Foundations of Physics III
Quantum and Particle Physics
Lecture 9

Frank Krauss

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Historical isospin: Discovery of neutrons

- Rutherford's experiment: Lightest atom $= H$ (p-e-bound state)
  But: next lightest atom (He) four times as heavy as hydrogen, with only two electrons. Similar for Li (three electrons, seven times as heavy), etc.. Why so heavy?
- Answer: Nuclei are bound states of protons and neutrons.
- Discovery of the Neutron by J.Chadwick (1932)
  Bombard Beryllium with α-particles, very penetrating non-ionising radiation emerges. Send through paraffin, in turn protons are emitted. Measure speed of protons: original radiation cannot be γ's.

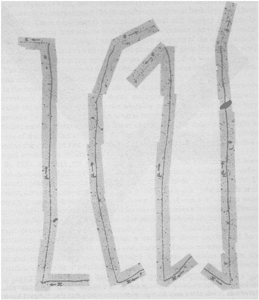
  Therefore new particle ("**neutron**") with nearly the same mass as the proton but no charge.
- Heisenberg's proposal (1932):
  Neutron and proton are two manifestations of same state, **Nucleon**.
  $$|\mathrm{Nucleon}\rangle = \left|\begin{array}{c}\mathrm{proton}\\ \mathrm{neutron}\end{array}\right\rangle \text{ or } |N\rangle = \left|\begin{array}{c}p\\ n\end{array}\right\rangle$$
- **Symmetry** relating them: **Isospin** (very similar to spin).

Proposing mesons

- H.Yukawa (1934): First prediction of **mesons** bind protons and neutron in nucleus.
- Yukawa's underlying assumption: Introduce a new force, short-ranged, thus mediated by massive mesons.
- Estimate: 3-400 times the electron mass.
  From uncertainty principle $\Delta E \Delta t \geq 1$ with time given by nucleon radius as $\Delta t \approx 1/r_0$. Assume r_0 of order $\mathcal{O}(1\text{fm})$, then
  $\Delta E \approx m_{\rm meson} \approx 200$ MeV
  (Note: natural units used in this estimate).

The first "mesons": The muon & the pion

- Two groups (1937): Anderson & Neddermeyer, Street & Stevenson: Finding such particles in cosmic rays using cloud chambers.
- But: wrong lifetime (too long, indicating weaker interaction), and inconsistent mass measurements
- Two decisive experiments to clarify the situation (Rome, 1946 & Powell et al. in Bristol, 1947) with photo emulsions.



- Result: In fact two new particles:
  - One **weakly interacting, the muon**, μ (a lepton),
  - one **strongly interacting, the pion**, π (a meson).
    Three differently charged versions, π^+, π^-, π^0;
    main decays: charged ones into muons plus a neutrino, neutral one into two γ's.

Mesons and baryons

- Have two kinds of strongly interacting particles:
  - **Mesons** = bound states of a quark and an antiquark
  - **Baryons** = bound states of 3 quarks or 3 antiquarks
- Quarks a point-like spin-1/2 particles
- **Quarks and gluons always in bound states** (strong interaction!)
- To accommodate for isospin: two quark-types, u and d:
  $\left|\begin{array}{c} up \\ down \end{array}\right\rangle$ with isospin and third component $I_{u,d} = \frac{1}{2}$, $I^3_{u,d,} = \pm\frac{1}{2}$.
- Then: $|p\rangle = |uud\rangle$, $|n\rangle = |udd\rangle$, $|\bar{p}\rangle = |\bar{u}\bar{u}\bar{d}\rangle$, $|\bar{n}\rangle = |\bar{u}\bar{d}\bar{d}\rangle$,
  and $|\pi^+\rangle = |u\bar{d}\rangle$, $|\pi^-\rangle = |d\bar{u}\rangle$, $|\pi^0\rangle = |\bar{\pi}^0\rangle = \frac{1}{\sqrt{2}}\left[|u\bar{u}\rangle - |d\bar{d}\rangle\right]$.
- This also fixes the charges: $Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$.
- Obviuos question: Are there more states?
  - yes, can have $|uuu\rangle$ and similar
  - yes, can supply the quarks with relative angular momentum etc..

Reminder: (Iso-)Spins and their addition

- Spin-1/2 systems are often studied in physics.
- Spin-statistics theorem suggests that such systems are fermionic in nature, i.e. respect Pauli exclusion.
- Interesting in the context of this lecture:
  Basic building blocks of matter (quarks & leptons) are spin-1/2.
- Simple representation:
  $$|\uparrow\rangle = \left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle \text{ and } |\downarrow\rangle = \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle.$$
  Important: Distinguish total spin s and its projection, $s_z = s^3$ on a measurement axis (here the z-axis, could also be I and I^3).
- Examples: electron and its spin, isospin, ...
- Note: Spin can also occur as spin-1 etc..
  Isopsin assignments for the hadrons above:
  $$|p, n\rangle = \left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle \text{ and } |\pi^+, \pi^0, \pi^-\rangle = \left|\frac{1}{2}, \{1, 0, -1\}\right\rangle$$

