ME+PS matching and merging at NLO

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Reminder: Ingredients of simulations

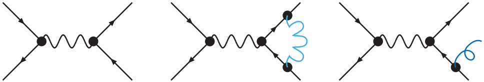
Cross sections at the LHC: Born approximation

$$\mathrm{d}\sigma_{ab\to N} = \int_0^1 \mathrm{d}x_a \mathrm{d}x_b f_a(x_a,\, \mu_F) f_b(x_a,\, \mu_F) \int_{\mathrm{cuts}} \mathrm{d}\Phi_N \frac{1}{2\hat{s}} \left|\mathcal{M}_{p_a p_b \to N}(\Phi_N;\, \mu_F,\, \mu_R)\right|^2$$

- parton densities $f_a(x,\, \mu_F)$ (PDFs)
- phase space Φ_N for N-particle final states
- incoming current $1/(2\hat{s})$
- squared matrix element $\mathcal{M}_{p_a p_b \to N}$ (summed/averaged over polarisations)
- renormalisation and factorisation scales μ_R and μ_F
- complexity demands numerical methods for large N

Higher orders: some general thoughts

- obtained from adding diagrams with additional:
  loops (virtual corrections) or
  legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F
  NLO allows for meaningful estimate of uncertainties
- additional difficulties when going NLO:
  ultraviolet divergences in virtual correction
  infrared divergences in real and virtual correction

  enforce
  UV regularisation & renormalisation
  IR regularisation & cancellation

(Kinoshita–Lee–Nauenberg–Theorem)

Structure of an NLO calculation

- sketch of cross section calculation

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \underbrace{\mathrm{d}\Phi_N \mathcal{B}_N}_{\substack{\text{Born}\\ \text{approximation}}} + \underbrace{\mathrm{d}\Phi_N \mathcal{V}_N}_{\substack{\text{renormalised}\\ \text{virtual correction}\\ \text{IR-divergent}}} + \underbrace{\mathrm{d}\Phi_{N+1} \mathcal{R}_{N+1}}_{\substack{\text{real correction}\\ \text{IR-divergent}}}$$

$$= \mathrm{d}\Phi_N \left[\mathcal{B}_N + \mathcal{V}_N + \mathcal{B}_N \otimes \mathcal{S}\right] + \mathrm{d}\Phi_{N+1} \left[\mathcal{R}_{N+1} - \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}\right]$$

- subtraction terms $\mathcal{S}$ (integrated) and $\mathrm{d}\mathcal{S}$:
  exactly cancel IR divergence in $\mathcal{R}$ – process-independent structures
- result: terms in both brackets separately infrared finite

Probabilistic treatment of emissions

- Sudakov form factor

$$\Delta_{ij,k}(t, t_0) = \exp\left[-\int\limits_{t_0}^{t} \mathrm{d}\Gamma_{ij,k}(t)\right]$$

  yields probability for **no decay** between scales t_0 and t
- decay width for parton $i(j) \to ik(j)$ (spectator j)

$$\mathrm{d}\Gamma_{ij,k}(t) = \frac{\mathrm{d}t}{t}\,\frac{\alpha_S}{2\pi}\int \mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\underbrace{\mathcal{K}_{ij,k}(t,\, z,\, \phi)}_{\text{splitting kernel}}$$

- evolution parameter t defined by kinematics

  generalised angle (HERWIG ++) or transverse momentum (PYTHIA, SHERPA)
- scale choice for strong coupling: $\alpha_S(k_\perp^2)$

  resums classes of higher logarithms
- regularisation through cut-off t_0

Emissions off a Born matrix element

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_S}{2\pi} \sum_{\text{all }\{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}) \,, \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp\left[-\int_{t_0}^{t} \mathrm{d}\Phi_1 \, \mathcal{K}_n(\Phi_1) \right]$$

