Foundations of Physics III
Quantum and Particle Physics
Lecture 10

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Symmetries in classical physics

- From classical physics it is known that

  invariance of a system under certain transformations enforces the conservation of corresponding quantities.

  (*Noether's theorem*)

Examples:

Invariance under		Conserved quantity
rotations	$\Longleftrightarrow$	angular momentum
time translations	$\Longleftrightarrow$	energy
space translations	$\Longleftrightarrow$	momentum

Symmetries in Quantum Physics

- In Quantum Physics also internal symmetries (spin, isospin, ...)
- Example: Invariance under phase transformations of the fields

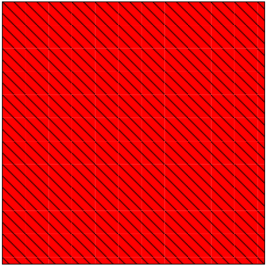
$$\psi(x,\, t) \to \psi'(x,\, t) = \exp(i\theta)\psi(x,\, t) \Longleftrightarrow |\psi|^2 = |\psi'|^2$$

  yields conserved charges like, e.g., the electrical charge.
- Note: global changes in phase cannot be observed (because typically squares are taken), but **phase differences are observable**.

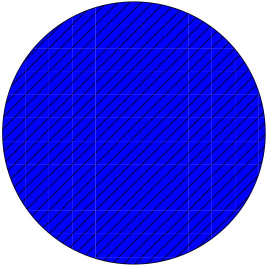
  (Aharonov-Bohm effect.)

Discrete vs. continuous symmetries

- Consider two slabs with quadratic and round cross section.
- The quadratic one has a discrete symmetry w.r.t. rotation along its axis, while the round one enjoys a continuous symmetry.



only multiples of 90 degrees



all angles

- More physical examples: parity vs. angular momenutm

Classical gauge invariance: Fields in electrodynamics

(Details not examinable)

- Remember Maxwells equation:

$$\begin{aligned} \underline{\nabla}\cdot\underline{E} &= 4\pi\rho & \underline{\nabla}\cdot\underline{B} &= 0 \\ \underline{\nabla}\times\underline{E}+\frac{\partial\underline{B}}{\partial t} &= 0 & \underline{\nabla}\times\underline{B}-\frac{\partial\underline{E}}{\partial t} &= 4\pi\underline{j}\,. \end{aligned}$$

- Implicit: conservation of current, $\dot{\rho}+\underline{\nabla}\cdot\underline{j}=0$.
- Can introduce potentials Φ and $\underline{A}$ such that

$$\underline{E} = -\underline{\nabla}\Phi-\frac{\partial\underline{A}}{\partial t} \quad\text{and}\quad \underline{B} = \underline{\nabla}\times\underline{A}\,.$$

(Can read them off from homogenous equations, i.e. equations of the form l.h.s.=0.)

- **Gauge invariance**: Fields will not change under

$$\Phi \Longrightarrow \Phi'=\Phi+\frac{\partial\Lambda}{\partial t} \quad\text{and}\quad \underline{A} \Longrightarrow \underline{A}'=\underline{A}-\underline{\nabla}\Lambda$$

(This is the gauge transformation of classical electrodynamics with an **arbitrary scalar function** Λ.)

Example: Invariance of Lorentz force

- Lorentz-force reads:

$$\underline{F} = e\left[\underline{E} + \frac{\mathrm{d}\underline{x}}{\mathrm{d}t} \times \underline{B}\right] = e\left[-\underline{\nabla}\Phi - \frac{\partial \underline{A}}{\partial t} + \frac{\mathrm{d}\underline{x}}{\mathrm{d}t} \times \underline{\nabla} \times \underline{A}\right]$$
$$= e\left[-\underline{\nabla}\Phi - \frac{\mathrm{d}\underline{A}}{\mathrm{d}t} + \underline{\nabla} \cdot \left(\frac{\mathrm{d}\underline{x}}{\mathrm{d}t} \cdot \underline{A}\right)\right].$$

To see this, use that $\frac{\mathrm{d}A_x}{\mathrm{d}t} = \frac{\partial A_x}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial A_x}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial A_x}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial A_x}{\partial z} = \frac{\partial A_x}{\partial t} + v_x\frac{\partial A_x}{\partial x} + v_y\frac{\partial A_x}{\partial y} + v_z\frac{\partial A_x}{\partial z}$

and that $(\underline{v} \times \underline{\nabla} \times \underline{A})_x = v_y\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + v_z\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + v_x\left(\frac{\partial A_x}{\partial x} - \frac{\partial A_x}{\partial x}\right)$

and that, since $\frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial x} = \frac{\partial v_z}{\partial x} = 0$, $\underline{v} \cdot (\underline{\nabla} \cdot \underline{A}) = \underline{\nabla}(\underline{v} \cdot \underline{A})$

- This can be used to construct a Lagrange function, rederive E.o.M. with Euler-Lagrange method & confirm the force, assess symmetries, construct a Hamilton function to handle the quantum mechanical problem ....

