Foundations of Physics III
Quantum and Particle Physics
Lecture 1

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February 3, 2012

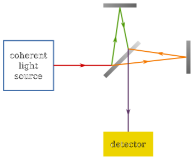
1. Review of special relativity
2. Relativistic kinematics

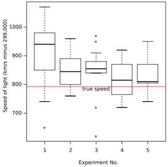
Experimental foundation: c is constant

- Idea: Check existence of ether. Use earth's velocity orbiting around the sun. Leads to different relative velocities.
- Send coherent light along two directions, check for shifts in interference pattern due to different run times:

$$t_\parallel = \frac{2L}{c(1-v_E^2/c^2)} \text{ and } t_\parallel = \frac{2L}{c\sqrt{1-v_E^2/c^2}}.$$

- But: No variation seen.
- Conclusion: Speed of light is a constant, verified by a number of similar experiments.
- Recently, a neutrino experiment (Opera) hinted at neutrinos being faster than c
- This is not confirmed by other experiments (neutrinos from supernovae etc.), still under discussion.





Lorentz transformations

- Basic idea: Space and time are connected.
- Consequence: Relative velocities between reference frames affect both space and time coordinates.

$$\underline{x} \to \underline{x}' = \frac{\underline{x} - \underline{u}t}{\sqrt{1 - \frac{\underline{u}^2}{c^2}}} \quad \text{and} \quad t \to t' = \frac{t - \frac{\underline{u}\underline{x}}{c^2}}{\sqrt{1 - \frac{\underline{u}^2}{c^2}}}\,.$$

- From now on: Will set $\boxed{c = 1}$:

$$\underline{x} \to \underline{x}' = \frac{\underline{x} - \underline{u}t}{\sqrt{1 - \underline{u}^2}} \quad \text{and} \quad t \to t' = \frac{t - \underline{u}\underline{x}}{\sqrt{1 - \underline{u}^2}}\,.$$

Adding velocities

- Consequence of this: Velocities are always below c and can **never** add up to a result larger than c:

$$\underline{v}_{\rm tot} = \frac{\underline{v}_1 + \underline{v}_2}{1 + \underline{v}_1 \cdot \underline{v}_2} .$$

- Remark: This limits the maximal transmission velocity of information to c, therefore a perfectly rigid body cannot exist.
- Remark: Space-time is divided into regions that are either
    - causally connected, "time-like distances", $\Delta_{12} = (t_1 - t_2)^2 - (\underline{x}_1 - \underline{x}_2)^2 > 0$ or
    - disconnected, "space-like distances", $\Delta_{12} = (t_1 - t_2)^2 - (\underline{x}_1 - \underline{x}_2)^2 < 0$

Manifest Lorentz-invariance: Four-vectors

- Since time and space on identical footing: Introduce "four-vectors"

$$x^\mu = (t, \underline{x}) \text{ and } p^\mu = (E, \underline{p}).$$

- Introduce **Lorentz-invariant** norms

$$x^2 = t^2 - \underline{x}^2 \quad \text{and} \quad p^2 = E^2 - \underline{p}^2 \,.$$

and scalar products

$$x_1 \cdot x_2 = x_1^\mu x_{2\mu} = x_{1\mu} x_2^\mu = x_1^\mu x_2^\nu g_{\mu\nu} = t_1 t_2 - \underline{x}_1 \cdot \underline{x}_2$$

- Achieved through a **metric** (metric tensor)

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Remark: Gravity acts by distorting the metric $g^{\mu\nu}$.

Lorentz transformations, once more

- Rewrite z-component of

$$\underline{x} \to \underline{x}' = \frac{\underline{x} - \underline{u}t}{\sqrt{1-\underline{u}^2}} \quad \text{and} \quad t \to t' = \frac{t - \underline{u}\underline{x}}{\sqrt{1-\underline{u}^2}}$$

in matrix form – acting on vector $(t, \underline{x})$:

$$\hat{M}_{u_z} = \begin{pmatrix} \cosh\eta & 0 & 0 & -\sinh\eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh\eta & 0 & 0 & \cosh\eta \end{pmatrix}$$

with $\tanh\eta = u_z$ for a "boost" in z-direction.

(Remember $\cosh x = 1/\sqrt{1-\tanh^2 x}$ and $\sinh x = \sqrt{\tanh^2 x/(1-\tanh^2 x)}$.)

- Note the similarity to a rotation in space!

(Gets even better when you remember $\cos(i\eta) = \cosh\eta$ and $\sin(i\eta) = i\sinh\eta$.)

Mass, momentum and energy

- Demand conservation of mass, momentum and energy to be invariant under Lorentz transformations:

  $$\text{For mass: } m(v) = \frac{m_0}{\sqrt{1-v^2}} = m_0 + m_0\frac{v^2}{2} + \ldots.$$

  This is the original reason for the identification $E = mc^2$ - the second term in the expansion is just the kinetic energy.
- Using $\underline{p} = m\underline{v}$ therefore $E^2 = m_0^2 + \underline{p}^2$.
- This implies that for particles with no rest mass $E/|\underline{p}| = 1$.
- From now on: will only call the rest mass the mass of a particle.

