

The Edge of Precision in Simulations for the LHC

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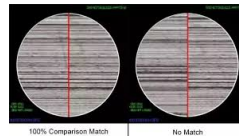
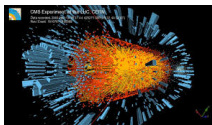
- why precision tools?
- current precision
- improving parton showers
- persistent problems
- summary & outlook

why precision

(carrying coal to Newcastle)

motivation: the need for (more) accurate tools

- to date no discovery of new physics (BSM) (a pity, but that's Nature)
- hope for “simple” discoveries is waning (don't expect anything glaringly obvious)
- push into precision tests of the Standard Model (find it or constrain “subtle”!)
- statistical uncertainties approach zero (because of fantastic work of accelerator, DAQ, etc.)
- systematic exp. uncertainties decrease (because of ingenious experimental work)
- theoretical uncertainties are or become dominant (obstacle to full exploitation of LHC)



CSI LHC: need precise & accurate tools for precision physics

how to build an event generator

- paradigm: “divide et impera”
- divide simulation in distinct phases, with (logarithmically) separated scales
- start with **signal event**

(fixed order perturbation theory)

- dress partons with **parton shower**

(resummed perturbation theory)

- add **underlying event**

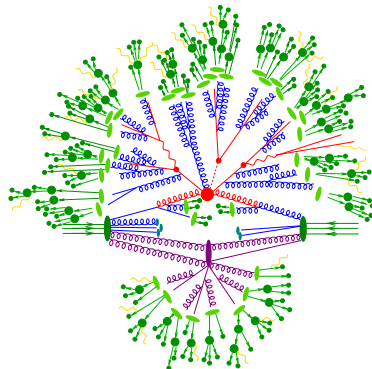
(phenomenological models)

- **hadronize** partons

(phenomenological models)

- **decay** hadrons

(effective theories, simple symmetries & data)



current precision

(where we are)

fixed-order accuracy

(apologies for any omissions in active field with ≈ 100 publications/past 5 years)

- $N^3\text{LO}$ for single-boson production (1503.06056 ... 1802.00833, 1807.11501)
for DIS, and for VBF H -production in double DIS (1803.09973; 1606.00840)
- NNLO for practically all $2 \rightarrow 2$ (and some $2 \rightarrow 3$) processes:
 - jj (1705.10271, 1905.09047, ...)
 - $Vj, \gamma j, Hj$ (1408.5325, 1504.02131, 1504.07922, 1505.03893, 1705.04664, 1901.11041, 1905.13738, ...)
 - $t\bar{t}$ & single top (1303.6254, 1511.00549; 1404.7116, ...)
 - VV and $\gamma\gamma$ (1408.5243, 1504.01330, 1507.06257, 1604.08576, 1605.02716, 1708.02925, 1711.06631, ...)
 - VBF (1506.02660, 1802.02445, ...)
 - dijets in DIS (1804.05663, ...)
- virtual $2 \rightarrow \geq 3$ amplitudes (1511.05409, 1511.09404, 1604.06631, 1712.02229, 1811.11699, ...)
- relative size argument: $\alpha_s^2 \approx \alpha_W$:
must include NLO EW corrections for $\mathcal{O}(1 - 10\%)$ accuracy
 \implies automated in OPENLOOPS, RECOLA, aMC@NLO, MADGRAPH

(1705.00598, 1704.05783, 1405.0301)

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SM precision simulation in a nutshell: Drell-Yan

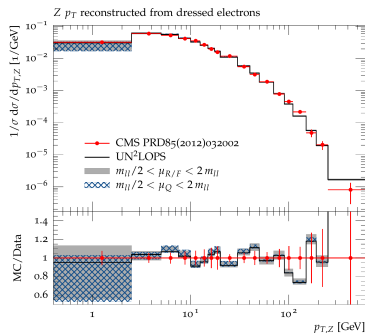
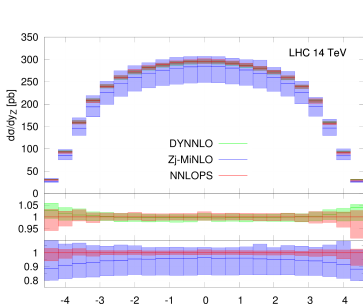
- current “accuracy standard(s)”:
 - matching: NNLOPS for inclusive V
 - merging: MEPS@NLO for $V + \leq 2$ jets at NLO $V + \geq 3$ jets at LO
- dominating QCD effects: $\mathcal{O}(10\text{-}30\%)$
 - low- p_\perp region dominated by parton shower
 - high- p_\perp region dominated by (multi-) jet topologies
 - higher accuracy in rate (and some shapes) through NNLO matching
- must add EW corrections for %-level precision
 - EW correction at large scales $\mathcal{O}(10\%)$
 - QED FSR + EW for V line shapes at $\mathcal{O}(1\%)$

matching at NLO and NNLO

- avoid double-counting of emissions
- two schemes at NLO: MC@NLO and POWHEG
- two schemes at NNLO: MINLO & UNNLOPS (singlets S only)
- MINLO:
 - use POWHEG for $S + j$ with $p_T^{(S)} \rightarrow 0$,
 - capture divergences by reweighting with analytic Sudakov form factor
 - NNLO accuracy by reweighting with full NNLO calculation
- UNNLOPS:
 - subtract and add parton shower terms at FO from $S + j$ contributions
 - maintaining unitarity using zero- p_\perp bin
- both available for two simple processes only
- **common limitation: accuracy of parton showers**

