Complexity to Simplicity and Back:

Challenges in Theoretical (Particle) Physics

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- what the talk is about
- finding simplicity in complexity
- re-emerging complexity
- lesson(s) learnt

what the talk is about

deciphering Nature

The aim of particle physics is to

understand Nature at the smallest scales
in terms of
fundamental constituents and their interactions

we pursue this goal by

building models in mathematical formulation
Simplicity

and test them through experimental scrutiny
Complexity

guiding principle for building models

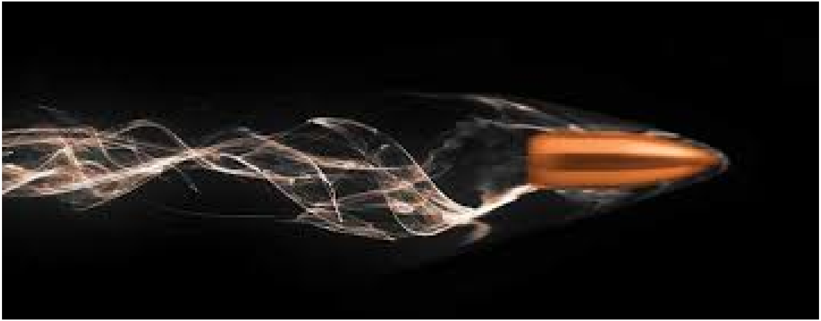
Occam's razor — economy of thought

or

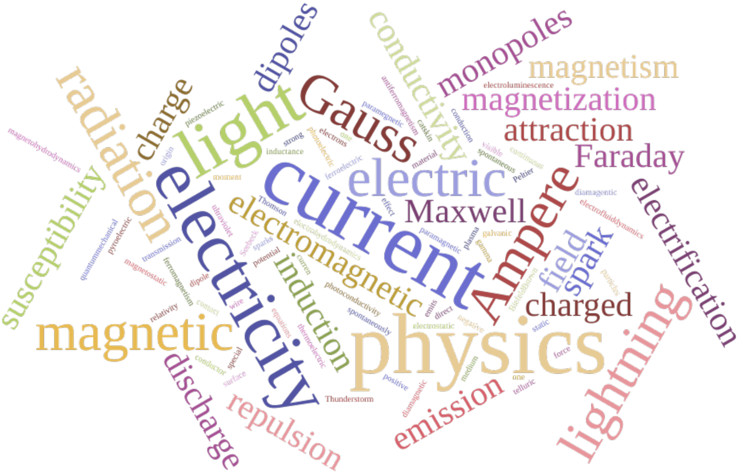
keep it simple — recycle successful ideas

finding simplicity in complexity

reductionism at work



electromagnetic phenomena



abstraction: simplicity from complexity

- many "laws" for electric/magnetic effects
- electromagnetic interactions summarised in Maxwell's equations:

(first unification of forces/phenomena, constructed bottom-up)

$$\boxed{\text{electricity} + \text{magnetism} = \text{electromagnetism}}$$

- solutions for electric and magnetic fields from potentials

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A} \quad \text{and} \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

- Lorentz force for fields acting on charge q

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

abstraction: top-down

- structure of Maxwell's equations:
  - two vector fields with opposite parity: vector $\vec{E}$ and axial-vector $\vec{B}$
  - first derivatives only (spatial and temporal)

  yields four equations, organised by spin & parity:

$$\begin{array}{lcccl} \text{scalar} & : & \text{charge} & = & \vec{\nabla}\cdot\vec{E} \\ \text{pseudo-scalar} & : & 0 & = & \vec{\nabla}\cdot\vec{B} \\ \text{vector} & : & \text{current} & = & \vec{\nabla}\times\vec{B}\mp\frac{\partial\vec{E}}{\partial t} \\ \text{axial-vector} & : & 0 & = & \vec{\nabla}\times\vec{E}\mp\frac{\partial\vec{B}}{\partial t} \end{array}$$

- arrive at Maxwell's equations with:
  - allow electric monopoles (charges), disallow magnetic monopoles
  - enforce electromagnetism = theory of light
- the first relativistic theory!