Global gauge invariance: generalised

- Copy idea underlying QED to construct dynamical picture of strong interactions.
  Basic idea: local gauge invariance
  - Start with a Lagrangian/Hamiltonian

    $$\partial_\mu \Psi^*(t, \underline{x}) \partial^\mu \Psi(t, \underline{x}) - m^2 \Psi^*(t, \underline{x}) \Psi(t, \underline{x})$$

    which is invariant under **global phase transformations** of the non-interacting electron/quark fields Ψ

    $$\Psi(t, \underline{x}) \longrightarrow \Psi'(t, \underline{x}) = e^{-i\alpha} \Psi(t, \underline{x}) \quad \text{with} \quad \alpha \in \mathbf{R} \text{ (a real constant).}$$

  - This enforces **conserved charges** of the fields.
  - Can now n arrange fields Ψ_i into one "vector of fields".
    Global gauge invariance now through phase factor $\exp(i\alpha_a \tau_a)$ with unitary $n \times n$ matrices τ_a

Local gauge invariance

- **Demand** invariance under **local phase transformations**:

$$\alpha \longrightarrow \alpha(t, \underline{x}) .$$

  This makes α identical in spirit to the Λ in classical electrodynamics.
- Then: **Must** introduce vector fields $A^\mu = (A^0, \underline{A})$ to compensate for terms of the kind $\partial\alpha$ that emerge in Hamiltonian.
- Vector fields must transform with α, identical to gauge transformation in electrodynamics.
- Previously non-interacting Ψ **coupled** to the $A \Longrightarrow$ **interacting**:

  Coupling/invariance achieved by replacing $\partial \longrightarrow (\partial - eA)$, reproduces the Lorentz force in electrodynamics.
- Similar feature in matrix form, but one A_a field per matrix τ_a.

Local gauge invariance for strong interactions

(details not examinable)

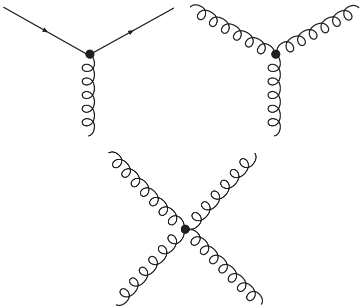
- Start with free, non-interacting quark fields Ψ_q
- For **global gauge invariance**: need a suitable **charge**.
  Cannot be the electrical charge - already taken for electrodynamics.
- Answer: It is "colour".

  (proposed 1973 by Fritzsch, Leutwyler and Gell-Mann)

  Quarks come in three colours, "red", "green", and "blue"
- To see why: Consider the Δ^{++} particle.
  - As a **fermion**, it **must** have an **antisymmetric wavefunction**.
  - It is constituted by three up-quarks, *uuu* in an *s*-wave, symmetric.
  - Its spin-3/2 state $|3/2, \pm 3/2\rangle$ has all quark spins aligned, symmetric.
  - Therefore: Need a new quantum number (or charge), in which the quarks are completely antisymmetric, i.e. $\propto \epsilon_{ijk}$.
- Demand: quarks form a triplet in the "strong charge" a.k.a. color: $\Psi_q = (\psi_q^r, \psi_q^g, \psi_q^b)$.

Feynman rules and colour factors

- Example: Gluon transform a “red” quark into “blue”.
- Charge conservation $\Longrightarrow$ **gluons carry a colour and an anti-colour.**
- Because of gluon charges: self-interactions (ggg and $gggg$ vertex).
- QED: γ's uncharged no 3γ or 4γ vertices.



- In calculations, colours are encoded in Gell-Mann matrices, representing the algebraic structure of the symmetry group.

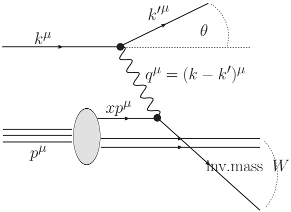
(Like Pauli matrices in spin/isospin group $SU(2)$.)

- There are 8 such matrices, therefore 8 gluons.

