Simulations in High-Energy Physics

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PART I: Introduction

1 Introduction: improving event generators

2 QCD Basics: Scales & Kinematics
PART II: Monte Carlo for Perturbative QCD

3 Parton–level Monte Carlo

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5 First improvements

6 Matching

7 Multijet merging

8 Electroweak corrections
PART IV: Monte Carlo for Non–Perturbative QCD

9 Hadronisation

10 Underlying Event
PART I: INTRODUCTION
IMPROVING EVENT GENERATORS
Strategy of event generators

principle: divide et impera

- hard process:
  fixed order perturbation theory
  traditionally: Born-approximation

- bremsstrahlung:
  resummed perturbation theory

- hadronisation:
  phenomenological models

- hadron decays:
  effective theories, data

- "underlying event":
  phenomenological models
... and possible improvements

possible strategies:

- improving the phenomenological models:
  - “tuning” (fitting parameters to data)
  - replacing by better models, based on more physics
    (my hot candidate: “minimum bias” and “underlying event” simulation)

- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:
    “NLO-Matching” & “Multijet-Merging”
  - systematic improvement of the parton shower:
    next-to leading (or higher) logs & colours
Motivation – precision edge of particle physics

- after Higgs discovery: time for precision studies
  is it the SM Higgs boson or something else?
  relevant: spin/parity (✓), couplings to other particles

- Higgs signal suffers from different backgrounds, depending on
  production and decay channel considered in the analysis

- decomposing in bins of different jet multiplicities yields
  - different signal composition (e.g. WBF vs. ggF)
  - different backgrounds (most notably: $t\bar{t}$ in $WW$ final states)

- to this end: must understand jet production in big detail
  name of the game: uncertainties and their control

  despite far-reaching claims: analytic resummation and fixed-order calculations will not be sufficient

- same reasoning also true for new resonances/phenomena
Motivation – BSM edge of particle physics

- to date no survivors in searches for new physics & phenomena (a pity, but that’s what Nature hands to us)
- push into precision tests of the Standard Model (find it or constrain it!)
- statistical uncertainties approach zero (because of the fantastic work of accelerator, DAQ, etc.)
- systematic experimental uncertainties decrease (because of ingenious experimental work)
- theoretical uncertainties are or become dominant (it would be good to change this to fully exploit LHC’s potential)

⇒ more accurate tools for more precise physics needed!
Aim of the lectures

- review the state of the art in precision simulations (celebrate success)
- highlight missing or ambiguous theoretical ingredients (acknowledge failure)
- (maybe) suggest some further studies – experiment and theory (…)
QCD BASICS

SCALES & KINEMATICS
Contents

2.a) Factorisation: an electromagnetic analogy
2.b) QED Initial and Final State Radiation
2.c) Hadrons in initial state: DGLAP equations of QCD
2.d) Hadron production: Scales
An electromagnetic analogy

- consider a charge \( Z \) moving at constant velocity \( v \)

\[ v = 0 \]

\[ v \cong c \]

- at \( v = 0 \): radial \( E \) field only
- at \( v = c \): \( B \) field emerges: \( \vec{E} \perp \vec{B}, \vec{B} \perp \vec{v}, \vec{E} \perp \vec{v} \),
  
  energy flow \( \sim \) Poynting vector \( \vec{S} \sim \vec{E} \times \vec{B} \parallel \vec{v} \)
- approximate classical fields by “equivalent quanta”: photons
spectrum of photons:

\[ dn_\gamma = \frac{Z^2 \alpha}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{db^2_\perp}{b^2_\perp} \]  

(electron (Z=1) \rightarrow \frac{\alpha}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{db^2_\perp}{b^2_\perp})

- Fourier transform to transverse momenta \( k_\perp \):

\[ dn_\gamma = \frac{\alpha}{\pi} \cdot \frac{d\omega}{\omega} \cdot \frac{dk^2_\perp}{k^2_\perp} \]

note: divergences for \( k_\perp \rightarrow 0 \) (collinear) and \( \omega \rightarrow 0 \) (soft)

- therefore: Fock state for lepton = superposition (coherent):

\[ |e\rangle_{\text{phys}} = |e\rangle + |e\gamma\rangle + |e\gamma\gamma\rangle + |e\gamma\gamma\gamma\rangle + \ldots \]

photon fluctuations will “recombine”
QED Initial and Final State Radiation

- consider final state radiation in $\gamma^* \rightarrow \ell\bar{\ell}$
  (electron velocities/momenta labelled as $v$ and $v'/p$ and $p'$)
- classical electromagnetic spectrum from radiation function:
  (this is from Jackson or any other reasonable book on ED)

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2} \left| \epsilon^* \cdot \left( \frac{\vec{v}}{1 - \vec{v} \cdot \vec{n}} - \frac{\vec{v}'}{1 - \vec{v}' \cdot \vec{n}} \right) \right|^2,$$

with $\epsilon$ the polarisation vector and $\vec{n}(\Omega)$ the direction of the radiation
- recast with four–momenta, equivalent photon spectrum:

$$dN = \frac{d^3 k}{(2\pi)^3 2k_0} \frac{\alpha}{\pi} \left| \epsilon^*_\mu \left( \frac{p^\mu}{p \cdot k} - \frac{p'^\mu}{p' \cdot k} \right) \right|^2 = \frac{d^3 k}{(2\pi)^3 2k_0} \frac{\alpha}{\pi} \left| W_{pp';k} \right|^2$$

with the eikonal $W_{pp';k}$
• repeat exercise in QFT, Feynman diagrams:

\[ \mathcal{M}_{X \rightarrow e^+e^-\gamma} = e \bar{u}(p) \left[ \Gamma \frac{p' - k}{(p' - k)^2} \gamma^\mu - \frac{p + k}{(p + k)^2} \gamma^\mu \right] u(p') \epsilon^*_\mu(k) \]

\[ \xrightarrow{\text{soft}} e \epsilon^*_\mu(k) \left[ \frac{p^\mu}{p \cdot k} - \frac{p'^\mu}{p' \cdot k} \right] \bar{u}(p') \Gamma u(p) = e \mathcal{M}_{X \rightarrow e^+e^-\gamma} \cdot W_{pp';k} \]

• manifestation of Low’s theorem:
  soft radiation independent of spin \( \rightarrow \) classical

(radiation decomposes into soft, classical part with logs – i.e. dominant – and hard collinear part)
DGLAP equations for QED

(Dokshitser–Gribov–Lipatov–Altarelli–Parisi Equations)

- define probability to find electron or photon in electron:

  at LO in $\alpha$ (no emission):
  $$\ell(x, k^2_\perp) = \delta(1 - x)$$
  and
  $$\gamma(x, k^2_\perp) = 0$$

  (introduced $x = \text{energy fraction w.r.t. physical state}$)

- including emissions:
  - probabilities change
  - energy fraction $\xi$ of lepton parton w.r.t. the physical lepton object reduced by some fraction $z = x/\xi$
  - reminder: differential of photon number w.r.t. $k^2_\perp$:

  $$dn_\gamma = \frac{\alpha}{\pi} \frac{dk^2_\perp}{k^2_\perp} \frac{d\omega}{\omega} \leftrightarrow \frac{dn_\gamma}{d\log k^2_\perp} = \frac{\alpha}{\pi} \frac{dx}{x}$$
evolution equations (trivialised)

\[
\frac{d\ell(x, k_\perp^2)}{d \log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{\ell\ell} \left( \frac{x}{\xi}, \alpha(k_\perp^2) \right) \ell(\xi, k_\perp^2)
\]

\[
\frac{d\gamma(x, k_\perp^2)}{d \log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{\gamma\ell} \left( \frac{x}{\xi}, \alpha(k_\perp^2) \right) \ell(\xi, k_\perp^2).
\]

- $k_\perp^2$ plays the role of “resolution parameter”
- the $P_{ab}(z)$ are the splitting functions, encoding quantum mechanics of the “splitting cross section”, for example (at LO)

\[
P_{\ell\ell}(z) = \left( \frac{1 + z^2}{1 - z} \right)_+ + \frac{3}{2} \delta(1 - z)
\]

- if $\gamma \rightarrow \ell\ell$ splittings included, have to add entries/splitting functions into evolution equations above
Running of $\alpha_s$ and bound states

- quantum effect due to loops: couplings change with scale
- running driven by $\beta$–function

$$\beta(\alpha_s) = \mu^2 R \frac{\partial \alpha_s(\mu^2_R)}{\partial \mu^2_R}$$

$$= \frac{\beta_0}{4\pi} \alpha_s^2 + \frac{\beta_1}{(4\pi)^2} \alpha_s^3 + \ldots$$

with

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R n_f$$

$$\beta_1 = \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_R n_f - 4 C_F T_R n_f$$

April 2012

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<th>Legend</th>
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<td>$\alpha_s(M_Z) = 0.1184 \pm 0.0007$</td>
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F. Krauss
Simulations in High-Energy Physics
Casimir operators in the fundamental and adjoint representation:

\[ C_F = \frac{N_c^2 - 1}{2N_c} \quad \text{and} \quad C_A = N_c \]

with \( N_c = 3 \) colours and \( T_R = 1/2 \).

