Simulation of High Energy QCD

Jeppe R. Andersen

IPPP, Durham University

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Overview of Lectures

Define what we mean by “High Energy”

The phrase “High Energy” is used to describe **mutually exclusive situations**:
small-x, large $\hat{s}$ (large x), ...
BFKL

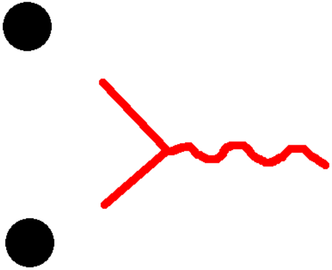
Hard Scattering at Large (Partonic) Energy

The (all-order) behaviour of the **hard scattering matrix element** at large partonic centre-of-mass energies ($\hat{s} \to \infty$, p_t fixed)
Connection to the **BFKL equation**
Benefits and **short-comings** of BFKL

Implementation in “High Energy Jets”

All-order approximations, Merging with **full fixed order**
Theory vs. Data. Hard, higher order effects beyond NLO (no surprise they exists - but they can be important even at Tevatron energies)

"High Energy" can mean slightly different things



Consider first the **production of W-boson** in a hadronic collision. **One-scale** partonic process: m_W.

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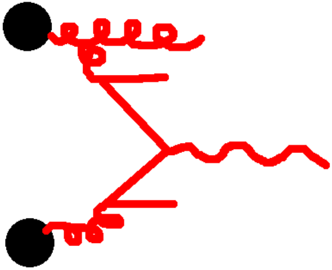

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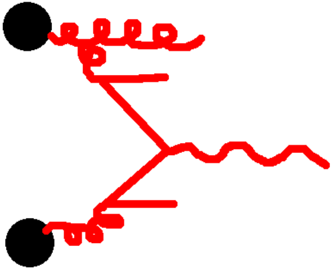

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This is **not** the “High Energy Limit” we will be discussing. Even at 14TeV, Wjj receives only a small perturbative contribution from gg-states.

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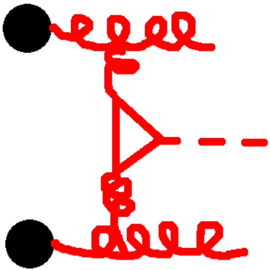

Will instead be discussing the limit of large **partonic** centre of mass energy: $s > \hat{s}(\gg p_t^2)$. Relevant for e.g. *hjj* (where cuts on large m_{jj} is often imposed). But what really is the difference of the two "High Energy Limits?" The diagrams look the same!

For "$\sqrt{s} \gg m_H$": Emissions of gluons considered process-independent. Fundamental process: off-shell gluon fusion

For the limit $\hat{s} \to \infty$, p_t fixed: Standard DGLAP pdfs. Describe the on-shell scattering matrix element at large invariant mass. (hjj dominated by qg-initial states!)

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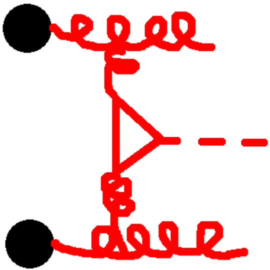

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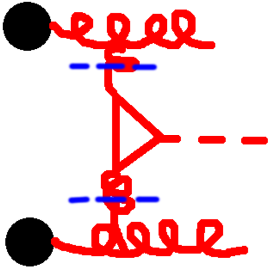

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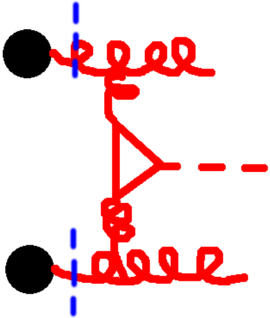

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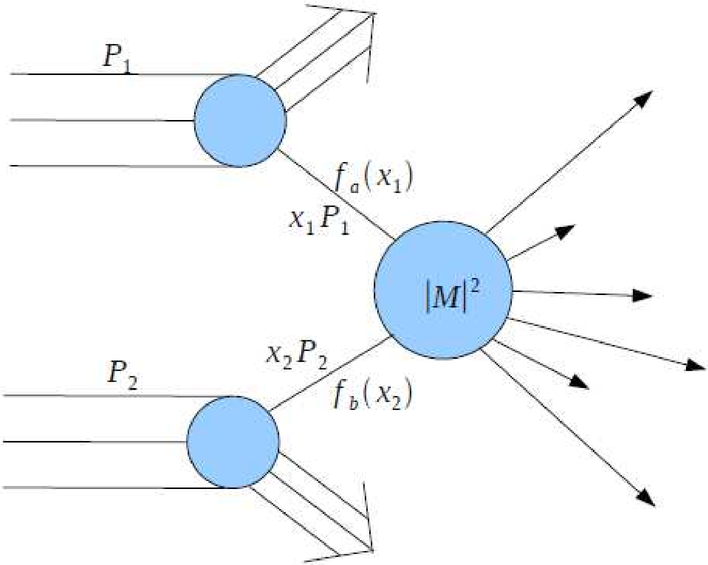
The Perturbative Description



Jets at the LHC

The age old hunt. . .

Effects beyond NLO DGLAP?
. . . apart from the obvious soft and collinear regions (shower profile)
Do we need more than NLO DGLAP to describe the hard jet events at the LHC?

The News

Data from Tevatron and LHC already show effects beyond pure **NLO** DGLAP. . .