abstraction: cranked up (Lagrangian)

- recast as Lagrangian for four-potential $A^\mu = (\phi, \vec{A})$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j^\nu A_\nu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

(transverse polarizations only: $\partial_\mu A^\mu = 0$ from Fourier transform $p_\mu \epsilon^\mu = 0$)

- arrive at equations of motion

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= j^\nu \quad \text{(scalar and vector from above)} \\ \partial_\mu \tilde{F}^{\mu\nu} &= 0 \quad \text{(``funny parities'')} \end{aligned}$$

- gauge invariance: invariant under (arbitrary Λ)

$$A^\mu(x) \to A'^\mu(x) = A^\mu(x) + \partial^\mu \Lambda(x)$$

constructing QED, top-down

- start with Dirac particles (spin-$\frac{1}{2}$, mass m)

$$\mathcal{L} = \bar{\psi}\,[i\gamma^\mu\partial_\mu - m]\,\psi\,,$$

invariant under global phase transformations $\psi \to \psi' = e^{iq\Theta}\psi$

(Noether's theorem: conserved currents $\partial_\mu j^\mu = 0$ demand conserved charges)

- demand invariance under local phase transformations: $\Theta \to \Theta(x)$
- compensate non-invariant terms through "gauge" field $A_\mu(x)$:

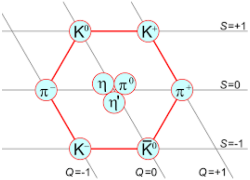
$$\partial_\mu \to D_\mu = \partial_\mu - iqA_\mu(x)$$

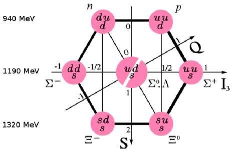
- identify A_μ with Maxwell's four-vector potential
- add kinetic term $\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ for gauge field & arrive at

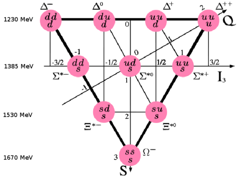
$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\,[i\gamma^\mu D_\mu - m]\,\psi$$

order from chaos: group theory

- in 50's and 60's: lots of new particles (incl. "strange" ones)
- introduce quarks as mnemonic device
- sorting them with group theory: $SU(3)_F$







recycling: interactions from more complicated symmetries

- recycle gauge idea for more complicated charges: isospin, colour, ...

  (e.g. $p \longleftrightarrow n$, $e \longleftrightarrow \nu_e$, 3 quark colours)

- necessitates multiplets $\psi \to \psi_i \longrightarrow$ symmetry transformations:

$$\psi_i \to \psi'_i = \exp\left(i\sum_a \Theta_a T^a_{ij}\right)\psi_j$$

  with generators T^a (matrices)

- classify interactions by symmetry group (the T^a_{ij}) and charge

Standard Model $= SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

encodes strong, weak, & electromagnetic interactions

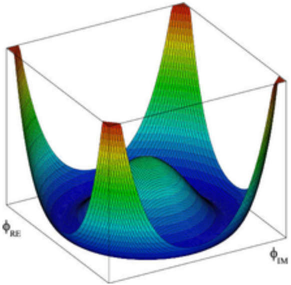
breaking symmetries

- for local phase transitions:
  need massless gauge bosons

  one for each generator
  $\longrightarrow$ 3 for $SU(2)_L$, 8 for $SU(3)_c$

  (also: Dirac fermions can only be massless)
- but: have massive particles
  (top, W, Z, Higgs, ...)
- must break/hide electroweak symmetry
- most efficient way: Higgs mechanism

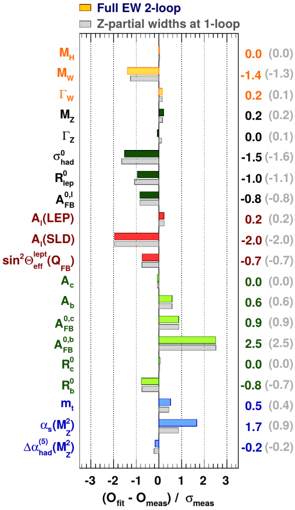
(another instance of Occam's razor)

