

Phenomenology at collider experiments [Part 1: QCD]

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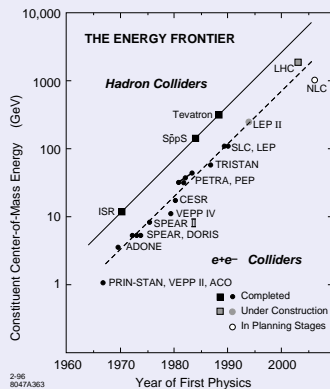
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Outline

- 1 **Introductory remarks**
 - Status of particle physics
 - Design considerations for LHC
- 2 **Cross section calculations at hadron colliders**
 - Matrix elements at leading and next-to leading order
 - PDFs and factorisation
- 3 **QCD radiation**
 - Pattern of QCD radiation: Infrared region rules
 - Parton showers: Simulating QCD radiation
- 4 **Hard QCD processes: Jets**
 - Basic considerations: Definitions and IR safety
 - Modern jet definitions
- 5 **Summary**

View of the 1990's ...



Phenomenology at colliders

The past up to LEP and Tevatron

- 1950's: The particle zoo
Discovery of hadrons, but no order criterion
- 1960's: Strong interactions before QCD
Symmetry: Chaos to order
- 1970's: The making of the Standard Model:
Gauge symmetries, renormalisability, asymptotic freedom
Also: November revolution and third generation
- 1980's: Finding the gauge bosons
Non-Abelian gauge theories are real!
- 1990's: The triumph of the Standard Model at LEP and Tevatron
Precision tests for precision physics

The present: LHC

- Historical trend: **Hadron colliders** for **discovery physics**
Lepton colliders for **precision physics**.
- Historical trend: **Shape your searches** - know what you're looking for.
This has never been truer.
- In last decades: Theory triggers, experiment executes.
Also true for the LHC?!

Setting the scene

Reminder: The Standard Model

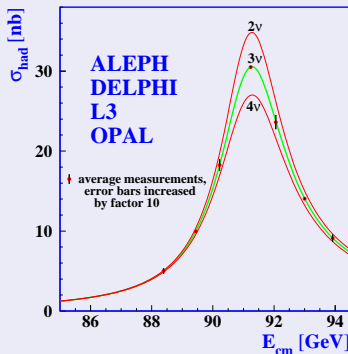
- 3 generations of matter fields:
left-handed doublets, right-handed singlets

Quarks			Leptons		
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$
u_R d_R	c_R s_R	t_R b_R	e_R	μ_R	τ_R

- (Broken) gauge group: $SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$:
8 gluons, 3 (massive) weak gauge bosons, 1 photon
- Electroweak symmetry breaking (EWSB) by introducing a complex scalar doublet (Higgs doublet) with a vacuum expectation value (vev) \Rightarrow 1 physical Higgs scalar

How we know what we know (examples)

Generations



EW precision data



(from LEP EWWG public page, winter 2009)

plot)

Open questions (private preference)

- True mechanism of EWSB: Higgs mechanism in its minimal or an extended version or something different?
- Generations: Three or more?
- More symmetry: Is there low-scale Supersymmetry?
- Space-time: How many dimensions? Four or more?
- Cosmology: Any candidates for dark matter?

LHC - The energy frontier

Design defines difficulty

- Design paradigm for LHC:
 - ① Build a hadron collider
 - ② Build it in the existing LEP tunnel
 - ③ Build it as competitor to the 40 TeV SSC
- Consequence:
 - ① LHC is a pp collider
 - ② LHC operates at 10-14 TeV c.m.-energy
 - ③ LHC is a high-luminosity collider: $100 \text{ fb}^{-1}/\text{y}$

Trade energy vs. lumi, thus pp
- Physics:
 - ① Check the EWSB scenario & search for more
 - ② Fight with overwhelming backgrounds, QCD always a stake-holder
 - ③ Consider niceties such as pile-up, underlying event etc..

Some example cross sections

Or: Yesterdays signals = todays backgrounds

Process	Evts/sec.
Jet, $E_{\perp} > 100$ GeV	10^3
Jet, $E_{\perp} > 1$ TeV	$1.5 \cdot 10^{-2}$
$b\bar{b}$	$5 \cdot 10^5$
$t\bar{t}$	1
$Z \rightarrow \ell\ell$	2
$W \rightarrow \ell\nu$	20
$WW \rightarrow \ell\nu\ell\nu$	$6 \cdot 10^{-3}$

Rates at "low" luminosity, $\mathcal{L} = 10^{33}/\text{cm}^2\text{s} = 10^{-1}\text{fb}^{-1}/\text{y}$, and $s = 14$ TeV.

Cross sections at hadron colliders

Master formula

Production cross section for final state Φ in AB collisions:

$$\sigma_{AB \rightarrow \Phi + X} = \sum_{ab} \int_0^1 dx_1 dx_2 f_{a/A}(x_1, \mu_F^2) f_{b/B}(x_2, \mu_F^2) \hat{\sigma}_{ab \rightarrow \Phi}(\hat{s}, \mu_F^2, \mu_R^2)$$

where

- $x_{1,2}$ are momentum fractions w.r.t. the hadron, $\hat{s} = x_1 x_2 s$;
- $\hat{\sigma}_{ab \rightarrow \Phi}(\hat{s}, \mu_F^2, \mu_R^2)$ is the parton-level cross section,
- and where $f_{a/A}(x, Q^2)$ is the parton distribution function (**PDF**).

Tree-level matrix elements

Simple scattering cross sections

- Detailed look into master formula above:
Convolution of **parton-level cross section** $\hat{\sigma}$ with **PDFs**.
- Must evaluate $\hat{\sigma}$ as **phase-space integral**, respecting **four-momentum conservation** of **amplitude squared**:

$$d\hat{\sigma}_{ab \rightarrow \Phi} = \frac{1}{4\sqrt{(p_a p_b)^2 - p_a^2 p_b^2}} |\mathcal{M}_{ab \rightarrow \Phi}(p_a, p_b, p_1, \dots, p_N)|^2$$

$$\prod_{i=1}^{N_\Phi} \left[\frac{d^4 p_i}{(2\pi)^4} (2\pi) \delta(p_i^2 - m_i^2) \theta(E_i) \right] (2\pi)^4 \delta^4(p_a + p_b - \sum p_i).$$

- Note: Have to normalise on **Lorentz-invariant flux**.
- Smart choices for phase space integration helpful.

Generic Lorentz-invariant quantities

- Use Mandelstam variables (especially for $2 \rightarrow 2$ scatterings):

$$\hat{s} = (p_a + p_b)^2 = (p_1 + p_2)^2$$

$$\hat{t} = (p_a - p_1)^2 = (p_b - p_2)^2$$

$$\hat{u} = (p_a - p_2)^2 = (p_b - p_1)^2.$$

- Relation to masses

$$\hat{s} + \hat{t} + \hat{u} = m_a^2 + m_b^2 + m_1^2 + m_2^2 \xrightarrow{\text{massless}} 0.$$

- In the massless case

$$\frac{d\hat{\sigma}_{ab \rightarrow 12}}{d\hat{t}} = \frac{1}{2\hat{s}} \frac{|\mathcal{M}_{ab \rightarrow 12}|^2}{8\pi\hat{s}}.$$

Kinematics at hadron colliders

- Typically, at hadron colliders: **transverse momentum** p_\perp and **rapidity** y characterise kinematics.
- Note **rapidity** y vs. **pseudorapidity** η (identical for $m = 0$ only):

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \longleftrightarrow \eta = -\ln \tan \frac{\vartheta}{2}.$$

- Rewrite four-momentum ($m_\perp^2 = p_\perp^2 + m^2$)

$$p^\mu = (E, p_x, p_y, p_z) = (m_\perp \cosh y, p_\perp \sin \phi, p_\perp \cos \phi, m_\perp \sinh y).$$

- One-particle phase space element:

$$\frac{d^4 p}{(2\pi)^4} (2\pi) \delta(p^2 - m^2) \theta(E) = \frac{d^3 p}{(2\pi^3) 2E} = \frac{dy}{4\pi} \frac{d^2 p_\perp}{(2\pi)^2}.$$

Resonance production ($2 \rightarrow 1$ processes)

- Assume **massless incoming partons**: $p_{a,b} = x_{1,2}(E, 0, 0, \pm E)$.

(Here, E is beam energy in c.m. system of collider, $s = 4E^2$.)

- Special form of cross section: $\hat{\sigma}_{ab \rightarrow \Phi} = g_\sigma(\hat{s}, m_\Phi^2) \delta(\hat{s} - m_\Phi^2)$.
- Example: $q\bar{q} \rightarrow V$ with vector and axial vector coupling V and A .

