Foundations of Physics III
Quantum and Particle Physics
Lecture 13

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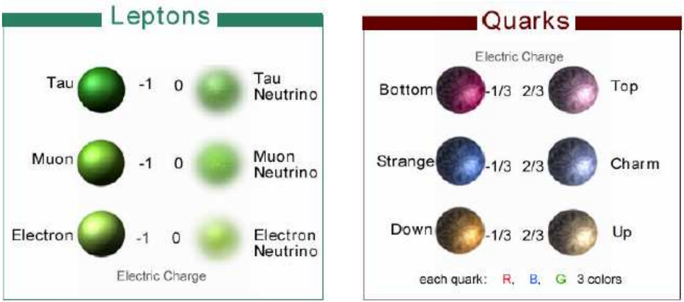
The Standard Model: Matter sector

- **Three generations** of **spin-1/2 leptons** and **quarks**:



- In addition, a **scalar (spin-0) Higgs boson** as remnant of **spontaneous symmetry breaking of electro-weak interactions** (not yet seen).

Charges and interactions in the matter sector

- Example: The first generation (u, d, ν_e and e):

(The other generations are copies with larger masses.)

$$\begin{array}{ccc} \text{particles} & \text{weak charges} & \text{e.m. charges} \\ \left(\begin{array}{c} \left(\begin{array}{ccc} u^{(r)} & u^{(b)} & u^{(g)} \end{array}\right) \\ \left(\begin{array}{ccc} d^{(r)} & d^{(b)} & d^{(g)} \end{array}\right) \end{array}\right) & \begin{array}{lcl} I_u & = & \left|\frac{1}{2};+\frac{1}{2}\right\rangle \\ I_d & = & \left|\frac{1}{2};-\frac{1}{2}\right\rangle \end{array} & \begin{array}{lcl} q_u & = & \frac{2}{3} \\ q_d & = & -\frac{1}{3} \end{array} \\ \left(\begin{array}{c} \nu_e \\ e^- \end{array}\right) & \begin{array}{lcl} I_\nu & = & \left|\frac{1}{2};+\frac{1}{2}\right\rangle \\ I_e & = & \left|\frac{1}{2};-\frac{1}{2}\right\rangle \end{array} & \begin{array}{lcl} q_\nu & = & 0 \\ q_d & = & -1 \end{array} \end{array}$$

- Fermions arranged in weak isodoublets, **all interacting weakly**.

  (More precisely: left-handed iso-doublets with weak interactions and right-handed iso-singlets without them.)

- **Quarks** come as colour-triplets, **coupling to the strong force**. **Leptons** do not carry strong charge, therefore **no strong interaction**.

- E.m. charges for **quarks** and **charged leptons** $\rightarrow$ **e.m. interactions**; **neutrinos** do **not interact electromagnetically** (charge is 0).

Interactions and generations

- Strong, e.m., and neutral weak interactions **do not change particle type** (flavour); **only charged weak interactions do**:
  In the lepton sector, generation number is conserved (lepton-type conservation), in the quark sector it isn't.

  (No transitions between generations in lepton sector, in quark sector $s \to u$ and similar allowed in charged weak interactions.)

- In the Standard Model, neutrinos *by definition* massless.
  Other fermion masses:

$1^{\rm st}$ generation	$2^{\rm nd}$ generation	$3^{\rm rd}$ generation
$m_u \approx 5$ MeV	$m_c \approx 1.5$ GeV	$m_t \approx 175$ GeV
$m_d \approx 5$ MeV	$m_s \approx 200$ MeV	$m_b \approx 4.5$ GeV
$m_{\nu_e} = 0 MeV$	$m_{\nu_\mu} = 0 MeV$	$m_{\nu_\tau} = 0 MeV$
$m_e \approx 0.511$ MeV	$m_\mu \approx 105$ MeV	$m_\tau \approx 1.75$ GeV

- Note: Recent experiments show that neutrinos mix.
  They also show that they have (tiny) masses, in the eV-range.
  A **first hint** for physics **beyond the SM**

Quark mixing & CKM matrix

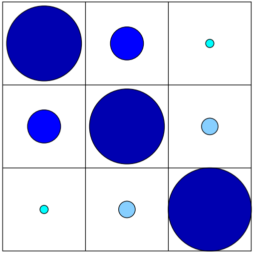
- Inter-generation transitions dominated by mass spectrum and CKM matrix:
- dominant: $t \to b$, $b \to c$, $\ldots$.
- Source of CP-violation in complex elements, mainly V_{13}.
- Unitarity of CKM matrix: triangles $(V_{ik}V^*_{kj} = \delta_{ij})$

(Size of CP-violation in SM given by area of the triangle.)