NNLOs for Z production: MINLO & UNNLOs

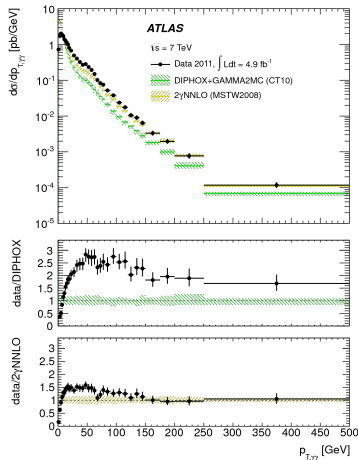
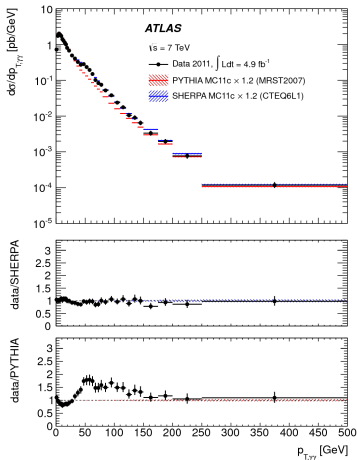
(1407.2904, 1405.3607)



- different logic of achieving NNLO precision
- available for H , V production (both) and VV production (MINLO)

merging example: $p_{\perp, \gamma\gamma}$ in MEPS@LO vs. NNLO

(arXiv:1211.1913 [hep-ex])



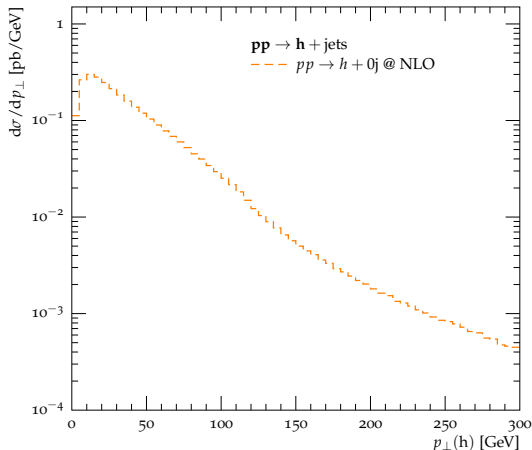
multijet-merging at NLO

- sometimes “more legs” wins over “more loops”
- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting
- maintain NLO and LL accuracy of ME and PS
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities
- different implementations, parametric accuracy not always clear

(MEPs@NLO, FxFx, UNLOPs)

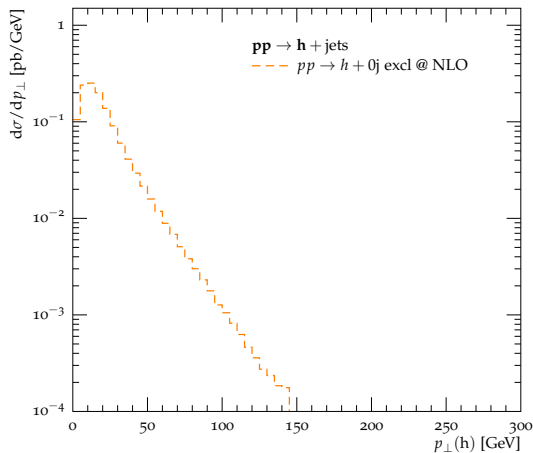
- starts being used, still lacks careful cross-validation

illustration: p_{\perp}^H in MEPS@NLO



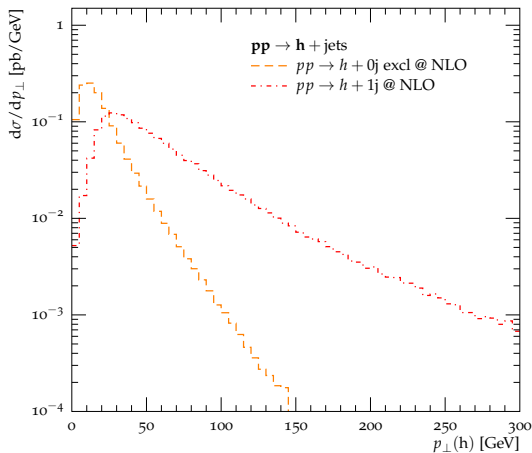
- first emission by Mc@NLO

illustration: p_{\perp}^H in MEPS@NLO



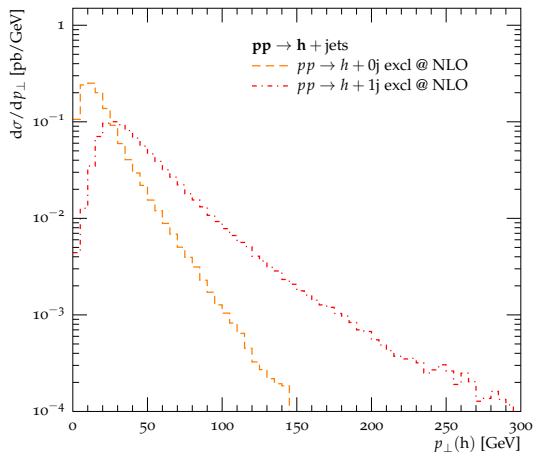
- first emission by MC@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$