- \( n_f = \) the number of (quark) flavours
- the Casimirs correspond to quark and gluon colour charges
- explicit expression for strong coupling

\[ \alpha_s(\mu_R^2) \equiv \frac{g_s^2(\mu_R^2)}{4\pi} = \frac{1}{\frac{\beta_0}{4\pi} \log \frac{\mu_R^2}{\Lambda_{QCD}^2}} \]

with \( \Lambda_{QCD} \) the Landau pole of QCD, \( \Lambda_{QCD} \approx 250\text{MeV} \).
borrowed from QED: lifetime of electron–photon fluctuations:
\[ e(P) \rightarrow e(p) + \gamma(k) \]

- estimate: use uncertainty relation and Lorentz time dilation
  \[ P^2 = (p + k)^2 = M_{\text{virt}}^2 \] the virtual mass of the incident electron
  \[ \text{life time} = \text{life time in rest frame} \cdot \text{time dilation} \]

\[ \tau \sim \frac{1}{M_{\text{virt}}} \cdot \frac{E}{M_{\text{virt}}} = \frac{E}{(p + k)^2} \sim \frac{E}{2Ek(1 - \cos \theta)} \approx \frac{k}{k^2 \sin^2 \theta/2} \approx \frac{\omega}{k^2} \]

- lifetime larger with smaller transverse momentum (i.e. with larger transverse distance)

- same pattern also in QCD
- **physical interpretation:**
  equivalent quanta = quantum manifestation of accompanying fields

- **in absence of interaction:** recombination enforced by coherence

- **but:** hard interaction possibly “kicks out” quantum
  → coherence broken
  → equivalent (virtual) quanta become real
  → emission pattern unravels

- **alternative idea:**
  initial state radiation of photons off incident electron
Hadrons in initial state: DGLAP equations of QCD

- define **probabilities** (at LO) to find a parton $q$ – quark or gluon – in hadron $h$ at energy fraction $x$ and resolution parameter/scale $Q$:
  
  parton distribution function (PDF) $f_{q/h}(x, Q^2)$

- scale-evolution of PDFs: DGLAP equations

\[
\frac{\partial}{\partial \log Q^2} \left( \begin{array}{c}
    f_{q/h}(x, Q^2) \\
    f_{g/h}(x, Q^2)
\end{array} \right) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \left( \begin{array}{cc}
    P_{qq} \left( \frac{x}{z} \right) & P_{qg} \left( \frac{x}{z} \right) \\
    P_{gq} \left( \frac{x}{z} \right) & P_{gg} \left( \frac{x}{z} \right)
\end{array} \right) \left( \begin{array}{c}
    f_{q/h}(z, Q^2) \\
    f_{g/h}(z, Q^2)
\end{array} \right),
\]
QCD splitting functions:

\[
P_{qq}^{(1)}(x) = C_F \left[ \frac{1 + x^2}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right] = \left[ P_{qq}^{(1)}(x) \right]_+ + \gamma_q^{(1)} \delta(1 - x)
\]

\[
P_{qg}^{(1)}(x) = T_R \left[ x^2 + (1 - x)^2 \right] = P_{qg}^{(1)}(x)
\]

\[
P_{gq}^{(1)}(x) = C_F \left[ \frac{1 + (1 - x)^2}{x} \right] = P_{gq}^{(1)}(x)
\]

\[
P_{gg}^{(1)}(x) = 2C_A \left[ \frac{x}{(1 - x)_+} + \frac{1 - x}{x} + x(1 - x) \right] + \frac{11C_A - 4n_f}{6} T_R \delta(1 - x) = \left[ P_{gg}^{(1)}(x) \right]_+ + \gamma_g^{(1)} \delta(1 - x).
\]

remark: IR regularisation by +–prescription & terms \(\sim \delta(1 - x)\) from physical conditions on splitting functions

(flavour conservation for \(q \rightarrow qg\) and momentum conservation for \(g \rightarrow gg, q\bar{q}\))
Hadron production: Scales

- consider QCD final state radiation
- pattern for \( q \to qg \) similar to \( \ell \to \ell\gamma \) in QED:

\[
dw^{q\to qg} = \frac{\alpha_s(k_{\perp}^2)}{2\pi} C_F \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{d\omega}{\omega} \left[ 1 + \left( \frac{1 - \omega}{E} \right)^2 \right]
\]

\[
\omega = E(1-z) \quad \Rightarrow \quad \frac{\alpha_s(k_{\perp}^2)}{2\pi} C_F \frac{dk_{\perp}^2}{k_{\perp}^2} dz \frac{1 + z^2}{1 - z} = \frac{\alpha_s(k_{\perp}^2)}{2\pi} C_F \frac{dk_{\perp}^2}{k_{\perp}^2} dz P_{qg}^{(1)}(z).
\]

- divergent structures for:
  - \( z \to 1 \) (soft divergence) \( \leftrightarrow \) infrared/soft logarithms
  - \( k_{\perp}^2 \to 0 \) (collinear/mass divergence) \( \leftrightarrow \) collinear logarithms
- cut regularise with cut-off \( k_{\perp,\text{min}} \sim 1\text{GeV} > \Lambda_{\text{QCD}} \)
find two perturbative regimes:

- a regime of **jet production**, where $k_\perp \sim k_\parallel \sim \omega \gg k_{\perp,\text{min}}$ and emission probabilities scale like $w \sim \alpha_s(k_\perp) \ll 1$; and
- a regime of **jet evolution**, where $k_{\perp,\text{min}} \leq k_\perp \ll k_\parallel \leq \omega$ and therefore emission probabilities scale like $w \sim \alpha_s(k_\perp) \log^2 k_{\perp}^2 \gtrsim 1$.

- in jet production:
  standard fixed–order perturbation theory

- in jet evolution regime,
  perturbative parameter not $\alpha_s$ any more but rather towers of $\exp[\alpha_s \log k_{\perp}^2 \log k_\parallel]$

- induces counting of leading logarithms (LL), $\alpha_s L^{2n}$, next-to leading logarithms (NLL), $\alpha_s L^{2n-1}$, etc.
PART II: MONTE CARLO
FOR PERTURBATIVE QCD
MONTE CARLO FOR

PARTON LEVEL
Contents

3.a) Calculating matrix elements efficiently
3.b) Phase spacing for professionals
3.c) Including higher order corrections
3.d) Cancellation of IR divergences
3.e) Tools for LHC physics
Simulating hard processes (signals & backgrounds)

- Simple example: \( t \to bW^+ \to b\bar{\nu}_l: \)
  \[
  |\mathcal{M}|^2 = \frac{1}{2} \left( \frac{8\pi \alpha}{\sin^2 \theta_W} \right)^2 \frac{p_t \cdot p_{\nu} \cdot p_{b} \cdot p_{l}}{(p_{W}^2 - M_{W}^2)^2 + \Gamma_{W}^2 M_{W}^2}
  \]

- Phase space integration (5-dim):
  \[
  \Gamma = \frac{1}{2m_t} \frac{1}{128\pi^3} \int dp_{W}^2 \frac{d^2\Omega_{W}}{4\pi} \frac{d^2\Omega}{4\pi} \left( 1 - \frac{p_{W}^2}{m_t^2} \right) |\mathcal{M}|^2
  \]

- 5 random numbers \( \Rightarrow \) four-momenta \( \Rightarrow \) "events".
- Apply smearing and/or arbitrary cuts.
- Simply histogram any quantity of interest - no new calculation for each observable
stating the problem(s):
- multi-particle final states for signals & backgrounds.
- need to evaluate $d\sigma_N$:

$$\int_{\text{cuts}} \prod_{i=1}^{N} \frac{d^3 q_i}{(2\pi)^3 2E_i} \delta^4 \left( p_1 + p_2 - \sum_i q_i \right) |M_{p_1p_2\rightarrow N}|^2.$$ 

- problem 1: factorial growth of number of amplitudes.
- problem 2: complicated phase-space structure.
- solutions: numerical methods.
example for factorial growth: $e^+e^- \rightarrow q\bar{q} + ng$
obvious: traditional textbook methods (squaring, completeness relations, traces) fail
→ result in proliferation of terms ($M_i M_j^*$)

better ideas of efficient ME calculation:
→ realise: amplitudes just are complex numbers,
→ add them before squaring!

remember: spinors, gamma matrices have explicit form could be evaluated numerically (brute force)
but: Rough method, lack of elegance, CPU-expensive

can do better with smart basis for spinors (see detour)
this is still on the base of traditional Feynman diagrams!
Phase spacing for professionals

("Amateurs study strategy, professionals study logistics")

- democratic, process-blind integration methods:
  - Rambo/Mambo: Flat & isotropic
    
  
  - HAAG/Sarge: Follows QCD antenna pattern
    

- multi-channelling: each Feynman diagram related to a phase space mapping (= "channel"), optimise their relative weights

  

- main problem: practical only up to $\mathcal{O}(10^k)$ channels.

- some improvement by building phase space mappings recursively: more channels feasible, efficiency drops a bit.
basic idea of multichannel sampling (again):
use a sum of functions $g_i(\vec{x})$ as Jacobean $g(\vec{x})$.

$$\Rightarrow g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x});$$
$$\Rightarrow \text{condition on weights like stratified sampling;}$$
(“combination” of importance & stratified sampling).

algorithm for one iteration:
- select $g_i$ with probability $\alpha_i \rightarrow \vec{x}_j$.
- calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$
- add $f(\vec{x}_j)/g(\vec{x}_j)$ to total result and $f(\vec{x}_j)/g_i(\vec{x}_j)$ to partial (channel-) results.
- after $N$ sampling steps, update a-priori weights.

this is the method of choice for parton level event generation!
quality measure for integration performance: **unweighting efficiency**

want to generate events “as in nature”.

basic idea: use hit-or-miss method;

- generate $\vec{x}$ with integration method,
- compare actual $f(\vec{x})$ with maximal value during sampling
  \[ \Rightarrow “Unweighted events”. \]

comments:

- **unweighting efficiency**, $w_{\text{eff}} = \langle f(\vec{x}_j)/f_{\text{max}} \rangle = \text{number of trials for each event.}$
- expect $\log_{10} w_{\text{eff}} \approx 3 - 5$ for good integration of multi-particle final states at tree-level.
- maybe acceptable to use $f_{\text{max,eff}} = K f_{\text{max}}$ with $K < 1$.
  
  problem: what to do with events where $f(\vec{x}_j)/f_{\text{max,eff}} > 1$?

  answer: Add $\int [f(\vec{x}_j)/f_{\text{max,eff}}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\text{max,eff}} = k$. 