1. for some observables based on **hard jets**
2. in certain regions of phase space

Scope of this talk

Will not discuss several interesting effects:

- jet broadening (shower profiles)
- impact of underlying event on the jet energy

These are (well?) described by a tunable shower MC.

Will instead focus on the description of the **hard event**, and in particular on observables not well described by pure **NLO** DGLAP. Specifically **not** discussing a breakdown of DGLAP factorisation - only the fixed (NL-) order description.

*Which regions of phase space receive large corrections from **hard perturbative corrections** (= additional jet activity)*

Compare the description of hard jet activity from NLO, NLO+shower, High Energy Jets.
Dijets, W+Dijets, H+Dijets; Similarities in Jet Activity

Multiple (≥ 2) hard jets...

Smaller number of jets solved satisfactory (?) already...(POWHEG, MC@NLO, NNLO,...)

Special radiation pattern from **current-current** scattering

Look into **higher order corrections beyond** "inclusive K-factor"

Concentrate on the **hard, perturbative corrections** relevant for a description of the final state **in terms of jets**.

Goal

Build framework for **all-order summation** (virtual+real emissions). Exact in another limit than the usual soft&collinear. Better suited for describing **radiation relevant for multi-jet** production.

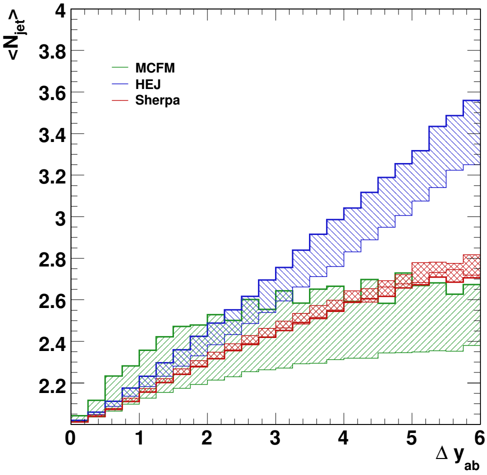
Insight

Can use the insight gained from studying the relevant limit to **guide and improve** analyses: *CP*-properties of the Higgs-boson couplings

Drivers of Emission

1. Collinear (jet profile)
2. Soft (p_t-hierarchies)
3. Opening of phase space (semi-hard emissions - not related to a divergence of $|M|^2$).
   Think (e.g.) multiple jets of fixed p_t, with increasing rapidity span (span=max difference in rapidity of two hard jets=Δy).
   All calculations will agree that number of additional jets increases - but the amount of radiation will differ (wildly) - e.g. due to **limitations** on the **number** (NLO) or **hardness** (shower) of additional radiation **imposed by theoretical assumptions.**

Increasing Rapidity Span → Increasing Number of Jets



J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

h+dijets (at least 40GeV).
Δy_{ab}: Rapidity difference between most forward and backward hard jet

Compare NLO (green), CKKW matched shower (red), and High Energy Jets (blue).

All models show a clear increase in the number of hard jets as the rapidity span Δy_{ab} increases.

Please recall this plot when I discuss the results of the ATLAS study of $\langle N_{\text{jets}} \rangle$

HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but. . .

Sufficiently **accurate** that the description is relevant

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Collinear limit $\rightarrow$ enters many resummation formalisms, parton showers....

Like all good limits, the collinear approximation is applied **outside its strict region of validity.**

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider...

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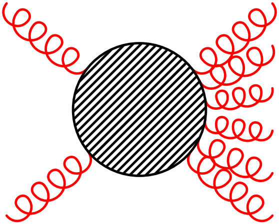
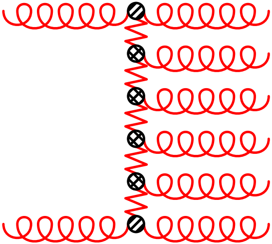


High Energy Limit
$\longrightarrow$
$|\hat{t}|$ fixed, $\hat{s} \to \infty$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\,\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right) e^{\omega(q_{n+1})(y_n-y_{n+1})}\,\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet hard scattering matrix elements

Universal behaviour of the hard scattering matrix element in the High energy (MRK) limit:

$$\forall i \in \{2, \ldots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}_{gg\to g\cdots g}\right|^2 \longrightarrow \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_A}{|p_{n\perp}|^2}.$$

$$\left|\overline{\mathcal{M}}_{qg\to qg\cdots g}\right|^2 \longrightarrow \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_A}{|p_{n\perp}|^2},$$

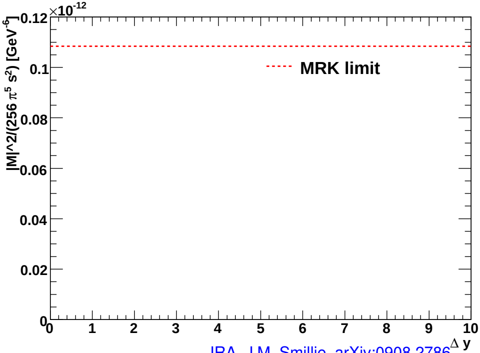
$$\left|\overline{\mathcal{M}}_{qQ\to qg\cdots Q}\right|^2 \longrightarrow \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Comparison of 3-jet hard scattering matrix elements

Study just a slice in phase space, and compare full tree-level with α_s^3-approximation from resummation:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