illustration: p_{\perp}^H in MEPS@NLO



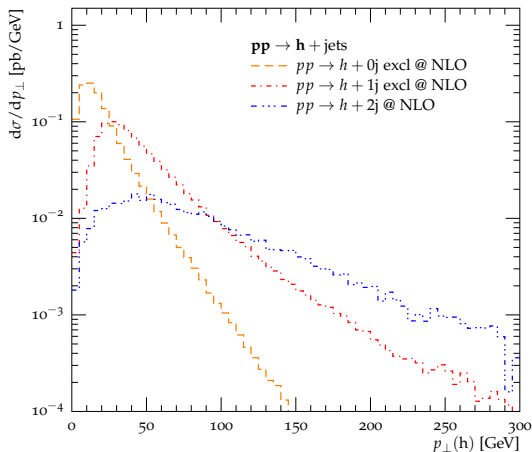
- first emission by MC@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$

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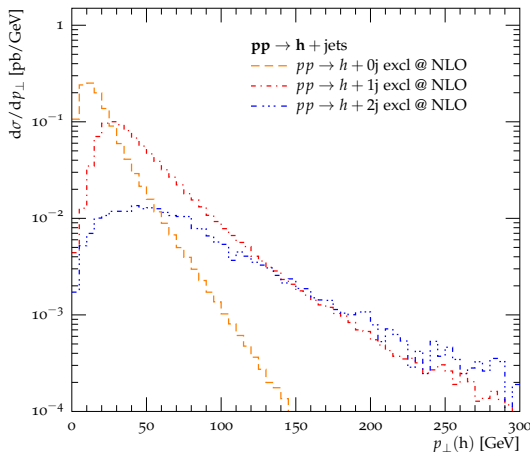
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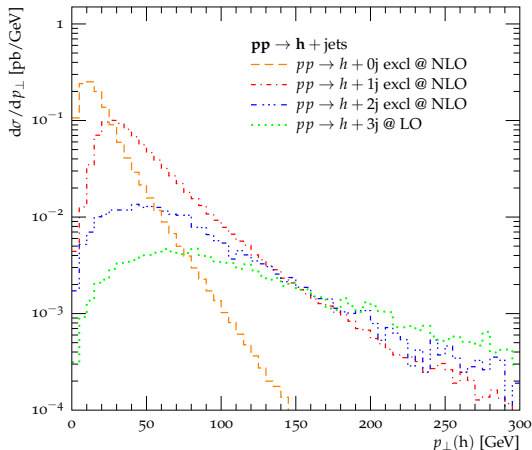
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- MC@NLO $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$

illustration: p_{\perp}^H in MEPS@NLO



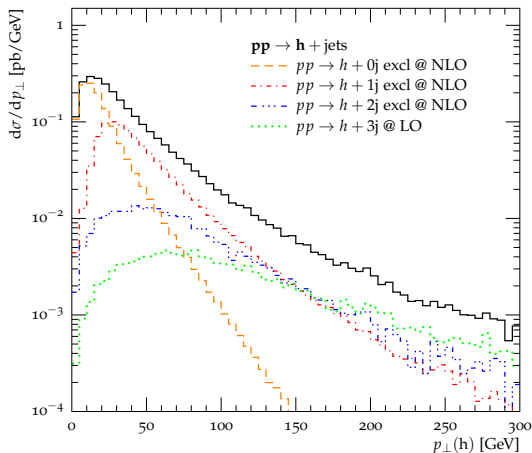
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- iterate

illustration: p_{\perp}^H in MEPS@NLO



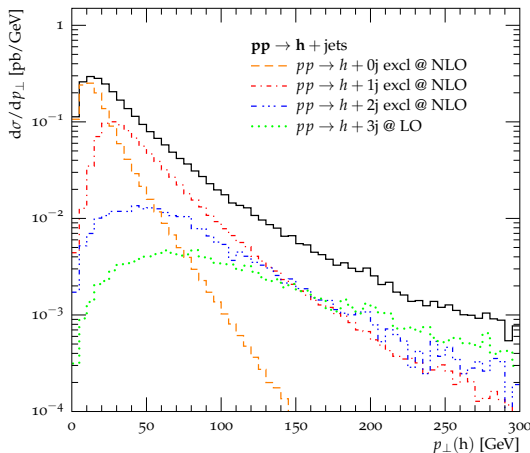
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- iterate
- sum all contributions

illustration: p_{\perp}^H in MEPS@NLO

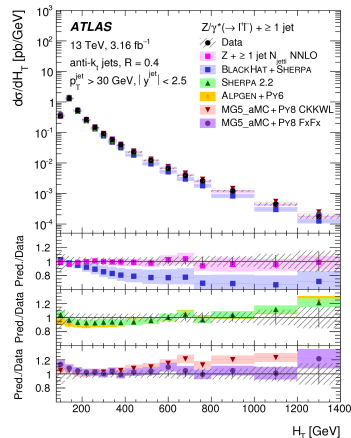
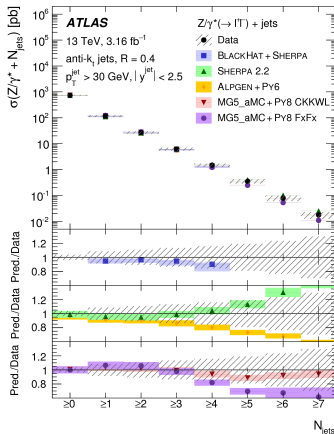


