Foundations of Physics III
Quantum and Particle Physics
Lecture 12

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1. Generation structure of fermions

Generation structure of fermions

- A simple question: Are the neutrinos emitted in neutron β-decay the same as those emitted in proton β-decay?
- Compare β decays of n and p: $n \to p + e^- + \nu$ and $p \to n + e^+ + \nu$. Suggests that in the former case an anti-neutrino $\bar{\nu}$ is emitted.

  (By moving a particle from left to right, need to "bar" it - replace with antiparticle.)
- Allows to formulate a new law: **lepton-number conservation** Assigning a lepton number of $+1$ to electron, muons and neutrinos, and of -1 to their anti-particles, i.e. positrons, anti-muons and anti-neutrinos, and a lepton number of 0 to all other particles, then in any reaction lepton number is conserved.
- Examples:
  - $\bar{\nu} + p \to e^+ + n$ is allowed
  - $\bar{\nu} + n \to e^- + p$ is forbidden.

The absence of $\mu \to e\gamma$

- Further: How do muons, strange quarks etc. fit into the picture?
- Naively, reactions of the type $\mu^- \to e^- + \gamma$ would be allowed: A Feynman diagram related to this would look like the muon decaying into a neutrino plus a charged W-boson, which recombine to form an electron, emitting a photon on the way.
  This process has never been observed - experimentally, its (probability to happen) branching ratio $\mathcal{BR} < 10^{-14}$.
- Solution to this puzzle: Postulate **two types of neutrinos** such that the corresponding doublets read

$$\bar{\Psi}_e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \text{ and } \bar{\Psi}_e = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}.$$

  In fact, there are three types of neutrinos ....

Conservation laws for lepton numbers

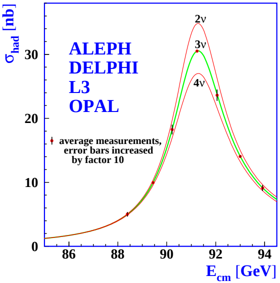
- **Lepton-number $\longrightarrow$ lepton-type number conservation**:
  - electron number $= +1$ for e^- and ν_e,
    $= -1$ for e^+ and $\bar{\nu}_e$,
    and $= 0$ for all other particles
  - similar for muon number (and tau number, see later).
- Genius experimenting: The two-neutrino experiment
  - The main problem when experimenting with neutrinos is to produce them in large quantities. One option, at low energies, is by using the huge fluxes coming from nuclear reactors (electron neutrinos). At large energies, accelerators must be used.
  - To this end, for instance, high-energy protons are collided with a beryllium target, producing large numbers of, e.g. pions. They in turn decay into muons and muon-neutrinos. The latter can be isolated by large amounts of material - iron. If there was only one neutrino species, the remaining neutrinos in the experiment could, in principle, initiate two kinds of reactions with equal likelihood, namely

$$\nu + n \to \mu^- + p \text{ and } \nu + n \to e^- + p.$$

  - However, in the first experiment of this kind L.Lederman, M.Schwartz and J.Steinberger used 25 years of accelerator time, producing around 10^{14} neutrinos yielding 51 muons and no electrons, establishing the existence of two kinds of neutrinos.

The number of generations

- Found lepton "generations "
  $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$
- ... quark generations:
  $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$
- Heavier copies of the same underlying structure.



- Weak interactions do not mediate between different lepton families (no transition $\mu^- \to \nu_e$ and similar) but mediate between quarks (transitions $s \to u$, $c \to d$, ... exist).
- Experimental evidence for three "regular" generations: The width (lifetime) of Z bosons suggests the existence of three neutrinos species available as decay products.

- **Strong, e.m., and neutral weak interactions do not change particle type** (flavour); **charged weak interactions do**:
  In the lepton sector, generation number is conserved (lepton-type conservation), in the quark sector it isn't.

  (No transitions between generations in lepton sector, in quark sector $s \longrightarrow u$ and similar allowed in charged weak interactions.)

- In the Standard Model, neutrinos *by definition* massless.

  (In reality: tiny mass differences measured . . . . Related to a mixing of neutrinos - beyond the aim of the course. A *first hint* for physics *beyond the SM*.)

Fermion masses:

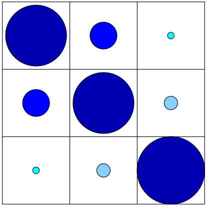
$1^{\rm st}$ generation	$2^{\rm nd}$ generation	$3^{\rm rd}$ generation
$m_u \approx 5$ MeV	$m_c \approx 1.5$ GeV	$m_t \approx 175$ GeV
$m_d \approx 5$ MeV	$m_s \approx 200$ MeV	$m_b \approx 4.5$ GeV
$m_{\nu_e} \approx 0$ MeV	$m_{\nu_\mu} \approx 0$ MeV	$m_{\nu_\tau} \approx 0$ MeV
$m_e \approx 0.511$ MeV	$m_\mu \approx 105$ MeV	$m_\tau \approx 1.75$ GeV

Quark mixing & CKM matrix

(CKM = Cabibbo-Kobayashi-Maskawa)

- Inter-generation transitions dominated by mass spectrum and CKM matrix
- dominant transitions: $t \to b$, $b \to c$, $c \to s$, ...
- Source of CP-violation in V_{13}. Need at least three generations for such complex entries.
  But: CKM alone not sufficient for observed matter-antimatter asymmetry in universe (Sakharov conditions).