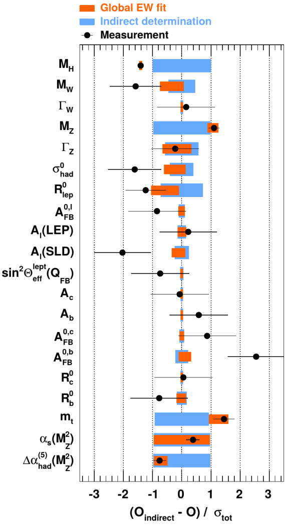


Standard Model in its full glory

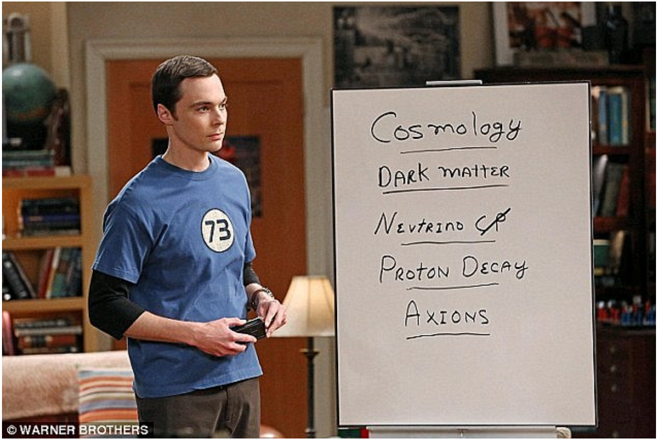
gauge group $\mathcal{G} = SU(3) \otimes SU(2) \otimes U(1)$

$$
\begin{aligned}
\mathcal{L} \; = \; & \sum_{F\in\mathcal{G}} \underbrace{\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right]}_{\text{gauge kinetic terms and self-interactions}} \\
& + \sum_i \underbrace{\left[\bar\psi_i\,\gamma^\mu\,D_\mu\psi_i\right]}_{\substack{\text{matter fermions:}\\ \text{kinetic terms and}\\ \text{gauge interactions}}} + \sum_{i,j} \underbrace{\left[y_{ij}\bar\psi_i\Phi\psi_j \; + \; \text{h.c.}\right]}_{\substack{\text{matter interactions}\\ \text{with Higgs doublet and}\\ \text{fermion masses}}} \\
& + \underbrace{\left[(D_\mu\Phi^\dagger)(D^\mu\Phi)\right]}_{\substack{\text{kinetic terms and gauge}\\ \text{interactions of Higgs dou-}\\ \text{blet, gauge boson mass}\\ \text{terms}}} + \underbrace{\left[V(\Phi^\dagger\Phi)\right]}_{\substack{\text{vev of Higgs doublet and}\\ \text{Higgs boson mass and}\\ \text{self-interactions}}}
\end{aligned}
$$


Full EW 2-loop
Z-partial widths at 1-loop
M_H 0.0 (0.0)
M_W -1.4 (-1.3)
Γ_W 0.2 (0.1)
M_Z 0.2 (0.2)
Γ_Z 0.0 (0.1)
σ⁰_had -1.5 (-1.6)
R⁰_lep -1.0 (-1.1)
A^{0,l}_FB -0.8 (-0.8)
A_l(LEP) 0.2 (0.2)
A_l(SLD) -2.0 (-2.0)
sin²Θ^{lept}_{eff}(Q_FB) -0.7 (-0.7)
A_c 0.0 (0.0)
A_b 0.6 (0.6)
A^{0,c}_FB 0.9 (0.9)
A^{0,b}_FB 2.5 (2.5)
R⁰_c 0.0 (0.0)
R⁰_b -0.8 (-0.7)
m_t 0.5 (0.4)
α_s(M²_Z) 1.7 (0.9)
Δα^{(5)}_{had}(M²_Z) -0.2 (-0.2)
-3 -2 -1 0 1 2 3
(O_fit - O_meas) / σ_meas


