QCD & Monte Carlo Event Generators

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PART I: INTRODUCTION
QCD BASICS

SCALES & KINEMATICS
Contents

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An electromagnetic analogy

- consider a charge $Z$ moving at constant velocity $v$

- at $v = 0$: radial $E$ field only
- at $v = c$: $B$ field emerges: $\vec{E} \perp \vec{B}$, $\vec{B} \perp \vec{v}$, $\vec{E} \perp \vec{v}$,
  energy flow $\sim$ Poynting vector $\vec{S} \sim \vec{E} \times \vec{B}$, $\parallel \vec{v}$
- approximate classical fields by “equivalent quanta”: photons
• spectrum of photons:
  (in dependence on energy $\omega$ and transverse distance $b_\perp$)

\[
dn_\gamma = \frac{Z^2 \alpha}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{db_\perp^2}{b_\perp^2} \xrightarrow{\text{electron (Z=1)}} \frac{\alpha}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{db_\perp^2}{b_\perp^2}
\]

• Fourier transform to transverse momenta $k_\perp$:

\[
dn_\gamma = \frac{\alpha}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{dk_\perp^2}{k_\perp^2}
\]

note: divergences for $k_\perp \to 0$ (collinear) and $\omega \to 0$ (soft)

• therefore: Fock state for lepton = superposition (coherent):

\[
|e\rangle_{\text{phys}} = |e\rangle + |e\gamma\rangle + |e\gamma\gamma\rangle + |e\gamma\gamma\gamma\rangle + \ldots
\]

photon fluctuations will “recombine”
lifetime of electron–photon fluctuations: $e(P) \rightarrow e(p) + \gamma(k)$

first estimate: use uncertainty relation and Lorentz time dilation

- $P^2 = (p + k)^2 = M_{\text{virt}}^2$ the virtual mass of the incident electron
- life time = life time in rest frame $\cdot$ time dilation

$$\tau \sim \frac{1}{M_{\text{virt}}} \cdot \frac{E}{M_{\text{virt}}} = \frac{E}{(p + k)^2} \sim \frac{E}{2Ek(1 - \cos \theta)} \approx \frac{k}{k^2 \sin^2 \theta / 2} \approx \frac{\omega}{k^2}$$

second estimate: use uncertainty relation and assume only photon off-shell

- energy balance of photon

$$P^2 = 2p \cdot k + k^2, \quad \text{therefore} \quad k^2 \approx -k^2_\perp \approx -2p \cdot k < 0.$$  

- assume photon momentum to be $k^\mu = (\omega, \vec{k}_\perp, k_\parallel)$, shift in energy for photon going on-shell: $\delta \omega \sim k^2_\perp / \omega$, therefore

$$\tau_\gamma \sim \frac{1}{\delta \omega} \approx \frac{\omega}{k^2_\perp} \approx \frac{\omega}{\omega^2 \sin^2 \theta} \approx \frac{1}{\omega \theta^2}$$

lifetime larger with smaller transverse momentum (i.e. with larger transverse distance)
QED Initial and Final State Radiation

- **physical interpretation:**
  equivalent quanta = quantum manifestation of accompanying fields

- in absence of interaction: recombination enforced by coherence

- but: hard interaction possibly “kicks out” quantum
  $\rightarrow$ coherence broken
  $\rightarrow$ equivalent (virtual) quanta become real
  $\rightarrow$ emission pattern unravels

- alternative idea:
  initial state radiation of photons off incident electron
- consider final state radiation in $\gamma^* \rightarrow \ell\ell$
  (electron velocities/momenta labelled as $v$ and $v'/p$ and $p'$)
- classical electromagnetic spectrum from radiation function:
  
  \[
  \frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2} \left| \epsilon^* \cdot \left( \frac{\vec{v}}{1 - \vec{v} \cdot \vec{n}} - \frac{\vec{v}'}{1 - \vec{v}' \cdot \vec{n}} \right) \right|^2,
  \]
  
  with $\epsilon$ the polarisation vector and $\vec{n}(\Omega)$ the direction of the radiation
- recast with four–momenta, equivalent photon spectrum:
  
  \[
  dN = \frac{d^3 k}{(2\pi)^3 2k_0} \frac{\alpha}{\pi} \left| \epsilon^*_\mu \left( \frac{p_\mu}{p \cdot k} - \frac{p'_\mu}{p' \cdot k} \right) \right|^2
  \]
  \[
  = \frac{d^3 k}{(2\pi)^3 2k_0} \frac{\alpha}{\pi} \left| W_{pp';k} \right|^2
  \]
  
  with the eikonal $W_{pp';k}$
repeat exercise in QFT, Feynman diagrams:

\[
\mathcal{M}_{X \rightarrow e^+e^-\gamma} = e\bar{u}(p) \left[ \Gamma \frac{p' - k}{(p' - k)^2} \gamma^\mu - \Gamma \frac{p + k}{(p + k)^2} \right] u(p') \epsilon^*_{\mu}(k)
\]

\[
\text{soft} \quad e\epsilon^*_{\mu}(k) \left[ \frac{p^\mu}{p \cdot k} - \frac{p'^\mu}{p' \cdot k} \right] \bar{u}(p')\Gamma u(p) = e\mathcal{M}_{X \rightarrow e^+e^-\gamma} \cdot W_{pp';k}
\]

manifestation of **Low's theorem**: 
soft radiation independent of spin (\(\rightarrow\) classical)

(radiation decomposes into soft, classical part with logs – i.e. dominant – and hard collinear part)
DGLAP equations for QED

(Dokshitser–Gribov–Lipatov–Altarelli–Parisi Equations)

- define probability to find electron or photon in electron:

  at LO in $\alpha$(noemission): $\ell(x, k^2_\perp) = \delta(1 - x)$
  and $\gamma(x, k^2_\perp) = 0$

  (introduced $x =$ energy fraction w.r.t. physical state)

- including emissions:
  - probabilities change
  - energy fraction $\xi$ of lepton parton w.r.t. the physical lepton object
    reduced by some fraction $z = x/\xi$
  - reminder: differential of photon number w.r.t. $k^2_\perp$:

    $\frac{d n_\gamma}{d \log k^2_\perp} = \frac{\alpha}{\pi} \frac{dx}{x}$
evolution equations (trivialised)

\[
\frac{d\ell(x, k_\perp^2)}{d\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} \mathcal{P}_{\ell\ell} \left( \frac{x}{\xi}, \alpha(k_\perp^2) \right) \ell(\xi, k_\perp^2)
\]

\[
\frac{d\gamma(x, k_\perp^2)}{d\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} \mathcal{P}_{\gamma\ell} \left( \frac{x}{\xi}, \alpha(k_\perp^2) \right) \ell(\xi, k_\perp^2).
\]

\(^\cdot\) \text{ } k_\perp^2 \text{ plays the role of “resolution parameter”}

\(^\cdot\) \text{ } \mathcal{P}_{ab}(z) \text{ are the splitting functions, encoding quantum mechanics of the “splitting cross section”, for example (at LO)}

\[
\mathcal{P}_{\ell\ell}(z) = \left( \frac{1 + z^2}{1 - z} \right)_+ + \frac{3}{2} \delta(1 - z)
\]

\(^\cdot\) \text{ if } \gamma \rightarrow \ell\bar{\ell} \text{ splittings included, have to add entries/splitting functions into evolution equations above}
Running of $\alpha_s$ and bound states

- quantum effect due to loops: couplings change with scale
- running driven by $\beta$–function

\[
\beta(\alpha_s) = \mu_R^2 \frac{\partial \alpha_s(\mu_R^2)}{\partial \mu_R^2} = \frac{\beta_0}{4\pi} \alpha_s^2 + \frac{\beta_1}{(4\pi)^2} \alpha_s^3 + \ldots
\]

with

\[
\begin{align*}
\beta_0 &= \frac{11}{3} C_A - \frac{4}{3} T_R n_f \\
\beta_1 &= \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_R n_f - 4 C_F T_R n_f
\end{align*}
\]
Casimir operators in the fundamental and adjoint representation:

\[ C_F = \frac{N_c^2 - 1}{2N_c} \quad \text{and} \quad C_A = N_c \]

with \( N_c = 3 \) colours and \( T_R = 1/2 \).

- \( n_f = \) the number of (quark) flavours
- the Casimirs correspond to quark and gluon colour charges
- explicit expression for strong coupling

\[
\alpha_s(\mu_R^2) \equiv \frac{g_s^2(\mu_R^2)}{4\pi} = \frac{1}{\frac{\beta_0}{4\pi} \log \frac{\mu_R^2}{\Lambda_{QCD}^2}}
\]

with \( \Lambda_{QCD} \) the Landau pole of QCD, \( \Lambda_{QCD} \approx 250 \text{MeV} \).
Hadrons in initial state: DGLAP equations of QCD

- similar to QED case:
  define probabilities (at LO) to find a parton $q$ – quark or gluon – in hadron $h$ at energy fraction $x$ and resolution parameter/scale $Q$:
  parton distribution function (PDF) $f_{q/h}(x, Q^2)$

- scale-evolution of PDFs: DGLAP equations

\[
\frac{\partial}{\partial \log Q^2} \begin{pmatrix} f_{q/h}(x, Q^2) \\ f_{g/h}(x, Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq}(\frac{x}{z}) & \mathcal{P}_{qg}(\frac{x}{z}) \\ \mathcal{P}_{gq}(\frac{z}{x}) & \mathcal{P}_{gg}(\frac{z}{x}) \end{pmatrix} \begin{pmatrix} f_{q/h}(z, Q^2) \\ f_{g/h}(z, Q^2) \end{pmatrix},
\]
QCD Basics: Scales & Kinematics

Hadrons in initial state: DGLAP equations of QCD

- **QCD splitting functions:**

\[
P_{qq}^{(1)}(x) = C_F \left[ \frac{1 + x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right] = \left[ P_{qq}^{(1)}(x) \right]_+ + \gamma_q^{(1)} \delta(1-x)
\]

\[
P_{qg}^{(1)}(x) = T_R \left[ x^2 + (1-x)^2 \right] = P_{qg}^{(1)}(x)
\]

\[
P_{gq}^{(1)}(x) = C_F \left[ \frac{1 + (1-x)^2}{x} \right] = P_{gq}^{(1)}(x)
\]

\[
P_{gg}^{(1)}(x) = 2C_A \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \frac{11C_A - 4n_f}{6} T_R \delta(1-x) = \left[ P_{gg}^{(1)}(x) \right]_+ + \gamma_g^{(1)} \delta(1-x).
\]

- **remark:** IR regularisation by $+\varepsilon$-prescription &
terms \( \sim \delta(1-x) \) from physical conditions on splitting functions

  (flavour conservation for $q \rightarrow qg$ and momentum conservation for $g \rightarrow gg, q\bar{q}$)
• simple idea: proton = \>|uud\rangle
  (valence quarks) only

• naively: no interactions
  \[ f_{u,d}/p \sim \delta (x - \frac{1}{3}) \]

• elastic interactions
  \[ \rightarrow \text{Gaussian smearing} \]

• strong interactions:
  develop “sea” = soft partons
  will depend on resolution scale
  remember: \(dn \propto \log \omega \log k_{\perp}^2\)

• in fact, due to \(g \rightarrow gg\), sea increases much faster,
  \[ f_{\text{sea}}/p(x, Q^2) \sim x^{-\lambda}, \quad \lambda \approx 1. \]
Hadrons in initial state: DGLAP equations of QCD
Hadron production: Scales

- Consider QCD final state radiation
- Pattern for $q \rightarrow qg$ similar to $\ell \rightarrow \ell\gamma$ in QED:

$$dW_{q \rightarrow qg} = \frac{\alpha_s(k^2_\perp)}{2\pi} \frac{dk^2_\perp}{k^2_\perp} \frac{d\omega}{\omega} \left[ 1 + \left( 1 - \frac{\omega}{E} \right)^2 \right]$$

$$\omega = E(1-z) \quad \frac{\alpha_s(k^2_\perp)}{2\pi} \frac{dk^2_\perp}{k^2_\perp} dz \frac{1 + z^2}{1 - z} = \frac{\alpha_s(k^2_\perp)}{2\pi} C_F \frac{dk^2_\perp}{k^2_\perp} dz P_{qg}^{(1)}(z).$$

- Divergent structures for:
  - $z \rightarrow 1$ (soft divergence) $\quad \leftrightarrow \quad$ infrared/soft logarithms
  - $k^2_\perp \rightarrow 0$ (collinear/mass divergence) $\quad \leftrightarrow \quad$ collinear logarithms
- Cut regularise with cut-off $k_{\perp,\text{min}} \sim 1\text{GeV} > \Lambda_{\text{QCD}}$
Hadrons in the final state

- find two perturbative regimes:
  - a regime of **jet production**, where $k_\perp \sim k_\parallel \sim \omega \gg k_{\perp,\min}$ and emission probabilities scale like $w \sim \alpha_s(k_\perp) \ll 1$; and
  - a regime of **jet evolution**, where $k_{\perp,\min} \leq k_\perp \ll k_\parallel \leq \omega$ and therefore emission probabilities scale like $w \sim \alpha_s(k_\perp) \log^2 k_\perp \sim 1$.

