The Higgs(-. . .)-Mechanism

I: The Power of Gauge Theories

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Overview of Lectures

The Standard Model

The Uncompromising Gauge Principle

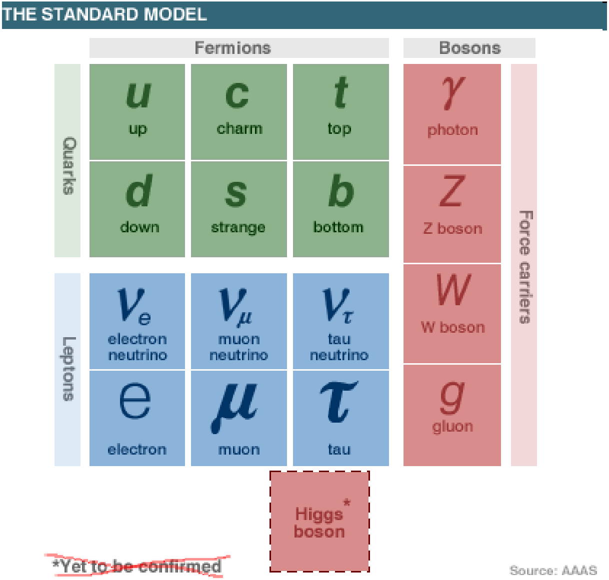
The Higgs(-. . .)-Mechanism

Mass Generation
Stability of the Standard Model
Characteristics of the SM Higgs Boson

The Higgs Boson Discovery and Phenomenology

Discovery of the Resonance, Mass
Couplings
CP-Properties

The Particle Content of the Standard Model



The Content of the Standard Model

- Particle Physics not just "stamp collecting", hunting new resonances.
- The study of the **fundamental** particles and their **interactions** is interesting...
- ...but what makes it **really interesting** is that the **fundamental interactions** are **dictated** by a **few theoretical guidelines** and the **fundamental dynamics** can be directly related to **observations.**
  (fundamental: related to terms in the Lagrangian)

The Standard Model

Standard Model of Particle Physics

Unified theoretical framework describing

1. Electromagnetism
2. Weak Nuclear Force
3. Strong Nuclear Force
4. Matter Particles

Forces

Electromagnetism: massless photon. Electric charge

Weak nuclear force: massive $W^{\pm}, Z$-boson. Interactions only with left-handed matter and the force carriers themselves. Weak isospin.

Strong nuclear force: massless gluons. Interactions with quarks and with gluons. Colour charge.

The Forces of the Standard Model

All the forces can be quantised by the use of **gauge field theory**, based on the groups

$$\boldsymbol{SU_c(3) \times SU_L(2) \times U_Y(1)}$$

A straightforward introduction of masses - **not only** for the **gauge bosons**, but **also for the matter particles** - would break the gauge invariance of the electro-weak component of the theory.

We will now explore the construction of the SM - in order to expose the **mass generation** through **dynamical electro-weak symmetry breaking** (aka the **Higgs(-...)-mechanism**), and the appearance of the **Standard Model Higgs Boson**.

Building the Standard Model Langrangian

Start from the Lagrangian of a free (Dirac Fermion) field

$$\mathcal{L} = \bar{\psi} i \partial_\mu \gamma^\mu \psi$$

Introduce **interactions**: Left/right-handed chiral fermion fields: $f_{L,R} = \frac{1}{2}(1 \mp \gamma_5) f$.
Left-handed fermions are in weak **iso-doublets**, **right-handed** fermions in weak **iso-singlets**

$$L_1 = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \; e^-_R, \; Q_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$

Similar for the other two generations.