- consider first emission only off Born configuration

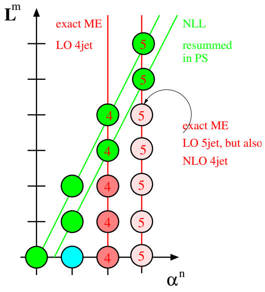
$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N \, \mathcal{B}_N(\Phi_N) \cdot \underbrace{\left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_N(\Phi_1) \Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}}_{\text{integrates to unity} \longrightarrow \text{"unitarity" of parton shower}}$$

- further emissions by recursion with $\mu_N^2 \longrightarrow t$ of previous emission

NLO improvements: Matching & Merging

NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- adjust ("match") terms:
  - cross section at NLO accuracy
  - correct hardest emission in PS to exactly reproduce ME at order α_S
    ($\mathcal{R}$-part of the NLO calculation)



The POWHEG-trick: modifying the Sudakov form factor

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \xrightarrow{\text{IR}} \mathrm{d}\Phi_1 \frac{\alpha_S}{2\pi} \mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) = \exp\left[-\int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\right],$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_S to parton shower scale

Local K-factors

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$
\begin{aligned}
\mathrm{d}\sigma_N^{\mathrm{(NLO)}} &= \mathrm{d}\Phi_N\,\bar{\mathcal{B}}(\Phi_N)\\
&= \mathrm{d}\Phi_N\,\Big\{\mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N)\otimes\mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)}\\
&\qquad + \int \mathrm{d}\Phi_1\,\left[\mathcal{R}_N(\Phi_N\otimes\Phi_1) - \mathcal{B}_N(\Phi_N)\otimes\mathrm{d}\mathcal{S}(\Phi_1)\right]\Big\}
\end{aligned}
$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}$

(relevant for MC@NLO)

NLO accuracy in radiation pattern

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0)$:

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\, \bar{\mathcal{B}}(\Phi_N)$$
$$\times \underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_N \otimes \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, k_\perp^2(\Phi_1))\right\}}_{\text{integrating to yield 1 - “unitarity of parton shower”}}$$

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2 (this is vanilla POWHEG!)
- apart from logs: which configurations enhanced by local K-factor

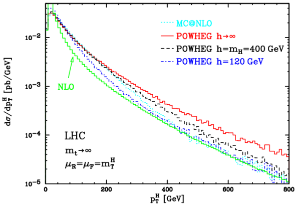
(K-factor for inclusive production of X adequate for $X+$ jet at large $p_\perp$?)

Improved POWHEG

(S. Alioli, P. Nason, C. Oleari, & E. Re, JHEP 0904 (2009) 002)

- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_\perp^2+h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2+h^2}}_{\mathcal{R}^{(F)}}\right)$$



- can "tune" h to mimick NNLO - or maybe resummation result
- differential event rate up to first emission

$$\begin{aligned} \mathrm{d}\sigma \;=\;& \mathrm{d}\Phi_B \bar{\mathcal{B}}^{(\mathrm{R}^{(\mathrm{S})})}\left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, t_0) + \int\limits_{t_0}^{s} \mathrm{d}\Phi_1 \frac{\mathcal{R}^{(S)}}{\mathcal{B}}\, \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, k_\perp^2)\right] \\ & + \mathrm{d}\Phi_R\, \mathcal{R}^{(F)}(\Phi_R) \end{aligned}$$

Resummation in MC@NLO

(S. Frixione & B. Webber, JHEP 0602 (2002) 029)

(S. Hoeche, F. Krauss, M. Schoenherr, & F. Siegert, JHEP 1209 (2012) 049)

- divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}_1 + \mathcal{H}_N$$

- identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum\limits_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1st emission to account for colour)

$$\begin{aligned} \mathrm{d}\sigma_N \;=\; & \mathrm{d}\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1\, \mathcal{K}_{ij,k}(\Phi_1)\, \Delta_N^{(\mathcal{K})}(\mu_N^2,\, k_\perp^2) \right] \\ & + \mathrm{d}\Phi_{N+1}\, \mathcal{H}_N \end{aligned}$$

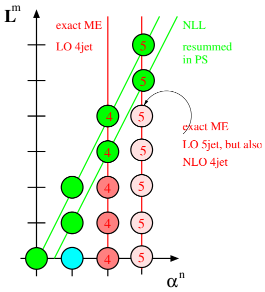
- effect: only resummed parts modified with local K-factor

