p-brane Solutions and Dynamics of a Brane Probe

Based on the paper "*Brane Solutions in Supergravity*" by K.S. Stelle

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Outline

- Start with a general simple action and derive equations of motion
- Derive the *p*-brane equations
- Study the specific case of D=11
- Study the dynamics of a light brane moving in a heavy membrane background

Single-Charge Action in D dimensions

$$I = \int d^D x \sqrt{-g} \left[R - \frac{1}{2} \nabla_M \phi \nabla^M \phi - \frac{1}{2n!} e^{a\phi} F_{[n]}^2 \right] \tag{1}$$

Contains:

- Curvature Component R (Ricci Scalar)
- Scalar Field ϕ
- A Single Field Strength $F_{[n]}$

  Where the antisymmetric tensor field $F_{[n]} = dA_{[n-1]}$

Equations of Motion

Varying the action with respect to the different variables we obtain:

$$\delta I = 0$$

$$R_{MN} = \frac{1}{2}\,\partial_M\phi\partial_N\phi + S_{MN} \tag{2a}$$

$$S_{MN} = \frac{1}{2(n-1)!}e^{a\phi}\left(F_{M\ldots}F_N^{\cdots} - \frac{n-1}{n(D-2)}F^2 g_{MN}\right) \tag{2b}$$

$$\nabla_{M_1}(e^{a\phi}F^{M_1\ldots M_N}) = 0 \tag{2c}$$

$$\Box\phi = \frac{a}{2n!}e^{a\phi}F^2 \tag{2d}$$

Ansätze

In order to find explicit solutions, we need to make some simplifying assumptions:

- $(\text{Poincaré})_d \times SO(D-d)$ symmetry of space-time
- Ansätze related to the field strength $\Rightarrow$ Electric or magnetic cases
- Linearity conditions

Spatial Ansatz

A metric with $(\text{Poincaré})_d \times SO(D-d)$ symmetry:

$$
\begin{aligned}
&ds^2 = e^{2A(r)} dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B(r)} dy^m dy^n \delta_{mn} \\
&\mu = 0, 1, ..., p = d-1 \qquad\qquad m = p+1, ..., D-1
\end{aligned} \tag{3}
$$

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The scalar field becomes:

$\phi = \phi(r)$ with $r = \sqrt{y^m y_m}$

This single r dependence will be common for all the functions (e.g. A(r) and B(r))

Field Strength Ansätze

One can consider two possibilities:

- **Electric/Elementary case:** $A_{[n-1]}$ couples to the worldvolume of a $p = d_{el} - 1 = (n-1) - 1$ dimensional "charged" extended object

$$A_{\mu_1 \dots \mu_{n-1}} = \epsilon_{\mu_1 \dots \mu_{n-1}} e^{C(r)}, \quad \text{others zero.} \tag{4a}$$

$$F^{(el)}_{m\mu_1 \dots \mu_{n-1}} = \epsilon_{\mu_1 \dots \mu_{n-1}} \partial_m e^{C(r)}, \quad \text{others zero.} \tag{4b}$$

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- **Magnetic/Solitonic case:** Consider ${}^*F^{el}$ which is a $(D-n)$ form and will couple to a $d_{so} = D - n - 1$ dimensional worldvolume:

$$F^{(mag)}_{m_1 \dots m_n} = \lambda \epsilon_{m_1 \dots m_n p} \frac{y^p}{r^{n+1}}, \quad \text{others zero.} \tag{5}$$

Curvature Components

Use of vielbeins which leave the metric as:

$$g_{MN} = e_M{}^{\underline{N}} e_N{}^{\underline{F}} \eta_{\underline{NF}} \tag{6}$$

Where the worldvolume/transverse 1-forms:

$$e^{\underline{\mu}} = e^{A(r)} dx^{\mu}, \qquad e^{\underline{m}} = e^{B(r)} dy^{m} \tag{7}$$