Gauge Groups of the Electroweak Sector

Matter fields in the fundamental representation; acted on by the generators of the adjoint

$$T^a = \frac{1}{2}\tau^a; \quad \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Commutation relations for weak isospin and hypercharge

$$[T^a, T^b] = i\epsilon^{abc} T_c \quad \text{and} \quad [Y, Y] = 0$$

Local Gauge Transformations in Electroweak Sector

Arrange the Lagrangian as

$$\mathcal{L} = \bar{L}_i i\partial_\mu \gamma^\mu L_i + \bar{e}_{R_i} i\partial_\mu \gamma^\mu e_{R_i}$$

plus the similar terms for the weak iso-doublets and -singlets of the quark sector.
Already then is invariant under global gauge transformations. **Require** the Lagrangian to be invariant under **local** gauge transformations:

$$L(x) \to L'(x) = e^{i\alpha_a(x)T^a + i\beta(x)Y} L(x)$$
$$R(x) \to R'(x) = e^{i\beta(x)Y} R(x)$$

Local Gauge Transformations in Electroweak Sector

The Lagrangian of the massless free field can be made invariant under these **local** gauge transformations by a minimal modification: introduce the **gauge fields** to counteract the effect of the derivatives on the gauge transform functions. **Covariant derivative**:

$$\partial_\mu \to D_\mu = \partial_\mu - ig_2 T_a W^a_\mu - ig_1 \frac{Y}{2} B_\mu$$

Local Gauge Transformations in Electroweak Sector

The gauge fields must then transform as

$$\vec{W}_\mu(x) \to \vec{W}'_\mu(x) = \vec{W}_\mu(x) - \frac{1}{g_2}\partial_\mu\vec{\alpha}(x) - \vec{\alpha}(x) \times \vec{W}_\mu(x)$$
$$B_\mu(x) \to B'_\mu(x) = B_\mu(x) - \frac{1}{g_1}\partial_\mu\beta(x)$$

This **minimal** modifications **uniquely** defines the coupling between the fermion fields and the gauge fields:

$$-g_i\bar{\psi}V_\mu\gamma^\mu\psi$$

Dynamics of the Gauge Fields

The dynamics of the gauge fields themselves are encoded in the **field strength tensors**

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Adding terms of $-\frac{1}{4} W^{\mu\nu}_a W^a_{\mu\nu}$ to the Lagrangian introduces **unique** triple and quartic gauge boson couplings

$$\text{triple} : i g_2 \mathrm{Tr}(\partial_\nu W_\mu - \partial_\mu W_\nu)[W_\mu, W_\nu]$$
$$\text{quartic} : \frac{1}{2} g_2^2 \mathrm{Tr}[W_\mu, W_\nu]^2$$

The (massless) SM Lagrangian

$$\mathcal{L} = -\frac{1}{4} W_a^{\mu\nu} W^a_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{L}_i i D_\mu \gamma^\mu L_i + \bar{e}_{R_i} i D_\mu \gamma^\mu e_{R_i} \ldots$$

Unique, starting from free fermion fields, require **local gauge invariance** of $SU(2)_L \times U_1(Y)(\times SU(3)_c)$.

The Problem

Deals so far only with **massless** fields. But both fermions and the weak gauge fields are **heavy**. A standard mass term for a fermion **breaks gauge invariance**, which was the guiding principle in constructing the interactions!

The Problem of Mass-terms and Gauge Invariance

Standard mass terms would break gauge invariance

$$-m_f\bar{\psi}\psi = -m_f\bar{\psi}\left(\frac{1}{2}(1-\gamma_5)+\frac{1}{2}(q+\gamma_5)\right)\psi = -m_f(\bar{\psi}_R\psi_L+\bar{\psi}_L\psi_R)$$

Obviously not gauge invariance, since e.g. only left handed fields transform under $SU(2)$.
Also mass terms for the gauge bosons would cause trouble, e.g.

$$\frac{1}{2}M_W^2 W_\mu W^\mu$$

would generate additional terms under a gauge transformation.