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- MC@NLO $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions
- eg. $p_{\perp}(h) > 200$ GeV has contributions fr. multiple topologies

MEPs@NLO for Z +jets: ATLAS data (13 TeV)

(arXiv:1702.05725 [hep-ex])

- various merging codes at LO and NLO

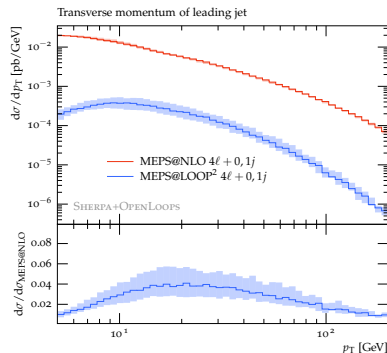
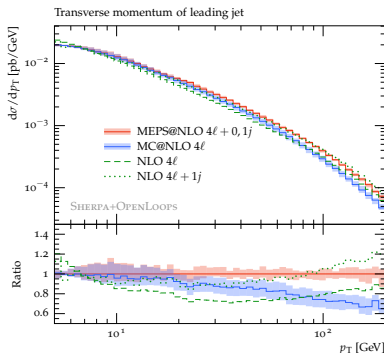


adding loop-induced processes: WW production

(arXiv:1309.0500 [hep-ph])

- combine MEPS@NLO for “direct” WW production with LO merging for $gg \rightarrow WW$

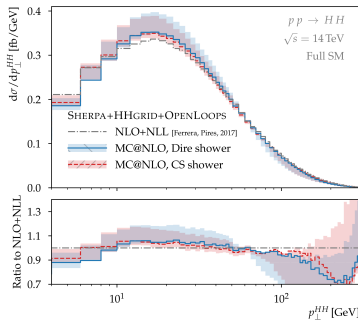
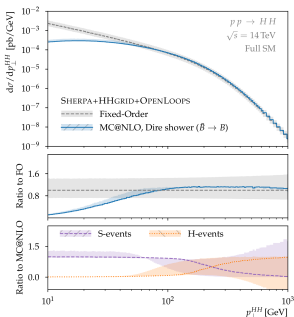
(“tagged” by light-quark box)



MC@NLO for loop-induced processes (HH production)

(arXiv:1703.09252 & 1711.03319 [hep-ph])

- technology ready for loop-induced NLO (effectively parts of NNLO) combined with parton shower
- two implementations: aMC@NLO, MADGRAPH & SHERPA



EW corrections

- EW corrections sizeable $\mathcal{O}(10\%)$ at large scales: **must include them!**
- but: more painful to calculate
- need EW showering & possibly corresponding PDFs

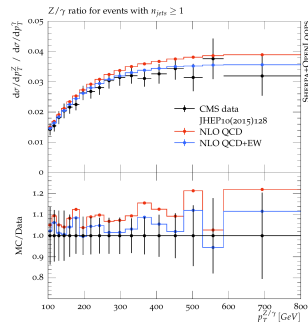
(somewhat in its infancy: chiral couplings)

- example: Z/γ vs. p_T (right plot)

(handle on p_{\perp}^Z in $Z \rightarrow \nu\bar{\nu}$)

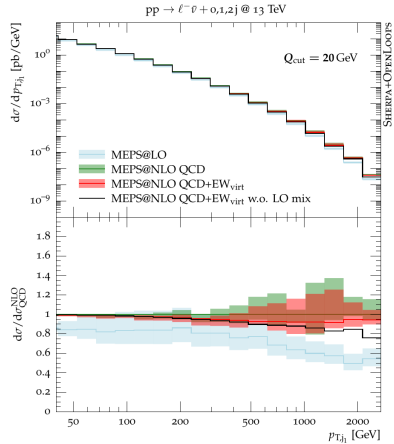
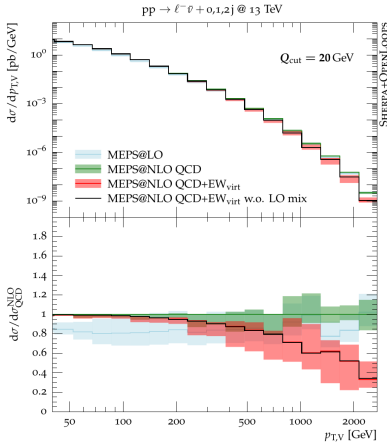
(Kallweit, Lindert, Pozzorini, Schoenherr for LH'15)

- difference due to EW charge of Z
- no real correction (real V emission)
- improved description of $Z \rightarrow \ell\ell$



EW corrections in $pp \rightarrow \ell^- \bar{\nu} + \text{jets}$

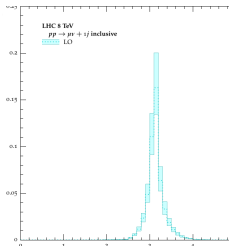
(arXiv:1511.08692 [hep-ph])