Including higher order corrections

- obtained from adding diagrams with additional:
  - loops (virtual corrections) or
  - legs (real corrections)

- effect: reducing the dependence on $\mu_R$ & $\mu_F$
NLO allows for meaningful estimate of uncertainties

- additional difficulties when going NLO:
  - ultraviolet divergences in virtual correction
  - infrared divergences in real and virtual correction

  enforce

  UV regularisation & renormalisation
  IR regularisation & cancellation

(Kinoshita–Lee–Nauenberg–Theorem)
Structure of NLO calculations

\[ d\sigma = d\Phi_B B_N(\Phi_B) + d\Phi_B V_N(\Phi_B) + d\Phi_R R_N(\Phi_R) \]

\[ = d\Phi_B \left( B_N + V_N + I_N^{(S)} \right) + d\Phi_R \left( R_N - S_N \right) \]

after adding a zero: \[ I_N^{(S)} = \int d\Phi_R / B S_N \]
Parton-level Monte Carlo

Parton showers – the basics

Including higher order corrections

- phase space factorisation assumed here ($\Phi_R = \Phi_B \otimes \Phi_1$)

$$\int d\Phi_1 S_N(\Phi_B \otimes \Phi_1) = I^{(S)}_N(\Phi_B)$$

- process independent, universal subtraction kernels

$$S_N(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes S_1(\Phi_B \otimes \Phi_1)$$

$$I^{(S)}_N(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes I^{(S)}_1(\Phi_B),$$

and invertible phase space mapping (e.g. Catani-Seymour)

$$\Phi_R \leftrightarrow \Phi_B \otimes \Phi_1$$
Aside: choices . . .

- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:

  unphysical scale choices will yield unphysical results

- more ways of botching it at higher orders
Availability of exact calculations (hadron colliders)

- fixed order matrix elements ("parton level") are exact to a given perturbative order.
- important to understand limitations: only tree-level and one-loop level fully automated, beyond: prototyping

\[ \text{done} \quad \text{for some processes} \quad \text{first solutions} \]
Survey of existing parton-level tools @ tree–level

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### Survey of existing parton-level tools @ NLO

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GOING MONTE CARLO

PARTON SHOWERS – THE BASICS
Contents

4.a) An analogy: radioactive decays
4.b) The pattern of QCD radiation
4.c) Quantum improvements
4.d) Compact notation
An analogy: Radioactive decays

- consider radioactive decay of an unstable isotope with half-life \( \tau \).

- “survival” probability after time \( t \) is given by
  \[
  S(t) = P_{\text{nodec}}(t) = \exp\left[-\frac{t}{\tau}\right]
  \]

  (note “unitarity relation”: \( P_{\text{dec}}(t) = 1 - P_{\text{nodec}}(t) \).)

- probability for an isotope to decay at time \( t \):
  \[
  \frac{dP_{\text{dec}}(t)}{dt} = -\frac{dP_{\text{nodec}}(t)}{dt} = \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right)
  \]

- now: connect half-life with width \( \Gamma = 1/\tau \).
- probability for isotope decay at any fixed time \( t \) determined by \( \Gamma \).
• spice things up now: add time-dependence, $\Gamma = \Gamma(t')$

• rewrite

$$\Gamma t \longrightarrow \int_0^t dt' \Gamma$$

• decay-probability at a given time $t$ is given by

$$\frac{dP_{\text{dec}}(t)}{dt} = \Gamma(t) \exp \left[ - \int_0^t dt' \Gamma(t') \right] = \Gamma(t) P_{\text{nodec}}(t)$$

(unitarity strikes again: $dP_{\text{dec}}(t)/dt = -dP_{\text{nodec}}(t)/dt$)

• interpretation of l.h.s.:
  • first term is for the actual decay to happen.
  • second term is to ensure that no decay before $t$
    $\implies$ conservation of probabilities.
the exponential is - of course - called the \textbf{Sudakov form factor}. 

F. Krauss
Simulations in High-Energy Physics
The pattern of QCD radiation

- a detour: Altarelli-Parisi equation, once more
- AP describes the scaling behaviour of the parton distribution function

\[
\frac{dq(x, Q^2)}{d \ln Q^2} = \int_x^1 \frac{dy}{y} \left[ \alpha_s(Q^2) P_q(x/y) \right] q(y, Q^2)
\]

- term in square brackets determines the probability that the parton emits another parton at scale \( Q^2 \) and Bjorken-parameter \( y \)
  (after the splitting, \( x \rightarrow yx + (1 - y)x \)).
- driving term: Splitting function \( P_q(x) \)
  important property: universal, process independent
Rederiving the splitting functions

- differential cross section for gluon emission in $e^+ e^- \rightarrow$ jets

$$\frac{d\sigma_{ee\rightarrow 3j}}{dx_1 dx_2} = \sigma_{ee\rightarrow 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

**singular** for $x_1, x_2 \rightarrow 1$.

- rewrite with opening angle $\theta_{qg}$ and gluon energy fraction $x_3 = 2E_g/E_{c.m.}$:

$$\frac{d\sigma_{ee\rightarrow 3j}}{d\cos \theta_{qg} dx_3} = \sigma_{ee\rightarrow 2j} \frac{C_F \alpha_s}{\pi} \left[ \frac{2}{\sin^2 \theta_{qg}} \frac{1 + (1 - x_3)^2}{x_3} - x_3 \right]$$

**singular** for $x_3 \rightarrow 0$ (**soft**), $\sin \theta_{qg} \rightarrow 0$ (**collinear**).
re-express collinear singularities

\[
\frac{2d \cos \theta_{qg}}{\sin^2 \theta_{qg}} = \frac{d \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{d \cos \theta_{qg}}{1 + \cos \theta_{qg}}
\]

\[
= \frac{d \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{d \cos \theta_{\bar{q}g}}{1 - \cos \theta_{\bar{q}g}} \approx \frac{d \theta_{qg}^2}{\theta_{qg}^2} + \frac{d \theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2}
\]

independent evolution of two jets (q and \(\bar{q}\))

\[
d\sigma_{ee \rightarrow 3j} \approx \sigma_{ee \rightarrow 2j} \sum_{j \in \{q, \bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{d \theta_{jg}^2}{\theta_{jg}^2} P(z)
\]
note: same form for any $t \propto \theta^2$:
- transverse momentum $k_\perp^2 \approx z^2(1 - z)^2E^2\theta^2$
- invariant mass $q^2 \approx z(1 - z)E^2\theta^2$

\[
\frac{d\theta^2}{\theta^2} \approx \frac{dk_\perp^2}{k_\perp^2} \approx \frac{dq^2}{q^2}
\]

- parametrisation-independent observation: (logarithmically) divergent expression for $t \to 0$.
- practical solution: cut-off $Q_0^2$.
  \[\implies\] divergence will manifest itself as $\log Q_0^2$.
- similar for $P(z)$: divergence for $z \to 0$ cured by cut-off.
what is a parton?
collinear pair/soft parton recombine!
introduce resolution criterion $k_\perp > Q_0$.

combine virtual contributions with unresolvable emissions:
cancels infrared divergences $\implies$ finite at $O(\alpha_s)$

(Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

unitarity: probabilities add up to one
$P(\text{resolved}) + P(\text{unresolved}) = 1$. 

$\begin{align*} 
&\begin{array}{c}
\text{collinear pair/soft parton recombine!} \\
\text{introduce resolution criterion } k_\perp > Q_0.
\end{array} \\
&\begin{array}{c}
\text{combine virtual contributions with unresolvable emissions:} \\
\text{cancels infrared divergences } \implies \text{finite at } O(\alpha_s)
\end{array} \\
&\begin{array}{c}
\text{(Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)}
\end{array} \\
&\begin{array}{c}
\text{unitarity: probabilities add up to one} \\
\text{$P(\text{resolved}) + P(\text{unresolved}) = 1$.}
\end{array}
\end{align*}$
• the Sudakov form factor, once more
• differential probability for emission between $q^2$ and $q^2 + dq^2$:

$$d\mathcal{P} = \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} \int_{z_{\text{min}}}^{z_{\text{max}}} dz P(z) =: dq^2 \Gamma(q^2)$$

• from radioactive example: evolution equation for $\Delta$

$$-\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2) \frac{d\mathcal{P}}{dq^2} = \Delta(Q^2, q^2) \Gamma(q^2)$$

$$\Rightarrow \Delta(Q^2, q^2) = \exp \left[ -\int_{q^2}^{Q^2} dk^2 \Gamma(k^2) \right]$$
- maximal logs if emissions ordered
- impacts on radiation pattern: in each emission $t$ becomes smaller

\[ q_1^2 > q_2^2 > q_3^2, \quad q_1^2 > q_2'^2 \]
Quantum improvements

- improvement: inclusion of various quantum effects
- trivial: effect of summing up higher orders (loops) $\alpha_s \rightarrow \alpha_s(k^2_\perp)$

- much faster parton proliferation, especially for small $k^2_\perp$.
- avoid Landau pole: $k^2_\perp > Q^2_0 \gg \Lambda^2_{\text{QCD}} \implies Q^2_0 = \text{physical parameter}$.
• soft limit for single emission also universal
• problem: soft gluons come from all over (not collinear!)
  quantum interference? still independent evolution?
• answer: not quite independent.
• consider case in QED
assume photon into $e^+ e^-$ at $\theta_{ee}$ and photon off electron at $\theta$

- photon momentum denoted as $k$

- energy imbalance at vertex: $k^\gamma_\perp \sim k_\parallel \theta$, hence $\Delta E \sim k^2_\perp / k_\parallel \sim k_\parallel \theta^2$.

- formation time for photon emission:
  $\Delta t \sim 1/\Delta E \sim k_\parallel / k^2_\perp \sim 1/(k_\parallel \theta^2)$.