A Job for the Higgs(-…)-mechanism

In simple words, the Higgs mechanism solves the problem of introducing mass terms by coupling $\bar{\psi}_f\psi_f$ and the relevant $W_\mu W^\mu$ to a **new scalar field**, which **counters the effect** of a gauge transformation. This will leave the terms gauge invariant.

Masses are then **generated by dynamically** (mysteriously?) by requiring this new Higgs field to **acquire a vacuum expectation value** (vev).

The Higgs Mechanism of the Standard Model

Need to generate masses to three gauge bosons $W^{\pm}$ and Z, but the photon should remain massless for exact QED gauge symmetry. Need at least three degrees of freedom for the scalar field introduced.

Simplest choice: complex $SU(2)$ doublet of scalar fields

$$\Phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}, \quad Y_{\phi} = +1$$

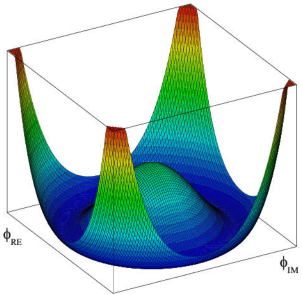
Add the following terms to the Lagrangian (free scalar field plus simplest Φ^4 potential to generate vev (for $\mu^2 < 0$).

$$\mathcal{L}_S = (D^{\mu}\Phi)^{\dagger}(D_{\mu}\Phi) - \mu^2\Phi^{\dagger}\Phi - \lambda(\Phi^{\dagger}\Phi)^2$$

The Higgs Mechanism of the Standard Model

The vev should not be in the direction of the charged field, for the photon to remain massless ($U(1)_{\mathrm{QED}}$ to survive unbroken)

$$\langle\Phi\rangle_0 = \langle 0|\Phi|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad \text{with} \qquad v = \sqrt{-\frac{\mu^2}{\lambda}}$$



The Higgs Mechanism of the Standard Model

To investigate the terms generated, expand the field around the new minimum at v:

$$\Phi(x) = \left(\begin{array}{c} \theta_2 + i\theta_1 \\ \frac{1}{\sqrt{2}}(v+H) - i\theta_3 \end{array} \right) = e^{i\theta_a(x)\tau^a} \left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{array} \right)$$

Transform to the Unitary Gauge:

$$\Phi(x) \to e^{-i\theta_a(x)\tau^a}\Phi(x) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + H(x) \end{array} \right)$$

The Higgs Mechanism of the Standard Model

Gauge Boson masses: The covariant derivative of the scalar field generates mass terms for linear combinations of the fields W, B:

$$W^{\pm} = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), Z_\mu = \frac{g_2 W^3_\mu - g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}}, A_\mu = \frac{g_2 W^3_\mu + g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}}$$

of

$$M_W = \frac{1}{2} v g_2, M_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2}, M_A = 0$$

The relationship between the masses are **uniquely** predicted.

$$M_W = \frac{1}{2} g_2 v = \left(\frac{\sqrt{2} g_2^2}{8 G_\mu}\right)^2 \therefore v = \frac{1}{(\sqrt{2} G_\mu)^{1/2}} \approx 246 \text{ GeV}.$$

The Higgs Mechanism of the Standard Model

Fermion masses: Can be generated using the same scalar field Φ, and the iso-doublet $\tilde{\Phi} = i\tau_2\Phi^*$

$$\begin{aligned}\mathcal{L}_F &= -\lambda_2 \bar{L}\Phi e_R + \cdots + h.c.\\ &= -\frac{1}{\sqrt{2}}\lambda_e(v+H)\bar{e}_L e_R + \cdots\end{aligned}$$

so we identify

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

One parameter per fermion mass: **fermion masses not predicted**, **no special relationship between fermion masses.**

The Higgs Boson of the Standard Model

Kinetic part (interactions with the other fields etc.) arises from the covariant derivative, whereas the mass of the Higgs boson itself comes from the scalar potential $V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$.
Using $v^2 = -\mu^2/\lambda$ one obtains

$$V = -\frac{1}{2}\lambda v^2(v+H)^2 + \frac{1}{4}\lambda(v+H)^4$$

so the Lagrangian of the Higgs field can be written

$$\begin{aligned}\mathcal{L}_H &= \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V\\ &= \frac{1}{2}(\partial H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4.\end{aligned}$$

