Introduction to particle physics
Lecture 4

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Outline

Nuclei, nucleons, and mesons

Neutrons

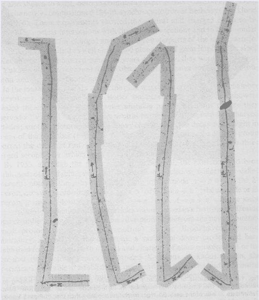
- Rutherford's experiment: Lightest atom $= H$ (p-e-bound state)
  But: next lightest atom (He) four times as heavy as hydrogen, with only two electrons. Similar for Li (three electrons, seven times as heavy), etc.. Why so heavy?
- Discovery of the Neutron by J.Chadwick (1932): Bombard Beryllium with α-particles, very penetrating non-ionising radiation emerges. Send through paraffin, in turn protons are emitted.
  Measure speed of protons: original radiation cannot be γ's.
  Therefore new particle ("**neutron**") with nearly the same mass as the proton but no charge.
- Heisenberg's proposal (1932): Both neutron and proton are two manifestations of the same state, the **Nucleon**.
- Symmetry relating them: **Isospin** (very similar to spin).

Proposing mesons

- H.Yukawa (1934): First prediction of **mesons**.
- Answer to why neutrons and protons bind together in nucleus.
- Yukawa's underlying assumption: Introduce a new force, short-ranged, thus mediated by massive mesons.
- Estimate: 3-400 times the electron mass.
  From uncertainty principle $\Delta E \Delta t \geq 1$ with time given by nucleon radius as $\Delta t \approx 1/r_0$. Assume r_0 of order $\mathcal{O}(1\text{fm})$, then
  $\Delta E \approx m_{\rm meson} \approx 200$ MeV
  (Note: natural units used in this estimate).

The first “mesons”: The muon & the pion

- Two groups (1937): Anderson & Neddermeyer, Street & Stevenson: Finding such particles in cosmic rays using cloud chambers.
- But: wrong lifetime (too long, indicating weaker interaction), and inconsistent mass measurements
- Two decisive experiments to clarify the situation (Rome, 1946 & Powell et al. in Bristol, 1947) with photo emulsions.



- Result: In fact two new particles.
  One **weakly interacting, the muon**, μ, one **strongly interacting, the pion**, π. The latter comes in three versions, π^+, π^-, π^0, where the charged ones mainly decay into muons plus a neutrino, while the neutral one decays mainly into two γ's.

Detour: Spins and their addition

Spin-1/2 systems: General remarks

- Spin-1/2 systems are often studied in physics.
- Spin-statistics theorem suggests that such systems are fermionic in nature, i.e. respect Pauli exclusion.
- Interesting in the context of this lecture:
  Basic building blocks of matter (quarks & leptons) are spin-1/2.
- Simple representation:

$$|\uparrow\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle \text{ and } |\downarrow\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle.$$

  Important: Distinguish total spin s and its projection, s_z on a measurement axis (here the z-axis).
- Examples: electron and its spin, isospin, ...
- Note: Spin can also occur as spin-1 etc..

Adding two spin-1/2 objects

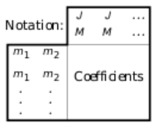
- Often two spin-1/2 objects form a compound.
  Examples: bound states of fermions, spin- orbit coupling, etc..
- If two spin-1/2 systems are added, the following objects can emerge:
  $$|\uparrow\uparrow\rangle,\ |\uparrow\downarrow\rangle,\ |\downarrow\uparrow\rangle,\ \text{and}\ |\downarrow\downarrow\rangle.$$
  Naively, they have spin 1, 0, or -1, respectively.
  But: **Need to distinguish total spin s and its projection onto the measurement axis s_z** (here, z has been chosen for simplicity)
- Then, truly relevant states are $s = 1$ (triplet, symmetric)
  $$|1,1\rangle = |\uparrow\uparrow\rangle,\quad |1,0\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right],\quad |1,-1\rangle = |\downarrow\downarrow\rangle$$
  and $s = 0$ (singlet, anti-symmetric):
  $$|0,0\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$
- Catchy way of writing this: $\mathbf{2}\otimes\mathbf{2} = \mathbf{3}\oplus\mathbf{1}$

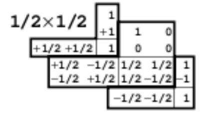
Clebsch-Gordan coefficients

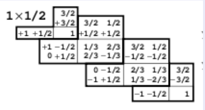
- The **Clebsch-Gordan coefficients** in front of the new compound states can be calculated (or looked up).
- Formally speaking, they are defined as follows:

$$\left\langle s^{(1)},\, s_z^{(1)};\, s^{(2)},\, s_z^{(2)} \middle| s^{(1)},\, s^{(2)};\, s,\, s_z \right\rangle$$

indicating that two spin systems $s^{(1)}$ and $s^{(2)}$ are added to form a new spin system with total spin s (or J). Obviously, it is not only the total spin of each system that counts here, but also its orientation. This is typically indicated through "magnetic" quantum numbers, m, replacing the s_z in the literature.







