Foundations of Physics III
Quantum and Particle Physics
Lecture 2

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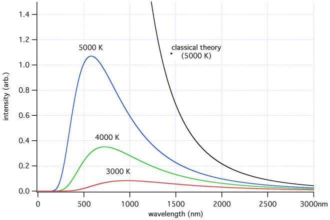
1 Reminder: Quantum mechanics

Quantising the electromagnetic field: Planck's hypothesis

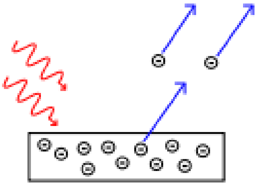
- Problem in 1900: The electromagnetic spectrum emitted by a hot black body.
- Statistical physics (classical) failed completely in explanation, predicting the total energy emitted to be infinite.



- Planck's ad hoc proposal: electromagnetic radiation is "quantised".
- Relation of energy E and frequency ν is $E = h\nu$
- From now on: units changed such that $\boxed{\hbar = 1}$.
- Remark: Most "perfect" black-body radiation is observed in cosmic microwave background.

Substantiating Planck's claim: Photoelectric effect

- Quantisation is a natural, intrinsic property of electromagnetic radiation.
- Explains the photoelectric effect: Electromagnetic radiation "kicks" electrons out of metal. Process **depends on frequency of light only, not on intensity**.



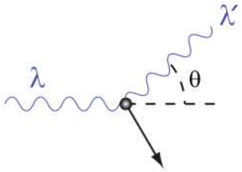
- Therefore it is the effect of a single photon.
- Energy of leaving electrons: $E_e = \omega - W_{\rm out}$.
  $W_{\rm out}$ is a material-specific energy neccessary for the electrons to leave the metal.
- Remark: One of four papers in Einstein's "annus mirabilis".
  (Others: Brownian motion and special and general relativity)

Substantiating Planck's claim: Compton effect

- Light scattered off a particle with mass m at rest changes wavelength:

$$\lambda \to \lambda' = \lambda + \frac{1-\cos\theta}{m}$$

- Exactly the behaviour of a massless particle in relativistic physics (energy-momentum conservation).



- Quanta of electromagnetic field are photons, symbolised by γ.
- First example of:

Interactions are mediated by exchange particles.

de Broglie's matter waves

- Hypothesis: Waves (light) has particle character, therefore particles may have wave character, undergoing interference etc..
- Wavelength proportional to inverse momentum, $\lambda = 1/|\underline{p}|$.
- This applies to **all** particles, including us.
- Observation of the wave-like character of particles by diffraction of electrons on a lattice and emerging interference patterns.
- de Broglie's hypothesis motivates (a posteriori) Bohr's model of the atom: Only such orbits are allowed that can be filled with an integer number of wavelengths.

Essence of quantum mechanics: Schrödinger equation

- Using the matter wave idea of de Broglie, Schrödinger formulated a full theory based on wave mechanics for them.
- In his framework the wave of a particle with mass m (denoted by ψ) develops in space and time as

$$-\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\frac{\partial\psi}{\partial t}\,.$$

- Applying this equation to the hydrogen atom, he was able to reproduce Bohr's findings of discrete energy levels.
- Interpretation of the wave function: Its absolute value squared $|\psi(t,x)|^2$ gives the probability of finding the particle at x and t.
- Note: Such probabilities are invariant under

$$\psi(t,x) \longrightarrow \psi'(t,x) = e^{i\alpha}\psi(t,x)\,.$$

Heisenberg's uncertainty principle

- Alternative formulation by Heisenberg, centred around **observation**.
- Wave functions are replaced by (infinite dimensional) state vectors, observables are operators acting on them.
- Measurements are identified with expectation values of operators.
- Consequence: statistical/probabilistic treatment inherent.
- Uncertainty relations

$$\Delta p \Delta x \geq \frac{1}{2} \quad \text{and} \quad \Delta E \Delta t \geq 1\,.$$

- Example: Confine a particle in a small volume $\Delta x = 1$ fm.
  Then: Uncertainty of position yields undirected random movement

$$\Delta p = \frac{1}{2\Delta x} \approx 100\mathrm{MeV}\,.$$

(Used $\hbar c \approx 200$ MeV fm.)