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Free-torsion condition("tetrad postulate"): $de^{\underline{E}} + w^{\underline{E}}{}_{\underline{F}} \wedge e^{\underline{F}} = 0$
Can calculate the 2-form curvature components by:

$$R_{[2]}^{\underline{EF}} = dw^{\underline{EF}} + w^{\underline{ED}} \wedge w_{\underline{D}}{}^{\underline{F}} \tag{8}$$

Explicit Curvature Components

Requires careful calculation:

For example, $R^{\underline{mn}} = dw^{\underline{mn}} + w^{\underline{m\alpha}} \wedge w_{\underline{\alpha}}{}^{\underline{n}} + w^{\underline{mp}} \wedge w_{\underline{p}}{}^{\underline{n}}$ (9)

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Leads:

$$\begin{aligned}
R^{\underline{\mu\nu}}{}_{\mu\nu} &= -e^{2(A-B)}\partial_\mu A\cdot\partial^\mu A(\delta^{\underline{\mu}}_{\mu}\delta^{\underline{\nu}}_{\nu} - \delta^{\underline{\mu}}_{\nu}\delta^{\underline{\nu}}_{\mu}) \\
R^{\underline{\mu n}}{}_{p\mu} &= e^{(A-B)}\delta^{\underline{\mu}}_{\mu}\Big(\partial_p(A-B)\partial^n A + \partial_p\partial^n A - \partial_p A\cdot\partial^n B + \partial_p A\cdot\partial^p B\delta_{\underline{np}}\Big) \\
R^{\underline{mn}}{}_{ps} &= \partial_p\partial^n B\delta^{\underline{m}}_{s}\delta^{\underline{n}}_{n} - \partial_s\partial^n B\delta^{\underline{m}}_{p}\delta^{\underline{n}}_{n} - \partial_p\partial^m B\delta^{\underline{n}}_{s}\delta^{\underline{m}}_{m} + \partial_s\partial^m B\delta^{\underline{m}}_{m}\delta^{\underline{n}}_{p} \qquad (10)\\
&\quad + \partial_s B\partial^n B\delta^{\underline{n}}_{n}\delta^{\underline{m}}_{p} - \partial_p B\partial^n B\delta^{\underline{n}}_{n}\delta^{\underline{m}}_{s} + \partial_m B\cdot\partial^m B(\delta^{\underline{m}}_{s}\delta^{\underline{n}}_{p} - \delta^{\underline{m}}_{p}\delta^{\underline{n}}_{s}) \\
&\quad + \partial_p B\partial^m B\delta^{\underline{n}}_{s}\delta^{\underline{m}}_{m} - \partial_s B\partial^m B\delta^{\underline{n}}_{p}\delta^{\underline{m}}_{m}
\end{aligned}$$

Ricci Tensor

Contract $\Rightarrow$ Ricci Tensor:

$$R_{\mu\nu} = R^{\underline{n}\underline{\nu}}{}_{n\mu} e^{n}_{\underline{n}} e_{\nu\underline{\nu}} + R^{\underline{\beta}\underline{\nu}}{}_{\beta\mu} e^{\beta}_{\underline{\beta}} e_{\nu\underline{\nu}} \qquad (11a)$$

$$R_{mn} = R^{\underline{\mu}\underline{n}}{}_{\mu m} e^{\mu}_{\underline{\mu}} e_{n\underline{n}} + R^{\underline{p}\underline{n}}{}_{pm} e^{p}_{\underline{p}} e_{n\underline{n}} \qquad (11b)$$