The Higgs Boson of the Standard Model

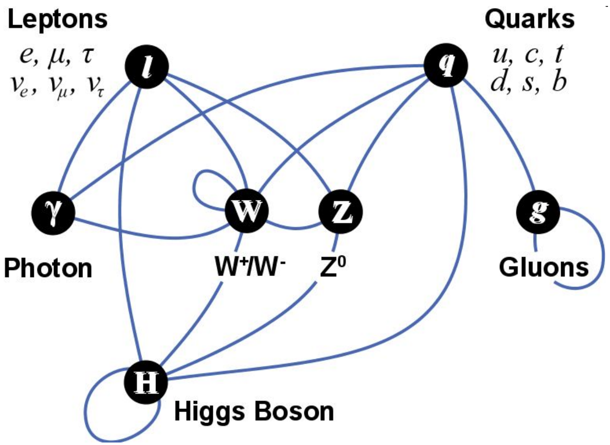
From this, we get

$$M_H^2 = 2\lambda v^2 = -2\mu^2.$$

Higgs mass not predicted, but **knowing** M_H **fixes** λ and μ (since v is fixed by e.g. the mass of the W).
Furthermore, we find the existence of three and four-Higgs-boson vertices, with couplings

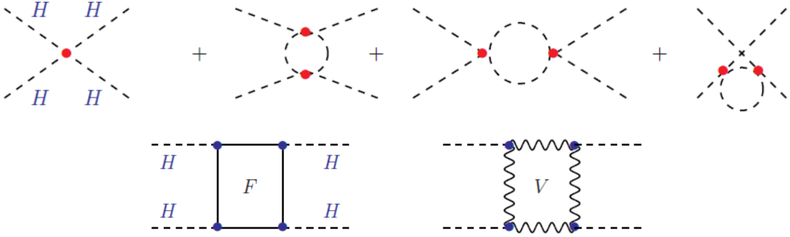
$$g_{H^3} = (3!)i\lambda v = 3i\frac{M_H^2}{v}, \quad g_{H^4} = (4!)i\frac{\lambda}{4} = 3i\frac{M_H^2}{v^2}.$$

The Higgs Boson of the Standard Model



Evolution of Higgs Self Coupling

Triviality Bounds: The value of the Higgs boson self coupling varies with energy (like all other couplings), determined by RGE.



$$\frac{d\lambda}{d\log Q^2} = \frac{1}{16\pi^2}[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_1^2 + g_2^2)^2)]$$

Evolution of the Higgs Self Coupling

Triviality Bounds: Require that the ϕ^4-theory remains perturbative, i.e. $\lambda \leq 1$ up to a high scale Λ: Put bounds on λ at the EW scale, and therefore puts an **upper limit** on the Higgs boson mass.

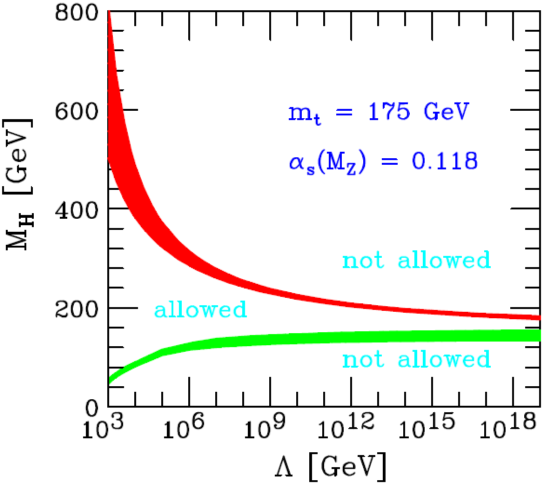
$$\begin{aligned}\frac{d\lambda}{d\log Q^2} =& \frac{1}{16\pi^2}[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) \\ &+ \frac{3}{16}(2g_2^4 + (g_1^2 + g_2^2)^2)]\end{aligned}$$