(Add normalisation: average over incoming degrees of freedom, include incoming flux.)

$$|\overline{\mathcal{M}}_{q\bar{q} \rightarrow V}|^2 = \frac{1}{3} M_V^2 (V^2 + A^2)$$

$$\hat{\sigma}_{q\bar{q} \rightarrow V} = \frac{\pi}{3} (V^2 + A^2) \delta(\hat{s} - M_V^2).$$

- Trivial relation to partial decay widths of the produced particles:

($|\overline{\mathcal{M}}_{V \rightarrow q\bar{q}}|^2 = 36/3 |\overline{\mathcal{M}}_{q\bar{q} \rightarrow V}|^2$.)

$$d\Gamma = \frac{1}{8\pi M} |\overline{\mathcal{M}}_{V \rightarrow q\bar{q}}|^2.$$

Resonance production (cont'd)

- Then

$$\hat{s} = x_1 x_2 s \quad \text{and} \quad \hat{y} = \frac{1}{2} \ln \frac{x_1 + x_2 + x_1 - x_2}{x_1 + x_2 - x_1 + x_2} = \frac{1}{2} \ln \frac{x_1}{x_2}.$$

- Relation of Bjorken- x and rapidity:

$$x_{1,2} = \sqrt{\frac{\hat{s}}{s}} e^{\pm \hat{y}} \quad \text{and} \quad \hat{y} = \frac{1}{2} \ln \frac{x_1^2 s}{m_\phi^2} \leq \ln \frac{2E}{m_\phi} = \hat{y}_{\max}.$$

- Together ($sd x_1 dx_2 = d\hat{s} d\hat{y}$):

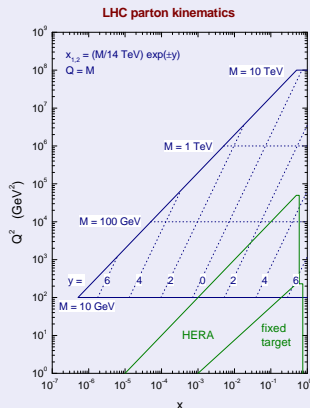
$$\sigma_{AB \rightarrow \Phi} = \sum_{ab} \int_{-\hat{y}_{\max}}^{\hat{y}_{\max}} d\hat{y} x_1 f_{a/A}(x_1, \mu_F^2) x_2 f_{b/B}(x_2, \mu_F^2) \frac{g_\sigma(m_\phi^2, m_\phi^2)}{m_\phi^2}.$$

Resonance production (cont'd)

- Note: Only dependence on rapidity through the PDFs
 \Rightarrow rapidity distribution of Φ contains information on the PDFs of partons a and b .

(Remember: $x_{1,2} = m_\Phi / \sqrt{s} e^{\pm y}$.)

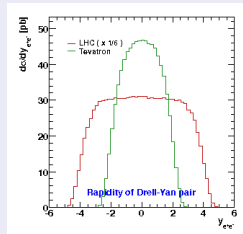
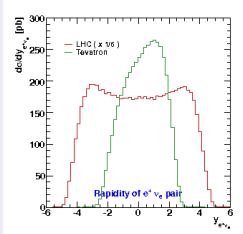
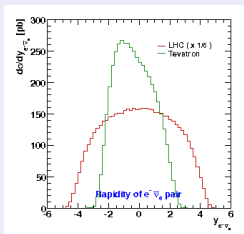
- Obvious consequence: The higher the mass of the produced system the more central it is.



(Plot from MSTW homepage)

Aside: Rapidities of gauge bosons

From the Tevatron to the LHC



Kinematics of $2 \rightarrow 2$ processes

- Use transverse momenta and (pseudo-) rapidities: p_{\perp} , y_3 , y_4 .
- Introduce average (centre-of-mass) rapidity and rapidity distance,

$$\bar{y} = (y_3 + y_4)/2 \text{ and } y^* = (y_3 - y_4)/2.$$

- Relate rapidities to Bjorken- x :

$$x_{1,2} = \frac{p_{\perp}}{\sqrt{2}} (e^{\pm y_3} + e^{\pm y_4}) = \frac{p_{\perp}}{2\sqrt{s}} e^{\pm \bar{y}} \cosh y^*.$$

Therefore: $\hat{s} = M_{12}^2 = 4p_{\perp}^2 \cosh y^*.$

Similarly $\hat{t}, \hat{u} = -\frac{\hat{s}}{2} (1 \mp \tanh y^*).$

Kinematics of $2 \rightarrow 2$ processes (cont'd)

- Partonic cross section (keep all massless) reads

$$\begin{aligned}\hat{\sigma}_{ab \rightarrow 12} &= \frac{1}{2\hat{s}} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} |\overline{\mathcal{M}}_{ab \rightarrow 12}|^2 \\ &\quad (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2) \\ &= \frac{1}{2\hat{s}^2} \int \frac{d^2 p_\perp}{(2\pi)^2} |\overline{\mathcal{M}}_{ab \rightarrow 12}|^2.\end{aligned}$$

- Fold in the PDFs (sum over a, b , integrate over $x_{1,2}$):

$$\sigma_{AB \rightarrow 12} = \sum_{ab} \int \frac{dy_1 dy_2 d^2 p_\perp}{16\pi^2 s^2} \frac{f_a(x_1, \mu_F) f_b(x_2, \mu_F)}{x_1 x_2} |\overline{\mathcal{M}}_{ab \rightarrow 12}|^2.$$

- Note: Do not forget a factor $1/(1 + \delta_{12})$ for identical final states.

QCD matrix elements

- Common feature: t -channel dominance

(If existing, "elastic" scattering wins.)

- Note: Typically
 $t \rightarrow 0 \iff p_{\perp}^2 \rightarrow 0$.
- Consequence: parton-parton cross section grows fast for $p_{\perp} \rightarrow 0$.
- Effect further enhanced by running α_s .

(Would use $\mu_R = p_{\perp}$ as scale.)

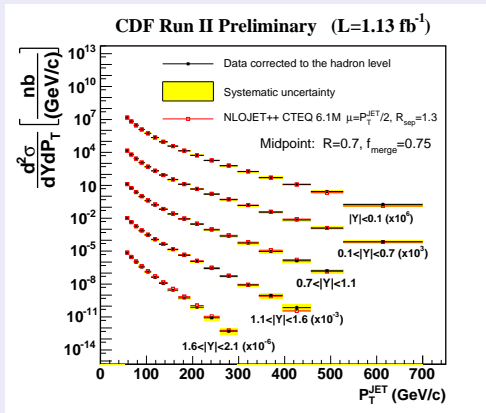
Examples:

$qq' \rightarrow qq'$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \rightarrow q'\bar{q}'$	$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \rightarrow gg$	$\frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$qg \rightarrow qg$	$\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - \frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}}$
$gg \rightarrow q\bar{q}$	$\frac{1}{6} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$gg \rightarrow gg$	$\frac{9}{2} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right)$
$q\bar{q} \rightarrow g\gamma$	$\frac{8}{9} \frac{\hat{t}^2 + \hat{u}^2 + 2\hat{s}(\hat{s} + \hat{t} + \hat{u})}{\hat{t}\hat{u}}$
$qg \rightarrow q\gamma$	$-\frac{1}{3} \frac{\hat{s}^2 + \hat{u}^2 + 2\hat{t}(\hat{s} + \hat{t} + \hat{u})}{\hat{s}\hat{u}}$

Note: For real photons $\hat{t} + \hat{u} + \hat{s} = 0$

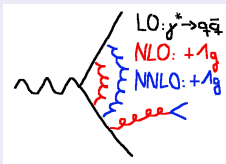
(multiply with couplings, e.g. $g^4 = (4\pi\alpha_s)^2$, $g^2 e^2 e_q^2$)

Jet production at Tevatron



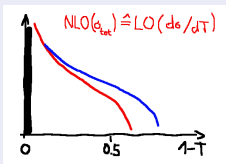
Higher-order corrections

Specifying higher-order corrections: $\gamma^* \rightarrow \text{hadrons}$



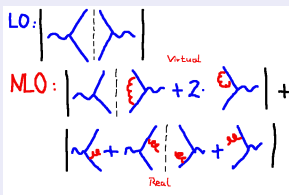
- In general: $N^n\text{LO} \leftrightarrow \mathcal{O}(\alpha_s^n)$
- But: only for inclusive quantities
(e.g.: total xsecs like $\gamma^* \rightarrow \text{hadrons}$).

Counter-example: thrust distribution



- In general, distributions are HO.
- Distinguish real & virtual emissions:
Real emissions \rightarrow mainly distributions,
virtual emissions \rightarrow mainly normalisation.

