Hard Jets and Higgs Bosons

HEJ: All-Order Perturbative Corrections to Hard Multi-Jet Processes

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IPPP, Durham University

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Overview of Talk

Elements of Proton Collisions

Hard scattering, **shower**, **matching to fixed order**
multiple interactions, underlying event. . .

Jets to the rescue!

Multi-Jet Predictions

Why we **must** care about HO corrections (in some situations). . .
A new approach to multiple, wide-angle emissions from the **hard scattering**:

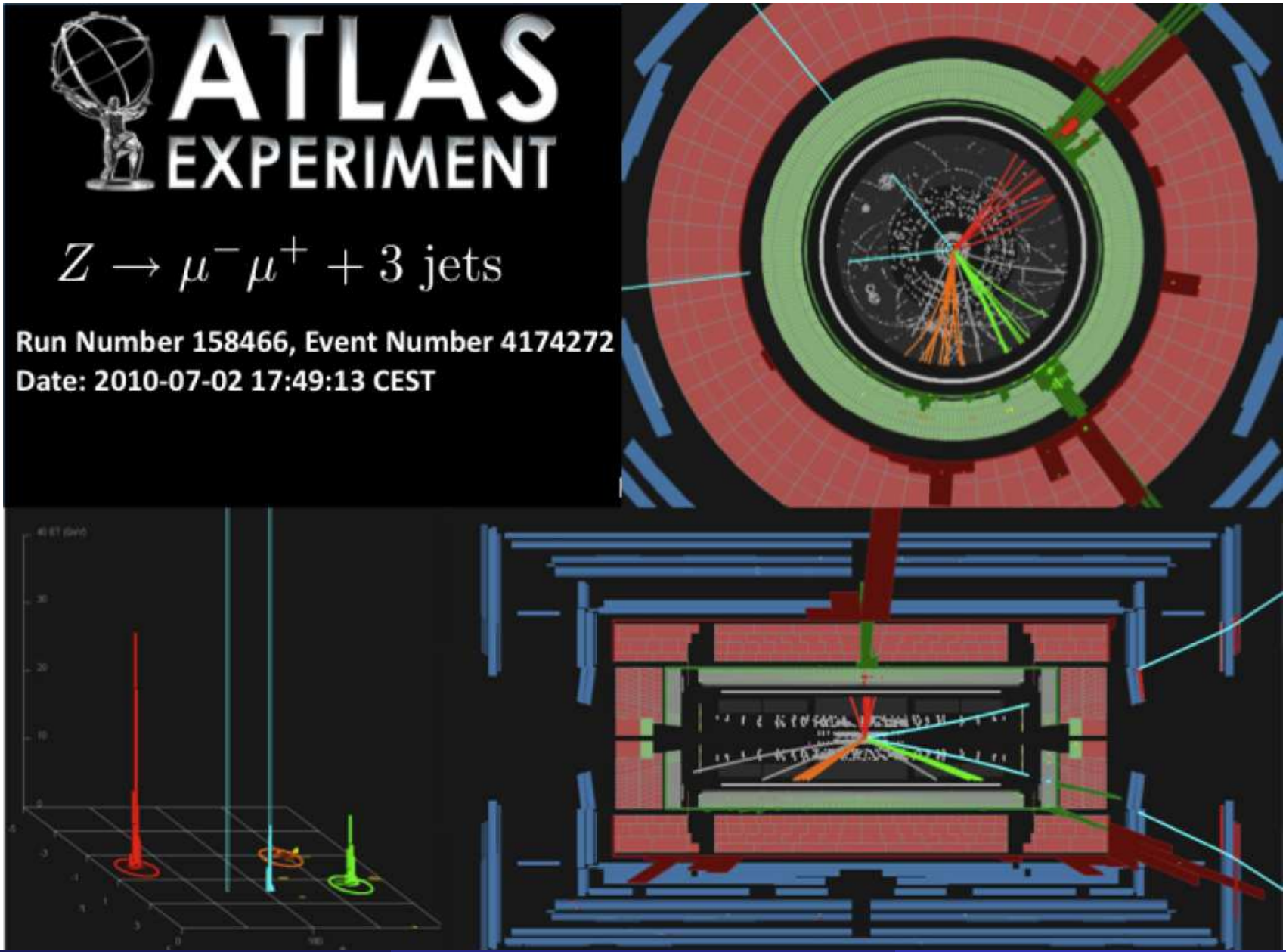
High Energy Jets

Predictions for dijets, W+jets, H+jets,. . .