- $ee$-separation: $\Delta b \sim \theta_{ee} \Delta t$

- must be larger than transverse wavelength of photon:
  $\theta_{ee} / (k_\parallel \theta^2) > 1 / k_\perp = 1 / (k_\parallel \theta)$

- thus: $\theta_{ee} > \theta$ must be satisfied for photon to form

- angular ordering as manifestation of quantum coherence
pictorially:

GLUONS AT LARGE ANGLE FROM COMBINED COLOUR CHARGE!
experimental manifestation:
\( \Delta R \) of 2\textsuperscript{nd} & 3\textsuperscript{rd} jet in multi-jet events in pp-collisions

\[ \begin{align*}
E_{T1} > 110 \text{ GeV}, \ E_{T2} > 10 \text{ GeV}.
\end{align*} \]
Parton showers, compact notation

- Sudakov form factor **(no-decay probability)**

\[ \Delta_{ij,k}^{(\mathcal{K})}(t, t_0) = \exp \left[ - \int_{t_0}^{t} \frac{dt}{t} \frac{\alpha_s}{2\pi} \int dz \frac{d\phi}{2\pi} \mathcal{K}_{ij,k}(t, z, \phi) \right] \]

- evolution parameter \( t \) defined by kinematics
  - generalised angle (HERWIG ++) or transverse momentum (PYTHIA, SHERPA)
  - will replace \( \frac{dt}{t} dz \frac{d\phi}{2\pi} \rightarrow d\Phi_1 \)
- scale choice for strong coupling: \( \alpha_s(k_\perp^2) \)
- regularisation through cut-off \( t_0 \)

F. Krauss
Simulations in High-Energy Physics

Parton–level Monte Carlo

Parton showers – the basics

Parton showers, compact notation
"compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off $n$-particle final state:

$$
\mathcal{K}_n(\Phi_1) = \frac{\alpha_s}{2\pi} \sum_{\text{all } \{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}), \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp \left[ - \int_{t_0}^{t} d\Phi_1 \mathcal{K}_n(\Phi_1) \right]
$$

consider first emission only off Born configuration

$$d\sigma_B = d\Phi_N B_N(\Phi_N)$$

$$
\cdot \left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \left[ \mathcal{K}_N(\Phi_1) \Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}
$$

integrates to unity $\rightarrow$ "unitarity" of parton shower

further emissions by recursion with $Q^2 = t$ of previous emission
Connection to resummation

- consider standard Collins-Soper-Sterman \(Q_T\)-formalism (CSS):

\[
\frac{d\sigma_{AB\to X}}{dy dQ_{\perp}^2} = d\Phi_X B_{ij}(\Phi_X) \cdot \int \frac{d^2 b_{\perp}}{(2\pi)^2} \exp(i\vec{b}_{\perp} \cdot \vec{Q}_{\perp}) \tilde{W}_{ij}(b; \Phi_X)
\]

guarantee 4-mom conservation \hspace{1cm} \text{higher orders}

with

\[
\tilde{W}_{ij}(b; \Phi_X) = \underbrace{C_i(b; \Phi_X, \alpha_s)C_j(b; \Phi_X, \alpha_s)}_{\text{collinear bits}} + \underbrace{H_{ij}(\alpha_s)}_{\text{loops}}
\]

\[
\exp \left[ - \int^{Q_X^2}_{1/b_{\perp}^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \left( A(\alpha_s(k_{\perp}^2)) \log \frac{Q_X^2}{k_{\perp}^2} + B(\alpha_s(k_{\perp}^2)) \right) \right]
\]

\[
\text{Sudakov form factor, } A, B \text{ expanded in powers of } \alpha_s
\]
- analyse structure of emissions above
- logarithmic accuracy in \( \log \frac{\mu N}{k_\perp} \) (a la CSS)
  possibly up to next-to leading log,
  - if evolution parameter \( \sim \) transverse momentum,
  - if argument in \( \alpha_s \) is \( \propto k_\perp \) of splitting,
  - if \( K_{ij,k} \rightarrow \) terms \( A_{1,2} \) and \( B_1 \) upon integration
    (OK, if soft gluon correction is included, and if \( K_{ij,k} \rightarrow \) AP splitting kernels)

- in CSS \( k_\perp \) typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions
- resummation scale \( \mu_N \approx \mu_F \) given by (Born) kinematics –
  simple for cases like \( q\bar{q}' \rightarrow V, gg \rightarrow H, \ldots \)
  tricky for more complicated cases
Example: achievable precision of shower alone in DY

\[ p_T \] spectrum, \( Z \to ee \) (dressed)

\[ \phi^*_\eta \] spectrum, \( Z \to ee \) (dressed)
Another systematic uncertainty

- Parton showers are approximations, based on leading colour, leading logarithmic accuracy, spin-averaged.
- Parametric accuracy by comparing Sudakov form factors:

$$\Delta = \exp \left\{ - \int \frac{d k_\perp^2}{k_\perp^2} \left[ A \log \frac{k_\perp^2}{Q^2} + B \right] \right\},$$

where $A$ and $B$ can be expanded in $\alpha_s(k_\perp^2)$.
- Showers usually include terms $A_{1,2}$ and $B_1$ (NLL).
- $A_2$ realised by pre-factor multiplying scale $\mu_R \simeq k_\perp$.

(CMW rescaling: Catani, Marchesini, Webber, Nucl Phys B,349 635)

- Fixed-order precision necessitates to consistently assess uncertainties from parton showers.
- Maybe improve by including higher orders?

(F. Krauss  IPPP  Simulations in High-Energy Physics)
Event generation (on-the-fly scale variations)

- basic idea: want to vary scales to assess uncertainties
- simple reweighting in matrix elements straightforward
- reweighting in parton shower more cumbersome
  - shower is probabilistic, concept of weight somewhat alien
  - introduce relative weight
  - evaluate (trial-)emission by (trial-)emission
Implementation in HERWIG7
Weight variation for $W + \text{jets}$ with MEPS@NLO

- **uncertainties in $p_T^W$**
  - $W p_T$ uncertainty bands
    - Sherpa MEPS@NLO
      - $pp \rightarrow W(\ell\nu)$, $\sqrt{s} = 13 \text{ TeV}$
      - $n_{\text{NLOPS}} = 1$, $n_{\text{PS}} = 2$
    - Sherpa CT14
      - $pp \rightarrow W(\ell\nu)$, $\sqrt{s} = 13 \text{ TeV}$
      - $n_{\text{NLOPS}} = 1$, $n_{\text{PS}} = 2$

- **CPU budget**
  - Sherpa MEPS@LO
    - $n_{\text{NLOPS}} = 0$
  - Sherpa MEPS@NLO
    - $n_{\text{NLOPS}} = 1$, $n_{\text{PS}} = 0$
  - Sherpa NLOPS
    - parton-level only
    - + non-perturbative effects
    - + unweighting
    - $n_{\text{NLOPS}} = 1$, $n_{\text{PS}} = 0$
Going beyond leading colour

- start including next-to leading colour

(first attempts by Platzer & Sjodahl; Nagy & Soper)

average transverse momentum w.r.t. $\vec{n}_3$

- also included in 1st emission in SHERPA’s MC@NLO
Towards higher logarithmic accuracy

- reproduce DGLAP evolution at NLO
  - include all NLO splitting kernels
- corrections to standard $1 \rightarrow 2$ trivial
  - 2-loop cusp term subtracted &
  - combined with LO soft contribution
- use weighting algorithms

(Hoeche, Schumann, Siegert, 0912.3501)

- new topology at NLO from
  - $q \rightarrow \bar{q}$ and $q \rightarrow q'$ splittings
- generic $1 \rightarrow 3$ process in parton shower
- implementation complete and
  - cross-checked (PYTHIA vs. SHERPA)
Comparison with data: $e^- e^+ \rightarrow \text{hadrons}$

(Hoeche, FK & Prestel, 1705.00982)
Comparison with data: $e^- e^+ \rightarrow \text{hadrons}$

(Hoeche, FK & Prestel, 1705.00982)
Comparison with data: DY at LHC

(Hoeche, FK & Prestel, 1705.00982)
ROUND III: PRECISION MONTE CARLO
FIRST IMPROVEMENTS:

ME CORRECTIONS
Contents

5.a) Improving event generators
5.b) Matrix-element corrections
Improving event generators

The inner working of event generators
... simulation: divide et impera

- **hard process:**
  fixed order perturbation theory
  traditionally: Born-approximation

- **bremsstrahlung:**
  resummed perturbation theory

- **hadronisation:**
  phenomenological models

- **hadron decays:**
  effective theories, data

- **"underlying event":**
  phenomenological models
... and possible improvements

possible strategies:

- improving the phenomenological models:
  - “tuning” (fitting parameters to data)
  - replacing by better models, based on more physics
    
    (my hot candidate: “minimum bias” and “underlying event” simulation)

- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:
    
    “NLO-Matching” & “Multijet-Merging”
  - systematic improvement of the parton shower:
    next-to leading (or higher) logs & colours
• remember structure of NLO calculation for $N$-body production

$$d\sigma = d\Phi_B B_N(\Phi_B) + d\Phi_B V_N(\Phi_B) + d\Phi_R R_N(\Phi_R)$$

$$= d\Phi_B \left(B_N + V_N + I_N^{(S)}\right) + d\Phi_R (R_N - S_N)$$

• phase space factorisation assumed here ($\Phi_R = \Phi_B \otimes \Phi_1$)

$$\int d\Phi_1 S_N(\Phi_B \otimes \Phi_1) = I_N^{(S)}(\Phi_B)$$

• process independent subtraction kernels

$$S_N(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes S_1(\Phi_B \otimes \Phi_1)$$

$$I_N^{(S)}(\Phi_B \otimes \Phi_1) = B_N(\Phi_B) \otimes I_1^{(S)}(\Phi_B)$$

with universal $S_1(\Phi_B \otimes \Phi_1)$ and $I_1^{(S)}(\Phi_B)$
Matrix element corrections

- Parton shower ignores interferences typically present in matrix elements
- Pictorially:
  \[
  \text{ME} : \left| \begin{array}{c}
  \text{process 1} \\
  \text{process 2}
  \end{array} \right|^2 + \left| \begin{array}{c}
  \text{process 1} \\
  \text{process 2}
  \end{array} \right|^2
  \]
  \[
  \text{PS} : \left| \begin{array}{c}
  \text{process 1} \\
  \text{process 2}
  \end{array} \right|^2 + \left| \begin{array}{c}
  \text{process 1} \\
  \text{process 2}
  \end{array} \right|^2
  \]