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Tracing:

$$\begin{aligned}
R_{\mu\nu} &= -\eta_{\mu\nu} e^{2(A-B)}\left(A'' + d(A')^2 + \tilde{d}A'B' + \frac{(\tilde{d}+1)}{r}A'\right) \\
R_{mn} &= -\delta_{mn}\left(B'' + dA'B' + \tilde{d}(B')^2 + \frac{(2\tilde{d}+1)}{r}B' + \frac{d}{r}A'\right) \\
&\quad - \frac{y^m y^n}{r^2}\left(\tilde{d}B'' + dA'' - 2dA'B' + d(A')^2 - \tilde{d}(B')^2 - \frac{\tilde{d}}{r}B' - \frac{d}{r}A'\right)
\end{aligned} \qquad (12)$$

$$\tilde{d} = D - d - 2$$

Equations of Motion (Again)

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$$\nabla_{M_1}(e^{a\phi}F^{M_1\ldots M_N}) = 0 \tag{2c}$$

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Equations of Motion (Again!!)

We have expressions for F and R, substitute back into equations of motion:

$$
\begin{aligned}
&A'' + d(A')^2 + \tilde{d}A'B' + \frac{(\tilde{d}+1)}{r}A' &&= \frac{\tilde{d}}{2(D-2)}S^2 && \{\mu\nu\} \\
&B'' + dA'B' + \tilde{d}(B')^2 + \frac{(2\tilde{d}+1)}{r}B' + \frac{d}{r}A' &&= -\frac{d}{2(D-2)}S^2 && \{\delta_{mn}\} \\
&\tilde{d}B'' + dA'' - 2dA'B' + d(A')^2 - \tilde{d}(B')^2 && && \\
&\qquad - \frac{\tilde{d}}{r}B' - \frac{d}{r}A' + \frac{1}{2}(\phi')^2 &&= \frac{1}{2}S^2 && \{y_m y_n\} \\
&\phi'' + dA'\phi' + \tilde{d}B'\phi' + \frac{(\tilde{d}+1)}{r}\phi' &&= -\frac{1}{2}\varsigma a S^2 && \{\phi\}
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\tag{13}
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Equations of Motion (Again!!)

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$$A'' + d(A')^2 + \tilde{d}A'B' + \frac{(\tilde{d}+1)}{r}A' = \frac{\tilde{d}}{2(D-2)}S^2 \qquad \{\mu\nu\}$$

$$B'' + dA'B' + \tilde{d}(B')^2 + \frac{(2\tilde{d}+1)}{r}B' + \frac{d}{r}A' = -\frac{d}{2(D-2)}S^2 \qquad \{\delta_{mn}\}$$

$$\tilde{d}B'' + dA'' - 2dA'B' + d(A')^2 - \tilde{d}(B')^2 - \frac{\tilde{d}}{r}B' - \frac{d}{r}A' + \frac{1}{2}(\phi')^2 = \frac{1}{2}S^2 \qquad \{y_m y_n\}$$

$$\phi'' + dA'\phi' + \tilde{d}B'\phi' + \frac{(\tilde{d}+1)}{r}\phi' = -\frac{1}{2}\varsigma a S^2 \qquad \{\phi\}$$

$$\varsigma = \pm 1 \tag{13}$$

$$S = \begin{cases} (e^{\frac{1}{2}a\phi - dA + C})C' & \text{electric: } d_{el} = n-1 \\ \lambda(e^{\frac{1}{2}a\phi - \tilde{d}B})r^{-\tilde{d}-1} & \text{magnetic: } d_{so} = D-n-1 \end{cases} \tag{14}$$

Simplification

Noticing Laplace operator for $\phi(r)$:

$$\nabla^2\phi = \phi'' + (\tilde{d}+1)r^{-1}\phi' \tag{15}$$

Using linearity conditions:

- $dA' + \tilde{d}B' = 0 \qquad (16a)$

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Equations of motion are reduced to:

$\nabla^2\phi = -\frac{1}{2}\varsigma a S^2 \qquad \{\phi\}$

$\nabla^2 A = \frac{\tilde{d}}{2(D-2)}S^2 \quad \{\mu\nu\}$

$d(D-2)(A')^2 + \frac{1}{2}\tilde{d}(\phi')^2 = \frac{1}{2}\tilde{d}S^2 \quad \{y_m y_n\}$