Multijet merging @ leading order

Multijet merging: basic idea

(S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111 (2001) 063,
L. Lonnblad, JHEP 0205 (2002) 046, & F. Krauss, JHEP 0208 (2002) 015)

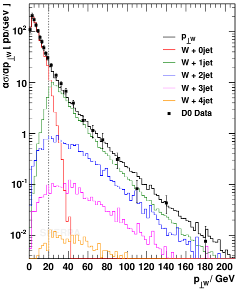
- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- combine ("merge") both:
  result: "towers" of MEs with increasing number of jets evolved with PS
  - multijet cross sections at Born accuracy
  - maintain (N)LL accuracy of parton shower



Separating jet evolution and jet production

- separate regions of jet production and jet evolution with jet measure Q_J

  ("truncated showering" if not identical with evolution parameter)
- matrix elements populate hard regime
- parton showers populate soft domain



Multijet-merging at NLO: FxFx, MEPS@NLO, & UNLOPS

(arXiv: 1207.5030, 1207.5031 [hep-ph])

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

maintain NLO and LL accuracy of ME and PS

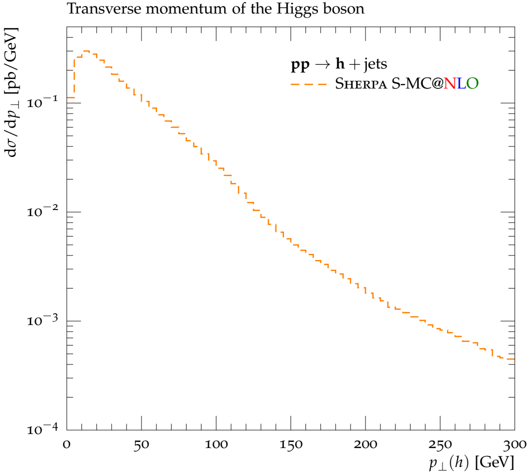
- towers of MC@NLO simulations without double-counting

(possibly further supplemented with LO simulations for even higher final state multiplicities)

First emission(s) in NLO-merged samples

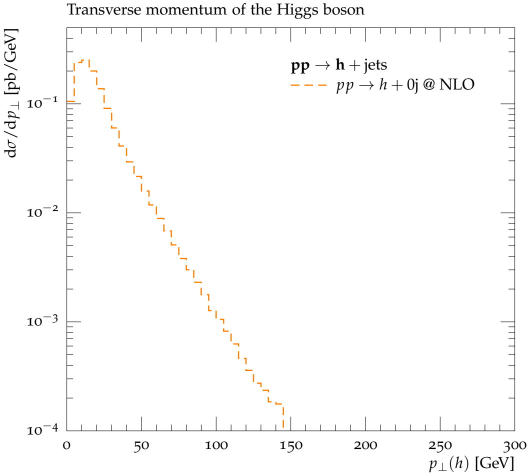
$$
\begin{aligned}
\mathrm{d}\sigma \;=\; & \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right] \\
& +\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1}) \\
& +\mathrm{d}\Phi_{N+1}\,\tilde{\mathcal{B}}_{N+1}\left(1+\frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int\limits_{t_{N+1}}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\right)\Theta(Q_{N+1}-Q_J) \\
& \quad\cdot\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\cdot\left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_0)+\int\limits_{t_0}^{t_{N+1}}\mathrm{d}\Phi_1\,\mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\right] \\
& +\mathrm{d}\Phi_{N+2}\,\mathcal{H}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\Theta(Q_{N+1}-Q_J)+\ldots
\end{aligned}
$$

