Simulations in High-Energy Physics

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INTRODUCTION



motivation: the quest for precision

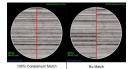
- LHC (and particle physics in general) in phase of SM "crash-testing": confirm minutiae of EWSB and gauge structure
- QCD effects often limiting factor: $p_{\perp}^{W,Z,H}$, m_{top} , $g \to Q\bar{Q}$, boosted objects & substructure
- necessary: work on better understanding of parton shower

(based on perturbtion theory: theory-driven with experimental validation)

also: multi-parton interactions, hadronization

(very phenomenological - parially or totally experiment-driven)





CSI LHC: need precise & accurate tools for precision physics

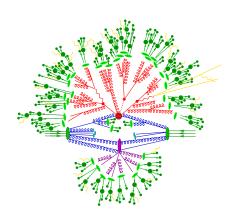
strategy of event generators

principle: divide et impera

 hard process: fixed order perturbation theory

traditionally: Born-approximation

- bremsstrahlung: resummed perturbation theory
- hadronisation: phenomenological models
- hadron decays: effective theories, data
- "underlying event": phenomenological models

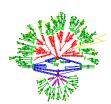


... and possible improvements

possible strategies:

- improving the phenomenological models:
 - "tuning" (fitting parameters to data)
 - replacing by better models, based on more physics

(my hot candidate: "minimum bias" and "underlying event" simulation)



- improving the perturbative description:
 - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

"NLO-Matching" & "Multijet-Merging"

 systematic improvement of the parton shower: next-to leading (or higher) logs & colours



aim of the lectures

review the state of the art in precision simulations

(celebrate success)

highlight missing or ambiguous theoretical ingredients

(acknowledge failure)

• (maybe) suggest some further studies – experiment and theory

(...)

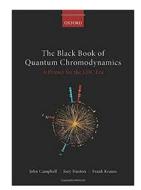
Plan of the Lectures

- Perturbative QCD
 - Parton Level
 - Parton Showers
- Precision Simulations
 - Matching
 - Merging
- Non-Perturbative QCD
 - Hadronization
 - Underlying Event

Shameless promotion:

(as instructed by my co-author J.Huston)

material of lectures covered in





MONTE CARLO FOR PERTURBATIVE QCD

simulating hard processes (signals & backgrounds)

• simple example: $t \to bW^+ \to b\bar{l}\nu_l$:

$$|\mathcal{M}|^2 = \frac{1}{2} \left(\frac{8\pi\alpha}{\sin^2\theta_W} \right)^2 \frac{p_t \cdot p_\nu \ p_b \cdot p_l}{(p_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2}$$



phase space integration (5-dim):

$$\Gamma = \tfrac{1}{2m_t} \tfrac{1}{128\pi^3} \int \mathrm{d}\rho_W^2 \tfrac{\mathrm{d}^2\Omega_W}{4\pi} \tfrac{\mathrm{d}^2\Omega}{4\pi} \left(1 - \tfrac{\rho_W^2}{m_t^2}\right) |\mathcal{M}|^2$$

- ullet 5 random numbers \Longrightarrow four-momenta \Longrightarrow "events".
- apply smearing and/or arbitrary cuts.
- Simply histogram any quantity of interest no new calculation for each observable

calculating matrix elements efficiently

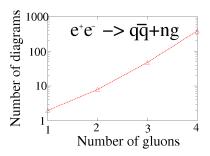
- stating the problem(s):
 - multi-particle final states for signals & backgrounds.
 - need to evaluate $d\sigma_N$:

$$\int\limits_{
m cuts} \left[\prod_{i=1}^N rac{{
m d}^3 q_i}{(2\pi)^3 2 E_i}
ight] \delta^4 \left(
ho_1 +
ho_2 - \sum_i q_i
ight) \left| \mathcal{M}_{
ho_1
ho_2
ightarrow N}
ight|^2.$$

- problem 1: factorial growth of number of amplitudes.
- problem 2: complicated phase-space structure.
- solutions: numerical methods.

ullet example for factorial growth: $e^+e^ightarrow qar q+ng$

n	$\#_{diags}$
0	1
1	2
2	8
3	48
4	384



- obvious: traditional textbook methods (squaring, completeness relations, traces) fail
 - \implies result in proliferation of terms $(\mathcal{M}_i \mathcal{M}_i^*)$
- better ideas of efficient ME calculation:
 - ⇒ realise: amplitudes just are complex numbers,
 - ⇒ add them before squaring!
- remember: spinors, gamma matrices have explicit form could be evaluated numerically (brute force)
 but: Rough method, lack of elegance, CPU-expensive
- can do better with smart basis for spinors (see detour)
- this is still on the base of traditional Feynman diagrams!

phase spacing for professionals

("Amateurs study strategy, professionals study logistics")

- democratic, process-blind integration methods:
 - Rambo/Mambo: Flat & isotropic

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R.Kleiss, W.J.Stirling & S.D.Ellis, Comput. Phys. Commun. 40 (1986) 359;
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HAAG/Sarge: Follows QCD antenna pattern

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A.van Hameren & C.G.Papadopoulos, Eur. Phys. J. C 25 (2002) 563.
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 multi-channelling: each Feynman diagram related to a phase space mapping (= "channel"), optimise their relative weights

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R.Kleiss & R.Pittau, Comput. Phys. Commun. 83 (1994) 141.
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- ullet main problem: practical only up to $\mathcal{O}(10\mathrm{k})$ channels.
- some improvement by building phase space mappings recursively: more channels feasible, efficiency drops a bit.

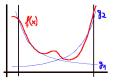
basic idea of multichannel sampling (again): use a sum of functions $g_i(\vec{x})$ as Jacobean $g(\vec{x})$.

$$\implies$$
 $g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x});$

condition on weights like stratified sampling; ("combination" of importance & stratified sampling).

algorithm for one iteration:

- select g_i with probability $\alpha_i \rightarrow \vec{x_j}$.
- lacktriangledown calculate total weight $g(\vec{x_j})$ and partial weights $g_i(\vec{x_j})$
- add $f(\vec{x_j})/g(\vec{x_j})$ to total result and $f(\vec{x_j})/g_i(\vec{x_j})$ to partial (channel-) results.
- after N sampling steps, update a-priori weights.



this is the method of choice for parton level event generation!

- quality measure for integration performance: unweighting efficiency
- want to generate events "as in nature".
- basic idea: use hit-or-miss method;
 - generate \vec{x} with integration method,
 - compare actual $f(\vec{x})$ with maximal value during sampling \implies "Unweighted events".

comments:

- unweighting efficiency, $w_{\rm eff} = \langle f(\vec{x_j})/f_{\rm max} \rangle =$ number of trials for each event.
- expect $\log_{10}w_{\rm eff}\approx 3-5$ for good integration of multi-particle final states at tree-level.
- maybe acceptable to use $f_{\mathrm{max,eff}} = K f_{\mathrm{max}}$ with K < 1. problem: what to do with events where $f(\vec{x_j})/f_{\mathrm{max,eff}} > 1$? answer: Add $\mathrm{int}[f(\vec{x_j})/f_{\mathrm{max,eff}}] = k$ events and perform hit-or-miss on $f(\vec{x_j})/f_{\mathrm{max,eff}} k$.

including higher order corrections

obtained from adding diagrams with additional:

loops (virtual corrections) or legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F NLO allows for meaningful estimate of uncertainties
- additional difficulties when going NLO:
 ultraviolet divergences in virtual correction
 infrared divergences in real and virtual correction

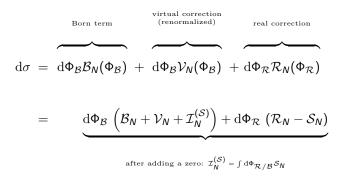
enforce

UV regularisation & renormalisation IR regularisation & cancellation

 $({\sf Kinoshita-Lee-Nauenberg-Theorem})$



structure of NLO calculations



• phase space factorisation assumed here $(\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1)$

$$\int \mathrm{d}\Phi_1 \mathcal{S}_{\mathcal{N}}(\Phi_{\mathcal{B}} \otimes \Phi_1) \, = \, \mathcal{I}_{\mathcal{N}}^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

process independent, universal subtraction kernels

$$\mathcal{S}_{N}(\Phi_{\mathcal{B}} \otimes \Phi_{1}) = \mathcal{B}_{N}(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_{1}(\Phi_{\mathcal{B}} \otimes \Phi_{1})$$

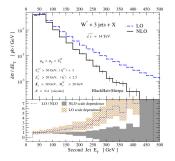
$$\mathcal{I}_{N}^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_{1}) = \mathcal{B}_{N}(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_{1}^{(\mathcal{S})}(\Phi_{\mathcal{B}}),$$

and invertible phase space mapping (e.g. Catani-Seymour)

$$\Phi_{\mathcal{R}} \; \longleftrightarrow \; \Phi_{\mathcal{B}} \otimes \Phi_1$$

aside: choices ...

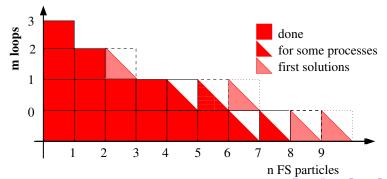
- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:
 unphysical scale choices will yield unphysical results



more ways of botching it at higher orders



- fixed order matrix elements ("parton level") are exact to a given perturbative order.
- important to understand limitations: only tree-level and one-loop level fully automated, beyond: prototyping



a simple analogy

• school book example: radiactive decay of isotopes with half-life $au=1/\Gamma$

$$\mathcal{N}(t) = \mathcal{N}(0) \exp[-t/\tau] = \mathcal{N}(0) \exp[-\Gamma t]$$

$$= \mathcal{N}(0) \exp\left[-\int_0^t dt' \Gamma(t')\right] = \mathcal{N}(0) \underbrace{\Delta(t, 0)}_{\text{Sudakov form factor}}$$

- Sudakov form factor = "survival" probability
- ullet decay probability for individual isotope at given time t

$$\frac{\mathrm{d}\mathcal{P}_{\mathrm{decay}}(t)}{\mathrm{d}t} = \overbrace{\Gamma(t)}^{\mathrm{decays}} \underbrace{\Delta(t,0)}_{\mathrm{didn't}} = -\frac{\mathrm{d}\mathcal{P}_{\mathrm{no}\,\mathrm{decay}}(t)}{\mathrm{d}t}$$

the pattern of QCD radiation

- a detour: Altarelli-Parisi equation
- AP describes the scaling behaviour of the parton distribution function

(which depends on Bjorken-parameter and scale \mathcal{Q}^2)

$$\frac{\mathrm{d}q(x, Q^2)}{\mathrm{d}\ln Q^2} = \int\limits_{x}^{1} \frac{\mathrm{d}y}{y} \left[\alpha_s(Q^2) P_q(x/y)\right] q(y, Q^2)$$

ullet term in square brackets determines the probability that the parton emits another parton at scale Q^2 and Bjorken-parameter y

(after the splitting,
$$x \rightarrow yx + (1 - y)x$$
.)

