November 24, 2004, CLNS 04/1894

WHY POLARIZED POSITRONS SHOULD BE IN THE BASE LINE OF LINEAR COLLIDER[1]

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Abstract

International Linear Collider with both polarized beams delivers higher luminosity at lower backgrounds with unique possibilities in discovery of new phenomena, especially in collisions with transversely polarized beams. Polarized positrons, regenerated from primary beam with help of gammas radiated in helical undulator can be obtained in quantities satisfying conversion efficiency one to one. Scheme with undulator is the only practical way for SC linear collider because it allows drastic reduction of power deposition in a target. Undulator based scheme is much cheaper, than the traditional one. Undulator must be *helical*, as it is much easier to fabricate. Polarized positrons can be obtained by *selecting* the energetic ones.

Polarized *electrons* for SC Linear Collider better to be generated by the same method also.

INTRODUCTION

I began to write this paper when the warm and SC variants of Linear Collider were under competitive discussion. I planned to write the paper in a way it shows at the end that the method of positron production with circularly polarized gammas from helical undulator is preferable. Meanwhile decision was made in favor of SC variant, so the necessity to convince anyone in undulator-based conversion system disappeared naturally, as SC variant simply can not work with the other, than undulator based conversion system. So now my plan is to convince physical community, that *polarized* positrons must be present in base line of SC Linear Collider. For this undulator must be *helical* in first place. There must be a room reserved allowing installation of ~100-long undulator[2]. All beam optics must be designed allowing accommodation of such undulator. If energy selection is not applied, positrons are not polarized.

Polarized electrons are the non discussable part of Linear Collider as it was demonstrated by SLC. It is interesting, that I found, that there is no *engineering* solution published about *polarized electron* production in quantities, required for SC linear collider, just estimations. So the method with helical undulator can be recommended for polarized electrons production also. With the other words, the way for obtaining polarized electrons with photons from helical undulator is a preferable one here too as much cheaper, than arrays of lasers.

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[1] Work supported by NSF. Electronic version is available at http://www.lns.cornell.edu/public/CLNS/2004/CLNS04-1894/CLNS 04-1894.pdf

[2] Undulator is *sectioned* with ~4 m long sections interlaced by focusing and beam control elements. In [1] static magnetic fields, circularly polarized electromagnetic waves and helical crystals were considered. The method with static-field helical undulator was found to be the only practical one.

The way to obtain polarized photons with back-scattered laser photons, actively developed by KEK[3], lost its actuality–it is not working for SC machine for the reason of energetic and complexity.

Main subject of discussion now is about utilization in method either planar or helical undulator. So basically here I am analyzing a comment floating around, that the planar undulator will be used in first stage and helical undulator will be installed in second stage, later on. Supposed that first planar undulator will have short, *L*~30 m long, *K*~1 undulator/wiggler[4] and for polarized positrons required somewhat a 100 m long wiggler with the *K*~0.4.

When I tried to understand why this question exists in first place, results of my investigation emerge to be rather surprising, see “One additional argument” below. Similar to original dilemma-traditional method of positron production versus the one with undulator-the way how it looks now is the same type mythology based on misunderstanding of real technical and scientific nuances. Helical undulator can have high *K* factor and serve for unpolarized positron production as well as a planar one. The point I am defending is that undulator ***must be helical,*** even polarized positrons might be thought at later stage.

Other interesting comment emerges here is that some speakers who for whatever reason representing undulator based scheme, saying that to create positrons, the electron beam of linear collider must be used, so these electron-positron beams become inter-coupled (cross talked). This is a mistake. Positron bunch can be regenerated at positron side of linear collider itself by introduction very simple auto-regulating procedure. In case if positron beam is lost, small low energy ***electron*** linac must just illuminate the same positron target and positrons can be restored up to full number in few circles. This will take fraction of a second only and considered as a rare event.

So the right strategy, according to my understanding is the following: undulator must be fabricated as a helical one and must be long enough to satisfy even polarized positron production. In first stage the energy selection can be minimal and this spare length delivers positrons in quantities with lager safety margins.

This plan is a reflection of the fact that basic difference between polarized and unpolarized positron production is in selection (or not selection) of secondary particles by energy. That is because the method of polarized positron production is a combination of circularly polarized photons ***and*** energy selection. Some speakers or writers are missing this nuance. So that is why in first place this selection reduces yield of secondary particles if polarized energetic ones are selected. And that is why undulator here must be longer. Selection, however, affects about half of all secondary particles concentrated in phase space. So roughly speaking the length of undulator for polarized positrons must be ~twice of the length of undulator for unpolarized particles generation. If this is helical or planar undulator–does not matter at all. We will see that for the magnetic field strength under interest, helical undulator generates ~twice more photons, than the planar one, so utilization of helical undulator is preferable from this

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[3] Laser as a particular case of electromagnetic wave was mentioned by E. Bessonov at 15th International Conference on High Energy Accelerators, Hamburg, 1992, Proceedings, p.138.

[4] $K = eH_u \lambda_u / 2\pi mc^2$ is so called undulatority factor, H_u, λ_u stand for magnetic field and period respectively. *K*~1 is a transition region between wiggler und undulator, and this is not a linguistic difference, but referrers the field strength when radiation can not be treated as a dipole one anymore.

point too. Even more-helical undulator is much easier object to manufacture. Unfortunately wide audience misinformed about this fact.

During the years passed since the time of invention the method of polarized positron generation for Linear Collider by circularly polarized gammas, a quarter century ago [1], it was interesting to look on how some theorists changed theirs opinion about necessity of polarized positrons at all. Initially the brief answer could be decoded as: it is not necessary, polarized electrons only are enough. Now they are telling that polarized positrons will be useful, but in second stage. Looks to me that this category of people simply is not informed properly that the scheme with polarized positrons makes SC Collider much cheaper.

Once again, nobody showed that polarized electrons could be obtained in quantities required for SC Linear Collider practically. Moreover in first stage unpolarized electrons will be used too, as it could be seen from description of Guns for TESLA— they contain ordinary thermo/RF guns without polarization. This is done for guaranteed operation, if polarized electron generation with (strained) photocathode-method will be not successful.

It is true, that for NLC such analyses done indicates, that polarized electrons somehow could be obtained, basically with significant engineering efforts applied. I would like just remind that SC linear collider requires ~ 40 times more particles *per single train*, and ~3 times more particles *in single bunch* than the NLC. So simply there might be no alternatives to undulator based conversion system for polarized electron production at all. Physical community is absolutely sure now, that polarized electrons are necessary addition to linear collider. My opinion is more radical in the sense that I believe that there are *no alternatives* to polarized positrons in first stage by the reason of simplicity and reduction of cost of linear collider. This must be done in expectation of new results or simply speaking, linear collider without polarized positrons might be not interesting for high energy physics.

Again, I am sure that polarized electrons for SC Linear Collider could be generated by gammas obtained from helical undulator as well –in the same manner as polarized positrons.