- Form many processes $R_N < B_N \times K_N$
- Typical processes: $q\bar{q}' \rightarrow V$, $e^- e^+ \rightarrow q\bar{q}$, $t \rightarrow bW$
- Practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $P = R_N / (B_N \times K_N)$
analyse **first** emission, given by

\[ d\sigma_B = d\Phi_N B_N(\Phi_N) \]

\[
\left\{ \Delta_N^{(R/B)}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \left[ \frac{R_N(\Phi_N \times \Phi_1)}{B_N(\Phi_N)} \Delta_N^{(R/B)}(\mu_N^2, t(\Phi_1)) \right] \right\}
\]

once more: integrates to unity \(\rightarrow\) “unitarity” of parton shower

- radiation given by \( \mathcal{R}_N \) (correct at \( O(\alpha_s) \))
  - (but modified by logs of higher order in \( \alpha_s \) from \( \Delta_N^{(R/B)} \))
- emission phase space constrained by \( \mu_N \)
- also known as “soft ME correction”
  - hard ME correction fills missing phase space
- used for “power shower”:
  \( \mu_N \rightarrow E_{pp} \) and apply ME correction
PRECISION MONTE CARLO

(N)NLO MATCHING
Contents

6.a) Basic idea
6.b) Powheg
6.c) MC@NLO
6.d) NNLO - the new frontier
NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution
  \textit{(where the logs are large)}

- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production
  \textit{(where the logs are small)}

- adjust ("match") terms:
  - cross section at \textbf{NLO accuracy} &
    correct hardest emission in PS to exactly
    reproduce ME at order $\alpha_s$
    ($\mathcal{R}$-part of the NLO calculation)
    \textit{(this is relatively trivial)}

  - maintain \textbf{(N)LL-accuracy} of parton shower
    \textit{(this is not so simple to see)}
PowHeg

- reminder: $K_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/B_N$ in soft/collinear regions of phase space

$$d\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{B_N(\Phi_N)} \overset{\text{IR}}{\to} d\Phi_1 \frac{\alpha_s}{2\pi} K_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta^{(\mathcal{R}/B)}_N(\mu_N^2, t_0) = \exp \left[ - \int_{t_0}^{\mu_N^2} d\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{B_N(\Phi_N)} \right],$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of $\alpha_s$ to parton shower scale
1. define local $K$-factors
2. start from Born configuration $\Phi_N$ with NLO weight:

\[
\frac{d\sigma^{(\text{NLO})}}{d\Phi_N} = d\Phi_N \, \tilde{B}(\Phi_N)
\]

\[
= d\Phi_N \left\{ B_N(\Phi_N) + \tilde{\nu}_N(\Phi_N) + B_N(\Phi_N) \otimes S \tilde{\nu}_N(\Phi_N) \right\} + \int d\Phi_1 \left[ R_N(\Phi_N \otimes \Phi_1) - B_N(\Phi_N) \otimes dS(\Phi_1) \right] \right\}
\]

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $R_N \equiv B_N \otimes dS$

(relevant for MC@NLO)
analyse accuracy of radiation pattern

generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0)$:

$$d\sigma_N^{(NLO)} = d\Phi_N \tilde{B}(\Phi_N) \times \left\{ \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \frac{\mathcal{R}_N(\Phi_N \otimes \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, k_{\perp}^2(\Phi_1)) \right\}$$

integrating to yield $1 - \text{"unitarity of parton shower"}$

radiation pattern like in ME correction

pitfall, again: choice of upper scale $\mu_N^2$

apart from logs: which configurations enhanced by local $K$-factor

(this is vanilla POWHEG!)

(K-factor for inclusive production of $X$ adequate for $X$+ jet at large $p_\perp$?)
• large enhancement at high $p_{T,h}$
• can be traced back to large NLO correction
• fortunately, NNLO correction is also large $\rightarrow \sim$ agreement
- improving POWHEG
- split real-emission ME as

\[ \mathcal{R} = \mathcal{R} \left( \frac{h^2}{p_\perp^2 + h^2} + \frac{p_\perp^2}{p_\perp^2 + h^2} \right) \]

- can “tune” \( h \) to mimick NNLO - or other (resummation) result
- differential event rate up to first emission

\[ d\sigma = d\Phi B^{(S)} \left( \Delta^{(S)/B}(s, t_0) + \int_{t_0}^{s} d\Phi_1 \frac{\mathcal{R}^{(S)}}{B} \Delta^{(S)/B}(s, k_\perp^2) \right) \]

\[ + d\Phi_R \mathcal{R}^{(F)}(\Phi_R) \]
MC@NLO

- **MC@NLO paradigm**: divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:
  \[ \mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes dS_1 + \mathcal{H}_N \]

- Identify subtraction terms and shower kernels $dS_1 \equiv \sum_{\{ij,k\}} K_{ij,k}$
  
  (modify $K$ in 1st emission to account for colour)

\[
d\sigma_N = d\Phi_N \tilde{B}_N(\Phi_N) \left[ \Delta_N^{(K)}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 K_{ij,k}(\Phi_1) \Delta_N^{(K)}(\mu_N^2, k_{\perp}^2) \right] + d\Phi_{N+1} \mathcal{H}_N
\]

- **Effect**: only resummed parts modified with local $K$-factor
- phase space effects: shower vs. fixed order

- problem: impact of subtraction terms on local $K$-factor
  (filling of phase space by parton shower)
- studied in case of $gg \rightarrow H$ above
- proper filling of available phase space by parton shower paramount
NNLOPS in the MINLO approach: merging without $Q_J$


- based on POWHEG + shower from PYTHIA or HERWIG
- up to today only for singlet $S$ production, gives NNLO + PS
- basic idea:
  - use $S$+jet in POWHEG
  - push jet cut to parton shower IR cutoff
  - apply analytical NNLL Sudakov rejection weight for intrinsic line in Born configuration
  - (kills divergent behaviour at order $\alpha_s$)
- don’t forget double-counted terms
- reweight to NNLO fixed order
NNLOPS for $H$ production

NNLOPS– the new frontier

NNLOPS for $Z$ production: UNNLOPS

S. Hoche, Y. Li, & S. Prestel, Phys.Rev.D90 & D91

- also available for $H$ production
NNLOPS: shortcomings/limitations

- **MINLO** relies on knowledge of $B_2$ terms from analytic resummation
  $\rightarrow$ to date only known for colour singlet production

- **MINLO** relies on reweighting with full NNLO result
  $\rightarrow$ one parameter for $H (y_H)$, more complicated for $Z$, ...

- **UNNLOPS** relies on integrating single- and double emission to low scales and combination of unresolved with virtual emissions
  $\rightarrow$ potential efficiency issues, need NNLO subtraction

- **UNNLOPS** puts unresolved & virtuals in “zero-emission” bin
  $\rightarrow$ no parton showering for virtuals (?)
PRECISION MONTE CARLO

MULTIJET MERGING
Contents

7.a) Basic idea
7.b) Multijet merging at LO
7.c) Multijet merging at NLO
Multijet merging: basic idea

- Parton shower resums logarithms
- Fair description of collinear/soft emissions
- Jet evolution (where the logs are large)
- Matrix elements exact at given order
- Fair description of hard/large-angle emissions
- Jet production (where the logs are small)
- Combine ("merge") both:
  - Result: "towers" of MEs with increasing number of jets evolved with PS
    - Multijet cross sections at Born accuracy
    - Maintain (N)LL accuracy of parton shower
• separate regions of jet production and jet evolution with jet measure $Q_J$
  
  (“truncated showering” if not identical with evolution parameter)

• matrix elements populate hard regime

• parton showers populate soft domain
Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \rightarrow \text{hadrons}$
  - Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor $\alpha_s$ and up to $\log^2 \frac{E_{\text{c.m.}}}{Q_J}$ per jet
- use Sudakov form factor for resummation & replace approximate fixed order by exact expression:

$$R_2(Q_J) = \left[ \Delta_q(E_{\text{c.m.}}^2, Q_J^2) \right]^2$$

$$R_3(Q_J) = 2\Delta_q(E_{\text{c.m.}}^2, Q_J^2) \int_{Q_j^2}^{E_{\text{c.m.}}^2} \frac{dk_\perp^2}{k_\perp^2} \left[ \frac{\alpha_s(k_\perp^2)}{2\pi} \right] d z K_q(k_\perp^2, z) \times \Delta_q(E_{\text{c.m.}}^2, k_\perp^2) \Delta_q(k_\perp^2, Q_J^2) \Delta_g(k_\perp^2, Q_J^2)$$
Multijet merging at LO

- expression for first emission

\[ d\sigma = d\Phi_N B_N \left[ \Delta_N^{(K)}(\mu_N^2, t_0) \right. \]

\[ + \int_{t_0}^{\mu_N^2} d\Phi_1 K_N \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \]

\[ + d\Phi_{N+1} B_{N+1} \Delta_N^{(K)}(\mu_{N+1}^2, t_{N+1}) \Theta(Q_{N+1} - Q_J) \]

- note: \( N+1 \)-contribution includes also \( N+2, N+3, \ldots \)
  (no Sudakov suppression below \( t_{N+1} \), see further slides for iterated expression)

- potential occurrence of different shower start scales: \( \mu_N, N+1, \ldots \)

- “unitarity violation” in square bracket: \( B_N K_N \rightarrow B_{N+1} \)
  (cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &
\[ d\sigma = \sum_{n=N}^{n_{\text{max}}-1} \left\{ d\Phi_n B_n \left[ \prod_{j=N}^{n-1} \Theta(Q_{j+1} - Q_J) \right] \right\} \left[ \prod_{j=N}^{n-1} \Delta_j^{(K)}(t_j, t_{j+1}) \right] \]

\[ \times \left[ \Delta_n^{(K)}(t_n, t_0) + \int_{t_0}^{t_n} d\Phi_1 K_n \Delta_n^{(K)}(t_n, t_{n+1}) \Theta(Q_J - Q_{n+1}) \right] \]

\[ + d\Phi_{n_{\text{max}}} B_{n_{\text{max}}} \left[ \prod_{j=N}^{n_{\text{max}}-1} \Theta(Q_{j+1} - Q_J) \right] \left[ \prod_{j=N}^{n_{\text{max}}-1} \Delta_j^{(K)}(t_j, t_{j+1}) \right] \]

\[ \times \left[ \Delta_{n_{\text{max}}}^{(K)}(t_{n_{\text{max}}}, t_0) + \int_{t_0}^{t_{n_{\text{max}}}} d\Phi_1 K_{n_{\text{max}}} \Delta_{n_{\text{max}}}^{(K)}(t_{n_{\text{max}}}, t_{n_{\text{max}}+1}) \right] \]
Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])
Aside: Comparison with higher order calculations
Multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