- in jet production:
  standard fixed–order perturbation theory

- in jet evolution regime,
  perturbative parameter **not** $\alpha_s$ any more
  but rather **towers** of $\exp[\alpha_s \log k_\perp \log k_\parallel]$

- induces counting of **leading logarithms (LL)**, $\alpha_s L^{2n}$,
  next-to leading logarithms (NLL), $\alpha_s L^{2n-1}$, etc.
consider spatio-temporal structure in classical QED case
  
  assume a charge comes into existence at \( t = 0 \) with \( \nu = 0 \): in its rest frame radial \( \vec{E} \) spreads out in sphere \( r' \leq t' \)
  
  assume charge moves with \( \nu \to 1 \) and Lorentz factor \( E/m \): then in lab frame field at radial distance \( r_\perp \) will arrive at \( t = \gamma t' = Er_\perp/m \)

translate to classical QCD:

  light quarks with constituent mass \( m \approx \Lambda_{QCD} \approx 1/R \)
  or \( m = m_Q \) for heavy quarks

  identify \( r_\perp \) with typical hadronic size \( R \)

  then: hadronization time

\[
 t^{(\text{had})} \approx \begin{cases} 
 ER^2 & \text{for light quarks} \\
 ER \quad \frac{1}{m_Q} & \text{for heavy quarks.} 
\end{cases}
\]
repeat exercise in quantum mechanics

- confining forces associated with gluons with 
  \( k \approx k_\perp \approx k_\parallel \approx 1/R \approx m \) in hadronic rest frame

- demand hadronization time \( \geq \) formation time:
  \[
  t^{(\text{form})} \approx \frac{k_\parallel}{k_\perp^2} \leq k_\parallel R^2 \approx t^{(\text{had})}
  \]

- therefore \( k_\perp \geq 1/R = \mathcal{O} (\text{few } \Lambda_{\text{QCD}}) \)

- therefore: breakdown of perturbative picture at scales/transverse momenta \( \mathcal{O} (\text{few } \Lambda_{\text{QCD}}) \)

- “gluers” replace gluons

- transition to bound states (phase transition)

- no first-principle understanding: \( \implies \) models
Summary
GOING MONTE CARLO

GENERAL IDEAS & TECHNIQUES
Contents

2.a) Prelude: selecting from a distribution
2.b) Monte Carlo integration: basic idea
2.c) Traditional MC simulation
Prelude: Selecting from a distribution

- a typical Monte Carlo/simulation problem:
  - wanted: random numbers \( x \in [x_{\text{min}}, x_{\text{max}}] \), distributed according to (probability) density \( f(x) \), i.e.
    \[
    \mathcal{P}(x \in [x', x' + dx']) = f(x')dx'
    \]
  - but: only “usual” random numbers \# available: “flat” in [0, 1]
  - exact solution:
    - must know integral \( F \) of density \( f \) and its inverse \( F^{-1} \)
    - \( x \) given by
      \[
      \int_{x_{\text{min}}}^{x} dx' f(x') = \# \int_{x_{\text{min}}}^{x_{\text{max}}} dx' f(x')
      \]
      and therefore
      \[
      x = F^{-1}[F(x_{\text{min}}) + \#(F(x_{\text{max}}) - F(x_{\text{min}}))]
      \]
• case above is very untypical – integral sometimes known, inverse practically never
• need a work-around solution: “Hit-or-miss” (solution, if exact case does not work.)
• construct good “over-estimator” \( g(x) \) \( (G \text{ and } G^{-1} \text{ known}) \):

\[
g(x) > f(x) \quad \forall x \in [x_{\text{min}}, x_{\text{max}}]
\]

• exact algorithm to select a trial-\( x \) according to over-estimator \( g \)
• accept \( x \) with probability \( \frac{f(x)}{g(x)} \)
• obvious fall-back choice for \( g(x) \):

\[
g(x) = \text{Max}_{[x_{\text{min}}, x_{\text{max}}]} \{ f(x) \}.
\]
Monte Carlo integration

- underlying idea: determination of $\pi$ with random number generator

\[
\text{Hits} + \text{Misses} \rightarrow \frac{\pi}{4}
\]

Throw random points $(x, y)$, with $x, y$ in $[0, 1]$.

For hits: $(x^2 + y^2) < r^2 = 1$
MC integration: estimate integral by $N$ probes

\[ I_f^{(a,b)} = \int_a^b \, dx \, f(x) \]

\[ \rightarrow \langle I_f^{(a,b)} \rangle = \frac{b - a}{N} \sum_{i=1}^N f(x_i) = \langle f \rangle_{a,b} \]

where $x_i$ homogeneously distributed in $[a, b]$

error estimate from statistical sample \(\rightarrow\) standard deviation

\[ \langle E_f^{(a,b)}(N) \rangle = \sigma = \left[ \frac{\langle f^2 \rangle_{a,b} - \langle f \rangle_{a,b}^2}{N} \right]^{1/2} \]

\(\rightarrow\) method of choice for high-dimensional integrals.
Monte Carlo integration: basic idea

- **Evaluating π**
  - Number of steps vs. Estimate

- **Errors in evaluating π**
  - Number of steps vs. Relative error

- Variance vs. True value
Monte Carlo integration: refinements

- want to minimise number of potentially expensive function calls
  \[\implies\text{need to improve convergence of MC integration.}\]
- first basic idea: sample in regions, where \(f\) largest

(\[\implies\text{corresponds to a Jacobean transformation of integral}\])

- alternative algorithm: minimise error by “smoothing” integrand
  (”importance sampling”)
  - assume a function \(g(x)\) similar to \(f(x)\).
  - \(f(x)/g(x)\) smooth \[\implies\langle E(f/g)\rangle\text{ small}\]
  - must sample according to \(dx\cdot g(x)\) rather than \(dx\):
    - \(g(x)\) plays role of probability distribution; we know already how to deal with this!

- works, if \(f(x)\) is well-known, but hard to generalise.
- importance sampling
- consider \( f(x) = \cos \frac{\pi x}{2} \) and \( g(x) = 1 - x^2 \):

\[
I = \int_0^1 dx \cos \frac{\pi x}{2} \\
= 0.637 \pm 0.308/\sqrt{N}
\]

\[
I = \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi x}{2}}{1 - x^2} \\
= \int d\rho \frac{\cos \frac{\pi x}{2}}{1 - x^2} [x(\rho)] \\
= 0.637 \pm 0.032/\sqrt{N}
\]
yet another idea: decompose integral in $M$ sub-integrals

$$
\langle I(f) \rangle = \sum_{j=1}^{M} \langle I_j(f) \rangle
$$

$$
\langle E(f) \rangle^2 = \sum_{j=1}^{M} \langle E_j(f) \rangle^2
$$

overall variance smallest, if “equally distributed”.

(⇒ sample, where the fluctuations are.)

"stratified sampling"

algorithm:

- divide interval in bins (variable bin-size or weight);
- adjust such that variance identical in all bins.
stratified sampling

consider \( f(x) = \cos \frac{\pi x}{2} \) and \( g(x) = 1 - x^2 \):

\[
\langle I \rangle = 0.637 \pm 0.147/\sqrt{N}
\]
• a hybrid of stratified and importance sampling:
  replace independent bins of stratified sampling with independent
  functions of importance sampling
• “bins” – with weight $\alpha_i$ – of “eigenfunctions” – $g_i(x)$:
  $\Rightarrow g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x})$.
• in particle physics, this is the method of choice for parton level event
generation:
  • translate each Feynman diagram into one or more channels
  • optimise interplay of channels, cuts, etc. through weights $\alpha_i$
  • optional: add VEGAS to “best” channels
Traditional MC simulation

- a classical example: two-dimensional Ising model:

  \[ \mathcal{H} = -J \sum_{\langle ij \rangle} s_i s_j \]

  (spins \( s_i \) fixed on 2-D lattice with nearest neighbour interactions.)

- evaluation of observable \( \mathcal{O} \) by summing over all micro states \( \phi_{\{i\}} \), given as spin ensembles

  \[ \langle \mathcal{O} \rangle = \int D\phi_{\{i\}} \text{Tr} \left\{ \mathcal{O}(\phi_{\{i\}}) \exp \left[ -\frac{\mathcal{H}(\phi_{\{i\}})}{k_B T} \right] \right\} \]

  (similar to path integral in QFT.)

- typical problem in such calculations (integrations!):

  phase space too large \( \Rightarrow \) need to sample.
Metropolis algorithm simulates the canonical ensemble, summing/integrating over micro-states with MC method.

necessary ingredient: interactions among spins in probabilistic language

algorithm:

- go over the spins,
- check whether they flip:
  - compare $P_{\text{flip}}$ with random number
  - $P_{\text{flip}}$ from energies of the two micro-states (before and after flip) and Boltzmann factors
- repeat to equilibrium.
- evaluate observables directly during run & take thermal average (average over many steps).
why does this work? **detailed balance**!
- consider one spin flip, connecting micro-states 1 and 2.
- rate of transitions given by the transition probabilities $\mathcal{W}$
- if $E_1 > E_2$ then $\mathcal{W}_{1\rightarrow 2} = 1$ and $\mathcal{W}_{2\rightarrow 1} = \exp\left(-\frac{E_1 - E_2}{k_B T}\right)$
- in thermal equilibrium, both transitions equally often:

$$P_2 \mathcal{W}_{2\rightarrow 1} = P_1 \mathcal{W}_{1\rightarrow 2}$$

which takes into account that the respective states are occupied according to their Boltzmann factors.

in principle, all systems in thermal equilibrium can be studied with Metropolis - just need to write transition probabilities in accordance with detailed balance, as above $\implies$ general simulation strategy in thermodynamics.
example results on a $10 \times 10$ lattice
PART II: MONTE CARLO

FOR PERTURBATIVE QCD
MONTE CARLO FOR PARTON LEVEL
Contents

3.a) Calculating matrix elements efficiently
3.b) Phase spacing for professionals
3.c) Including higher order corrections
3.d) Cancellation of IR divergences
3.e) Tools for LHC physics
Simulating hard processes (signals & backgrounds)

• Simple example: \( t \rightarrow bW^+ \rightarrow b\bar{l}\nu_l \):

\[
|\mathcal{M}|^2 = \frac{1}{2} \left( \frac{8\pi\alpha}{\sin^2 \theta_W} \right)^2 \frac{p_t \cdot p_\nu p_b \cdot p_l}{(p_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2}
\]

• Phase space integration (5-dim):

\[
\Gamma = \frac{1}{2m_t} \frac{1}{128\pi^3} \int d^2 p_W^2 \frac{d^2 \Omega_W}{4\pi} \frac{d^2 \Omega}{4\pi} \left( 1 - \frac{p_W^2}{m_t^2} \right) |\mathcal{M}|^2
\]

• 5 random numbers \( \rightarrow \) four-momenta \( \rightarrow \) “events”.

