	e^-	e^+		
$\sigma_{\mathbf{RR}}$	$\Rightarrow$	$\Leftarrow$	$\frac{1+P_{e^-}}{2} \cdot \frac{1+P_{e^+}}{2}$	J = 0
$\sigma_{\mathbf{LL}}$	$\Leftarrow$	$\Rightarrow$	$\frac{1-P_{e^-}}{2} \cdot \frac{1-P_{e^+}}{2}$	
$\sigma_{\mathbf{RL}}$	$\Rightarrow$	$\Rightarrow$	$\frac{1+P_{e^-}}{2} \cdot \frac{1-P_{e^+}}{2}$	J = 1
$\sigma_{\mathbf{LR}}$	$\Leftarrow$	$\Leftarrow$	$\frac{1-P_{e^-}}{2} \cdot \frac{1+P_{e^+}}{2}$	

Table 1: Graphical representation of the various spin configurations in e^+e^- collisions. The thick arrow represents the direction of motion of the particle and the double-arrow its spin direction. The first column indicates the corresponding cross section, the third column the fraction of this configuration and the last column the total spin assuming a zero orbital angular momentum.

2.2.4 Effective Polarization

In general the cross section of any process in an e^+e^- collider can be subdivided according to the initial helicity states see 1:

$$\sigma = \sigma_{\mathrm{RR}} + \sigma_{\mathrm{LL}} + \sigma_{\mathrm{RL}} + \sigma_{\mathrm{LR}}. \tag{1}$$

In the case of e^+e^- annihilation into a vector particle (in the SM this would be $e^+e^- \to \gamma/Z^0$) only the two J=1 configurations σ_{RL} and σ_{LR} contribute and the cross section for arbitrary beam polarizations is given by

$$\begin{aligned}
\sigma &= \frac{1+\mathrm{P}_{e^-}}{2}\,\frac{1-\mathrm{P}_{e^+}}{2}\,\sigma_{\mathrm{RL}} + \frac{1-\mathrm{P}_{e^-}}{2}\,\frac{1+\mathrm{P}_{e^+}}{2}\,\sigma_{\mathrm{LR}} \\
&= (1-\mathrm{P}_{e^-}\mathrm{P}_{e^+})\,\frac{\sigma_{\mathrm{RL}}+\sigma_{\mathrm{LR}}}{4}\left[1 + \frac{\mathrm{P}_{e^+}-\mathrm{P}_{e^-}}{1-\mathrm{P}_{e^+}\mathrm{P}_{e^-}}\,\frac{\sigma_{\mathrm{LR}}-\sigma_{\mathrm{RL}}}{\sigma_{\mathrm{LR}}+\sigma_{\mathrm{RL}}}\right] \\
&= (1-\mathrm{P}_{e^+}\mathrm{P}_{e^-})\,\sigma_0\,[1 + \mathrm{P}_{\mathrm{eff}}\,\mathrm{A}_{\mathrm{LR}}]
\end{aligned} \tag{2}$$

with the unplaced cross section $$\sigma_0 = \frac{\sigma_{\mathrm{RL}}+\sigma_{\mathrm{LR}}}{4} \tag{3}$$

the forward backward asymmetry $$\mathrm{A}_{\mathrm{LR}} = \frac{\sigma_{\mathrm{LR}}-\sigma_{\mathrm{RL}}}{\sigma_{\mathrm{LR}}+\sigma_{\mathrm{RL}}} \tag{4}$$

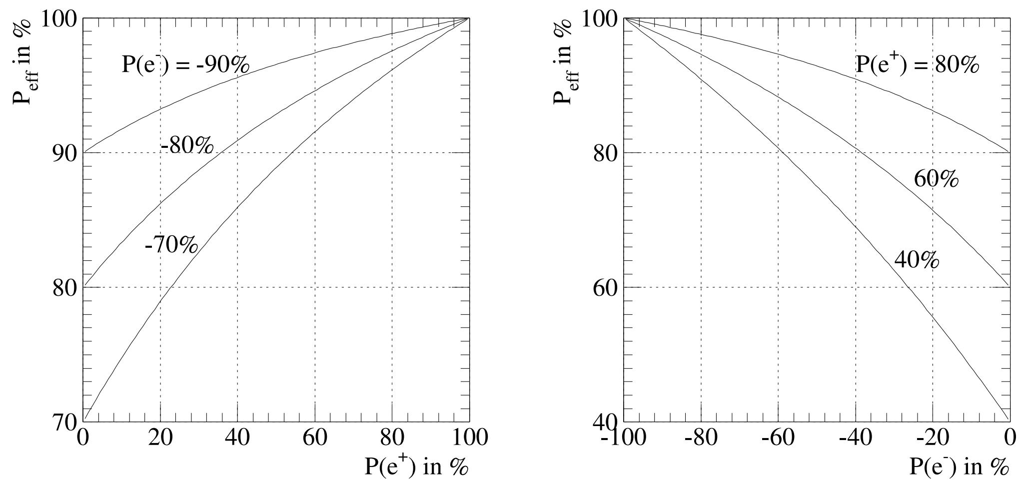

Figure 1: Effective polarization as a function of the beam polarization.

