Subleading processes in High Energy Jets

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IP3
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MCnet

Table of Contents

An Introduction to HEJ

High Energy Jets

- A Partonic Monte Carlo Generator which aims to describe **high multiplicity events**.
- Provides perturbative predictions at LL accuracy ($\log(\hat{s}/\hat{t})$) with **resummation of hard corrections to all orders**.

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Multi Regge Kinematic (MRK) Limit

The MRK Limit:

large $\hat{s}$; small P_T ; **strongly ordered jet rapidities** $(\mathbf{y_j})$:

$$y_1 \ll y_2 \ll ... \ll y_i \ll .. \ll y_{n-1} \ll y_n$$

Some nice relations:

$$\begin{aligned} \hat{s}^2 &\sim -\hat{u}^2 \to large \\ \hat{t}_i &\sim -p^2_{\perp j_i} \sim -p^2_{\perp} \\ \log\left(\frac{\hat{s}_{ij}}{\hat{t}_{ij}}\right) &\approx |y_j - y_i| \end{aligned}$$

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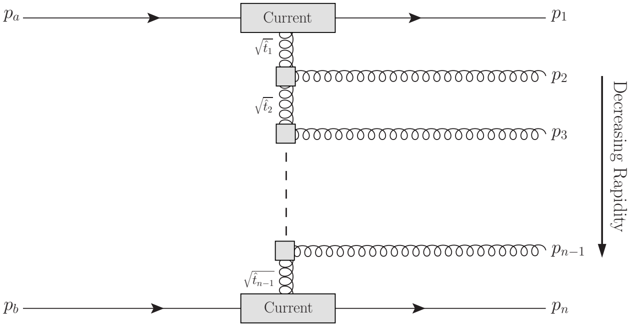
FKL Contributions

FKL configurations are the leading contributions in the MRK limit.

- $(2 \to n)$ amplitudes with strong rapidity ordering in final state
- **Internal jets** (by rapidity) required to be **gluons**



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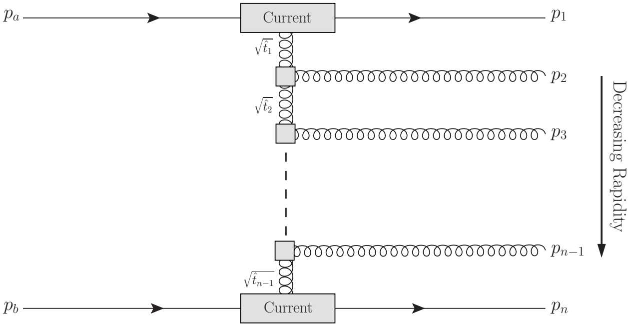

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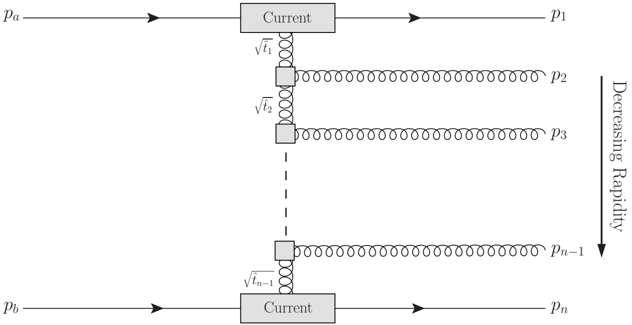

Resum via effective **Lipatov Vertices** and the **Lipatov Ansatz**.

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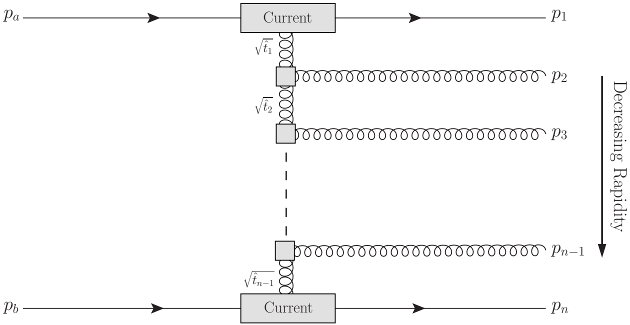
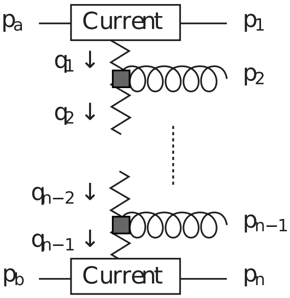


Resum via effective **Lipatov Vertices** and the **Lipatov Ansatz**.