**maintain NLO and (N)LL accuracy of ME and PS**

- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities
First emission(s), once more

\[
d\sigma = d\Phi_N \tilde{B}_N \left[ \Delta^{(K)}_N (\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 K_N \Delta^{(K)}_N (\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \\
+ d\Phi_{N+1} H_N \Delta^{(K)}_N (\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \\
+ d\Phi_{N+1} \tilde{B}_{N+1} \left( 1 + \frac{B_{N+1}}{\tilde{B}_{N+1}} \int_{t_{N+1}}^{\mu_N^2} d\Phi_1 K_N \right) \Theta(Q_{N+1} - Q_J) \\
\cdot \left[ \Delta^{(K)}_{N+1} (t_{N+1}, t_0) + \int_{t_0}^{t_{N+1}} d\Phi_1 K_{N+1} \Delta^{(K)}_{N+1} (t_{N+1}, t_{N+2}) \right] \\
+ d\Phi_{N+2} H_{N+1} \Delta^{(K)}_N (\mu_N^2, t_{N+1}) \Delta^{(K)}_{N+1} (t_{N+1}, t_{N+2}) \Theta(Q_{N+1} - Q_J) + \ldots
\]
First improvements

Matching

Multijet merging

EW corrections

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp_H^\perp} \quad \text{[pb/GeV]}
\]

\[
pp \rightarrow h + \text{jets}
\]

\[
\text{SHERPA S-MC@NLO}
\]

first emission by Mc@NLO

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Simulations in High-Energy Physics
First improvements
Matching
Multijet merging
EW corrections

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

\[
\frac{d\sigma}{dp^\perp} \quad [\text{pb/GeV}]
\]

\( pp \to h + \text{jets} \)

\( pp \to h + 0j \) @ NLO

- first emission by MC@NLO, restrict to \( Q_{n+1} < Q_{\text{cut}} \)

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Simulations in High-Energy Physics
First improvements

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Multijet merging

EW corrections

---

Multijet merging at NLO

$p_H$ in MEPS@NLO

Transverse momentum of the Higgs boson

\[ \frac{d\sigma}{dp_{\perp}} \text{[pb/GeV]} \]

- first emission by MC@NLO, restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- MC@NLO \( pp \to h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)

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Simulations in High-Energy Physics
First improvements
Matching
Multijet merging
EW corrections

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \rightarrow h + \text{jet} \) to \( Q_{n+2} < Q_{\text{cut}} \)

Transverse momentum of the Higgs boson \( p_\perp(h) \) [GeV]

\[ \frac{d\sigma}{dp_\perp} [\text{pb/GeV}] \]

\( pp \rightarrow h + \text{jets} \)
- \( pp \rightarrow h + 0j \) @ NLO
- \( pp \rightarrow h + 1j \) @ NLO

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Multijet merging at NLO

$\mathcal{O}_{\text{MEPs@NLO}}$

Transverse momentum of the Higgs boson

$\frac{d\sigma}{dp_{\perp}}$ [pb/GeV]

- $pp \rightarrow h + \text{jets}$
- $pp \rightarrow h + 0j @ \text{NLO}$
- $pp \rightarrow h + 1j @ \text{NLO}$
- $pp \rightarrow h + 2j @ \text{NLO}$

- first emission by MC@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \rightarrow h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$

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Multijet merging

EW corrections

Multijet merging at NLO

\( p_H^\perp \) in MEPS@NLO

Transverse momentum of the Higgs boson

\[
pp \rightarrow h + \text{jets} \\
\quad \text{d} \sigma / \text{d} p_H^\perp \ [\text{pb/GeV}] \\
\begin{array}{c}
0.5 \\
10^{-4} \\
10^{-3} \\
10^{-2} \\
10^{-1} \\
\end{array}
\]

- first emission by \( \text{MC@NLO} \), restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + \text{jet} \) for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \rightarrow h + \text{jet} \) to \( Q_{n+2} < Q_{\text{cut}} \)
- \( \text{MC@NLO} \) \( pp \rightarrow h + 2\text{jets} \) for \( Q_{n+2} > Q_{\text{cut}} \)
- iterate
First improvements

Matching

Multijet merging

EW corrections

Multijet merging at NLO

$p_H^\perp$ in MEPS@NLO

Transverse momentum of the Higgs boson

Transverse momentum of the Higgs boson $p_{\perp}(h)$ [GeV]

$\frac{d\sigma}{dp_{\perp}}$ [pb/GeV]

$pp \rightarrow h + \text{jets}$

- $pp \rightarrow h + 0j$ @ NLO
- $pp \rightarrow h + 1j$ @ NLO
- $pp \rightarrow h + 2j$ @ NLO
- $pp \rightarrow h + 3j$ @ LO

- first emission by $\text{MC@NLO}$, restrict to $Q_{n+1} < Q_{\text{cut}}$
- $\text{MC@NLO}$ $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \rightarrow h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- $\text{MC@NLO}$ $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate

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**\( p_\perp^H \) in MEPS@NLO**

Transverse momentum of the Higgs boson

- **first emission by** MEPS@NLO, restrict to \( Q_{n+1} < Q_{\text{cut}} \)
- **MC@NLO** \( pp \to h + \text{jet} \)
  - for \( Q_{n+1} > Q_{\text{cut}} \)
- restrict emission off \( pp \to h + \text{jet} \) to \( Q_{n+2} < Q_{\text{cut}} \)
- **MC@NLO**
  - \( pp \to h + 2\text{jets} \) for \( Q_{n+2} > Q_{\text{cut}} \)
- iterate
- sum all contributions

---

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Simulations in High-Energy Physics
$p_H^\perp$ in MEPS@NLO

Transverse momentum of the Higgs boson

- First emission by MC@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- Restrict emission off $pp \rightarrow h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- Iterate
- Sum all contributions
- Eg. $p_\perp(h) > 200$ GeV has contributions from multiple topologies
Example: **MEPS@NLO for $W$+jets**

(up to two jets @ NLO, from BLACKHAT, see arXiv: 1207.5031 [hep-ex])

![Inclusive Jet Multiplicity](image)
First improvements

Matching

Multijet merging

EW corrections

Simulations in High-Energy Physics

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IPPP
FxFx: validation in $Z$+jets

(Data from ATLAS, 1304.7098, aMC@NLO _MADGRAPH with HERWIG++)

(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: Mc@NLO)
FxFx: validation in $Z+\text{jets}$

(Data from ATLAS, 1304.7098, aMC@NLO _MADGRAPH_ with HERWIG++)

(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: Mc@NLO)
### Differences between MEPS@NLO, UNLOPS & FxFx

<table>
<thead>
<tr>
<th></th>
<th>FxFx</th>
<th>MEPS@NLO</th>
<th>UNLOPS</th>
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</thead>
<tbody>
<tr>
<td><strong>ME</strong></td>
<td>all internal</td>
<td>( \nabla ) external</td>
<td>all external</td>
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<tr>
<td></td>
<td>\textit{aMc@NLO .MadGraph}</td>
<td>\textit{Comix or AMEGIC++}</td>
<td>\textit{Comix or AMEGIC++}</td>
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<td>external</td>
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<td>\textit{HERWIG or PYTHIA}</td>
<td>\textit{PYTHIA}</td>
<td>\textit{PYTHIA}</td>
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<tr>
<td>( \Delta_N )</td>
<td>\textit{analytical}</td>
<td>from PS per emission</td>
<td>from PS per emission</td>
</tr>
<tr>
<td>( \Theta(Q_J) )</td>
<td>\textit{a-posteriori}</td>
<td>( &gt; ) Sudakov regime</td>
<td>( \approx ) Sudakov regime</td>
</tr>
<tr>
<td><strong>Q_J-range</strong></td>
<td>relatively high</td>
<td>( \approx 10% )</td>
<td>( \approx 10% )</td>
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<tr>
<td></td>
<td>(but changed)</td>
<td>\textit{Θ((Q_J))}</td>
<td>\textit{Θ((Q_J))}</td>
</tr>
</tbody>
</table>

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*Simulations in High-Energy Physics*
Higgs-$p_{\perp}$: exclusive over inclusive rate

- \approx 20\% of Higgs with $p_{\perp} = 60$ GeV are not accompanied by a jet
PRECISION MONTE CARLO

ELECTROWEAK CORRECTIONS
Contents

8.a) Motivation
8.b) Multijet merging at LO
8.c) Multijet merging at NLO
Motivation: the size of EW corrections

- EW corrections sizeable $\mathcal{O}(10\%)$ at large scales: must include them!
- but: more painful to calculate
- need EW showering & possibly corresponding PDFs
  (somewhat in its infancy: chiral couplings)

- example: $Z\gamma$ vs. $p_T$ (right plot)
  (handle on $p_T^Z$ in $Z \rightarrow \nu \bar{\nu}$)
  (Kallweit, Lindert, Pozzorini, Schoenherr for LH'15)
- difference due to EW charge of $Z$
- no real correction (real $V$ emission)
- improved description of $Z \rightarrow \ell\ell$

\[ Z/\gamma \text{ ratio for events with } n_{jets} \geq 1 \]
Inclusion of electroweak corrections in simulation

- incorporate approximate electroweak corrections in MEPS@NLO
  1. using electroweak Sudakov factors
     \[ \tilde{B}_n(\Phi_n) \approx \tilde{B}_n(\Phi_n) \Delta_{EW}(\Phi_n) \]
  2. using virtual corrections and approx. integrated real corrections
     \[ \tilde{B}_n(\Phi_n) \approx \tilde{B}_n(\Phi_n) + V_{n,EW}(\Phi_n) + I_{n,EW}(\Phi_n) + B_{n,mix}(\Phi_n) \]
- real QED radiation can be recovered through standard tools
  (parton shower, YFS resummation)
- simple stand-in for proper QCD⊕EW matching and merging
  → validated at fixed order, found to be reliable,
    difference \( \lesssim 5\% \) for observables not driven by real radiation
Results: $pp \rightarrow \ell^- \bar{\nu} + \text{jets}$

(Kallweit, Lindert, Maierhöfer, Pozzorini, Schoenherr JHEP04(2016)021)

⇒ particle level events including dominant EW corrections
NLO EW predictions for $\Delta R(\mu, j_1)$

measure collinear $W$ emission?