Note: Square-roots around the coefficients are understood in the table above

Using spin-algebra

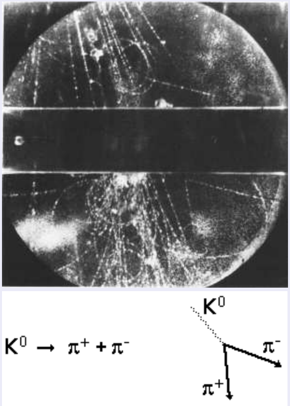
- Identify: $|p\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle$ and $|n\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$.
  Heisenberg's proposal: Call this isospin (rather than spin).
- Also, the three kinds of pions can be written as:
  $|\pi^+\rangle = |1, +1\rangle$, $|\pi^0\rangle = |1, 0\rangle$, and $|\pi^-\rangle = |1, -1\rangle$.
- Catch: **Isospin conserved in strong interactions**!
- Dynamical implications: Bound states (here the deuteron).
  Add two nucleons: can in principle have iso-singlet and iso-triplet.
  But: No pp, nn-bound states, therefore $|d\rangle = |0, 0\rangle$ (deuteron = iso-singlet).
- Consider processes (+ their isospin amplitudes, below):

$$p + p \to d + \pi^+ \qquad p + n \to d + \pi^0 \qquad n + n \to d + \pi^-$$
$$\mathcal{A}_{\rm iso} \propto 1 \qquad \mathcal{A}_{\rm iso} \propto 1/\sqrt{2} \qquad \mathcal{A}_{\rm iso} \propto 1$$

Strangeness. Who ordered that?

Finding strange particles

- Rochester & Butler (1947): Cloud chamber experiment with cosmic rays. Unusual "fork" of a π^+ and a π^-.
- Interpretation: Cosmic ray particles, mass between π and p, the **kaon**, **K**.
- Like pions, but strangely long lifetime (typically decay to pions or a muon-neutrino pair), again hinting at weak interactions being responsible.



Finding more strangeness

- Anderson (1950): Another "strange" particle, decaying into proton and π^-, the **hyperon**: $\Lambda \to p\pi^-$.

Why are they "strange"?

- With the advent of the Bevatron it became clear: Strange particles (kaons and lambdas) are copiously produced, but decay slowly (strong interaction in production, weak interaction in decay)!
- Also: In strong reactions, strangeness only pairwise produced.

Cataloguing strangeness

- Gell-Mann and Nishijima propose a new quantum number (1953):

  Strangeness.

- **Conserved by strong interactions, but not by weak interactions.**
  Allowed: $p + \pi^- \to K^0\Lambda,\ \ \Sigma^+K^-,\ \ldots$
  Forbidden: $p^+ + \pi^- \to K^0 n,\ \ \Sigma^+\pi^-,\ \ldots$

  Side remark: Baryon number (B) is also conserved. (More on baryons and mesons later)

- Relation of strangeness S, electric charge Q, and isospin I:

$$Q = e\left(I_3 + \frac{B+S}{2}\right).$$

(Gell-Mann-Nishijima relation)

Here I_3 is the third component ($= \pm 1/2$ for p, n) of the isospin, $S = \pm 1$ for kaons, Λ's, and Σ's, B is the baryon number ($= 1$ for baryons like p, n, Λ, Σ and $= 0$ for mesons like π, K).

Kaons

- Found four varieties, of kaons K^+, K^-, K^0, $\bar{K}^0$.
  All are pseudo-scalars (i.e. spin-0, negative parity), just like pions.
  All have the same mass, about three times m_π ($m_\pi \approx 140$ MeV, $m_K \approx 495$ MeV) $\Longrightarrow$ "relatives"?
- Apparent problem: Different multiplet structure.
  Pions come in one iso-triplet (3 states with same isospin $I = 1$ but different $I_3 = +1, 0, -1$ for π^+, π^0, π^- - see the Gell-Man-Nishijima formula).
  The kaons in contrast have either $S = +1$ (K^+, K^0) or $S = -1$ (K^-, $\bar{K}^0$), and they do not form an iso-triplet - they are organised in two iso-doublets.
- Also, while for pions the antiparticles are $\bar{\pi}^+ = \pi^-$ and $\bar{\pi}^0 = \pi^0$, for the kaons $\bar{K}^+ = K^-$ but $\bar{K}^0 \neq K^0$!