• driving term: Splitting function $P_q(x)$ important property: universal, process independent



first implementations used DGLAP splitting kernels:

$$\mathcal{K}_{ijk}(\Phi_1) = \frac{\mathrm{d}t\mathrm{d}\phi}{t} \frac{\alpha_S(\mu)}{2\pi} \hat{P}_{\{ij\} \to ij}(z)$$

with colour factors C and $\hat{P}_{\{ji\} \to ji}$ given by

$$P_{q \to qg}(z) = C_F \left[\frac{2}{1-z} - (1+z) \right]$$

$$P_{q \to gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right]$$

$$P_{g \to q\bar{q}}(z) = T_R \left[z^2 + (1-z)^2 \right]$$

$$P_{g \to gg}(z) = C_A \left[\frac{2}{1-z} + \frac{2}{z} - 2(z^2 - z + 2) \right]$$

 refinements of splitting kernels: Catani-Seymour subtraction kernels or symmetrised eikonal kernels, both including recoil factor (see later)

rederiving the splitting functions (pedestrianized)

ullet differential cross section for gluon emission in $e^+e^ightarrow$ jets

$$\frac{\mathrm{d}\sigma_{\mathsf{ee}\to3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{\mathsf{ee}\to2j} \frac{C_\mathsf{F}\alpha_\mathsf{s}}{\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

singular for $x_{1,2} \to 1$.

• rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\rm c.m.}$:

$$\frac{\mathrm{d}\sigma_{\mathsf{ee}\to3j}}{\mathrm{d}\cos\theta_{\mathsf{qg}}\mathrm{d}x_3} = \sigma_{\mathsf{ee}\to2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{\mathsf{qg}}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

singular for $x_3 \to 0$ ("soft"), $\sin \theta_{qg} \to 0$ ("collinear").

re-express collinear singularities

$$\begin{split} \frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} &= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} \\ &= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta_{qg}^2}{\theta_{qg}^2} + \frac{\mathrm{d}\theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2} \end{split}$$

• independent evolution of two jets $(q \text{ and } \bar{q})$

$$\mathrm{d}\sigma_{\mathrm{ee}\to3j} \approx \sigma_{\mathrm{ee}\to2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{\mathrm{d}\theta_{jg}^2}{\theta_{jg}^2} P(z) \;,$$

- note: same form for any $t \propto \theta^2$:
- ullet transverse momentum $k_\perp^2pprox z^2(1-z)^2E^2 heta^2$
- invariant mass $q^2 pprox z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

- parametrisation-independent observation: (logarithmically) divergent expression for t o 0.
- practical solution: cut-off Q_0^2 . \implies divergence will manifest itself as log Q_0^2 .
- similar for P(z): divergence for $z \to 0$ cured by cut-off.

- what is a parton? collinear pair/soft parton recombine!
- introduce resolution criterion $k_{\perp} > Q_0$.



• combine virtual contributions with unresolvable emissions: cancels infrared divergences \Longrightarrow finite at $\mathcal{O}(\alpha_s)$

(Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

• unitarity: probabilities add up to one $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1$.



- the Sudakov form factor, once more
- differential probability for emission between q^2 and $q^2 + dq^2$:

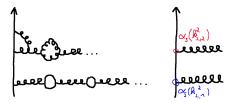
$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi} \frac{\mathrm{d}q^2}{q^2} \int_{z_{\mathrm{min}}}^{z_{\mathrm{max}}} \mathrm{d}z P(z) =: \mathrm{d}q^2 \, \Gamma(q^2)$$

ullet from radioactive example: evolution equation for Δ

$$-rac{\mathrm{d}\Delta(Q^2,\,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)rac{\mathrm{d}\mathcal{P}}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\Gamma(q^2)$$
 $\implies \Delta(Q^2,\,q^2) = \exp\left[-\int\limits_{q^2}^{Q^2}\mathrm{d}k^2\Gamma(k^2)
ight]$

quantum improvements: running coupling

- improvement: inclusion of various quantum effects
- trivial: effect of summing up higher orders (loops) $\alpha_s \to \alpha_s(k_\perp^2)$



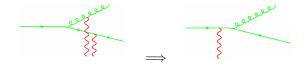
- much faster parton proliferation, especially for small k_{\perp}^2 .
- avoid Landau pole: $k_{\perp}^2 > Q_0^2 \gg \Lambda_{\rm QCD}^2 \Longrightarrow Q_0^2 = \text{physical parameter.}$

- consider two subsequent emissions and effect of interference: leads to (calculable) cancellations in parts of phase space
- QM considerations:
 - assume splittings $\gamma \to e^+ e^-$ with θ_{ee} and $e^- \to e^- \gamma$ at θ , with photon momentum k
 - energy imbalance at vertex: $k_\perp^\gamma \sim k_\parallel \theta$, hence $\Delta E \sim k_\perp^2/k_\parallel \sim k_\parallel \theta^2$.
 - formation time for photon emission: $\Delta t \sim 1/\Delta E \sim k_{\parallel}/k_{\perp}^2 \sim 1/(k_{\parallel}\theta^2)$.
 - ullet ee-separation: $\Delta b \sim heta_{ee} \Delta t$
 - must be larger than transverse wavelength of photon: $\theta_{\rm ee}/(k_\parallel \theta^2) > 1/k_\perp = 1/(k_\parallel \theta)$
 - thus: $\theta_{\rm ee}>\theta$ must be satisfied for photon to form



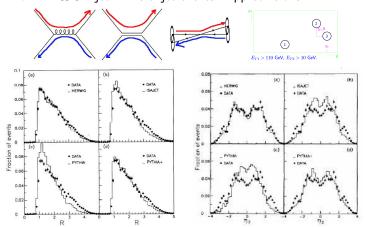
angular ordering (or similar) as manifestation of quantum coherence

- QCD: all quanta are colured
- pictorial solution



gluons at large angle from combined colour charge!

• experimental manifestation: ΔR of $2^{\rm nd}$ & $3^{\rm rd}$ jet in multi-jet events in pp-collisions



parton showers, compact notation

Sudakov form factor (no-decay probability)

$$\Delta_{ij,k}^{(\mathcal{K})}(t,t_0) = \exp\left[-\int\limits_{t_0}^t \frac{\mathrm{d}t}{t} \, \frac{\alpha_{\rm s}}{2\pi} \int \mathrm{d}z \frac{\mathrm{d}\phi}{2\pi} \quad \underbrace{\mathcal{K}_{ij,k}(t,z,\phi)}_{\text{splitting kernel for}}\right]$$
splitting kernel for
$$(ij) \to ij \text{ (spectator }k)$$

evolution parameter t defined by kinematics

generalised angle (HERWIG ++) or transverse momentum (PYTHIA, SHERPA)

- will replace $\frac{\mathrm{d}t}{t}\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi}\longrightarrow\mathrm{d}\Phi_1$
- scale choice for strong coupling: $\alpha_{\rm s}(k_{\perp}^2)$

resums classes of higher logarithms

ullet regularisation through cut-off t_0



• "compound" splitting kernels \mathcal{K}_n and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_{\textit{n}}(\Phi_1) = \frac{\alpha_{\mathsf{s}}}{2\pi} \sum_{\mathsf{all}\,\{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k})\,, \quad \Delta_{\textit{n}}^{(\mathcal{K})}(t,t_0) = \exp\left[-\int\limits_{t_0}^t \mathrm{d}\Phi_1\,\mathcal{K}_{\textit{n}}(\Phi_1)\right]$$

consider first emission only off Born configuration

$$\mathrm{d}\sigma_{\mathcal{B}} = \mathrm{d}\Phi_{\mathcal{N}}\,\mathcal{B}_{\mathcal{N}}(\Phi_{\mathcal{N}})$$

$$\cdot \left\{ \Delta_{\mathcal{N}}^{(\mathcal{K})}(\mu_{\mathcal{N}}^2, t_0) + \int\limits_{t_0}^{\mu_{\mathcal{N}}^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_{\mathcal{N}}(\Phi_1) \Delta_{\mathcal{N}}^{(\mathcal{K})}(\mu_{\mathcal{N}}^2, t(\Phi_1)) \right] \right\}$$
integrates to unity \(\to \) "unitarity" of parton shower

• further emissions by recursion with $Q^2 = t$ of previous emission

the link to resummation

- origin of observables such as $p_{\perp}^{W,z,H}$: (multiple) initial state parton emissions, boson "kicked" out by recoil
- resum emissions with Sudakov form factor
- build Sudakov form factor from "parton splitting kernels", in Q_T resummation:

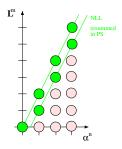
$$\Delta(Q^2, Q_0^2) = \exp \left[- \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}k_{\perp}^2}{k_{\perp}^2} \left(A(k_{\perp}^2) \log \frac{Q^2}{k_{\perp}^2} + B(k_{\perp}^2) \right) \right]$$

both A and B have expansion in α_S

• various schemes available: Q_T , SCET, etc.