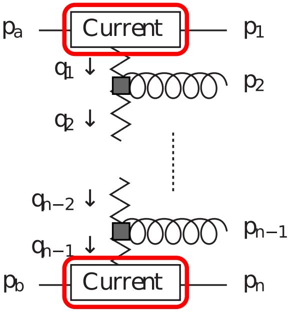
HEJ Matrix element



$$
= \frac{1}{4\,(N_C^2-1)} \left\| S_{f_a f_b \to f_1 f_n} \right\|^2
\cdot \left(g_s^2 \; K_{f_1} \; \frac{1}{t_1} \right) \cdot \left(g_s^2 \; K_{f_n} \; \frac{1}{t_{n-1}} \right)
\cdot \prod_{i=1}^{n-2} \left(\frac{-\, g_s^2 C_A}{t_i t_{i+1}} \; V^{\mu}(q_i, q_{i+1}) V_{\mu}(q_i, q_{i+1}) \right)
\cdot \prod_{j=1}^{n-1} \exp\left[\omega^0(q_{j\perp})(y_{j+1}-y_j) \right]
$$

Processes $\Leftrightarrow$ currents, e.g. $\mathcal{S}_{f_1 f_2 \to f_1 H f_2} = j_\mu(p_1, p_a) V_H^{\mu\nu}(q_j, q_{j+1}) j_\nu(p_b, p_n)$.

arxiv:1706.01002



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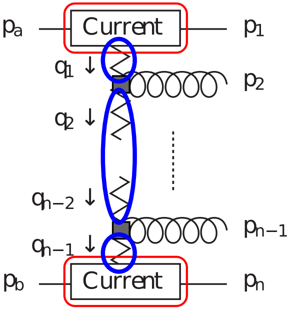

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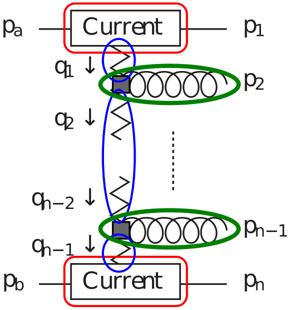

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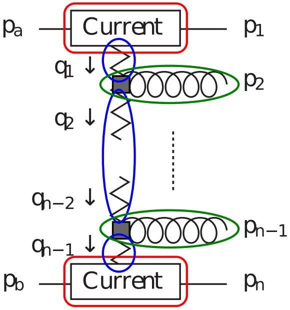

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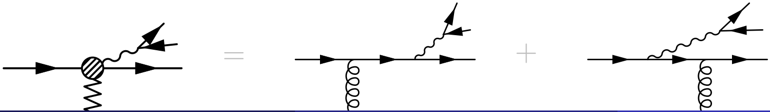

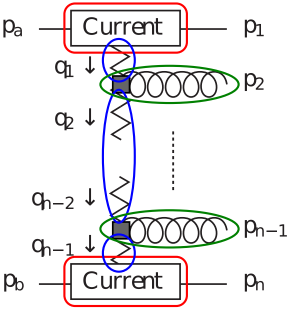

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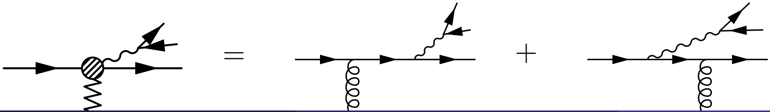
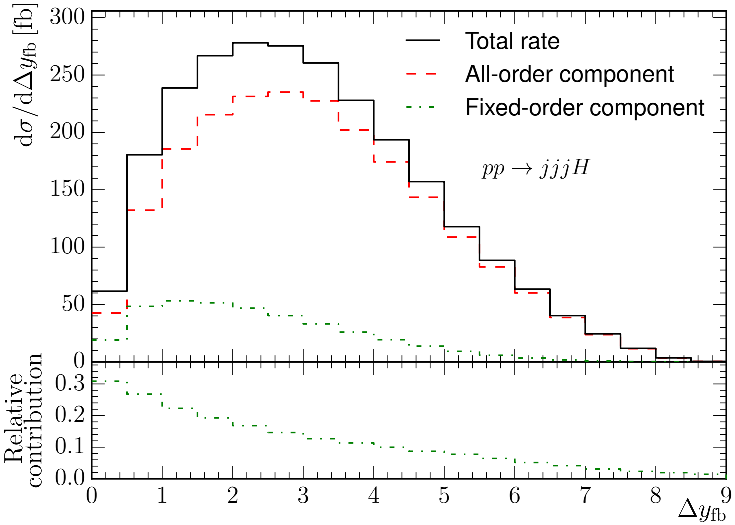
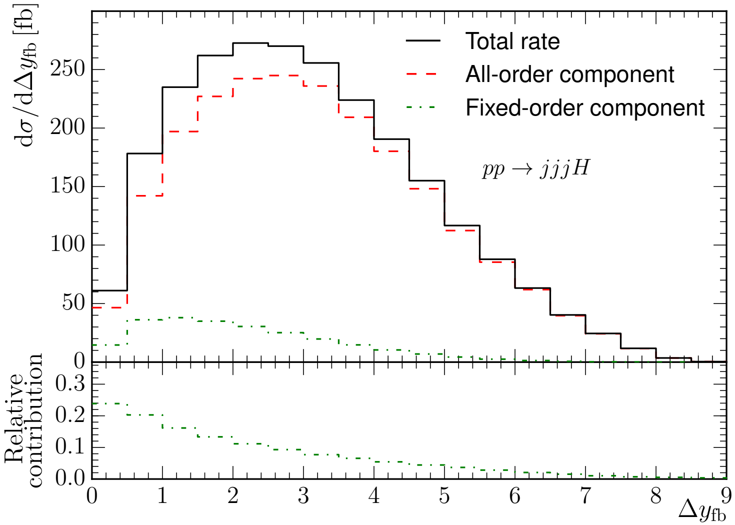