(CKM=Cabibbo-Kobayashi-Maskawa

Relative size of CKM Matrix

(not to scale)



- Expand in small parameter $V_{us} \approx \lambda$
- Up to $\mathcal{O}(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Carriers of the interactions

- **Unified construction principle of interactions** in the Standard Model:

  gauge invariance:

  - Invariance of the Lagrangian under global gauge transformations ensures conserved charges (electrical charge, weak isospin, colour),
  - the demand for local gauge invariance generates the interactions, mediated by spin-1 gauge bosons (may carry charge themselves).
  - Local gauge invariance ensures massless gauge bosons: $m_\gamma = m_g = 0$.
    Spontaneous symmetry breaking generates masses for the weak gauge bosons ($m_W \approx 80.4$ GeV, $m_Z \approx 91.2$ GeV).

Spontaneous symmetry breaking: The Higgs boson

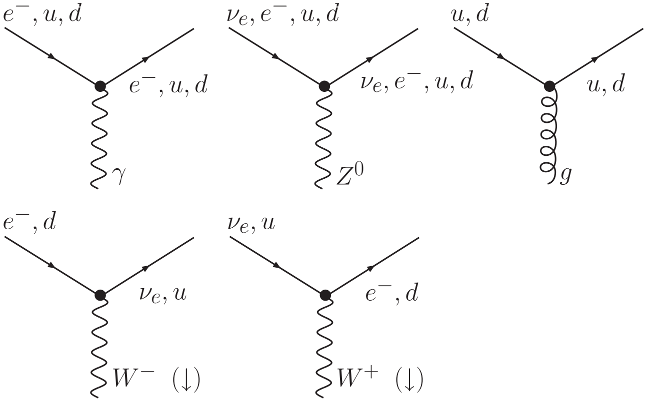
- Basic idea: Have a **symmetric theory** with **infinitely many, symmetric potential ground states**.

  (Degenerate states of minimal energy as candidates for ground states, infinitely many number prevents tunneling.)

- Pick one of them & expand fields around it (breaks symmetry).
- Realisation in the Standard Model:
  - Add a complex scalar doublet, Φ;
  - couple it to the weak gauge bosons of $U(1)_Y$ and $SU(2)_L$;
  - equip Φ with a non-trivial potential ("Mexican hat") with infinitely many minima $\langle\Phi_0\rangle^2 = v^2$ (v = vacuum expectation value);
  - generates three massless "angular" modes ($\to$ Goldstone bosons) and a massive "radial" one ($\to$ Higgs boson);
  - absorb the Goldstones in the gauge bosons, become massive ($\propto v$) – they now have 3 instead of 2 d.o.f.;
  - massive remainder (the Higgs boson) couples to all other particles and itself proportional to their mass.
- Leads to a highly testable theory (some examples later).

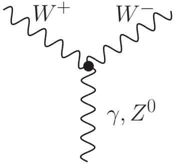
Feynman rules in the Standard Model

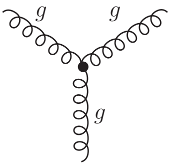
- Fermion-fermion-gauge boson interactions (1st generation only):



Important note: The charged bosons ($W^\pm$) only allow for inter-generation mixing of the quarks (us, ub, dc, dt, cb, st).

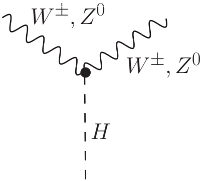
- Gauge boson self-interactions (omitted the 4-boson vertices):

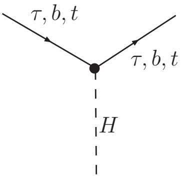




- Interactions of the Higgs boson (omitted self-interactions):

(Interactions proportional to mass, therefore only heavy particles.)





The Standard Model: Tests and status

- There are many relations between the parameters of the Standard Model, especially in the (weak) gauge sector:
  - For example, the masses of the bosons are directly related with the Weinberg angle by $M_Z = M_W/\cos\theta_W$,
  - also related with the Fermi-constant governing the muon decay,
  - loop corrections in addition include quark masses, especially the heaviest one, the top quark. Therefore relations between, e.g. m_W, m_t and m_H.
- These relations have very precisely been measured and calculated, the agreement is astonishing!
- However, despite this apparent success, the building rests on the existence of the Higgs mechanism to give a mass to the particles. Its ultimate test is the existence of the Higgs boson, not yet found.
- Note also: Neutrino oscillations indicate that neutrinos have masses: a first sign that the Standard Model is not complete!

