Foundations of Physics III
Quantum and Particle Physics
Lecture 6

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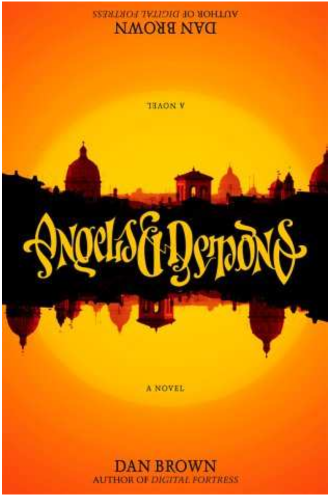
1. Antimatter

2. Quantum Field Theory

Anti-matter

. . . and some misconceptions



Merging special relativity and quantum mechanics

- The **Schrödinger equation is non-relativistic**, predicting the correct Newtonian relationship between energy and momentum for a particle described by ψ (identifying $E = i\partial/\partial t$ and $p_x = i\partial/\partial x$):
$$E = \frac{\vec{p}^2}{2m} + V.$$
- But for a Lorentz-invariant description, rather fulfil the relativistic relation of energy and momentum
$$E^2 = \vec{p}^2 + m^2 \qquad \text{(for a free particle).}$$
- This leads to a **quadratic** equation in E (or $\partial/\partial t$) with positive and negative energies (or advanced and retarded waves) as solutions:
$$E = \pm\sqrt{\vec{p}^2 + m^2} \qquad \text{(for a free particle).}$$

Problem of the solution

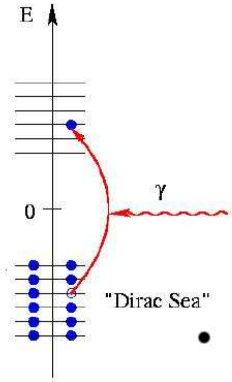
- Negative energy solutions are bad:
  no stable ground state!
  Every state would decay further “down”, a unique source of energy.
- This also contradicts the non-relativistic limit:
  Harmonic oscillator in QM has minimal energy.

The Dirac equation

- Dirac realised that this is potentially problematic and that naively spin could not be included with ψ a simple complex number.
- To get rid of the negative energies, he linearised the equation in E $(\partial/\partial t)$ - this was possible only with ψ forced to have at least two components.
- Identify the two components with spin up and down: $\psi = (\psi_\uparrow, \psi_\downarrow)$. Seemingly special relativity enforces spin!
- But how about the **negative energy solutions?** Dirac's suggestion: **hitherto unseen anti-particles!**
- As a result, he finally wrote down an equation with ψ having four components, two for the two spins of positive and two for negative energies.

Anti-particles

- Dirac's proposal for solutions with $E < 0$: Fill the (Dirac-)"sea" of negative energy states, (Fermi-character prevents double fillings and therefore guarantees the stability of the vacuum).
- Can excite them with, e.g., photons.
- Then anti-particles are just "holes" in the sea: **absence of negative energy looks like net positive energy**.
- The related particle (the anti-particle) must have
  same mass as ordinary particles, but **opposite charge**.

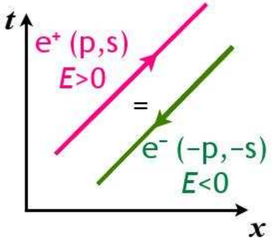


Stueckelberg-Feynman interpretation

- Stueckelberg-Feynman antimatter-interpretation (1947): Negative energy solutions are indeed positive energy solutions of a new particle, moving backwards in time (advanced vs. retarded waves).
- Motivation: Time evolution operator,

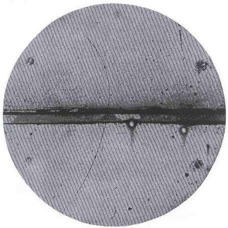
$$U(t,\, t_0) = \exp[-iE(t - t_0)]$$

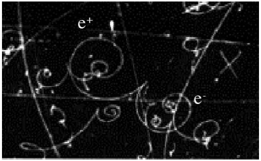
  for unperturbed free particle of energy E.
- Benefit of this interpretation: treating electrons and positrons on equal footing (no more holes).



Evidence for anti-particles (Andersson, 1932)

- Finding a particle electron's mass but opposite charge: the electron's antiparticle, "positron".
- "On August 2 1932 during the course of photographing cosmicray tracks produced in a vertical Wilson chamber (magnetic field 15,000 gauss) designed in the summer of 1930 by Prof R A Millikan and the writer the track shown in fig 1 was obtained which seemed to be interpretable only on the basis of a particle carrying a positive charge but having the same mass of the same order of magnitude as that normally possessed by a free electron."





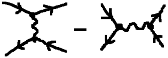
Anti-matter etc. in Quantum Field Theory (QFT)

- Relativistic version of QM, **represents particles as fields** (functions of position x - quantised in QM - and time t).
- Wave functions of, say, individual electrons are excitations of the electron field with a given frequency and wave vector. Summing over these excitations in Fourier space yields the field.
- While "**first quantisation**" recognises the wave nature of particles and the particle nature of waves, this "**second quantisation**" allows for the presence of anti-particles and, accordingly, the possibility to create and annihilate particles.
- Interpretation: Fields can be thought of as harmonic oscillators filling the entire space, one at each position.