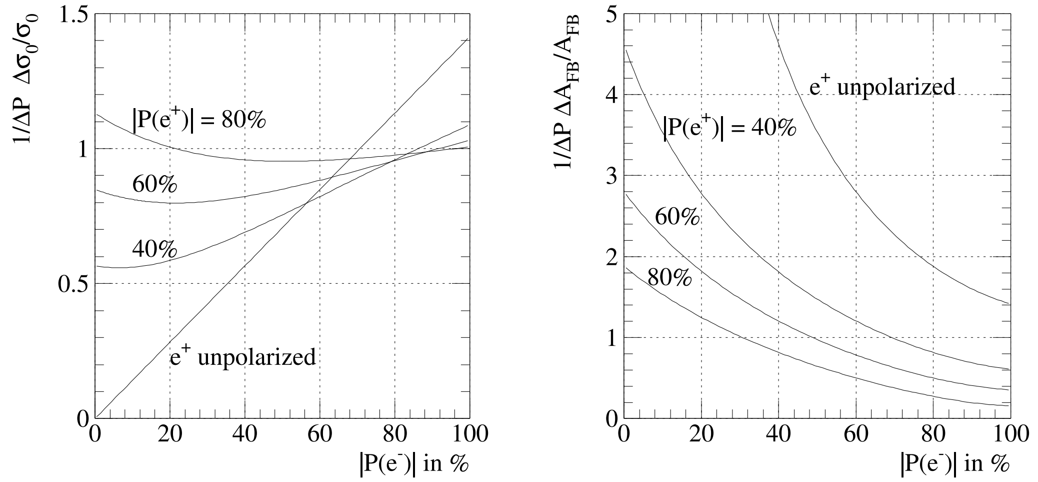

Figure 2: Left: Contribution of the uncertainty on beam polarization on the measurement of the unpolarized cross section. Right: Same for the forward-backward asymmetry. Both plots are normalized to the polarimeter resolution ΔP which is assumed to be identical for both beams.

$$\text{and the effective polarization} \qquad \mathrm{P}_{\mathrm{eff}} = \frac{\mathrm{P}_{e^+} - \mathrm{P}_{e^-}}{1 - \mathrm{P}_{e^+}\mathrm{P}_{e^-}} \tag{5}$$

The values of the effective polarization can be read from fig. 1. Notice that the effective polarization is closer to 100 % than any of the two beam polarizations.

In the experiment one would like to extract the two quantities σ_0 and A_{LR}. This can be done by running the experiment with two different polarizations. One would choose one setup with the electron beam predominantly left-handed and the positron beam right-handed and in the second setup one would reverse both spin directions. The cross sections measured with the two setups are denoted as σ_{-+} and σ_{+-}. It is then

$$\begin{aligned} \sigma_0 &= (1 + |\mathrm{P}_{e^+}||\mathrm{P}_{e^-}|) \, \frac{\sigma_{-+} + \sigma_{+-}}{2} \\ \mathrm{A}_{\mathrm{LR}} &= \frac{1}{|\mathrm{P}_{\mathrm{eff}}|} \, \frac{\sigma_{-+} - \sigma_{+-}}{\sigma_{-+} + \sigma_{+-}} \end{aligned} \tag{6}$$

Both quantities depend on the beam polarizations. The contribution of the uncertainty of the polarization measurement to the error is

$$\begin{aligned} \frac{\Delta\sigma_0}{\sigma_0} &= \frac{\sqrt{2}\,\Delta P}{1 + |\mathrm{P}_{e^+}||\mathrm{P}_{e^-}|} \sqrt{|\mathrm{P}_{e^+}|^2 + |\mathrm{P}_{e^-}|^2} \\ \frac{\Delta\mathrm{A}_{\mathrm{LR}}}{\mathrm{A}_{\mathrm{LR}}} &= -\frac{\Delta\mathrm{P}_{\mathrm{eff}}}{|\mathrm{P}_{\mathrm{eff}}|} \end{aligned} \tag{7}$$

with

$$\frac{\Delta\mathrm{P}_{\mathrm{eff}}}{|\mathrm{P}_{\mathrm{eff}}|} = \frac{\sqrt{2}\,\Delta P}{(|\mathrm{P}_{e^+}| + |\mathrm{P}_{e^-}|)\ (1 + |\mathrm{P}_{e^+}||\mathrm{P}_{e^-}|)} \sqrt{(1 - |\mathrm{P}_{e^-}|^2)^2 + (1 - |\mathrm{P}_{e^+}|^2)^2} \tag{8}$$

Equal precision ΔP for the measurement of the two beam polarizations is assumed.

