

Foundations of Physics III

Quantum and Particle Physics

Lecture 11

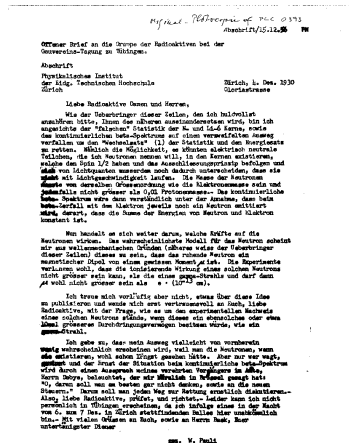
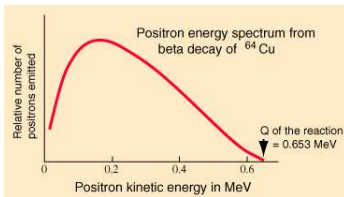
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- 1 Gauge theory of weak interactions
- 2 Discrete symmetries
- 3 Spontaneous symmetry breaking

The “invention” of neutrinos

- Hypothesised by Pauli 1930:
Famous letter to the “Dear radioactive ladies and gentlemen” (see left).
- Reason: In β -decay only two particles seen, but continuous energy spectrum of the electron. Impossible in two-body decays.



Gauge theory of weak interactions

- As before, want to copy the success of QED/QCD:
 - Introduce **conserved charges** through **global phase invariance**
 - In QED: Write electron field ψ , attach charge $q_e = -1$
 - In QCD: Write quark field Ψ_q as colour triplet ($\psi_q^{(r)}$, $\psi_q^{(g)}$, $\psi_q^{(b)}$)
 - Promote global phase invariance to **local phase invariance**
 - Introduce “gauge fields” to ensure the invariance
 - Free fields ψ get coupled with “gauge fields” A ,
the A must be massless to preserve gauge invariance (See lecture 10.)
 - Effect of gauge fields in QCD: $\psi_q^{(r)} \xrightarrow{A\vec{g}} \psi_q^{(g)}$
 \implies gluons carry a “charge” (a colour and an anti-colour)
- In QED/QCD interaction does not change “type” of particle if possible, only charge quantum number gets changed.
- In contrast: Weak interactions trigger β -decays ($n \rightarrow p$ or $d \rightarrow u$)
Must embed this in a “gauge structure”, putting different particle species in one multiplet, with charges.

A simplified model (for β -decay)

- Today's knowledge: weak interactions incorporate transitions $e^- \rightarrow \nu_e$, $d \rightarrow u \dots$ (different from QED and QCD!).

- Natural to group the leptons and quarks into

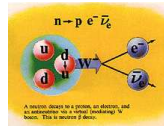
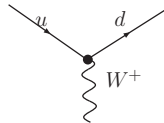
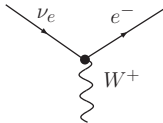
$$\text{weak iso-doublets } l_e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, l_q = \begin{pmatrix} u \\ d \end{pmatrix}$$

each with isospin $|\frac{1}{2}, \pm\frac{1}{2}\rangle$ for upper/lower entries.

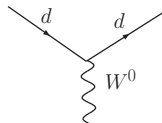
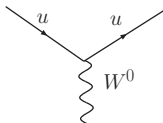
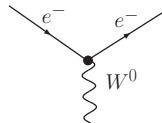
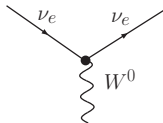
- Define **weak isospin**, make Lagrangian globally phase invariant, promote to local phase invariance and introduce corresponding gauge fields $\vec{W} = W^{+,-,0}$ – one for each Pauli matrix.
- Gauge fields carry isospin, namely $W^\pm : |1, \pm 1\rangle$ and $W^0 : |1, 0\rangle$.
- Note: Charged weak interactions change flavour, neutral ones don't. Other quantum numbers (colour!) is not altered.

Naive Feynman rules in weak interactions

- Charged interactions (W^+ goes “down”, similar for charge-conjugated):



- Neutral interactions (similar for other leptons and quarks):



Parity violation: Decays of charged kaons

- Two decay modes of the charged kaons massively different. Thus, two different particles assumed in the 1950's:

$$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^- \text{ and } \theta^+ \rightarrow \pi^+ + \pi^0.$$

Lifetime in both modes: $\tau \approx 10^{-8}$ s, fairly long; therefore assumed (correctly) to be weak decays.

- Difference: Parity.

Reasoning: Two- and three-pion final states have different parities, if both in s-wave. Therefore either the decaying particle(s) have different parities or the interaction responsible must violate parity!

- T.D.Lee and C.N.Yang postulate (1956):

Weak interactions violate parity,

i.e. can distinguish between left- and right-handed coordinate systems. This is not the case for strong or electromagnetic interactions - weak interactions are very different here!

Experimental check: Electron polarisation in β -decays

- \mathcal{P} violation induces an asymmetry in the e^- -direction in β decay.
- It also affects the helicities of the electrons. With \hat{P} conserved as many plus as minus helicity electrons would be emitted.
- Defining a polarisation

$$P = \frac{N_+ - N_-}{N_+ + N_-}$$

$P = \pm 1$ translates into **all** electrons being helicity ± 1 . Therefore, if all electrons are left-handed, $P = -v/c$.

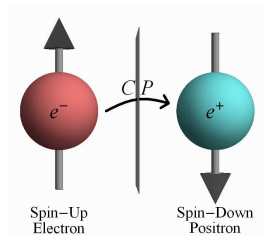
- Experimentally: **all electrons emitted in β -decay are left-handed.**
- Therefore: Only left-handed components of fermions take part in weak interactions. Neutrinos are completely left-handed in the Standard Model.