• Apply smearing and/or arbitrary cuts.

• Simply histogram any quantity of interest - no new calculation for each observable
Calculating matrix elements efficiently

- stating the problem(s):
  - multi-particle final states for signals & backgrounds.
  - need to evaluate $d\sigma_N$:
    \[
    \int_{\text{cuts}} \prod_{i=1}^{N} \frac{d^3 q_i}{(2\pi)^3 2E_i} \delta^4 \left( p_1 + p_2 - \sum_i q_i \right) |M_{p_1p_2\rightarrow N}|^2.
    \]
  - problem 1: factorial growth of number of amplitudes.
  - problem 2: complicated phase-space structure.
  - solutions: numerical methods.
example for factorial growth: $e^+e^- \rightarrow q\bar{q} + ng$

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obvious: traditional textbook methods (squaring, completeness relations, traces) fail
\[ \Rightarrow \] result in proliferation of terms \((M_i M_j^*)\)

better ideas of efficient ME calculation:
\[ \Rightarrow \] realise: amplitudes just are complex numbers,
\[ \Rightarrow \] add them before squaring!

remember: spinors, gamma matrices have explicit form could be evaluated numerically (brute force)
but: Rough method, lack of elegance, CPU-expensive

can do better with smart basis for spinors (see next slide)

this is still on the base of traditional Feynman diagrams!
helicity method:

- introduce basic helicity spinors (needs to “gauge”-vectors)
- write everything as spinor products, e.g.
  \[ \bar{u}(p_1, h_1)u(p_2, h_2) = \text{complex numbers.} \]
- have completeness relations such as

\[ (p + m) \implies \frac{1}{2} \sum_h \left[ \left( 1 + \frac{m^2}{p^2} \right) \bar{u}(p, h)u(p, h) \right. \]
\[ + \left. \left( 1 - \frac{m^2}{p^2} \right) \bar{v}(p, h)v(p, h) \right] \]

- there are other genuine expressions . . .
- translate Feynman diagrams into “helicity amplitudes”: complex-valued functions of momenta & helicities.
- spin-correlations etc. nearly come for free.
taming the factorial growth in the helicity method
- by reusing pieces: calculate only once!
- factoring out: reduce number of multiplications!

can be implemented as a-posteriori manipulations of amplitudes.

- better method: recursion relations (recycling built in).
  best candidate so far: off-shell recursions

(Dyson-Schwinger, Berends-Giele etc.)
improvement: off-shell recursion relations

general idea: recursively construct generalised currents $J_\alpha(\pi)$ for a set $\pi$ of external particles on their mass shell plus one internal one

\[
J_\alpha(\pi) = P_\alpha(\pi) \left\{ \sum_{P_2(\pi)} \sum_{\nu_1^{\alpha_1 \alpha_2}} \left[ S(\pi_1, \pi_2) \nu_1^{\alpha_1 \alpha_2} J_{\alpha_1}(\pi_1) J_{\alpha_2}(\pi_2) \right] + \sum_{P_3(\pi)} \sum_{\nu_1^{\alpha_1 \alpha_2 \alpha_3}} \left[ S(\pi_1, \pi_2, \pi_3) \nu_1^{\alpha_1 \alpha_2 \alpha_3} J_{\alpha_1}(\pi_1) J_{\alpha_2}(\pi_2) J_{\alpha_3}(\pi_3) \right] \right\}
\]

$P_\alpha(\pi)$ denotes the propagator denominator
$S(\pi_1, \pi_2)$ and $S(\pi_1, \pi_2, \pi_3)$ for symmetry factors
$\nu_\alpha$ for three- and four-particle vertices
go over all permutations of external particles $\pi$
recursion relations particularly powerful due to massive recycling as integral part of structure → bookkeeping problem only

there are sub–classes of particularly simple amplitudes: maximally helicity violating (MHV) amplitudes

all-gluon amplitudes with helicities given

\[
A(1^+, 2^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = ig_s^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle (n-1)n \rangle \langle n1 \rangle} \\
A(1^-, 2^-, \ldots, i^+, \ldots, j^+, \ldots n^-) = ig_s^{n-2} \frac{[ij]^4}{[12][23] \ldots [(n-1)n][n1]}.
\]

terms \([ij]\) etc. are products of two–component left– and right–handed Weyl spinors (particularly simple)

note: all-sign identical amplitudes vanish due to the conservation of angular momentum
in principle also factorial growth with number of colours
sampling over colours improves situation.

(but still, e.g. naively \(\approx (n - 1)!\) permutations/colour-ordering for \(n\) external gluons).

improved scheme: colour dressing

\[
T_{ij}^a T_{k\bar{l}}^a = \delta_{ij} \delta_{k\bar{l}} - \frac{1}{N_c} \delta_{ij} \delta_{k\bar{l}} - \frac{1}{N_c} \delta_{i\bar{l}} \delta_{j\bar{k}}
\]

works very well with Berends-Giele recursions

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<th>CSW</th>
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Time [s] for the evaluation of \(10^4\) phase space points, sampled over helicities & colour.
Phase spacing for professionals

("Amateurs study strategy, professionals study logistics")

- democratic, process-blind integration methods:
  - Rambo/Mambo: Flat & isotropic
    
  - HAAG/Sarge: Follows QCD antenna pattern
    

- multi-channelling: each Feynman diagram related to a phase space mapping (= ”channel”), optimise their relative weights
  

- main problem: practical only up to $O(10^k)$ channels.

- some improvement by building phase space mappings recursively: more channels feasible, efficiency drops a bit.
basic idea of multichannel sampling (again):
use a sum of functions $g_i(\vec{x})$ as Jacobean $g(\vec{x})$.

$$g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x});$$

condition on weights like stratified sampling;
(“combination” of importance & stratified sampling).

algorithm for one iteration:
- select $g_i$ with probability $\alpha_i \rightarrow \vec{x}_j$.
- calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$
- add $f(\vec{x}_j)/g(\vec{x}_j)$ to total result and $f(\vec{x}_j)/g_i(\vec{x}_j)$ to partial (channel-) results.
- after $N$ sampling steps, update a-priori weights.

this is the method of choice for parton level event generation!
quality measure for integration performance: **unweighting efficiency**

want to generate events “as in nature”.

basic idea: use hit-or-miss method;

- generate $\vec{x}$ with integration method,
- compare actual $f(\vec{x})$ with maximal value during sampling
  $\implies$ “Unweighted events”.

comments:

- **unweighting efficiency**, $w_{\text{eff}} = \langle f(\vec{x}_j)/f_{\text{max}} \rangle =$ number of trials for each event.
- expect $\log_{10} w_{\text{eff}} \approx 3 - 5$ for good integration of multi-particle final states at tree-level.
- maybe acceptable to use $f_{\text{max,eff}} = Kf_{\text{max}}$ with $K < 1$.

  problem: what to do with events where $f(\vec{x}_j)/f_{\text{max,eff}} > 1$?

  answer: Add $\int [f(\vec{x}_j)/f_{\text{max,eff}}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\text{max,eff}} - k$. 
Including higher order corrections

- obtained from adding diagrams with additional:
  - loops (virtual corrections) or
  - legs (real corrections)

- effect: reducing the dependence on $\mu_R \& \mu_F$

NLO allows for meaningful estimate of uncertainties

- additional difficulties when going NLO:
  - ultraviolet divergences in virtual correction
  - infrared divergences in real and virtual correction

enforce

UV regularisation & renormalisation
IR regularisation & cancellation

(Kinoshita–Lee–Nauenberg–Theorem)
traditional bottleneck of higher–order calculations: virtual parts

algorithm before about 2005:

- Passarino–Veltman reduction of tensors in numerator

\[ 2p \cdot k = (p + k)^2 - p^2 - k^2 \]

- reduce to scalar master integrals of the form

\[
\int \frac{d^D k}{[(p_1 + k)^2(p_2 + k)^2 \ldots]}
\]

- further reduce to integrals with up to four propagators only

  (but careful: introduces instabilities through “Gram determinants”)

F. Krauss
QCD & Monte Carlo Event Generators
about 2005: begin of “NLO revolution”

basic idea: reduce to master integrals numerically by cutting

coefficients of master integrals emerge as solutions of linear equations
Cancellation of infrared divergences

- need a mechanism to cancel IR divergences for higher multiplicities in final states
- toy model in one dimension:

\[ |\mathcal{M}_{m+1}^R|^2 = \frac{1}{x} R(x) \quad \text{and} \quad |\mathcal{M}_m^V|^2 = \frac{1}{\epsilon} V, \]

where \( x \) = gluon energy & regularised in \( d = 4 - 2\epsilon \) dimensions.

Cross section in \( d \) dimensions with jet measure \( F^J \):

\[
\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} R(x) F^J_1(x) + \frac{1}{\epsilon} V F^J_0
\]

- infrared safety of jet measure: \( F^J_1(0) = F^J_0 \)
  \( \implies \) "a soft/collinear parton has no effect."
  (tricky issue - without it, no reliable NLO calculation!)
- KLN theorem: \( R(0) = V. \)
rewrite toy-model cross section as

\[
\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} R(x) F^J_1(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} VF^J_0 + \int_0^1 \frac{dx}{x^{1+\epsilon}} VF^J_0 + \frac{1}{\epsilon} VF^J_0
\]

\[
= \int_0^1 \frac{dx}{x^{1+\epsilon}} (R(x) F^J_1(x) - VF^J_0) + O(1) VF^J_0.
\]

two separately finite integrals, with no large numbers to be added/subtracted.

subtraction terms are universal (analytic bit can be calculated once and for all).

this has been automated in two schemes: Catani-Seymour and Frixione-Kunszt-Signer
general structure of NLO calculation for \( N \)-body production

\[
d\sigma = d\Phi_B B_N(\Phi_B) + d\Phi_B \nu_N(\Phi_B) + d\Phi_R R_N(\Phi_R)
\]

\[
= d\Phi_B \left( B_N + \nu_N + I_N^{(S)} \right) + d\Phi_R \left( R_N - S_N \right)
\]

phase space factorisation assumed here \((\Phi_R = \Phi_B \otimes \Phi_1)\)

\[
\int d\Phi_1 S_N(\Phi_B \otimes \Phi_1) = I_N^{(S)}(\Phi_B)
\]

process independent subtraction kernels

\[
S_N(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes S_1(\Phi_B \otimes \Phi_1)
\]

\[
I_N^{(S)}(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes I_1^{(S)}(\Phi_B)
\]

with universal \( S_1(\Phi_B \otimes \Phi_1) \) and \( I_1^{(S)}(\Phi_B) \)

in Catani–Seymour invertible phase space mapping

\[
\Phi_R \longleftrightarrow \Phi_B \otimes \Phi_1
\]
Aside: choices . . .

- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:
  unphysical scale choices will yield unphysical results

so maybe we have to be a bit smarter than just running NLO code
Availability of exact calculations (hadron colliders)

- fixed order matrix elements ("parton level") are exact to a given perturbative order.

- important to understand limitations:
  only tree-level and one-loop level fully automated, beyond: prototyping

(and often quite a pain!)