The resulting uncertainties are shown in fig. 2. The error contribution from the polarimeter to the unpolarized cross section is rather small. For a polarimeter precision of 0.05 %, it only becomes relevant for data samples with more than $4 \cdot 10^6$ signal events. For an electron beam polarization of 80 % there is a small improvement in the extraction of the unpolarized cross section due to positron beam polarization.

The error introduced in A_{LR} by the polarization measurement is larger. Without positron beam polarization one is limited by the polarimeter (0.05 % precision) for samples with more

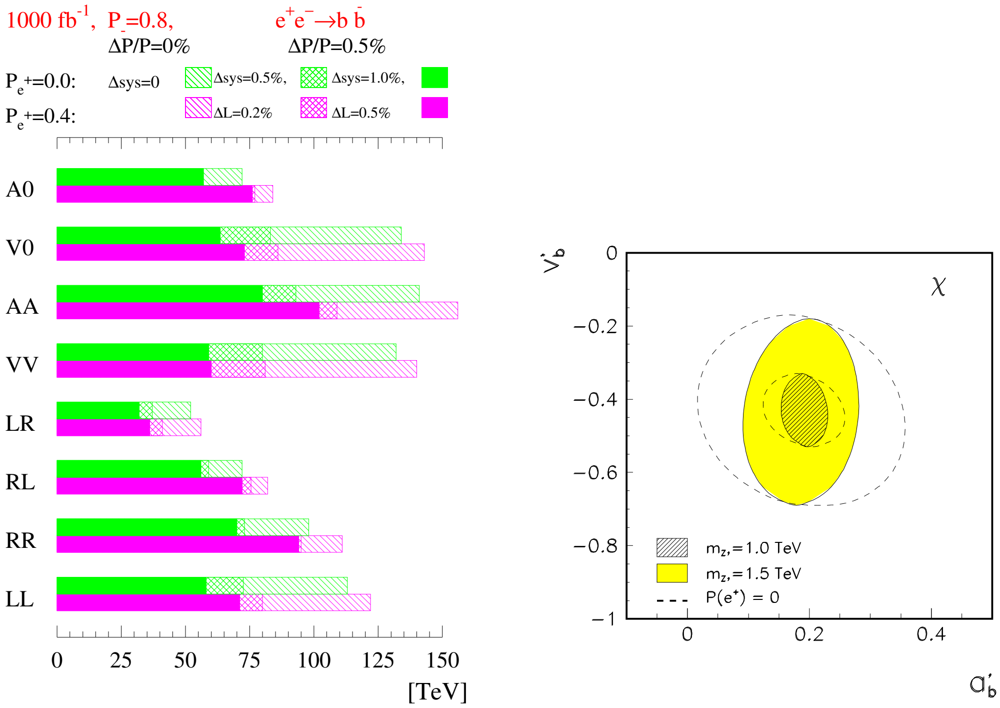

Figure 3: Left: Limits on contact interactions from $e^+e^- \to b\bar{b}$ without positron polarization (upper bars) and with 40 % polarization [1]. Right: Accuracy of the Z' couplings in $e^+e^- \to b\bar{b}$ without positron polarization (dashed line) and with 60 % polarization [2].

than 10^6 events. The improvement due to positron beam polarization is substantial. For a positron polarization of 60 % the error on A_{LR} is reduced by a factor of 3.8.

In fig. 3 we show an example of the improvements that can be achieved with positron polarization. The left figure shows the achievable limits on contact interactions in $e^+e^- \to b\bar{b}$ from the measurements of the cross section, A_{LR}, A_{FB}, and A_{FB}^{pol} at a center-of-mass energy of 800 GeV [1]. The different pairs of bars show contact interactions with different helicity structures. For each pair the upper bar shows the achievable limits without positron polarization and the lower with a positron polarization of 40 %.

Once new physics is detected positron polarization is even more important to understand the nature of the new interaction. The right plot of fig. 3 shows the accuracy with which the couplings of a new vector boson Z' could be measured in $e^+e^- \to b\bar{b}$ (observables as above, center-of-mass energy 500 GeV). The dashed line indicates the achievable precision without positron polarization for two different Z' masses while the solid lines assume 60 % positron polarization.

References

[1] S. Riemann, *Fermion-Pair Production at a Linear Collider - A Sensitive Tool for New Physics Searches*, TESLA note LC-TH-2001-007.

[2] R. Casalbuoni et al., *Z' indication from new APV data in Cesium and searches at linear colliders*, TESLA note LC-TH-2000-006.