Positrons are not easy objects to get; *polarized* positrons production always was thought as a much more difficult task. There are few myths circulating in physical community about polarized positron production (with undulator).

Myth #1: As polarized positrons could be obtained in smaller quantities, than unpolarized ones, hence *luminosity* reached in Linear Collider with polarized positrons is lower.

Myth #2: Source of polarized positrons is much more expensive, than the traditional one, so polarized positrons option increase the cost of LC.

Myth #3: Positrons in undulator conversion scheme could be obtained from electron beam only, so positron/electron part is inter-looped.

Each of these myths is wrong. Truth lies exactly in opposite statement for each of these ones.

It is impossible to find the source of these myths now. As on my best knowledge, the Novosibirsk team only made evaluations with such type of conversion system up to technical realization of critical components. Conversion system using undulator was found to be cheap and could deliver polarization-65-70%[5]. As the conclusions were positive in each instance, the source of polarized positrons *and electrons* with undulator was immediately implemented into VLEPP project. *Polarized electrons* were planned to obtain in the same manner as polarized positrons. Strayed photocathode technique was not invented yet. Polarized electrons importance was confirmed for SLC soon after.

Desire to have polarized positrons in a second stage might reflect some doubts about vital importance for both polarized electrons and positrons compared with polarized electrons only (responsibility of theorists), doubts about the cost and complexity (responsibility of engineers) and, hence, by assurance about guarantee of success. Meanwhile possible *unsuccess* with polarized positron production scheme could be only in lowered degree of polarization, *not* in the number of positrons. In that sense, *unsuccess* delivers the same results as the success in unpolarized positrons. Even in this case conversion system with undulator looks like the only possible one for the beam intensity requested by SC Linear collider. The only minus is that this scheme with undulator is practical with primary beam energy ~100-150 GeV. This is the only disadvantage however.

Other question–which place is better for installation of undulator– before IP or after. Beam after collision looks disrupted, while the beam before IP, indeed, has small emittance, which could be disrupted easily inside undulator, where magnetic field is rather nonlinear. The last brought doubts about possible coupling between horizontal and vertical degrees of freedom. One additional point of concern was perturbation of spin inside helical undulator.

All these questions have positive answers in proposed scheme for positron production [1]. Technology of helical undulator tested indicates that it is much simpler and cheaper, than the planar one. So there are no apparent limitations for implementation of hardware required for polarized positron in first place.

POLARIZED POSITRON PRODUCTION

Positron always generated by high-energy photon in the field of nuclei accompanied by electron. Appropriate diagram is represented in Fig. 1 below. Primary high-energy electron beam initiates a cascade generating these photons in traditional conversion system. So that is why in traditional scheme target is rather thick, going to be in few radiation lengths. Positron carries energy E_+, which's maximal value is simply $E_{+\max} = \hbar\omega - 2mc^2$. Parameter $x = E_+ / E_{+\max} \cong E_+ / \hbar\omega$ used as independent variable here. One can see, that for mostly energetic secondary particle, positrons or electrons, degree of polarization ζ_1 reaches its maximal value $\vec{\zeta}_1 \cong \bar{\xi}_2 \vec{n}$, where $\bar{\xi}_2$ stands for average circular polarization of primary photon beam.

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[5] Level of polarization with *L*~150 m – long undulator having period 1 cm and *single* target. Longer undulator-higher polarization could be reached. With 300-m long undulator polarization for electrons/positrons expected to be ~85% [1].

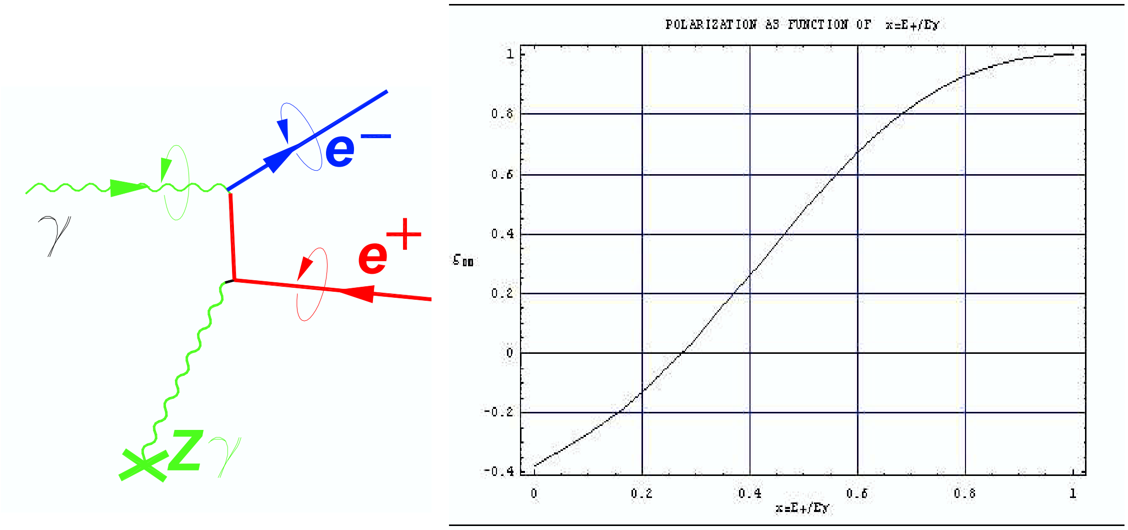

Figure 1: The way to create circularly polarized positron, left. Cross-diagram is not shown. At the right–the graph of $\zeta_{\|}$ –longitudinal polarization–as function of particle's fractional energy is represented, [2].

High-energy circularly polarized photon must be created in first place, Fig.2. The last can be done by shaking electron with help of *electromagnetic wave*. This might be a laser wave or undulator (wiggler) field among the possibilities. The last is the subject of this paper, basically. *By selecting secondary particles by energy, one can select particles by polarization.* Namely the last procedure is a key element of the method. If the photon is not circularly polarized then secondary positron (or electron) is not polarized too.

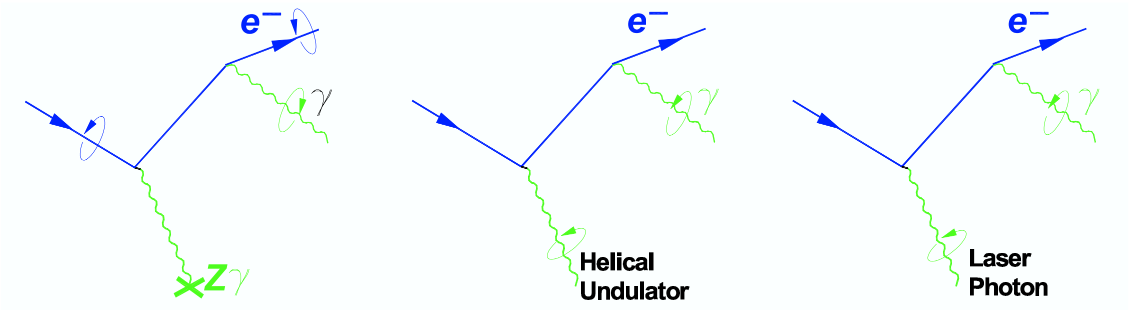

Figure 2: The ways to create circularly polarized photon. At the left diagram electron has longitudinal polarization. In the middle- the photons created by helical undulator. At the right –laser radiation is circularly polarized.