Evolution of the Higgs Self Coupling

Stability Bounds: For small values of lambda, the evolution is determined from the (negative) top quark contribution. λ can go negative! This would mean the minimum of the Higgs potential is moved back to the origin, no spontaneous symmetry break, and no particle masses. This puts a **lower limit** on M_H.

$$\frac{d\lambda}{d\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_1^2 + g_2^2)^2)\right]$$

Stability of the Higgs Self Coupling



Conclusion (2005): 130 GeV $\leq M_H \leq$ 180 GeV

Bounds very dependent on e.g. top-quark mass, α_s, matching...

Naturalness...

Where did the Higgs potential come from?
How stable is the Higgs mass against radiative corrections?
For a long time, people argued that since, with a cut-off regularisation, the Higgs mass had quadratic divergences, whatever value one found had to be very fine-tuned.
However, cut-off regularisation may be the very source of these problems (violates Lorentz invariance,...)! Certainly, there are no quadratic divergences in dimensional regularisation.

Recent Ideas

Perhaps the Higgs Mass and EWSB is dynamically generated from a condition of $\lambda(Q) = 0$ at some large scale.

The **Standard Model Lagrangian** was constructed, starting from a **free fermion**, and by imposing the relevant **local gauge invariances.** All interactions determined from the gauge principle. The Higgs field is postulated, in order to introduce gauge invariant mass terms.

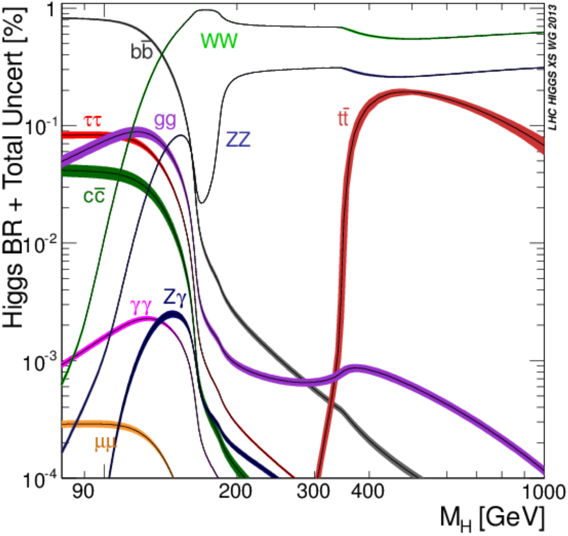
Gauge Theories Rule!

Nature would rather introduce a new particle, than violate gauge invariance!

Higgs Boson Physics

Characteristics of the Higgs Boson

The masses of all other particles are well established; the characteristics of Higgs boson production and decay can be calculated as a function of the Higgs boson mass only.
We can calculate the frequency with which a Higgs boson decays to specific SM particles. **The only parameter is the Higgs boson mass.**