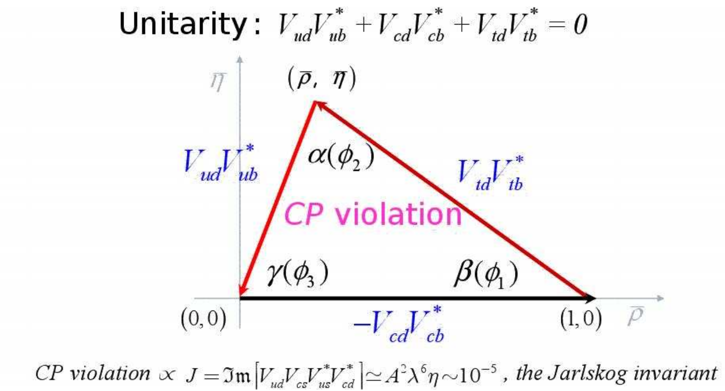
Relative size of CKM Matrix (not to scale)



- Expand in small parameter $V_{us} \approx \lambda \approx 0.22$, up to $\mathcal{O}(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

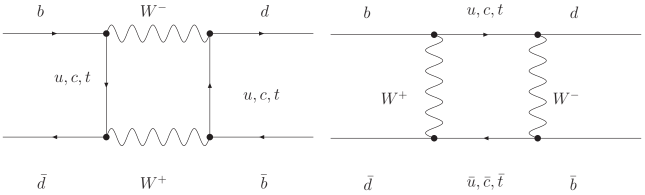
“The” unitarity triangle



D.Hitlin, Talk at “Flavor in the Era of LHC”, 2005)

More on $\hat{C}\hat{P}$ violation

- Observed in mixing and decays of neutral mesons.
- Consider mixing only:
  $|K^0\rangle = |\bar{s}d\rangle$ mixes with $|\bar{K}^0\rangle = |s\bar{d}\rangle$,
  $|D^0\rangle = |c\bar{u}\rangle$ mixes with $|\bar{D}^0\rangle = |\bar{c}u\rangle$,
  $|B^0\rangle = |\bar{b}d\rangle$ mixes with $|\bar{B}^0\rangle = |b\bar{d}\rangle$.
- Relevant diagram (exemplified for $B\bar{B}$-mixing):



- Quantum mechanically speaking: Have two flavour eigenstates (e.g. for B-mesons $|\bar{b}d\rangle$ and $|b\bar{d}\rangle$), but also two $\hat{C}\hat{P}$/mass eigenstates (e.g. for kaons K_S and K_L) which decay into different numbers of pions etc. and therefore have a different lifetime.
- This will lead to a (unitary) matrix relating the two bases:

$$\hat{H}\left|\begin{array}{c} B^0 \\ \bar{B}^0 \end{array}\right\rangle = \left(\begin{array}{cc} M & \Delta M/2 \\ \Delta M/2 & M \end{array}\right)\left|\begin{array}{c} B^0 \\ \bar{B}^0 \end{array}\right\rangle$$

  such that the mass eigenstates M_{LH} of the neutral B mseons have a mass difference of $\Delta m_B = \Delta M$.
- ΔM can be calculated from diagrams on previous slide. Estimate:

$$\Delta M \propto (V_{tb}V_{td})^2 \frac{\alpha^2}{\sin^4\theta_W}\frac{m_B f_B^2 m_t^2}{m_W^4}$$

  where m_B and f_B are the mass and decay width of the B-meson, yielding a small number for Δ_M!

Time evolution of two states

- Assume a case where a $b\bar{b}$ quark pair is produced. They would *hadronize* into a $B\bar{B}$ meson pair - so you know that there is one B and one $\bar{B}$ meson.
- Quantum-mechanically speaking, *b*eauty is a conserved quantum number, so if you know that one of the mesons is carrying a b you know that the other must carry a $\bar{b}$ – this is a prime example for **entanglement**.
- Now, if you measure the decay of one of them – and, say, by the charge of a lepton emitted in this decay – you know, it was the b-quark, then you know that, at the same time, the other meson carried the $\bar{b}$.
- However, the time development of the surviving meson state $|\psi\rangle$ is governed by $\mathrm{d}|\psi\rangle/\mathrm{d}t = \hat{H}|\psi\rangle$ and this fluctuates back and forth, $\propto \sin(\Delta m_B t)$.

Learning outcomes

- Generation/family structure of fermions
- Weak interactions conserve lepton- and lepton-family number, also conserve quark number, but not quark-family number.
- The CKM-picture of quark mixing
- Discrete symmetries and their breaking in weak interactions. Most notably: $\hat{P}$, $\hat{C}$, and $\hat{C}\hat{P}$-violation.
- Heavy quarks and their bound states.