- Often two spin-1/2 objects form a compound.
  Examples: bound states of fermions, spin-orbit coupling, etc..
- If two spin-1/2 systems are added, the following objects can emerge:

  $$|\uparrow\uparrow\rangle,\ |\uparrow\downarrow\rangle,\ |\downarrow\uparrow\rangle,\ \text{and}\ |\downarrow\downarrow\rangle.$$

  Naively, they have spin 1, 0, or -1, respectively.
  But: **Need to distinguish total spin s and its projection onto the measurement axis s_z** (here, z has been chosen for simplicity)
- Then, truly relevant states are $s = 1$ (triplet, symmetric)

  $$|1,1\,\rangle = |\uparrow\uparrow\rangle, \quad |1,0\,\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right], \quad |1,-1\,\rangle = |\downarrow\downarrow\rangle$$

  and $s = 0$ (singlet, anti-symmetric):

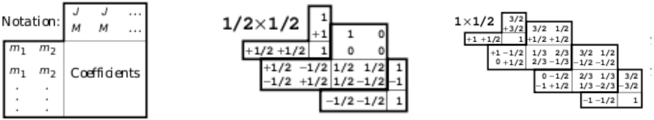
  $$|0,0\,\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$
- Catchy way of writing this: $\mathbf{2}\otimes\mathbf{2} = \mathbf{3}\oplus\mathbf{1}$

Clebsch-Gordan coefficients

- The **Clebsch-Gordan coefficients** in front of the new compound states can be calculated (or looked up).
- Formally speaking, they are defined as follows:

$$\left\langle s^{(1)},\, s_z^{(1)};\, s^{(2)},\, s_z^{(2)} \middle| s^{(1)},\, s^{(2)};\, s,\, s_z \right\rangle$$

  indicating that two spin systems $s^{(1)}$ and $s^{(2)}$ are added to form a new spin system with total spin s (or J). Obviously, it is not only the total spin of each system that counts here, but also its orientation. This is typically indicated through "magnetic" quantum numbers, m, replacing the s_z in the literature.



Square-roots around the coefficients are implicit

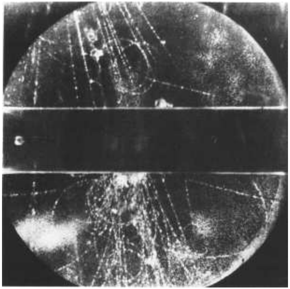
Using spin-algebra

- Identify: $|p\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle$ and $|n\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$.
  Heisenberg's proposal: Call this isospin (rather than spin).
- Also, the three kinds of pions can be written as:
  $|\pi^+\rangle = |1, +1\rangle$, $|\pi^0\rangle = |1, 0\rangle$, and $|\pi^-\rangle = |1, -1\rangle$.
- Catch: **Isospin conserved in strong interactions**!
- Dynamical implications: Bound states (here the deuteron).
  Add two nucleons: can in principle have iso-singlet and iso-triplet.
  But: No *pp*, *nn*-bound states, therefore $|d\rangle = |0, 0\rangle$ (deuteron = iso-singlet).
- Consider processes (+ their isospin amplitudes, below):

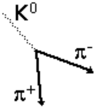
$$\begin{array}{ccc} p+p \to d+\pi^+ & p+n \to d+\pi^0 & n+n \to d+\pi^- \\ \mathcal{A}_{\rm iso} \propto 1 & \mathcal{A}_{\rm iso} \propto 1/\sqrt{2} & \mathcal{A}_{\rm iso} \propto 1 \end{array}$$

Strangeness. Who ordered that?

- Rochester & Butler (1947): Cloud chamber experiment with cosmic rays. Unusual "fork" of a π^+ and a π^-.
- Interpretation: Cosmic ray particles, mass between π and p, the **kaon**, **K**.
- Like pions, but strangely long lifetime (typically decay to pions or a muon-neutrino pair), again hinting at weak interactions being responsible.
- Ultimately, this lead to extending isospin ($SU(2)$) to a larger symmetry group, $SU(3)$ to catalogue mesons.



$K^0 \to \pi^+ + \pi^-$



Finding more strangeness

- Anderson (1950): Another "strange" particle, decaying into proton and π^-, the **hyperon**: $\Lambda \to p\pi^-$.

- Copious production of such particles $\longrightarrow$ strong interaction
- But: Slow decays $\longrightarrow$ weak interaction

Cataloguing strangeness

- A new quantum number proposed in 1953 by Gell-Mann and Nishijima
- Emerges as another quark type, strange, s:
  $I_s = 0,\ Q_s = Q_d = -\frac{1}{3},\ S_s = -1\ (S_{u,d} = 0)$
- Conserved in strong interactions: only produced pairwise $s\bar{s}$, violated in weak interactions (s decays into u)
- Gell-Mann-Nishijima relation:

$$Q = e\left(I_3 + \frac{B+S}{2}\right).$$

(I_3 is the third component ($= \pm 1/2$ for p, n) of the isospin, $S = \pm 1$ for kaons, Λ's, and Σ's, B is the baryon number ($= 1$ for baryons like p, n, Λ, Σ and $= 0$ for mesons like π, K).)