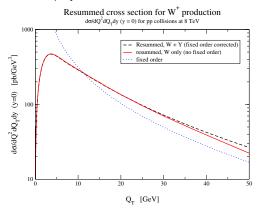
- analyse structure of emissions above
- logarithmic accuracy in $\log \frac{\mu_N}{k_\perp}$ (a la CSS) possibly up to next-to leading log,
 - ullet if evolution parameter \sim transverse momentum,
 - if argument in $lpha_{
 m s}$ is \propto \emph{k}_{\perp} of splitting,
 - ullet if $K_{ij,k}
 ightarrow$ terms $A_{1,2}$ and B_1 upon integration

(OK, if soft gluon correction is included, and if $K_{ij,k} o ext{AP}$ splitting kernels)



- in CSS k_{\perp} typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions
- resummation scale $\mu_N \approx \mu_F$ given by (Born) kinematics simple for cases like $q\bar{q}' \rightarrow V$, $gg \rightarrow H$, ... tricky for more complicated cases

• example result: interplay of fixed order and resummation



ullet note: parton shower will act similar to Q_T resummation

currently best realisation:

• evolution and splitting parameter $((ij) + k \rightarrow i + j + k)$:

$$\kappa_{j,jk}^2 \; = \; \frac{4(p_j p_j)(p_j p_k)}{Q^4} \quad \text{and} \quad z_j \; = \; \frac{2(p_j p_k)}{Q^2} \; .$$

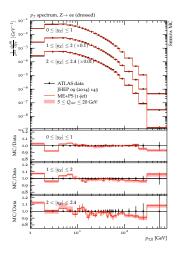
splitting functions including IR regularisation

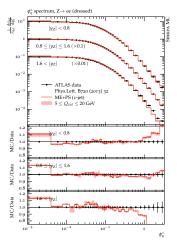
(a la Curci, Furmanski & Petronzio, Nucl.Phys. B175 (1980) 27-92)

$$\begin{split} P_{qq}^{(0)}(z,\kappa^2) &=& 2C_F \left[\frac{1-z}{(1-z)^2+\kappa^2} - \frac{1+z}{2} \right] \,, \\ P_{qg}^{(0)}(z,\kappa^2) &=& 2C_F \left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2} \right] \,, \\ P_{gg}^{s(0)}(z,\kappa^2) &=& 2C_A \left[\frac{1-z}{(1-z)^2+\kappa^2} - 1 + \frac{z(1-z)}{2} \right] \,, \\ P_{gq}^{(0)}(z,\kappa^2) &=& T_R \left[z^2 + (1-z)^2 \right] \end{split}$$

- ullet renormalisation/factorisation scale given by $\mu^2=\kappa^2Q^2$
- ullet combine gluon splitting from two splitting functions with different spectators k o accounts for different colour flows

example: achievable precision of shower alone in DY





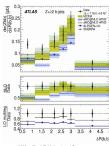
massive quarks are tricky

- parton showers geared towards collinear & soft emissions of gluons (double log structure)
- $g \rightarrow q\bar{q}$ only collinear, beyond "shower-approximation" \longrightarrow no soft gluon
- old measurements at of inclusive $g \rightarrow bb$ and $g \rightarrow c\bar{c}$ rate
- fix this at LHC for modern showers

(important for $t\bar{t}b\bar{b}$)

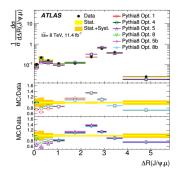
• questions: kernel, scale in α_s

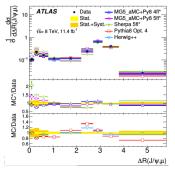
(example: k_{\parallel} vs. m_{bb})





- ullet ATLAS measurement in $bar{b}$ production
- use decay products in $B \to J/\Psi(\mu\mu) + X$ and $B \to \mu + X$
- ullet use muons as proxies, most obvious observable $\Delta R(J\Psi,\mu)$





massive quarks are tricky - encore

- heavy quarks also problematic in initial state:
 - no PDF support for $Q^2 \leq m_Q^2 \longrightarrow$ quarks stop showering
- possible solutions:
 - naive: ignore and leave for beam remnants (SHERPA)
 - better: enforce splitting in region around m_Q^2 (PYTHIA)
 - \longrightarrow effectively produces collinear Q and gluon in IS
- ullet will need to check effect on precision obsevables: $p_{\perp}^{(W)}/p_{\perp}^{(Z)}$

another systematic uncertainty

- parton showers are approximations, based on leading colour, leading logarithmic accuracy, spin-averaged
- parametric accuracy by comparing Sudakov form factors:

$$\Delta = \exp\left\{-\int \frac{\mathrm{d}k_{\perp}^2}{k_{\perp}^2} \left[A\log\frac{k_{\perp}^2}{Q^2} + B\right]\right\}\,,$$

where A and B can be expanded in $\alpha_s(k_{\perp}^2)$

- showers usually include terms $A_{1,2}$ and B_1 (NLL)
- ullet A_2 realised by pre-factor multiplying scale $\mu_R \simeq k_\perp$

(CMW rescaling: Catani, Marchesini, Webber, Nucl Phys B,349 635)

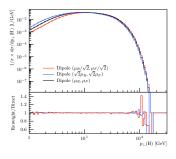
- fixed-order precision necessitates to consistently assess uncertainties from parton showers
- maybe improve by including higher orders?

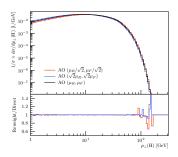


event generation (on-the-fly scale variations)

- basic idea: want to vary scales to assess uncertainties
- simple reweighting in matrix elements straightforward
- reweighting in parton shower more cumbersome
 - shower is probabilistic, concept of weight somewhat alien
 - introduce relative weight
 - evaluate (trial-)emission by (trial-)emission

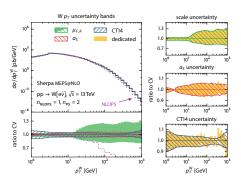
implementation in HERWIG7



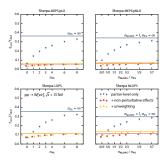


weight variation for W+jets with MEPs@NLO

• uncertainties in p_{\perp}^{W}



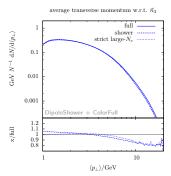
CPU budget



going beyond leading colour

start including next-to leading colour

(first attempts by Platzer & Sjodahl; Nagy & Soper)



also included in 1st emission in SHERPA's Mc@NLO

including NLO splitting kernels

(Hoeche, FK & Prestel, 1705.00982, and Hoeche & Prestel, 1705.00742)

expand splitting kernels as

$$P(z, \kappa^2) = P^{(0)}(z, \kappa^2) + \frac{\alpha_s}{2\pi} P^{(1)}(z, \kappa^2)$$

- aim: reproduce DGLAP evolution at NLO include all NLO splitting kernels
- three categories of terms in $P^{(1)}$:
 - cusp (universal soft-enhanced correction)

(already included in original showers)

- ullet corrections to 1 o 2
- new flavour structures (e.g. $q \rightarrow q'$), identified as $1 \rightarrow 3$
- new paradigm: two independent implementations



subtle symmetry factors

- observations for LO PS in final state:
 - only $P_{qq}^{(0)}$ used but not $P_{qg}^{(0)}$
 - $P_{gg}^{(0)}$ comes with "symmetry factor" 1/2
- challenge this way of implementing symmetry through:

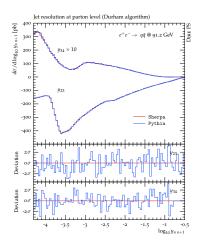
 $(\mathsf{Jadach}\ \&\ \mathsf{Skrzypek},\ \mathsf{hep\text{-}ph}/0312355)$

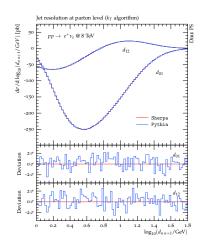
$$\sum_{i=q,g} \int_{0}^{1-\epsilon} dz \, z \, P_{qi}^{(0)}(z) = \int_{\epsilon}^{1-\epsilon} dz \, P_{qq}^{(0)}(z) + \mathcal{O}(\epsilon)$$

$$\sum_{i=q,g} \int_{0}^{1-\epsilon} dz \, z \, P_{gi}^{(0)}(z) = \int_{\epsilon}^{1-\epsilon} dz \, \left[\frac{1}{2} P_{gg}^{(0)}(z) + n_f P_{gq}^{(0)}(z) \right] + \mathcal{O}(\epsilon)$$

net effect: replace symmetry factors by parton marker z

validation of $1 \rightarrow 3$ splittings





physical results: DY at LHC

0.04

 $Z \rightarrow ee$ "dressed", Inclusive

- NLO

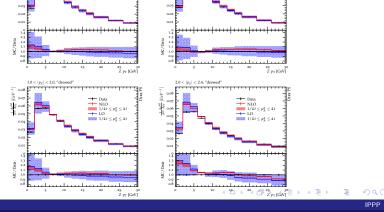
- LO

 $1/4t \le \mu_R^2 \le 4t$

 $1/4t \le \mu_R^2 \le 4t$

(untuned showers vs. 7 TeV ATLAS data, optimistic scale variations)

0.04



 $0.0 < |\psi_Z| < 1.0$, "dressed"

- NLO

- LO

 $1/4t \le \mu_R^2 \le 4t$

 $1/4t \le \mu_R^2 \le 4t$

leading colour differential two-loop soft corrections

(Dulat, Hoeche & Prestel, 1805.03757)

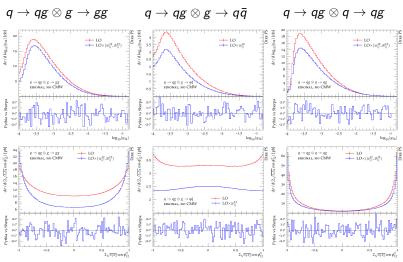
- analyse two-emission soft contribution and compare with iterated single emissions
- subtract double-counted terms and endpoint contributions
- capture residual effect by reweighting original parton shower, with
 - · accounting for finite recoil
 - including first $1/N_c$ corrections