(We will sea that later, when we discuss weak interactions and $\mathcal{CP}$-violation)

Detour: Resonances

. . . and how they manifest themselves

- Up to now: Most particles have lifetimes $\tau \geq 10^{-12}$ s, long enough to observe them **directly** in bubble chambers etc..
- But: There are many particles with shorter lifetimes. $\Longrightarrow$ direct detection mostly impossible, existence must be inferred **indirectly**.
- These transient particles appear as "intermediate" ones. They typically form when colliding two particles, and decay very quickly. They respect conservation laws: If, e.g., isospin of colliding particles is $3/2$, resonance must have isospin $3/2$. $\Longrightarrow$ a Δ-resonance.
- Indication for their emergence: Strongly peaking cross section σ (i.e. probability for the process $ab \to cd$ to happen), when plotting σ vs. c.m. energy of the collision. The mean is then at $E_{ab}^{\rm c.m.}$, with a width given by $\Delta E = 1/\tau$, the lifetime of the **resonance**.
- Will look at this in more detail in homework assignment.

Comparison with driven, damped oscillators

- For oscillators, intensity I is defined as the square of the amplitude.
- Consider a linear oscillator with a resonance frequency Ω, driven with a frequency ω. The intensity of oscillations then reads

$$I(\omega) \propto \frac{\frac{\Gamma}{2}}{(\omega-\Omega)^2+\left(\frac{\Gamma}{2}\right)^2}.$$

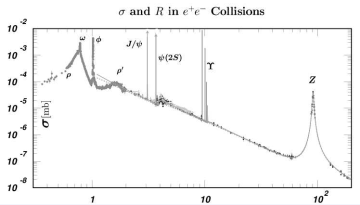
  Here, Γ, the width parametrises the dampening of the oscillator. It is also known as the (line-) width of the resonance.
- In particle physics, cross sections for resonances are very similar:

$$\sigma(s) \propto \frac{1}{(s-M^2)^2+(M\Gamma)^2},$$

  where $s=(p_a+p_b)^2$ is the c.m. energy squared of the incoming particles a and b, M is the mass and $\Gamma=1/\tau$ is the lifetime of the resonance.

Resonances in $e^+e^- \to$ hadrons

$$\sigma_{ee\to\text{hadrons}}(E_{ee})$$

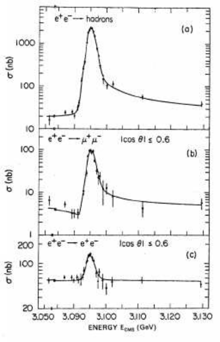


- Note the more or less sharp resonances on a comparably flat "continuum", coming from $e^+e^- \to q\bar{q}$

(We will discuss this in more detail!)

- They are (apart from the Z) all related to $q\bar{q}$-bound states.

Zoom into J/Ψ



- Note: Here width around 3 MeV completely determined by detector ($\Gamma_{J/\Psi} = 87$ keV)

The quark model

The particle zoo

- In the early 60's it was clear that hundreds of "elementary" resonances exist. Each had well-defined quantum numbers such as spin, isospin, strangeness, baryon number etc.. Typically, widths increased with mass, or, reversely, lifetimes decreased with mass of the resonance.
- Obvious task: Need a classification scheme

  (similar to Mendeleev's periodic table).
- Obvious question: Are all these particles "elementary" or are they composed of even more fundamental objects.

Internal symmetries, once more

- Such a classification scheme is provided by internal symmetries.
- Proposed independently by M.Gell-Mann and Y.Ne'eman (1961). Starting point: Isospin

  (from charge independence of strong interactions).

  In symmetry language: p and n are in a two-dimensional representation of the group $SU(2)$ (rotations in two-dimensional complex space). Hamiltonian governing their strong interactions is invariant under transformations of the form

$$\begin{pmatrix} p \\ n \end{pmatrix} \longrightarrow \hat{\mathcal{G}}^{SU(2)} \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} p' \\ n' \end{pmatrix}$$

- Similarly, the pions (π^+, π^- and π^0) and the Delta-resonances (Δ^{++}, Δ^+, Δ^0, and Δ^-) are in three- and four-dimensional representations of this group.

  (Note: Despite different dimensions the number of real angles to characterise these $SU(2)$-rotations is the same, namely 3. The rotations, i.e. the matrices $\hat{\mathcal{G}}$ are linear combinations of the three Pauli-matrices in the respective representation.)