Of cause these facts were known for a long time. It was important to show, however, that these theoretically predicted properties could be *combined* so that efficiency of polarized positron/electron generation could satisfy requirements of Linear Collider business. That was done taking into account real hardware such as collecting optics, target design, undulator design, electro-optics, spin rotators and so on.

Total number of positrons at the exit of the target having thickness d can be expressed in terms of characteristic length of interaction l_γ as

$$N_{+} \cong N_{\gamma} \cdot d / l_{\gamma}, \qquad (1)$$

Characteristic length of interaction for gammas can defined by cross section of interaction σ_{tot} and density of material as the following

$$l_{\gamma}^{-1} = \sigma_{tot} n \cong \frac{7}{9} \frac{\rho}{X_0}, \qquad (2)$$

where ρ –is specific weight of the target material, and radiation length X_0 defined as

$$X_0^{-1} \cong 4 r_0^2 \alpha \frac{N_0}{A} Z(Z+1) \ln(\frac{183}{Z^{1/3}})[cm^2 / g], \qquad (3)$$

A –is atomic weight of target substance, $N_0 \cong 6.022 \cdot 10^{23}$ is the Avohadro number, Z is atomic number, $\alpha = e^2 / \hbar c \cong 1/137$, r_0 is classic electron radius. For Tungsten $X_0 \cong 6.8$ *g/cm*^{2}, $\rho = 19.1$ *g/cm*^{3}, so $l_{\gamma} \cong 0.45 cm$. Meanwhile the length corresponding to X_0 is $\cong 0.35$ *cm*. Typically, target is half of this length, so $N_{+} \cong 0.38 N_{\gamma}$ in full spectra. Strictly speaking factor 7/9 in (2) defined for $\hbar\omega_{\gamma} / mc^2 >> 1$. For gammas with intermediate energy it is~30% from this one. Typically number of quantas required ~10 per each initial particle. This leads the full number of periods in undulator $\sim M = L / \lambda_u \cong 10^4$, where λ_u stands for period of undulator. The last number well described by fact, that electron radiates $\sim \alpha = e^2 / \hbar c$ photons at the formation length, what is $\sim \lambda_u$ in this case.

CONVERSION SYSTEM WITH UNDULATOR

Let me remind briefly how helical field could be generated. One way uses for these purposes two helixes enclosed one into other with a shift equal to a half of winding period. In these helixes the currents running in opposite directions, Fig 3.

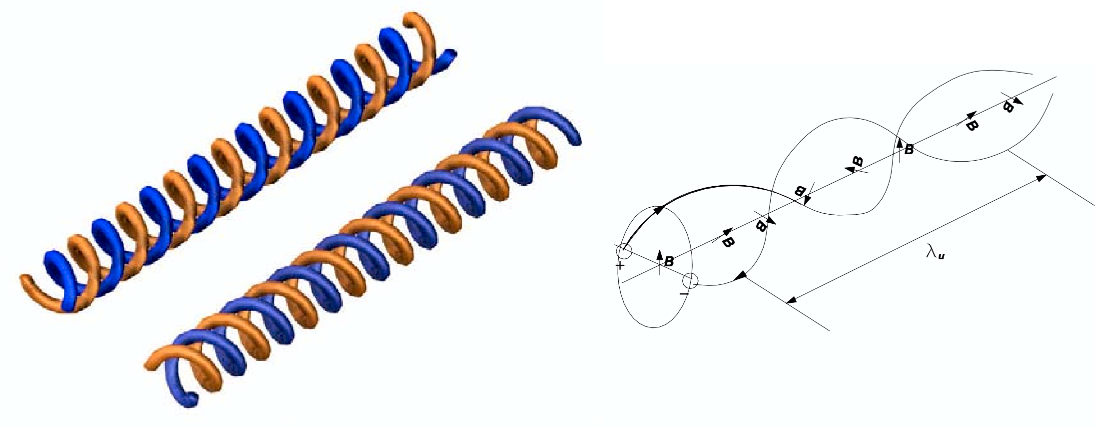

Figure 3: Bifilar helixes with opposite helicities, left. Period of variation of helical field is equal to spatial period of each helix.

Undulator has two helixes shifted in longitudinal direction by half-period. So basically this is double helix construction, similar to DNA[6]. Direction of helix twist (left/right handed) defines helicity of radiation in undulator. Although in high-energy physics (in contrast to optics) the observer is looking *towards* direction of propagation, the way on how helicity is defined is independent on this.

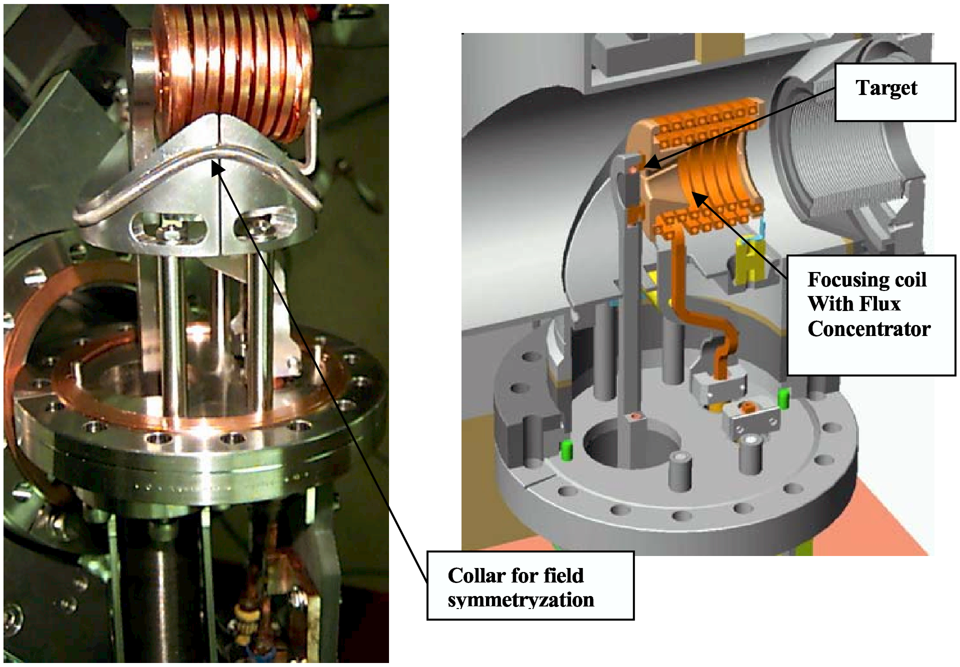

Figure 4: Prototype of conversion unit successfully tested at CESR.

Powerful optics with short focusing lens used for collection of positrons after the target. Example of such lens is represented in Fig.4. Other example of such lens is under test in framework of E-166 experiment which is under way at SLAC[7].