(another way to solve "problem" in Dasgupta et al., 1805.09327)

incorporating spin correlations



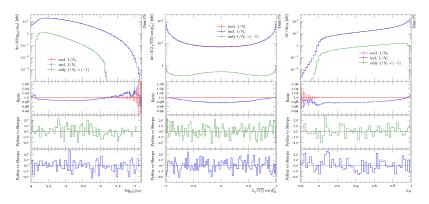
reweighting



Simulations in High-Energy Physics

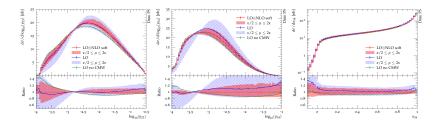
including $1/N_c$ effects

• capturing the difference of $C_F - C_A/2$ in assigning the correct emitter in the admixture of soft and collinear limits



scale uncertainties

ullet varying κ in the soft-enhanced terms, including NLO explicit corrections



PRECISION MONTE CARLO



• remember structure of NLO calculation for N-body production

$$\begin{split} \mathrm{d}\sigma \;&= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_{N}(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_{N}(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_{N}(\Phi_{\mathcal{R}}) \\ &= \mathrm{d}\Phi_{\mathcal{B}}\,\left(\mathcal{B}_{N} + \mathcal{V}_{N} + \mathcal{I}_{N}^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\,\left(\mathcal{R}_{N} - \mathcal{S}_{N}\right) \end{split}$$

• phase space factorisation assumed here ($\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1$)

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_\mathcal{B} \otimes \Phi_1) \,=\, \mathcal{I}_N^{(\mathcal{S})}(\Phi_\mathcal{B})$$

process independent subtraction kernels

$$\mathcal{S}_{N}(\Phi_{\mathcal{B}} \otimes \Phi_{1}) = \mathcal{B}_{N}(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_{1}(\Phi_{\mathcal{B}} \otimes \Phi_{1})$$

$$\mathcal{I}_{N}^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_{1}) = \mathcal{B}_{N}(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_{1}^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

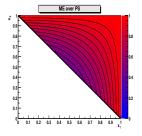
with universal $S_1(\Phi_{\mathcal{B}} \otimes \Phi_1)$ and $\mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})$



matrix element corrections

- parton shower ignores interferences typically present in matrix elements
- pictorially

ME:
$$\left| \sqrt{v^{vv}} + \sqrt{v^{uv}} \right|^{2}$$
PS:
$$\left| \sqrt{v^{vv}} \right|^{2} + \left| \sqrt{v^{uv}} \right|^{2}$$



- form many processes $\mathcal{R}_N < \mathcal{B}_N \times \mathcal{K}_N$
- typical processes: $q\bar{q}' \rightarrow V$, $e^-e^+ \rightarrow q\bar{q}$, $t \rightarrow bW$
- practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $\mathcal{P} = \mathcal{R}_N/(\mathcal{B}_N \times \mathcal{K}_N)$

analyse first emission, given by

$$d\sigma_{B} = d\Phi_{N} \mathcal{B}_{N}(\Phi_{N})$$

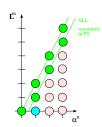
$$\cdot \left\{ \Delta_{N}^{(\mathcal{R}/\mathcal{B})}(\mu_{N}^{2}, t_{0}) + \int_{t_{0}}^{\mu_{N}^{2}} d\Phi_{1} \left[\frac{\mathcal{R}_{N}(\Phi_{N} \times \Phi_{1})}{\mathcal{B}_{N}(\Phi_{N})} \Delta_{N}^{(\mathcal{R}/\mathcal{B})}(\mu_{N}^{2}, t(\Phi_{1})) \right] \right\}$$

once more: integrates to unity \rightarrow "unitarity" of parton shower

• radiation given by \mathcal{R}_N (correct at $\mathcal{O}(\alpha_s)$)

(but modified by logs of higher order in α_s from $\Delta_N^{(\mathcal{R}/\mathcal{B})}$)

- emission phase space constrained by μ_N
- also known as "soft ME correction" hard ME correction fills missing phase space
- used for "power shower": $\mu_N \to E_{pp}$ and apply ME correction



NLO matching: Basic idea

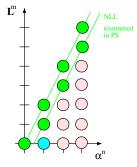
 parton shower resums logarithms fair description of collinear/soft emissions iet evolution (where the logs are large)

- matrix elements exact at given order fair description of hard/large-angle emissions jet production (where the logs are small)
- adjust ("match") terms:
 - cross section at NLO accuracy & correct hardest emission in PS to exactly reproduce ME at order α_s (R-part of the NLO calculation)

(this is relatively trivial)

• maintain (N)LL-accuracy of parton shower

(this is not so simple to see)



PowHea

• reminder: $\mathcal{K}_{ii,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1 \, \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \, \stackrel{\mathsf{IR}}{\longrightarrow} \, \mathrm{d}\Phi_1 \, \frac{\alpha_\mathsf{s}}{2\pi} \, \mathcal{K}_{ij,k}(\Phi_1)$$

define modified Sudakov form factor (as in ME correction)

$$\Delta_{\mathcal{N}}^{(\mathcal{R}/\mathcal{B})}(\mu_{\mathcal{N}}^2,t_0) = \exp \left[-\int\limits_{t_0}^{\mu_{\mathcal{N}}^2} \mathrm{d}\Phi_1 \, rac{\mathcal{R}_{\mathcal{N}}(\Phi_{\mathcal{N}+1})}{\mathcal{B}_{\mathcal{N}}(\Phi_{\mathcal{N}})}
ight] \; ,$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_s to parton shower scale



- define local K-factors
- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$\begin{split} \mathrm{d}\sigma_N^{(\mathrm{NLO})} &= \mathrm{d}\Phi_N \, \bar{\mathcal{B}}(\Phi_N) \\ &= \mathrm{d}\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N) \otimes \mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)} \right. \\ &+ \int \mathrm{d}\Phi_1 \left[\mathcal{R}_N(\Phi_N \otimes \Phi_1) - \mathcal{B}_N(\Phi_N) \otimes \mathrm{d}\mathcal{S}(\Phi_1) \right] \left. \right\} \end{split}$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes \mathrm{d}S$

(relevant for MC@NLO)



• generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0)$:

$$d\sigma_{N}^{(\text{NLO})} = d\Phi_{N} \,\bar{\mathcal{B}}(\Phi_{N}) \times \left\{ \Delta_{N}^{(\mathcal{R}/\mathcal{B})}(\mu_{N}^{2}, t_{0}) + \int_{t_{0}}^{\mu_{N}^{2}} d\Phi_{1} \frac{\mathcal{R}_{N}(\Phi_{N} \otimes \Phi_{1})}{\mathcal{B}_{N}(\Phi_{N})} \Delta_{N}^{(\mathcal{R}/\mathcal{B})}(\mu_{N}^{2}, k_{\perp}^{2}(\Phi_{1})) \right\}$$

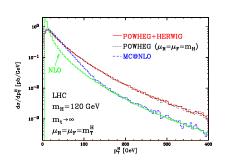
integrating to yield 1 - "unitarity of parton shower"

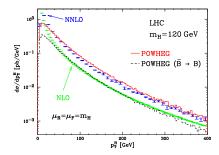
- radiation pattern like in ME correction
- ullet pitfall, again: choice of upper scale μ_N^2

(this is vanilla POWHEG!)

• apart from logs: which configurations enhanced by local K-factor

(K-factor for inclusive production of X adequate for X+ jet at large p | ?)

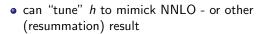


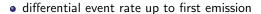


- large enhancement at high p_{T,h}
- can be traced back to large NLO correction
- ullet fortunately, NNLO correction is also large $ightarrow \sim$ agreement

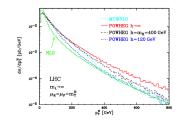
- improving POWHEG
- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_{\perp}^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_{\perp}^2}{p_{\perp}^2 + h^2}}_{\mathcal{R}^{(F)}}\right)$$





$$d\sigma = d\Phi_{B} \bar{\mathcal{B}}^{(R^{(S)})} \left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s, t_{0}) + \int_{t_{0}}^{s} d\Phi_{1} \frac{\mathcal{R}^{(S)}}{\mathcal{B}} \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s, k_{\perp}^{2}) \right] + d\Phi_{R} \mathcal{R}^{(F)}(\Phi_{R})$$



MC@NLO

• MC@NLO paradigm: divide \mathcal{R}_N in soft ("S") and hard ("H") part:

$$\mathcal{R}_{N} = \mathcal{R}_{N}^{(S)} + \mathcal{R}_{N}^{(H)} = \mathcal{B}_{N} \otimes d\mathcal{S}_{1} + \mathcal{H}_{N}$$

ullet identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum_{\{ij,k\}} \mathcal{K}_{ij,k}$

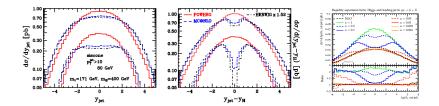
(modify \mathcal{K} in $\mathbf{1}^{\text{st}}$ emission to account for colour)

$$d\sigma_{N} = d\Phi_{N} \underbrace{\tilde{\mathcal{B}}_{N}(\Phi_{N})}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{0}) + \int_{t_{0}}^{\mu_{N}^{2}} d\Phi_{1} \, \mathcal{K}_{ij,k}(\Phi_{1}) \, \Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, k_{\perp}^{2}) \right] + d\Phi_{N+1} \, \mathcal{H}_{N}$$

• effect: only resummed parts modified with local K-factor



phase space effects: shower vs. fixed order



- problem: impact of subtraction terms on local K-factor (filling of phase space by parton shower)
- studied in case of $gg \rightarrow H$ above
- proper filling of available phase space by parton shower paramount



NNLOPS in the MINLO approach: merging without Q_J

(K.Hamilton, P.Nason, C.Oleari & G.Zanderighi, JHEP 1305 (2013) 082)