Theory vs. Data

Results of **first data** compared to HEJ
Hard, higher order effects beyond NLO

$pp \to \mu^+\mu^-$+3 jets



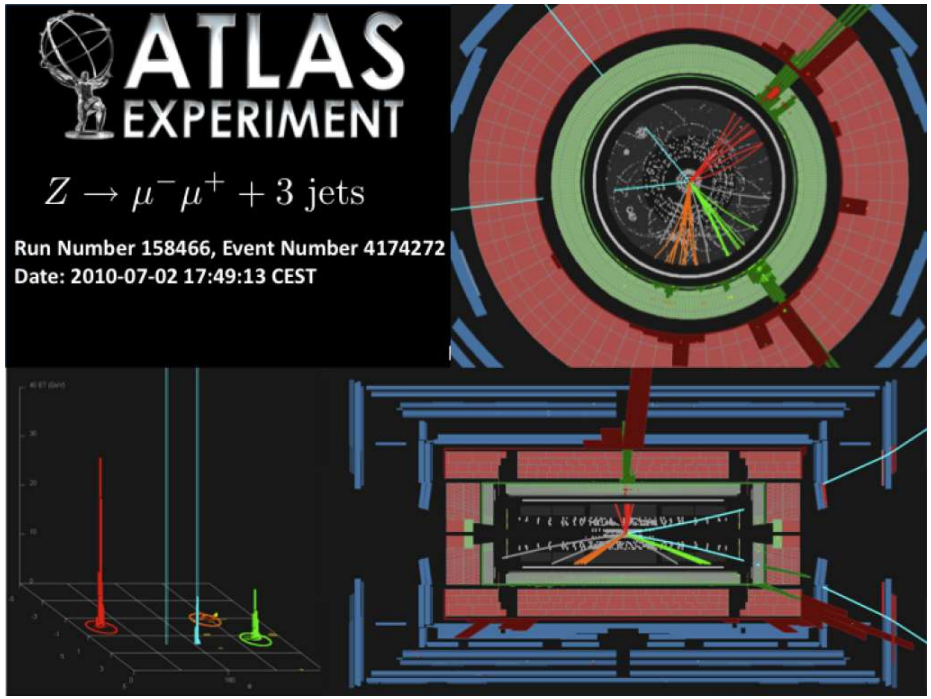
Jet (algorithms) to the Rescue!

Depending on the question we want to answer, we **may not need** to describe all the **stages of the collision**.

The notion of jets allow us to **compare pure perturbation theory** (few partons) to **experimental observation** (many hadrons)

Transverse Momentum Rapidity: $y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$

still need to ensure (relative) insensitivity to underlying event, multiple interactions. . . ask questions only about relatively hard jets ($p_\perp > 30$ GeV?)