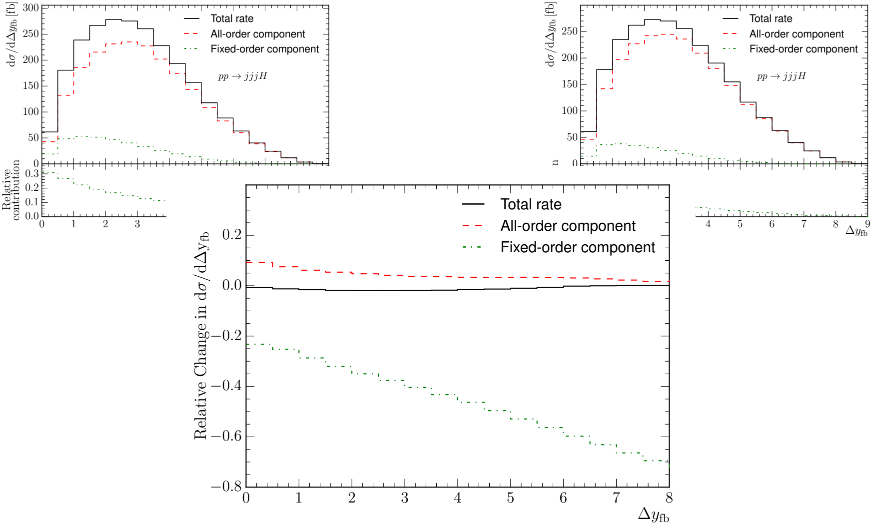
H+Jets FKL Only



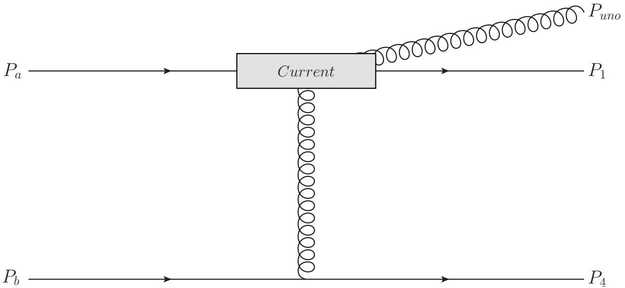
H+Jets Including Unordered



Change due to Unordered



Unordered Contributions

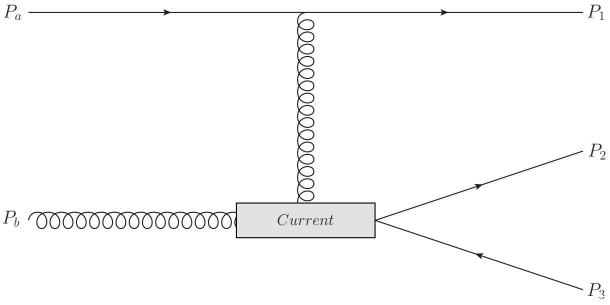


A gluon outside of FKL rapidity ordering is known as an **Unordered emission**.

In HEJ this is modelled as a modified current. Where we now allow that $y_{uno} \sim y_1$ and $y_1 >> y_2$. (QMRK Limit)

$$\mathcal{M}^{\text{uno}}_{qQ \to gqQ} \sim \frac{j^{\mu}_{\text{uno}}(p_a, p_1, p_{uno}) j_{\mu}(p_b, p_2)}{\hat{t}}$$

Extremal $q\bar{q}$



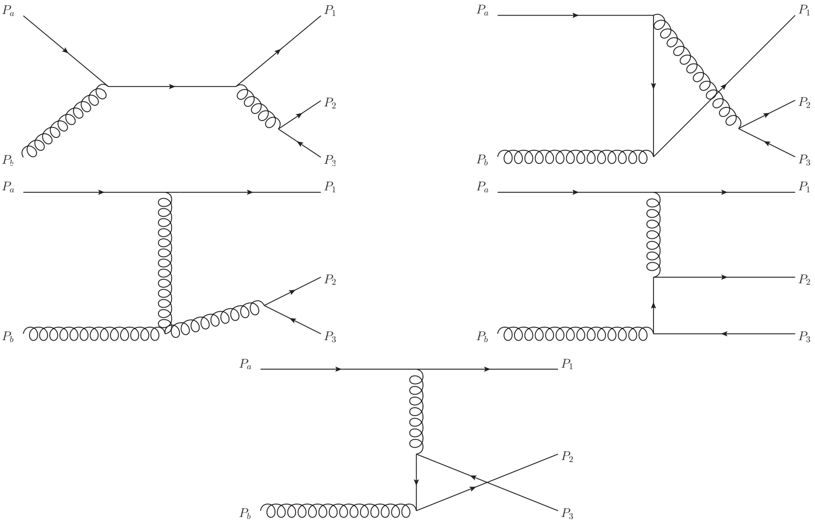
The **Extremal q$\bar{\text{q}}$** case is an incoming gluon splitting to $q\bar{q}$.

In HEJ use a a modified current (related by crossing symmetry to Uno case) in the scattering.