![Diagram showing the availability of exact calculations](Image)

- done
- for some processes
- first solutions
## Survey of existing parton-level tools @ tree–level

<table>
<thead>
<tr>
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# Survey of existing parton-level tools @ NLO

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<tr>
<th>Tool</th>
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<td>McFM</td>
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GOING MONTE CARLO

PARTON SHOWERS – THE BASICS
Contents

4.a) An analogy: radioactive decays
4.b) The pattern of QCD radiation
4.c) Quantum improvements
4.d) Compact notation
An analogy: Radioactive decays

- consider radioactive decay of an unstable isotope with half-life $\tau$.

  (and ignore factors of ln 2.)

- “survival” probability after time $t$ is given by

  $$S(t) = \mathcal{P}_{\text{nodec}}(t) = \exp[-t/\tau]$$

  (note “unitarity relation”: $\mathcal{P}_{\text{dec}}(t) = 1 - \mathcal{P}_{\text{nodec}}(t)$.)

- probability for an isotope to decay at time $t$:

  $$\frac{d\mathcal{P}_{\text{dec}}(t)}{dt} = -\frac{d\mathcal{P}_{\text{nodec}}(t)}{dt} = \frac{1}{\tau} \exp(-t/\tau)$$

- now: connect half-life with width $\Gamma = 1/\tau$.

- probability for the isotope to decay at any fixed time $t$ determined by $\Gamma$. 

F. Krauss
IPPP
QCD & Monte Carlo Event Generators
An analogy: radioactive decays

- spice things up now: add time-dependence, \( \Gamma = \Gamma(t') \)

- rewrite

\[
\Gamma t \rightarrow \int_0^t d t' \Gamma(t')
\]

- decay-probability at a given time \( t \) is given by

\[
\frac{d \mathcal{P}_{\text{dec}}(t)}{d t} = \Gamma(t) \exp \left[-\int_0^t d t' \Gamma(t')\right] = \Gamma(t) \mathcal{P}_{\text{nodec}}(t)
\]

(unitarity strikes again: \( d \mathcal{P}_{\text{dec}}(t)/d t = -d \mathcal{P}_{\text{nodec}}(t)/d t \).)

- interpretation of l.h.s.:

  - first term is for the actual decay to happen.
  - second term is to ensure that no decay before \( t \)
    \( \implies \) conservation of probabilities.

the exponential is - of course - called the **Sudakov form factor**.
The pattern of QCD radiation

- a detour: Altarelli-Parisi equation, once more

- AP describes the scaling behaviour of the parton distribution function

\[
\frac{dq(x, Q^2)}{d \ln Q^2} = \int_x^1 \frac{dy}{y} \left[ \alpha_s(Q^2) P_q(x/y) \right] q(y, Q^2)
\]

- term in square brackets determines the probability that the parton emits another parton at scale \(Q^2\) and Bjorken-parameter \(y\)

- driving term: Splitting function \(P_q(x)\)

  important property: universal, process independent

(after the splitting, \(x \rightarrow yx + (1 - y)x\)\.)
differential cross section for gluon emission in $e^+ e^- \to$ jets

$$\frac{d\sigma_{ee\to 3j}}{dx_1 dx_2} = \sigma_{ee\to 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

singular for $x_1, 2 \to 1$.

rewrite with opening angle $\theta_{qg}$ and gluon energy fraction $x_3 = 2E_g / E_{c.m.}$:

$$\frac{d\sigma_{ee\to 3j}}{d \cos \theta_{qg} dx_3} = \sigma_{ee\to 2j} \frac{C_F \alpha_s}{\pi} \left[ \frac{2}{\sin^2 \theta_{qg}} \frac{1 + (1 - x_3)^2}{x_3} - x_3 \right]$$

singular for $x_3 \to 0$ ("soft"), $\sin \theta_{qg} \to 0$ ("collinear").
re-express collinear singularities

\[
\frac{2d \cos \theta_{qg}}{\sin^2 \theta_{qg}} = \frac{d \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{d \cos \theta_{qg}}{1 + \cos \theta_{qg}} = \frac{d \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{d \cos \theta_{\bar{q}g}}{1 - \cos \theta_{\bar{q}g}} \approx \frac{d \theta_{qg}^2}{\theta_{qg}^2} + \frac{d \theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2}
\]

independent evolution of two jets \((q\text{ and }\bar{q})\)

\[
d\sigma_{ee \rightarrow 3j} \approx \sigma_{ee \rightarrow 2j} \sum_{j \in \{q, \bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{d \theta_{jg}^2}{\theta_{jg}^2} P(z)
\]
Parton–level Monte Carlo

The pattern of QCD radiation

- note: same form for any $t \propto \theta^2$
- transverse momentum $k^2_\perp \approx z^2 (1 - z)^2 E^2 \theta^2$
- invariant mass $q^2 \approx z (1 - z) E^2 \theta^2$

$$\frac{d \theta^2}{\theta^2} \approx \frac{d k^2_\perp}{k^2_\perp} \approx \frac{d q^2}{q^2}$$

- parametrisation-independent observation: (logarithmically) divergent expression for $t \to 0$.
- practical solution: cut-off $Q_0^2$.
  $\implies$ divergence will manifest itself as $\log Q_0^2$.
- similar for $P(z)$: divergence for $z \to 0$ cured by cut-off.
- what is a parton?
  collinear pair/soft parton recombine!
- introduce resolution criterion $k_\perp > Q_0$.

- combine virtual contributions with unresolvable emissions:
cancels infrared divergences $\implies$ finite at $O(\alpha_s)$

(Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- unitarity: probabilities add up to one
  $P(\text{resolved}) + P(\text{unresolved}) = 1$.  

$\begin{align*}
\text{[Diagram]} & + \text{[Diagram]} + \text{[Diagram]} = 1.
\end{align*}$
the Sudakov form factor, once more

differential probability for emission between $q^2$ and $q^2 + dq^2$:

$$dP = \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} \int_{z_{\text{min}}}^{z_{\text{max}}} dz P(z) = dq^2 \Gamma(q^2)$$

from radioactive example: evolution equation for $\Delta$

$$-\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2) \frac{dP}{dq^2} = \Delta(Q^2, q^2)\Gamma(q^2)$$

$$\implies \Delta(Q^2, q^2) = \exp \left[ -\int_{q^2}^{Q^2} dk^2 \Gamma(k^2) \right]$$
- maximal logs if emissions ordered
- impacts on radiation pattern: in each emission $t$ becomes smaller

$q_1^2 > q_2^2 > q_3^2, \quad q_1^2 > q_2'^2$
Quantum improvements

- improvement: inclusion of various quantum effects
- trivial: effect of summing up higher orders (loops) $\alpha_s \to \alpha_s(k^2_{\perp})$

- much faster parton proliferation, especially for small $k^2_{\perp}$.
- avoid Landau pole: $k^2_{\perp} > Q^2_0 \gg \Lambda^2_{\text{QCD}} \implies Q^2_0 = \text{physical parameter.}$
- soft limit for single emission also universal
- problem: soft gluons come from all over (not collinear!) quantum interference? still independent evolution?
- answer: not quite independent.
- consider case in QED

\[ e^{-} \rightarrow \gamma \rightarrow e^{+} \]
• assume photon into $e^+ e^-$ at $\theta_{ee}$ and photon off electron at $\theta$
  photon momentum denoted as $k$

• energy imbalance at vertex: $k_\perp \sim k_\parallel \theta$, hence $\Delta E \sim k_\perp^2 / k_\parallel \sim k_\parallel \theta^2$.

• formation time for photon emission: $\Delta t \sim 1 / \Delta E \sim k_\parallel / k_\perp^2 \sim 1 / (k_\parallel \theta^2)$.

• ee-separation: $\Delta b \sim \theta_{ee} \Delta t$

• must be larger than transverse wavelength of photon:
  $\theta_{ee} / (k_\parallel \theta^2) > 1 / k_\perp = 1 / (k_\parallel \theta)$

• thus: $\theta_{ee} > \theta$ must be satisfied for photon to form

• angular ordering as manifestation of quantum coherence
pictorially:

\[
\text{gluons at large angle from combined colour charge!}
\]
• experimental manifestation:
  $\Delta R$ of $2^{nd}$ & $3^{rd}$ jet in multi-jet events in pp – collisions

$E_{T1} > 110$ GeV, $E_{T3} > 10$ GeV.
Parton showers, compact notation

- Sudakov form factor (no-decay probability)

\[
\Delta^{(\mathcal{K})}_{ij,k}(t, t_0) = \exp \left[ - \int_{t_0}^{t} \frac{dt}{t} \frac{\alpha_s}{2\pi} \int dz \frac{d\phi}{2\pi} \mathcal{K}_{ij,k}(t, z, \phi) \right]
\]

- Evolution parameter \( t \) defined by kinematics

  generalised angle (HERWIG ++) or transverse momentum (PYTHIA, SHERPA)

- Will replace \( \frac{dt}{t} dz \frac{d\phi}{2\pi} \rightarrow d\Phi_1 \)

- Scale choice for strong coupling: \( \alpha_s(k_{\perp}^2) \)

- Regularisation through cut-off \( t_0 \)
“compound” splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off $n$-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_s}{2\pi} \sum_{\text{all } \{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}), \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp\left[ - \int_{t_0}^{t} d\Phi_1 \mathcal{K}_n(\Phi_1) \right]$$

consider first emission only off Born configuration

$$d\sigma_B = d\Phi_N B_N(\Phi_N)$$

$$\cdot \left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \left[ \mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}$$

integrates to unity $\rightarrow$ “unitarity” of parton shower

further emissions by recursion with $Q^2 = t$ of previous emission
analyse connection to $Q_T$ resummation formalism

consider standard Collins-Soper-Sterman formalism (CSS):

\[
\frac{d\sigma_{AB \rightarrow X}}{dy dQ_{2\perp}^2} = d\Phi_X B_{ij}(\Phi_X) \cdot \int \frac{d^2 b_{\perp}}{(2\pi)^2} \exp(i\vec{b}_{\perp} \cdot \vec{Q}_{\perp}) \tilde{W}_{ij}(b; \Phi_X)
\]

guarantee 4-mom conservation higher orders

with

\[
\tilde{W}_{ij}(b; \Phi_X) = \text{collinear bits} \underbrace{C_i(b; \Phi_X, \alpha_s)C_j(b; \Phi_X, \alpha_s)}_{\text{loops}}\underbrace{H_{ij}(\alpha_s)}\exp \left[ -\int_{1/b_{\perp}^2}^{Q_X^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \left( A(\alpha_s(k_{\perp}^2)) \log \frac{Q_X^2}{k_{\perp}^2} + B(\alpha_s(k_{\perp}^2)) \right) \right]
\]

Sudakov form factor, $A, B$ expanded in powers of $\alpha_s$
analyse structure of emissions above

logarithmic accuracy in \( \log \frac{\mu N}{k_\perp} \) (a la CSS)
possibly up to next-to leading log,

- if evolution parameter \( \sim \) transverse momentum,
- if argument in \( \alpha_s \) is \( \propto k_\perp \) of splitting,
- if \( K_{ij,k} \rightarrow \) terms \( A_{1,2} \) and \( B_1 \) upon integration

(OK, if soft gluon correction is included, and if \( K_{ij,k} \rightarrow \) AP splitting kernels)

in CSS \( k_\perp \) typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions

resummation scale \( \mu_N \approx \mu_F \) given by (Born) kinematics – simple for cases like \( q \bar{q}' \rightarrow V, \; gg \rightarrow H, \ldots \)
tricky for more complicated cases
<table>
<thead>
<tr>
<th>First improvements</th>
<th>Matching</th>
<th>Multijet merging</th>
</tr>
</thead>
</table>

ROUND III: PRECISION MONTE CARLO
FIRST IMPROVEMENTS:

ME CORRECTIONS
Contents

5.a) Improving event generators

5.b) Matrix-element corrections
Improving event generators

The inner working of event generators
... simulation: divide et impera

- **hard process:**
  - fixed order perturbation theory
    - traditionally: Born-approximation

