Foundations of Physics III
Quantum and Particle Physics
Lecture 7

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1. Stable vs. Unstable vs. Virtual

Particles and propagators

- Up to now, two kinds of particles/lines in Feynman diagrams:

  (Remember: $q^2 = E^2 - \underline{q}^2$.)

  - internal lines $\propto \frac{i}{q^2 - M^2}$
    for particles with four-momentum q and mass M;
  - external lines $\propto 1$
    for particles with four-momentum q and mass M with $q^2 = M^2$.
- In both cases, the spin of the particles has been ignored.

Particles as plane waves

(details not examinable)

- up to now, all particles are assumed to be free & stable. This allows to represent external particles as plane waves,

$$\psi_\pm(\underline{r},\,t) \propto e^{\mp iEt \pm i\underline{q}\cdot\underline{r}}\,,$$

with $\psi_\pm$ representing particles $(+)$ and anti-particles (-).

- The term $e^{\mp iEt}$ is nothing but the time evolution operator for a free particles, in their rest frame therefore particles with mass M can be represented as $\psi_\pm(\underline{0},\,t) = e^{\mp iMt}$.

Propagators

(details not examinable)

- To obtain the propagator S_F, the idea is that it fulfils the differential equation (it is a Green's function) given by the **free Hamiltonian**

  (Remember: four vectors!)

$$HS_F(x_1,\, x_0) = \left(-\frac{\partial^2}{\partial t_1^2} + \underline{\nabla}_1^2 - m^2\right) S_F(x_1,\, x_0) = i\delta^4(x_1 - x_0)$$

  and Fourier transformation yields

$$\left(E^2 - \underline{p}^2 - m^2\right)\, S_F(p) = i \quad \longrightarrow \quad S_F(p) = \frac{i}{p^2 - m^2}\,.$$

- Therefore: Feynman diagrams are given by building blocks obtained from free particles with interactions in between.

Unstable particles: non-relativistic case

(details not examinable)

- Consider some unstable system, like, e.g. radioactive isotopes with a life time τ, proportional to the conventional half-life. The number of isotopes at some time t is given by

$$N(t) = N(t=0)e^{-t/\tau} = N_0 e^{-\Gamma t}$$

with "**width**" or "**decay rate**" $\Gamma = 1/\tau$.

- In Quantum Mechanics, the number of "surviving" isotopes is given by absolute value squared of their wave function,

$$N(t) = \int \mathrm{d}^3 x |\psi(\underline{x},\, t)|^2 \,.$$

(details not examinable)

- If this wave function has a stable particle in the limit of infinite life-time or zero decay rate, then

$$\psi(\underline{x},\, t) \propto e^{-\Gamma/2t} e^{\mp iMt}\,.$$

In other words the original sinusoidal free wave function is modulated to have decreasing amplitudes. Squaring then gives the correct $e^{-\Gamma t}$-behaviour.

(details not examinable)

- Consider now the nergy dependence of a process, where such a wave function enters. To this end, one must Fourier transform the wave function from time to energy, by

$$\psi(E) = \int \mathrm{d}t\psi(t)e^{iEt} \propto \int \mathrm{d}te^{[i(E-M)-\Gamma/2]t} \propto \frac{1}{E-M+i\Gamma/2},$$

  and therefore

$$|\psi(E)|^2 \propto \frac{1}{(E-M)^2+(\Gamma/2)^2}$$

  where position/momentum dependence have been ignored.

- This looks pretty much like the resonance structure of a driven dampened linear oscillator with an eigenfrequency of Ω and a driving frequency ω yielding an intensity of oscillations given by

$$I(\omega) \propto \frac{1}{(\omega-\Omega)^2+(\Gamma/2)^2}$$

Relativistic generalisation

(details not examinable)

- The structure encountered is known as **Breit-Wigner resonance**.
- The relativistic generalisation can be obtained by replacing

$$\psi(E) = \frac{1}{E^2 - \underline{p}^2 - M^2 + iM\Gamma}$$

leading to a factor

$$\frac{i}{(q^2 - M^2)^2 + M^2\Gamma^2}$$

in the squared amplitude.