- Four mesons: kaons (K^+, K^-, K^0, $\bar{K}^0$).
  - Pseudo-scalars (i.e. spin-0, negative parity), just like pions.
  - All have the same mass, about three times m_π ($m_\pi \approx 140$ MeV, $m_K \approx 495$ MeV) $\Longrightarrow$ "relatives"?

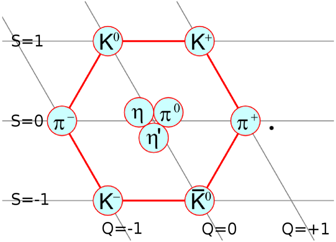
The quark model

- In 1964 Gell-Mann and Zweig proposed three "hypothetical" quarks, *u*p, *d*own, *s*trange, as a mnemonic help to sort hadrons.
- Knowledge now: All known hadrons are their "bound states":
  - **Mesons** are made from a $q\bar{q}$-pair, $q\bar{q} \equiv \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$
  - **baryons** from three quarks, $qqq \equiv \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$;

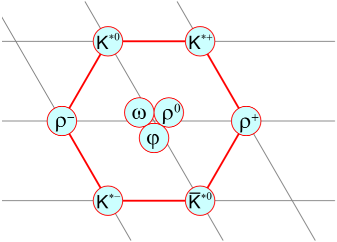
  i.e. one singlet of mesons and baryons, one octet of mesons, and two octets and one decuplet of baryons.
- Later more quark types were found: charm, bottom, top.
  Masses of up, down, strange below about 250 MeV, a typical scale for the strong interaction becoming non-perturbative,
  masses of charm, bottom and top much higher (about 1.5, 5, and 175 GeV), thus top quark about as heavy as a gold atom.

Meson multiplets

Pseudoscalars



Vectors



- Quark content of the mesons:

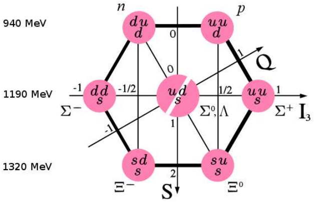
$$|K^+\rangle = |u\bar{s}\rangle\,,\quad |K^0\rangle = |d\bar{s}\rangle\,,\quad |K^-\rangle = |s\bar{u}\rangle\,,\quad |\bar{K}^0\rangle = |s\bar{d}\rangle\,,$$

$$|\pi^+\rangle = |u\bar{d}\rangle\,,\quad |\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle\,,\quad |\pi^-\rangle = |d\bar{u}\rangle\,,$$

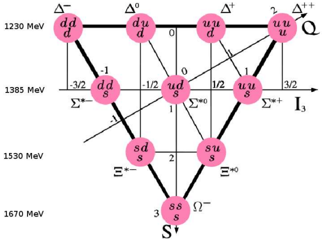
$$|\eta'\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle\,,\quad |\eta\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle\,.$$

Baryon multiplets

Octet



Decuplet



The discovery of the missing link: Ω^-

- In 1961, "tip" of the decuplet not yet found.
  M.Gell-Mann's prediction: m = 1672 MeV, plus the right production mechanism and a long lifetime.
  - Decay chain:

$$K^- + p \to \Omega^- + K^+ + K^{*0}$$

(strangeness conserving)

$$\Omega^- \to \Lambda^0 + K^-$$

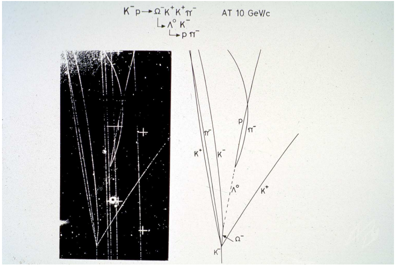
($\Delta S = 1$ weak decay)

$$\Lambda^0 \to \pi^- + p$$

($\Delta S = 1$ weak decay)

$$K^{*0} \to \pi^- + K^+$$

($\Delta S = 0$ strong decay)



The postulate of colour

- In the decuplet, one problem appears: Some states like for instance the Δ^{++} are composed from three identical quarks (u's for the Δ^{++}). Since the decuplet baryons are spin-3/2 objects they are fermions, i.e. their wave function must be antisymmetric. With three identical quarks, in identical spin states (spin-3/2 implies the spin-1/2's point into the same direction), this is possible only by invoking a new quantum number, **colour**.
- It will turn out that this quantum number acts as strong interaction "charge" of the quarks, triggering their strong interactions, like the electromagetic charge triggering the eletromagnetic interactions.

Learning outcomes

- Hadrons as bound states of quarks and antiquarks
- Two types of hadrons: mesons & baryons
- Isospin, strangeness as quantum numbers of u, d and s.
- Adding spins: Clebsch-Gordan coefficients
- Multiplet structure
- Interplay of charge $\longleftrightarrow$ interaction