- **bremsstrahlung:**
  - resummed perturbation theory

- **hadronisation:**
  - phenomenological models

- **hadron decays:**
  - effective theories, data

- "**underlying event**":
  - phenomenological models
... and possible improvements
possible strategies:

- improving the phenomenological models:
  - "tuning" (fitting parameters to data)
  - replacing by better models, based on more physics
    (my hot candidate: "minimum bias" and "underlying event" simulation)

- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:
    - "NLO-Matching" & "Multijet-Merging"
  - systematic improvement of the parton shower:
    next-to leading (or higher) logs & colours
First improvements

Matching

Multijet merging

Improving event generators

- remember structure of NLO calculation for $N$-body production
  \[ d\sigma = d\Phi_B B_N(\Phi_B) + d\Phi_B \nu_N(\Phi_B) + d\Phi_R \mathcal{R}_N(\Phi_R) \]
  \[ = d\Phi_B \left( B_N + \nu_N + \mathcal{I}_N^{(S)} \right) + d\Phi_R \left( \mathcal{R}_N - S_N \right) \]

- phase space factorisation assumed here ($\Phi_R = \Phi_B \otimes \Phi_1$)
  \[ \int d\Phi_1 S_N(\Phi_B \otimes \Phi_1) = \mathcal{I}_N^{(S)}(\Phi_B) \]

- process independent subtraction kernels
  \[ S_N(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes S_1(\Phi_B \otimes \Phi_1) \]
  \[ \mathcal{I}_N^{(S)}(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes \mathcal{I}_1^{(S)}(\Phi_B) \]

  with universal $S_1(\Phi_B \otimes \Phi_1)$ and $\mathcal{I}_1^{(S)}(\Phi_B)$

- in Catani–Seymour invertible phase space mapping
  \[ \Phi_R \leftrightarrow \Phi_B \otimes \Phi_1 \]
Matrix element corrections

- Parton shower ignores interferences typically present in matrix elements.

- Pictorially:
  \[
  \begin{align*}
  \text{ME} & : | \begin{array}{c}
  \text{parton graph 1} \\
  \text{parton graph 2}
  \end{array} |^2 + | \begin{array}{c}
  \text{parton graph 3} \\
  \text{parton graph 4}
  \end{array} |^2 \\
  \text{PS} & : | \begin{array}{c}
  \text{parton graph 1} \\
  \text{parton graph 2}
  \end{array} |^2 + | \begin{array}{c}
  \text{parton graph 3} \\
  \text{parton graph 4}
  \end{array} |^2
  \end{align*}
  \]

- Form many processes $\mathcal{R}_N < \mathcal{B}_N \times \mathcal{K}_N$.

- Typical processes: $q\bar{q}' \rightarrow V$, $e^- e^+ \rightarrow q\bar{q}$, $t \rightarrow bW$.

- Practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $P = \mathcal{R}_N / (\mathcal{B}_N \times \mathcal{K}_N)$.
analyse **first** emission, given by

\[ d\sigma_B = d\Phi_N \mathcal{B}_N(\Phi_N) \]

\[
\begin{align*}
\mathcal{N}_N(\mathcal{R}/\mathcal{B})(\mu^2_N, t) + & \int_{t_0}^{\mu^2_N} d\Phi_1 \left[ \frac{\mathcal{R}_N(\Phi_N \times \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N(\mathcal{R}/\mathcal{B})(\mu^2_N, t(\Phi_1)) \right] \\
\end{align*}
\]

once more: integrates to unity → “unitarity” of parton shower

radiation given by \( \mathcal{R}_N \) (correct at \( \mathcal{O}(\alpha_s) \))

(but modified by logs of higher order in \( \alpha_s \) from \( \Delta_N(\mathcal{R}/\mathcal{B}) \))

emission phase space constrained by \( \mu_N \)

also known as “soft ME correction”

hard ME correction fills missing phase space

used for “power shower”:

\( \mu_N \rightarrow E_{pp} \) and apply ME correction
PRECISION MONTE CARLO

NLO MATCHING
Contents

6.a) Basic idea
6.b) Powheg
6.c) MC@NLO
NLO matching: Basic idea

- parton shower resums logarithms
- fair description of collinear/soft emissions
- jet evolution (where the logs are large)

- matrix elements exact at given order
- fair description of hard/large-angle emissions
- jet production (where the logs are small)

- adjust ("match") terms:
  - cross section at NLO accuracy &
    correct hardest emission in PS to exactly
    reproduce ME at order $\alpha_s$
    ($\mathcal{R}$-part of the NLO calculation)
    (this is relatively trivial)

- maintain (N)LL-accuracy of parton shower
  (this is not so simple to see)
PowHeg

- reminder: $K_{ij,k}$ reproduces process-independent behaviour of $R_N/B_N$ in soft/collinear regions of phase space

$$\frac{d\Phi_1}{B_N(\Phi_N)} \xrightarrow{\text{IR}} \frac{d\Phi_1}{2\pi} \frac{\alpha_s}{K_{ij,k}(\Phi_1)}$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta^{(R/B)}_{N}(\mu^2_N, t_0) = \exp \left[ - \int_{t_0}^{\mu^2_N} d\Phi_1 \frac{R_N(\Phi_{N+1})}{B_N(\Phi_N)} \right],$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$

- typically will adjust scale of $\alpha_s$ to parton shower scale
• define local $K$-factors

• start from Born configuration $\Phi_N$ with NLO weight:

\[
d\sigma_N^{(NLO)} = d\Phi_N \tilde{B}(\Phi_N) = d\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N) \otimes S \right\}_{\tilde{\mathcal{V}}_N(\Phi_N)} + \int d\Phi_1 \left[ \mathcal{R}_N(\Phi_N \otimes \Phi_1) - \mathcal{B}_N(\Phi_N) \otimes dS(\Phi_1) \right] \right\}
\]

• by construction: exactly reproduce cross section at NLO accuracy

• note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes dS$ (relevant for MC@NLO)
analyse accuracy of radiation pattern

generate emissions with $\Delta_N^{(R/B)}(\mu^2_N, t_0)$:

$$d\sigma^{(NLO)}_N = d\Phi_N \bar{B}(\Phi_N)$$

$$\times \left\{ \Delta_N^{(R/B)}(\mu^2_N, t_0) + \int_{t_0}^{\mu^2_N} d\Phi_1 \frac{R_N(\Phi_N \otimes \Phi_1)}{B_N(\Phi_N)} \Delta_N^{(R/B)}(\mu^2_N, k^2_{\perp}(\Phi_1)) \right\}$$

integrating to yield $1 - \text{“unitarity of parton shower”}$

radiation pattern like in ME correction

pitfall, again: choice of upper scale $\mu^2_N$

apart from logs: which configurations enhanced by local $K$-factor

$(K$-factor for inclusive production of $X$ adequate for $X+$ jet at large $p_{\perp}$?)
First improvements matching multijet merging

- large enhancement at high $p_{T,h}$
- can be traced back to large NLO correction
- fortunately, NNLO correction is also large $\rightarrow \sim$ agreement
First improvements

Matching

Multijet merging

improving POWHEG

- split real-emission ME as

\[ \mathcal{R} = \mathcal{R} \left( \frac{h^2}{p_2^2 + h^2} + \frac{p_2^2}{p_2^2 + h^2} \right) \]

- can “tune” \( h \) to mimick NNLO - or other (resummation) result

- differential event rate up to first emission

\[
d\sigma = d\Phi_B \bar{B}^{(R(S))} \left[ \Delta^{(R(S)/B)}(s, t_0) + \int_{t_0}^{s} d\Phi_1 \frac{\mathcal{R}^{(S)}}{B} \Delta^{(R(S)/B)}(s, k_2^2) \right] + d\Phi_R \mathcal{R}^{(F)}(\Phi_R)
\]
MC@NLO

- MC@NLO paradigm: divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = B_N \otimes dS_1 + \mathcal{H}_N$$

- identify subtraction terms and shower kernels $dS_1 \equiv \sum_{\{ij,k\}} K_{ij,k}$

(modify $K$ in 1st emission to account for colour)

$$d\sigma_N = d\Phi_N \underbrace{\tilde{B}_N(\Phi_N)}_{B+\tilde{\nu}} \left[ \Delta_N^{(K)}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 K_{ij,k}(\Phi_1) \Delta_N^{(K)}(\mu_N^2, k_\perp^2) \right]$$

$$+ d\Phi_{N+1} \mathcal{H}_N$$

- effect: only resummed parts modified with local $K$-factor
phase space effects: shower vs. fixed order

problem: impact of subtraction terms on local $K$-factor (filling of phase space by parton shower)

studied in case of $gg \rightarrow H$ above

proper filling of available phase space by parton shower paramount
MC@NLO for light jets: jet-$p_\perp$

Inclusive jet transverse momenta in different rapidity ranges

**MC@NLO**

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---

**First improvements**

**Matching**

**Multijet merging**
MC@NLO for light jets: dijet mass

Dijet invariant mass spectra in different rapidity ranges

- ATLAS data
- MC@NLO
  \( \mu_R = \mu_F = \frac{1}{2} H_T \), \( \mu_Q = \frac{1}{2} p_\perp \)
- MC@NLO
  \( \mu_R = \mu_F = \frac{1}{2} H_T^{(y)} \), \( \mu_Q = \frac{1}{2} p_\perp \)

\[ \frac{d^2 \sigma}{d m_{12}^2} \text{[pb/TeV]} \]

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MC@NLO for light jets: azimuthal decorrelations

Dijet azimuthal decorrelation in various \( p_{\text{lead}} \) bins

CMS data

\begin{align*}
\mu_R = \mu_F &= \frac{1}{2} H_T, \quad \mu_Q = \frac{1}{2} p_{\perp} \\
\end{align*}


\[ \frac{1}{\sigma} \frac{d\sigma}{d\Delta \phi} \text{[pb]} \]

MC/data

SHERPA MC@NLO

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MC@NLO for light jets: $R_{32}$ & forward energy flow

3 jets over 2 jets ratio (anti-kt R=0.5)

Forward energy flow in dijet events, $p_{\perp}^{\text{jets}} > 20$ GeV
MC@NLO for light jets: jet vetoes

Inclusive jet transverse momenta in different rapidity ranges

Forward-backward selection

$\mu_F, \mu_R$ variation
$\mu_Q$ variation
MPI variation

$\Delta y$

MC/data
PRECISION MONTE CARLO

MULTIJET MERGING
## Contents

7.a) Basic idea  
7.b) Multijet merging at LO  
7.c) Multijet merging at NLO
Multijet merging: basic idea

- Parton shower resums logarithms
- Fair description of collinear/soft emissions
- Jet evolution
  (where the logs are large)

- Matrix elements exact at given order
- Fair description of hard/large-angle emissions
- Jet production
  (where the logs are small)

- Combine ("merge") both:
  Result: "towers" of MEs with increasing number of jets evolved with PS
- Multijet cross sections at Born accuracy
- Maintain (N)LL accuracy of parton shower

\[ L^m \]

\[ \alpha^n \]

exact ME
LO 4jet

NLL resummed in PS

exact ME
LO 5jet, but also NLO 4jet
• separate regions of jet production and jet evolution with jet measure $Q_J$
  
  ("truncated showering" if not identical with evolution parameter)