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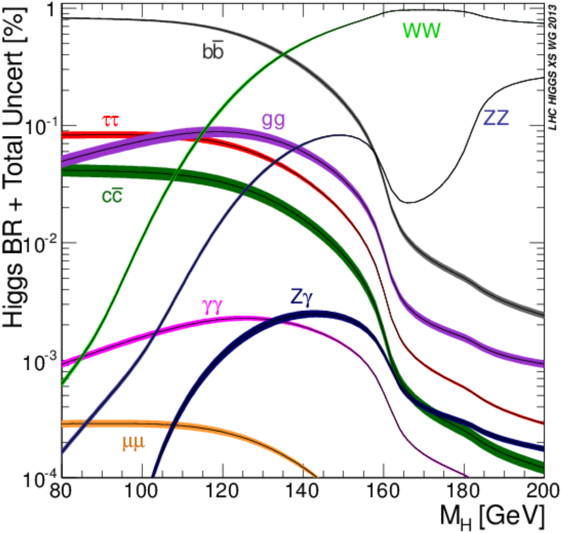
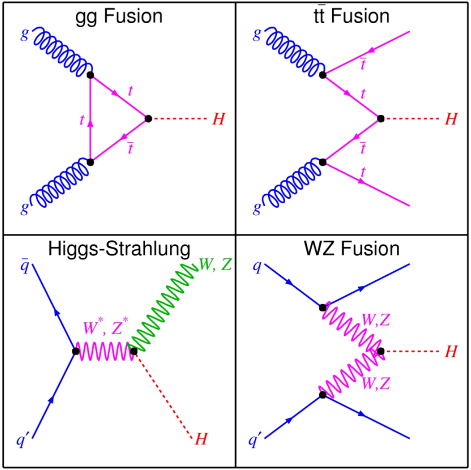


Production of Higgs Bosons at Hadron Colliders

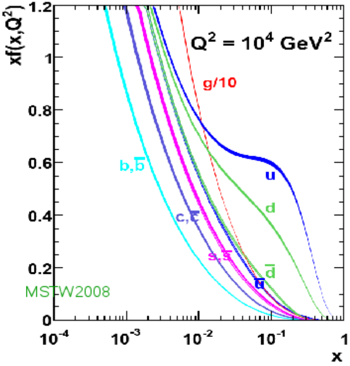
- Gluon-Fusion through top (+bottom+$\cdots$)-loop is the dominant production process. Coupling is small, but gluon flux is large!
- Weak Boson Fusion (WBF) second largest production method



Predicting Perturbative Hadronic Processes

$$\sigma_{ab\to 1\cdots N} = \int \prod_{i=1}^{N} \left(\frac{\mathrm{d}^2\mathbf{p}_{\perp i}\, \mathrm{d}y_i}{(2\pi)^3\, 2} \right) (2\pi)^4 \delta^2 \left(\sum_{i=1}^{N} \mathbf{p}_{\perp i} \right) \frac{1}{\hat{s}^2}$$
$$\times\, x_1 f_1(x_1, Q_1^2)\, x_2 f_2(x_2, Q_2^2)\, \left|\mathcal{M}_{ab\to 1\cdots N}\right|^2$$

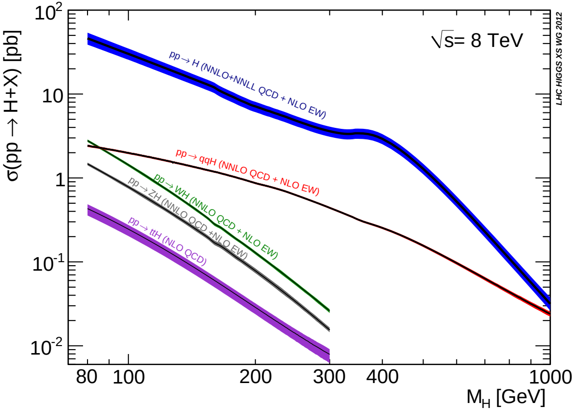
The parton density functions are determined using other processes and earlier experiments (non-perturbative **low-scale input**, **perturbative evolution**).
Hard scattering matrix element calculated in **perturbation theory.**



Production cross sections

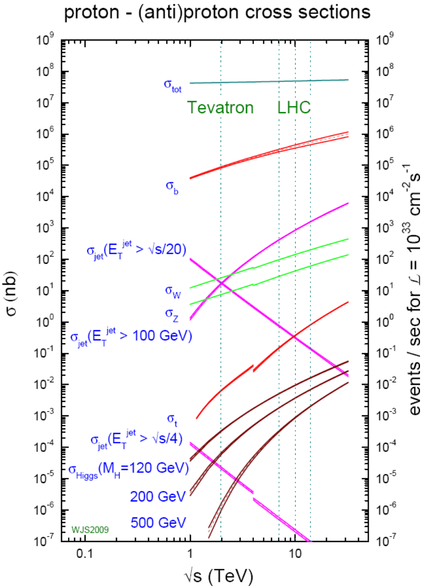
The four main production channels:

- Gluon Fusion
- Weak Boson Fusion
- Associated production/Higgs Strahlung
- $t\bar{t}H$



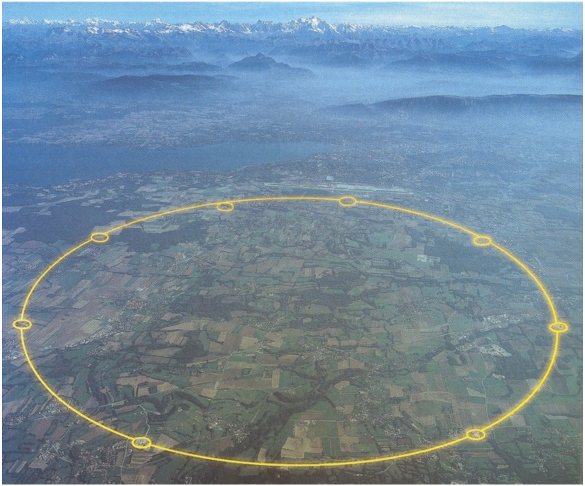
Part of the challenge

At first it may look hopeless. Higgs production happens much less frequently than e.g. dijet.
Calculate the cross section for several simple processes, spanning 12 orders of magnitude.
Requires very detailed calculations, and often special refinements for specific processes and channels.

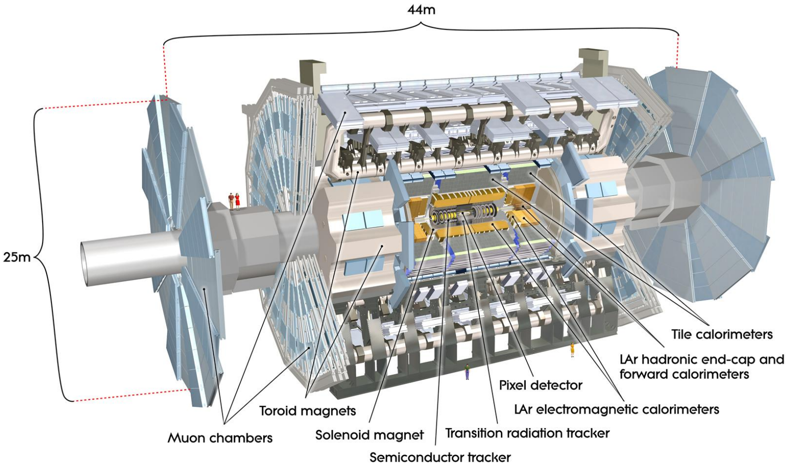


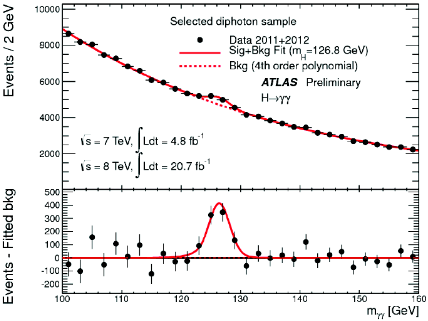
CERN Large Hadron Collider

- 27km in circumference
- proton-proton collider
- "Roughly" 14TeV of centre-off-mass (hadronic) energy



A Detector at the LHC (ATLAS)