$p_\perp^H$ in MEPS@NLO



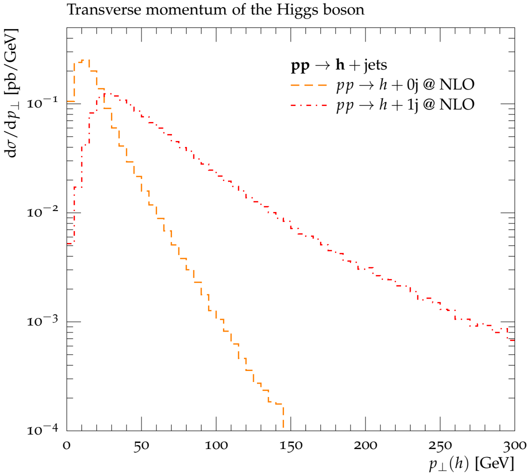
- first emission by MC@NLO

$p_\perp^H$ in MEPS@NLO



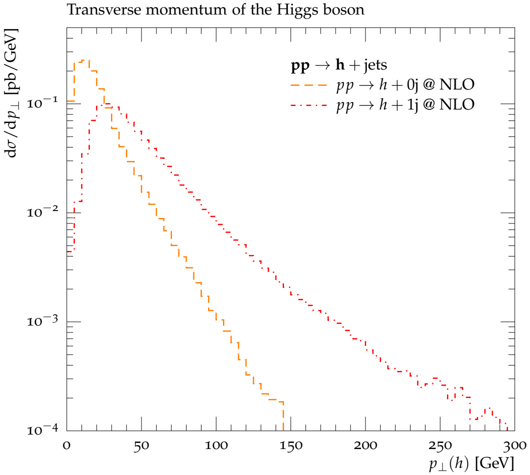
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



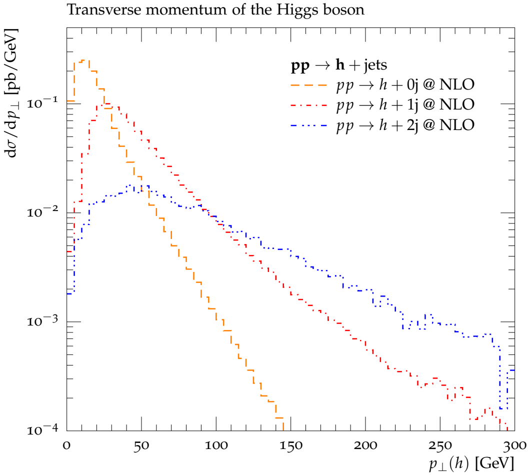
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$

$p_\perp^H$ in MEPS@NLO



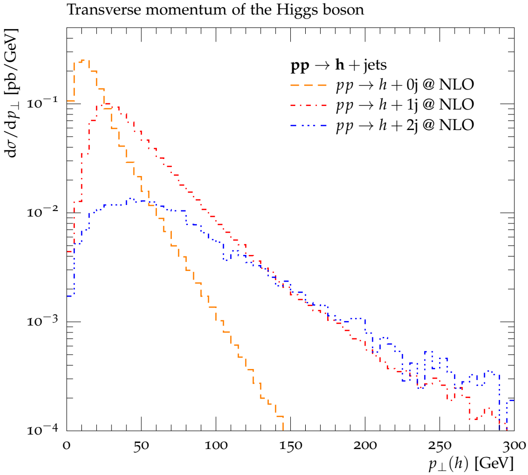
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h + {\rm jet}$ for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h + {\rm jet}$ to $Q_{n+2} < Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



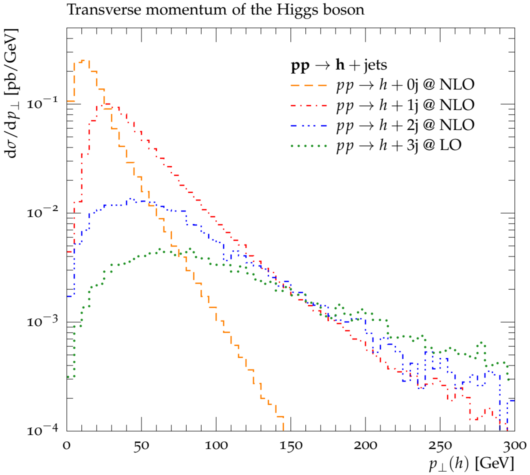
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$

