A new approach to color-coherent parton evolution: ALARIC

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- Color Coherence, encore
- New Kinematics Mapping
- Checking the Maps
- Comparison with LEP data
- NLO Subtraction Terms for ALARIC
- Outlook

color coherence

(the story never gets old)

factorization of amplitudes

- collinear:

$$ {}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n \stackrel{i\|j}{\longrightarrow} \sum_{\lambda,\lambda'=\pm} {}_{n-1}\left\langle 1,\ldots,\not{i}(ij),\ldots,\not{j},\ldots,n\left|\frac{8\pi\alpha_s}{2p_ip_j}P^{\lambda\lambda'}_{(ij)i}(z)\right|1,\ldots,\not{i}(ij),\ldots,\not{j},\ldots,n\right\rangle_{n-1}, $$

with spin-dependent splitting function $P^{\lambda\lambda'}_{(ij)i}(z)$

- soft:

$$ {}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n \stackrel{p_j\to 0}{\longrightarrow} -8\pi\alpha_s\sum_{i,k\neq j} {}_{n-1}\langle 1,\ldots,\not{j},\ldots,n|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}|1,\ldots,\not{j},\ldots,n\rangle_{n-1} $$

with colour-insertion operators $\mathbf{T}_{i,k}$ & soft eikonal

$$ w_{ik,j} = \frac{p_ip_k}{(p_ip_j)(p_jp_k)} = \frac{W_{ik,j}}{E_j^2} = \frac{1}{E_j^2}\,\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} $$

(obviously, frame-dependent when expressed by energies & angles)

soft eikonals, decomposed

- textbook decomposition (pink bible): $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$
  with "radiator functions" $\tilde{W}^i_{ik,j}$: (identify "splitters" to combine with collinear terms)

$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

- express θ_{jk} for use in i-splitter term:

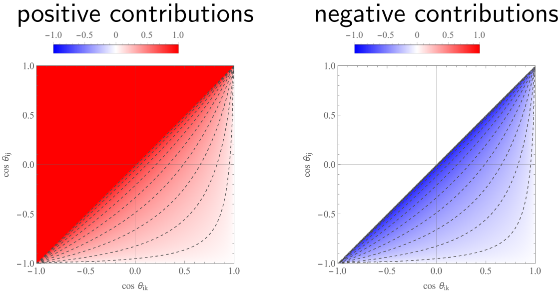
$$\cos\theta_{jk} = \cos\theta_{ij}\cos\theta_{ik} + \sin\theta_{ij}\sin\theta_{ik}\cos\phi^i_{jk} \;\ldots$$

- …and average over azimuth ϕ^i_{jk}:

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi^i_{jk}\,\tilde{W}^i_{ik,j} = \frac{\tilde{I}^i_{ik,j}}{1-\cos\theta^i_j}\,, \qquad \text{where} \qquad \tilde{I}^i_{ik,j} = \begin{cases} 1 & \text{if} \quad \theta^i_j < \theta^i_k \\ 0 & \text{else} \end{cases}$$

(this is the well-known source of angular ordering)

- azimuthally integrated radiator function (normalised to 2π):



- need to include azimuthal modulation, if observables sensitive to it
- but: naive inclusion bound to fail (MC efficiency $\to 0$)

soft eikonals, decomposed again

- define **positive definite** radiators:

(borrowing from Catani & Seymour, Nucl. Phys. **B485** (1997) 291)

$$\bar{W}^{i}_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{jk})}$$

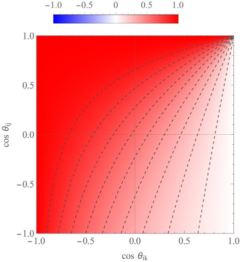
- same result after azimuthal averaging, but $\tilde{I}^{i}_{ik,j} \longrightarrow \bar{I}^{i}_{ik,j}$ with

$$\bar{I}^{i}_{ik,j} = \frac{1}{\sqrt{(\bar{A}^{i}_{ij,k})^2-(\bar{B}^{i}_{ij,k})^2}}$$

where

$$\bar{A}^{i}_{ij,k} = \frac{2-\cos\theta^{i}_{j}(1+\cos\theta^{i}_{k})}{1-\cos\theta^{i}_{k}}\,,\ \bar{B}^{i}_{ij,k} = \frac{\sqrt{(1-\cos^2\theta^{i}_{j})(1-\cos^2\theta^{i}_{k})}}{1-\cos\theta^{i}_{k}}$$

integrated radiator, $\bar{I}^{i}_{ik,j}$



matching with collinear terms

- collinear limit of eikonal factors:

$$w_{ik,j} \xrightarrow{i||j} w_{ik,j}^{(\mathrm{coll})}(z) = \frac{1}{2p_ip_j}\frac{2z}{1-z}\,, \qquad \text{where} \qquad z \xrightarrow{i||j} \frac{E_i}{E_i+E_j}$$