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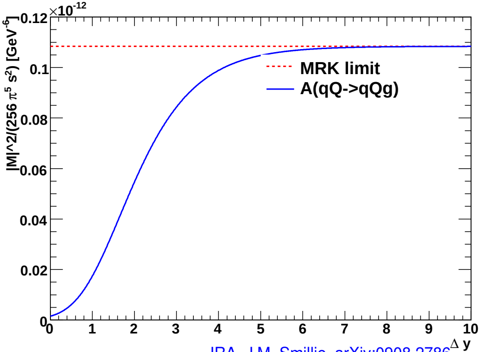

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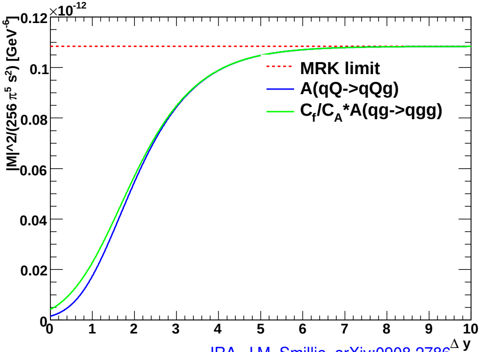

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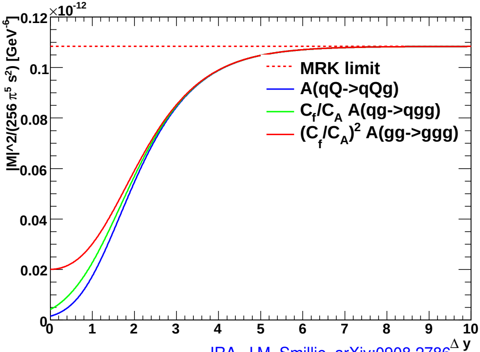

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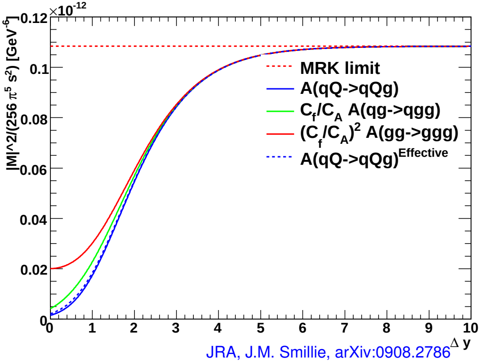


High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: dominance by *t*-channel colour octet exchange
2) No kinematic approximations in invariants
3) Accurate definition of currents (coupling through *t*-channel exchange)
4) Gauge invariance. Not just asymptotically.

Sources of inaccuracies

qQ scattering:

$$|M|^2 = g^4 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

In the strict limit $\hat{s} \to \infty, \hat{t}$ fixed, $s^2 = u^2$.
However, in the LHC phase space, these are not good approximations (as indicated on the previous plot).
Only one t-channel diagram. Need the starting approximation to get this right.
$\hat{s}$: scattering of same-helicity states
$\hat{u}$: scattering of opposite-helicity states
$\hat{t}$: square of full t-channel propagator momentum
Need to study helicity states independently.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^- Q^- \to q^- Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

***t*-channel factorised**: Contraction of (local) currents across *t*-channel pole

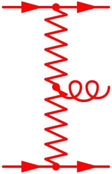
$$\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = \frac{1}{4\,(N_C^2-1)} \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\ C_F\ \frac{1}{t_1}\right) \cdot \left(g^2\ C_F\ \frac{1}{t_2}\right).$$

Extend to $2 \to n$ ...

J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

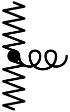


q

$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

q^ν

q_{i-1}

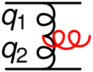


q_i

μ $V^\mu(q_{i-1},q_i)$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$

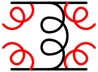
p_A p_1



q_1 q_2 p_2

p_B p_3

$$\begin{aligned} V^\rho(q_1,q_2) =& -(q_1+q_2)^\rho \\ &+\frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A}+\frac{p_2\cdot p_B}{p_A\cdot p_B}+\frac{p_2\cdot p_n}{p_A\cdot p_n}\right)+p_A\leftrightarrow p_1 \\ &-\frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B}+\frac{p_2\cdot p_A}{p_B\cdot p_A}+\frac{p_2\cdot p_1}{p_A\cdot p_1}\right)-p_B\leftrightarrow p_3. \end{aligned}$$



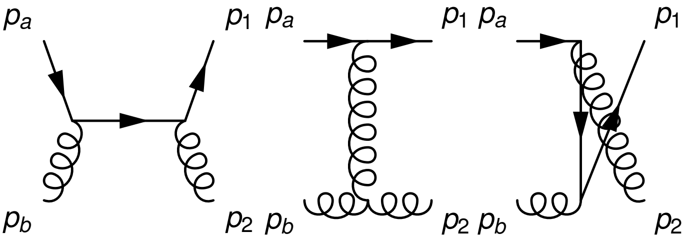
Building Blocks for an Amplitude

$p_g \cdot V = 0$ can easily be checked (**exact** gauge **invariance**)
The approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^t_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\ C_F\ \frac{1}{t_1}\right)\cdot\left(g^2\ C_F\ \frac{1}{t_2}\right) \cdot \left(\frac{-g^2\,C_A}{t_1\,t_2}\ V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right).$$

Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \ \times \langle 1|\sigma|a\rangle.$$

Complete t-channel factorisation!