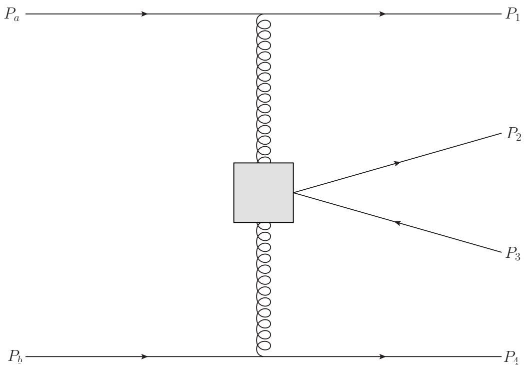
$$\mathcal{M}^{q\bar{q}}_{qg\to qQQ} \sim \frac{j^{\mu}_{q\bar{q}}(p_b, p_2, p_3) j_{\mu}(p_a, p_1)}{\hat{t}}$$

There are **5 possible diagrams** which contribute.

Extremal $q\bar{q}$: Possibilities



Central $q\bar{q}$

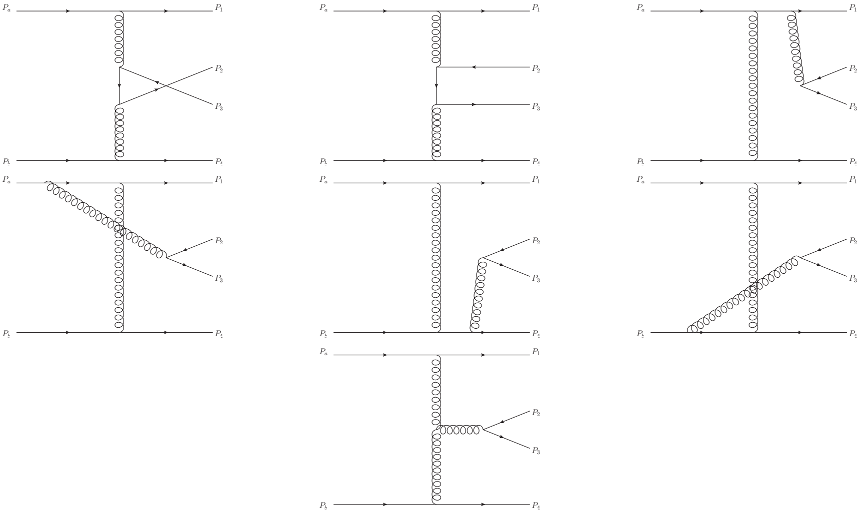


In the case a **Central q$\bar{\text{q}}$** pair is produced, we use an effective vertex which fits the form:

$$\mathcal{M}_{qq\to qQ\overline{Q}q} \sim \frac{\langle 1|\mu|a\rangle\, X^{\mu\nu}\, \langle 4|\nu|b\rangle}{\hat{t}_1\,\hat{t}_3}$$

There are **7 possible diagrams** which contribute.

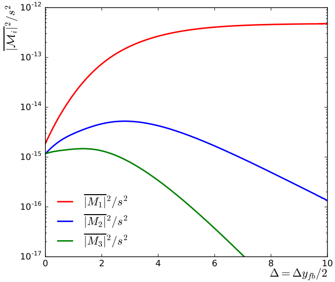
Central $q\bar{q}$

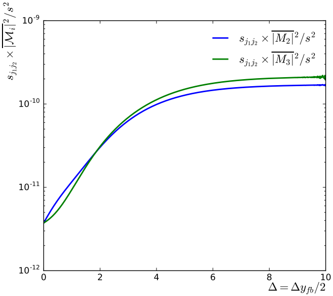


Scaling of the Matrix Elements

Higgs+3j: $qQ \to qgHQ$

Gluon Exchange (FKL) | Quark Exchange (Unordered) | Higgs Outside (Unordered)





In Multi Regge Theory:

$$|\mathcal{M}| \sim (\hat{s}_{j_i j_j})^{spin}$$

Swapping propagator (gluon $\to$ quark) suppresses ME by $(\hat{s}_{j_i j_j})^{1/2}$.

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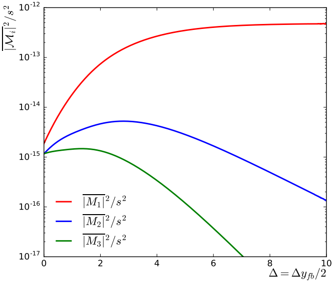

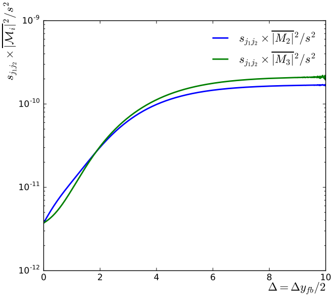

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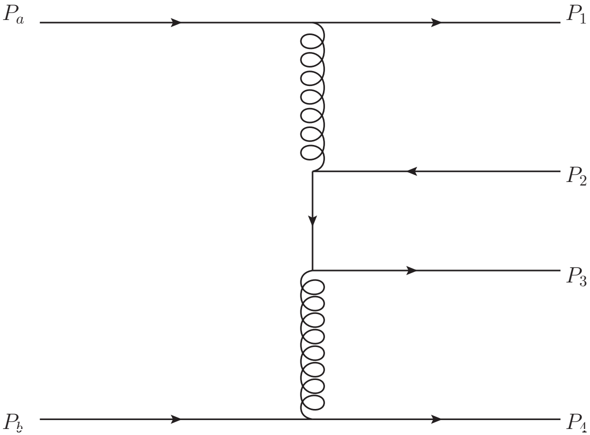
Reducing Dependence on Matching