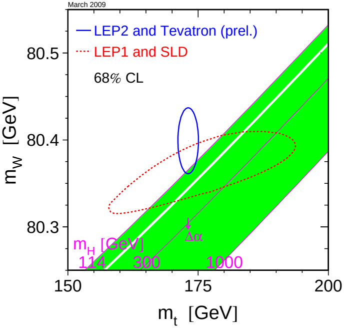
Summary of precision tests

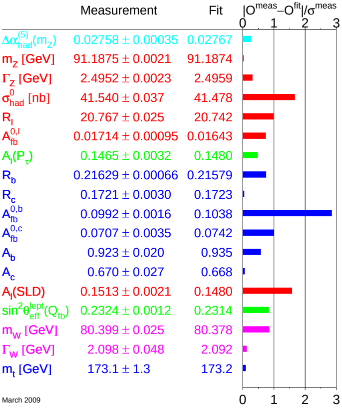
- Right: Overall consistency

  (Fit of fundamental parameters from various data.)

- Below: Consistency of m_W/m_t.

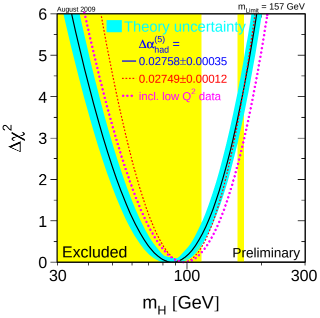
  (Higher order corrections by Higgs boson etc..)





The quest for the Higgs boson: Basic findings

- Higgs boson not yet found, therefore $m_H \geq 114$ GeV.
- Precision data favour light m_H (see fit right)
- Higgs boson couples to heavy objects - all couplings proportional to mass.
- Problem: No heavy objects inside typically used beams ($e^{\pm}$, p), production processes become non-trivial.



Why go beyond the Standard Model?

- SM is a model with $18(+1)$ parameters, can this be reduced?
- Somewhat related: Can a GUT be constructed -
  a theory with only one interaction rather than three?
- If there is a GUT, it presumably lives at scales $\mathcal{O}(10^{16}\mathrm{GeV})$.
  A big desert from μ_{EWSB} to μ_{GUT}?
  (The "philosophical" hierarchy problem)
- How can gravity be incorporated at all?
  Gauge constructions of gravity are tricky.
- If dark matter is fundamental, where is it?
  The SM has no viable candidates.
- Let's not even start with dark energy/cosmological constant.

Another nasty feature: The technical hierarchy problem

- Consider two corrections to the mass of the Higgs boson:

$\propto \lambda_H \Lambda^2$ $\qquad$ $\propto -\lambda_t^2 \Lambda^2$

- Each of them is quadratically divergent, with a brute-force cutoff Λ.

  (Think of it as limit of validity of SM, $\mu_{\rm GUT}$, or scale of new physics kicking in)

  Remark: In QED, the fermion self-energy is only log-divergent due to gauge symmetry. Not a help here.

- Huge fine-tuning of renormalisation mandatory to keep $m_H \approx vev$.

  (One-loop correction terms alone $\propto \mu^2_{\rm GUT}$)

- Two solutions: Lower Λ (idea behind extra dimensions)
  or introduce a symmetry, e.g. $\lambda_H = \lambda_t^2$ (SUSY)

Supersymmetry

- Remember quantisation through operators:
  - Have creation and annihilation operators $\hat{a}^{(\dagger)}$: $\hat{a}^\dagger|n\rangle \propto |n+1\rangle$, $\hat{a}|n\rangle \propto |n-1\rangle$, and $\hat{a}|0\rangle = 0$.
  - Quantisation achieved through fixing their relation Commutator: $[\hat{a},\, \hat{a}^\dagger] \propto i$, $[\hat{a},\, \hat{a}] = [\hat{a}^\dagger,\, \hat{a}^\dagger] = 0$
- **Commutator for bosonic degrees of freedom.**
- **Anticommutator** $\{f_1,\, f_2\} = f_1 f_2 + f_2 f_1$ **for fermionic d.o.f.**
- Supersymmetry:
  - Construct operation $\hat{Q}$ **linking bosonic and fermionic states**: $\hat{Q}|b\rangle = |f\rangle$ & $\hat{Q}^\dagger|f\rangle = |b\rangle$.
  - Demand invariance under this operation
  - Therefore: For each bosonic d.o.f. in your model a fermionic one is mandatory and vice versa $\Longrightarrow b, f \in$ one "superfield"

    (This is the symmetry from above: Scalar and fermion belong to same superfield, therefore same coupling)

Two “philosophical” in principle reasons, why we like SUSY:

1. The **Coleman-Mandula Theorem** states
   that the construction of a quantum theory of gravitation in form of a local gauge theory is feasible only in the framework of supersymmetric theories.
2. The **Haag-Sohnius-Lopuszanski Theorem** states
   that the maximal symmetry of a consistent QFT is given by the direct product of Lorentz-invariance, gauge symmetry and supersymmetry.