LHC@8TeV, $p_{j_1}^\perp > 500\ \text{GeV}$, central $\mu$ and jet

- LO $pp \rightarrow Wj$ with $\Delta \phi(\mu, j) \approx \pi$
NLO EW predictions for $\Delta R(\mu, j_1)$

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- LO $pp \rightarrow Wj$ with $\Delta \phi(\mu, j) \approx \pi$
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- NLO corrections neg. in peak
- large $pp \rightarrow Wjj$ component opening PS
- sub-leading Born ($\gamma$PDF) at large $\Delta R$
- restrict to exactly 1j, no $p_T^{j_2} > 100\text{ GeV}$
NLO EW predictions for $\Delta R(\mu, j_1)$

measure collinear $W$ emission?
LHC@8TeV, $p_{j_1}^\perp > 500$ GeV, central $\mu$ and jet

- LO $pp \rightarrow Wj$ with $\Delta \phi(\mu, j) \approx \pi$
- NLO corrections neg. in peak
- large $pp \rightarrow Wjj$ component opening PS
- sub-leading Born ($\gamma$PDF) at large $\Delta R$
- restrict to exactly 1$j$, no $p_{j_2}^\perp > 100$ GeV
- describe $pp \rightarrow Wjj$ @ NLO, $p_{j_2}^\perp > 100$ GeV
NLO EW predictions for $\Delta R(\mu, j_1)$

measure collinear $W$ emission?

LHC@8TeV, $p^j_\perp > 500$ GeV, central $\mu$ and jet

- LO $pp \to Wj$ with $\Delta \phi(\mu, j) \approx \pi$
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- restrict to exactly 1$j$, no $p^j_\perp > 100$ GeV
- describe $pp \to Wjj$ @ NLO, $p^j_\perp > 100$ GeV
- pos. NLO QCD, $\sim$ flat
NLO EW predictions for $\Delta R(\mu, j_1)$

measure collinear $W$ emission?

LHC@8TeV, $p_T^{j_1} > 500\text{ GeV}$, central $\mu$ and jet

- LO $pp \rightarrow Wj$ with $\Delta \phi(\mu, j) \approx \pi$
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- restrict to exactly 1j, no $p_T^{j_2} > 100\text{ GeV}$
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NLO EW predictions for $\Delta R(\mu, j_1)$

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- pos. NLO QCD, neg. NLO EW, $\sim$ flat
- sub-leading Born contribs positive
NLO EW predictions for $\Delta R(\mu, j_1)$

measure collinear $W$ emission?

LHC@8TeV, $p_{j_1}^\perp > 500\text{ GeV}$, central $\mu$ and jet
- LO $pp \to Wj$ with $\Delta \phi(\mu, j) \approx \pi$
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- describe $pp \to Wjj$ @ NLO, $p_{j_2}^\perp > 100\text{ GeV}$
- pos. NLO QCD, neg. NLO EW, $\sim$ flat
- sub-leading Born contribs positive
- sub$^2$leading Born (diboson etc) conts. pos. $\to$ possible double counting with BG
NLO EW predictions for $\Delta R(\mu, j_1)$

measure collinear $W$ emission?

LHC@8TeV, $p_T^{j_1} > 500$ GeV, central $\mu$ and jet
- LO $pp \rightarrow Wj$ with $\Delta \phi(\mu, j) \approx \pi$
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- restrict to exactly 1$j$, no $p_T^{j_2} > 100$ GeV
- describe $pp \rightarrow Wjj$ @ NLO, $p_T^{j_2} > 100$ GeV
- pos. NLO QCD, neg. NLO EW, $\sim$ flat
- sub-leading Born contribs positive
- sub$^2$leading Born (diboson etc) conts. pos. $\rightarrow$ possible double counting with BG
- merge using exclusive sums
NLO EW predictions for $\Delta R(\mu, j_1)$

Data comparison

(M. Wu ICHEP’16, ATLAS arXiv:1609.07045)

- **ALPGEN+PYTHIA**
  $pp \rightarrow W + \text{jets MLM merged}$
  (Mangano et.al., JHEP07(2003)001)

- **PYTHIA 8**
  $pp \rightarrow Wj + \text{QCD shower}$
  $pp \rightarrow jj + \text{QCD+EW shower}$
  (Christiansen, Prestel, EPJC76(2016)39)

- **SHERPA+OPENLOOPS**
  NLO QCD+EW+subLO
  $pp \rightarrow Wj/Wjj$ excl. sum
  (Kallweit, Lindert, Maierhöfer,)
  (Pozzorini, Schoenherr, JHEP04(2016)021)

- **NNLO QCD** $pp \rightarrow Wj$
  (Boughezal, Liu, Petriello, arXiv:1602.06965)
NLO EW predictions for $\Delta R(\mu, j_1)$

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SIMULATING SOFT QCD
SIMULATING SOFT QCD

HADRONISATION
Contents

9.a) Connection to QCD
9.b) General ideas
9.c) String model
9.d) Cluster model
9.e) Some questions
QCD radiation, once more

- remember QCD emission pattern

\[ \mathrm{d}w^{q \rightarrow qg} = \frac{\alpha_s(k^2)}{2\pi} C_F \frac{k^2}{k^2} \frac{\mathrm{d}\omega}{\omega} \left[ 1 + \left( 1 - \frac{\omega}{E} \right) \right]. \]

- spectrum cut-off at small transverse momenta and energies by onset of hadronization, at scales \( R \approx 1 \text{ fm}/\Lambda_{\text{QCD}} \)

- two (extreme) classes of emissions: gluons and gluers determined by relation of formation and hadronization times
• gluers formed at times $R$, with momenta $k_{∥} \sim k_{⊥} \sim \omega \gtrsim 1/R$

• assuming that hadrons follow partons,

$$dN_{(\text{hadrons})} \sim \int_{k_{⊥}>1/R}^{Q} \frac{dk_{⊥}^2}{k_{⊥}^2} \frac{C_F \alpha_s(k_{⊥}^2)}{2\pi} \left[ 1 + \left( 1 - \frac{\omega}{E} \right) \right] \frac{d\omega}{\omega}$$

$$\sim \frac{C_F \alpha_s(1/R^2)}{\pi} \log(Q^2R^2) \frac{d\omega}{\omega}$$

or - relating their energy with that of the gluers -

$$dN_{(\text{hadrons})}/d\log \epsilon = \text{const.},$$

a plateau in log of energy (or in rapidity)
- impact of additional radiation
- new partons must separate before they can hadronize independently
- therefore, one more time

\[ t^{\text{form}} \sim \frac{k_{\parallel}}{k_{\perp}^2} \]
\[ t^{\text{sep}} \sim R\theta \sim t^{\text{form}}(Rk_{\perp}) \]
\[ t^{\text{had}} \sim k_{\parallel}R^2 \sim t^{\text{form}}(Rk_{\perp})^2. \]

- for gluers \( Rk_{\perp} \approx 1 \): all times the same
- naively; new & more hadrons following new partons
- but: colour coherence
  primary and secondary partons not separated enough in

\[ \frac{1}{R} \sim \omega_{\text{(hadron)}} \sim \frac{1}{R\theta} \]

and therefore no independent radiation
Hadronisation: General thoughts

- confinement the striking feature of low-scale strong interactions
- transition from partons to their bound states, the hadrons
- the Meissner effect in QCD

QED:

QCD:
- linear QCD potential in Quarkonia – like a string
combine some experimental facts into a naive parameterisation

- in $e^+e^- \rightarrow$ hadrons: exponentially decreasing $p_\perp$, flat plateau in $y$ for hadrons

$$\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$$
use parameterisation to “guesstimate” hadronisation effects:

\[
E = \int_0^Y dy d\rho_\perp(p_\perp^2) p_\perp \cosh y = \lambda \sinh Y
\]

\[
P = \int_0^Y dy d\rho_\perp(p_\perp^2) p_\perp \sinh y = \lambda (\cosh Y - 1) \approx E - \lambda
\]

\[
\lambda = \int d\rho_\perp(p_\perp^2) p_\perp = \langle p_\perp \rangle.
\]

- estimate \( \lambda \sim 1/R_{\text{had}} \approx m_{\text{had}} \), with \( m_{\text{had}} \) 0.1-1 GeV.
- effect: jet acquire non-perturbative mass \( \sim 2\lambda E \) (\( \mathcal{O}(10\text{GeV}) \)) for jets with energy \( \mathcal{O}(100\text{GeV}) \).
similar parametrization underlying Feynman-Field model for independent fragmentation

- recursively fragment $q \rightarrow q' + \text{had}$, where
  - transverse momentum from (fitted) Gaussian;
  - longitudinal momentum arbitrary (hence from measurements);
  - flavour from symmetry arguments + measurements.