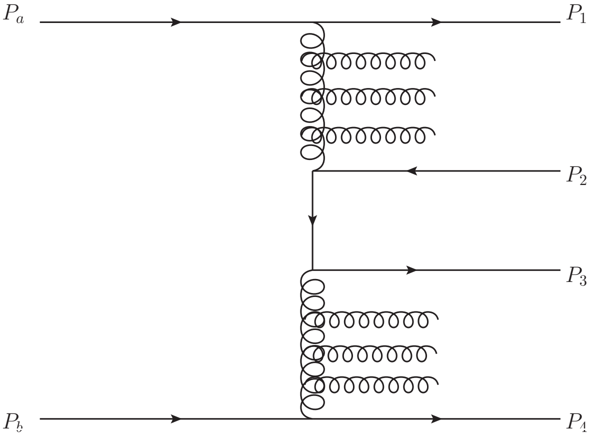
By Matching

α_s^4

Add Resummation

$(\alpha_s \Delta_y)^N$

Reducing Dependence on Matching



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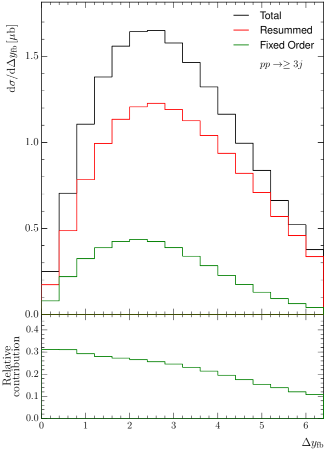
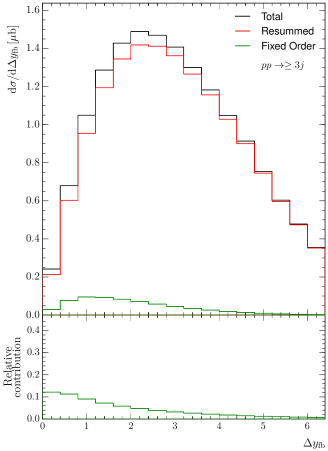
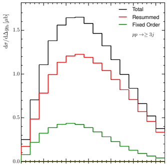
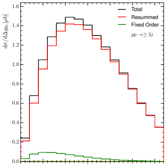
Pure Jets: FKL Only



Pure Jets: All subleading processes




Total
Resummed
Fixed Order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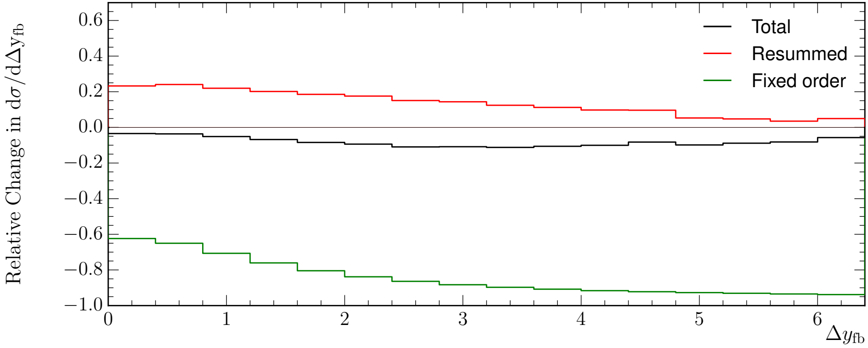
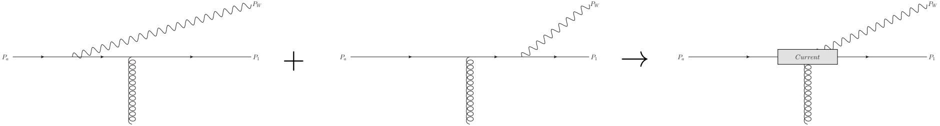

Total
Resummed
Fixed order
Relative Change in $\mathrm{d}\sigma/\mathrm{d}\Delta y_{\mathrm{fb}}$
Δy_{fb}
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
1
2
3
4
5
6

W+Jets at LL

In HEJ, W+Jets are usually calculated differently from Pure Jets by the use of a **modified current**.

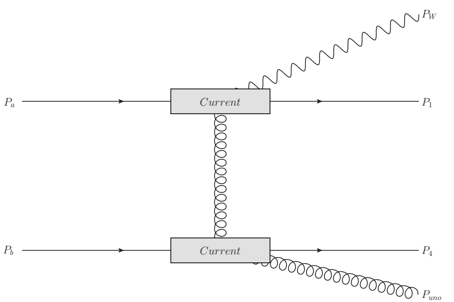


With the addition of the $q\bar{q}$ pairs we have additional places from which a W-Boson can be emitted.