NLO EW predictions for $\Delta R(\mu, j_1)$

(LHC@8TeV, $p_{\perp}^{j_1} > 500$ GeV, central μ and jet)

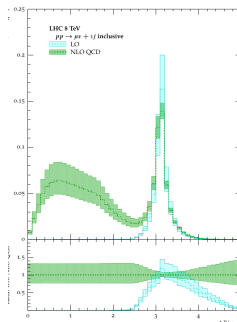
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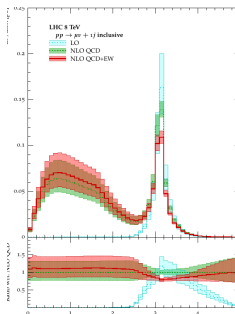
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- NLO corrections neg. in peak
large $pp \rightarrow Wjj$ component opening PS



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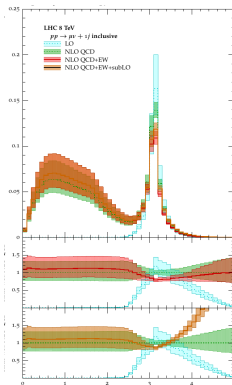
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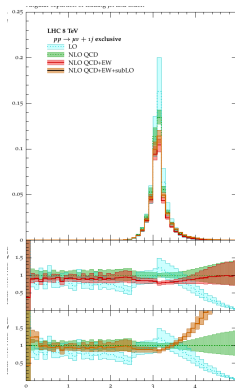
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- sub-leading Born (γ PDF) at large ΔR



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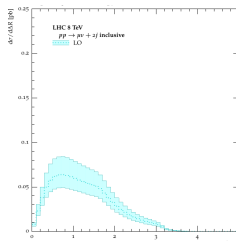
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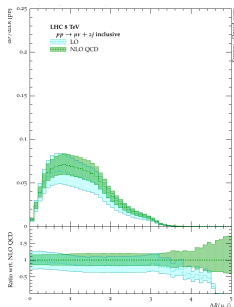
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- describe $pp \rightarrow Wjj$ @ NLO,
 $p_{\perp}^{j_2} > 100$ GeV



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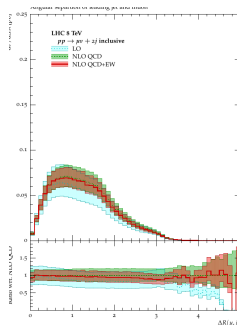
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- pos. NLO QCD, \sim flat



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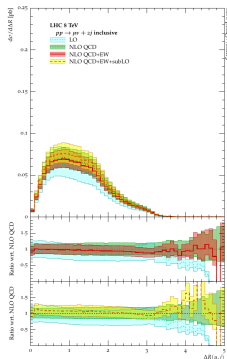
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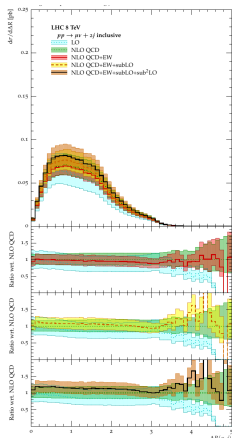
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- sub-leading Born contris positive



NLO EW predictions for $\Delta R(\mu, j_1)$

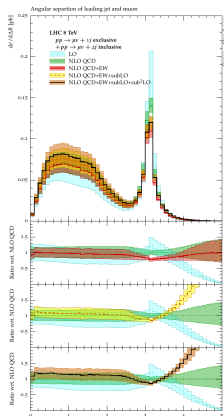
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- sub²leading Born (diboson etc) contrs.
pos.
→ possible double counting with BG

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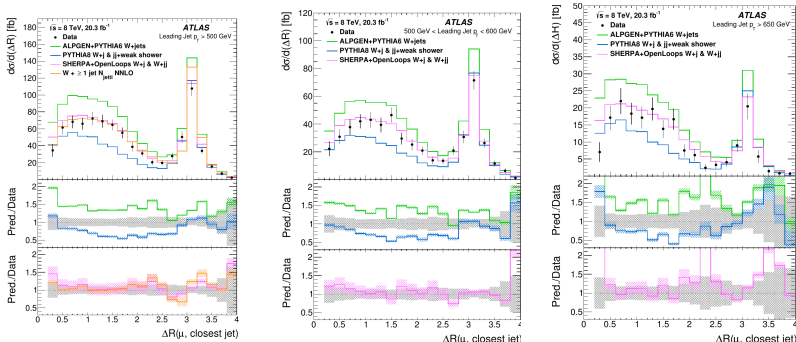


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pos.
→ possible double counting with BG
- merge using exclusive sums

... and the measurement

(arXiv:1609.07045 [hep-ex])

- different fixed order and simulation tools



another systematic uncertainty: parton showering

- parton showers are approximations, based on
leading colour, leading logarithmic accuracy, spin-average
- parametric accuracy by comparing Sudakov form factors:

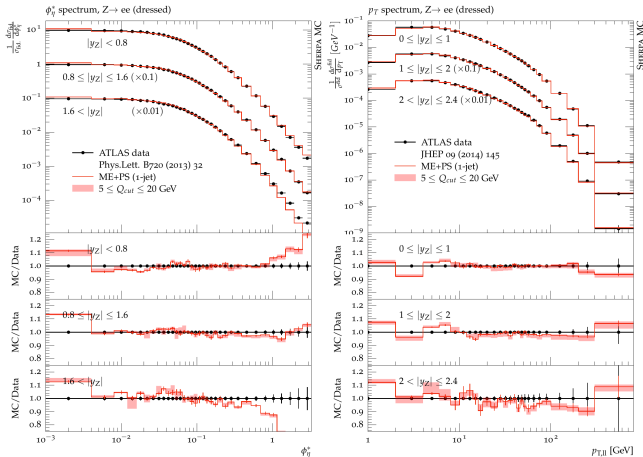
$$\Delta = \exp \left\{ - \int \frac{dk_{\perp}^2}{k_{\perp}^2} \left[A \log \frac{k_{\perp}^2}{Q^2} + B \right] \right\},$$

where A and B can be expanded in $\alpha_s(k_{\perp}^2)$

- Q_T resummation includes $A_{1,2,3}$ and $B_{1,2}$
(transverse momentum of Higgs boson etc.)
- showers usually include terms $A_{1,2}$ and B_1
 A = cusp terms (“soft emissions”), $B \sim$ anomalous dimensions γ

LO results for Drell-Yan

(example of accuracy in description of standard precision observable, 1506.05057)



improving parton showers

(going beyond “plumbing”)

including NLO splitting kernels

(1705.00982, 1705.00742)

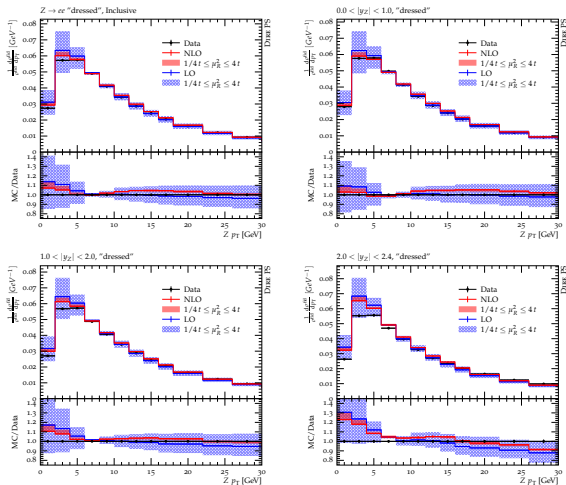
- expand splitting kernels as

$$P(z, \kappa^2) = P^{(0)}(z, \kappa^2) + \frac{\alpha_s}{2\pi} P^{(1)}(z, \kappa^2)$$

- aim: reproduce DGLAP evolution at NLO
include all NLO splitting kernels
- three categories of terms in $P^{(1)}$:
 - cusp (universal soft-enhanced correction) (already included in original showers)
 - corrections to $1 \rightarrow 2$
 - new flavour structures (e.g. $q \rightarrow q'$), identified as $1 \rightarrow 3$
- new paradigm: two independent implementations

physical results: DY at LHC

(untuned showers vs. 7 TeV ATLAS data, optimistic scale variations)

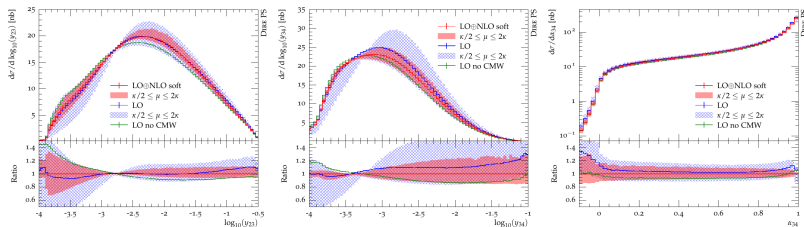


leading colour differential two-loop soft corrections

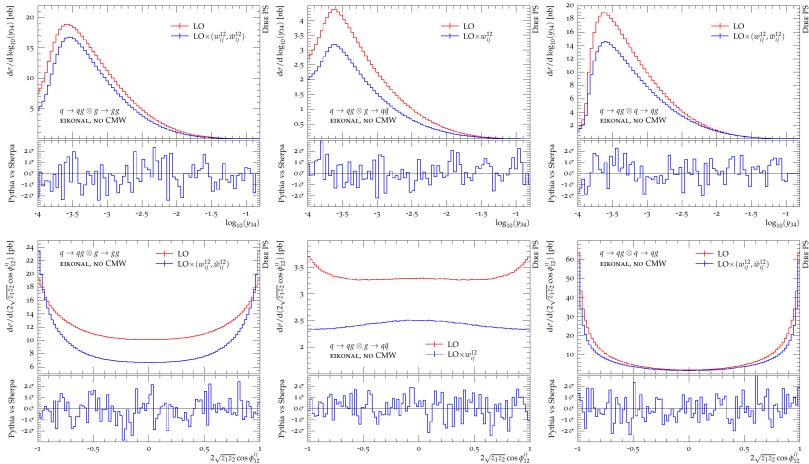
(1805.03757)

- compare two-emission soft contribution with iterated single emissions
- capture effect by reweighting original parton shower, with
 - accounting for finite recoil
 - including first $1/N_c$ corrections
 - incorporating spin correlations
- resulting scale dependence (pessimistic estimate) below