J.M.Smillie and JRA

Quark-Gluon Scattering

The t-channel current generated by a gluon in qg scattering is that genersated by a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

instead of C_F. Tends to C_A in the MRK limit.

Similar results for e.g. $g^+g^- \to g^+g^-$ (well-defined t-channel):
Exact, complete t-channel **factorisation.**

By using the formalism of **current-current scattering**, we get a better description of the t-channel pole than by using just the MRK kinematic limit of BFKL.

Performing the Explicit Resummation

Analytic subtraction of soft divergence from real radiation:

$$\left|\mathcal{M}_t^{p_a p_b \to p_0 p_1 p_2 p_3}\right|^2 \xrightarrow{\mathbf{p}_1^2 \to 0} \left(\frac{4 g_s^2 C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_t^{p_a p_b \to p_0 p_2 p_3}\right|^2$$

Integrate over the soft part $\mathbf{p}_1^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

$$\int_0^\lambda \frac{\mathrm{d}^{2+2\varepsilon}\mathbf{p}\ \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon}\ 4\pi} \left(\frac{4 g_s^2 C_A}{\mathbf{p}^2}\right) \mu^{-2\varepsilon}$$
$$= \frac{4 g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi}\ \Delta y_{02}\ \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^\varepsilon$$

Pole in ε cancels with that from the **virtual corrections**

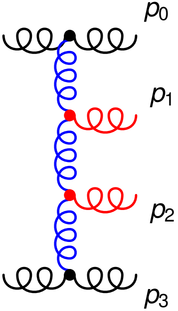
$$\frac{1}{t_1} \to \frac{1}{t_1} \exp\left(\hat{\alpha}(t) \Delta y_{02}\right) \qquad \hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^\varepsilon.$$

Expression for the Regularised Amplitude

$$\overline{\left|\mathcal{M}_{\rm HEJ}^{\rm reg}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{f_1 f_2\to f_1 f_2}\right\|^2 \cdot \left(g^2\ K_{f_1}\ \frac{1}{t_1}\right)\cdot\left(g^2\ K_{f_2}\ \frac{1}{t_{n-1}}\right)$$

$$\cdot \prod_{i=1}^{n-2}\left(g^2 C_A\ \left(\frac{-1}{t_i t_{i+1}} V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\ \theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right)$$

$$\cdot \prod_{j=1}^{n-1} \exp\left[\omega^0(q_j,\lambda)(y_{j-1}-y_j)\right], \qquad \omega^0(q_j,\lambda) = -\frac{\alpha_s N_C}{\pi}\log\frac{\mathbf{q}_j^2}{\lambda^2}.$$



All-Order Summed (and Matched) Cross Section

The cross section is calculated as the sum over the phase space integrals of the explicit *n*-body phase space

$$\sigma_{2j}^{\text{sum,match}} = \sum_{n=2}^{\infty} \sum_{f_1,f_2} \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{\left|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g\cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2}$$

$$\times \quad \mathcal{O}_{2j}(\{p_i\}) \times \sum_m \mathcal{O}_{mj}^e(\{p_i\})\, w_{m-\text{jet}}$$

$$\times \quad x_a f_{A,f_1}(x_a, Q_a)\, x_2 f_{B,f_2}(x_b, Q_b)\, (2\pi)^4\, \delta^2\!\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp} \right).$$

Matching to fixed order (tree-level so far) is obtained by clustering the *n*-parton phase space point into *m*-jet momenta and multiply by the ratio of full to approximate matrix element:

$$w_{m-\text{jet}} \equiv \frac{\overline{\left|\mathcal{M}^{f_1 f_2 \to f_1 g\cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}{\overline{\left|\mathcal{M}^{t,f_1 f_2 \to f_1 g\cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}.$$

Summary: All-Orders, Regularisation, etc.

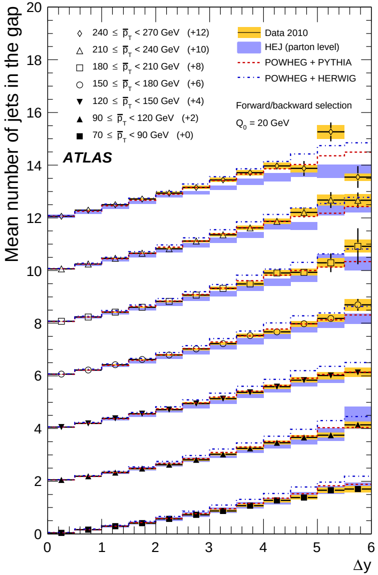
- Have prescription for $2 \to n$ matrix element, including virtual corrections: Lipatov Ansatz $1/t \to 1/t \exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can **calculate** the **sum over the *n*-particle phase space** explicitly ($n \sim 30$) to get the **all-order corrections** (just as if one had provided all the $N^{30}LO$ matrix elements and a regularisation procedure)
- **Merge** *n*-jet tree-level MEs (by merging *m*-parton momenta to *n* hard jet-momenta) where these can be evaluated in reasonable time
  Extension of merging mechanism to **NLO** ongoing
- HEJ recently merged with a **dipole shower** (Ariadne)

Comparison to data

Two drivers for multi-jet production:

- large ratio of transverse scales (shower resummation)
- Colour exchange over a range in rapidity

Both the Tevatron and the LHC has the energy to explore the second mechanism.
Several interesting studies already, and more to come!