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Using linearity conditions:

- $dA' + \tilde{d}B' = 0 \qquad (16a)$
- $\phi' = \frac{-\varsigma a(D-2)}{\tilde{d}}A' \quad (16b)$

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Introducing notation $\Delta = a^2 + \frac{2d\tilde{d}}{(D-2)}$ $\qquad \Rightarrow S^2 = \frac{\Delta(\phi')^2}{a^2} \qquad \{y_m y_n\}$

Brane Equations

We are left to solve $\nabla^2\phi + \frac{\varsigma\Delta}{2a}(\phi')^2 = 0 \equiv \nabla^2 e^{\frac{\varsigma\Delta}{2a}\phi} = 0$ (17)

$$e^{\frac{\varsigma\Delta}{2a}\phi} \equiv H(y) = 1 + \frac{k}{r^{\tilde{d}}} \qquad k > 0 \qquad \phi_{|r\to\infty} = 0 \tag{18}$$

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Using H(y) and setting the integration constant $A_\infty = B_\infty = 0$

$$\boxed{\begin{aligned} ds^2 &= H^{\frac{-4\tilde{d}}{\Delta(D-2)}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{\frac{4d}{\Delta(D-2)}} dy^m dy^m \\ H(y) &= 1 + \frac{k}{r^{\tilde{d}}} \\ \varsigma &= \pm 1 \end{aligned}} \tag{19}$$

p-brane solutions

Electric case: $e^C = \frac{2}{\sqrt{\Delta}} H^{-1}$ Magnetic case: $k = \frac{\sqrt{\Delta}}{2\tilde{d}}\lambda$

Solutions for D=11

The action in D=11:

$$I_{11} = \int d^{11}x \Big\{ \sqrt{-g}(R - \frac{1}{48}F_{[4]}^2) - \frac{1}{6}F_{[4]} \wedge F_{[4]} \wedge A_{[3]} \Big\} \tag{20}$$

No scalar field $\phi \Rightarrow a = 0$

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No scalar field $\phi \Rightarrow a = 0$

For $F_{[4]}$ two cases:

- Electric: $d_{el} = n - 1 = 3 \Rightarrow p = d_{el} - 1 = 2$-brane
- Magnetic: $d_{so} = \tilde{d}_{el} = 11 - 3 - 2 = 6 \Rightarrow p = d_{so} - 1 = 5$-brane

$\Delta = 2d\tilde{d}/(D-2) = (2 \cdot 3 \cdot 6)/9 = 4$

D=11: Electric Case

$$ds^2 = (1+\frac{k}{r^6})^{-2/3} dx^\mu dx^\nu \eta_{\mu\nu} + (1+\frac{k}{r^6})^{1/3} dy^m dy^m$$

$$A_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda}(1+\frac{k}{r^6})^{-1}, \qquad \text{other components zero} \qquad (21)$$

electric 2-brane: isotropic coordinates

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electric 2-brane: isotropic coordinates

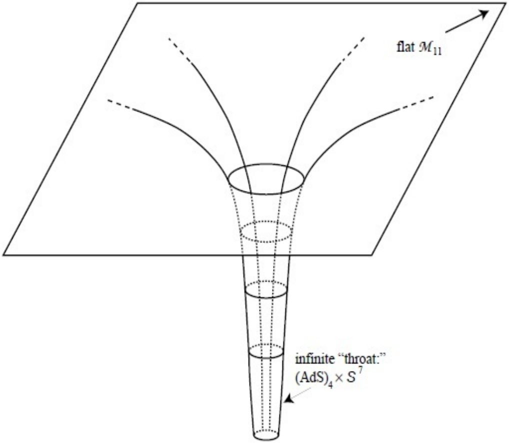
If $r = \frac{k^{1/6} R^{1/2}}{(1-R^3)^{1/6}}$

$$\begin{aligned} ds^2 =& \{R^2(-dt^2 + d\sigma^2 + d\rho^2) + \frac{1}{4} k^{1/3} R^{-2} dR^2\} + k^{1/3} d\Omega_7^2 \\ &+ \frac{1}{4} k^{1/3} [(1 - R^3)^{-7/3} - 1] R^{-2} dR^2 + k^{1/3} [(1 - R^3)^{-1/3} - 1] d\Omega_7^2 \\ A_{\mu\nu\lambda} =& R^3 \epsilon_{\mu\nu\lambda}, \qquad \text{other components zero} \end{aligned} \tag{22}$$