$p_\perp^H$ in MEPS@NLO



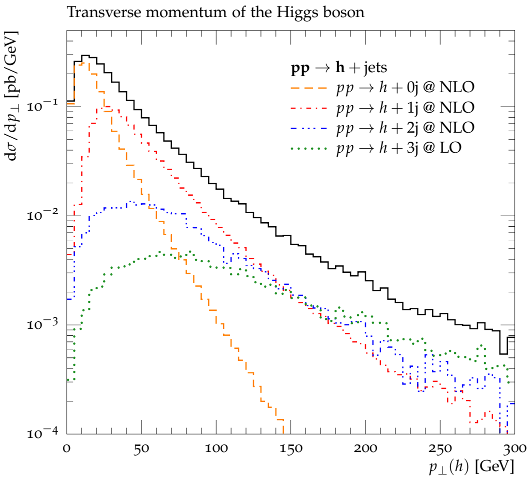
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+\text{jet}$ for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+\text{jet}$ to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h+2\text{jets}$ for $Q_{n+2} > Q_{\rm cut}$
- iterate

$p_\perp^H$ in MEPS@NLO



- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+$ jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+$ jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h+$ 2jets for $Q_{n+2} > Q_{\rm cut}$
- iterate

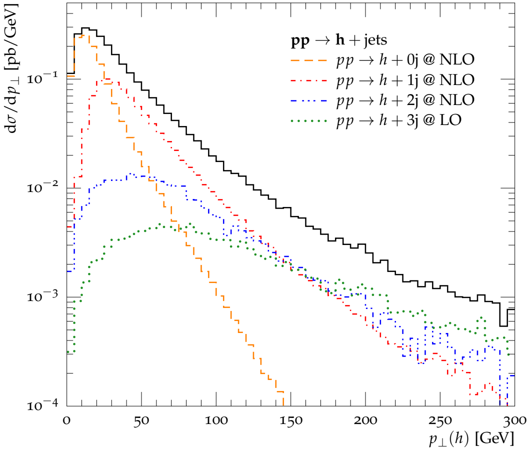
$p_\perp^H$ in MEPS@NLO



- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h +$ jet for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h +$ 2jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

$p_\perp^H$ in MEPS@NLO

Transverse momentum of the Higgs boson



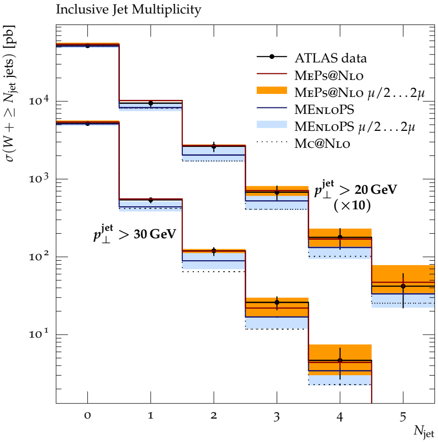
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+$jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+$jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h+2$jets for $Q_{n+2} > Q_{\rm cut}$
- iterate
- sum all contributions
- eg. $p_\perp(h)>200$ GeV has contributions fr. multiple topologies

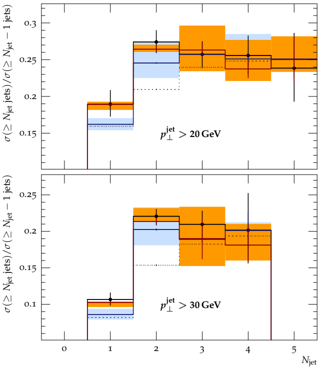
Differences between MEPS@NLO, UNLOPS & FxFx