Relativistic two-body decay

- Consider the decay of a massive particle into two lighter ones, such that **rest masses** satisfy $M > m_1 + m_2$.
- To calculate energies and momenta of decay products use:

Rest frame of decaying particle:	$E = M,\ \underline{P} = 0;$
Energy conservation:	$E = E_1 + E_2;$
Momentum conservation:	$\underline{P} = \underline{p}_1 + \underline{p}_2 \Longrightarrow \lvert\underline{p}_1\rvert = \lvert\underline{p}_2\rvert;$
Energy-momentum relation:	$E_i^2 = m_i^2 + p_i^2.$

- Case 1: $m_1 = m_2 = 0$

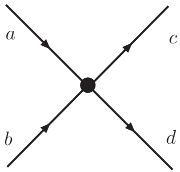
$$E_1 = p_1 = p_2 = E_2 = M/2$$

- Case 2: Arbitrary masses, $m_1 \neq 0$, $m_2 \neq 0$:

$$E_{1,2} = \frac{M^2 \pm (m_1^2 - m_2^2)}{2M} \quad \text{and} \quad p_{1,2} = \frac{\sqrt{(M^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}}{2M}\,.$$

Relativistic two-body reactions

- Central reaction type in particle physics:
  2-body scattering: $a + b \to c + d$
- Convenient frame of inertia for description:
  centre-of momentum frame,
  characterised by $\underline{p}_a + \underline{p}_b = \underline{p}_c + \underline{p}_d = 0$



- Calculate Lorentz-invariant mass (energy) from:

$$\begin{aligned} s = M^2_{\rm inv} &= (E_a+E_b)^2 - (\underline{p}_a + \underline{p}_b)^2 \\ &= (E_c+E_d)^2 - (\underline{p}_c + \underline{p}_d)^2 \,. \end{aligned}$$

- This is the energy squared in the c.m.-frame: $s = E^2_{\rm c.m.}$.

Relativistic two-body reactions (cont'd)

- Can also calculate the (Lorentz-invariant) momentum transfer from a to c, called t and from a to d, called u:

$$\begin{aligned} t &= (E_a - E_c)^2 - (\underline{p}_a - \underline{p}_c)^2 = (E_b - E_d)^2 - (\underline{p}_b - \underline{p}_d)^2 \\ u &= (E_a - E_d)^2 - (\underline{p}_a - \underline{p}_d)^2 = (E_b - E_c)^2 - (\underline{p}_b - \underline{p}_c)^2 \,. \end{aligned}$$

- Properties:
  - $s > 0$, and $t,\ u \leq 0$
  - $s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2$.

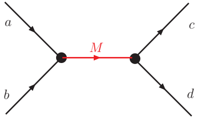
  Therefore, for massless particles $s + t + u = 0$.
- In the c.m.-frame, and for massless particles:

$$t = -\frac{E_{\rm c.m.}^2}{2}\left(1 - \cos\theta_{ac}\right) \quad \text{and} \quad u = -\frac{E_{\rm c.m.}^2}{2}\left(1 + \cos\theta_{ac}\right).$$

  θ_{ac} is called the "scattering angle".

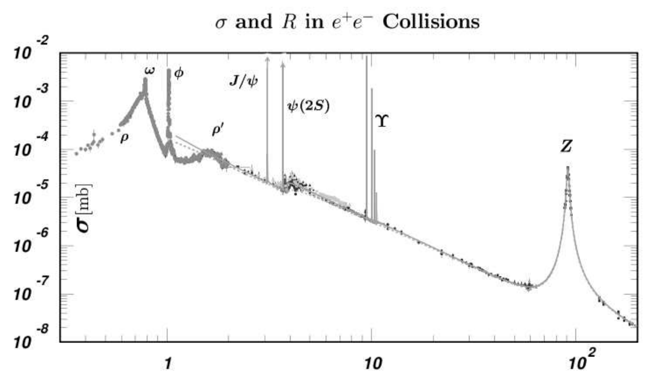
Particle creation and decay

- Consider a special case of $2 \to 2$-scattering:
  Production of intermediate particle:
  $$a + b \to M \to c + d$$
- Energy and momentum of M in c.m.-frame:
  $$E = E_a + E_b,\ \underline{P} = 0$$



- We will see that the probability for this process "resonates", if $s = E^2_{\mathrm{c.m.}} = M^2$ (resonance production).
  The production cross section will yield a peak.
- Note: Cross section is a way to quantify the probability for a process to happen, more on this in Lecture 3.

Example for resonance production: $e^+e^- \to$ hadrons



Learning outcomes

- Lorentz transformations: time dilation, length contraction.
  Very examinable!
- Will use a system of units where $c = 1$ (please, get used to it)
- Four vectors (not examinable)
- Energy-momentum mass relation $E^2 - \underline{p}^2 = m^2$, mass in the following *always* the rest mass.
- Kinematics of two-body decays and two-body reactions.
  Very examinable!