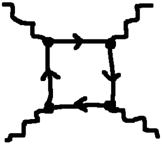
Perturbative expansion

- Replace the potentials with **interactions between particles**
- Basic idea: Particles as carriers of force (photon carrier of electromagnetic force)
- Couplings in interactions parametrise interaction strengths and act as small perturbation parameter λ.
- Assuming the interaction strength between particles is small, **transition amplitudes** $\mathcal{M}$ between particle states can be computed perturbatively.
- Each term of the perturbative amplitude can be represented graphically: **Feynman diagrams**

Interference of amplitudes in e^-e^+-scattering



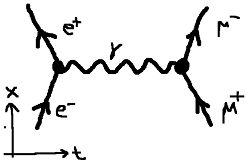
A QM effect: Light-by-light scattering



Virtual particles

- How about the internal lines/propagators? Total four-momentum conservation gives them a virtual mass: $E^2 - \underline{p}^2 = Q^2 \neq m^2$.
- Go to Lorentz-frame where $\underline{p} = 0$ and recall Heisenberg's uncertainty relation: $\Delta E \Delta t \geq 1$ (with $\Delta E \propto \sqrt{Q^2 - m^2}$).
- This allows to create unphysical (anti-)particles with lifetime $\tau \simeq \Delta t \leq 1/\Delta E$

  (diagram to the right, the photon is called virtual.)



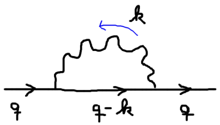
- No problem, if (local) conservation of energy-momentum guaranteed.
- Such processes are known as a **virtual processes**. They typically form the intermediate states of Feynman diagrams.
- But: Can also "borrow" energy from the vacuum for a time $\tau \simeq 1/\Delta E \longrightarrow$ vacuum fluctuations.

- Consider the reaction $e^+e^- \to \gamma \to \mu^+\mu^-$ (previous slide).
  $$E_{e^+e^-} = E_+ + E_- = \sqrt{m^2 + \vec{p}_+^2} + \sqrt{m^2 + \vec{p}_-^2} \geq 0.$$
  Energy-momentum conservation ensures that
  $$E_\gamma = E_{e^+e^-} \quad \text{and} \quad \vec{p}_\gamma = \vec{p}_+ + \vec{p}_-.$$
  However, due to the electron's rest mass it is impossible to satisfy
  $$E_\gamma^2 - \vec{p}_\gamma^2 = m_\gamma^2 = 0,$$
  the photon is **off its mass shell!** $(E^2 - p^2 \neq 0!)$
- This implies that the lifetime of the photon is limited: $\Delta t < 1/\Delta E$ in the centre-of-mass frame of the photon $(\vec{p}_\gamma = 0)$
  For a photon with $E_{c.m.} > 2m_\mu \approx 210 MeV$, $\tau \leq 1\text{fm}/c \approx 10^{-24}$s.
- The photon cannot be observed at all - they remain intermediate.

Renormalisation: Sketching the problem

(Not examinable)

- Virtual particles also emerge by, e.g., an electron emitting and re-absorbing a photon.
- Then the four-momenta of the intermediate particles is not fixed by energy-momentum conservation: an integration over the four-momentum inside the loop becomes mandatory.



- In the case above, this **quantum correction** is related to the integral

$$\int_0^\infty \mathrm{d}^4 k \, \frac{k}{k^2((q-k)^2-m^2)}$$

and diverges - naively linearly.

(Not examinable)

- The infinities stemming from diagrams like the one above are cured by **redefining** the fields and their interaction strengths to include the quantum corrections, **renormalisation**.
- This is done by adding counter-terms to the theory, which have exactly the same divergence structure.
- In so doing, the quantities in the theory are replaced by "bare" quantities, including all diagrams, including the counter-terms yields finite, physical results.
- The beauty of this concept is that it can be proven to be in principle mathematically well-defined and without ambiguities.
- The catch is, though, that in practical calculations the perturbation series is truncated, leading to residual ambiguities (see later).

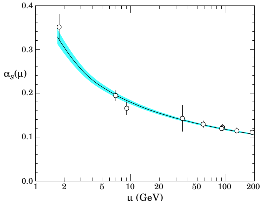
Experimental evidence: Running couplings

- On of the manifestations of the quantum corrections above is that the couplings (interaction strengths) become scale-dependent!

  (Dependence would vanish after all perturbative orders are calculated.)

- This comes from calculating an observable to a given perturbative order and comparing the result with experiment to extract the coupling strength.
- In particle physics, scales are given in units of energy (inverse lengths).
- Similarly, also masses vary with scale.

Strong coupling strength $\alpha_s = g_s^2/(4\pi)$



Learning outcomes

- Dirac equation and antiparticles
- Perturbative expansion, once more, and its representation by Feynman diagrams
- Internal lines and virtual particles.