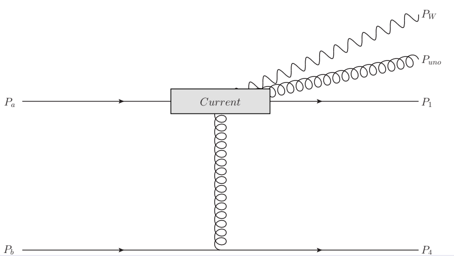
W+Jets at NLL: Unordered

Complications to Unordered

No New Objects



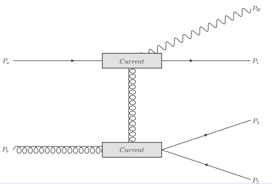
New Objects Required

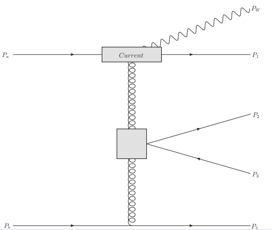


W+Jets at NLL: $q\bar{q}$

Complications to $q\bar{q}$

No New Objects





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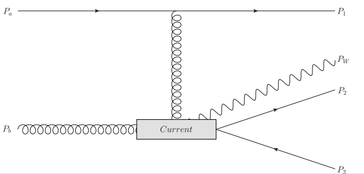
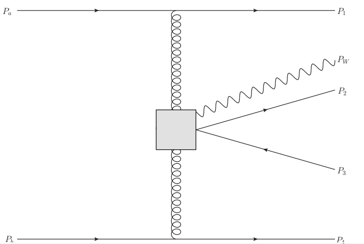




W+Jets at NLL: Extremal $q\bar{q}$

Consider Process: $qg \to qQ\overline{Q}W$

AIM:

Factorise the t channel exchanges and the current scattering, resulting in a new effective current at either end of the FKL chain.

Need to find an amplitude for the process $qg \to qQ\overline{Q}W$ of the form:

$$M_{qg\to qQ\overline{Q}W} \sim \frac{\langle 1|\mu|a\rangle \, Q^{\mu\nu\rho}\,(p_2, p_w, p_3, p_b)\,\varepsilon_\nu(p_b)\varepsilon^*_\rho(p_w)}{\hat{t}_1} \qquad (1)$$

Where $Q^{\mu\nu\rho}$ is this effective current.

W+Jets at NLL: Central $q\bar{q}$

Consider Process: $qq \to qQ\overline{Q}Wq$

AIM:

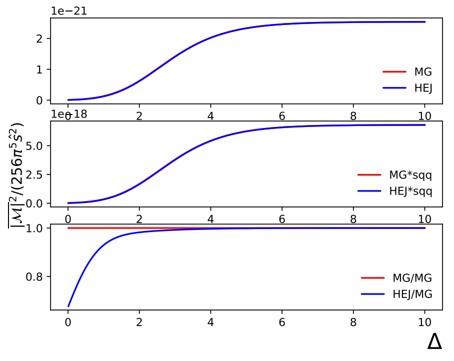
Factorise Currents and effective $q\bar{q}$ vertex. As in the extremal $q\bar{q}$

We therefore search for an expression of the form:

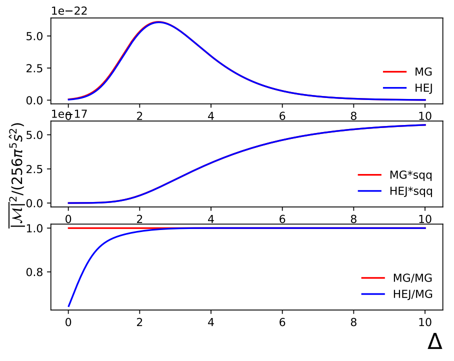
$$M_{qq\to qQ\overline{Q}qW} \sim \frac{\langle 1|\mu|a\rangle X^{\mu\nu} \langle 4|\nu|b\rangle}{\hat{t}_1 \hat{t}_3} \quad (2)$$

W+Jets Status Matrix Element Comparison

$q\bar{q}$ at fixed Δ_y



$q\bar{q}$ at increasing Δ_y

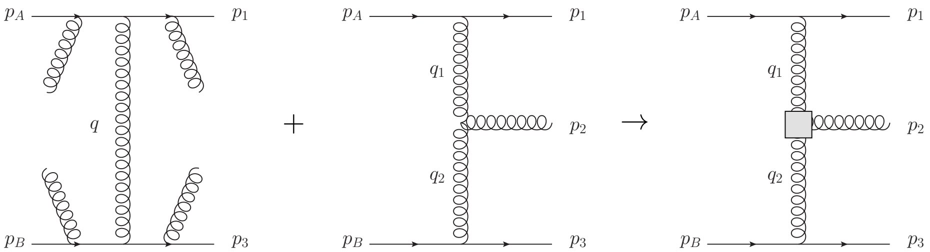


Conclusions and Further Considerations

- Added resummation for Unordered, Extremal and Central $q\bar{q}$
- HEJ is now leading log accurate for all sub-leading processes
- Verification process underway for W+Jets
- Next steps for Next-to-Leading Log:
  - Virtual Corrections
  - Remove need for the contributions to be hard enough to form jets

Backup slides

Lipatov Vertices



$$
\begin{aligned}
V^\rho(q_1, q_2) = &- (q_1 + q_2)^\rho \\
&+ \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2 \cdot p_A} + \frac{p_2 \cdot p_B}{p_A \cdot p_B} + \frac{p_2 \cdot p_3}{p_A \cdot p_3}\right) + p_A \leftrightarrow p_1 \\
&- \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2 \cdot p_B} + \frac{p_2 \cdot p_A}{p_B \cdot p_A} + \frac{p_2 \cdot p_1}{p_B \cdot p_1}\right) - p_B \leftrightarrow p_3.
\end{aligned}
$$