CERN-PH-EP-2011-100

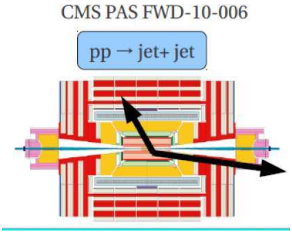
This Atlas analysis tests **both** of the two "drivers" of jet production. (cut on $\bar{p}_t$ induces large p_t-hierarchy on forward/backward jet, besides the hierarchy between large $\bar{p}_t$ and Q_0, the general jet scale)

HEJ slightly undershoots the jet activity when large ratios of transverse scales are imposed (shower region).

Very good agreement in the most important regions of phase space

Obviously **beyond** NLO (more than one extra jet **on average** at $\Delta y \geq 3$!)

CMS: Simultaneous prod. of central and forward jet

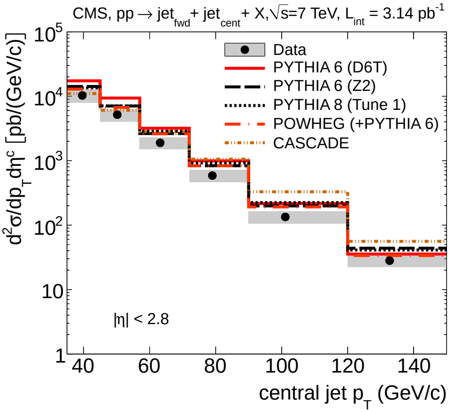


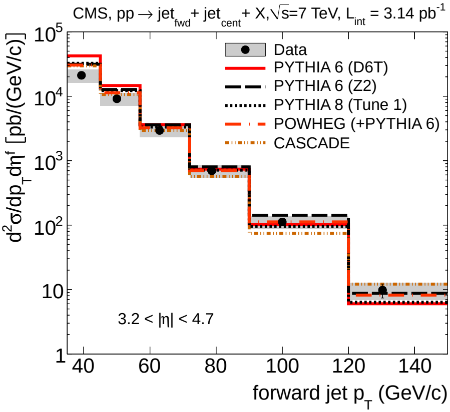
Jets: anti-kt, R=.5, $p_t > 35$GeV

$$\begin{aligned} \text{central} &: |\eta| < 2.8 \\ \text{forward} &: 3.2 < |\eta| < 4.7 \end{aligned}$$

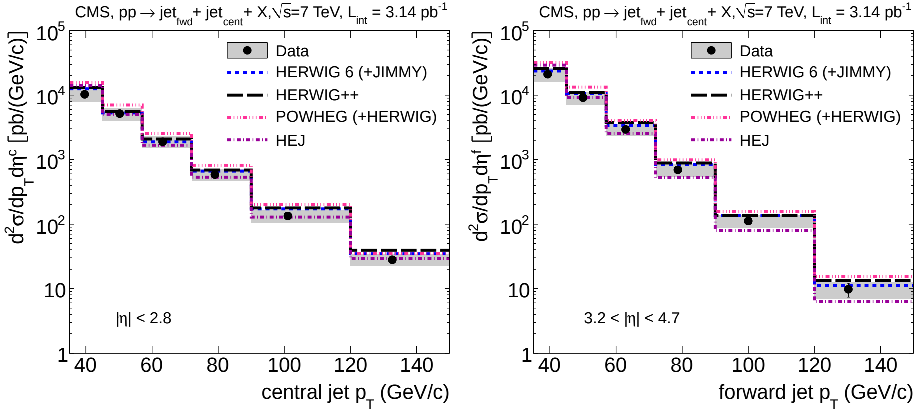
(not particularly large rapidity spans, typically 1 unit).
Measure the p_t-spectrum of the central and the forward jet. Any difference is obviously due to additional radiation.

Comparison to Theory, I





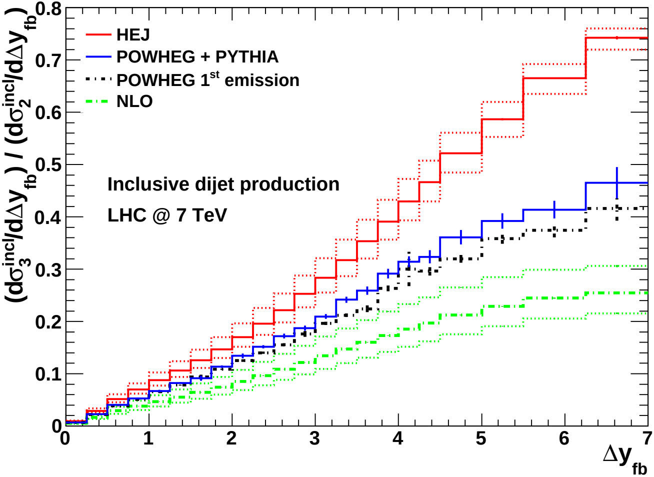
Comparison to Theory, II



This event selection does not probe particularly large rapidity separations (peaking around 1 unit of rapidity between the dijets). HEJ gives good description of the pt-spectrum.