Quarks

- But there's also strangeness: Maybe go to $SU(3)$?
- In 1964 Gell-Mann and Zweig pointed out that this fits the bill: They proposed three "hypothetical" quarks, *u*p, *d*own and *s*trange, could built all known particles as their "bound states".
- Similar to combining spins in $SU(2)$. Two kinds of bound states: **Mesons** are made from a $q\bar{q}$-pair, **baryons** from three quarks. In the group theory notation from before they have:

$$\text{Mesons: } q\bar{q} \equiv \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$
$$\text{Baryons } qqq \equiv \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10},$$

  i.e. one singlet of mesons and baryons, one octet of mesons, and two octets and one decuplet of baryons.
- This would repeat itself for higher spin states.

Hadron multiplets

- With only u, d, and s quarks, the hadrons are characterised by strangeness and electrical charge (or third component of isospin).

(For some graphs see next transparencies)

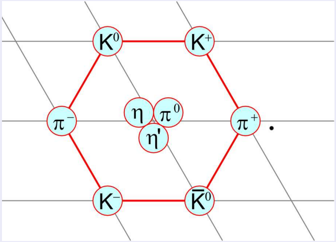
- Implies the following quark charge assignments (added as scalars):
$$q_u = 2/3 \text{ and } q_d = q_s = -1/3.$$
- Also isospin assignments (isospin added with Clebsch-Gordans's)
$$I_{3,u} = 1/2,\ I_{3,d} = -1/2, \text{ and } I_{3,s} = 0.$$

(Result: Δ's form an isospin 3/2 multiplet, nucleons an isospin-1/2 doublet.)

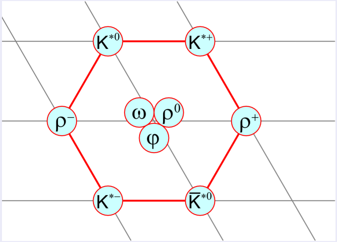
- The mesons (bound $q\bar{q}'$-states) come in multiplets of nine particles, which differ by their spin and occupy different mass regions. The most important ones are the two lightest ones: a pseudo-scalar multiplet (including pions and kaons), a vector multiplet (including ρ's and the ϕ-meson).
- The two lowest lying baryon multiplets are an octet and a decuplet. The former includes, e.g., the proton and neutron, while the latter includes the Δ-resonances and the Ω^-.

Meson multiplets

Pseudoscalars

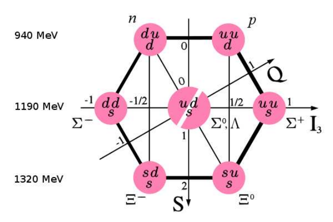


Vectors

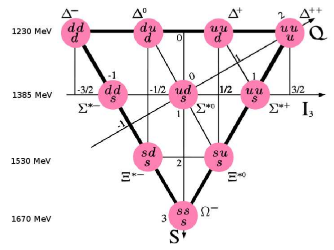


Baryon multiplets

Octet



Decuplet



The discovery of the Ω^-

- In 1961, “tip” of the decuplet not yet found.
  M.Gell-Mann’s prediction: m = 1672 MeV, plus the right production mechanism and a long lifetime.
  - Decay chain:

$$K^- + p \to \Omega^- + K^+ + K^{*0}$$

(strangeness conserving)

$$\Omega^- \to \Lambda^0 + K^-$$

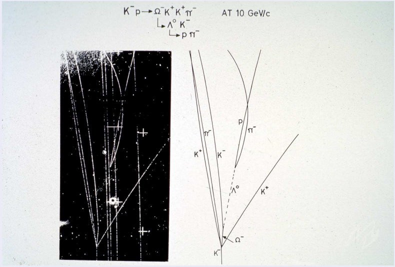
($\Delta S = 1$ weak decay)

$$\Lambda^0 \to \pi^- + p$$

($\Delta S = 1$ weak decay)

$$K^{*0} \to \pi^- + K^+$$

($\Delta S = 0$ strong decay)



The postulate of colour

- In the decuplet, one problem appears: Some states like for instance the Δ^{++} are composed from three identical quarks (u's for the Δ^{++}). Since the decuplet baryons are spin-3/2 objects they are fermions, i.e. their wave function must be antisymmetric. With three identical quarks, in identical spin states (spin-3/2 implies the spin-1/2's point into the same direction), this is possible only by invoking a new quantum number, **colour**.

We will discuss this when we encounter the strong interaction again.

Summary

- More particles in the zoo.
- First encounter with isospin as a first symmetry.
- Emergence of strangeness - giving rise to the quark model: $SU(3)$ or "the eightfold way".
- Symmetry as **the** method of choice to gain control.
- Resonances as intermediate states.
- To read: Coughlan, Dodd & Gripaios, "The ideas of particle physics", Sec 7-10.