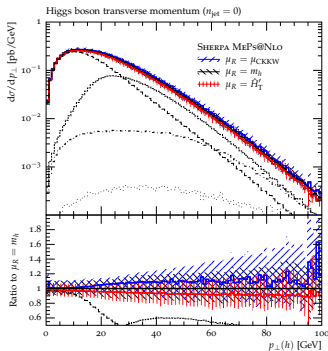
Results for Higgs boson production through gluon fusion

- parton-shower level, Higgs boson does not decay
- setup & cuts:
 - jets: anti-kt, $p_{\perp} \geq 20$ GeV, $R = 0.4$, $|\eta| \leq 4.5$
 - dijet cuts: at least 2 jets with $p_{\perp} \geq 25$ GeV
 - WBF cuts: $m_{jj} \geq 400$ GeV, $\Delta y_{jj} \geq 2.8$
- jet multiplicity plots:
 - 0-jet excl.: no jet with $p_{\perp} \geq \{20, 25, 30\}$ GeV
 - 2-jet incl.: at least two jets with $p_{\perp} \geq \{20, 25, 30\}$ GeV
- SHERPA with $H + \{0, 1, 2\}^{(NLO)} + \{3\}^{(LO)}$ jets, $Q_{\text{cut}} = 20 \text{ GeV}$

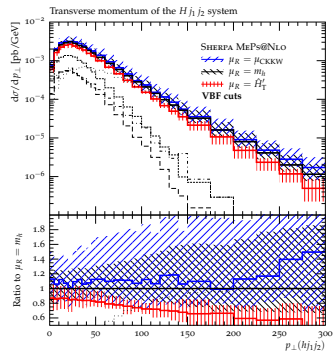
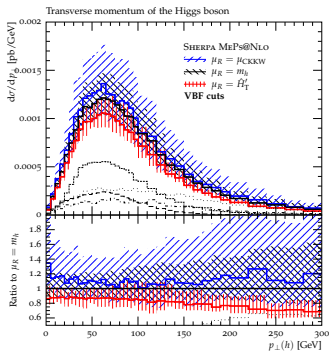
Inclusive observables for $gg \rightarrow H$



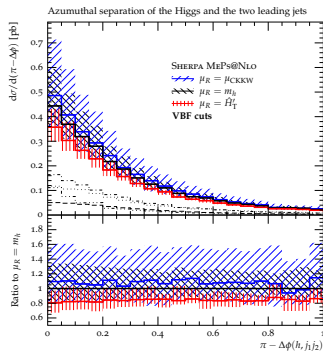
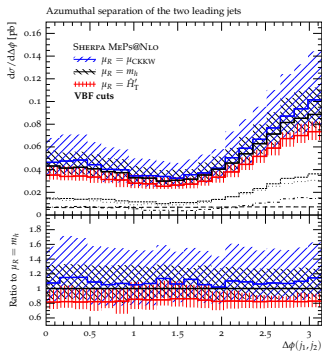
Exclusive observables for $gg \rightarrow H$



$gg \rightarrow H$ after WBF cuts

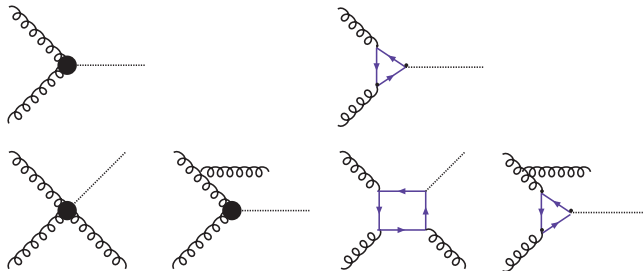


$gg \rightarrow H$ after WBF cuts



Quark mass effects

- include effects of quark masses

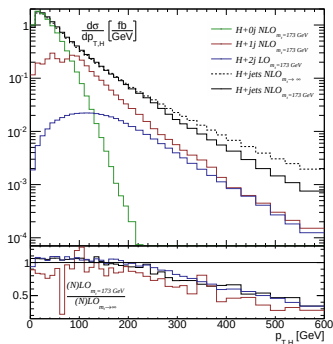


- reweight NLO HEFT with LO ratio:

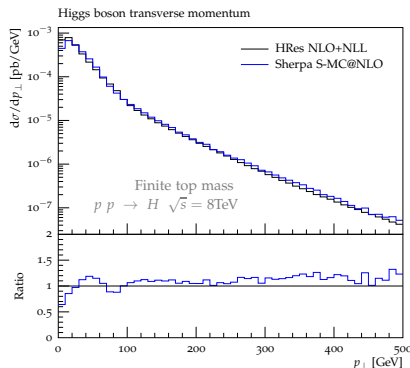
$$d\sigma_{\text{mass}}^{(\text{NLO})} \approx d\sigma_{\text{HEFT}}^{(\text{NLO})} \times \frac{d\sigma_{\text{mass}}^{(\text{LO})}}{d\sigma_{\text{HEFT}}^{(\text{LO})}}$$

Quark mass effects – results

- top mass effect in MEPS@NLO (on Higgs- $p_{\perp}$)

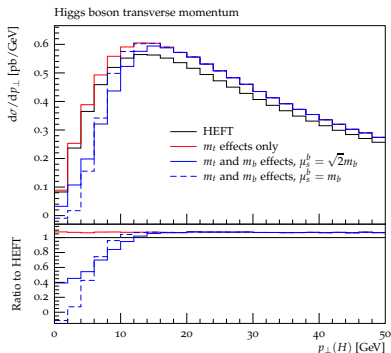


- comparison S-MC@NLO– HRES (top-loop only)

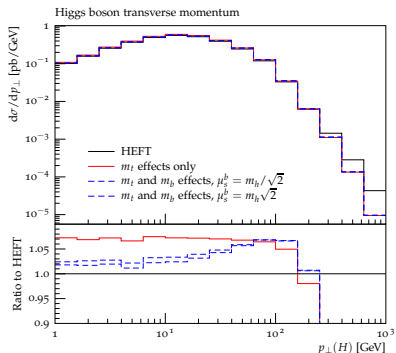


b -mass effects: playtime

vary around $\mu_Q = m_b$

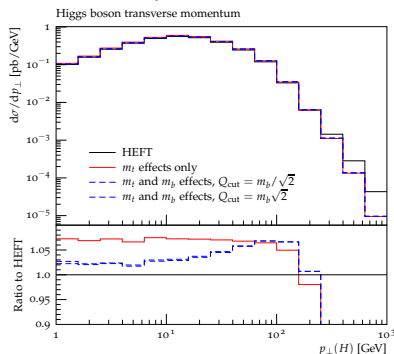


vary around $\mu_Q = m_h$ with $Q_{\text{cut}} = m_b$

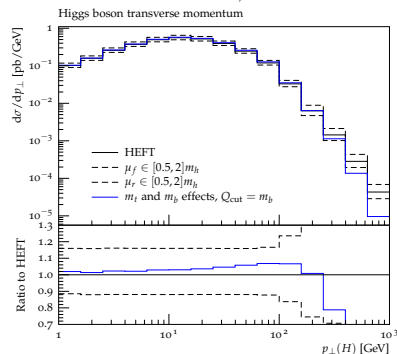


b -mass effects: playtime (cont'd)

vary around $\mu_Q = m_h$ with $Q_{\text{cut}} = m_b$

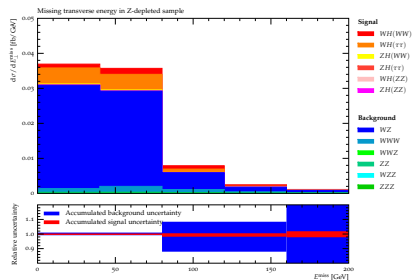
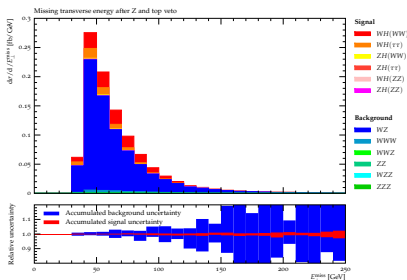


vary $\mu_{F,R}$

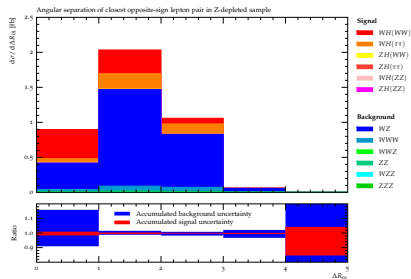
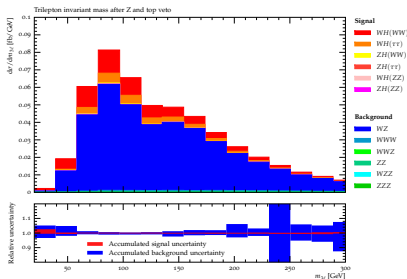