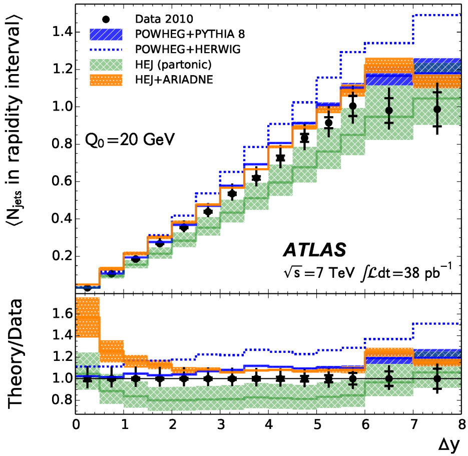
Ratio of Inclusive Jet Rates vs. Rapidity

S. Alioli, E. Re, J.M. Smillie, C. Oleari, JRA; arXiv:1202.1475



Clear differences: NLO, POWHEG, HEJ

DiJets



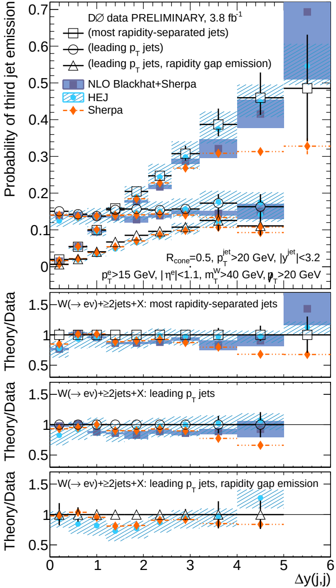
ATLAS: arXiv:1407.5756. $p_{t1} > 60\text{GeV}$, $p_{t2} > 50\text{GeV}$. Average number of jets (above 20GeV) in-between the two hardest jets. Ariadne shower improves upon the HEJ-predictions.

D0: W+Jets Phys. Rev. D 88, 092001

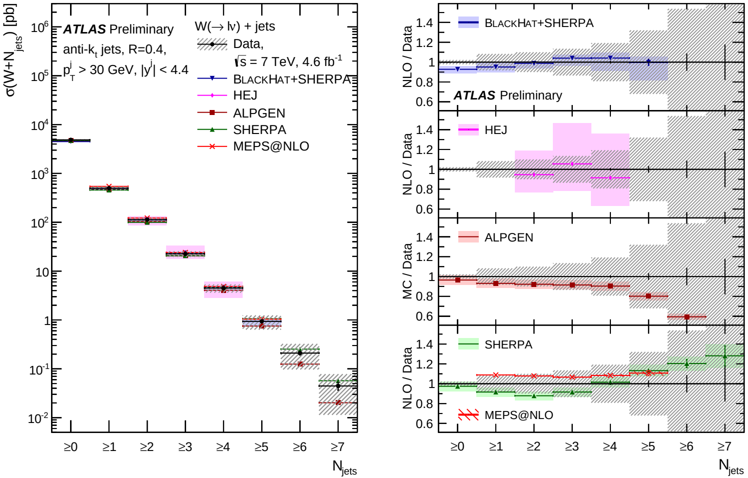
D0 measurement of the probability of at least one additional jet when requiring just a *W* in association with two jets. Probability measured vs. rapidity separation of

1. the two most rapidity separated jets
2. the two hardest (in pt) jets
3. the two hardest (in pt) jets, counting additional jets only in the rapidity interval between the two hardest jets

Good agreement between data and HEJ for all observables - effects will be even more pronounced at the LHC.

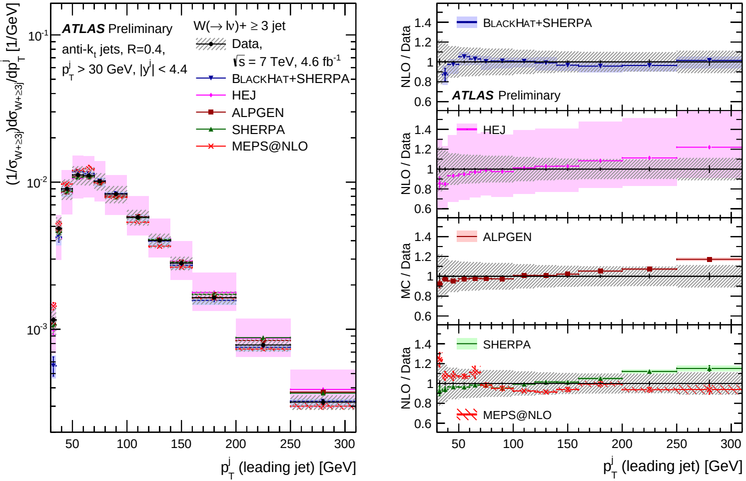


W+DiJets

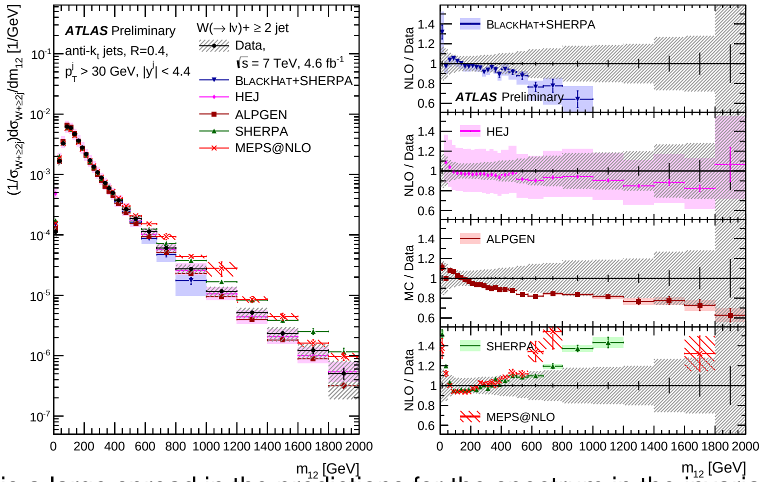


Good agreement between all predictions and data - on inclusive quantities.