Anatomy of HO calculations: Virtual and real corrections



NLO corrections: $\mathcal{O}(\alpha_s)$

Virtual corrections = extra loops

Real corrections = extra legs

- UV-divergences in virtual graphs \rightarrow renormalisation
- But also: IR-divergences in real & virtual contributions
Must cancel each other (Kinoshita-Lee-Nauenberg & Bloch-Nordsieck theorems),
non-trivial to see: N vs. $N + 1$ particle FS, divergence in PS vs. loop

Cancelling the IR divergences: Subtraction method

- Total NLO xsec: $\sigma_{\text{NLO}} = \sigma_{\text{Born}} + \int d^D k |\mathcal{M}|_V^2 + \int d^4 k |\mathcal{M}|_R^2$
- IR div. in real piece \rightarrow regularise: $\int d^4 k |\mathcal{M}|_R^2 \rightarrow \int d^D k |\mathcal{M}|_R^2$

- Construct **subtraction term with same IR structure**:

$$\int d^D k (|\mathcal{M}|_R^2 - |\mathcal{M}|_S^2) = \int d^4 k |\mathcal{M}|_{RS}^2 = \text{finite.}$$

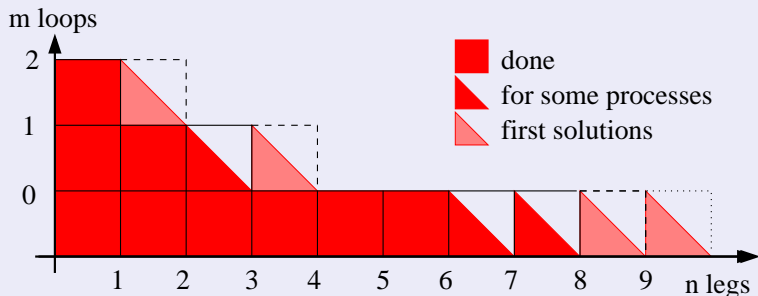
$$\text{Possible: } \int d^D k |\mathcal{M}|_S^2 = \sigma_{\text{Born}} \int d^D k |\tilde{\mathcal{S}}|^2, \text{ **universal** } |\tilde{\mathcal{S}}|^2.$$

- $\int d^D k |\mathcal{M}|_V^2 + \sigma_{\text{Born}} \int d^D k |\tilde{\mathcal{S}}|^2 = \text{finite}$ (analytical)
- Has been automated in various programs.
- Remark: Part of the collinear divergences in initial state absorbed in PDFs.

(This introduces scheme dependence and spoils probabilistic interpretation of PDFs.)

Cross sections @ hadron colliders

Availability of exact calculations



Tree-level tools (publicly available)

	Models	$2 \rightarrow n$	Ampl.	Integ.	lang.
Alpgen	SM	$n = 8$	rec.	Multi	Fortran
Amegic	SM,MSSM,ADD	$n = 6$	hel.	Multi	C++
CompHep	SM,MSSM	$n = 4$	trace	1Channel	C
COMIX	SM	$n = 8$	rec.	Multi	C++
HELAC	SM	$n = 8$	rec.	Multi	Fortran
MadEvent	SM,MSSM,UED	$n = 6$	hel.	Multi	Fortran
O'Mega	SM,MSSM,LH	$n = 8$	rec.	Multi	O'Caml

(One-)Loop-level tools (publicly available)

	Processes	lang.
MCFM	SM, 3-particle FS	Fortran
NLOJET++	up to 3 light jets	C++
Prospino	MSSM pair production	Fortran

PDFs and factorisation

Parton picture

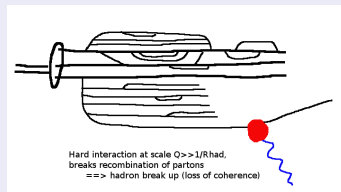
- Parton picture: Hadrons made from partons.
- Distribution(s) of partons in hadrons:
not from first principles, only from measurements.
- First idea: probability to find parton a in hadron h only dependent on Bjorken- x ($x = E_a/E_h$ or similar) – “Bjorken-scaling”
 $\mathcal{P}(a|h) = f_a^h(x)$ (LO interpretation of PDF).
- But QCD: Partons in partons in partons
 \implies scaling behaviour of PDFs: $f = f(x, Q^2)$.
- Still: PDFs must be measured, but scaling in Q^2 from theory (DGLAP, resums large logs of Q^2)

Space-time picture of hard interactions

Partons “collinear” with hadron: $k_{\perp} \ll 1/R_{\text{had}}$.

Lifetime of partons $\tau \sim 1/x$, $r \sim 1/Q$.

Hard interaction at scales $Q_{\text{hard}} \gg 1/R_{\text{had}}$.



- Too “fast” for colour field - **only one parton takes part.**
- Other partons feel absence only when trying to recombine.
- Universality (process-independence) of PDFs.
- Collinear factorisation.

Revealing the inner structure: *ep*-scattering

A detour: Elastic scattering & Form factors

- Extended objects have a matter density $\rho(\vec{r})$.

$$\text{Normalisation: } \int d^3r \rho(\vec{r}) = 1$$

- Its Fourier transform is called a form factor:

$$F(\vec{q}) = \int d^3r \exp[-i\vec{q}\vec{r}] \rho(\vec{r}) \implies F(0) = 1$$

- Naive modification of cross sections for scattering on such objects:

$$\left. \frac{d\sigma}{d^2\Omega} \right|_{\text{ptlike}} \implies \left. \frac{d\sigma}{d^2\Omega} \right|_{\text{extended}} \approx \left. \frac{d\sigma}{d^2\Omega} \right|_{\text{ptlike}} |F(q)|^2$$

Elastic ep scattering and the Rosenbluth formula

- Simple test of proton's charge distribution: elastic ep scattering (exchange of a photon). Elastic: **Nucleon remains intact**.
- Rosenbluth-formula (E and E' are energies of electron before and after scattering, M is the proton mass, q^2 is the space-like momentum transfer, and θ is the scattering angle):

$$\frac{d\sigma}{d^2\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\left(F_1^2(q^2) - \frac{\kappa^2 q^2}{4M^2} F_2^2(q^2) \right) - \frac{q^2}{2M^2} (F_1(q^2) + \kappa F_2(q^2))^2 \tan^2 \frac{\theta}{2} \right]$$

Compare with Rutherford scattering (on very massive objects):

$$\frac{d\sigma}{d^2\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}$$

Elastic ep scattering and charge radius of the proton

- Differences due to relativistic kinematics plus recoil of the protons (in Rutherford scattering, the nuclei stay at rest).
- Also inner structure: there are two form factors $F_{1,2}$. They are related to the electric and magnetic form factors, and are parametrised as

$$F_{1,2} \approx \left[\frac{1}{1 - q^2/0.71\text{GeV}^2} \right]^2$$

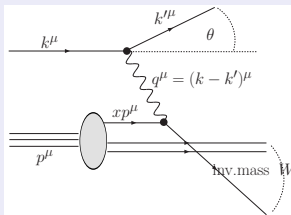
- Connection to charge radius: Assume $F_1 = F_2$ and

$$F(q^2) = \int d^3r \rho(\vec{r}) \exp[-i\vec{q}\vec{r}] \approx 1 - \frac{\vec{q}^2}{6} \langle r^2 \rangle + \dots$$

- Therefore: $r_{\text{proton}} \equiv \langle r^2 \rangle^{1/2} \approx 0.75 \pm 0.25 \text{ fm}$.

Deep-inelastic scattering: The process

- Terminology arises because in contrast to elastic scattering the **nucleon nearly always disintegrates**.
- Typically in DIS proton is probed with γ 's.
From $p \approx 1/\lambda$: If momentum transfer larger than 1 GeV, ($\approx 1/0.2\text{fm}$) then inner structure revealed.
- Kinematics:



$$\nu = \frac{2pq}{m_p} \longrightarrow E - E' \quad (\text{energy transfer})$$

$$x = \frac{Q^2}{2pq} \longrightarrow \frac{Q^2}{E - E'} \quad (\text{momentum fraction of parton})$$

$$Q^2 = -q^2 = -2EE'(1 - \cos \theta) \quad (\text{momentum transfer squared})$$

Two basic ideas

- Typically, the behaviour of the cross section with varying x (or, alternatively ν) and Q^2 is being measured.
In addition, νp -scattering with W exchange is considered.
- Two basic ideas:
 - The **parton model** (by R.Feynman):
The nucleon is made of smaller bits (partons). Later knowledge: Can be identified with quarks and gluons. But: In addition to the three **valence** quarks, carrying the quantum numbers (e.g. $|p\rangle = |uud\rangle$), there are virtual quarks and gluons, the **sea**.
 - The **scaling hypothesis** (by J.D.Bjorken):
At large energies and momentum transfers, the cross section depends on one variable only. Reason: The photon ceases to scatter coherently off the nucleon, but solely sees the individual, point-like partons.