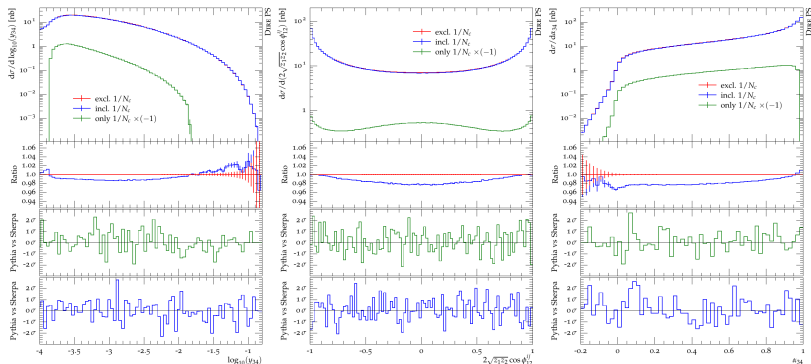
(another way to solve problems in 1805.09327)



reweighting



including $1/N_c$ effects



how to assess formal precision?

- PS proven to be NLL accurate for simple observables, provided

[Catani, Marchesini, Webber, NPB349 \(1991\) 635](#)

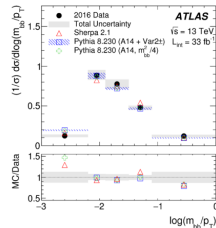
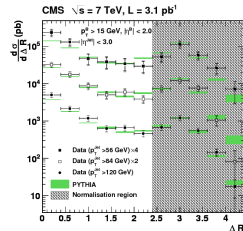
- soft double-counting removed (\nearrow before) and
 - 2-loop cusp anomalous dimension included
- not entirely clear what this means numerically, because
 - parton shower is momentum conserving, NLL is not
 - parton shower is unitary, NLL approximations break this
- differences can be quantified by
 - designing an MC that reproduces NLL exactly
 - removing NLL approximations one-by-one
 - employ well-established NLL result as example ([technical discussion in 1711.03497](#))
- recent study: issues with 2nd emission in modern showers ([1805.09327](#))
known problem ([and solved for ARIADNE in, e.g., Nucl.Phys. B392 \(1993\) 251](#))

persistent problems

(not everything is rosy)

$g \rightarrow Q\bar{Q}$ — a systematic nightmare

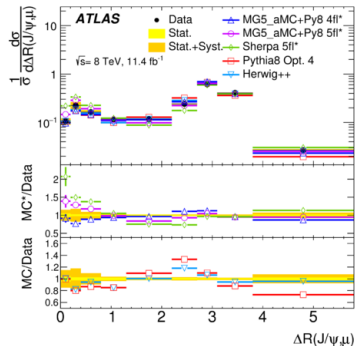
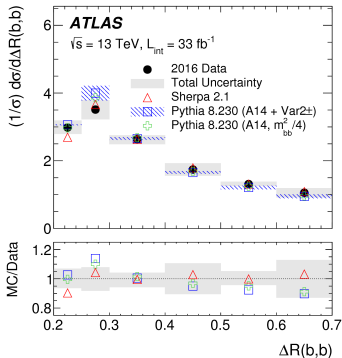
- parton showers geared towards collinear & soft emissions of gluons
(double log structure)
- $g \rightarrow q\bar{q}$ only collinear
- old measurements at LEP of $g \rightarrow b\bar{b}$ and $g \rightarrow c\bar{c}$ rate
- fix this at LHC for modern showers
(important for $t\bar{t}b\bar{b}$)
- questions: kernel, scale in α_s
(example: k_\perp vs. $m_{b\bar{b}}$)



latest ATLAS measurements

(arXiv:1812.09283, 1705.03374 [hep-ex])

- use b -tagged jets with $R = 0.2$ (left)
- use muons in $B \rightarrow J/\psi(\mu\mu) + X$ and $B \rightarrow \mu + X$ as proxies (right)

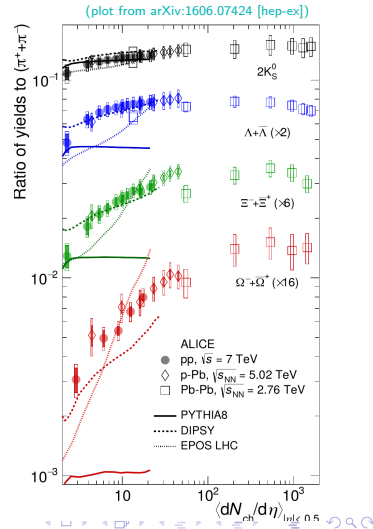


massive quarks are tricky - encore

- heavy quarks also problematic in initial state:
no PDF support for $Q^2 \leq m_Q^2 \rightarrow$ quarks stop showering
- possible solutions:
 - naive: ignore and leave for beam remnants (SHERPA)
 - better: enforce splitting in region around m_Q^2 (PYTHIA)
 \rightarrow effectively produces collinear Q and gluon in IS
- will need to check effect on precision observables: $p_{\perp}^{(W)}/p_{\perp}^{(Z)}$