• matrix elements populate hard regime

• parton showers populate soft domain
Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \rightarrow$ hadrons
  Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor $\alpha_s$ and up to $\log^2 \frac{E_{c.m.}}{Q_J}$ per jet
- use Sudakov form factor for resummation & replace approximate fixed order by exact expression:

\[
\mathcal{R}_2(Q_J) = \left[ \Delta_q(E_{c.m.}^2, Q_J^2) \right]^2
\]

\[
\mathcal{R}_3(Q_J) = 2\Delta_q(E_{c.m.}^2, Q_J^2) \int_{Q_J^2}^{E_{c.m.}^2} \frac{dk_\perp^2}{k_\perp^2} \left[ \frac{\alpha_s(k_\perp^2)}{2\pi} dz K_q(k_\perp^2, z) \right] \times \Delta_q(E_{c.m.}^2, k_\perp^2) \Delta_q(k_\perp^2, Q_J^2) \Delta_g(k_\perp^2, Q_J^2)
\]
Multijet merging at LO

- expression for first emission

\[
\begin{align*}
\frac{d\sigma}{d\Phi} &= \frac{d\phi}{d\Phi} B_N \left[ \Delta_N^{(K)}(\mu^2_N, t_0) + \int_{t_0}^{\mu^2_N} d\Phi_1 \kappa_N \Delta_N^{(K)}(\mu^2_N, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \\
&\quad + \frac{d\phi}{d\Phi} B_{N+1} \Delta_{N+1}^{(K)}(\mu^2_{N+1}, t_{N+1}) \Theta(Q_{N+1} - Q_J)
\end{align*}
\]

- note: \( N + 1 \)-contribution includes also \( N + 2, N + 3, \ldots \)

  (no Sudakov suppression below \( t_{N+1} \), see further slides for iterated expression)

- potential occurrence of different shower start scales: \( \mu_{N,N+1}, \ldots \)

- “unitarity violation” in square bracket: \( B_N \kappa_N \rightarrow B_{N+1} \)

  (cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &
  
First improvements

Matching

Multijet merging

Multijet merging at LO

\[d\sigma = \sum_{n=N}^{n_{\text{max}}-1} \left\{ d\Phi_n B_n \left[ \prod_{j=N}^{n-1} \Theta(Q_{j+1} - Q_J) \right] \prod_{j=N}^{n-1} \Delta^{(K)}_j(t_j, t_{j+1}) \right\} \]

\[\times \left[ \Delta^{(K)}_n(t_n, t_0) + \int_{t_0}^{t_n} d\Phi_1 K_n \Delta^{(K)}_n(t_n, t_{n+1}) \Theta(Q_J - Q_{n+1}) \right] \]

\[+ d\Phi_{n_{\text{max}}} B_{n_{\text{max}}} \left[ \prod_{j=N}^{n_{\text{max}}-1} \Theta(Q_{j+1} - Q_J) \right] \prod_{j=N}^{n_{\text{max}}-1} \Delta^{(K)}_j(t_j, t_{j+1}) \]

\[\times \left[ \Delta^{(K)}_{n_{\text{max}}}(t_{n_{\text{max}}}, t_0) + \int_{t_0}^{t_{n_{\text{max}}}} d\Phi_1 K_{n_{\text{max}}} \Delta^{(K)}_{n_{\text{max}}}(t_{n_{\text{max}}}, t_{n_{\text{max}}+1}) \right] \]
Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])
Aside: Comparison with higher order calculations
A step towards multijet-merging at NLO: MENLOPs

- combine matching for lowest multiplicity with multijet merging
- interpolating local $K$-factor for reweighting hard emissions

\[ k_N(\Phi_{N+1}) = \frac{\tilde{B}_N}{B_N} \left( 1 - \frac{H_N}{B_{N+1}} \right) + \frac{H_N}{B_{N+1}} \rightarrow \begin{cases} \tilde{B}_N/B_N \quad \text{for soft emission} \\ 1 \quad \text{for hard emission} \end{cases} \]

\[
d\sigma = d\Phi_N \tilde{B}_N \left[ \Delta^{(K)}_N (\mu_N^2, t_0) + \int_{t_0} d\Phi_1 K_N \Delta^{(K)}_N (\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \\
+ d\Phi_{N+1} H_N \Delta^{(K)}_N (\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \\
+ d\Phi_{N+1} k_N B_{N+1} \Delta^{(K)}_N (\mu_N^2, t_{N+1}) \Theta(Q_{N+1} - Q_J) \]
Transverse momentum of $W$ & $Z$ boson

**Z+jets: inclusive quantities**

ATLAS, arXiv:1111.2690
**Z+jets: jet transverse momenta**

**ATLAS, arXiv:1111.2690**

First improvements

Matching

Multijet merging

MELOPS

Z+jets: jet transverse momenta

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**Z+jets: jet transverse momenta**

\[ \sigma(\gamma Z) = \frac{1}{6 \times 10^{-3}} \text{ pb} \]

\[ \sigma(\gamma Z) = \frac{1}{6 \times 10^{-3}} \text{ pb} \]

**Events and Jets**

- ATLAS, ALPGEN, SHERPA, BLACKHAT
- Data 2011 (\(p_T = 7 \text{ TeV}\))
- MC / Data

**Graphs**

- **ATLAS**
- L dt = 4.6 fb\(^{-1}\)
- anti-\(k_T\) jets, \(R = 0.4\)
- \(p_T^{\text{jet}} > 30 \text{ GeV}, |y| < 4.4\)

**Legend**

- ALPGEN
- SHERPA
- BLACKHAT + SHERPA

\[ (1/\sigma_{Z\gamma-n}) \text{ d} \sigma/\text{d}p_T \]

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ATLAS, arXiv:1111.2690
Z+jets: correlation of leading jets

ATLAS, arXiv:1111.2690

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$Z+\text{jets: } \Delta\phi_{Zj} \text{ in unboosted sample}$

CMS, arXiv:1301.1646
$Z + \text{jets: } \Delta \phi_{Zj}$ in boosted sample

CMS, arXiv:1301.1646
Multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting
  maintain NLO and (N)LL accuracy of ME and PS
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities
First emission(s), once more

\[ d\sigma = \Phi_N \tilde{B}_N \left[ \Delta_N^{(K)}(\mu_N^2, t_0) + \int_{t_0}^{\mu^2_N} d\Phi_1 \, \mathcal{K}_N \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \]

\[ + d\Phi_{N+1} \mathcal{H}_N \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \]

\[ + d\Phi_{N+1} \tilde{B}_{N+1} \left( 1 + \frac{\mathcal{B}_{N+1}}{\tilde{B}_{N+1}} \int_{t_{N+1}}^{\mu^2_N} d\Phi_1 \mathcal{K}_N \right) \Theta(Q_{N+1} - Q_J) \]

\[ \cdot \left[ \Delta_{N+1}^{(K)}(t_{N+1}, t_0) + \int_{t_0}^{t_{N+1}} d\Phi_1 \mathcal{K}_{N+1} \Delta_{N+1}^{(K)}(t_{N+1}, t_{N+2}) \right] \]

\[ + d\Phi_{N+2} \mathcal{H}_{N+1} \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Delta_{N+1}^{(K)}(t_{N+1}, t_{N+2}) \Theta(Q_{N+1} - Q_J) + \ldots \]
First improvements

Matching

Multijet merging

Multijet merging at NLO

\[ p_H \] in MEPS@NLO

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp_{\perp}} \ [\text{pb/GeV}]
\]

\[ pp \rightarrow h + \text{jets} \]

SHERPA S-MC@NLO

first emission by Mc@NLO
First improvements

Matching

Multijet merging

Multijet merging at NLO

\[ p^H_\perp \text{ in MEPS@NLO} \]

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp_\perp} \text{ [pb/GeV]} \]

\[
pp \rightarrow h + \text{jets} \]

\[ pp \rightarrow h + 0j \text{ @ NLO} \]

- first emission by MC@NLO, restrict to \( Q_{n+1} < Q_{\text{cut}} \)
First improvements

Matching

Multijet merging

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

\[
d\sigma/dp(H)^\perp \text{ [pb/GeV]} \\
pp \rightarrow h + \text{jets} \\
- pp \rightarrow h + 0j @ NLO \\
- pp \rightarrow h + 1j @ NLO
\]

- first emission by Mc@NLO, restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- Mc@NLO \( pp \rightarrow h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
First improvements

Matching

Multijet merging

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

\( \frac{d\sigma}{dp^\perp} \) [pb/GeV]

\( pp \rightarrow h + \text{jets} \)

\( pp \rightarrow h + 0\text{j} @ \text{NLO} \)

\( pp \rightarrow h + 1\text{j} @ \text{NLO} \)

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \rightarrow h + \text{jet} \) to \( Q_{n+2} < Q_{\text{cut}} \)
First improvements

Matching

Multijet merging

Multijet merging at NLO

\[ p_H^\perp \text{ in MEPS@NLO} \]

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp_H^\perp} \text{ [pb/GeV]}
\]

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \to h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \to h + \text{jet} \) to \( Q_{n+2} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \to h + 2\text{jets} \) for \( Q_{n+2} > Q_{\text{cut}} \)

\[ pp \to h + \text{jets} \]
- \( pp \to h + 0\text{j} @ \text{NLO} \)
- \( pp \to h + 1\text{j} @ \text{NLO} \)
- \( pp \to h + 2\text{j} @ \text{NLO} \)
First improvements

Matching

Multijet merging

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp_\perp} \text{[pb/GeV]}
\]

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \rightarrow h + \text{jet} \) to \( Q_{n+2} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \)
  \( pp \rightarrow h + 2\text{jets} \) for \( Q_{n+2} > Q_{\text{cut}} \)
- iterate

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QCD & Monte Carlo Event Generators
First improvements

Matching

Multijet merging

Multijet merging at NLO

\( p_\perp^H \) in MEPS@NLO

Transverse momentum of the Higgs boson

- **pp** \( \rightarrow h + \text{jets} \)
- \( pp \rightarrow h + 0j @ \text{NLO} \)
- \( pp \rightarrow h + 1j @ \text{NLO} \)
- \( pp \rightarrow h + 2j @ \text{NLO} \)
- \( pp \rightarrow h + 3j @ \text{LO} \)

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \rightarrow h + \text{jet to} \) \( Q_{n+2} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + 2\text{jets for} \) \( Q_{n+2} > Q_{\text{cut}} \)
- iterate

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First improvements

Matching

Multijet merging

Multijet merging at NLO

$p_H^\perp$ in MEPS@NLO

Transverse momentum of the Higgs boson

$d\sigma/dp_\perp$ [pb/GeV]

pp → h + jets
pp → h + 0j @ NLO
pp → h + 1j @ NLO
pp → h + 2j @ NLO
pp → h + 3j @ LO

- first emission by MC@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \rightarrow h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

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First improvements

Matching

Multijet merging at NLO

\( p_{\perp}^H \) in MEPS@NLO

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp_{\perp}} \text{[pb/GeV]}
\]

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + \text{jet} \)
  for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \rightarrow h + \text{jet} \) to
  \( Q_{n+2} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \)
  \( pp \rightarrow h + 2\text{jets} \) for
  \( Q_{n+2} > Q_{\text{cut}} \)
- iterate
- sum all contributions
- eg. \( p_{\perp}(h) > 200 \text{ GeV} \)
  has contributions fr. multiple topologies

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**MEPS@NLO**: example results for $e^-e^+ \rightarrow$ hadrons

---

**Durham jet resolution 3 → 2 ($E_{CMS} = 91.2$ GeV)**

- ALEPH data
- MePS@NLO
- MePS@NLO $\mu/2$ → $2\mu$
- ME@LOPS
- ME@LOPS $\mu/2$ → $2\mu$
- Mc@NLO

**Durham jet resolution 4 → 3 ($E_{CMS} = 91.2$ GeV)**

- ALEPH data
- MePS@NLO
- MePS@NLO $\mu/2$ → $2\mu$
- ME@LOPS
- ME@LOPS $\mu/2$ → $2\mu$
- Mc@NLO

**Durham jet resolution 5 → 4 ($E_{CMS} = 91.2$ GeV)**

- ALEPH data
- MePS@NLO
- MePS@NLO $\mu/2$ → $2\mu$
- ME@LOPS
- ME@LOPS $\mu/2$ → $2\mu$
- Mc@NLO