Bjorken-scaling

- Equation for cross section (cf. elastic scattering, replacing form factors $F_{1,2}(q^2)$ with **structure functions** $W_{1,2}(\nu, Q^2)$):

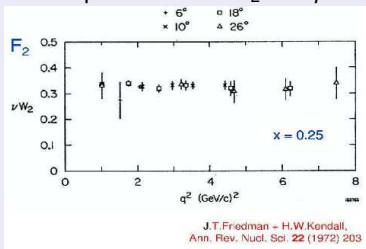
$$\frac{d\sigma}{d^2\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} [W_2(\nu, Q^2) + 2W_1(\nu, Q^2)]$$

- Bjorken scaling implies that with no special scale present in the dynamics of the scattering the $W_{1,2}(\nu, Q^2)$ can be replaced:

$$m_p W_1(\nu, Q^2) \longrightarrow F_1(x)$$

$$\frac{Q^2}{2m_p x} W_2(\nu, Q^2) \longrightarrow F_2(x),$$

Independence of W_2 on q^2 :



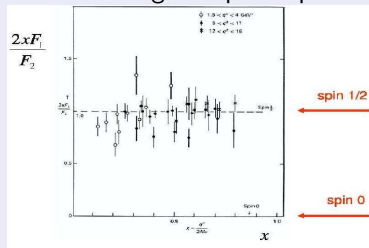
The spin of the quarks: The Callan-Gross relation

- Bjorken scaling established that DIS in fact must be described in terms of parton-photon processes.
But what are the properties of these point-like constituents?
- In 1969 Callan and Gross suggested that Bjorken's scaling functions are related:

$$2xF_1(x) = F_2(x).$$

- This reflects the assumption that the **partons inside the proton are indeed quarks**, i.e. **spin-1/2 particles** (spin-0 for example would lead to $2xF_1(x)/F_2(x) = 0$.)

Measuring the quark spin



Deriving the Callan-Gross relation

- Basic idea: Compare eq -scattering cross section (free quark) with the DIS ep cross section and assume identity:

$$\frac{d^2\sigma_{eq}}{d^2\Omega dE'} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \left[1 + \frac{Q^2}{2m_p^2} \tan^2 \frac{\theta}{2} \right] \delta \left(\nu - \frac{Q^2}{2m_p x} \right)$$

$$\frac{d^2\sigma_{ep}}{d^2\Omega dE'} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \left[\frac{1}{\nu} F_2(x) + \frac{2}{m_p} \tan^2 \frac{\theta}{2} F_1(x) \right]$$

Parton distributions and sum rules

- Define **probabilities** (possible at LO only) $f_a(x)$ to find a parton of type a with energy fraction between x and $x + dx$:

$$F_1(x) = \sum_a q_a^2 f_a(x), \quad q_a = \text{parton's charge.}$$

- The parton momenta must add to the proton momentum:

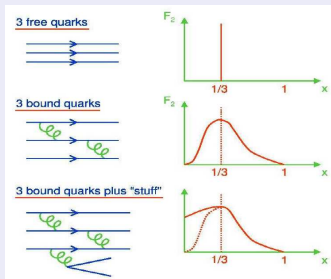
$$\int_0^1 dx \, x [f_u(x) + f_{\bar{u}}(x) + f_d(x) + f_{\bar{d}}(x) + f_s(x) + f_{\bar{s}}(x) + \dots] = 1.$$

- The parton types must yield a “net proton”, $p\rangle = |uud\rangle$:

$$\begin{aligned} \int_0^1 dx [f_u(x) - f_{\bar{u}}(x)] &= 2 & \int_0^1 dx [f_d(x) - f_{\bar{d}}(x)] &= 1 \\ \int_0^1 dx [f_s(x) - f_{\bar{s}}(x)] &= 0 & \int_0^1 dx [f_c(x) - f_{\bar{c}}(x)] &= 0. \end{aligned}$$

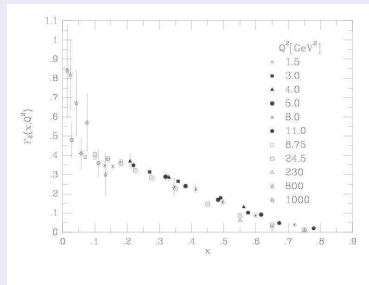
QCD effect on structure functions: Scaling violations

- Now it is possible to quantify the picture of “proton = quarks + stuff”



- Leads to evolution equations: “Russian dolls”

- This implies dependence of $F_{1,2}$ on the momentum transfer.
- Therefore: $F_{1,2}$ depend on both x and Q^2 - not constant in Q^2 any more.



Quantifying scaling violations: Evolution equations

- Explanation: As the proton is hit harder and harder (i.e. at larger Q^2), the virtual photon starts resolving gluons and quark-antiquark fluctuations (partons in partons!).
- The scale Q^2 plays the role of a “resolution parameter”.
- Described by the DGLAP equations. Basic structure:

$$\frac{dq(x, Q^2)}{d \ln Q^2} = \alpha_s(Q^2) \int_x^1 dy \left[q(y, Q^2) P_{q \rightarrow qg} \left(\frac{x}{y} \right) + g(y, Q^2) P_{g \rightarrow q\bar{q}} \left(\frac{x}{y} \right) \right]$$

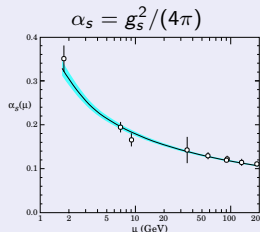
Here the quark at x can come from a **quark** (**gluon**) at y , the functions P encode the details of the decays $q \rightarrow qg$ ($g \rightarrow q\bar{q}$) responsible for it.

Aside: The “running” coupling in QCD & asymptotic freedom

- Reassuring: Can understand the proton structure at large Q^2 in terms of perturbative objects (quarks and gluons). This implies that the coupling g_s is sufficiently small there:

Asymptotic freedom.

- But measurements (left) and calculation show that the coupling becomes stronger the lower the scale ($\simeq Q^2$), i.e. the larger the distance.
- In fact, the perturbative α_s diverges for $\mu = \Lambda_{\text{QCD}} \approx 300$ MeV, signalling the breakdown of the expansion.



- Non-perturbative regime, where **only colour-less states** can exist:
- Confinement.
- Therefore, only hadrons (no quarks or gluons) as observable initial and final states in experiments.

Fitting PDFs: Strategy in a nutshell

- Ansatz $g(x)$ for PDFs at some fixed value of $Q_0^2 = Q^2 \approx 1\text{GeV}^2$.
For example, MRST/MSTW: (personal Durham bias)

$$xu_V = A_U x^{\eta_1} (1-x)^{\eta_2} (1 + \varepsilon_U \sqrt{x} + \gamma_U x)$$

$$xd_V = A_D x^{\eta_2} (1-x)^{\eta_4} (1 + \varepsilon_D \sqrt{x} + \gamma_D x)$$

$$xs = A_S x^{-\lambda_S} (1-x)^{\eta_S} (1 + \varepsilon_S \sqrt{x} + \gamma_S x)$$

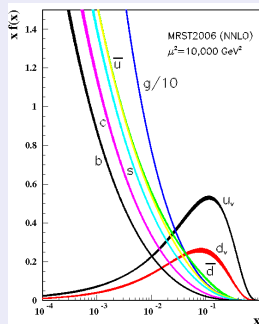
$$xg = A_G x^{-\lambda_G} (1-x)^{\eta_G} (1 + \varepsilon_G \sqrt{x} + \gamma_G x)$$

- Collect data at various x , Q^2 , use DGLAP equation to evolve down to Q_0^2 , also fix α_s .
- Order of fit \iff order of kernels.
- Enforce sum rules (momentum, ...)

(Partially relaxed for LO^* and LO^{**} .)

Generic structure

- Large sea for $x \rightarrow 0$
- Valence at $x \approx 0.15$



Determination of PDFs: Data input

Example: MSTW parameterisation and their effect:

New data included.

NuTeV and Chorus data on $F_2^{\nu,\theta}(x, Q^2)$ and $F_3^{\nu,\theta}(x, Q^2)$ replacing CCFR.

NuTeV and CCFR dimuon data included directly. Leads to a direct constraint on $s(x, Q^2) + \bar{s}(x, Q^2)$ and on $s(x, Q^2) - \bar{s}(x, Q^2)$. Affects other partons.

CDFII lepton asymmetry data in two different E_T bins – $25\text{GeV} < E_T < 35\text{GeV}$ and $35\text{GeV} < E_T < 45\text{GeV}$.