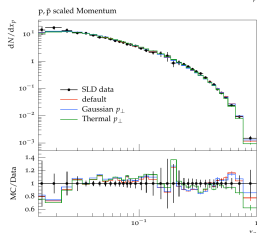
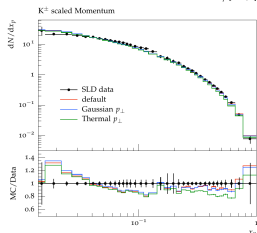
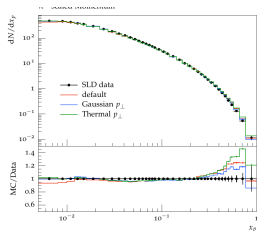
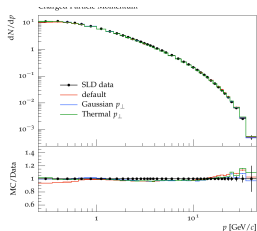
soft physics: strange strangeness

- universality of hadronization assumed
- parameters tuned to LEP data
in particular: strangeness suppression
- for strangeness: flat ratios
but data do not reproduce this
- looks like $SU(3)$ restoration
not observed for protons
- needs to be investigated (see next)



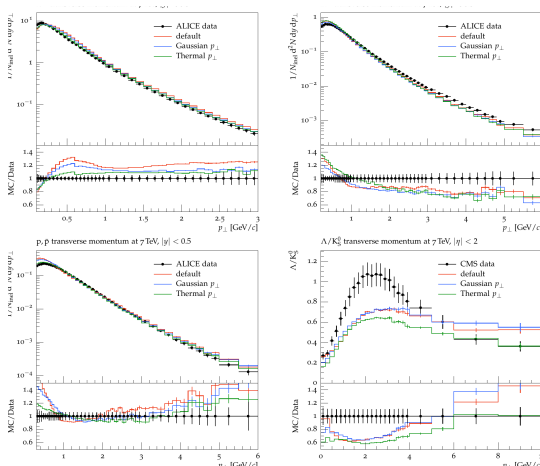
hadronization issues

(illustrative plots from arXiv:1610.09818 [hep-ph])



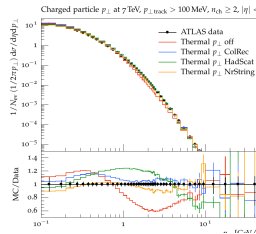
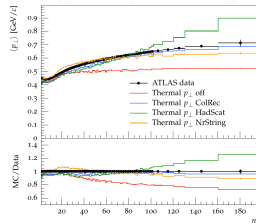
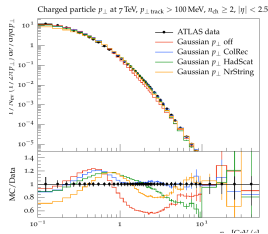
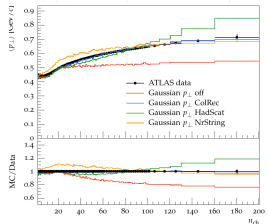
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hadronization issues

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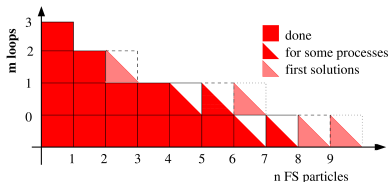


summary & outlook

(successes, wild dreams, & heretical thoughts)

successes & questions

- program of precision calculations (HO QCD) successful:



- NNLO QCD calculations consolidated for $2 \rightarrow 2$ processes

(but: not yet available in full simulations, necessary for investigations at $\mathcal{O}(< 10\%)$ accuracy)

- combine (N)NLO QCD and NLO EW corrections?

(investigate additive vs. multiplicative, maybe with calculations like 1511.08016)

- consolidated MC simulation at NLO

(Mc@NLO, MEPS@NLO & friends, addition of EW effects)

(but: still steep learning curve ahead)

wild dreams for upcoming LHC run(s)

(things that I think are feasible in next 5 years)

- NNLO (QCD) \oplus NLO (EW) for all $2 \rightarrow 2$ SM processes
- NNLO (QCD) for first “real” $2 \rightarrow 3$ SM processes
- parton shower at $\mathcal{O}(\alpha_s^2)$

(interesting interplay with subtraction at NNLO)

- “proper” NNLOPs (Mc@NNLO) for all $2 \rightarrow 2$ processes

(plus multijet merging with (N)NLOs)

- $\mathcal{O}(1\%)$ control over inclusive/precision observables:
inclusive xsecs; p_\perp spectrum of $W, Z, H; \dots$
- fix treatment of heavy flavours in FS & IS

(important for Higgs precision/BSM searches, “higher-twist” corrections to simple factorisation, role of PDFs)

(problems at $\mu_F^2 < m_Q^2 \longrightarrow$ forced transitions to gluons at/around mass threshold)

summary: some heretical thoughts

- massive efficiency issues with HO calculations

(must learn to use tools in smarter ways)

- is there a limit to our perturbative precision programme?

- discuss non-perturbative effects: compare Λ_{QCD}/Q -effects with α_s
- improvement scales like ratio of “exponent” n in $N^n\text{LO}/N^{n-1}\text{LO}$?

(= ∞ for $\text{LO} \rightarrow \text{NLO}$, 100% for $\text{NLO} \rightarrow \text{NNLO}$, 50% for $\text{NNLO} \rightarrow \text{NNNLO}$, ...)

- soft/non-perturbative physics will be the biggest uncertainty for many observables/measurements

(but practically nobody works on it)