W+DiJets



For standard p_t-based observables, all predictions give a reasonable description (NLO is very good!).



There is a large spread in the predictions for the spectrum in the invariant mass between the two hardest jets. Here, the terms systematically dealt with in HEJ are important, and HEJ gives a good description.

Note: hjj interesting for $m_{jj} > 400 - 600$GeV.

CP Properties of Higgs-Boson Couplings from Hjj through Gluon Fusion

Stabilising the Extraction against Higher Order Corrections

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

. . . in a region of phase space where the **perturbative corrections are large.**

How do we deal with events with **three or more** jets?

The Solution

By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

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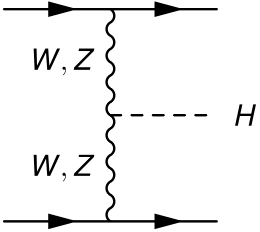
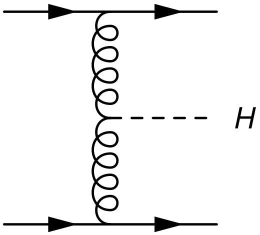
Higgs Couplings through Azimuthal Correllations



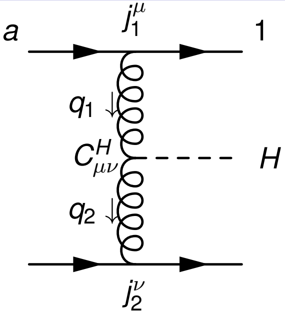
Considerations for Weak Boson Fusion

Higgs Couplings through Azimuthal Correllations



…and gluon fusion (Higgs coupling to gluons through top loop)

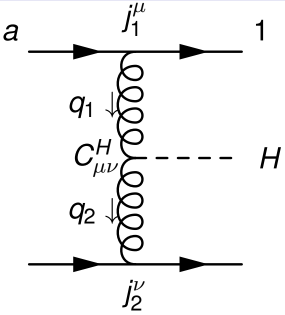
Higgs Couplings through Azimuthal Correllations



$$\mathcal{M} \propto \frac{j_1^\mu\ C^H_{\mu\nu}\ j_2^\nu}{t_1\ t_2}, \qquad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a$$

$$C_H^{\mu\nu} = a_2\,(q_1 q_2 g^{\mu\nu} \ - \ q_1^\nu q_2^\mu) + a_3\ \varepsilon^{\mu\nu\rho\sigma}\ q_{1\rho}\ q_{2\sigma}.$$

Higgs Couplings through Azimuthal Correllations



$$\mathcal{M} \propto \frac{j_1^\mu \; C^H_{\mu\nu} \; j_2^\nu}{t_1 \; t_2}, \qquad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a$$

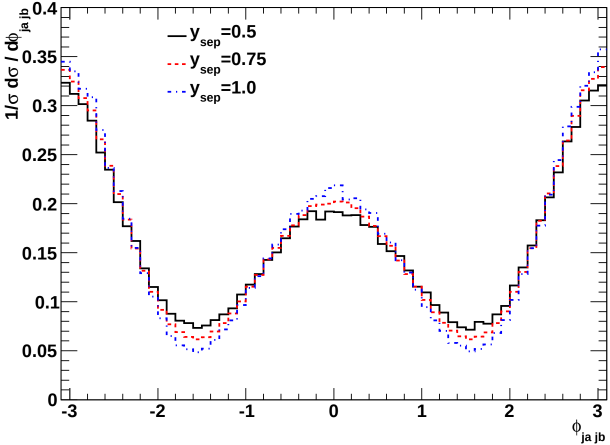
$$C_H^{\mu\nu} = a_2 \left(q_1 q_2 g^{\mu\nu} \; - \; q_1^\nu q_2^\mu\right) + a_3 \; \varepsilon^{\mu\nu\rho\sigma} \; q_{1\rho} \; q_{2\sigma}.$$

Take e.g. the term $\varepsilon^{\mu\nu\rho\sigma} \; q_{1\rho} \; q_{2\sigma}$: for $|p_{1,z}| \gg |p_{1,x,y}|$ and for small energy loss (i.e. $\overline{\psi}_1\gamma^\mu\psi_a \to 2p_a, \overline{\psi}_2\gamma^\mu\psi_b \to 2p_b, p_{a,e} \sim p_{1,e}$):

$$\left[j_1^0 \; j_2^3 - j_1^3 \; j_2^0\right] \left(\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp}\right).$$

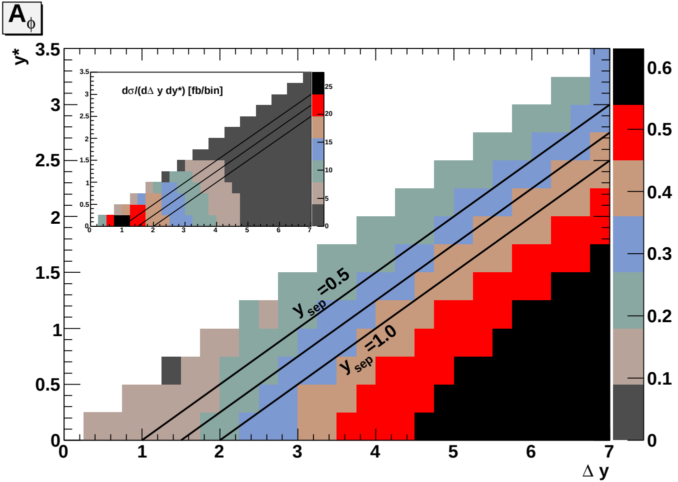
In this limit, the azimuthal dependence of the propagators is also suppressed: $|\mathcal{M}|^2$:$\sin^2(\phi)$ **(CP-odd)**, $\cos^2(\phi)$ **(CP-even)**.