Global EW fit
Indirect determination
Measurement
M_H
M_W
Γ_W
M_Z
Γ_Z
σ⁰_had
R⁰_lep
A^{0,l}_FB
A_l(LEP)
A_l(SLD)
sin²Θ^{lept}_{eff}(Q_FB)
A_c
A_b
A^{0,c}_FB
A^{0,b}_FB
R⁰_c
R⁰_b
m_t
α_s(M²_Z)
Δα^{(5)}_{had}(M²_Z)
-3 -2 -1 0 1 2 3
(O_indirect - O) / σ_tot

edge of the Standard Model



going beyond the edge

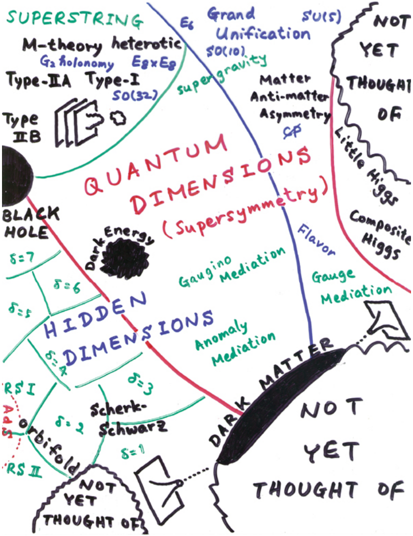
- despite success and self-consistency of SM: not complete theory
  - lacking: Gravitation, Dark Matter & Energy, ...
  - plus questions: nature of neutrinos, why 3 generations, ...
- aesthetic arguments
  - naturalness, hierarchy problem, unification of couplings, ...
  - Coleman-Mandula theorem

(maximal symmetry of S matrix = Lorentz $\otimes$ gauge $\otimes$ supersymmetry)

build more complete models

answering some of the (perceived) shortcomings

potentially with more symmetries that need to be broken

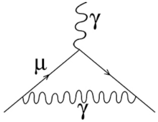

SUPERSTRING
M-theory heterotic
G2 holonomy E8xE8
Type-IIA Type-I
SO(32)
Type IIB
E6
Grand Unification
SU(5)
SO(10)
supergravity
Matter Anti-matter Asymmetry
CP
NOT YET THOUGHT OF
Little Higgs
Composite Higgs
QUANTUM DIMENSIONS
(Supersymmetry)
BLACK HOLE
Dark Energy
Flavor
Gaugino Mediation
Gauge Mediation
δ=7
δ=6
δ=5
HIDDEN DIMENSIONS
Anomaly Mediation
δ=4
δ=3
RS I
AdS5
orbifold
δ=2
Scherk-Schwarz
δ=1
RS II
NOT YET THOUGHT OF
DARK MATTER
NOT YET THOUGHT OF

re-emerging complexity:

calculational technology and concepts

complexity in calculating

example: magnetic moment $(g-2)$



$$m_S = -g \cdot \frac{\mu_B S}{\hbar}$$

- $S = \frac{1}{2}$ = spin, μ_B = Bohr's magneton and $g = 2$ (classical)
- measured through precession in a cylindrical Penning trap at 100 mK

("one-electron quantum cyclotron"; Hanneke et al., PRA 83 (2011) 052122)

- for electron

$$a_e = \frac{g-2}{2} = (1,159,652,180.73 \pm 0.28) \cdot 10^{-12}$$

yields most precise value of fine structure constant

$$\alpha^{-1} = 137.035,999,040(90)$$

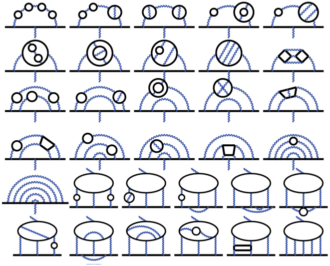
- reminder: Feynman diagram = part of QM transition amplitude
- for extraction of α need to calculate
  - $\approx$ 900 4-loop QED diagrams

    (every loop = one integral $\mathrm{d}^4 k$)

    results in semi-analytic form: harmonic polylogarithms, elliptic integrals, ...