- compare with leading $(1-z)$-terms of splitting functions

($1/z$ term in $g \longrightarrow gg$ captured with other "dipole")

$$P_{qq}(z) = C_F\left(\frac{2z}{1-z} + (1-z)\right)\,,$$
$$P_{gg}(z) = C_A\left(\frac{2z}{1-z} + z(1-z)\right)\,,$$
$$P_{gq}(z) = T_R\,(1-2z(1-z))\,.$$

$\longrightarrow$ defines "collinear remnant"

kinematics mapping

(don't change history)

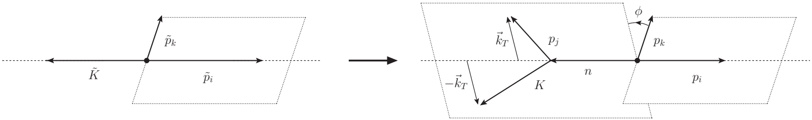
birds-eye view

- kinematics as main obstacle to NLL accuracy in dipole showers: recoil of subsequent soft emissions may change "NLL history"
- construct new mapping $\{\tilde{p}_l\} \longrightarrow \{p_l\}$

(inspired by Catani & Seymour's treatment of identified hadrons)

- logic: disentangle colour spectator $\tilde{p}_k$ and recoil partner $\tilde{K}$

(*i.e.* define a **global** recoil scheme, use spectator for eikonal/azimuth)



constructing the kinematics

- splitter $\tilde{p}_i \to p_i = z\tilde{p}_i$, spectator $\tilde{p}_k \to p_k$ (splitter and spectator keep direction)
- introduce orientation n to define splitting variable $z = \frac{p_i n}{(p_i + p_j)n}$
- with recoil momentum $\tilde{K}$: $n = \tilde{K} + (1-z)\tilde{p}_i$
- construct emitted momentum and recoil partner after splitting:

(demand $\tilde{K}^2 = K^2$ & $p_j^2 = m_j^2 = 0$)

$$\begin{aligned} p_j &= v\,\bar{n} + \frac{1}{v}\frac{\mathrm{k}_\perp^2}{2\tilde{p}_i\tilde{K}}\,\tilde{p}_i - k_\perp \\ K &= (1-v)\,\bar{n} + \frac{1}{1-v}\frac{\mathrm{k}_\perp^2 + \tilde{K}^2}{2\tilde{p}_i\tilde{K}}\,\tilde{p}_i + k_\perp\,. \end{aligned}$$

with $v = \frac{p_i p_j}{p_i \tilde{K}}$ and additional direction $\bar{n} = n - \frac{n^2}{2\tilde{p}_i n}\tilde{p}_i$

- transverse momentum vanishes for $p_i \parallel p_j$

$$\mathrm{k}_\perp^2 = v(1-v)\,2p_jK - v^2K^2 = v(1-v)(1-z)\,2\tilde{p}_i\tilde{K} - v^2\tilde{K}^2$$

constructing the kinematics

- boost every momentum in recoil partner to new system $\tilde{K} \to K$:

$$p_l^\mu \to \Lambda^\mu_{\ \nu}(K,\tilde{K})\, p_l^\nu \text{ with } \Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_{\ \nu} - \frac{2(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{(K+\tilde{K})^2} + \frac{2\tilde{K}^\mu K_\nu}{K^2}$$

- construct emission phase space:
  - obtain by factorising 3-body phase space, result:

$$\mathrm{d}\Phi^{(\mathrm{FI})}_{+1}(-\tilde{K};\tilde{p}_1,\ldots,\tilde{p}_{j-1},\tilde{p}_{j+1},\ldots,\tilde{p}_n;p_j) = \frac{-2\tilde{p}_i\tilde{K}}{16\pi^2}\,\mathrm{d}v\,\mathrm{d}z\,z\,\frac{\mathrm{d}\phi}{2\pi}$$