- Example: Lifetime of unstable particle $\tau = 10^{-18}$ s.
  Then: Uncertainty in mass given by limited time for measurement

$$\Delta E = \Gamma = 1/\tau \approx 0.65\mathrm{MeV}\,.$$

(Used $\hbar \approx 6.5 \cdot 10^{-19}$ MeV s.)

Detour: Some notation

- Quantum physical states characterised by quantum numbers ψ are described by wave functions $\psi(t, x)$ in position space and by wave functions $\chi(t, p)$ in momentum space.
- Want to introduce a notation, where all quantum numbers, apart from space/momentum are encoded in one time-dependent state vector $|\psi(t)\rangle$ (Schrödinger picture) such that

$$\psi(t, x) = \langle x|\psi(t)\rangle \quad \text{and} \quad \chi(t, p) = \langle p|\psi(t)\rangle\,.$$

  This is achieved by introducing position and momentum eigenstates:

$$\hat{x}|x\rangle = x|x\rangle \quad \text{and} \quad \hat{p}|p\rangle = p|p\rangle\,.$$

- Then Schrödinger equation reads (omitting the $\langle x|$)

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle = \left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]|\psi(t)\rangle\,.$$

Detour: Dealing with many particles

- For more than one particle: State vector $|\psi\rangle$ depends on all quantum numbers: spins, momenta, etc..
- Building a state vector through creation and annihilation operators.

  (Like ladder operators in the harmonic oscillator.)
- Examples (use only momenta to describe N identical particles):

$$|p_1 p_2 \ldots p_N\rangle = \hat{a}^\dagger(p_1)\hat{a}^\dagger(p_2)\ldots\hat{a}^\dagger(p_N)|0\rangle$$

$$\hat{a}(p_1)\hat{a}(p_2)\ldots\hat{a}(p_N)|p_1 p_2 \ldots p_N\rangle = |0\rangle\,,$$

  with vacuum state $|0\rangle$, and omitting symmetrisation factors.
- The $\hat{a}$ and $\hat{a}^\dagger$ have (anti-)commutation relations, encoding whether the particles obey Fermi-Dirac or Bose-Einstein statistics.

Going relativistic

- Starting point (compare Schrödinger equation):

$$\hat{H}|\psi\rangle = \left(\frac{\hat{p}^2}{2m} + \hat{V}\right)|\psi\rangle = i\frac{\partial}{\partial t}|\psi\rangle\,,$$

  with $p = -i\partial/\partial x$.
- Recognise: First term of $\hat{H}$ is non-relativistic kinetic energy,
  it is **always positive:** $E > 0$.
- Therefore: Try to replace with relativistic expression.
- Problem: Quadratic relation of E and p ($E^2 = p^2 + m^2$)
  energy is **not positive-definite** any more!
- Many far-reaching consequences:
  Antiparticles, varying number of particles, vacuum-fluctuations, etc..
  Seemed like a complete mess, good enough to abandon the idea.
  But there was a way out, will discuss this in a later lecture.

Aside: Spin

- Discovery of small splitting of spectral lines in hydrogen spectrum - not explained by Bohr's model.
- Explanation: electron has "intrinsic magnetic moment" (spin), interacting with magnetic field produced by orbiting around nucleus.
- Distinctively quantum: In classical physics, all spin orientations are allowed, leading to a range rather than two lines.
- Therefore: spin must be quantised as well.
- By convention: $s = \pm 1/2$ for electrons etc..
- Important: Spins always come in integers or half integers, at integer distance. Two different kinds of particles: bosons (integer spins) and fermions (half-integers), the latter enjoying the Pauli exclusion principle.

Learning outcomes

- Reminder of some basic quantum mechanics and phenomena
- de Broglie waves and uncertainty relation to estimate characteristic scales in a process.
- Get used to $\hbar = c = 1$

$$\boxed{\begin{gathered} c \approx 3 \cdot 10^8 \ \mathrm{m/s} = 3 \cdot 10^{23} \ \mathrm{fm/s} \\ \hbar \approx 6.6 \cdot 10^{-22} \ \mathrm{MeV\,s} \\ \hbar c \approx 200 \ \mathrm{MeV\,fm} = 0.2 \ \mathrm{GeV\,fm} \end{gathered}}$$