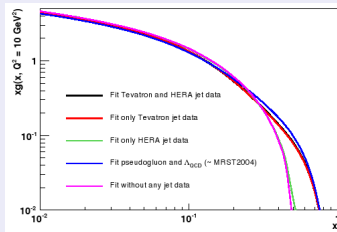
HERA inclusive jet data (in DIS).

New CDFII high- E_T jet data.

Direct high- x data on $F_L(x, Q^2)$.

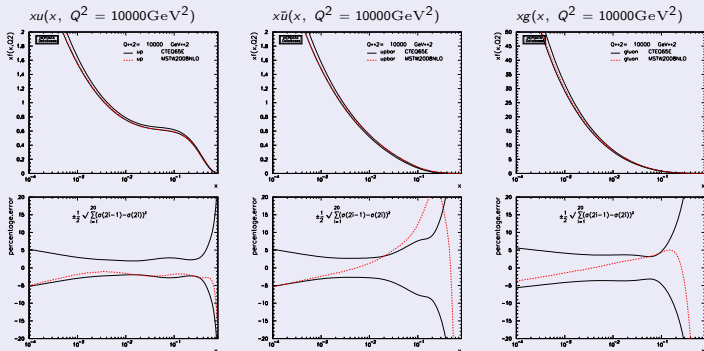
Update to include all recent charm structure function data.

Look at dependence of fit on m_c – defined as pole mass.



(From R.Thorne's talk at DIS 2007)

Uncertainties of global PDFs: CTEQ65E vs. MSTW2008 NLO



(From Hepdata base)

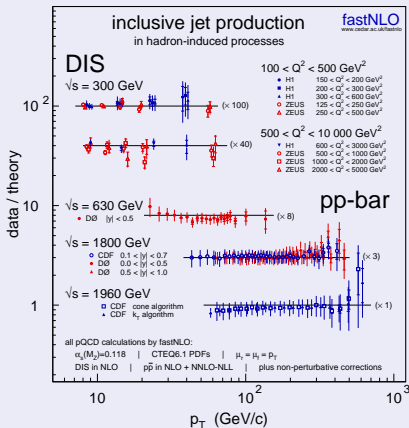
Remark on scales and PDF choices

- In perturbative calculations at hadron colliders, two (unphysical) scales enter:
 - Renormalisation scale μ_R (scale for coupling constants)
 - Factorisation scale μ_F (scale for PDFs)
- In **principle**, all-orders results would be independent, in **practise**, results shows a dependence on scales.
- This dependence decreases by adding more orders.
- Smart process-dependent choices can mimic some HO effects.
- A common recipe to estimate higher-order effects and the related uncertainty is to vary both scales by a factor (typically 2).

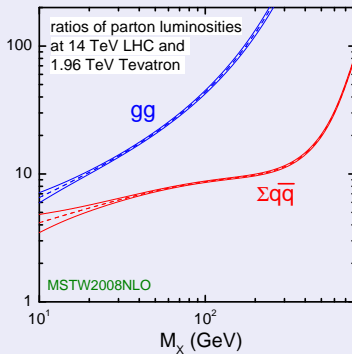
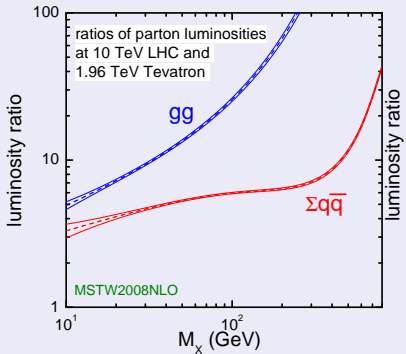
This is not always reliable \iff nothing replaces the true HO calculation

... especially if we want to know for sure ...

Understanding of perturbative QCD



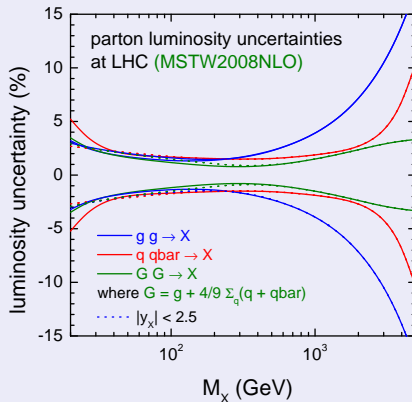
PDFs: From Tevatron to LHC



(From MSTW homepage.)

PDF uncertainties at LHC

(Propaganda plot by MSTW collaboration, CTEQ similar.)



From parton to hadron level

Limitations of parton level calculations

- Fixed order parton level (LO, NLO, ...) implies fixed multiplicity
 \Rightarrow no clean way toward exclusive final states.
- No control over potentially large logs
 (appear when two partons come close to each other).
- Parton level is parton level
experimental definition of observables relies on hadrons.

Therefore: **Need hadron level!**

Must dress partons with radiation!

(will also enable universal hadronisation)

Origin of radiation

Accelerated charges radiate

- Well-known: [Accelerated charges radiate](#)
- QED: Electrons (charged) emit photons
Photons split into electron-positron pairs
- QCD: Quarks (coloured) emit gluons
Gluons split into quark pairs
- Difference: Gluons are coloured (photons are not charged)
Hence: Gluons emit gluons!
- Cascade of emissions: [Parton shower](#)

Pattern of radiation

Leading logs: $e^+e^- \rightarrow \text{jets}$

- Differential cross section:

$$\frac{d\sigma_{ee \rightarrow 3j}}{dx_1 dx_2} = \sigma_{ee \rightarrow 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Singular for $x_{1,2} \rightarrow 1$.

- Rewrite with opening angle θ_{qg}
and gluon energy fraction $x_3 = 2E_g/E_{\text{c.m.}}$:

$$\frac{d\sigma_{ee \rightarrow 3j}}{d \cos \theta_{qg} dx_3} = \sigma_{ee \rightarrow 2j} \frac{C_F \alpha_s}{\pi} \left[\frac{2}{\sin^2 \theta_{qg}} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right]$$

Singular for $x_3 \rightarrow 0$ (“soft”), $\sin \theta_{qg} \rightarrow 0$ (“collinear”).

Leading logs: Collinear singularities

- Use

$$\frac{2d \cos \theta_{q\bar{q}}}{\sin^2 \theta_{q\bar{q}}} = \frac{d \cos \theta_{q\bar{q}}}{1 - \cos \theta_{q\bar{q}}} + \frac{d \cos \theta_{q\bar{q}}}{1 + \cos \theta_{q\bar{q}}} = \frac{d \cos \theta_{q\bar{q}}}{1 - \cos \theta_{q\bar{q}}} + \frac{d \cos \theta_{q\bar{q}}}{1 - \cos \theta_{q\bar{q}}} \approx \frac{d\theta_{q\bar{q}}^2}{\theta_{q\bar{q}}^2} + \frac{d\theta_{q\bar{q}}^2}{\theta_{q\bar{q}}^2}$$

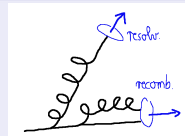
- Independent evolution of two jets (q and \bar{q}):

$$d\sigma_{ee \rightarrow 3j} \approx \sigma_{ee \rightarrow 2j} \sum_{j \in \{q, \bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{d\theta_{jg}^2}{\theta_{jg}^2} P(z),$$

where $P(z) = \frac{1+(1-z)^2}{z}$ (DGLAP splitting function)

Leading logs: Parton resolution

- What is a parton?
Collinear pair/soft parton recombine!
- Introduce resolution criterion $k_{\perp} > Q_0$.



- Combine virtual contributions with unresolvable emissions:
Cancels infrared divergences \implies Finite at $\mathcal{O}(\alpha_s)$

(Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- Unitarity: Probabilities add up to one
 $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1$.