Azimuthal distribution



JRA, K. Arnold, D. Zeppenfeld (JHEP 1006 (2010) 091)

$$CP\text{-even},\ p_{j\perp} > 40\ \text{GeV}, \quad y_{ja} < y_h < y_{jb},$$
$$|y_{ja,jb}| < 4.5, \min\left(|y_h - y_{ja}|, |y_h - y_{jb}|\right) > y_{\text{sep}}.$$

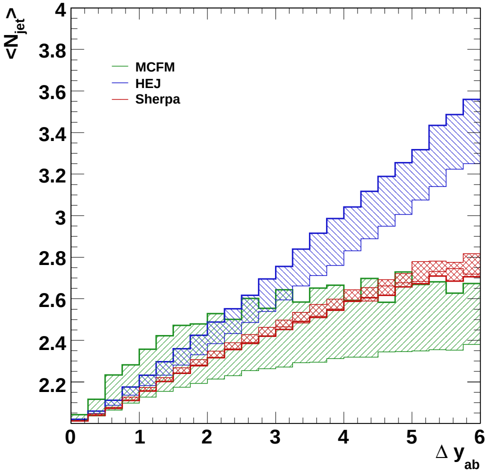


$$\Delta y = |y_{j_a} - y_{j_b}|, \quad y^* = y_h - \frac{y_{j_a}+y_{j_b}}{2}.$$

JRA, K. Arnold, D. Zeppenfeld

Rapidity separation between the jets and the Higgs Boson **enhance the azimuthal correlation.**

Increasing Rapidity Span → Increasing Number of Jets

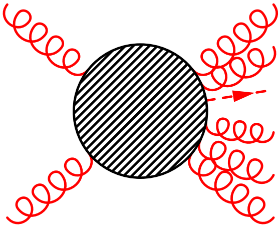


All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

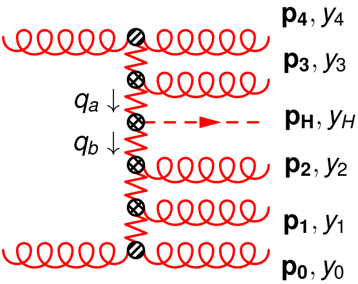
J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

Develop Insight Into the Perturbative Corrections



High Energy Limit $\longrightarrow$

$|p_{\perp,i}|$ fixed, $\hat{s}_{ij} \to \infty$

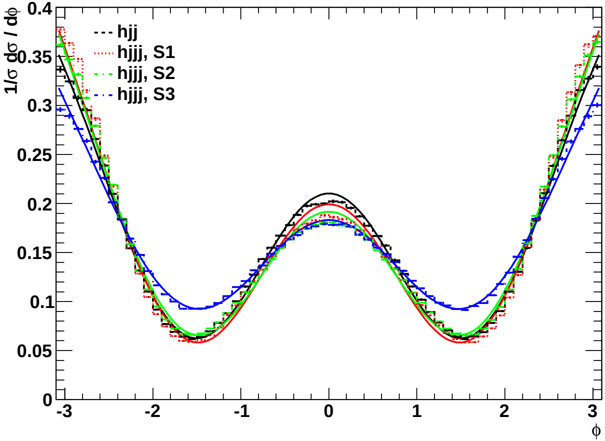


$$\left|\mathcal{M}_{gg\to g\cdots ghg\cdots g}\right|^2 \to \frac{4\hat{s}^2}{N_C^2-1}\left(\prod_{i=1}^{j}\frac{C_A\, g_s^2}{\mathbf{p}_{i\perp}^2}\right)\frac{|C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp})|^2}{\mathbf{q}_{a\perp}^2\ \mathbf{q}_{b\perp}^2}\left(\prod_{i=j+1}^{n}\frac{C_A\, g_s^2}{\mathbf{p}_{i\perp}^2}\right)$$

$$C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp}) \ = \ -i\,\frac{\alpha_s}{3\pi v}\,\mathbf{q}_{a\perp}\cdot\mathbf{q}_{b\perp}, \quad y_0 < \cdots < y_j < y_H < y_{j+1} < y_n$$

The **High Energy Limit** tells us to investigate the **azimuthal angle** between the **sum of the jet vectors** either side in rapidity of the Higgs Boson!

And It Even Works!



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

Three subsamples of tree-level three-jet events: two jets on same side of the Higgs boson parallel (S1), perpendicular (S2) or anti-parallel (S3). Azimuthal correlation almost unchanged from hjj.

Conclusions

- Hadron colliders probes hard (=jets) perturbative corrections beyond pure NLO … already at 2, 7TeV!
- ***High Energy Jets****[*]* provides a new approach to the perturbative description of proton collider physics
  … and compares favourably to data in several analyses
  … several ongoing improvements in the formal accuracy of the perturbative approximations

* http://cern.ch/hej