General scheme emerged is that high energy (~150 *GeV*) beam passes inside ~100 *m*-long helical undulator, with $\lambda_u \sim 1\text{-}cm$ period, where it radiates photons with energy ~20 *MeV* in narrow spectra. Further, these photons irradiate a $\cong X_0/2$-thick target, where in its turn these photons create electron-positron pairs. It was shown that this scheme might have one-to-one conversion.

This length of undulator can be reduced by 1/2 by using *few* targets and targets of appropriate shape, however. Heat deposition in target is reduced from few hundred kilowatts[8] in traditional scheme to a hundred watt level simply as there is no necessity to create a shower (cascade).

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[6] DNA was described first as a right handed helix of B-conformation, so called DNA-B or B-DNA. Mostly of living matter have this type of twist. Now the role of left-hand twisted DNA-Z molecule is under discussion. Although DNA-Z is not quite stereo-isomeric copy of DNA-B. Period of DNA helix is 34Å, diameter ~24Å, the longitudinal shift of these helixes is not a half period, however.

[7] See http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-tn-04-018.pdf

[8] Let me remind, that this energy converted into radioactive waste finally.

BEFORE OR AFTER IP

Some questions arise about location of this undulator first. Beam after collision becomes disrupted with emittance increased by $\Delta\gamma\varepsilon \propto a \cdot r_0 N$, where a is numerical factor ~0.1-1, N stands for the bunch population, r_0 is classic electron radius. Also, the energy spread can be estimated as

$$\Delta E / E \cong \frac{r_0^3 N^2 \gamma}{\sigma_{\|} \cdot (\sigma_x + \sigma_y)^2} \tag{4}$$

where $\sigma_{\|}$ stands for longitudinal dimension, σ_x, σ_y for horizontal and vertical dimensions respectively, it looks significant, depending on aspect ratio $R = \sigma_x / \sigma_y$. In principle it is possible to have undulator after IP, but this brings some troubles with design of powerful optics. With final emittance like just mentioned, the beam size in undulator can reach $x \cong \sqrt{r_0 N \beta / \gamma} \cong 1.7 \cdot 10^{-2} cm$, or 0.17 mm. Ten sigma full size will occupy ~3*mm* only. Here was substituted envelope function $\beta \cong 100m$, $N \cong 3 \cdot 10^{10}$, $\gamma = 3 \cdot 10^5$. Even with energy spread estimated as 10% there is a possibility to push the beam through undulator having aperture opening ~ 6 *mm* (see details of undulator design lower). Basically this is in line with safe evacuation of used beam from IP. Optics can be simplified to avoid bending magnets come to minimal two magnets to exclude chromaticity at undulator location. Deflection of the beam from the line of secondary gammas also can be seen as possible. One disadvantage of energy spread associated with quadratic dependence of quantum energy on the energy of particle and hence, energy spread in the beam. However this looks tolerable. So in principle, this remains as a possible solution.

If undulator is planned to be installed after the beam reaches ~100-150 *GeV*, before IP, this will require careful investigation of possible sources of emittance and polarization dilution, which may happen in undulator. Fortunately all these perturbations seem to be small, so undulator can be installed before IP as well. If collisions are planned to go at lower energy, say ~50 *GeV*, rest part of main linac just reduces energy to desirable level.

We mentioned 100-150 *GeV* for the beam energy. Optimization made in framework of VLEPP project shows that this is an optimal one for conversion.

SCHEMATICS

Let us now consider schematics of conversion system with undulator, Fig.5. The scheme with undulator installed *before* IP is shown there. So positron beam generates itself every time it passes through undulator. Some feedback must keep the number of particles fixed. This feedback can be thought as a set of very simple optical elements.

One question about cross talk between electron and positron lines solved by introduction of electron linac having low energy, ~200 MeV with possibility to irradiate the main target, i.e. the same target as in use with undulator radiation. Time pattern of the electron beam generated in this linac is the same as in main beam. Efficiency of conversion can reach ~5%. Positrons, naturally, will be not polarized. After filling damping ring with unpolarized positrons in ~20 shots, what will happen in fraction of a second, stored beam (of ***not*** polarized positrons) extracted into regular channel and going into linac and further into undulator. After normal circle of conversion, positron beam acquired by damping ring is polarized now. So the loop closed and the beam at IP will be absent only for this ~20 shots. Losses of beam might be considered as a rare event, however. Of cause some modifications of this basic idea can be proposed instantly. For example use of pre accelerator 6 in Fig.5 or fraction of main linac can be appointed for these purposes.

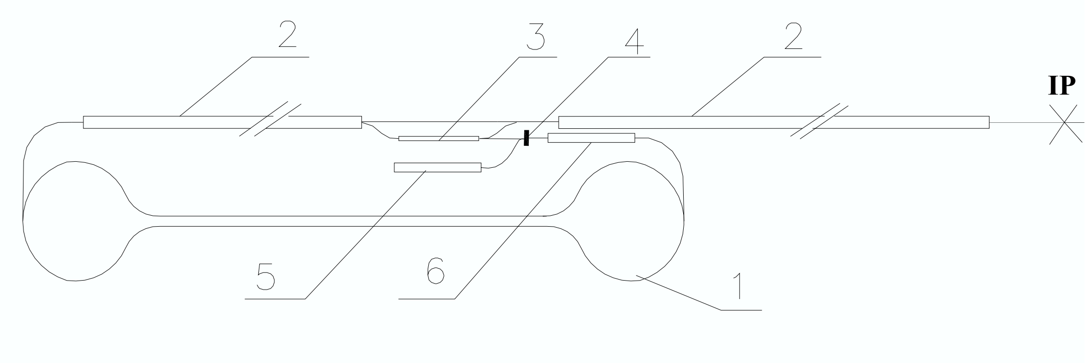

Figure 5: Scheme with undulator before IP. 1–damping ring, 2–linear accelerator, 3–undulator, 4–target, 5–electron linac with energy ~200MeV, 6–pre-accelerator. Spin rotators and some complementary electro-optical elements not shown in this figure.

So namely existence of additional low-energy linac, 5 in Fig.5, allows independent operation of electron and positron wings.