Multijet merging @ next-to leading order: $VH \rightarrow 3\ell$

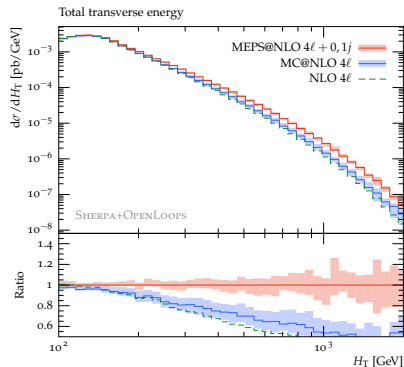
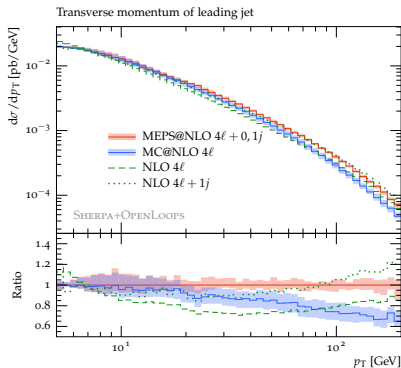
Relevant observables for $VH \rightarrow 3\ell$: $\cancel{E}_T$



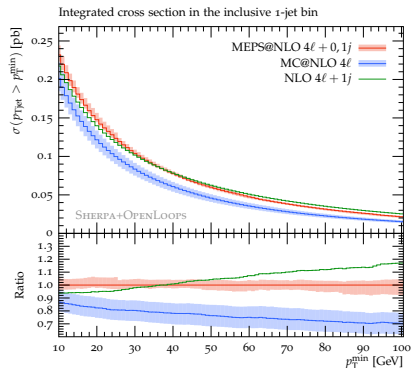
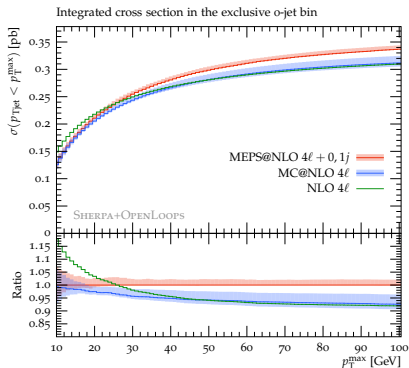
Relevant observables for $VH \rightarrow 3\ell$: m_{123} & ΔR_{01}



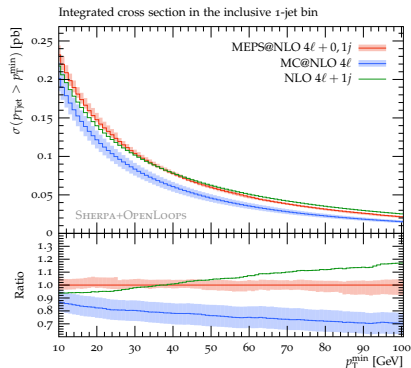
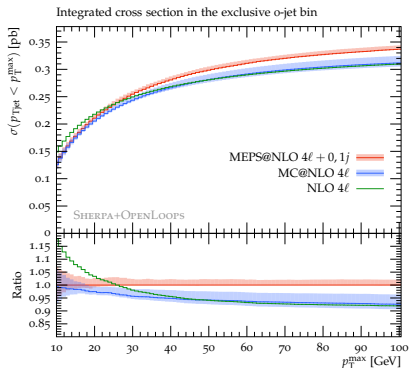
Higgs backgrounds: inclusive observables in $W^+W^- + \text{jets}$



Higgs backgrounds: jet vetoes in W^+W^-+jets

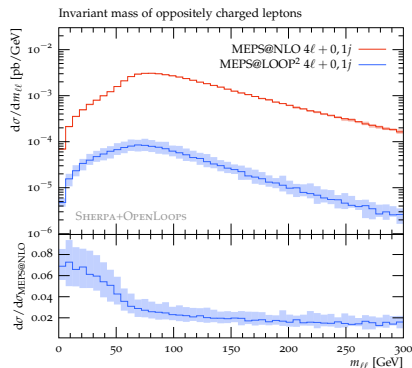
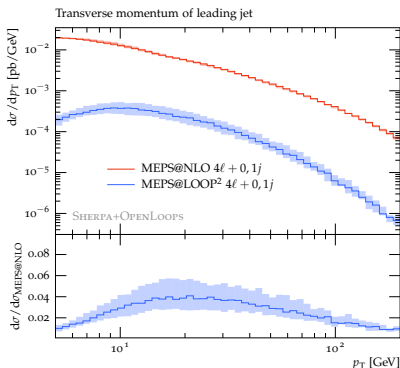