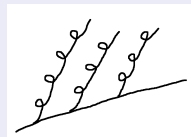


Occurrence of large logarithms

Many emissions: Parton parted partons

- Iterate emissions (jets)

Maximal result for $t_1 > t_2 > \dots t_n$:

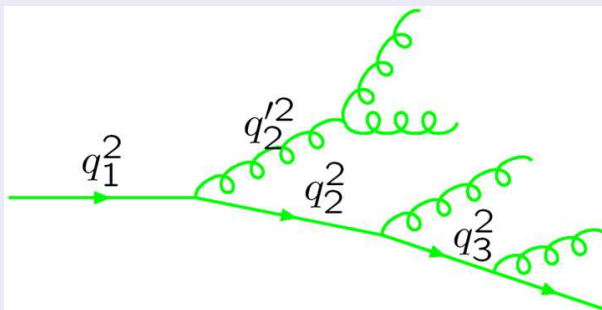


$$d\sigma \propto \sigma_0 \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

- How about Q^2 ? **Process-dependent!**

Towards a parton cascade/shower

Ordering the emissions : Pattern of parton parted partons

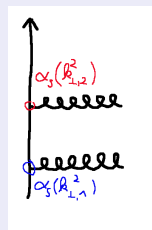
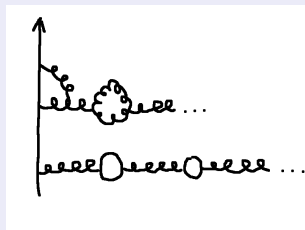


$$q_1^2 > q_2^2 > q_3^2, q_1^2 > q_2'^2$$

Aside: Inclusion of quantum effects

Running coupling

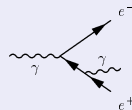
- Effect of summing up higher orders (loops): $\alpha_s \rightarrow \alpha_s(k_\perp^2)$



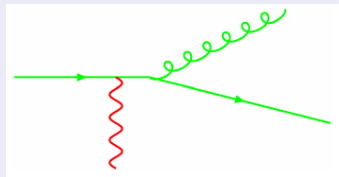
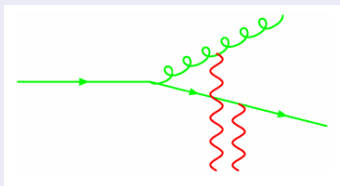
- Much faster parton proliferation, especially for small k_\perp^2 .
- Must avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda_{\text{QCD}}^2$
 $\Rightarrow Q_0^2 = \text{physical parameter.}$

Soft logarithms : Angular ordering

- In principle, independence on collinear variable:
 t (inv.mass), k_{\perp}^2 , θ all lead to same leading logs
- But: Soft limit for single emission also universal
- Problem: Soft gluons come from all over (not collinear!)
Quantum interference? Still independent evolution?
- Answer: Not quite independent.
 - Assume photon into e^+e^- at θ_{ee} and photon off electron at θ
 - Transverse momentum and wavelength of photon: $k_{\perp}^{\gamma} \sim zp\theta$, $\lambda_{\perp}^{\gamma} \sim 1/k_{\perp}^{\gamma} = 1/(zp\theta)$.
 - Formation time of photon: $\Delta t \sim 1/\Delta E$, $\Delta E \sim \theta/\lambda_{\perp}^{\gamma} \sim zp\theta^2$.
 - ee-separation: $\Delta b \sim \theta_{ee}\Delta t \sim \theta_{ee}/(zp\theta^2)$.
 - Must be larger than transverse wavelength: $\Delta b > \lambda_{\perp}^{\gamma} \Rightarrow \theta_{ee} > \theta$
- Thus: Angular ordering takes care of soft limit.



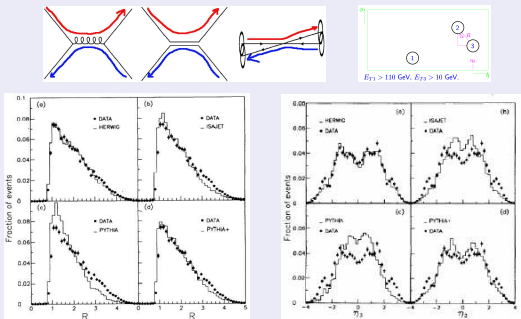
Soft logarithms : Angular ordering in pictures



Gluons at large angle from combined colour charge!

Experimental manifestation of angular ordering

ΔR of 2nd & 3rd jet in multi-jet events in pp-collisions @ Tevatron



(from CDF, Phys. Rev. D50 (1994) 5562)

Parton showers

Simulating parton radiation

- Catch: Can exponentiate all emissions due to universal log pattern.
- For parton showers use **Sudakov form factor**:

$$\begin{aligned}\Delta_q(Q^2, Q_0^2) &= \exp \left[- \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \int dz \frac{\alpha_s[k_\perp^2(z, k^2)]}{2\pi} P(z) \right] \\ &= \exp \left[- \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \bar{P}(k^2) \right] \approx \exp \left[- C_F \frac{\alpha_s}{2\pi} \log^2 \frac{Q^2}{Q_0^2} \right]\end{aligned}$$

- Interpretation: **No-emission probability between Q^2 and Q_0^2 .**

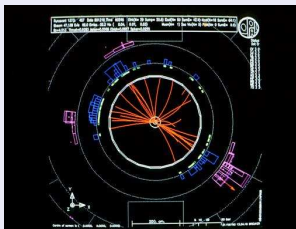
Parton showers

Tools

	Shower variable	A0?	lang.
Pythia	inv.mass: t	approx.	Fortran
Pythia8	transv.mom.: k_{\perp}^2	yes(?)	C++
Herwig	opening angle	yes	Fortran
Herwig++	mod.opening angle	yes	C++
Ariadne	dipole transv.mom.	yes	Fortran
Sherpa	2 showers: t and k_{\perp}^2	varying	C++

What are jets?

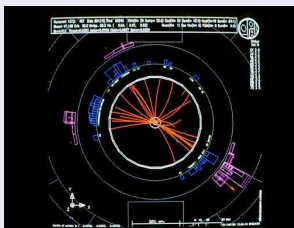
Jets = collimated hadronic energy



- Jets (unavoidably) happen in high-energy events: a collimated bunch of hadrons flying roughly in the same direction.
- Note: hundreds of hadrons contain a lot of information.
- More than we can hope to make use of.

What are jets?

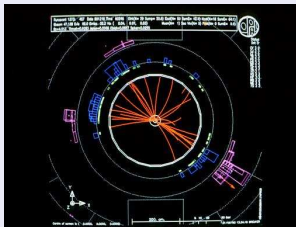
Jets = collimated hadronic energy



- Often you don't need a fancy algorithm to “see” the jets.
- But you do to give them a precise and quantitative meaning.

What are jets?

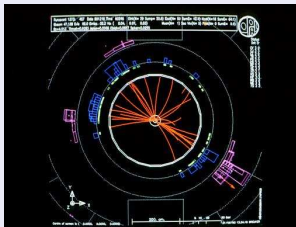
Jets = collimated hadronic energy



- Jets are usually related to some underlying perturbative dynamics (i.e. quarks and gluons).
- The purpose of a “jet algorithm” is then to reduce the complexity of the final state, simplifying many hadrons to simpler objects that one can hope to calculate.

What are jets?

Jets = collimated hadronic energy



- A jet algorithm maps the momenta of the final state particles into the momenta of a certain number of jets:

$$\{p_i\} \xleftrightarrow{\text{jet algo}} \{j_l\}$$

It can act on momenta, calo towers, etc..

- Most algorithms contain a resolution parameter, R , which controls the extension of the jet.

Linking partons and detector signals

Jets occur in decays of heavy objects:
Z, W^\pm bosons, tops, SUSY, ...

Example: top-decays

Fully hadronic: Jets	Tau + jets	Lepton + Jets
Tau + jets	Taus	Tau + lepton
Lepton + Jets	Tau + lepton	Leptons

Event rates for 10 fb^{-1} :

Process	Number
$t\bar{t}$	10^7
QCD Multijets	
3	$9 \cdot 10^8$
4	$7 \cdot 10^7$
5	$6 \cdot 10^6$
6	$3 \cdot 10^5$
7	$2 \cdot 10^4$
8	$2 \cdot 10^3$

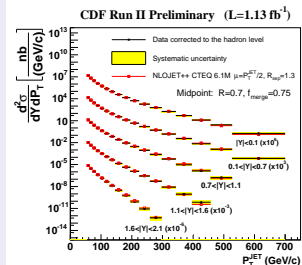
Tree-level (parton-level) numbers

$$p_\perp^{\text{jet}} > 60 \text{ GeV}, \theta_{ij} > \pi/6, |y_i| < 3$$

Draggiotis, Kleiss & Papadopoulos '02

But: Jets \neq partons!

- Jets are unavoidable whenever partons scatter.
- Perturbative picture well understood.
Example: Jet cross sections
- Partons fragment through multiple parton emissions:
 - Soft & collinear divergences dominate
 - Large logs overcome "small" coupling
- No quantitative understanding for transition to hadrons
(fate of non-perturbative QCD)
- But: Fragmentation & hadronisation dominated by low p_{\perp} .
- Therefore: Partons result in collimated bunches of hadrons



Jet definitions

General considerations

A jet definition is a set of rules to project large numbers of objects (dozens of partons, hundred's of hadrons, thousand's of calorimeter towers) onto a small number of parton-like objects with one well-defined four-momentum each.