INTENSITY OF RADIATION. PLANAR VERSUS HELICAL

First of all one can see that *intensity of radiation* in helical undulator is two times higher, than in planar undulator with the same period and *K* factor. This is because in helical undulator particle participates in acceleration in two geometrical planes, (vertical and horizontal) while the acceleration in planar undulator is going in one direction only. As radiated field with every polarization is proportional to acceleration in corresponding direction, this yields the intensity balance just mentioned. Spectral density of radiation is two times higher also. Numerically

$$\left.\frac{dI}{d\xi}\right|_{helical} = 2\left.\frac{dI}{d\xi}\right|_{planar}, \tag{5}$$

where $\xi = \frac{\hbar\omega_\gamma}{\hbar\omega_{\gamma\max}}$ is a ratio of current photon energy to the maximal possible one at first harmonic. Energy of quanta for the first harmonic in each case defined by

$$\hbar\omega_{\gamma\max}^{helical} = \hbar\frac{2\pi c}{\lambda_u}\frac{2\gamma^2}{\cdot\left(1+K^2\right)}\ , \quad \hbar\omega_{\gamma\max}^{planar} = \hbar\frac{2\pi c}{\lambda_u}\frac{2\gamma^2}{\cdot\left(1+\frac{K^2}{2}\right)}. \tag{6}$$

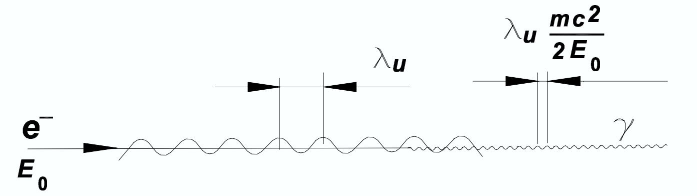

Figure 6: Relationship between wavelength of undulator and radiated waves.

From the formula for radiation

$$\frac{dN_\gamma^{helical}}{d\xi} = \left.\frac{dI}{d\xi}\right|_{helical} \times \frac{1}{\hbar\omega^{helical}} = \sum_n \frac{dN_{\gamma n}}{d\xi} \cong 4\pi\alpha M \frac{K^2}{1+K^2} \sum_n nF_n(K,\xi), \quad (7)$$

where M stands for the number of periods and n stands for harmonic number, $M = L/\lambda_u$, $\alpha = e^2/\hbar c$ –is a fine structure constant. Functions

$$F_1(K,\xi) \cong \frac{1}{2}(1-2\xi+2\xi^2), \quad F_2(K,\xi) \cong 2\xi(1-\xi)(1-\xi+2\xi^2)K^2. \quad (8)$$

Integrating over ξ one can obtain *the number of radiated photons* for the first harmonic

$$N_\gamma^{helical} \cong \int_0^1 \frac{dN_\gamma^{helical}}{d\xi} d\xi \cong 4\pi\alpha M \cdot \frac{1}{6} \cdot \frac{K^2}{1+K^2} \quad (9)$$

For planar undulator one has

$$N_\gamma^{planar} \cong 2\pi\alpha M \cdot \frac{1}{6} \cdot \frac{K^2}{1+K^2/2} \quad (10)$$

So the ratio of numbers is going to be

$$\eta = \frac{N^\gamma_{helical}}{N^\gamma_{planar}} \cong \frac{2\cdot(1+K^2/2)}{1+K^2} \ . \quad (11)$$

For small K values (dipole approximation used above; small amount of higher harmonics) this ratio is $\eta \cong 2$ in a favor of helical undulator. For $K \cong 1$ (beyond this value dipole approximation is not valid anymore), $\eta \cong 1.5$. Optimal K factor corresponds K~0.35. That is because for higher K values second (and higher) harmonics emerge. What is important here is that for small K, energy of quanta is the same for both undulators having the same period. So for optimal K value $\eta \cong 1.86$ and $\frac{\hbar\omega_{\gamma\max}^{helical}}{\hbar\omega_{\gamma\max}^{planar}} = \frac{1+K^2/2}{1+K^2} \cong 0.95$. So one can see why helical undulator must be used here.

EMITTANCE PERTURBATIONS

If undulator installed on the way of main beam, the vital questions arise about possible perturbations of *emittance* and *polarization*.

Emittance perturbation due to the field roll-off in undulator

Inside undulator particles developing tiny helixes, having pitch angle $\alpha \sim K/\gamma$, and the radius of helixes is $a \cong \lambdabar_u K/\gamma$, where $\lambdabar_u = \lambda/2\pi$, λ_u stands for period of undulator; we will be interested in $\lambda_u \cong 1$ *cm*. The numbers for angular spread and radius of helix for *E*=150 *GeV* ($\gamma \cong 3\cdot 10^5$), and *K*~1 coming to $\alpha \sim 3\cdot 10^{-6}$ rad and $a \cong 10^{-5}/6\pi \cong 5\cdot 10^{-7}\,cm$ respectively. Meanwhile natural vertical angular spread inside the beam and the beam size $y' \cong \sqrt{\gamma\varepsilon_z/\beta\gamma}$ and $\sqrt{\langle y^2\rangle} \cong \sqrt{\gamma\varepsilon_y\beta/\gamma}$, for invariant vertical emittance $\gamma\varepsilon_z \cong 2\cdot 10^{-8}\,m\cdot rad$ and the envelope function value $\beta \cong 100m$, go to be $y' \cong \sqrt{2\cdot 10^{-8}/3\cdot 10^5/100} \cong 2.6\cdot 10^{-8}$ rad and $\sqrt{\langle y^2\rangle} \cong \sqrt{2\cdot 10^{-8}\cdot 100/3\cdot 10^5} \cong 2.6\cdot 10^{-4}[cm]$ respectively. For horizontal motion, $\gamma\varepsilon_x \cong 2\cdot 10^{-6}\,m\cdot rad$ giving all numbers ten times bigger.

Angular spread in radiation	$\alpha \sim K/\gamma$	$3\cdot 10^{-6}$
Angular spread in beam, vert.	$y' \cong \sqrt{\gamma\varepsilon_z/\beta\gamma}$	$2.6\cdot 10^{-8}$
Radius of helix	$a \cong \lambdabar_u K/\gamma$	$5\cdot 10^{-7}\,cm$
Beam size, vertical	$\sqrt{\langle y^2\rangle} \cong \sqrt{\gamma\varepsilon_y\beta/\gamma}$	$2.6\cdot 10^{-4}\,cm$

One can see that angular wiggling is much bigger, than natural angular spread in the beam. Meanwhile radiuses of helixes are much smaller, than the beam size. First fact makes possible angular separation for reduction of contest of second harmonic and enhance average polarization as it becomes reversed under angle $\sim 1/\gamma$.

Important here is that all particles in cross-section are developing helixes with the same phase inside undulator, despite field dependence across aperture. The last generates ellipticity in helix. The field roll-off from the center of undulator, described in polar coordinates, is proportional to the square transverse radial displacement ρ

$$H_\varphi(\rho,\varphi,z) \cong H_0 \times \cos(\varphi - \varphi_0 - \frac{z}{\lambdabar_u}) \times \left[1 + \frac{1}{8}\left(\frac{\rho}{\lambdabar_u}\right)^2 + \frac{1}{768}\left(\frac{\rho}{\lambdabar_u}\right)^4 + \ldots\right], \qquad (12)$$

where $H_0 \propto I/\lambdabar_u$ is the field at the axis, *I* stands for the current running in helix [CBN 02-10]. So in our case the second term for the particles at the distance $\sqrt{\langle x^2\rangle} \cong 26\cdot 10^{-4}\,cm$ goes to be

$$\frac{1}{8}\left(\frac{\rho}{\lambdabar_u}\right)^2 \cong \frac{\langle x^2\rangle}{8\lambdabar_u^2} \cong 3.3\cdot 10^{-5}.$$

This means, that ellipticity is negligible, however. Important is that the difference in kicks obtained by the particles across the beam size will be different in the same proportion, if the helix is ended suddenly. As the field along the particle's trajectory oscillating, the nonlinear kicks are oscillating too. So resulting kick averaged along period will be different in accordance with formula (12), where for radius the value $\rho \cong ƛ_u K / \gamma$ needs to be substituted now, making the integral difference negligibly small.