Some more "technological" motivation:

- Quadratic divergences are cancelled.
  For each loop with bosonic d.o.f. (sign $= +$), there is one with fermionic d.o.f. (sign $=$ -) with exactly the same coupling, mass etc.: only difference is the sign!
  $\Longrightarrow$ Perfect cancellation of quadratic divergences.
- Extra particles may help in enforcing unification of couplings.
- The vacuum energy arising in second quantisation (zero-mode energy of harmonic oscillator) is exactly cancelled by fermions
  $\Longrightarrow$ Vacuum energy is exactly 0

  (Compare: Cosmological constant)
- Typically, SUSY models have a natural dark matter candidate (a stable WIMP=LSP) with reasonable mass for CDM.

  (Caveat: Only after SUSY-breaking)

Field content before EWSB/SUSY breaking: all massless

Matter fields: left-handed doublets right-handed singlets Weyl-spinors/complex scalars generations $J = 1, 2, 3$	$\begin{pmatrix} u^J \\ d^J \end{pmatrix}_L,\ u_R^J,\ d_R^J$ $\begin{pmatrix} \nu^J \\ \ell^J \end{pmatrix}_L,\ \ell_R^J$	$\begin{pmatrix} \tilde{u}^J \\ \tilde{d}^J \end{pmatrix}_L,\ \tilde{u}_R^J,\ \tilde{d}_R^J$ $\begin{pmatrix} \tilde{\nu}^J \\ \tilde{\ell}^J \end{pmatrix}_L,\ \tilde{\ell}_R^J$
Gauge fields: spin-1 bosons/ Weyl-spinors generators $a = 1 \dots n_g$	$G_\mu^a,\ W_\mu^{\pm,0},\ B_\mu$	$\tilde{\psi}_G^a,\ \tilde{\psi}_W^{\pm,0},\ \tilde{\psi}_B$
Higgs fields: 2 doublets (i=1,2) of Complex scalars/ Weyl-spinors	$\begin{pmatrix} H_i^1 \\ H_i^2 \end{pmatrix}_L$	$\begin{pmatrix} \tilde{\psi}_{H_i}^1 \\ \tilde{\psi}_{H_i}^2 \end{pmatrix}_L$

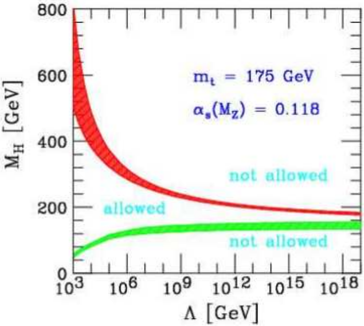
An unfortunate neccessity: Breaking SUSY

- Pattern: SUSY partners with quantum numbers as SM particles, differing just in spin by a half unit
- SUSY must be broken: no superpartner (with identical mass) found
- Various mechanisms advocated, barely tractable
- Way out: Breaking by hand through "soft term"

  (Terms that do not spoil the nice features, like absence of quadratic divergences)
- This introduces ≈ 100 new parameters in MSSM: mostly boiling down to all possible mixings.
- Typically imposed: R-**parity**
  Pictorial: SUSY particles **always** pairwise in vertex!
  Consequence: A lightest stable SUSY particle (LSP).

Aside: Could the Standard Model survive up to μ_{Planck}?

- Remember: $m_H^2 = \lambda v^2$
  (v = vev = 246 GeV)
- Two constraints on mass:
  1. Keep perturbativity: $\lambda \to \infty$ forbidden.
  2. Keep vacuum structure: $\lambda \to 0$ forbidden.
- Therefore: "Stable island" in the middle



Learning outcomes

- Summary of construction principles of the Standard Model

(Very examinable!)

- All types of vertices apart from 4-gauge boson vertices

(Very examinable!)

- Some ideas of what may be lie beyond the Standard Model

(Not examinable!)