- based on POWHEG + shower from PYTHIA or HERWIG
- up to today only for singlet S production, gives NNLO + PS
- basic idea:
 - use S+jet in POWHEG
 - push jet cut to parton shower IR cutoff
 - apply analytical NNLL Sudakov rejection weight for intrinsic line in Born configuration

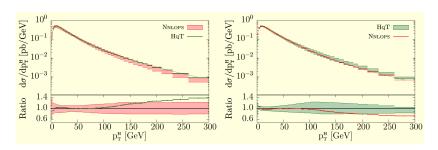
(kills divergent behaviour at order $\alpha_{\rm S}$)

- don't forget double-counted terms
- reweight to NNLO fixed order



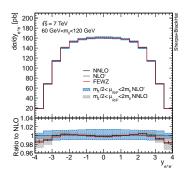
NNLOPS for H production

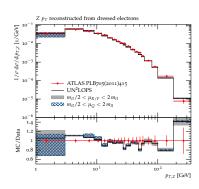
(K.Hamilton, P.Nason, E.Re & G.Zanderighi, JHEP 1310 (2013) 222)



NNLOPS for Z production: UNNLOPS

S. Hoche, Y. Li, & S. Prestel, Phys.Rev.D90 & D91





also available for H production



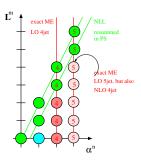
NNLOPS: shortcomings/limitations

- \bullet MINLO relies on knowledge of B_2 terms from analytic resummation --- to date only known for colour singlet production
- MINLO relies on reweighting with full NNLO result \longrightarrow one parameter for $H(y_H)$, more complicated for Z, \ldots
- UNNLOPS relies on integrating single- and double emission to low scales and combination of unresolved with virtual emissions → potential efficiency issues, need NNLO subtraction
- UNNLOPS puts unresolved & virtuals in "zero-emission" bin \longrightarrow no parton showering for virtuals (?)



multijet merging: basic idea

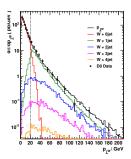
- parton shower resums logarithms fair description of collinear/soft emissions iet evolution (where the logs are large)
- matrix elements exact at given order fair description of hard/large-angle emissions jet production (where the logs are small)
- combine ("merge") both: result: "towers" of MEs with increasing number of jets evolved with PS
 - multijet cross sections at Born accuracy
 - maintain (N)LL accuracy of parton shower



 separate regions of jet production and jet evolution with jet measure Q_J

("truncated showering" if not identical with evolution parameter)

- matrix elements populate hard regime
- parton showers populate soft domain



why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \rightarrow hadrons$ Durham jet definition: relative transverse momentum $k_{\perp} > Q_{I}$
- fixed order: one factor α_S and up to $\log^2 \frac{E_{c.m.}}{Q_L}$ per jet
- use Sudakov form factor for resummation & replace approximate fixed order by exact expression:

$$\mathcal{R}_2(Q_J) = \left[\Delta_q(E_{\text{c.m.}}^2, Q_J^2)\right]^2$$

$$\mathcal{R}_{3}(Q_{J}) = 2\Delta_{q}(\boldsymbol{E}_{\text{c.m.}}^{2}, Q_{J}^{2}) \int_{Q_{J}^{2}}^{E_{\text{c.m.}}^{2}} \frac{dk_{\perp}^{2}}{k_{\perp}^{2}} \left[\frac{\alpha_{s}(k_{\perp}^{2})}{2\pi} dz \mathcal{K}_{q}(k_{\perp}^{2}, z) \right]$$

$$\times \Delta_q(E_{\text{c.m.}}^2, k_\perp^2) \Delta_q(k_\perp^2, Q_J^2) \Delta_g(k_\perp^2, Q_J^2) \bigg]$$



multijet merging at LO

expression for first emission

$$d\sigma = d\Phi_{N} \mathcal{B}_{N} \left[\Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{0}) + \int_{t_{0}}^{\mu_{N}^{2}} d\Phi_{1} \mathcal{K}_{N} \Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{N+1}) \Theta(Q_{J} - Q_{N+1}) \right] + d\Phi_{N+1} \mathcal{B}_{N+1} \Delta_{N}^{(\mathcal{K})}(\mu_{N+1}^{2}, t_{N+1}) \Theta(Q_{N+1} - Q_{J})$$

• note: N+1-contribution includes also N+2, N+3, ...

(no Sudakov suppression below t_{n+1} , see further slides for iterated expression)

- potential occurrence of different shower start scales: $\mu_{N,N+1,...}$
- "unitarity violation" in square bracket: $\mathcal{B}_N \mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &

S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])



$$\mathrm{d}\sigma = \sum_{n=N}^{n_{\max}-1} \left\{ \mathrm{d}\Phi_n \, \mathcal{B}_n \, \overline{\left[\prod_{j=N}^{n-1} \Theta(Q_{j+1} - Q_J) \right]} \, \overline{\left[\prod_{j=N}^{n-1} \Delta_j^{(\mathcal{K})}(t_j, \, t_{j+1}) \right]} \right.$$

$$\times \left[\Delta_n^{(\mathcal{K})}(t_n, t_0) + \int\limits_{t_0}^{t_n} \mathrm{d}\Phi_1 \, \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_n, t_{n+1}) \Theta(Q_J - Q_{n+1}) \right]$$
no emission no iet & below last ME emission

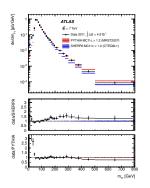
$$+\mathrm{d}\Phi_{n_{\mathsf{max}}}\,\mathcal{B}_{n_{\mathsf{max}}}\left[\prod_{j=N}^{n_{\mathsf{max}}-1}\,\Theta(Q_{j+1}-Q_J)
ight]\left[\prod_{j=N}^{n_{\mathsf{max}}-1}\,\Delta_j^{(\mathcal{K})}(t_j,\,t_{j+1})
ight]$$

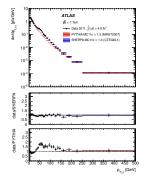
$$imes \left[\Delta_{n_{\mathsf{max}}}^{(\mathcal{K})}(t_{n_{\mathsf{max}}},t_0) + \int\limits_{t_0}^{\cdot_{n_{\mathsf{max}}}} \mathrm{d}\Phi_1 \, \mathcal{K}_{n_{\mathsf{max}}} \Delta_{n_{\mathsf{max}}}^{(\mathcal{K})}(t_{n_{\mathsf{max}}},t_{n_{\mathsf{max}}+1})
ight]$$

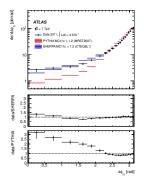
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di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

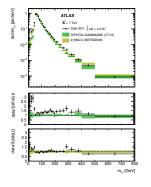
(arXiv:1211.1913 [hep-ex])

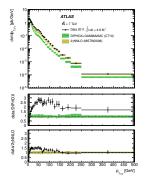


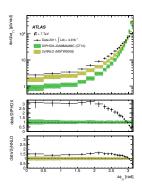




aside: Comparison with higher order calculations







multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting maintain NLO and (N)LL accuracy of ME and PS
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities



first emission(s), once more

$$d\sigma = d\Phi_{N} \tilde{\mathcal{B}}_{N} \left[\Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{0}) + \int_{t_{0}}^{\mu_{N}^{2}} d\Phi_{1} \mathcal{K}_{N} \Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{N+1}) \Theta(Q_{J} - Q_{N+1}) \right]$$

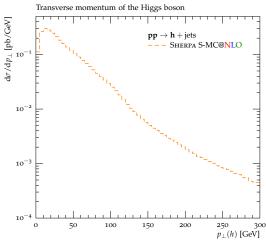
$$+ d\Phi_{N+1} \mathcal{H}_{N} \Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{N+1}) \Theta(Q_{J} - Q_{N+1})$$

$$+ d\Phi_{N+1} \tilde{\mathcal{B}}_{N+1} \left(1 + \frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}} \int_{t_{N+1}}^{\mu_{N}^{2}} d\Phi_{1} \mathcal{K}_{N} \right) \Theta(Q_{N+1} - Q_{J})$$

$$\cdot \left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{0}) + \int_{t_{0}}^{t_{N+1}} d\Phi_{1} \mathcal{K}_{N+1} \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2}) \right]$$

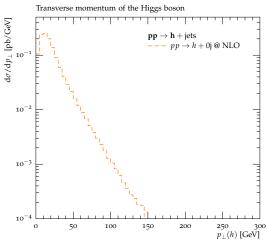
$$+ d\Phi_{N+2} \mathcal{H}_{N+1} \Delta_{N}^{(\mathcal{K})}(\mu_{N}^{2}, t_{N+1}) \Delta_{N+1}^{(\mathcal{K})}(t_{N+1}, t_{N+2}) \Theta(Q_{N+1} - Q_{J}) + \dots$$

Simulations in High-Energy Physics

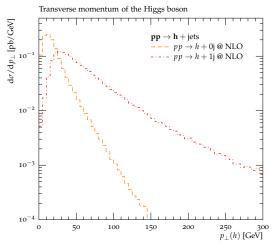


• first emission by Mc@NLo

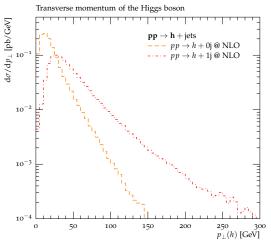




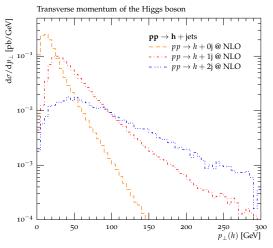
• first emission by Mc@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$



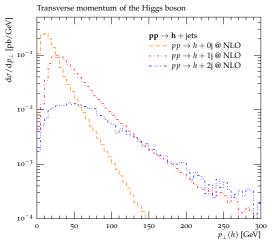
- first emission by Mc@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- Mc@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{cut}$