    (Laporta, PLB 772 (2017) 232)
  - $\approx$ 12,500 5-loop QED diagrams (results only in numerical form)

    (Aoyama et al., PRD91 (2015) 033006)



calculating cross sections for the LHC

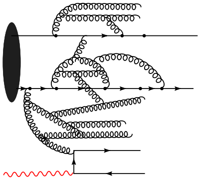
- master formula

$$\mathrm{d}\sigma_{pp\to X} = \sum_{ij}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2 f_i(x_1,\mu_F)\, f_j(x_2,\mu_F) \int \mathrm{d}\Phi_X\, \hat{\sigma}_{ij\to X}(\{p_X\};\mu_F,\mu_R)$$

relating parton-level $\hat{\sigma}$ with particle-level (observable) cross section σ

(partons = quarks and gluons)

- based on "factorization":
  parton distribution function $f_i(x,\,\mu_F)$
  process-independent if typical momentum scale $Q \approx \mu_F \gg m_{\rm proton}$
  (PDF = probability to find one parton of type i with energy fraction x at factorization scale μ_F in proton)



complex inputs I: parton distribution functions

- PDFs not known from first principles, only their scaling with μ_F

  (from the Altarelli-Parisi equations)

- fitted from data at different processes, at different, μ_F, at different experiments, with different systematics
- current accuracy: next-to–next-to–leading order (NNLO)

  (various collaborations: CTEQ, MMHT, NNPDF, ABM, GRV, HeraPDF)

- needs NNLO calculations and three-loop kernels driving the evolution

  (4-loop kernels partially known; Moch et al., 1707.08315)

- but: α_S^2 is $\mathcal{O}(1\%)$ – must also include electromagnetic and weak evolution at (N)LO

  (current frontier: LUXqed; Manohar PRL 117 (2016) 242002)

determining reliable PDFs — a complex endeavour

complex inputs II: partonic cross sections $\hat{\sigma}$

- automated evaluation of $\hat{\sigma}$ at LO and NLO,
  integral over phase space $\mathrm{d}\Phi_X$ with $3n-2$ dimensions for n particles
  $\longrightarrow$ Monte Carlo methods with involved sampling strategies
- problem beyond LO: occurrence of divergent structures when
  - momenta $k \to \infty$ ("ultraviolet divergence")
    regularization and renormalization
  - or $k \to 0$ ("infrared divergence")
    regularization and exact cancellation between contributions

  (Kinoshita–Lee–Nauenberg theorem)
- regularization by analytic continuation $\mathrm{d}^4k \to \mathrm{d}^Dk$ with $D = 4 + 2\epsilon$
  divergences manifest as poles $1/\epsilon$
- straightforward for UV divergences but tricky for IR divergences:
  cancellation is between contributions of different multiplicity

  (and phase space integrals are usually done with Monte Carlo methods)

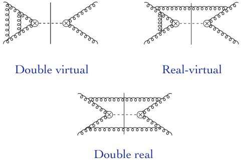
example: $gg \to H$

NLO $\mathcal{O}(\alpha_S)$

(Dawson; Djouadi et al.; 1990)

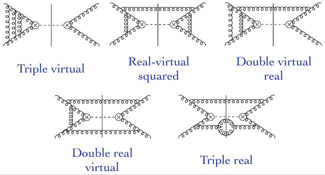
NNLO $\mathcal{O}(\alpha_S^2)$

(Harlander et al., Anastasiou et al.; 2002)



NNNLO $\mathcal{O}(\alpha_S^3)$

(Anastasiou et al.; 2015)



- $\sim$ 1,000 Feynman diagrams at NNLO
- $\sim$ 100,000 Feynman diagrams at N^{3}LO
  reduced to $\sim$ 1000 master integrals

symbols

- emergence of integrals of special functions (e.g. polylogarithms) that cannot be found in Mathematica or Standard integral tables