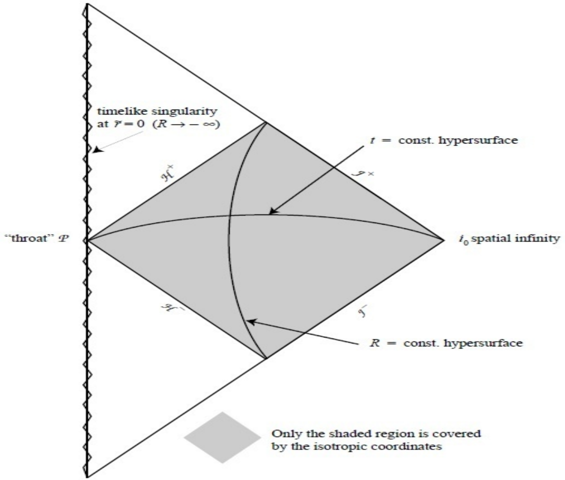
electric 2-brane: interpolating coordinates

$R \to 1 \Rightarrow$ Flat Space $\qquad$ $R \to 0 \Rightarrow$ Metric of $(AdS)_4 \times \mathcal{S}^7$

Interpolation Representation



Diagrams I


timelike singularity
at $\bar{r} = 0$ $(R \to -\infty)$
t = const. hypersurface
"throat" $\mathcal{P}$
i_0 spatial infinity
R = const. hypersurface
Only the shaded region is covered
by the isotropic coordinates

D=11: Magnetic Case

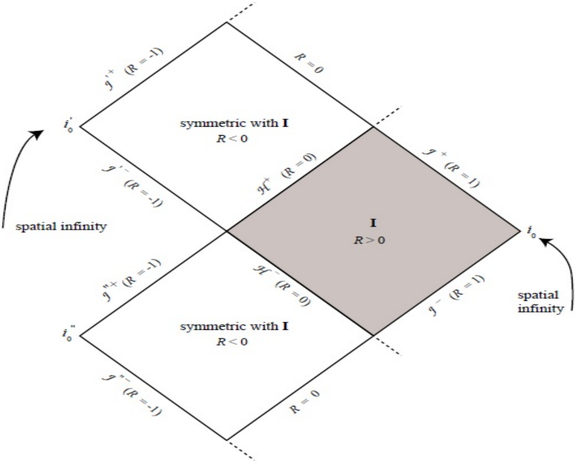
$$ds^2 = R^2 dx^\mu dx^\nu \eta_{\mu\nu} + k^{2/3}\left[\frac{4R^{-2}}{(1-R^6)^{8/3}}dR^2 + \frac{d\Omega_4^2}{(1-R^6)^{2/3}}\right] \qquad (23)$$

magnetic 5-brane: interpolating coordinates

$R \to 1 \Rightarrow$ Flat Space

$R \to 0 \Rightarrow$ Metric of $(AdS)_7 \times \mathcal{S}^4$

Diagrams II



$\mathscr{I}'^{+}$ $(R=-1)$
$R=0$
i_0'
symmetric with **I**
$R<0$
$\mathscr{I}'^{-}$ $(R=-1)$
$\mathscr{H}^{+}$ $(R=0)$
$\mathscr{I}^{+}$ $(R=1)$
spatial infinity
I
$R>0$
i_0
$\mathscr{I}''^{+}$ $(R=-1)$
$\mathscr{H}^{-}$ $(R=0)$
$\mathscr{I}^{-}$ $(R=1)$
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Probe brane