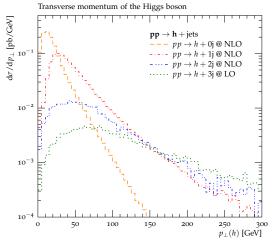
- first emission by Mc@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- Mc@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{cut}$
- restrict emission off $pp \rightarrow h + \text{jet to}$ $Q_{n+2} < Q_{\text{cut}}$



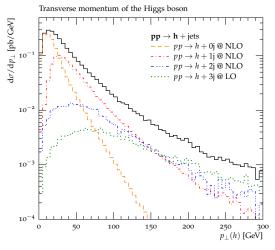
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- Mc@NLo $pp \rightarrow h + 2jets$ for $Q_{n+2} > Q_{\text{cut}}$



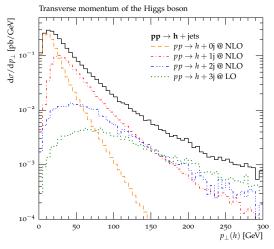
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- iterate



- first emission by Mc@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$
- Mc@NLO $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
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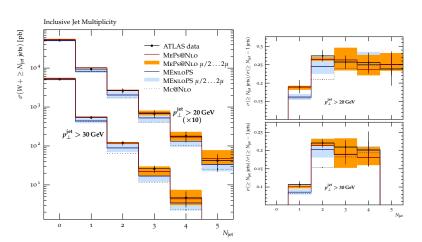
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- iterate
- sum all contributions

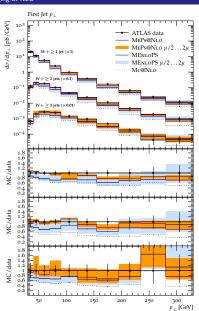


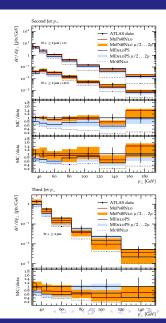
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- Mc@Nio $pp \rightarrow h + 2$ jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions
- eg. $p_{\perp}(h) > 200 \text{ GeV}$ has contributions fr multiple topologies

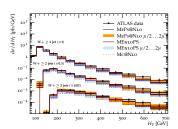
example: MEPS@NLO for W+jets

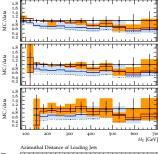
(up to two jets @ NLO, from BLACKHAT, see arXiv: 1207.5031 [hep-ex])

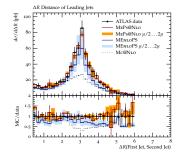


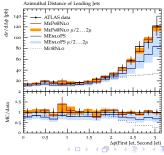








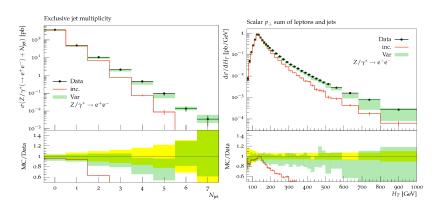




FxFx: validation in Z+jets

(Data from ATLAS, 1304.7098, aMC@NLO _MADGRAPH with HERWIG++)

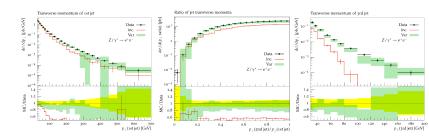
(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



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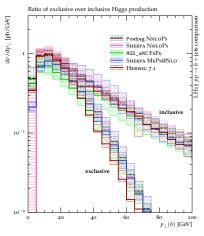


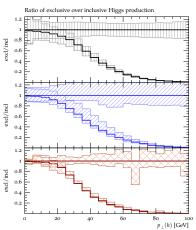
differences between MEPS@NLO, UNLOPS & FxFx

	FxFx	MePs@Nlo	UNLoPs
ME	all internal	${\cal V}$ external	all external
	aMc@NLO _MADGRAPH	COMIX or AMEGIC++	
		${\cal V}$ from OPENLOOPS, BLACKHAT, MJET,	
shower	external	intrinsic	intrinsic
	HERWIG or PYTHIA		Рутніа
Δ_N	analytical	from PS	from PS
$\Theta(Q_J)$	a-posteriori	per emission	per emission
Q_J -range	relatively high	> Sudakov regime	pprox Sudakov regime
	(but changed)		
		≈ 10%	≈ 10%



Higgs- p_{\perp} : exclusive over inclusive rate





• $\approx 20\%$ of Higgs with $p_{\perp} = 60 \,\text{GeV}$ are not accompanied by a jet

◆□ト ◆圖ト ◆重ト ◆重ト

motivation: the size of EW corrections

- EW corrections sizeable $\mathcal{O}(10\%)$ at large scales: must include them!
- but: more painful to calculate
- need EW showering & possibly corresponding PDFs

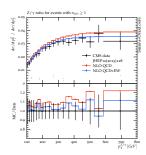
(somewhat in its infancy: chiral couplings)

• example: $Z\gamma$ vs. p_T (right plot)

(handle on
$$p_{\perp}^{Z}$$
 in $Z \to \nu \bar{\nu}$)

(Kallweit, Lindert, Pozzorini, Schoenherr for LH'15)

- difference due to EW charge of Z
- no real correction (real V emission)
- improved description of $Z \to \ell\ell$



inclusion of electroweak corrections in simulation

- incorporate approximate electroweak corrections in MEPs@NLO
 - using electroweak Sudakov factors

$$\tilde{\mathrm{B}}_{n}(\Phi_{n}) \, pprox \, \tilde{\mathrm{B}}_{n}(\Phi_{n}) \, \Delta_{\mathsf{EW}}(\Phi_{n})$$

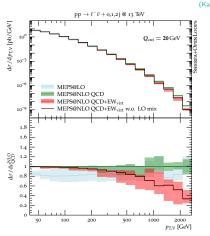
using virtual corrections and approx. integrated real corrections

$$\tilde{\mathrm{B}}_{n}(\boldsymbol{\Phi}_{n}) \; \approx \; \tilde{\mathrm{B}}_{n}(\boldsymbol{\Phi}_{n}) + \mathrm{V}_{n,\mathrm{EW}}(\boldsymbol{\Phi}_{n}) + \mathrm{I}_{n,\mathrm{EW}}(\boldsymbol{\Phi}_{n}) + \mathrm{B}_{n,\mathrm{mix}}(\boldsymbol{\Phi}_{n})$$

- real QED radiation can be recovered through standard tools (parton shower, YFS resummation)
- simple stand-in for proper QCD⊕EW matching and merging
 - \rightarrow validated at fixed order, found to be reliable, difference $\lesssim 5\%$ for observables not driven by real radiation

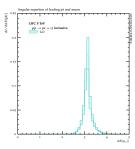


results: $pp \rightarrow \ell^- \bar{\nu} + \text{jets}$



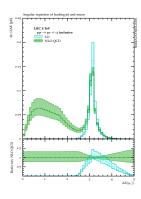
(Kallweit, Lindert, Maierhöfer, Pozzorini, Schoenherr JHEP04(2016)021) $pp \rightarrow \ell^- \bar{\nu} + 0,1,2j @ 13 \text{ TeV}$ lσ/dp_{Tjt} [pb/GeV $Q_{\text{cut}} = 20 \,\text{GeV}$ 10 MEPS@LO MEPS@NLO OCD MEPS@NLO QCD+EWvirt MEPS@NLO QCD+EWvirt w.o. LO mix 10-9 1.8 1.6 1.4 do/doNLO 1.2 0.8 0.6 0.4 0.2 o 50 200 500 1000 2000 100 p_{T,j_1} [GeV]

particle level events including dominant EW corrections



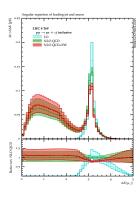
measure collinear W emission?

• LO
$$pp o Wj$$
 with $\Delta \phi(\mu,j) pprox \pi$



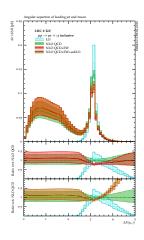
measure collinear W emission?

- LO $pp \rightarrow Wi$ with $\Delta \phi(\mu, j) \approx \pi$
- NLO corrections neg. in peak large $pp \rightarrow Wij$ component opening PS



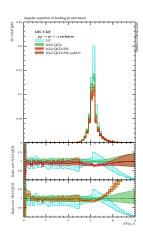
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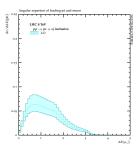
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- sub-leading Born (γ PDF) at large ΔR



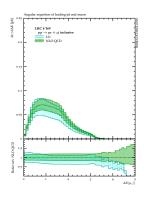
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- restrict to exactly 1j, no $p_{\perp}^{j_2} > 100 \, {\rm GeV}$



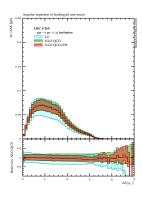
measure collinear W emission?

- LO pp o Wj with $\Delta \phi(\mu,j) pprox \pi$
- NLO corrections neg. in peak large pp o Wjj component opening PS
- ullet sub-leading Born (γ PDF) at large ΔR
- ullet restrict to exactly 1j, no $p_{\perp}^{j_2}>100\,{
 m GeV}$
- ullet describe pp o Wjj @ NLO, $p_{\perp}^{j_2}>100\,{
 m GeV}$



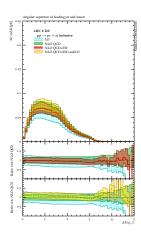
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- describe $pp \rightarrow Wjj$ @ NLO, $p_{\perp}^{j_2} > 100 \, {\rm GeV}$
- pos. NLO QCD, \sim flat



measure collinear W emission?

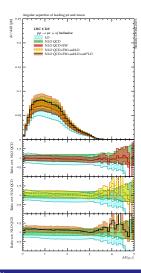
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- describe $pp \rightarrow Wjj$ @ NLO, $p_{\perp}^{j_2} > 100 \, {\rm GeV}$
- ullet pos. NLO QCD, neg. NLO EW, \sim flat



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- ullet pos. NLO QCD, neg. NLO EW, \sim flat
- sub-leading Born contribs positive

NLO EW predictions for $\Delta R(\mu, j_1)$

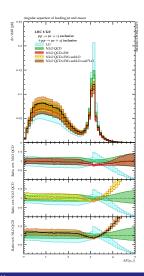


measure collinear W emission?