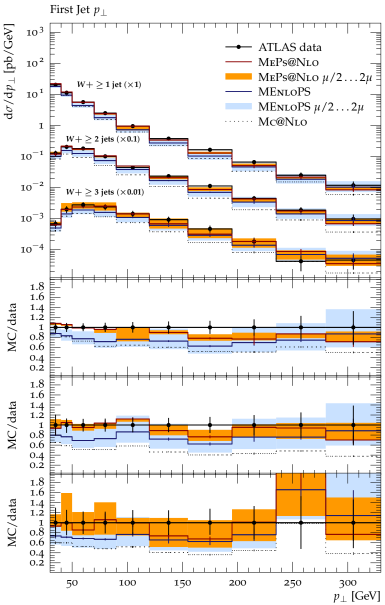
	FxFx	MEPS@NLO	UNLOPS
ME	all internal aMC@NLO_MADGRAPH	$\mathcal{V}$ external COMIX or AMEGIC++ $\mathcal{V}$ from OPENLOOPS, BLACKHAT, NJET, ...	all external
shower	external HERWIG or PYTHIA	intrinsic	intrinsic PYTHIA
Δ_N $\Theta(Q_J)$	analytical a-posteriori	from PS per emission	from PS per emission
Q_J-range	relatively high (but changed)	$>$ Sudakov regime $\approx$ 10%	$\approx$ Sudakov regime $\approx$ 10%

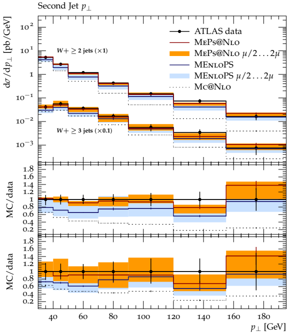
MEPS@NLO: validation in W+jets

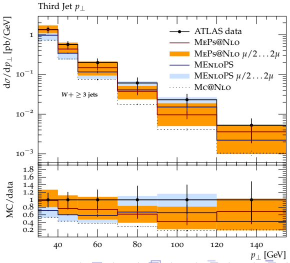
(S. Hoeche, F. Krauss, M. Schoenherr & F. Siegert, JHEP 1304 (2013) 027)

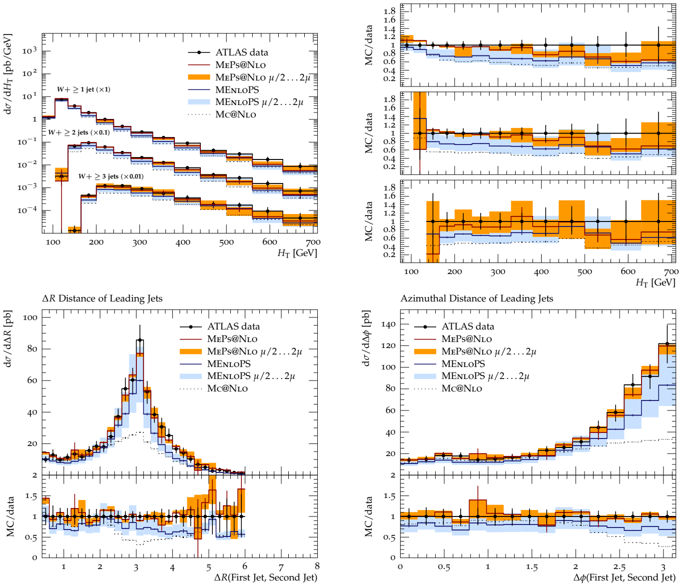




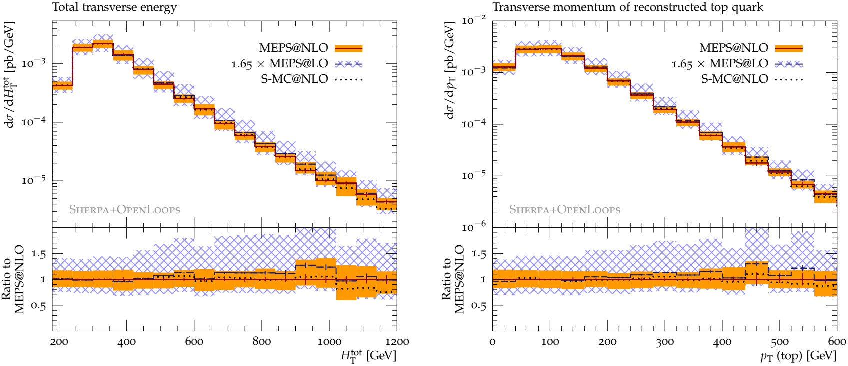

First Jet $p_\perp$
$d\sigma/dp_\perp$ [pb/GeV]
$W+\geq 1$ jet ($\times 1$)
$W+\geq 2$ jets ($\times 0.1$)
$W+\geq 3$ jets ($\times 0.01$)
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
MC/data
$p_\perp$ [GeV]