Handedness and CP -invariance

- This allows to establish the notion of left- and right-handed coordinate systems and to communicate **on physical grounds** which one is used.
- However, repeating a parity violation experiment in an antimatter world, the additional effect of the matter-antimatter symmetry kicks in. It is interesting to note that the weak interaction also violates \hat{C} , in such a way that invariance under the product $\hat{C}\hat{P}$ is preserved to a very high degree.
- Weak interactions trigger $\hat{C}\hat{P}$ violation

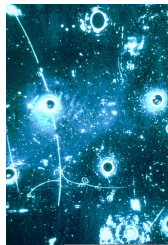
(Important for matter-antimatter asymmetry in universe, but not enough $\hat{C}\hat{P}$ violation in Standard Model.)

- Its existence is intimately related to ≥ 3 generations of quarks.



Problems of weak gauge theory (when invented)

- While charged weak interactions (of the $e^- \rightarrow \nu_e$ -type) have been known since 60's, evidence for neutral ones came in 1973 only.
- **Local gauge invariance dictates massless gauge bosons** (like in QED) but experimental evidence showed that they are in fact very massive:
 $m_W \approx 80.4 \text{ GeV}$, $m_Z \approx 91.2 \text{ GeV}$.
- Also: note that we've moved from W^0 to Z^0 ...
- Gargamelle at CERN (1973): first "photo" of a "*neutral current*" event.
- Neutrinos interact with matter in a 1200 litre bubble chamber. Here: A neutrino interacts with an electron (the horizontal line) and evades unseen.



Spontaneous symmetry breaking: Basic idea

- Common phenomenon: Asymmetric solutions to symmetric theory.
- Example: Magnet (e.g. Heisenberg/Ising model).
Theory: local spin-spin interactions with preference for alignment, but direction not fixed. Nevertheless: Preferred direction emerges. Symmetric state (no alignment) is not state of minimal energy.
- In QFT: Every particle related to a field.
Ground state of the field (a.k.a. “vacuum”) is state of minimal energy and no particles are present. For most fields, minimal energy equals 0, but cases can be constructed, where $E_{\min} = \langle E_0 \rangle = v < 0$.
- Add such a field with $\langle E_0 \rangle = v < 0$ and couple to gauge bosons.
Can show that v sets scale for mass of gauge bosons.

The Higgs mechanism: Making the gauge bosons massive

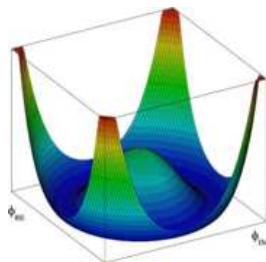
- Add a complex iso-doublet Φ to the theory.

(Iso-doublet to trigger interactions with \vec{W} .)

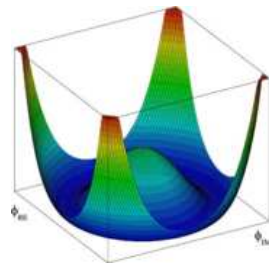
- Lagrangian invariant under $SU(2)$:

$$\mathbf{G}^{SU(2)_W} \mathcal{L}(\vec{W}, \Phi) \rightarrow \mathcal{L}(\vec{W}', \Phi')$$

- Give it a potential of the form $\mu^2 |\Phi|^2 + \lambda |\Phi|^4$
 \implies non-trivial minimum at $|\Phi|^2 = v^2 = -\frac{\mu^2}{\lambda}$.
- Picture to the right: Sketch for real rather than complex scalars.



- Pick a vacuum, for instance $\mathcal{R}(\Phi_1)_0 = v$.
- Expand in new fields around this vacuum:
Three orbital modes parallel to the minimum (the Goldstone modes $\vec{\theta}$), one radial mode (the Higgs field η), which “feels” the potential.
- The Lagrangian is not trivially $SU(2)$ invariant any longer!
The original symmetry has been hidden by the choice of a ground state - an orientation of v . This is similar to the magnet example above, where at low temperatures the magnetisation introduces a random direction.



- Absorb the three Goldstone bosons $\vec{\theta}$ into the gauge bosons (by “choosing a gauge”), gives their 3rd polarisation d.o.f.
 \implies **gauge bosons become massive!**

(Massless spin-1 bosons have two polarisation directions.)

- One real scalar η (“Higgs boson”) remains, interactions fixed.
- This mechanism fixes the interplay of gauge boson masses and their interactions with the Higgs boson (proportional to their mass!).

Testable non-trivial predictions (“Holy grail”)

- Note: The Higgs boson also gives mass to the fermions - an even more complicated story, again with keyword “gauge invariance”.

- Note: The neutral Goldstone boson is absorbed by a combination of the original $U(1)$ gauge boson (“ B ”) and W^0 , yields a massive Z^0 . The orthogonal linear combination becomes the photon, γ .

$$\begin{aligned} Z^0 &= \cos \theta_W W^0 - \sin \theta_W B^0 \\ \gamma &= \sin \theta_W W^0 + \cos \theta_W B^0. \end{aligned}$$

The mixing angle is known as the Weinberg angle, and $\sin \theta_W \approx 0.23$. The weak coupling is related to the electromagnetic one by $g_W = e/\sin \theta_W$.

- That's why electroweak symmetry breaking (EWSB) is

$$SU(2)_L \otimes U(1)_Y \xrightarrow{EWSB} U(1)_{e.m.}$$

Learning outcomes

- Introduced the gauge theory of weak interactions:
bases on doublets containing only left-handed fermions of different type.
- Discrete symmetries and their breaking in weak interactions.
(To be continued in lecture 12.)
- Introduce electroweak symmetry breaking: Higgs mechanism.
A subtle way of beating “symmetry enforces massless bosons”