LHC@8TeV, $p_{\perp}^{j_1} > 500 \, \mathrm{GeV}$, central μ and jet

- LO $pp \to Wj$ with $\Delta \phi(\mu, j) \approx \pi$
- NLO corrections neg. in peak large $pp \rightarrow Wij$ component opening PS
- sub-leading Born (γ PDF) at large ΔR
- restrict to exactly 1j, no $p_{\perp}^{j_2} > 100 \, \text{GeV}$
- describe $pp \rightarrow Wij$ @ NLO, $p_{\perp}^{j_2} > 100 \, \text{GeV}$
- ullet pos. NLO QCD, neg. NLO EW, \sim flat
- sub-leading Born contribs positive
- sub²leading Born (diboson etc) conts. pos.
 - \rightarrow possible double counting with BG

NLO EW predictions for $\Delta R(\mu, j_1)$



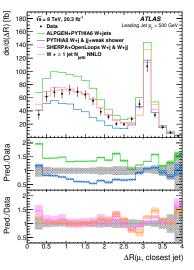
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- describe $pp \rightarrow Wij$ @ NLO, $p_{\perp}^{j_2} > 100 \, \text{GeV}$
- ullet pos. NLO QCD, neg. NLO EW, \sim flat
- sub-leading Born contribs positive
- sub²leading Born (diboson etc) conts. pos. \rightarrow possible double counting with BG
- merge using exclusive sums

FW corrections 00000

NLO EW predictions for $\Delta R(\mu, j_1)$



data comparison

(M. Wu ICHEP'16, ATLAS arXiv:1609.07045)

 ALPGEN+PYTHIA $pp \rightarrow W + \text{jets MLM merged}$ (Mangano et.al., JHEP07(2003)001)

PYTHIA 8

 $pp \rightarrow Wi + QCD$ shower $pp \rightarrow jj + QCD+EW$ shower (Christiansen, Prestel, EPJC76(2016)39)

 SHERPA+OPENLOOPS NLO QCD+EW+subLO $pp \rightarrow Wi/Wii$ excl. sum

(Kallweit, Lindert, Maierhöfer,)

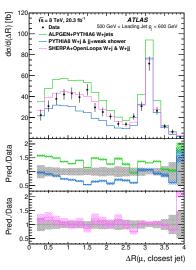
(Pozzorini, Schoenherr, JHEP04(2016)021)

• NNLO QCD $pp \rightarrow Wi$

(Boughezal, Liu, Petriello, arXiv:1602.06965)



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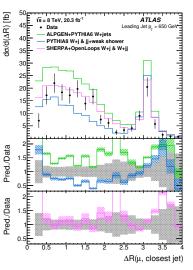
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SIMULATING SOFT QCD

QCD radiation, once more

• remember QCD emission pattern

$$\mathrm{d}w^{q\to qg} \;=\; \frac{\alpha_{\mathsf{s}}(k_{\perp}^2)}{2\pi}\; C_F \, \frac{\mathrm{d}k_{\perp}^2}{k_{\perp}^2} \, \frac{\mathrm{d}\omega}{\omega} \, \left[1 + \left(1 - \frac{\omega}{E}\right)\right] \;.$$

- spectrum cut-off at small transverse momenta and energies by onset of hadronization, at scales $R\approx 1\,{\rm fm}/\Lambda_{QCD}$
- two (extreme) classes of emissions: gluons and gluers determined by relation of formation and hadronization times

- Connection to QCD
 - ullet gluers formed at times R, with momenta $k_\parallel \sim k_\perp \sim \omega \stackrel{>}{\sim} 1/R$
 - assuming that hadrons follow partons,

$$\begin{split} \mathrm{d}\textit{N}_{\mathrm{(hadrons)}} &\sim \int\limits_{k_{\perp}>1/R}^{Q} \frac{\mathrm{d}k_{\perp}^{2}}{k_{\perp}^{2}} \, \frac{C_{F} \, \alpha_{\mathrm{s}}(k_{\perp}^{2})}{2\pi} \, \left[1 + \left(1 - \frac{\omega}{E}\right)\right] \, \frac{\mathrm{d}\omega}{\omega} \\ &\sim \frac{C_{F} \alpha_{\mathrm{s}}(1/R^{2})}{\pi} \log(Q^{2}R^{2}) \, \frac{\mathrm{d}\omega}{\omega} \end{split}$$

or - relating their energyn with that of the gluers -

$$dN_{(hadrons)}/d\log\epsilon = const.$$

a plateau in log of energy (or in rapidity)



- impact of additional radiation
- new partons must separate before they can hadronize independently
- therefore, one more time

- ullet for gluers $Rk_{\perp}pprox 1$: all times the same
- naively; new & more hadrons following new partons
- but: colour coherence primary and secondary partons not separated enough in

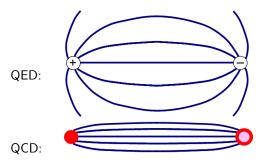
$$1/R \stackrel{<}{\sim} \omega_{({\rm hadron})} \stackrel{<}{\sim} 1/(R\theta)$$

and therefore no independent radiation

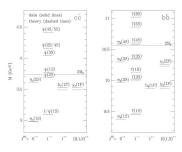


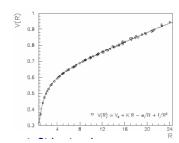
hadronisation: General thoughts

- confinement the striking feature of low-scale sotrng interactions
- transition from partons to their bound states, the hadrons
- the Meissner effect in QCD

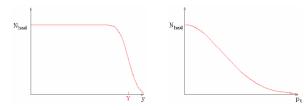


• linear QCD potential in Quarkonia – like a string





- combine some experimental facts into a naive parameterisation
- in $e^+e^- o$ hadrons: exponentially decreasing p_\perp , flat plateau in y for hadrons



ullet try "smearing": $ho(p_\perp^2)\sim \exp(-p_\perp^2/\sigma^2)$

• use parameterisation to "guesstimate" hadronisation effects:

$$\begin{split} E &= \int_0^Y \mathrm{d}y \mathrm{d}p_\perp^2 \rho(p_\perp^2) p_\perp \cosh y = \lambda \sinh Y \\ P &= \int_0^Y \mathrm{d}y \mathrm{d}p_\perp^2 \rho(p_\perp^2) p_\perp \sinh y = \lambda (\cosh Y - 1) \approx E - \lambda \\ \lambda &= \int \mathrm{d}p_\perp^2 \rho(p_\perp^2) p_\perp = \langle p_\perp \rangle \,. \end{split}$$

- estimate $\lambda \sim 1/R_{\rm had} \approx m_{\rm had}$, with $m_{\rm had}$ 0.1-1 GeV.
- effect: jet acquire non-perturbative mass $\sim 2\lambda E$ ($\mathcal{O}(10 \mathrm{GeV})$) for jets with energy $\mathcal{O}(100 \mathrm{GeV})$).

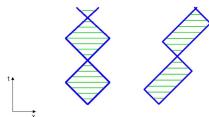
- similar parametrization underlying Feynman-Field model for independent fragmentation
- ullet recursively fragment q
 ightarrow q' + had, where
 - transverse momentum from (fitted) Gaussian;
 - longitudinal momentum arbitrary (hence from measurements);
 - \bullet flavour from symmetry arguments + measurements.
- problems: frame dependent, "last quark", infrared safety, no direct link to perturbation theory,

String model

string model

- a simple model of mesons: yoyo strings
 - light quarks $(m_q=0)$ connected by string, form a meson area law: $m_{\rm had}^2 \propto$ area of string motion

 - L=0 mesons only have 'yo-yo' modes:



- turn this into hadronisation model $e^+e^- o q\bar{q}$ as test case
- ullet ignore gluon radiation: qar q move away from each other, act as point-like source of string
- ullet intense chromomagnetic field within string: more $qar{q}$ pairs created by tunnelling and string break-up
- analogy with QED (Schwinger mechanism): $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t\exp\left(-\pi m_q^2/\kappa\right)$, $\kappa=$ "string tension".



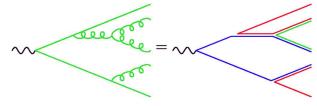
- string model = well motivated model, constraints on fragmentation (Lorentz-invariance, left-right symmetry, . . .)
- how to deal with gluons?
- ullet interpret them as kinks on the string \Longrightarrow the string effect



infrared-safe, advantage: smooth matching with PS.

cluster model

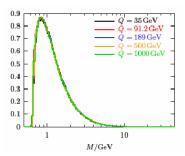
- underlying idea: preconfinement/LPHD
 - typically, neighbouring colours will end in same hadron
 - \bullet hadron flows follow parton flows \longrightarrow don't produce any hadrons at places where you don't have partons
 - ullet works well in large- N_c limit with planar graphs
- follow evolution of colour in parton showers



- paradigm of cluster model: clusters as continuum of hadron resonances
- ullet trace colour through shower in $\mathcal{N}_c o \infty$ limit
- force decay of gluons into $q\bar{q}$ or $\bar{d}d$ pairs, form colour singlets from neighbouring colours, usually close in phase space
- ullet mass of singlets: peaked at low scales $pprox Q_0^2$
- decay heavy clusters into lighter ones or into hadrons (here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like)
- if light enough, clusters will decay into hadrons
- naively: spin information washed out, decay determined through phase space only \to heavy hadrons suppressed (baryon/strangeness suppression)



- self-similarity of parton shower will end with roughly the same local distribution of partons, with roughly the same invairant mass for colour singlets
- adjacent pairs colour connected, form colourless (white) clusters.
- clusters ("≈ excited hadrons) decay into hadrons



- practicalities of hadronisation models: parameters
 - kinematics of string or cluster decay:
 - 2-5 parameters must "pop" quark or diquark flavours in string or cluster decay —
 - cannot be completely democratic or driven by masses alone
 - → suppression factors for strangeness, diquarks 2-10 parameters
 - transition to hadrons, cannot be democratic over multiplets
 - → adjustment factors for vectors/tensors etc. 2-6 parameters
- tuned to LEP data, overall agreement satisfying
- validity for hadron data not guite clear

(beam remnant fragmentation not in LEP.)