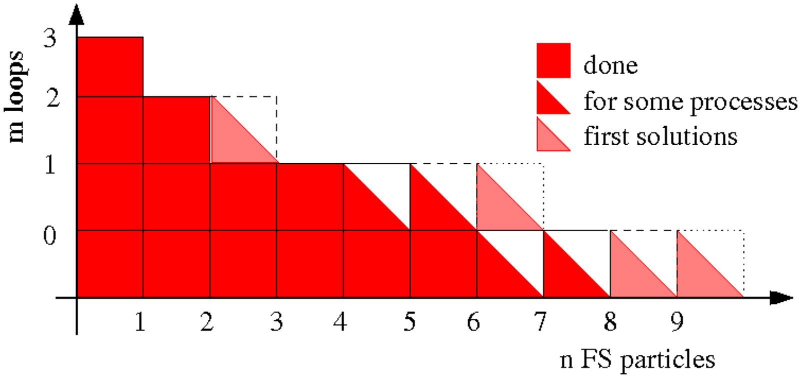
$$\text{example: } I = \int_0^\infty \mathrm{d}x \frac{\mathrm{Li}_2\left(\frac{ux}{(x+1)(v+x)}\right)}{x^2+(1-u+v)x+v}\,,$$

where u, v ratios of invariant masses, and dilogarithm

$$\mathrm{Li}_2(x) = -\int_0^z \frac{\mathrm{d}t}{t}\,\log(1-t)\,.$$

- with identities may be mapped onto known integrals but which identity? $\longrightarrow$ not all of them known
- trick: special functions follow algebraic structures (Hopf algebra) that allow to construct all identities

technological/complexity limit

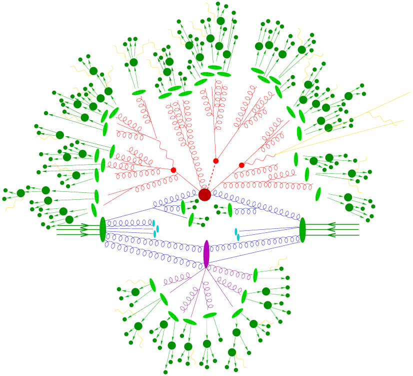


embedding in full simulations

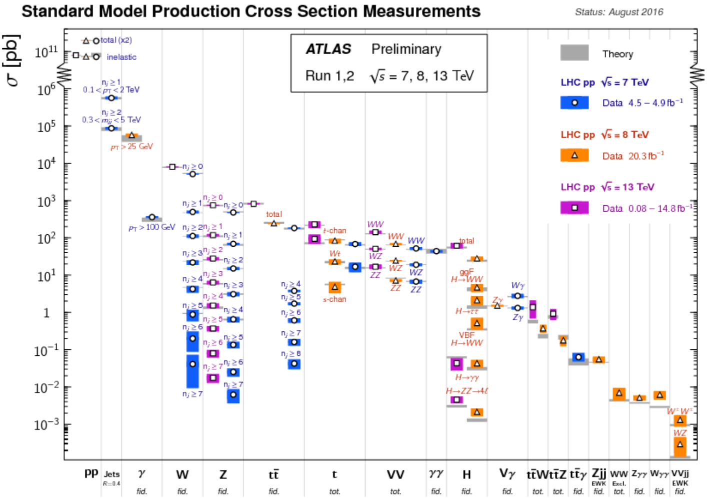
- fixed-order perturbation theory does not give full picture of *pp* collisions at LHC.
- more particles produced by
  - radiation of secondaries

    all-orders PT in approximation
  - transition from quarks & gluons to hadrons
  - decays of unstable hadrons & QED radiation
  - multiple interactions
- all in numerical simulation

  combination of first principles PT, effective theories, heavy modelling, and fitting to data