We will consider the dynamics of a light brane on a heavy brane background:

$$I_{total} = I_{background} + I_{probe} \tag{24}$$

Probe brane

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$$I_{total} = I_{background} + I_{probe} \tag{24}$$

$$I_{probe} = -T_\alpha \int d^{(p+1)}\xi \Big[-\det(\partial_\mu x^m \partial_\nu x^n g_{mn}) \Big]^{\frac{1}{2}} e^{-\frac{1}{2}\varsigma^{pr}\vec{a}_\alpha \cdot \vec{\phi}} + Q_\alpha \int \tilde{A}^\alpha_{[p+1]}$$

$$\tilde{A}^\alpha_{[p+1]} = \frac{1}{(p+1)!} \partial_{\mu_1} x^{m_1 \cdots} \partial_{\mu_{p+1}} x^{m_{p+1}} A^\alpha_{m_1 \ldots m_{p+1}} d\xi^{\mu_1} \wedge \cdots \wedge \xi^{\mu_{p+1}} \tag{25}$$

where $T_\alpha \equiv Q_\alpha$ so that the probe brane acts as a source.

Interaction between two membranes

For the case where we have two electric membranes interacting:

$$I_{probe} = -T \int d^3\xi \left[\sqrt{-\det(e^{2A(y)}\eta_{\mu\nu} + e^{2B(y)}\partial_\mu y^m \partial_\nu y^m)} - e^{C(y)} \right] \tag{26}$$

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so that

$$I_{probe} = -T \int d^3\xi \left[e^{3A(y)} \sqrt{1 + e^{2B(y)-2A(y)}\partial_\mu y^m \partial_\nu y^m \eta_{\mu\nu}} - e^{C(y)} \right] \tag{27}$$

First order

Expanding the square root to first order:

$$I^0_{probe} = -T \int d^3\xi \left[e^{3A(y)} - e^{C(y)} \right]$$
$$V_{probe} = -T\left(e^{3A(y)} - e^{C(y)}\right) \tag{28}$$

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From previous discussion:

$$e^{A(y)} = H(y)^{\frac{-2\tilde{d}}{\Delta(D-2)}} \qquad e^{C(y)} = \frac{2}{\sqrt{\Delta}} H^{-1} \tag{29}$$

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Substituting:

$$e^{A(y)} = H(y)^{-\frac{1}{3}} \qquad e^{C(y)} = H^{-1} \tag{30}$$

Therefore $e^{3A(y)} = e^C(y) \rightarrow V = 0 \rightarrow$ No Force condition.

Second order

Expanding to second order:

$$I^{(2)}_{probe} = -\frac{T}{2}\int d^3\xi e^{3A(y)} e^{2B(y)-2A(y)} \partial_\mu y^m \partial_\nu y^m \eta_{\mu\nu} \tag{31}$$

Second order

Expanding to second order:

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The induced metric becomes:

$$\gamma^{mn} = e^{A(y)+2B(y)}\delta^{mn}$$

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The induced metric becomes:

$$\gamma^{mn} = e^{A(y)+2B(y)} \delta^{mn}$$

Using the linearity condition
$dA' + \tilde{d}B' = 0 \rightarrow 3A(y) = -6B(y) \rightarrow A(y) = -2B(y)$

Therefore

$$\gamma^{mn} = \delta^{mn}$$

Flat due to the high degree of surviving supersymmetric parameters (too restrictive).

Conclusion

- Solutions are derived from a general simple action
- Electric and magnetic solutions present a simple form
- For D=11 solutions interpolate between flat space and $(AdS)_n \times \mathcal{S}^{D-n}$ space
- There is a no force interaction between heavy and light electric membranes and the induced metric is flat.

The End

THANKS FOR LISTENING

QUESTIONS?