For this jet definition to be useful,

- the rules must be the same, independent of the level of application: QCD resilience/robustness;
- the rules must be complete, with no ambiguities;
- the rules must be experimental feasible and theoretically sensible.
⇒ **Infrared safety** crucial!

Robustness

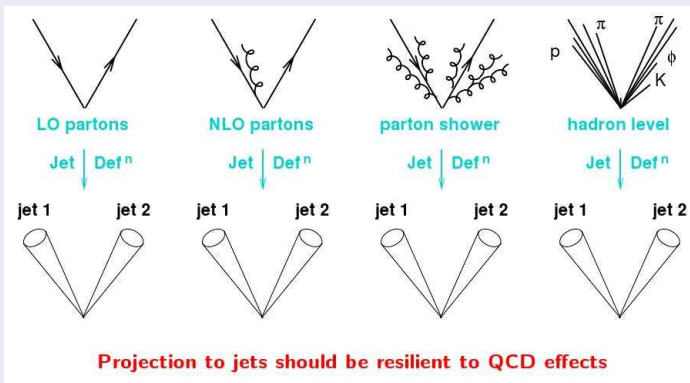
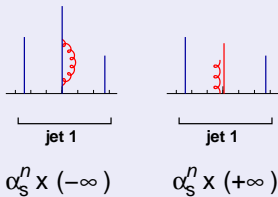


Figure from G.Salam

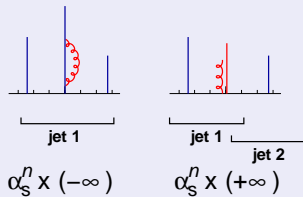
Collinear/infrared safety

Collinear Safe



Infinites cancel

Collinear Unsafe

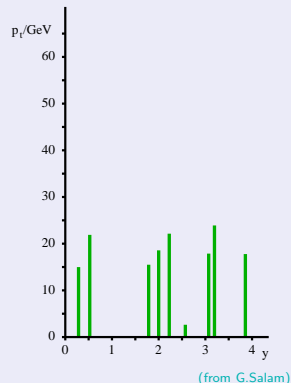


Infinites do not cancel

Figure from G.Salam

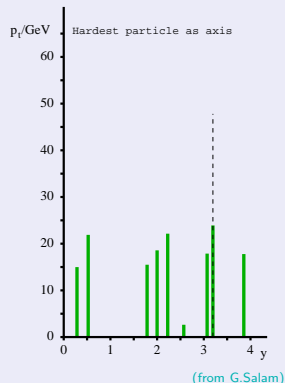
Cone jets: Fixed cone, progressive removal

- Main idea: Define jets geometrically, remove found jets.
- Take hardest particle = cone axis.
- Draw cone around it.
- Convert contents into a “jet” and remove them.
- Repeat until no particles left.
- Parameters: Cone-size, p_{\perp}^{\min}
- good feature: Simple.
- Bad feature: Infrared safe.



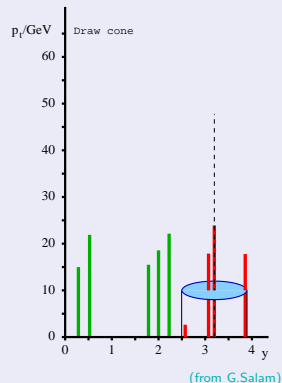
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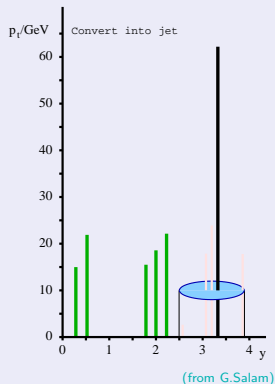
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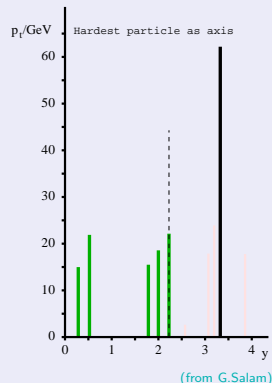
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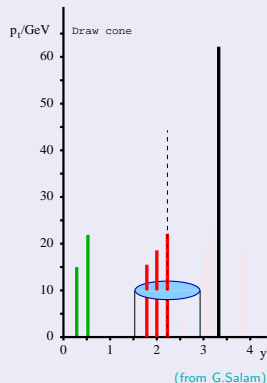
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- Parameters: Cone-size, p_{\perp}^{\min}
- good feature: Simple.
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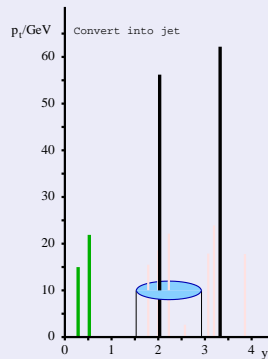
Cone jets: Fixed cone, progressive removal

- Main idea: Define jets geometrically, remove found jets.
- Take hardest particle = cone axis.
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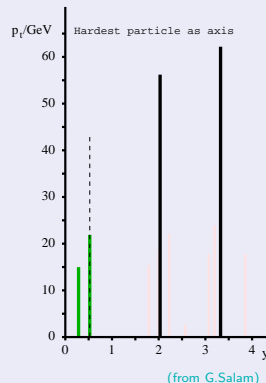
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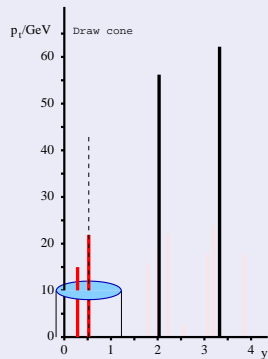
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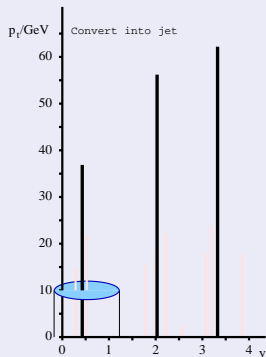
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(from G.Salam)

Cone jets: IR safety does matter

(stolen from M.Cacciari)

- All cone jets apart from SIS-cone are not infrared safe.
- The best ones typically fail at $(3+1)$ partons, others already at $(2+1)$.

Process	Last meaningful order			Known at
	JetClu, Atlas cone	MidPoint	CMS, it.cone	
incl.jets	LO	NLO	NLO	NLO (\rightarrow NNLO)
$V + 1$ jet	LO	NLO	NLO	NLO
3 jets	none	LO	LO	NLO
$V + 2$ jets	none	LO	LO	NLO
m_{jet} in $2j + X$	none	none	none	LO

- But: HO calculations cost real money

(100 theorists \times 15 years \approx 100 MEuro.)

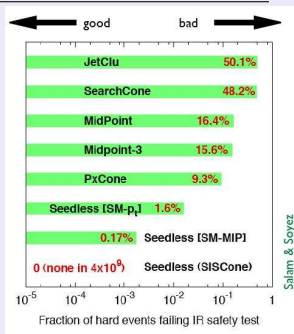
- Using unsafe tools makes them pretty much useless.

Cone jets: IR safety does matter

(stolen from M.Cacciari)

Question: How often are hard jets changes by soft stuff?

- Generate events with $2 < N < 10$ hard partons & find jets.
- Add $1 < N_{\text{soft}} < 5$ soft particles & repeat.
- How often do we end up with different jets?