Virtual vs. unstable

(details not examinable)

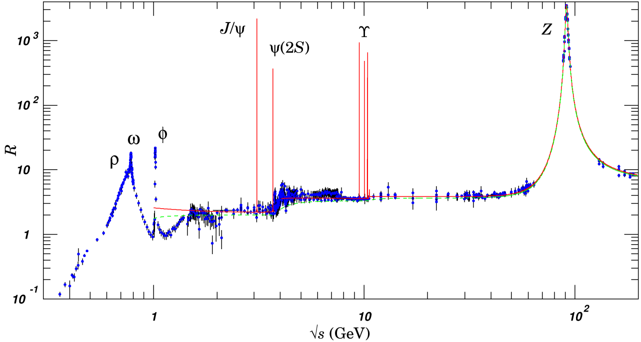
- Clearly, when $q^2 = E^2 - \underline{q}^2 \to M^2$ of such an internal line, the amplitude squared increases drastically, it "**resonates**".
- For stable particles, it is clear that, without loops, the propagator term can never approach $q^2 \to M^2$ - because then the decay of this particle would be allowed and hence it would not be stable.
- Virtual particles can enter a process as internal lines only, where their lifetimes are determined by the energy-time uncertainty relation. This in turn gives a measure for the characteristic time the process needs to take place. We have seen that such times usually are very short, of the order of 10^{-24} s or less.
- In contrast, unstable particles can in principle be treagetd as external lines, entering or leaving a process, if their lifetime is much larger than the characteristic times the process needs to take place at all. This lifetime is given by the decay rate or width Γ and directly enters their treatment.

Example plot: $R_{\rm had}(E)$

(will come back in a few lectures)

- Consider structures (resonances) in the R ratio of

$$R_{\rm had}(E) = \frac{\sigma_{e^+e^-\to{\rm hadrons}}(E)}{\sigma_{e^+e^-\to\mu^+\mu^-}(E)}$$



Calculating decay rates

- Similar to scattering processes, **partial decay rates**, i.e. the rate for aspecific decay of a particle, are determined by calculating the transition amplitude, squaring it, and summing or integrating over all final state degrees of freedom.
- For a two-body decay $M \to 12$, this then yields

$$\Gamma_{M\to 12} = |\mathcal{M}_{M\to 12}|^2 \frac{\mathrm{d}\Omega}{64\pi^2 M} \frac{(2|\underline{p}_1|)}{M}$$

$$\stackrel{m_1=m_2=0}{\longrightarrow} |\mathcal{M}_{AB\to 12}|^2 \frac{\mathrm{d}\Omega}{64\pi^2 M} .$$

- Note: since in $\hbar = c = 1$, Γ is measured in units of energy/mass (GeV), the transition amplitude $\mathcal{M}$ also is in units of energy.

Total decay rates and branching ratios

- The **total decay rate** is then given as the sum over all possible partial decay rates:

$$\Gamma_M = \sum_X \Gamma_{M\to X} \quad\longleftrightarrow\quad 1 = \sum_X \frac{\Gamma_{M\to X}}{\Gamma_M} = \sum_X BR(X)\,,$$

with branching ratios $BR(X)$ (relative probabilities for a specific decay channel to a final state X to happen).

Learning outcomes

- Ideas behind stable vs. unstable vs. virtual particles
- Breit-Wigner resonance structure
- Calculating partial decay rates - 2-body decays:

$$\begin{aligned}\Gamma_{M\to 12} &= |\mathcal{M}_{M\to 12}|^2 \frac{\mathrm{d}\Omega}{64\pi^2 M}\frac{(2|\underline{p}_1|)}{M}\\ &\stackrel{m_1=m_2=0}{\longrightarrow} |\mathcal{M}_{AB\to 12}|^2 \frac{\mathrm{d}\Omega}{64\pi^2 M}\,.\end{aligned}$$

- Total decay rates, partial decay rates and branching ratios.