Proton's inner structure: Deep inelastic scattering

(Details not examinable)

- Probe proton, allow its disintegration.
- Use uncertainty relation momentum $q \approx 1/\lambda$:
  Momentum transfer $q > 1$ GeV ($\approx 1/0.2\mathrm{fm}$) $\Longrightarrow$ reveal inner structure.
- Kinematics:



$$\nu = \frac{2pq}{m_p} \longrightarrow E - E'$$

(energy transfer)

$$x = \frac{Q^2}{2pq} \longrightarrow \frac{Q^2}{E - E'}$$

(momentum fraction of parton)

$$Q^2 = -q^2 = -2EE'(1 - \cos\theta)$$

(momentum transfer squared)

- Typically: Exchange of γ's, but sometimes also νp-scattering with W exchange considered.

The parton model

(Details not examinable)

- In DIS: Measure $\sigma_{\rm DIS}$ in dependence on x (or ν) and Q^2.
- Scaling hypothesis (by J.D.Bjorken):
  For $E \to \infty$ and $Q^2 \to \infty$, $\sigma_{\rm DIS}$ depends on one variable only.
  Reason: no more coherent scatter off the nucleon, photon starts seeing individual, point-like partons.
- Parton model (by R.Feynman):
  The nucleon is made of smaller bits (partons).

Later knowledge: Partons identified with quarks and gluons.

- Naively: Expect three valence quarks ($|p\rangle = |uud\rangle$),
  but: many more partons, the sea quarks and gluons.

Parton distributions and sum rules

- Define probabilities $f_a(x)$ to find a parton of type a with energy fraction between x and $x + dx$:

$$F_1(x) = \sum_a q_a^2 f_a(x) \,, \quad q_a = \text{parton's charge.}$$

- Parton momenta must add to the proton momentum:

$$\int_0^1 \mathrm{d}x\, x\, [f_u(x) + f_{\bar{u}}(x) + f_d(x) + f_{\bar{d}}(x) + f_s(x) + f_{\bar{s}}(x) + \ldots] = 1\,.$$

- Parton types must yield a "net proton", $|p\rangle = |uud\rangle$:

$$\begin{array}{rclrcl}
\int\limits_0^1 \mathrm{d}x\, [f_u(x) - f_{\bar{u}}(x)] & = & 2 & \int\limits_0^1 \mathrm{d}x\, [f_d(x) - f_{\bar{d}}(x)] & = & 1 \\
\int\limits_0^1 \mathrm{d}x\, [f_s(x) - f_{\bar{s}}(x)] & = & 0 & \int\limits_0^1 \mathrm{d}x\, [f_c(x) - f_{\bar{c}}(x)] & = & 0\,.
\end{array}$$

Partons and QCD: A bit of history

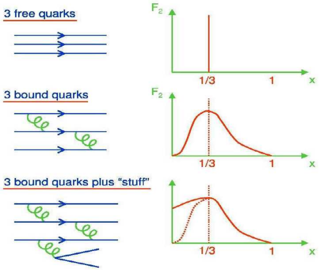
Experimental evidences in the late 60's and beginning 70's:

- Scaling behaviour of structure functions supports the assumption of point-like particles with relatively weak interactions between them at short distance/large momentum transfers - the partons in DIS behave nearly like free particles.
- Callan-Gross relation supports spin-1/2 fermions.
- Comparing *ep* and νp scattering supports assignment of fractional charges for the partons, like the quarks.
- Checking the momentum sum rule suggests that only about 50% of the momentum of the proton is carried by the quarks - the other half must be carried by charge neutral objects. These are identified with the gluons, the force carriers of the strong force, binding the quarks together.

Conclusion: Quarks are real objects, need to find an interacting theory.

Effect on structure functions: Scaling violations

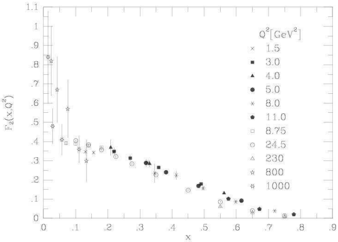
- In QCD: possible to quantify the picture of "proton = quarks + stuff"



- Leads to evolution equations: "Russian dolls"

- This implies dependence of $F_{1,2}$ on the momentum transfer.
- Therefore: $F_{1,2}$ depend on both x and Q^2

  Scaling violations



Learning outcomes

- Symmetries and conservation laws
- Continuous and discrete symmetries
- Internal symmetries in Quantum Mechanics: phase invariance
- Gauge symmetries as construction principle
- QCD as (non-Abelian) gauge symmetry: gluons have self-interactions
- Feynman rules of QCD