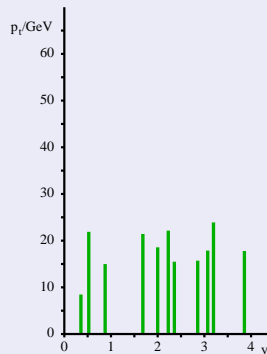
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- Main idea: Sequential recombination
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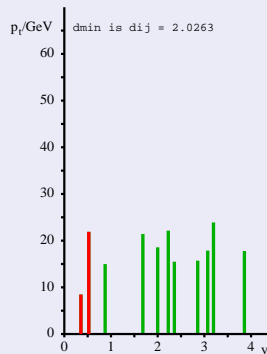
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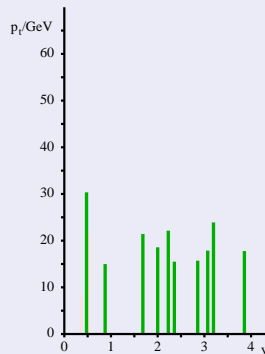
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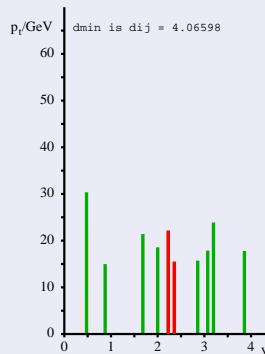
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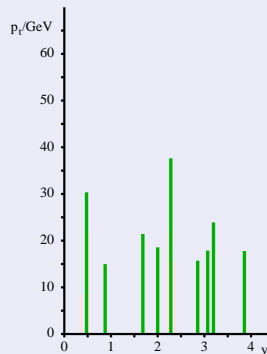
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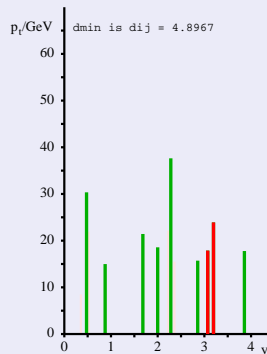
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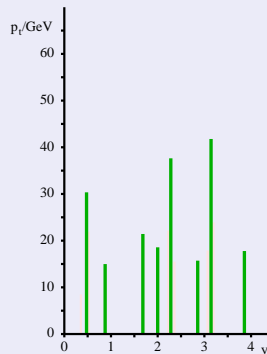
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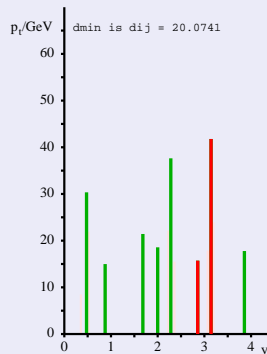
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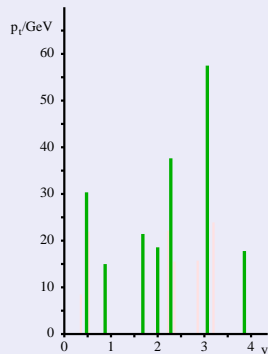
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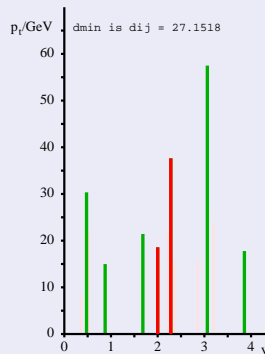
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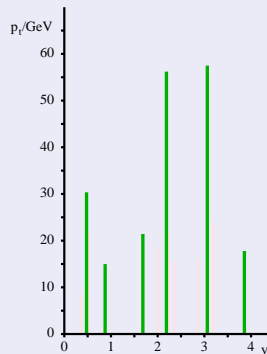
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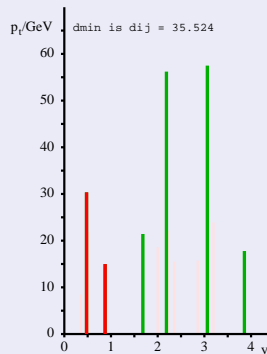
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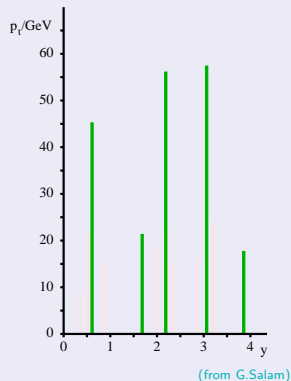
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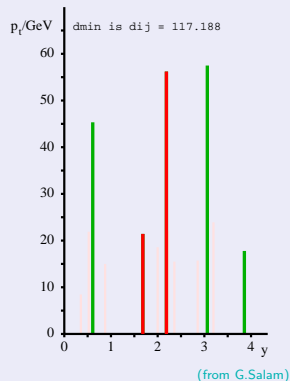
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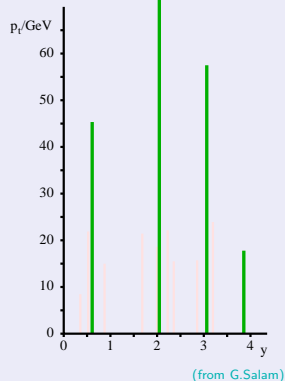
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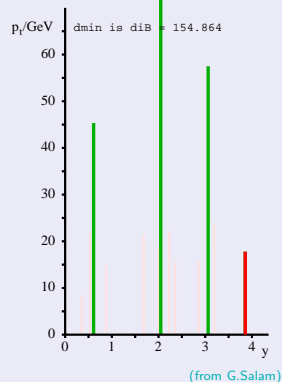
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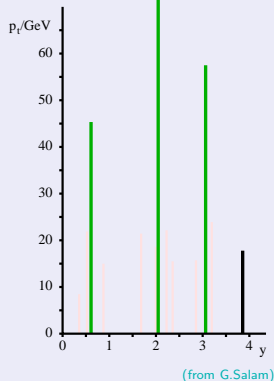
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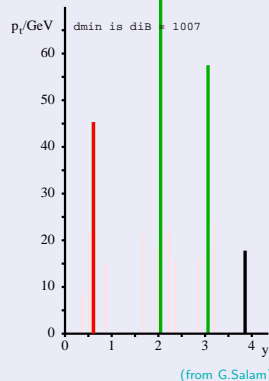
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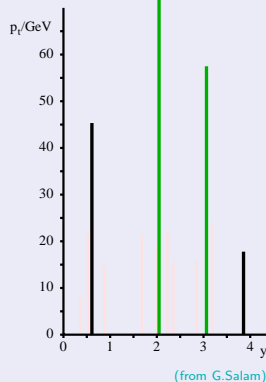
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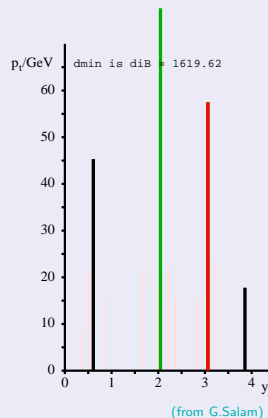
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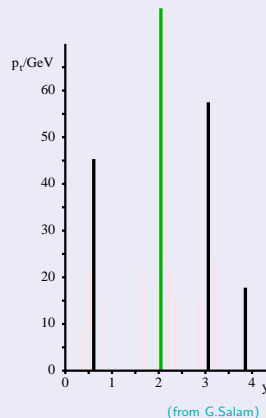
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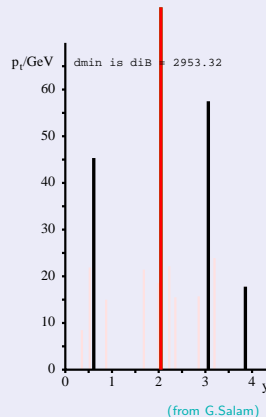
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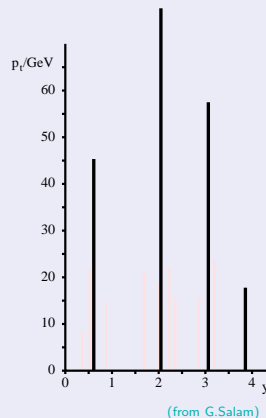
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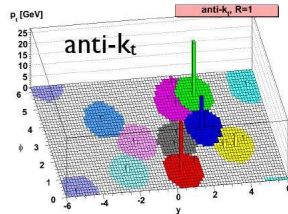
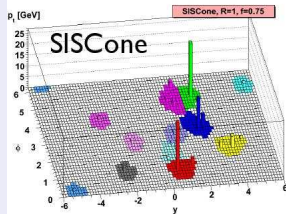
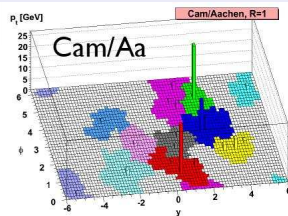
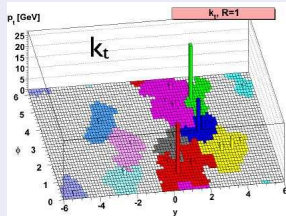
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Different jet algorithms

(stolen from M.Cacciari)



To take home

LHC, the QCD machine

- There are **no LHC events without QCD!!!**
- Perturbative expansion in α_s sufficiently well understood, but: hard to calculate beyond (N)LO.
- Important input to xsec calculations: PDFs
Must be taken from data, only scaling from QCD
- Order of an calculation is observable-dependent
make sure you know what you're talking about.

To take home

Parton-parted partons

- QCD radiation (bremsstrahlung) important
- Dominated by collinear & soft emissions
- Universal pattern of QCD bremsstrahlung
- Fills the phase space between large scales of signal creation and low scales of hadronisation
- Well understood in leading log approximation, gives rise to a probabilistic picture: parton showers.

To take home

A jet is (not) a jet is (not) a jet

- Jets are direct result of QCD in hard reactions - your primary experimental QCD entities.
- But: A parton is not a jet - a jet is what it is defined to be
- Jet definitions must match experimental and theoretical needs otherwise meaningless for comparison
- Infrared safety is a theoretical key requirement
- Many jet algorithms, presumably the “best” one does not exist