Virtual Corrections

Lipatov Ansatz

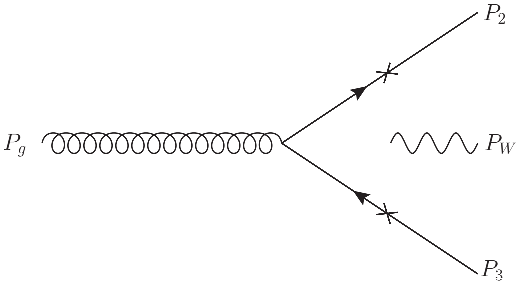
One can obtain the virtual corrections in the MRK limit with the Lipatov Ansatz, which is the following substitution within the analytic expression for the amplitudes:

$$\frac{1}{t_i} \to \frac{1}{t_i} \exp[\hat{\alpha}(q_i)(y_{i-1} - y_i)]$$

where

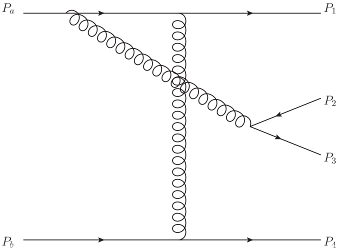
$$\hat{\alpha} = -g^2 C_A \frac{\Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\frac{q^2}{\mu^2}\right)^{\varepsilon}$$

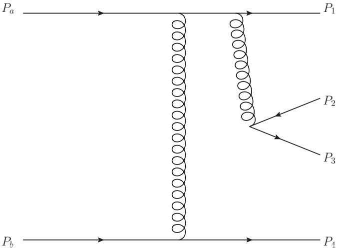
Building Blocks



$$J_V^\mu(p_2, p_A, p_B, p_3) = \left(\frac{\bar{u}_2 \gamma^\nu (\not{p}_2 + \not{p}_A + \not{p}_B) \gamma^\mu u_3}{s_{2AB}} + \frac{\bar{u}_2 \gamma^\mu (\not{p}_3 - \not{p}_A - \not{p}_B) \gamma^\nu u_3}{s_{3AB}} \right) [\bar{u}_A \gamma_\nu u_B]$$

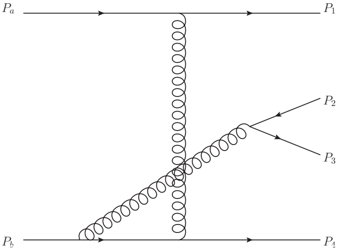
1a Contribution

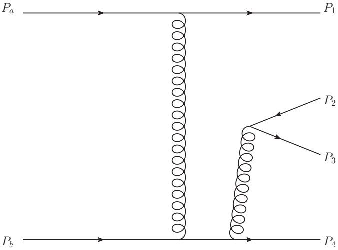




$$X_{1a}^{\mu\nu} = \frac{g^{\mu\nu} C_1 g_w g_s^4}{2\sqrt{2} s_{23AB} (s_{123AB})} (p_1^\rho) J_{V\rho}$$

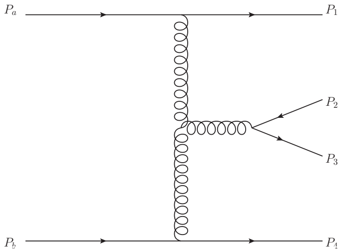
4b Contribution





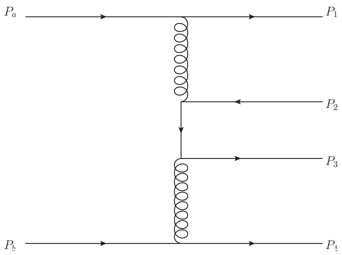
$$X^{\mu\nu}_{4b} = \frac{-g^{\mu\nu} C_1 g_w g_s^4}{2\sqrt{2} s_{23AB}(s_{234AB})} (p_4^{\rho}) J_{V\rho}$$

3 Gluon Contribution



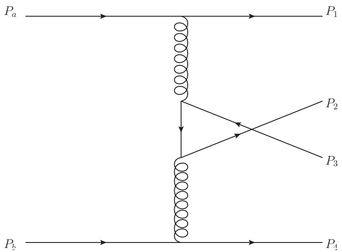
$$X_{3g}^{\mu\nu} = \frac{g_w g_s^2 \, T^{geg'} \, T_{23}^{e}}{2\sqrt{2}\, \hat{t}_1 s_{23AB} \, \hat{t}_3} \Big[(q_1)^\mu \, \eta^{\nu\rho} + (q_3)^\nu \, \eta^{\mu\rho} - (q_1 + q_3)^\rho \, \eta^{\mu\nu} \Big] J_{V\rho}(p_a, p_A, p_B, p_1)$$