  - note: azimuthal angle expressed through scalar products $\longrightarrow$ PS Lorentz-invariant
- IS kinematics from FS through crossing relations

parton evolution

- define evolution parameter:

$$t = 2E_j^2\,(1-\cos\theta_j{}^i) = v\,(1-z)\,2\tilde{p}_i\tilde{K}$$

and therefore soft evolution given by

$$\mathrm{d}P_{ik,j}^{i\,(\mathrm{soft})}(t,z,\phi) = \mathrm{d}\Phi_{+1}(\{\tilde{p}\},p_j)\,8\pi\alpha_s\,C_i\,\bar{w}_{ik,j}^i = \mathrm{d}t\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi\,t}\,2C_i\,\bar{W}_{ik,j}$$

- same for collinear evolution, but could evolve in virtuality or similar

checking the maps

(do they change history?)

analytic considerations

- analyse Lorentz boost (*i.e.* impact on previous emissions)
- decompose new recoil momentum as

$$K^\mu = \tilde{K}^\mu - X^\mu = \tilde{K}^\mu - [p_j - (1-z)\tilde{p}_i]^\mu$$

(X^μ will go to zero for soft/collinear emissions)

- write Lorentz transformation as

$$\Lambda^\mu_{\ \nu}(K,\tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

with

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right], \qquad \text{and} \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}$$

- follow CAESAR formalism and analyse behaviour under scaling

(A. Banfi, G. P. Salam, & G. Zanderighi, JHEP **03** (2005) 073)

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}\,, \quad \eta_l \to \eta'_l = \eta - \xi_l \frac{\ln\rho}{a+b}\,, \text{where } \xi = \frac{\eta}{\eta_{\max}}$$

- impact of recoil in Lund plane under global rescaling must vanish
- boost in $\rho \to 0$ limit:

$$A^\nu \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2} \quad \text{and} \quad B^\nu \xrightarrow{\rho\to 0} \frac{\tilde{K}^\nu}{\tilde{K}^2}$$

and

$$\Delta p_l^\mu = 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,\tilde{K}^\mu - \frac{\tilde{p}_lX}{\tilde{K}^2}\,\tilde{K}^\mu + \frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,X^\mu$$

- colour-singlet decay or production ($e^-e^+ \to$ hadrons, $q\bar{q} \to V$)
  $\longrightarrow \tilde{K} =$ c.m.-momentum, only energy component (not rescaled)
- assume emitter $\tilde{p}_i$ is soft (and for Alaric $a = 1$, $b = 0$)

$$\tilde{p}_l\tilde{K} \sim \rho^{1-\xi_l} \text{ and } \tilde{p}_l X \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)}$$

  and therefore, in components

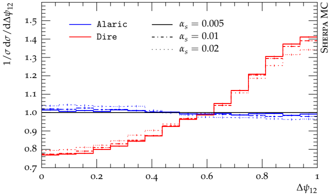
$$\begin{array}{lclcl} \Delta p_l^0 & \sim & \rho^{1-\xi_l}X^0 + \rho^{2-\xi_l-\max(\xi_i,\xi_j)}\tilde{K}^0 + \rho^{1-\xi_l}X^0 & \sim & \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \\ \Delta p_l^3 & \sim & \rho^{1-\xi_l}X^3 & \sim & \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \\ \Delta p_l^{1,2} & \sim & \rho^{1-\xi_l}X^{1,2} & \sim & \rho^{2-\xi_l} \end{array}$$

- therefore: impact of subsequent emissions vanishes with *rho*

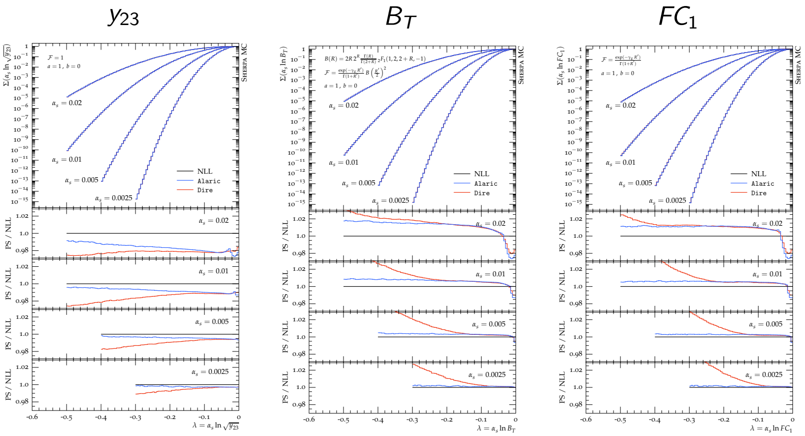
set-up of numerical tests

- compare results in $\alpha_S \to 0$ limit with NLL result
- set-up for checks
  - fixed α_S
  - leading colour $C_A = 2C_F = 3$
  - all partons massless
- example: azimuthal angle between two leading Lund-plane declusterings