- problems: frame dependent, “last quark”, infrared safety, no direct link to perturbation theory, . . . .
The string model

- a simple model of mesons: yoyo strings
  - light quarks \( (m_q = 0) \) connected by string, form a meson
  - area law: \( m_{\text{had}}^2 \propto \text{area of string motion} \)
  - \( L=0 \) mesons only have 'yo-yo' modes:
 • turn this into hadronisation model $e^+e^- \rightarrow q\bar{q}$ as test case
 • ignore gluon radiation: $q\bar{q}$ move away from each other, act as point-like source of string
 • intense chromomagnetic field within string: more $q\bar{q}$ pairs created by tunnelling and string break-up
 • analogy with QED (Schwinger mechanism): $d\mathcal{P} \sim dxdt \exp \left( -\pi m_q^2/\kappa \right)$, $\kappa = \text{“string tension”}$.
- string model = well motivated model, constraints on fragmentation (Lorentz-invariance, left-right symmetry, ...)
- how to deal with gluons?
- interpret them as kinks on the string \(\Rightarrow\) the string effect

- infrared-safe, advantage: smooth matching with PS.
The cluster model

- underlying idea: preconfinement/LPHD
  - typically, neighbouring colours will end in same hadron
  - hadron flows follow parton flows $\rightarrow$ don’t produce any hadrons at places where you don’t have partons
  - works well in large-$N_c$ limit with planar graphs

- follow evolution of colour in parton showers
paradigm of cluster model: clusters as continuum of hadron resonances

trace colour through shower in $N_c \to \infty$ limit

force decay of gluons into $q\bar{q}$ or $\bar{d}d$ pairs, form colour singlets from neighbouring colours, usually close in phase space

mass of singlets: peaked at low scales $\approx Q_0^2$

decay heavy clusters into lighter ones or into hadrons
(here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like)

if light enough, clusters will decay into hadrons

naively: spin information washed out, decay determined through phase space only $\to$ heavy hadrons suppressed (baryon/strangeness suppression)
self–similarity of parton shower will end with roughly the same local distribution of partons, with roughly the same invariant mass for colour singlets

adjacent pairs colour connected, form colourless (white) clusters.

clusters (≈ excited hadrons) decay into hadrons
Practicalities

- practicalities of hadronisation models: parameters
  - kinematics of string or cluster decay: 2-5 parameters
  - must “pop” quark or diquark flavours in string or cluster decay — cannot be completely democratic or driven by masses alone
    $\rightarrow$ suppression factors for strangeness, diquarks 2-10 parameters
  - transition to hadrons, cannot be democratic over multiplets
    $\rightarrow$ adjustment factors for vectors/tensors etc. 2-6 parameters
- tuned to LEP data, overall agreement satisfying
- validity for hadron data not quite clear

(beam remnant fragmentation not in LEP.)

- there are some issues with inclusive strangeness/baryon production
Collective flow observed in pp at LHC. Partly unexpected. New mechanisms required; could also (partly) replace CR.

Active field, e.g. N. Fischer & TS, arXiv:1610:09818 [hep-ph]:

- Thermal $\exp(-p_\perp/T) \rightarrow \exp(-m_\perp/T)$ hadronic spectrum.
- Close-packed strings $\Rightarrow$ increased string $\kappa$ or $T$.
- Dense hadronic gas $\Rightarrow$ hadronic rescattering.

(Slide stolen from Torbjorn Sjostrand)
Strange strangeness

- universality of hadronisation assumed
- parameters tuned to LEP data
- in particular: strangeness suppression
- for strangeness: flat ratios
- but data do not reproduce this
- looks like $SU(3)$ restoration
- not observed for protons
- needs to be investigated
SIMULATING SOFT QCD

UNDERLYING EVENT
Contents

10.a) Multiple parton scattering
10.b) Modelling the underlying event
10.c) Some results
10.d) Practicalities
Multiple parton scattering

- hadrons = extended objects!
- no guarantee for one scattering only.
- running of $\alpha_s$\
  $\Rightarrow$ preference for soft scattering.
first experimental evidence for double–parton scattering: events with $\gamma + 3$ jets:
- cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| < 1.3$;
- “clean sample”: two softest jets with $E_T < 7$ GeV;
- cross section for DPS

$$\sigma_{\text{DPS}} = \frac{\sigma_{\gamma j} \sigma_{jj}}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} \approx 14 \pm 4 \text{ mb.}$$
• more measurements, also at LHC
• ATLAS results from $W + 2$ jets
but: how to define the underlying event?

1. everything apart from the hard interaction, but including IS showers, FS showers, remnant hadronisation.
2. remnant-remnant interactions, soft and/or hard.
3. lesson: hard to define
• origin of MPS: parton–parton scattering cross section exceeds hadron–hadron total cross section

\[ \sigma_{\text{hard}}(p_{\perp, \text{min}}) = \int \frac{s/4}{p_{\perp, \text{min}}^2} dp_{\perp}^2 \frac{d\sigma(p_{\perp}^2)}{dp_{\perp}^2} > \sigma_{pp, \text{total}} \]

for low \( p_{\perp, \text{min}} \)

• remember

\[ \frac{d\sigma(p_{\perp}^2)}{dp_{\perp}^2} = \int_0^1 dx_1 dx_2 f(x_1, q^2)f(x_2, q^2) \frac{d\hat{\sigma}_{2 \rightarrow 2}}{dp_{\perp}^2} \]

• \( \left\langle \sigma_{\text{hard}}(p_{\perp, \text{min}}) / \sigma_{pp, \text{total}} \right\rangle \geq 1 \)

• depends strongly on cut-off \( p_{\perp, \text{min}} \) (energy-dependent)!
Modelling the underlying event

- take the old PYTHIA model as example:
  - start with hard interaction, at scale $Q_{\text{hard}}^2$.
  - select a new scale $p_{\perp}^2$ from
    \[
    \exp \left[ -\frac{1}{\sigma_{\text{norm}}} \int_{p_{\perp}^2}^{Q_{\text{hard}}^2} dp_{\perp}^2 \frac{d\sigma(p_{\perp}^2)}{dp_{\perp}^2} \right]
    \]
    with constraint $p_{\perp}^2 > p_{\perp,\text{min}}^2$.
  - rescale proton momentum (“proton-parton = proton with reduced energy”).
  - repeat until no more allowed $2 \to 2$ scatter.
Modelling the underlying event

- possible refinements:
  - may add impact-parameter dependence $\longrightarrow$ more fluctuations
  - add parton showers to UE
  - “regularisation” to dampen sharp dependence on $p_{\perp,\text{min}}$: replace $1/\hat{t}$ in MEs by $1/(t + t_0)$, also in $\alpha_s$.
  - treat intrinsic $k_{\perp}$ of partons ($\rightarrow$ parameter)
  - model proton remnants ($\rightarrow$ parameter)
Some results for MPS in Z production

- observables sensitive to MPS
- classical analysis: transverse regions in QCD/jet events
- idea: find the hardest system, orient event into regions:
  - toward region along system
  - away region back-to-back
  - transverse regions
- typically each in 120°
Underlying Event

Some results in $Z$ production

F. Krauss

Simulations in High-Energy Physics
Some results in $Z$ production

F. Krauss

Simulations in High-Energy Physics
Hadronisation

Some results in $Z$ production

F. Krauss

IPPP

Simulations in High-Energy Physics
see some data comparison in Minimum Bias
practicalities of underlying event models: parameters
  - profile in impact parameter space 2-3 parameters
  - IR cut-off at reference energy, its energy evolution, dampening parameter and normalisation cross section 4 parameters
  - treating colour connections to rest of event 2-5 parameters
  - tuned to LHC data, overall agreement satisfying
  - energy extrapolation not exactly perfect, plus other process categories such as diffraction etc.
SUMMARY
Summary of fixed order

- NLO (QCD) “revolution” consolidated:
  - lots of routinely used tools for large FS multis (4 and more)
  - incorporation in MC tools done, need comparisons, critical appraisals and a learning curve in their phenomenological use
  - to improve: description of loop–induced processes

- amazing success in NNLO (QCD) calculations:
  - emergence of first round of $2 \rightarrow 2$ calculations
  - next revolution imminent (with question marks)
  - first MC tools for simple processes ($gg \rightarrow H$, DY), more to be learnt by comparison etc. (see above)

- first $N^3$LO calculation in $gg \rightarrow H$, more to come (?)

- attention turning to NLO (EW)
  - first benchmarks with new methods ($V+3j$)
  - calculational setup tricky
  - need maybe faster approximation for high-scales (EW Sudakovs)
Limitations of fixed order

- practical limitations/questions to be overcome:
  - dealing with IR divergences at NNLO: slicing vs. subtracting
    (I’m not sure we have THE solution yet)
  - how far can we push NNLO? are NLO automated results stable enough for NNLO at higher multiplicity?
  - users of codes: higher orders tricky → training needed
    (MC = black box attitude problematic - a new brand of pheno/experimenters needed?)

- limitations of perturbative expansion:
  - breakdown of factorisation at HO (Seymour et al.)
  - higher-twist: compare \((\alpha_s/\pi)^n\) with \(\Lambda_{QCD}/M_Z\)

- limitations in analytic resummation: process- and observable-dependent
  - first attempts at automation (CAESAR and some others) – checks/cross-comparison necessary

- showering needs to be improved
  (for NNLO the “natural” accuracy is NNLL)
Summary for event generation

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "PowHEG")
  - MeNLOps NLO matching & merging
  - MePS@NLO ("SHERPA", "UNLOPs", "MINLO", "FxFx")

- multijet merging an important tool for many relevant signals and backgrounds - pioneering phase at LO & NLO over
- complete automation of NLO calculations done → must benefit from it!

(it's the precision and trustworthy & systematic uncertainty estimates!)
we have constructed lots of tools for precision physics at LHC
but we did not cross-validate them careful enough (yet)
but we did not compare their theoretical foundations (yet)
we also need unglamorous improvements:

- systematically check advanced scale-setting schemes (MINLO)
- automatic (re-)weighting for PDFs & scales (ME: ✓, PS: -)
- scale compensation in PS is simple (implement and check)
- PDFs: to date based on FO vs. data — will we have to move to resummed/parton showered?

(reminder: LO* was not a big hit, though)

... and maybe we will have to go to the “dirty” corners:
higher-twist, underlying event, hadronization, ...

(many of those driven by experiment)
LIMITATIONS

UNTIL YOU SPREAD YOUR WINGS,
YOU'LL HAVE NO IDEA HOW FAR YOU CAN WALK.