**Durham jet resolution 6 → 5 ($E_{CMS} = 91.2$ GeV)**

- ALEPH data
- MePS@NLO
- MePS@NLO $\mu/2$ → $2\mu$
- ME@LOPS
- ME@LOPS $\mu/2$ → $2\mu$
- Mc@NLO
MEPS@NLO: example results for $e^-e^+ \rightarrow$ hadrons
Example: MEPS@NLO for $W + \text{jets}$

(up to two jets @ NLO, from BlackHat, see arXiv: 1207.5031 [hep-ex])
First improvements

$\sigma_p^d\sigma_p/\text{pb/GeV}_{\perp}\text{per jet}$

MC/data

$p\perp$ [GeV]

First Jet

Second Jet

Third Jet

$W^+ \geq 1\text{ jet} (\times 1)$

$W^+ \geq 2\text{ jets} (\times 0.1)$

$W^+ \geq 3\text{ jets} (\times 0.01)$

$40 60 80 100 120 140$

$0.2 0.4 0.6 0.8 1$

$p\perp$ [GeV]

$\geq 2\text{ jets}$

$\geq 3\text{ jets}$

2 jets

3 jets

$\times 0.1$

$\times 0.01$

$\times 0.001$

$\times 1$

$\times 10$

$\times 100$

$\times 1000$
Results for Higgs boson production through gluon fusion

- parton-shower level, Higgs boson does not decay

- setup & cuts:
  - jets: anti-kt, $p_\perp \geq 20$ GeV, $R = 0.4$, $|\eta| \leq 4.5$
  - dijet cuts: at least 2 jets with $p_\perp \geq 25$ GeV
  - WBF cuts: $m_{jj} \geq 400$ GeV, $\Delta y_{jj} \geq 2.8$

- jet multiplicity plots:
  - 0-jet excl.: no jet with $p_\perp \geq \{20, 25, 30\}$ GeV
  - 2-jet incl.: at least two jets with $p_\perp \geq \{20, 25, 30\}$ GeV

- SHERPA with $H + \{0, 1, 2\}^{(NLO)} + \{3\}^{(LO)}$ jets, $Q_{cut} = 20$ GeV
Inclusive observables for $gg \rightarrow H$
Exclusive observables for $gg \rightarrow H$

- Higgs boson transverse momentum ($n_{\mu} = 0$)
- Transverse momentum of the $H_j^1$ system

Graphs showing the ratio of cross sections at different scales $\mu_R = \mu_{\text{CKKW}}, \mu_R = m_h, \mu_R = \hat{H}_T$.
$gg \rightarrow H$ after WBF cuts
$gg \rightarrow H$ after WBF cuts

Azimuthal separation of the two leading jets

Azimuthal separation of the Higgs and the two leading jets

Ratio to $\mu_R = m_h$

VBF cuts

Sherpa MePs@Nlo

$\mu_R = \mu_{CKKW}$

$\mu_R = m_h$

$\mu_R = \hat{H}'$

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Multijet merging at NLO

Quark mass effects

- include effects of quark masses

\[
d\sigma^{(NLO)}_{\text{mass}} \approx d\sigma^{(NLO)}_{\text{HEFT}} \times \frac{d\sigma^{(LO)}_{\text{mass}}}{d\sigma^{(LO)}_{\text{HEFT}}}
\]
Quark mass effects – results

- top mass effect in MEPS@NLO (on Higgs–$p_\perp$)

- comparison S-MC@NLO– HRES (top–loop only)
$b$–mass effects

- $b$–mass effects more tricky
- relevant only for (negative) interference of top– and bottom–loops
  (bottom$^2$ double Yukawa - supressed)
- but: cannot start shower at $m_H$
  radiation “sees” bottom at all scales above $m_b$
  $\implies$ must use full theory there
- $p_T$ spectrum naively “squeezed” – funny shapes
- LO multijet merging improves situation
Higgs backgrounds: inclusive observables in $W^+W^-+jets$
Higgs backgrounds: jet vetoes in $W^+W^-+\text{jets}$
Higgs backgrounds: gluon-induced processes $W^+W^-+jets$

- include (LO-) merged loop$^2$ contributions of $gg \to VV (+1 \text{ jet})$
Higgs backgrounds: jet vetoes in $W^+W^- +$jets

Transverse momentum of leading jet

$\frac{d\sigma}{dp_T}$ [pb/GeV]

Ratio

$\sigma$ [pb/GeV]

$p_T$ [GeV]

Sherpa+OpenLoops

MEPS@LOOP$^2$ $4\ell + 0,1j$

$4\ell + 0j$

$4\ell + 1j$

LOOP$^2$+PS $4\ell$

$pp \rightarrow 4\ell + 0,1j$

$gg \rightarrow 4\ell + 0,1g$

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Relevant observables for $VH \rightarrow 3\ell$: $E_T$
Relevant observables for $VH \rightarrow 3\ell$: $m_{123}$ & $\Delta R_{01}$
## Differences between MEPS@NLO, UNLOPS & FxFx

<table>
<thead>
<tr>
<th></th>
<th>FxFx</th>
<th>MEPS@NLO</th>
<th>UNLOPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ME</strong></td>
<td>all internal</td>
<td>$\forall$ external</td>
<td>all external</td>
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<tr>
<td></td>
<td></td>
<td>$\forall$ from OPENLOOPS, MJet, ...</td>
<td></td>
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<td></td>
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<td>Comix or AMEGIC++</td>
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<tr>
<td><strong>shower</strong></td>
<td>external</td>
<td>intrinsic</td>
<td>intrinsic</td>
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<td></td>
<td>HERWIG or PYTHIA</td>
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<td>PYTHIA</td>
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<tr>
<td><strong>$\Delta_N$</strong></td>
<td>analytical</td>
<td>from PS</td>
<td>from PS</td>
</tr>
<tr>
<td><strong>$\Theta(Q_J)$</strong></td>
<td>a-posteriori</td>
<td>per emission</td>
<td>per emission</td>
</tr>
<tr>
<td><strong>$Q_J$-range</strong></td>
<td>relatively high</td>
<td>$&gt; \text{Sudakov regime}$</td>
<td>$\approx \text{Sudakov regime}$</td>
</tr>
<tr>
<td></td>
<td>(but changed)</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>$\approx 10%$</td>
<td>$\approx 10%$</td>
</tr>
</tbody>
</table>
**FxFx: validation in $Z$+jets**

(Data from ATLAS, 1304.7098, with HERWIG++)

(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)
FxFx: validation in $Z+\text{jets}$

(Data from ATLAS, 1304.7098, with HERWIG++)

(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)
FxFx: $Q_J$ dependence in $t\bar{t}$

Aside: merging without $Q_J$ - the MINLO approach


- based on POWHEG + shower from PYTHIA or HERWIG
- up to today only for singlet $S$ production, gives NNLO + PS

basic idea:
- use $S$+jet in POWHEG
- push jet cut to parton shower IR cutoff
- apply analytical NNLL Sudakov rejection weight for intrinsic line in Born configuration
  (kills divergent behaviour at order $\alpha_S$)
- don’t forget double-counted terms
- reweight to NNLO fixed order
for $H$ production

ROUND IV: SIMULATING SOFT QCD
SIMULATING SOFT QCD

HADRONISATION
Contents

8.a) QCD radiation, once more
8.b) Hadronisation: General thoughts
8.c) The string model
8.d) The cluster model
8.e) Practicalities
QCD radiation, once more

- remember QCD emission pattern

\[
dW^{q\to qg} = \frac{\alpha_s(k_{\perp}^2)}{2\pi} C_F \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{d\omega}{\omega} \left[ 1 + \left(1 - \frac{\omega}{E}\right) \right].
\]

- spectrum cut-off at small transverse momenta and energies by onset of hadronization, at scales \( R \approx 1 \text{ fm}/\Lambda_{\text{QCD}} \)

- two (extreme) classes of emissions: gluons and gluers determined by relation of formation and hadronization times
**gluers formed at times** $R$, with momenta $k_{\parallel} \sim k_{\perp} \sim \omega \gtrsim 1/R$

**assuming that hadrons follow partons,**

\[
dN_{(\text{hadrons})} \sim \int_{k_{\perp}>1/R}^{Q} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{C_F \alpha_s(k_{\perp}^2)}{2\pi} \left[ 1 + \left( 1 - \frac{\omega}{E} \right) \right] \frac{d\omega}{\omega}
\]

\[
\sim \frac{C_F \alpha_s(1/R^2)}{\pi} \log(Q^2R^2) \frac{d\omega}{\omega}
\]

or - relating their energy with that of the gluers -

\[
dN_{(\text{hadrons})}/d\log \epsilon = \text{const.},
\]

a plateau in log of energy (or in rapidity)
impact of additional radiation

new partons must separate before they can hadronize independently

therefore, one more time

\[ t_{\text{form}} \sim \frac{k_{\parallel}}{k_{\perp}^2} \]

\[ t_{\text{sep}} \sim R\theta \sim t_{\text{form}} (Rk_{\perp}) \]

\[ t_{\text{had}} \sim k_{\parallel} R^2 \sim t_{\text{form}} (Rk_{\perp})^2. \]

for gluers \( Rk_{\perp} \approx 1 \): all times the same

naively; new & more hadrons following new partons

but: colour coherence

primary and secondary partons not separated enough in

\[ \frac{1}{R} \lesssim \omega_{(\text{hadron})} \lesssim \frac{1}{(R\theta)} \]

and therefore no independent radiation
Hadronisation: General thoughts

- confinement: the striking feature of low-scale strong interactions
- transition from partons to their bound states, the hadrons
- the Meissner effect in QCD

QED:

QCD:
linear QCD potential in Quarkonia – like a string
combine some experimental facts into a naive parameterisation

in $e^+e^- \rightarrow$ hadrons: exponentially decreasing $p_\perp$, flat plateau in $y$ for hadrons

- try “smearing”: $\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$
use parameterisation to “guesstimate” hadronisation effects:

\[
E = \int_0^\gamma dy dp_\perp^2 \rho(p_\perp^2) p_\perp \cosh y = \lambda \sinh \gamma
\]

\[
P = \int_0^\gamma dy dp_\perp^2 \rho(p_\perp^2) p_\perp \sinh y = \lambda (\cosh \gamma - 1) \approx E - \lambda
\]

\[
\lambda = \int dp_\perp^2 \rho(p_\perp^2) p_\perp = \langle p_\perp \rangle.
\]

estimate \( \lambda \sim 1/R_{\text{had}} \approx m_{\text{had}} \), with \( m_{\text{had}} \) 0.1-1 GeV.

effect: jet acquire non-perturbative mass \( \sim 2\lambda E \) 
\((O(10\text{GeV}) \text{ for jets with energy } O(100\text{GeV})).\)
similar parametrization underlying Feynman-Field model for independent fragmentation

recursively fragment \( q \rightarrow q' + \text{had} \), where

- transverse momentum from (fitted) Gaussian;
- longitudinal momentum arbitrary (hence from measurements);
- flavour from symmetry arguments + measurements.

problems: frame dependent, “last quark”, infrared safety, no direct link to perturbation theory, . . .
The string model

- a simple model of mesons: yoyo strings
  - light quarks \( (m_q = 0) \) connected by string, form a meson
  - area law: \( m_{\text{had}}^2 \propto \text{area of string motion} \)
  - \( L=0 \) mesons only have 'yo-yo' modes:
• turn this into hadronisation model $e^+ e^- \rightarrow q\bar{q}$ as test case

• ignore gluon radiation: $q\bar{q}$ move away from each other, act as point-like source of string

• intense chromomagnetic field within string: more $q\bar{q}$ pairs created by tunnelling and string break-up

• analogy with QED (Schwinger mechanism):
  \[ d\mathcal{P} \sim dxdt \exp\left(-\pi m_q^2/\kappa\right), \quad \kappa = \text{"string tension"}. \]
• string model = well motivated model, constraints on fragmentation (Lorentz-invariance, left-right symmetry, \ldots)

• how to deal with gluons?