To eliminate this effect completely, installation of sections of undulator in pairs can be used here. Neighboring undulators must be rotated azimuthally by 180°. Fringe field correction can also be used here, making smooth tapering for the field at the end. All together this eliminates dependence on the undulator entrance/exit. What important here, is that the plane of magnetic field can be controlled easily.

So finalizing one can say that the beam size is so small, that the difference in transverse kicks across the beam is small and perturbation of emittance is small too.

This however is the mostly important subject for concern while attempting installation of undulator before IP.

Focusing in the field of undulator

After passage a single pole at off-axis distance y, effective kick comes to be

$$y' \cong \frac{\alpha H y}{2(HR)} \cong \frac{K H y}{2(HR)\gamma}, \tag{13}$$

where (HR) stands for magnet rigidity, H is undulator magnetic field. So the focal distance for undulator having length L goes to be

$$F \cong \frac{y}{y'}\frac{\lambda_u}{L} = \frac{2(HR)\lambda_u \gamma}{KHL} \cong \frac{\lambda_u^2 \gamma^2}{K^2 L \pi}. \tag{14}$$

For 300 GeV beam $(HR) \cong 10^6 kG \cdot cm$, $\gamma \cong 6 \cdot 10^5$, $K \cong 1$, $L \cong 100\, m$,$\lambda_u \cong 1 cm$ focal distance goes to be $F \cong 10^7 cm$. So the angle obtained by side particle can be estimated as

$$x' \cong \frac{\sqrt{\gamma \varepsilon_x \beta / \gamma}}{F} \cong x' \cong \frac{2.6 \cdot 10^{-3}}{10^7} \cong 2.6 \cdot 10^{-10},$$

meanwhile angular spread in radial direction is $x' \cong 2.6 \cdot 10^{-6}$. So the effect of focusing is negligible. Also with reduction of envelope function by lenses installed between sections of undulator, these numbers can be improved.

Focusing by alternative sextupole field is much smaller as one can see from formula for magnetic field.

For planar undulator there is no focusing in direction across the poles (typically horizontal direction) at all. For this the pole must be wide enough. Practically, the pole width must be ~3-4 times the vertical pole gap size for these purposes. This is, probably, the only simplification in comparison with helical undulator.

Emittance perturbation due to radiation in disperse field of undulator

As that undulator field has dispersion, radiation of quanta may cause emittance growth. Equation describing this process is[9]

$$\frac{d\varepsilon_{x,y}}{dz} \cong \left(H_{x,y} + \frac{\beta_{x,y}}{\gamma^2} \right) \frac{\overline{d(E_\gamma / E)^2}}{dz} \tag{15}$$

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[9] Radiation damping is neglected.

$H(z)=\frac{1}{\beta(z)}\left(\eta^2(z)+(\beta\eta'-\frac{1}{2}\beta'\eta)^2\right)$, z—is a longitudinal coordinate here, $\overline{A}$ - means averaging over spectrum of radiation. Dispersion generated inside undulator is a periodic solution

$$\eta=\frac{K\bar{\lambda}_u}{\gamma}Sin\frac{z}{\bar{\lambda}_u}, \qquad \eta'(z)=\frac{K}{\gamma}Cos\frac{z}{\bar{\lambda}_u}, \tag{16}$$

so dispersion invariant becomes $H\cong\beta\eta'^2=\frac{\beta K^2}{\gamma^2}Cos^2\frac{z}{\bar{\lambda}_u}$.

$$\overline{\left(\frac{E_\gamma}{E}\right)^2}=\int\left(\frac{E_\gamma}{E}\right)^2\frac{dN_\gamma}{dE_\gamma}dE_\gamma=\left(\frac{\hbar\omega_{\max}}{E}\right)^2\int\xi^2\frac{dN_\gamma}{d\xi}d\xi\,, \tag{17}$$

and variable $\xi=\frac{E_\gamma}{E_{\gamma\max}}=\frac{1+K^2}{1+K^2+\gamma^2\vartheta^2}$ is the same as used in (1).

Using formulas for undulator radiation (7), (8) and taking into account that

$$\int_0^1\xi^2(1-2\xi+2\xi^2)d\xi=7/30\,, \qquad ,$$

one can obtain formula for emittance dilution as

$$\Delta\varepsilon\cong\frac{\overline{\beta}}{2}\frac{K^4}{(1+K^2)^3}\left(\frac{r_0}{\bar{\lambda}_u}\right)^2\cdot 2\pi M\cdot\frac{7}{30}\,, \tag{18}$$

where $\overline{\beta}$ stands for average envelop function in undulator. So perturbation of emittance due to this effect is negligible.

PERTURBATION OF POLARIZATION

Perturbations due to dynamical motion in helical field

As one can see that particles experience helical motion with pitch angle mentioned above $\alpha\sim K/\gamma$. Then vector of spin rotates relatively to momentum with the angle which is γ/γ_0 (γ_0 corresponds to ~440.65 MeV) times faster. Important thing here is that resulting angle is going to be

$$\theta_{spin}=\frac{K}{\gamma}\times\frac{\gamma}{\gamma_0}=\frac{K}{\gamma_0}\,, \tag{19}$$

i.e. this angle for each particle does not depend on its energy. The last means, that resulting angle for *all* particles in the bunch, having slightly different energies, is the same. This means, in its turn, that there is no depolarization at all. The only thing needs to be done –is just controlled proper orientation of spin before undulator, so that after undulator it is directed as needed.

Spin flip in undulator

Positron or electron may flip its spin direction while radiating in magnetic field. As primary beam is the beam of polarized particles (in case if undulator installed before IP), one can think about possible depolarization here. Let us examine formula for the probability of radiation transaction (spin flip) for the circular motion in magnetic field [2]

$$\frac{1}{\tau}[\sec^{-1}] = w_{flip} = \frac{5\sqrt{3}}{16}\frac{r_0^2}{\alpha}\frac{\omega_0^3}{c^2}\gamma^5\left(1-\frac{2}{9}\zeta_{\parallel}^2-\frac{8\sqrt{3}}{15}\frac{e}{|e|}\zeta_{\perp}\right), \tag{20}$$

where $\zeta_{\parallel}$, $\zeta_{\perp}$ stand for longitudinal and transverse components respectively, $\omega_0 = eH/mc\gamma$. Probability of radiation in undulator per second (which is responsible for radiation at all) can be written as