(should be $\Delta\Psi_{12} = 0$)

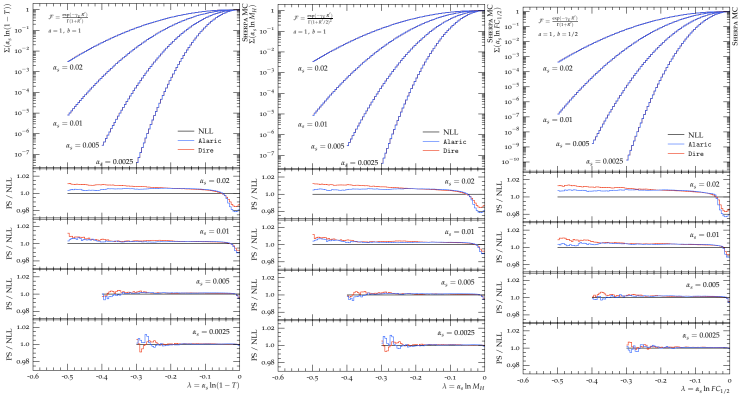


numerics: event shapes



more event shapes

$1-T$ $\quad M_H \quad$ $FC_{1/2}$



how about data?

(always nice to see practical impact, innit?)

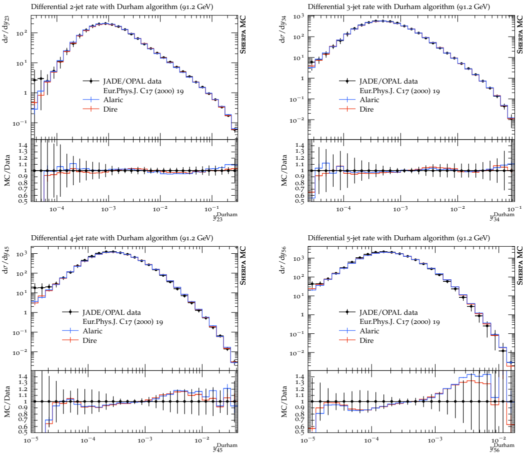
set-up of data comparison

- compare hadron-level results with LEP data
- perturbative set-up
  - no higher orders (no matching or merging)
  - running two-loop α_S with $\alpha_S(M_Z) = 0.118$
  - use CMW scheme for soft eikonal parts
  - all partons massless, masses emulated through simplistic thresholds
  - leading colour $C_A = N_c = 3,\ C_F = \frac{N_c^2-1}{2N_c}$
- non-perturbative set-up
  - need to use PYTHIA hadronization

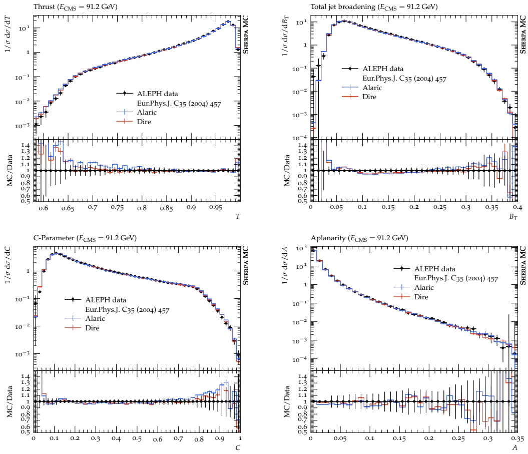
    (ALARIC not yet ready for heavy hadron decays)
  - default parameters of PYTHIA 6.4, but

$\texttt{PARJ(21)} = 0.3$, $\texttt{PARJ(41)} = 0.4$, $\texttt{PARJ(42)} = 0.36$(ALARIC)/0.45(DIRE)

differential jet rates

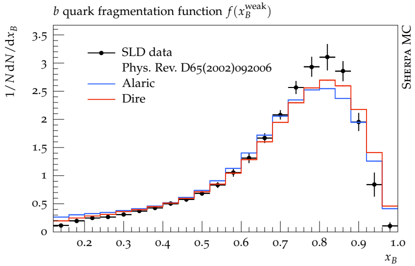


some event shapes: $1-T$, B_T, C, A



just for fun: *b*-quark fragmentation function

- emulation of quark masses with naive thresholds in $g \to q\bar{q}$: stop evolution if $t \leq (2m_q)^2$
  $\Longrightarrow$ must include quark masses also in $q \to qg$
- quick plausibility check: *b*-quark fragmentation function



shower = NLO subtraction

(bonus track: preparing for MC@NLO)

anatomy of NLO subtraction

(use FS subtraction as example, IS by crossing)