• there are some issues with inclusive strangeness/baryon production

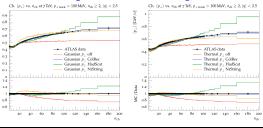
colour reconnections and friends

(Fischer, Sjostrand, 1610:09818)

Collective flow observed in pp at LHC. Partly unexpected. New mechanisms required; could also (partly) replace CR.

Active field, e.g. N. Fischer & TS, arXiv:1610:09818 [hep-ph]:

- ullet Thermal $exp(-p_{\perp}/T)
 ightarrow exp(-m_{\perp}/T)$ hadronic spectrum.
- \bullet Close-packed strings \Rightarrow increased string κ or ${\cal T}.$
- $\bullet \ \, \mathsf{Dense} \,\, \mathsf{hadronic} \,\, \mathsf{gas} \Rightarrow \mathsf{hadronic} \,\, \mathsf{rescattering}. \\$

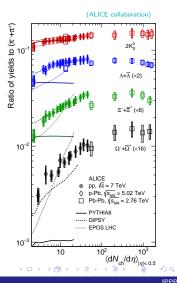


(slide stolen from Torbjorn Sjostrand)



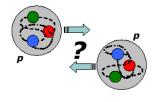
strange strangeness

- universality of hadronisation assumed
- parameters tuned to LEP data in particular: strangeness suppression
- for strangeness: flat ratios but data do not reproduce this
- looks like *SU*(3) restoration not observed for protons
- needs to be investigated



multiple parton scattering

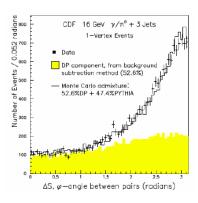
- hadrons = extended objects!
- no guarantee for one scattering only.
- running of α_S
 - ⇒ preference for soft scattering.



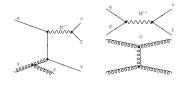
- first experimental evidence for double–parton scattering: events with $\gamma+3$ jets:
 - cone jets, R=0.7, $E_T>5$ GeV; $|\eta_j|<1.3$;
 - "clean sample": two softest jets with E_T < 7 GeV;
- cross section for DPS

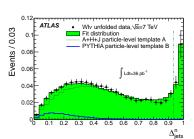
$$\sigma_{
m DPS} = rac{\sigma_{\gamma j} \sigma_{jj}}{\sigma_{
m eff}}$$

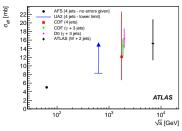
 $\sigma_{\rm eff} \approx$ 14 \pm 4 mb.

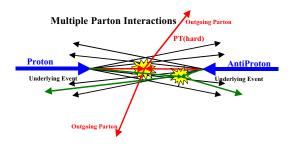


- more measurements, also at LHC
- ATLAS results from W + 2 jets









but: how to define the underlying event?

- everything apart from the hard interaction, but including IS showers, FS showers, remnant hadronisation.
- remnant-remnant interactions, soft and/or hard.
- lesson: hard to define



Multiple parton scattering

 origin of MPS: parton-parton scattering cross section exceeds hadron-hadron total cross section

$$\sigma_{
m hard}(p_{\perp,
m min}) \, = \int \limits_{p_{\perp,
m min}^2}^{s/4} {
m d} p_{\perp}^2 rac{{
m d} \sigma(p_{\perp}^2)}{{
m d} p_{\perp}^2} > \sigma_{pp,
m total}$$

for low $p_{\perp, \min}$

remember

$$\frac{d\sigma(p_{\perp}^{2})}{dp_{\perp}^{2}} = \int_{0}^{1} dx_{1} dx_{2} f(x_{1}, q^{2}) f(x_{2}, q^{2}) \frac{d\hat{\sigma}_{2 \to 2}}{dp_{\perp}^{2}}$$

- $ullet \left<\sigma_{
 m hard}(p_{\perp,
 m min})/\sigma_{pp,
 m total}
 ight> \geq 1$
- depends strongly on cut-off $p_{\perp,\min}$ (energy-dependent)!



modelling the underlying event

- take the old PYTHIA model as example:
 - start with hard interaction, at scale Q_{hard}^2 .
 - select a new scale p_{\perp}^2 from

$$\exp \left[-rac{1}{\sigma_{
m norm}} \int\limits_{
ho_{\perp}^2}^{Q_{
m hard}^2} {
m d}
ho_{\perp}'^2 rac{{
m d} \sigma(
ho_{\perp}^2)}{{
m d}
ho_{\perp}'^2}
ight]$$

with constraint $p_{\perp}^2 > p_{\perp, \min}^2$

- rescale proton momentum ("proton-parton = proton with reduced energy").
- \bullet repeat until no more allowed $2 \to 2$ scatter

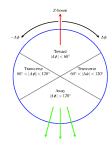
modelling the underlying event

- possible refinements:

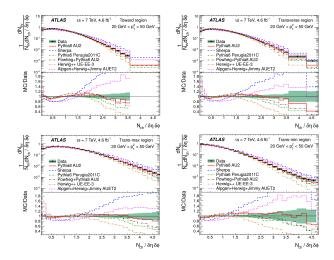
 - add parton showers to UE
 - "regularisation" to dampen sharp dependence on $p_{\perp, \min}$: replace $1/\hat{t}$ in MEs by $1/(t+t_0)$, also in α_s .
 - ullet treat intrinsic k_{\perp} of partons (ightarrow parameter)
 - $\bullet \ \ \mathsf{model} \ \mathsf{proton} \ \mathsf{remnants} \ (\to \mathsf{parameter})$

some results for MPS in Z production

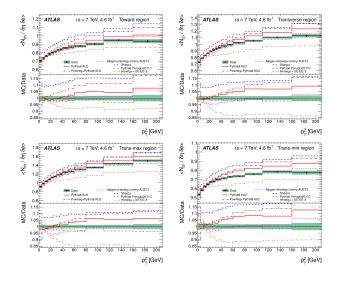
- observables sensitive to MPS
- classical analysis: transverse regions in QCD/jet events
- idea: find the hardest system, orient event into regions:
 - toward region along system
 - away region back-to-back
 - transverse regions
- typically each in 120°



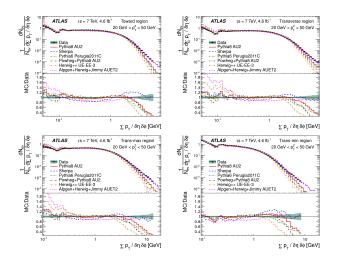
Some results in Z production



Some results in Z production



Some results in Z production



- see some data comparison in Minimum Bias
- practicalities of underlying event models: parameters
 - profile in impact parameter space
 - IR cut-off at reference energy, its energy evolution, dampening paramter and normalisation cross section
 - treating colour connections to rest of event 2-5 parameters
- tuned to LHC data, overall agreement satisfying
- energy extrapolation not exactly perfect, plus other process categories such as diffraction etc..

2-3 parameters

SUMMARY

Summary of fixed order

- NLO (QCD) "revolution" consolidated:
 - lots of routinely used tools for large FS multis (4 and more)
 - incorporation in MC tools done, need comparisons, critical appraisals and a learning curve in their phenomenological use
 - to improve: description of loop-induced processes
- amazing success in NNLO (QCD) calculations:
 - \bullet emergence of first round of $2 \to 2$ calculations
 - next revolution imminent (with question marks)
 - first MC tools for simple processes $(gg \rightarrow H, DY)$, more to be learnt by comparison etc. (see above)
- first N³LO calculation in $gg \rightarrow H$, more to come (?)
- attention turning to NLO (EW)
 - first benchmarks with new methods (V+3j)
 - calculational setup tricky
 - need maybe faster approximation for high-scales (EW Sudakovs)



Limitations of fixed order

- practical limitations/questions to be overcome:
 - dealing with IR divergences at NNLO: slicing vs. subtracting

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(I'm not sure we have THE solution yet)
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- how far can we push NNLO? are NLO automated results stable enough for NNLO at higher multiplicity?
- ullet users of codes: higher orders tricky o training needed

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(MC = black box attitude problematic - a new brand of pheno/experimenters needed?)
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- limitations of perturbative expansion:
 - breakdown of factorisation at HO (Seymour et al.)
 - higher-twist: compare $(\alpha_{\rm s}/\pi)^n$ with $\Lambda_{\rm QCD}/M_Z$
- limitations in analytic resummation: process- and observable-dependent
 - first attempts at automation (CAESAR and some others) checks/cross-comparison necessary
- showering needs to be improved

(for NNLO the "natural" accuracy is NNLL)



Summary for event generation

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
 - multijet merging ("CKKW", "MLM")
 - NLO matching ("MC@NLO", "PowHEG")
 - MENLOPS NLO matching & merging
 - MEPS@NLO ("SHERPA", "UNLOPS", "MINLO", "FxFx")



- multijet merging an important tool for many relevant signals and backgrounds - pioneering phase at LO & NLO over
- complete automation of NLO calculations done
 - → must benefit from it!

(it's the precision and trustworthy & systematic uncertainty estimates!)



Vision

- we have constructed lots of tools for precision physics at LHC but we did not cross-validate them careful enough (yet) but we did not compare their theoretical foundations (yet)
- we also need unglamorous improvements:
 - systematically check advanced scale-setting schemes (MINLO)
 - automatic (re-)weighting for PDFs & scales (ME: √, PS: -)
 - scale compensation in PS is simple (implement and check)
 - PDFs: to date based on FO vs. data will we have to move to resummed/parton showered?

(reminder: LO* was not a big hit, though)

... and maybe we will have to go to the "dirty" corners:
 higher-twist, underlying event, hadronization, ...

(many of those driven by experiment)



