



Relativistic Physics for Teachers SS 2006

Problems 5 11.5.2006

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1. Properties of the Lorentz transformation:

Prove the following relations for a boost along the x -axis with u_x :

- $\Lambda^\mu{}_\nu(-u_x)\Lambda^\nu{}_\kappa(+u_x) = \delta^\mu{}_\kappa$, i.e. $\Lambda(-u_x) = \Lambda^{-1}(u_x)$
- $g'_{\mu\nu} = \Lambda^\rho{}_\mu(-u_x)g_{\rho\sigma}\Lambda^\sigma{}_\nu(-u_x) = g_{\mu\nu}$
- $\Lambda_\mu{}^\sigma(u_x) = g_{\mu\nu}\Lambda^\nu{}_\kappa g^{\kappa\sigma} = \Lambda^\sigma{}_\mu(-u_x)$
- $\Lambda_\mu{}^\sigma(u_x)\Lambda^\mu{}_\nu(u_x) = \delta^\sigma{}_\nu$

2. Invariance of the metric:

Calculate $g'_{\mu\nu} = \Lambda_\mu{}^\kappa(u_x)\Lambda_\nu{}^\sigma(u_x)g_{\kappa\sigma}$ as well as $g'^{\mu\nu} = \Lambda^\mu{}_\kappa(u_x)\Lambda^\nu{}_\sigma(u_x)g^{\kappa\sigma}$.

3. Rotations:

The coordinate system (x, y, z) is rotated by Θ_z about the z -axis and thus forms the new system (x', y', z') . Determine the matrix D_{ik} in the relation $x'_i = D_{ik}x_k$ for this transformation.