$$w_{rad} \cong \frac{I}{\hbar\omega_0 2\gamma^2} = \frac{2}{3}\frac{e^4H^2\gamma^2}{m^2c^3}\frac{1}{\hbar\omega_0 2\gamma^2} = \frac{1}{3}\alpha\gamma^2\omega_0 \ , \tag{21}$$

so the ratio of these probabilities goes to be

$$\frac{w_{flip}}{w_{rad}} = \frac{15\sqrt{3}}{16}\frac{\lambda\!\!\!^{-}{}_c^2}{\lambda\!\!\!^{-}{}_u^2}\gamma^3\left(1-\frac{2}{9}\zeta_{\parallel}^2-\frac{8\sqrt{3}}{15}\frac{e}{|e|}\zeta_{\perp}\right) , \tag{22}$$

where $\lambda\!\!\!^{-}{}_c = r_0/\alpha = e^2/mc^2/\alpha \cong 3.8616\cdot 10^{-11} cm$ is Compton wavelength, and here was substituted $\lambda\!\!\!^{-}{}_u = \omega_0/c$ - normalized to 2π undulator wavelength. The last formula reflects the fact, that particle radiated a lot of photons and influence to polarization is small

Strictly speaking formula (20) represented here valid for circular motion in strong magnetic field (big K factor), where characteristic energy of photon is $\hbar\omega_c \cong \hbar\omega_0\gamma^3$, i.e. even higher, than used for undulator radiation in (21). So for simple circular motion this ratio is even less, than represented by factor γ

$$\frac{w_{flip}}{w_{rad}} \propto \gamma^2 .$$

It is by the way the similar ratio as the ratio of self polarization (third term in brackets in formula (22) is responsible for this process) to the damping time.

So for parameters of our interest, where $\gamma \cong 3\cdot 10^5$, this effect is small.

Perturbations due to radiation in a target

Similarly to the spin flip of polarized particles in undulator the spin might flip due to radiation of bremstrahlung quantas having energy $0 < \hbar\omega_\gamma \le E_1$, where E_1 stands for initial energy of positron. Depolarization after one single act of scattering defined as

$$D = 1 - \left|\frac{d\sigma_{\gamma e}(\zeta_1,\zeta_1) - d\sigma_{\gamma e}(\zeta_1,-\zeta_1)}{d\sigma_{\gamma e}}\right| , \tag{23}$$

where $d\sigma_{\gamma e}(\zeta_1,\zeta_1)$ stands for bremssrahlung cross section without spin flip, $d\sigma_{\gamma e}(\zeta_1,-\zeta_1)$–the cross section with spin flip and $d\sigma_{\gamma e}$ is total cross section. Substitute here formulas for each of these sections one can obtain [2]

$$D=\frac{\hbar^2\omega_\gamma^2\cdot[1-\frac{1}{3}\zeta_{1\|}^2\ ]}{E_1^2+E_2^2-\frac{2}{3}E_1E_2}, \tag{24}$$

E_2 stands for final energy of positron. As we are interesting in energetic secondary positrons, $\hbar\omega_\gamma/E_1 \ll 1$ and $D=\frac{\hbar^2\omega_\gamma^2\cdot[1-\frac{1}{3}\zeta_{1\|}^2\ ]}{E_1^2}\approx 0$. Although depolarization after single collision is small, one needs to calculated resulting depolarization after passage some distance. This defines the length of depolarization associated with spin-flip while multiply scattering on nuclei [2]

$$L_{dep}\cong\frac{1}{n\int D(\vec{p}_1,\zeta_1)d\sigma}=\frac{1}{2n\sigma_{flip}} \tag{25}$$

Substitute here formula for cross section with spin flip, one can obtain after [2] that

$$L_{dep}\cong\frac{2X_0}{1-\frac{1}{3}\zeta_\|^2}\cong 3X_0, \tag{26}$$

where X_0 stands for radiation length, (3). Optimal thickness of target is ~ ½X_0. As probability of positron creation is linearly growing with passage distance for gamma quanta inside the target, effective thickness is going to be ~1/6 X_0, so resulting depolarization is going to be $D\cong\exp(-1/18)\cong 1-1/18\cong 5\%$. So this effect is small and generally speaking is not associated with the choice of undulator – this is general effect.

Kinematical perturbations due to multiple scattering in a target

Let us consider the possible effect of *kinematical* depolarization associated with rotation of spin vector while particle experience multiple scattering in media of target before leaving. Typically polarized positron carries out ~(0.5-1)$\hbar\omega$ –energy of gamma quanta. As positrons/electrons created have longitudinal polarization, it is good to have assurance that during scattering in material of target polarization is not lost. Each act of scattering is Coulomb scattering in field of nuclei. So BMT equation describing the spin $\vec{\zeta}$ motion in electrical field of nuclei looks like

$$\frac{d\vec{\zeta}}{dt}=\frac{e}{mc^2\gamma}\left\{G\gamma+\frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\left(\vec{E}\times\vec{v}\right), \tag{27}$$

where $\vec{E}\sim Ze\vec{r}/r^3$ stands for repulsive (for positrons) electrical field of nuclei, factor $G=\frac{g-2}{2}\cong 1.1596\times 10^{-3}\approx\frac{\alpha}{2\pi}$. Deviation of momentum is simply $d\vec{p}/dt=e\vec{E}$.

So the spin equation becomes

$$\frac{d\vec{\zeta}}{dt} = \frac{1}{mc^2\gamma}\left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\left(\frac{d\vec{p}}{dt}\times\vec{v}\right). \tag{28}$$

We neglected variation of energy of particle during the act of scattering, so $\frac{d\vec{p}}{dt} \cong m\gamma\frac{d\vec{v}}{dt}$ and vector $\vec{p}$ just changes its direction. Introducing normalized velocity as usual $\vec{\beta} = \vec{v}/c$, equation of spin motion finally comes to the following

$$\frac{d\vec{\zeta}}{dt} = \left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\left(\dot{\vec{\beta}}\times\vec{\beta}\right) = \left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\frac{d\vec{\varphi}}{dt}, \tag{29}$$

where φ stands for the scattering angle and its vector directed normally to the scattering plane. For intermediate energy of our interest $\gamma \sim 40$, so the term in bracket ~1 and, finally

$$\frac{d\vec{\zeta}}{dt} = \vec{\zeta}\times\frac{d\vec{\varphi}}{dt}. \tag{30}$$

The last equation means that spin rotates to the same angle as the scattering one, i.e. spin follows the particle trajectory.

Depolarization at IP

Depolarization arises as the spin changes its direction in coherent magnetic field of incoming beam. Here again the deviation does not depend on energy, however it depends on location of particle in the bunch: central particles are not perturbed at all. Absolute value of angular rotation has opposite sign for particles symmetrically located around collision axes.

This topic was investigated immediately after the scheme for polarized positron production was invented. This effect is not associated with polarized positron production exclusively because this effect tolerates to the polarization of electrons at IP as well. Later many authors also considered this topic in detail. General conclusion here is that depolarization remains at the level ~5%.