Higgs backgrounds: jet vetoes in W^+W^-+jets

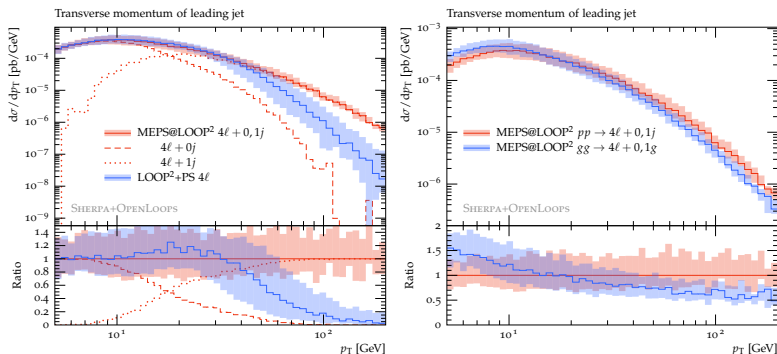


Higgs backgrounds: gluon-induced processes W^+W^-+jets

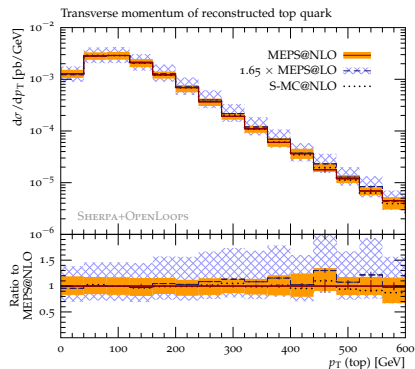
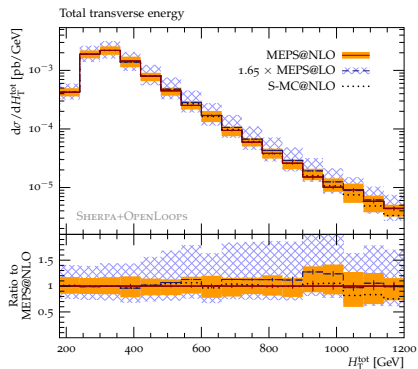
- include (LO-) merged loop² contributions of $gg \rightarrow VV$ (+1 jet)



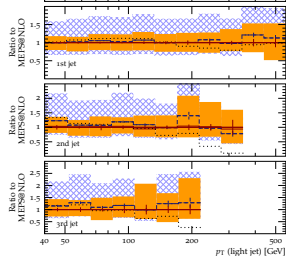
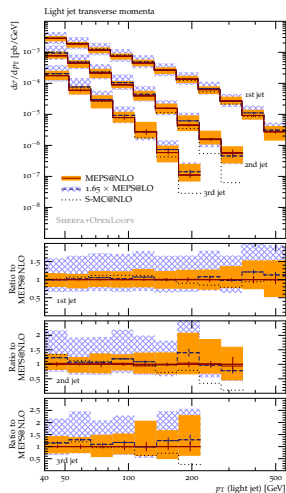
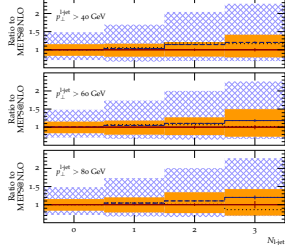
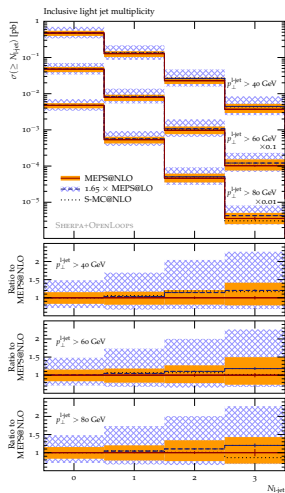
Higgs backgrounds: jet vetoes in $W^+W^- + \text{jets}$



Higgs backgrounds: $t\bar{t}$ + jets



Higgs backgrounds: light jets in $t\bar{t}$ + jets



Summary

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:

- multijet merging ("CKKW", "MLM")
- NLO matching ("MC@NLO", "POWHEG")
- MENLOPS NLO matching & merging
- MEPS@NLO ("SHERPA", "UNLOPS", "MINLO", "FxFx")

(first 3 methods are well understood and used in experiments)

(last method need validation etc.)



- multijet merging an important tool for many relevant signals and backgrounds - pioneered by SHERPA at LO & NLO
- complete automation of NLO calculations done
→ must benefit from it!

(it's the precision and trustworthy & systematic uncertainty estimates!)

Famous last screams

- in Run-II we'll be in for a ride:

more statistics
more energy
more channels
more precision
more fun