Uncrossed Contributions



$$X^{\mu\nu}_{\text{uncross}} = \frac{\langle A|\sigma|B\rangle}{(p_A + p_B)^2}\bar{u}_2 \left[\frac{\gamma^\sigma(\not{p}_2 + \not{p}_A + \not{p}_B)\gamma^\nu(\not{p}_3 + \not{p}_4 - \not{p}_b)\gamma^\mu}{(s_{2AB})(t_{int_2})} + \frac{\gamma^\nu(\not{p}_a - \not{p}_1 - \not{p}_2)\gamma^\sigma(\not{p}_3 + \not{p}_4 - \not{p}_b)\gamma^\mu}{(t_{int_1})(t_{int_2})} + \frac{\gamma^\nu(\not{p}_a - \not{p}_1 - \not{p}_2)\gamma^\mu(\not{p}_3 + \not{p}_A + \not{p}_B)\gamma^\sigma}{(t_{int_1})(s_{3AB})} \right] u_3$$

Crossed Contributions



$$X^{\mu\nu}_{cross} = \frac{\langle A|\sigma|B\rangle}{(p_A + p_B)^2}\bar{u}_2\left[\frac{\gamma^\sigma(\not{p}_2 + \not{p}_A + \not{p}_B)\gamma^\nu(\not{p}_a - \not{p}_1 - \not{p}_3)\gamma^\mu}{(s_{2AB})(t_{int_3})} + \frac{\gamma^\nu(\not{p}_2 - \not{p}_4 - \not{p}_b)\gamma^\sigma(\not{p}_a - \not{p}_1 - \not{p}_3)\gamma^\mu}{(t_{int_3})(t_{int_4})} + \frac{\gamma^\nu(\not{p}_2 + \not{p}_4 - \not{p}_b)\gamma^\mu(\not{p}_3 + \not{p}_A + \not{p}_B)\gamma^\sigma}{(t_{int_4})(s_{3AB})}\right]u_3$$

Matching with Fixed Order

$$\sigma_{2j}^{\mathrm{resum,match}} = \sum_{f_1,f_2} \sum_m \prod_{j=1}^{m} \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{\mathrm{d}^2\mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{\mathrm{d}y_j^B}{2} \right) (2\pi)^4 \, \delta^{(2)}\!\left(\sum_{k=1}^{m} \mathbf{p}_{k\perp}^B \right)$$
$$\cdot \; x_a^B \, f_a(x_a^B, Q_a^B) \; x_b^B \, f_b(x_b^B, Q_b^B) \, \frac{\overline{|\mathcal{M}^B|}^2}{(\hat{s}^B)^2}$$
$$\cdot \; \overline{|\mathcal{M}_{\mathrm{HEJ}}^{tree}|}^{-2} \, (2\pi)^{-4+3m} \, 2^m \, \frac{(\hat{s}^B)^2}{x_a^B \, f_{a,f_1}(x_a^B, Q_a^B) \; x_b^B \, f_{b,f_2}(x_b^B, Q_b^B)}$$
$$\cdot \; \sum_{n=2}^{\infty} \int_{p_{1\perp}=.9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=.9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \, (2\pi)^4 \, \delta^{(2)}\!\left(\sum_{k=1}^{n} \mathbf{p}_{k\perp} \right)$$
$$\cdot \; \mathbf{T}_y \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}y_i}{2} \right) \mathcal{O}_{mj}^e \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left(\prod_{l=1}^{m} \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\})$$
$$\cdot \; x_a f_{a,f_1}(x_a, Q_a) \; x_b f_{b,f_2}(x_b, Q_b) \, \frac{\overline{|\mathcal{M}_{\mathrm{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2} .$$

arxiv:1805.04446

Matching with Fixed Order

$$\sigma_{2j}^{\text{resum,match}} = \sum_{f_1,f_2} \sum_m \prod_{j=1}^m \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{\mathrm{d}^2\mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{\mathrm{d}y_j^B}{2} \right) (2\pi)^4 \, \delta^{(2)}\left(\sum_{k=1}^m \mathbf{p}_{k\perp}^B\right)$$

$$\cdot\; x_a^B \; f_a(x_a^B, Q_a^B) \; x_b^B \; f_b(x_b^B, Q_b^B) \; \frac{\overline{|\mathcal{M}^B|}^2}{(\hat{s}^B)^2}$$

Fixed Order

$$\cdot\; \overline{|\mathcal{M}_{\text{HEJ}}^{tree}|}^{-2} \; (2\pi)^{-4+3m} \; 2^m \; \frac{(\hat{s}^B)^2}{x_a^B \; f_{a,f_1}(x_a^B, Q_a^B) \; x_b^B \; f_{b,f_2}(x_b^B, Q_b^B)}$$

Overlap

HEJ

$$\cdot\; \sum_{n=2}^{\infty} \int_{p_{1\perp}=.9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=.9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \; (2\pi)^4 \; \delta^{(2)}\left(\sum_{k=1}^n \mathbf{p}_{k\perp}\right)$$

$$\cdot\; \mathbf{T}_y \prod_{i=1}^n \left(\int \frac{\mathrm{d}y_i}{2} \right) \mathcal{O}_{mj}^e \; \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \; \left(\prod_{l=1}^m \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \; \mathcal{O}_{2j}(\{p_i\})$$

$$\cdot\; x_a f_{a,f_1}(x_a, Q_a) \; x_b f_{b,f_2}(x_b, Q_b) \; \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2} .$$

arxiv:1805.04446