DESIGN

Design was performed in framework of VLEPP project. Two types of undulators were tested in 1986: pulsed one and the one with SC windings. SC undulator was recognized as having advantages as it is not dependent on repetition rate. Aperture of SC undulator 6*mm*- allows successful operation of this device in conversion scheme. Some details of cryostat for this cold mass are represented in Fig. 7 below. Photo of cold mass itself is represented in Fig. 8. Cold mass tested has 30 *cm* in length, period $\lambda_u = 1cm$ and wound with 15 turns of SC wire. Maximal value of *K* factor achieved was *K*~0.5 with feeding current~400 A/wire. Once again, optimal value of *K* factor for maximal yield of polarized positrons was found to be *K*~0.35 . So this over strength gives safety margins widening and can be useful for generation of positrons while tuning the system when spectral quality of radiation is less important.

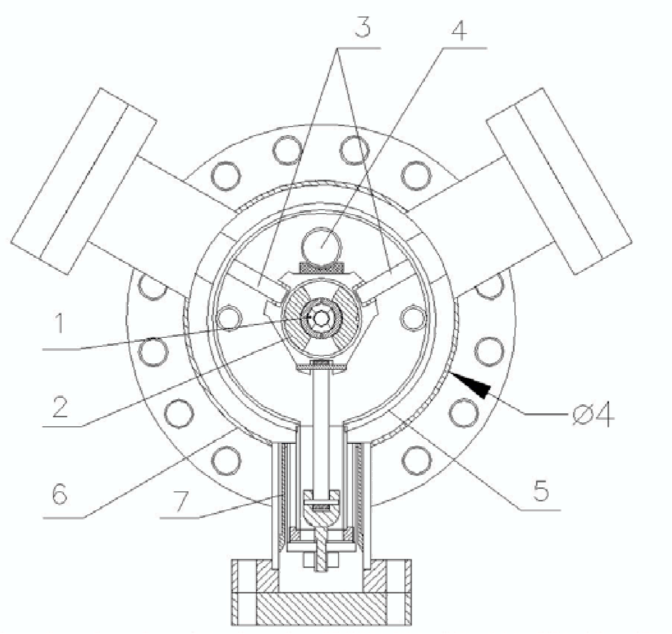

Figure 7: Cross-section of undulator with SC windings. Diameter of cryostat is 4 inches. 1–cold core, 2 –Copper semi-cylinders, 3–Strips, 4–Helium return, 5–LNitrogen shield, 6–outer vacuum chamber wall of cryostat, 7–LHe-LNitrogen shield transition.

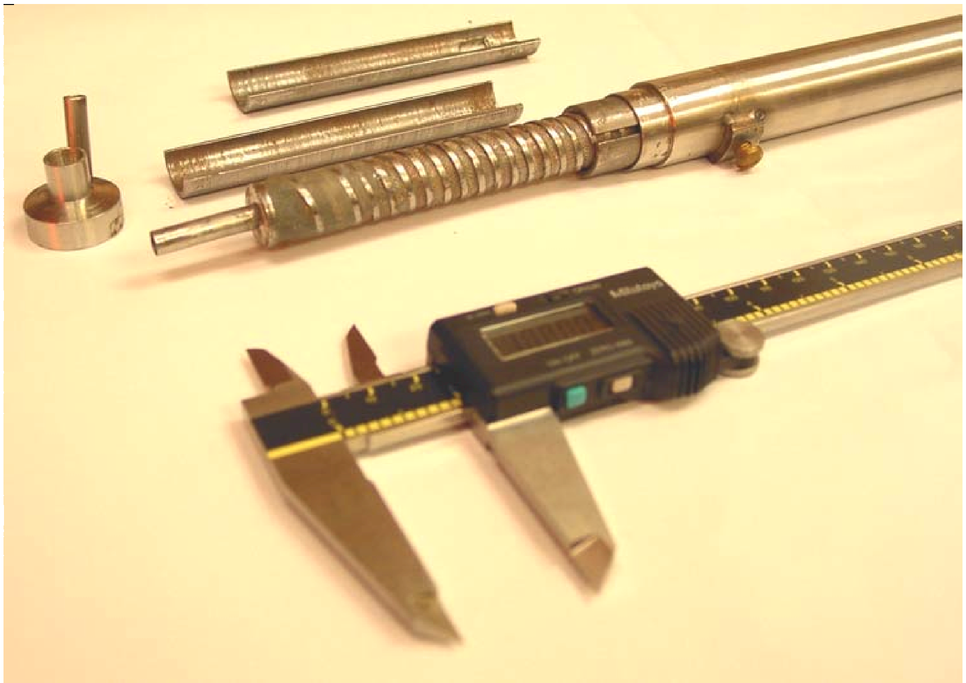

Figure 8: Central core of SC undulator (item 1 from previous figure) with period λ_u=10 *mm*, aperture a=6*mm*, length ~30*cm*, K~0.4 tested successfully in 1986. Revised at Cornell.

ONE ADDITIONAL ARGUMENT

We touched some scientific arguments in association with advantage of positron conversion with *helical* undulator. Now let me discuss one other reason why this question exists in first place. To me it looks like the mostly reasonable explanation.

It is in the following. After DESY have accepted conversion system with undulator, next step, naturally, could be associated with design and test of helical undulator in DESY Lab. This was not done, however, and Daresbury Laboratory finally was appointed for this purposes. Situation with undulator in this Laboratory can be seen from presentation at last Workshop on ILC at KEK (2004). My estimation is that Daresbury is at the very beginning. Despite the fact that helical undulator (which was designed and tested at Novosibirsk in 1986) was described in many publications, Daresbury even not coming close to these results. Some technological nuances remain not known there. As a result –wrong conclusion about aperture/period ratio and absence of adequate solution for the end region winding. As a sequence–absence of design for helical undulator in general. So that is why planar undulator is thought as a first one.

As far as SC linear collider design was a monopoly of DESY, of cause that was monopolistic conclusion. Right now solution here might be either Daresbury will quickly learn on how to fabricate helical undulator either to ***make Cornell responsible for conversion system of ILC.*** In last case the question about undulator will be vanished instantly.

CONCLUSION

There is no necessity to tell how important polarization in both beams for High energy physics is. The question is basically about the price of this enterprise. We are concluding that *if* Linear Collider will be built in some time at all, it must be equipped from the very beginning with helical undulator. This is a requirement of engineering mostly. The last asks for reduction of power deposition in a target from few hundred kilowatts (traditional scheme) to a hundred watts (undulator based scheme). Undulator needs to be made with sections ~4*m* each with total length ~100*m*. Longer undulator-higher polarization can be achieved. With 300 *m*-long undulator polarization can reach 85%. Polarized *electrons* for linear Collider could be obtained by the same way as polarized positrons.

Linear collider with undulator based scheme delivers higher luminosity and is cheaper, than traditional scheme –this is not even under discussion– but cheaper, than with planar undulator.

Experiment E-166, which is running at SLAC, will diffuse doubts about polarized positron production possibilities.

In conclusion author thanks Michael Woods who triggered writing this paper under its present title.

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