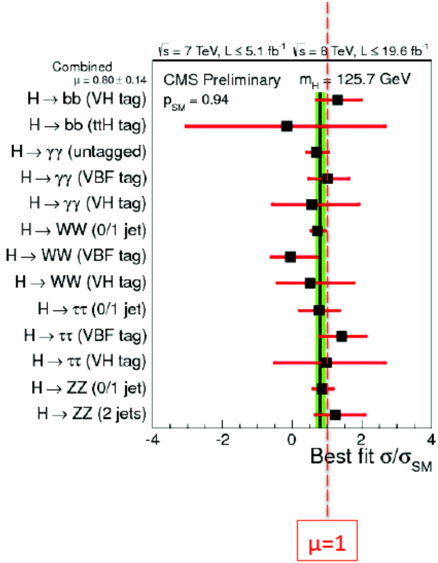




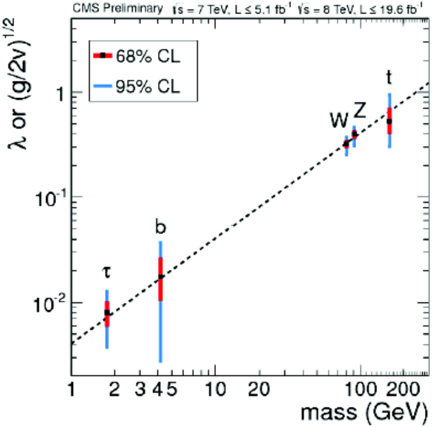
	ATLAS		CMS		
	expected	**observed**	expected	**observed**	observed
H→ZZ	4.4	**6.6**	7.1	**6.7**	
H→γγ	4.1	**7.4**	3.9	**3.2**	
H→WW	3.8	**3.8**	5.3	**3.9**	
H→ττ	1.6	**1.1**	2.6	**2.8**	**3.4**
H→bb	1.0	**0**	2.1	**2.1**	
combined	7.3	**10**	stopped computing		

Higgs Boson Precission Physics!

Current best CMS fit for signal strength compared to SM:
0.80 $\pm$ 0.14.
Current best ATLAS fit for signal strength compared to SM:
1.30 $\pm$ 0.20.

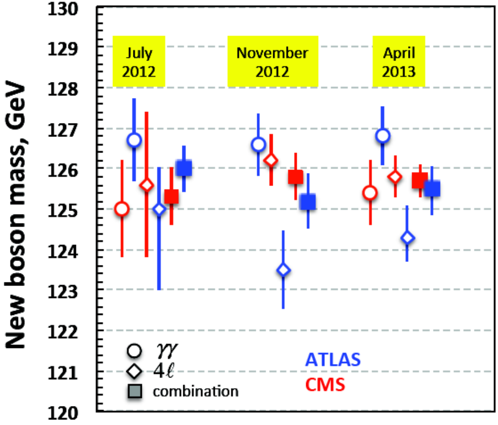


The Right Coupling Strength vs. Mass of Particles



Coupling strength depends on the particle masses as required by the SM Higgs mechanism.

Measurement of New Boson Mass Stable



Experimental Status

- In a **combined search** for the SM Higgs boson, **a significant excess of events near m_H=126 GeV** persists beyond any doubt and now has been **established in individual decay channels: ZZ, WW, γγ**

- **New boson's mass:**
  - **CMS:** **125.7 ± 0.4 GeV**
  - **ATLAS:** **125.5 ± 0.6 GeV**

- **Is X125 the SM Higgs boson?**
  - **event yields in all individual channels are consistent with the SM Higgs boson**
  - **couplings agree with the SM Higgs boson** with the current statistical accuracy: 20% (W & Z), 25% (t), 30% (τ), 60% (b)
  - **no significant modifications for loop-induced couplings (deviations < 2σ)**
  - **BR(H→BSM) < 0.5 (approx.) at 95%CL**
  - **100% pure $J^{CP} = 0^-$, $1^{\pm}$, 2^+_m states are excluded at >99% CL**
  - **CP-odd fractional contribution: *f*(0^-) < 0.58 at 95% CL**

. . . as presented at Les Houches, June 4.