• interpret them as kinks on the string \implies the string effect

• infrared-safe, advantage: smooth matching with PS.
The cluster model

- underlying idea: preconfinement/LPHD
  - typically, neighbouring colours will end in same hadron
  - hadron flows follow parton flows $\rightarrow$ don’t produce any hadrons at places where you don’t have partons
  - works well in large–$N_c$ limit with planar graphs

- follow evolution of colour in parton showers
- paradigm of cluster model: clusters as continuum of hadron resonances

- trace colour through shower in $N_c \to \infty$ limit

- force decay of gluons into $q\bar{q}$ or $\bar{d}d$ pairs, form colour singlets from neighbouring colours, usually close in phase space

- mass of singlets: peaked at low scales $\approx Q^2_0$

- decay heavy clusters into lighter ones or into hadrons (here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like)

- if light enough, clusters will decay into hadrons

- naively: spin information washed out, decay determined through phase space only $\rightarrow$ heavy hadrons suppressed (baryon/strangeness suppression)
The cluster model

- self–similarity of parton shower will end with roughly the same local distribution of partons, with roughly the same invariant mass for colour singlets
- adjacent pairs colour connected, form colourless (white) clusters.
- clusters (≈ excited hadrons) decay into hadrons
Observables

- in the following a selection of data from the LEP collaboration relevant for the tuning of hadronisation models
- all compared with an actual tune of SHERPA
- typically, PYTHIA does as good (or sometimes even slightly better)
- so, this is the level we talk about these days, agreement of 5% or better over large ranges of observables and scales
Hadronisation

Observables

1. Thrust, $1 - T$, at 91 GeV

$$\frac{1}{\sigma} \frac{d\sigma}{d(1 - T)}$$

2. Thrust major, $T_{maj}$, at 91 GeV

$$\frac{1}{\sigma} \frac{d\sigma}{dT_{maj}}$$

3. Thrust minor, $T_{min}$, at 91 GeV

$$\frac{1}{\sigma} \frac{d\sigma}{dT_{min}}$$

4. Oblateness, $O$, at 91 GeV

$$\frac{1}{\sigma} \frac{d\sigma}{dO}$$

Data Analysis LEP 91.2.5

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QCD & Monte Carlo Event Generators
Integrated 2-jet rate with Durham algorithm (91.2 GeV)

Integrated 3-jet rate with Durham algorithm (91.2 GeV)

Integrated 4-jet rate with Durham algorithm (91.2 GeV)

Integrated 5-jet rate with Durham algorithm (91.2 GeV)
Hadronisation

Durham jet resolution $2 \rightarrow 1 (E_{\text{CMS}} = 91.2 \text{ GeV})$

Durham jet resolution $3 \rightarrow 2 (E_{\text{CMS}} = 91.2 \text{ GeV})$

Durham jet resolution $4 \rightarrow 3 (E_{\text{CMS}} = 91.2 \text{ GeV})$

Durham jet resolution $5 \rightarrow 4 (E_{\text{CMS}} = 91.2 \text{ GeV})$
Charged multiplicity at a function of energy

$N_{ch}$

80 100 120 140 160 180 200

0.6 0.8 1 1.2 1.4

$E_{CMS}/GeV$

MC/Data

Charged multiplicity distribution

$1/NdN/dN_{ch}$

10 20 30 40 50

0.6 0.8 1 1.2 1.4

$N_{ch}$

MC/Data

All events scaled momentum

$1/\sigma d\sigma/dx_p$

0 0.2 0.4 0.6 0.8 1

0.6 0.8 1 1.2 1.4

$x_p$

MC/Data

Log of scaled momentum, $\log(1/x_p)$

$N d\sigma/d\log(1/x_p)$

0 1 2 3 4 5

0.6 0.8 1 1.2 1.4

$\log(1/x_p)$

MC/Data
Hadronisation

Observables

\[ \pi^+ \text{ scaled momentum} \]

\[ \rho \text{ spectrum} \]

\[ \Sigma^\pm (1385) \text{ scaled momentum} \]

\[ \frac{1}{\sigma} \frac{d\sigma}{dx_p} \]

Data Analysis

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QCD & Monte Carlo Event Generators

LEP 91.2.5
Hadronisation

Underlying Event

Observables

Scaled energy of $D^{±}$ in $e^+e^-\to Z\to\text{hadronic}$ at $\sqrt{s}=91.2\text{ GeV}$

$1/N\frac{dN}{dX_{E}}$

$0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6\ 0.7\ 0.8\ 0.9\ 1.0$

MC/Data

$I/\psi$ scaled momentum

$1/\sigma \frac{d\sigma}{dx_{B}}$

$0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6\ 0.7\ 0.8\ 0.9\ 1.0$

MC/Data

$b$ quark fragmentation function $f(x_{\eta^{weak}})$

$1/N\frac{dN}{dx_{B}}$

$0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6\ 0.7\ 0.8\ 0.9\ 1.0$

MC/Data

$1/N\frac{dN}{dx_{B}}$

$0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6\ 0.7\ 0.8\ 0.9\ 1.0$

MC/Data
practicalities of hadronisation models: parameters

- kinematics of string or cluster decay: 2-5 parameters
- must “pop” quark or diquark flavours in string or cluster decay — cannot be completely democratic or driven by masses alone → suppression factors for strangeness, diquarks 2-10 parameters
- transition to hadrons, cannot be democratic over multiplets → adjustment factors for vectors/tensors etc. 2-6 parameters

- tuned to LEP data, overall agreement satisfying
- validity for hadron data not quite clear (beam remnant fragmentation not in LEP.)
- there are some issues with inclusive strangeness/baryon production
SIMULATING SOFT QCD

UNDERLYING EVENT
Contents

8.a) Multiple parton scattering

8.b) Modelling the underlying event

8.c) Some results

8.d) Practicalities
Multiple parton scattering

- hadrons = extended objects!
- no guarantee for one scattering only.
- running of $\alpha_S$ 
  $\Rightarrow$ preference for soft scattering.
first experimental evidence for double–parton scattering: events with $\gamma + 3$ jets:
- cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| < 1.3$;
- “clean sample”: two softest jets with $E_T < 7$ GeV;
- cross section for DPS

$$\sigma_{\text{DPS}} = \frac{\sigma_{\gamma j} \sigma_{jj}}{\sigma_{\text{eff}}}$$

$\sigma_{\text{eff}} \approx 14 \pm 4$ mb.
• more measurements, also at LHC
• ATLAS results from $W + 2$ jets

More details can be found in the plots.
but: how to define the underlying event?

1. everything apart from the hard interaction, but including IS showers, FS showers, remnant hadronisation.

2. remnant-remnant interactions, soft and/or hard.

3. lesson: hard to define
• origin of MPS: parton–parton scattering cross section exceeds hadron–hadron total cross section

\[ \sigma_{\text{hard}}(p_{\perp,\text{min}}) = \int_{p_{\perp,\text{min}}^2}^{s/4} dp_{\perp}^2 \frac{d\sigma(p_{\perp}^2)}{dp_{\perp}^2} > \sigma_{pp,\text{total}} \]

for low \( p_{\perp,\text{min}} \)

• remember

\[ \frac{d\sigma(p_{\perp}^2)}{dp_{\perp}^2} = \int_0^1 dx_1 dx_2 f(x_1, q^2) f(x_2, q^2) \frac{d\hat{\sigma}_{2\rightarrow 2}}{dp_{\perp}^2} \]

• \( \langle \sigma_{\text{hard}}(p_{\perp,\text{min}})/\sigma_{pp,\text{total}} \rangle \geq 1 \)

• depends strongly on cut-off \( p_{\perp,\text{min}} \) (energy-dependent)!
Modelling the underlying event

- take the old PYTHIA model as example:
  - start with hard interaction, at scale $Q_{\text{hard}}^2$.
  - select a new scale $p_{\perp}^2$ from

$$\exp \left[ -\frac{1}{\sigma_{\text{norm}}} \int_{p_{\perp}^2}^{Q_{\text{hard}}^2} dp'_{\perp}^2 \frac{d\sigma(p_{\perp}^2)}{dp'_{\perp}^2} \right]$$

with constraint $p_{\perp}^2 > p_{\perp,\text{min}}^2$

- rescale proton momentum ("proton-parton = proton with reduced energy").
- repeat until no more allowed $2 \rightarrow 2$ scatter
Modelling the underlying event

- possible refinements:
  - may add impact-parameter dependence $\rightarrow$ more fluctuations
  - add parton showers to UE
  - “regularisation” to dampen sharp dependence on $p_{\perp,\text{min}}$: replace $1/\hat{t}$ in MEs by $1/(t + t_0)$, also in $\alpha_s$.
  - treat intrinsic $k_{\perp}$ of partons ($\rightarrow$ parameter)
  - model proton remnants ($\rightarrow$ parameter)
Some results for MPS in Z production

- observables sensitive to MPS
- classical analysis: transverse regions in QCD/jet events
- idea: find the hardest system, orient event into regions:
  - toward region along system
  - away region back-to-back
  - transverse regions
- typically each in 120°
Hadronisation

Underlying Event Summary

Some results in $Z$ production

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QCD & Monte Carlo Event Generators
Some results in $Z$ production

**ATLAS** 1s = 7 TeV, 4.6 fb$^{-1}$

**Toward region**

- $N_{ch}$ vs. $p_{T}$ [GeV]
- MC/Data vs. $p_{T}$ [GeV]

**Transverse region**

- $N_{ch}$ vs. $p_{T}$ [GeV]
- MC/Data vs. $p_{T}$ [GeV]

**Trans-max region**

- $N_{ch}$ vs. $p_{T}$ [GeV]
- MC/Data vs. $p_{T}$ [GeV]

**Trans-min region**

- $N_{ch}$ vs. $p_{T}$ [GeV]
- MC/Data vs. $p_{T}$ [GeV]

**ATLAS** 1s = 7 TeV, 4.6 fb$^{-1}$

- Data
- Pythia8 AU2
- Pythia8+Pythia8 AU2
- Powheg+Pythia8 AU2
- Herwig++ UE-EE-3
- Alpgen+Herwig++ Jimmy AUE2
- Sherpa
- Pythia8 + Powheg2011C
- Herwig++ UE-EE-3
Some results in $Z$ production
• see some data comparison in Minimum Bias

• practicalities of underlying event models: parameters

  • profile in impact parameter space 2-3 parameters
  • IR cut-off at reference energy, its energy evolution, dampening parameter and normalisation cross section 4 parameters
  • treating colour connections to rest of event 2-5 parameters

• tuned to LHC data, overall agreement satisfying

• energy extrapolation not exactly perfect, plus other process categories such as diffraction etc..
SUMMARY
Summary

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging (“CKKW”, “MLM”)
  - NLO matching (“MC@NLO”, “POWHEG”)
  - MeNLOps NLO matching & merging
  - MePS@NLO (“SHERPA”, “UNLOPS”, “MINLO”, “FxFx”)

- multijet merging an important tool for many relevant signals and backgrounds - pioneering phase at LO & NLO over complete automation of NLO calculations done
  → must benefit from it!

(it’s the precision and trustworthy & systematic uncertainty estimates!)

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QCD & Monte Carlo Event Generators
Famous last screams

- in Run-II we’ll be in for a ride:
  - more statistics
  - more energy
  - more channels
  - more precision
  - more fun

- ... and all with QCD ...

oh, and btw.: the first NNLO+PS are out!