overall agreement with data



complexity in concepts

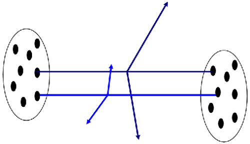
example: multi-parton interactions

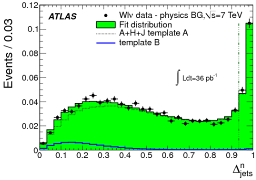
- protons = extended objects
  $\longrightarrow$ possibility of more parton pairs interacting
  resulting final states may be hard
  in "perturbative regime" $p_\perp \geq$ few GeV
- but: no factorization theorem available
  $\longrightarrow$ no first-principles theory
- simplistic parameterization

$$\sigma^{(DPS)}_{X+Y} = \frac{\sigma_X \otimes \sigma_Y}{\sigma_{\rm eff}}$$

with $\sigma_{\rm eff} \approx 15$ mb (measured)

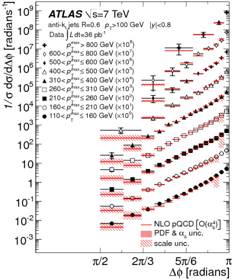
($\sigma_{\rm tot} \approx 100$ mb $= 10^{-29}$m^2 at LHC)



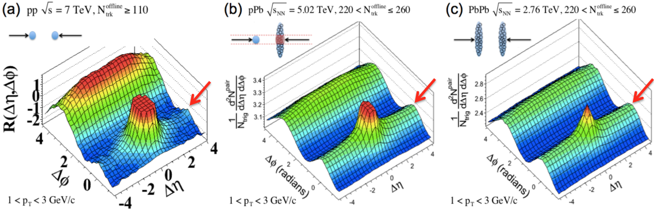


example: the ridge

- momentum conservation in transverse plane:
  $\longrightarrow$ in $2 \to 2$ collisions particles produced
  "back-to-back"
  $\longrightarrow$ decorrelation by additional radiation
- well understood in pert.QCD



- however: surprising structure in high-multiplicity *pp* cannot be reproduced in standard pert.QCD Monte Carlo



- typically explained as "collective effect" in heavy-ion collisions: "hydrodynamics" in *pp*: colour-ropes, "glasma"

(colour-glass condensate = non-pert. in weak coupling, glasma = Bose enhancement + Pauli blocking)

- conceptually different from textbook perturbation theory wrapping up

summary

- model building in particle theory driven by

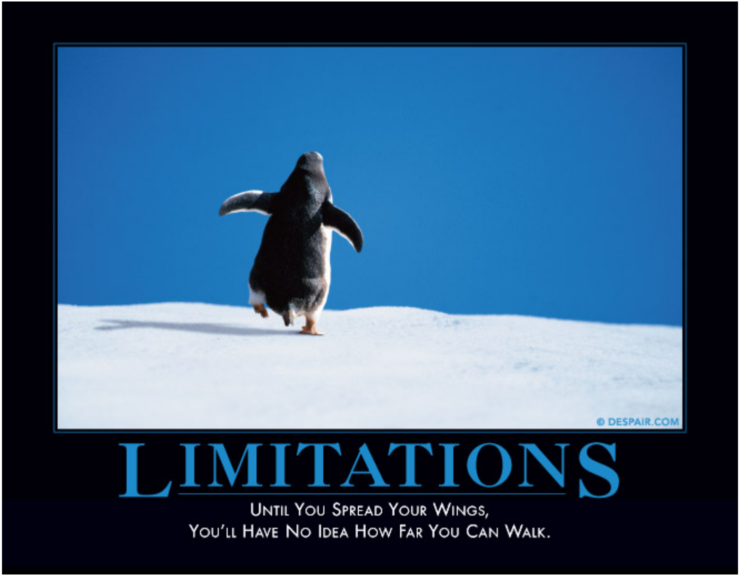
  symmetry considerations $\longrightarrow$ simplicity / elegance

  breaking mechanisms play crucial role

- phenomenological scrutiny of models reintroduces

  complexity in calculations and concepts

  technological breakthroughs in perturbation theory
  insufficiency of perturbative language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LIMITATIONS
Until You Spread Your Wings,
You'll Have No Idea How Far You Can Walk.