Second Jet $p_\perp$
$d\sigma/dp_\perp$ [pb/GeV]
$W+\geq 2$ jets ($\times 1$)
$W+\geq 3$ jets ($\times 0.1$)
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
MC/data
$p_\perp$ [GeV]

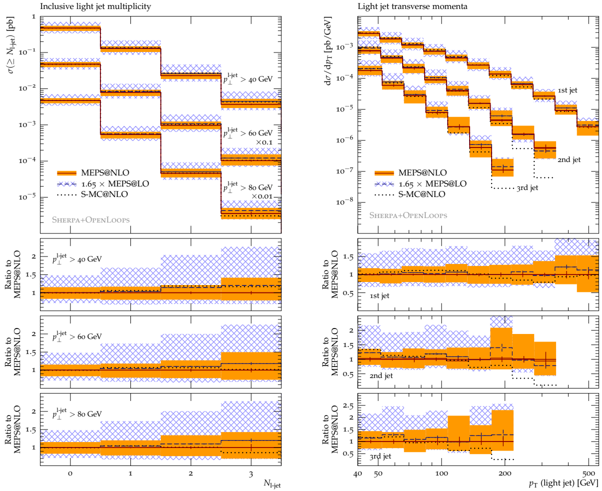

Third Jet $p_\perp$
$d\sigma/dp_\perp$ [pb/GeV]
$W+\geq 3$ jets
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
MC/data
$p_\perp$ [GeV]


ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 1$ jet $(\times 1)$
$W+\geq 2$ jets $(\times 0.1)$
$W+\geq 3$ jets $(\times 0.01)$
$d\sigma/dH_T$ [pb/GeV]
H_T [GeV]
MC/data
ΔR Distance of Leading Jets
$d\sigma/d\Delta R$ [pb]
ΔR(First Jet, Second Jet)
Azimuthal Distance of Leading Jets
$d\sigma/d\Delta\phi$ [pb]
$\Delta\phi$(First Jet, Second Jet)

$t\bar{t}$ + jets in MEPS@NLO



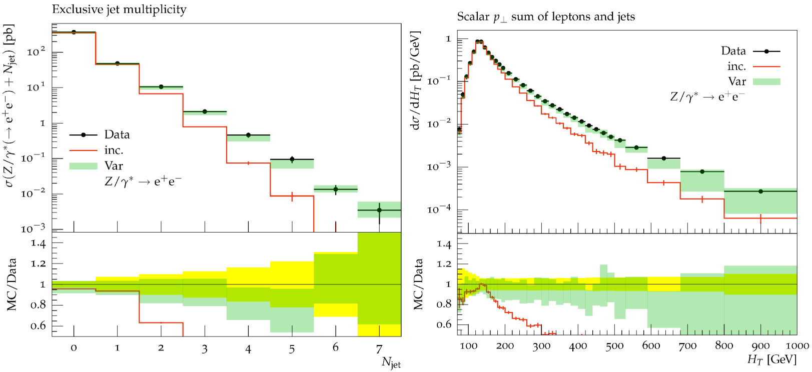
$t\bar{t}$ + jets in MEPS@NLO



FxFx: validation in Z+jets

(Data from ATLAS, 1304.7098, aMC@NLO_MADGRAPH with HERWIG++)

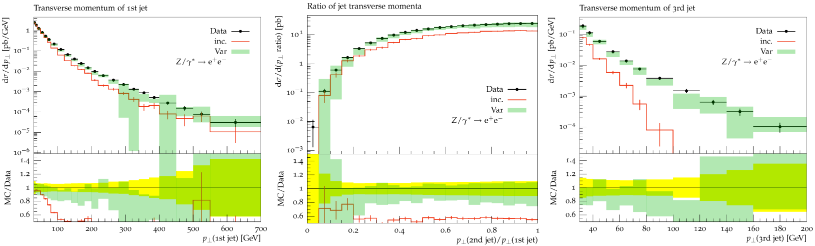
(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