- combined soft & collinear subtraction:

$$\int_{m+1} \mathrm{d}\sigma^S + \int_m \mathrm{d}\sigma^C = \frac{1}{2} \sum_{i=g,q,\bar{q}} \sum_{\tilde{\imath}=1}^{m} \int_0^1 \frac{\mathrm{d}z}{z^{2-2\epsilon}} \int_m \mathrm{d}\sigma^B(p_1, \dots, \frac{p_i}{z}, \dots, p_m) \otimes \hat{\mathbf{I}}^{(\mathrm{FS})}_{\tilde{\imath}i} \,,$$

- decompose $\hat{\mathbf{I}}^{(\mathrm{FS})}_{\tilde{\imath}i}$:

$$\hat{\mathbf{I}}^{(\mathrm{FS})}_{\tilde{\imath}i} = \underbrace{\delta(1-z)\mathbf{I}_{\tilde{\imath}i}}_{\text{standard dipole}} + \underbrace{\mathbf{P}_{\tilde{\imath}i}}_{\text{collinear part}} + \underbrace{\mathbf{H}_{\tilde{\imath}i}}_{\text{“identified partons”}}$$

- $\mathbf{H}_{\tilde{\imath}i}$ contains only tricky (new) term:

$$\mathbf{H}_{\tilde{\imath}i}(p_1,\dots,p_i,\dots,p_m;n;z)$$
$$= -\frac{\alpha_s}{2\pi}\sum_{k=1,k\neq\tilde{\imath}}^{m}\frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2}\left[\underbrace{\tilde{K}^{\tilde{\imath}i}(z)+\bar{K}^{\tilde{\imath}i}(z)+2P_{\tilde{\imath}i}(z)\ln z}_{\text{analytically integrable}}+\underbrace{\mathcal{L}^{\tilde{\imath}i}(z;p_i,p_k,n)}_{\text{recoil-dependent: 1-dim numerics}}\right]$$

- result under integration ($n = n(z)$!)

$$\int_0^1 \mathrm{d}z\,\mathbf{H}_{\tilde{\imath}i}(p_1,\dots,p_i,\dots,p_m;n;z)$$
$$= -\frac{\alpha_s}{2\pi}\sum_{k=1,k\neq\tilde{\imath}}^{m}\frac{\mathbf{T}_{\tilde{\imath}}\mathbf{T}_k}{\mathbf{T}_{\tilde{\imath}}^2}\left\{\mathcal{K}^{\tilde{\imath}i}+\delta_{\tilde{\imath}i}\,\mathrm{Li}_2(\beta^2)-\int_0^1\mathrm{d}z\,P_{\mathrm{reg}}^{qq}(z)\ln\frac{n^2\tilde{p}_ip_k}{2z(\tilde{p}_in)^2}\right\}$$

summary & next steps

(the now and the future)

summary

- re-visited color coherence, re-decomposed soft eikonal

  (correct in differential azimuthal angle)

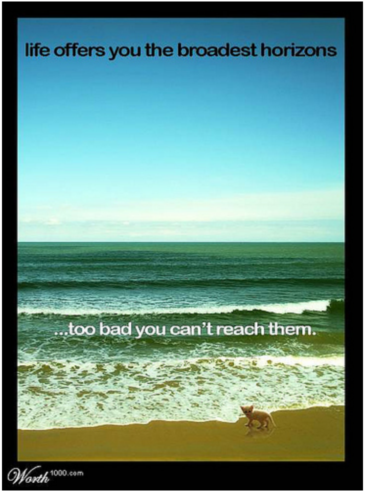
- designed new kinematics, IS and FS radiation on same footing

  (disentangle color- and recoil-partners)

- analytics: kinematics doesn't spoil "NLL history"
  numerics: kinematics reproduces NLL for $e^- e^+ \to$ hadrons
- for more details, especially IS, *cf.* arXiv on Monday

next steps

- near future:
  - include quark masses ($\longrightarrow$ wanna do pheno!)
  - checks and simple pheno for pp collisions ($q\bar{q} \to V$ & $gg \to H$)
  - LO multijet merging (need to implement parton shower history & rejections)
  - NLO matching (we know the terms . . .)
- further future:
  - add double–soft terms (F. Dulat, S. Hoeche & S. Prestel, Phys. Rev. **D98** (2018) 074013)
  - add triple–collinear terms (S. Hoeche, FK, & S. Prestel, JHEP **10** (2017) 093)
- far future:
  - spin correlations (have to implements CK algorithm for showers)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