FxFx: validation in Z+jets

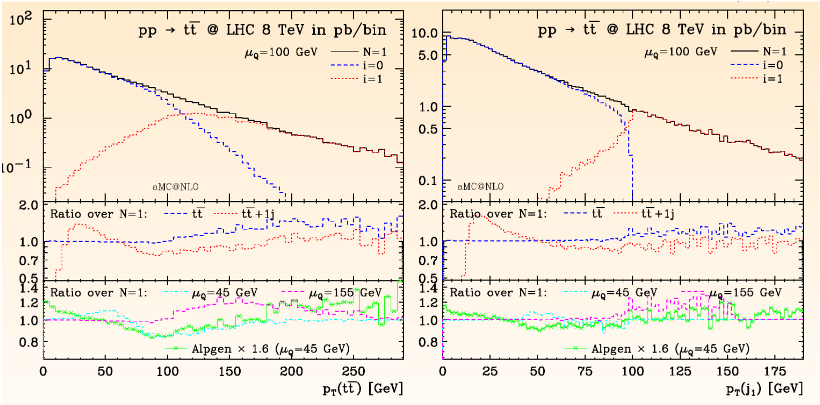
(Data from ATLAS, 1304.7098, aMC@NLO_MADGRAPH with HERWIG++)

(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



FxFx: Q_J dependence in $t\bar{t}$

(R.Frederix & S.Frixione, JHEP 1212 (2012) 061)



Aside: merging without Q_J - the MINLO approach

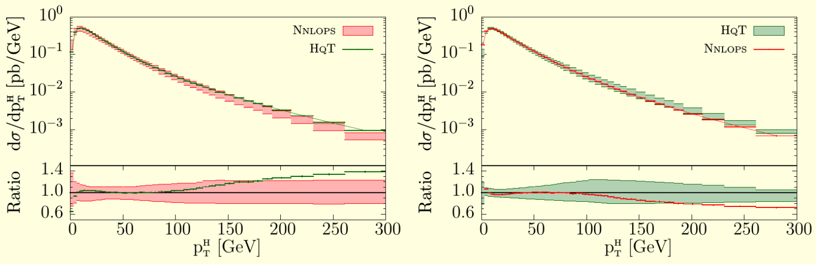
(K.Hamilton, P.Nason, C.Oleari & G.Zanderighi, JHEP 1305 (2013) 082)

- based on POWHEG + shower from PYTHIA or HERWIG
- up to today only for singlet S production, gives NNLO + PS
- basic idea:
  - use S+jet in POWHEG
  - push jet cut to parton shower IR cutoff
  - apply analytical NNLL Sudakov rejection weight for intrinsic line in Born configuration

    (kills divergent behaviour at order α_S)
  - don't forget double-counted terms
  - reweight to NNLO fixed order

NNLOPS for H production

(K.Hamilton, P.Nason, E.Re & G.Zanderighi, JHEP 1310 (2013) 222)

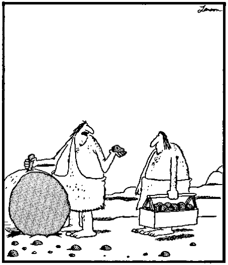


Summary

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "POWHEG")
  - MEPS@NLO ("SHERPA", "UNLOPS", "FXFX")
  - NNLOPS feasible for simple processes

(first 3 methods are well understood and used in experiments)

(last method need validation etc.)



"So what's this? I asked for a *hammer*! A hammer! *This* is a crescent wrench! ... Well, maybe it's a hammer. ... Damn these stone tools."

- multijet merging at NLO for relevant signals and backgrounds
- complete automation of NLO calculations done
  $\longrightarrow$ must benefit from it!

(it's the precision and trustworthy & systematic uncertainty estimates!)