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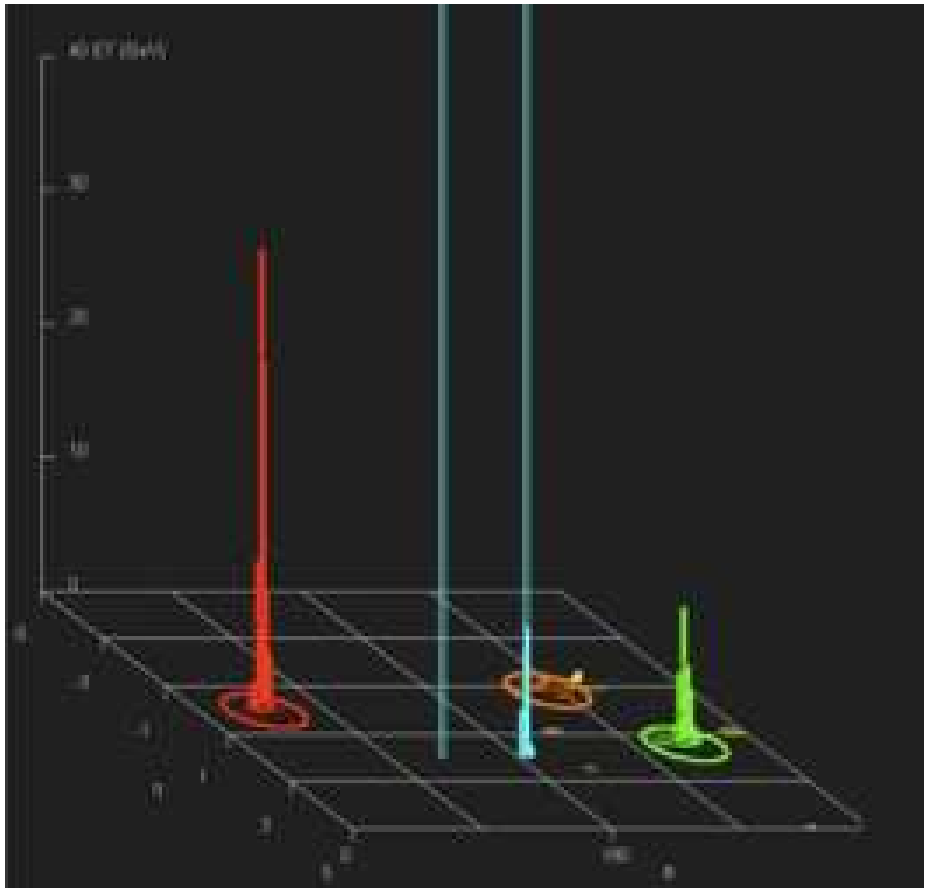
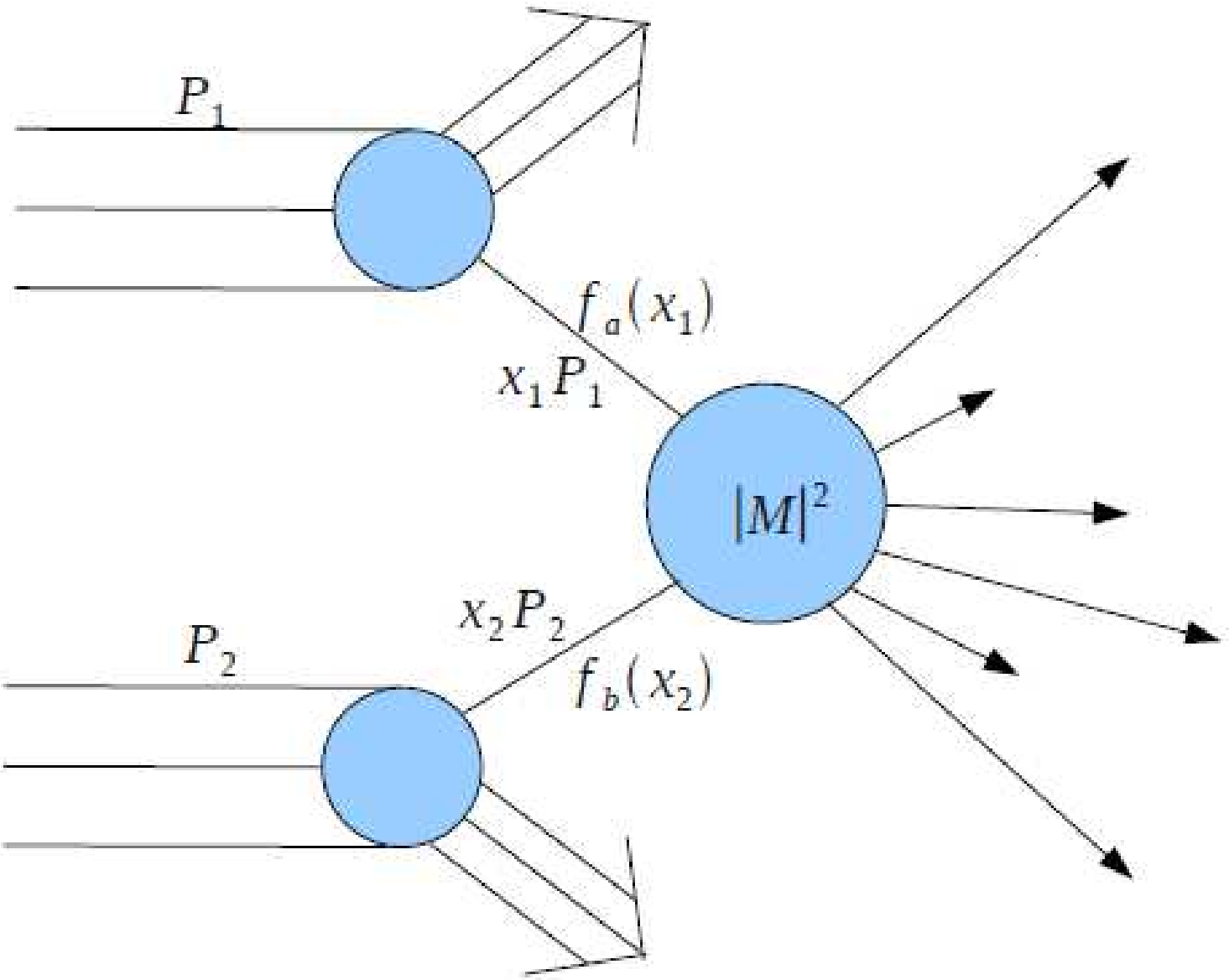


Obviously need the jet algorithms to be well-defined both experimentally (many discrete hits) and theoretically (probing singularity structure). Use fastjet!

The Perturbative Description



Why Study Multi-jet Observables?

We don't have a choice!

1. Many BSM (e.g. SUSY) particles will have *decay chains* involving the production of jets (e.g. 4 jets + $\not{p}_T$). Calculation of signal is easy (one process), SM contribution is very hard (several processes).
2. All LHC processes involves QCD-charged particles; sometimes the (n+1)-jet cross section is as large as the n-jet cross section!
3. It is a challenge we cannot ignore !

Jets at the LHC

The age old hunt...

Effects beyond NLO DGLAP?
...apart from the obvious soft and collinear regions (shower profile)
Do we need more than NLO DGLAP to describe the hard jet events at the LHC?

The News

Data from Tevatron and LHC already show effects beyond pure **NLO** DGLAP...

1. for some observables based on **hard jets**
2. in certain regions of phase space

Scope of this talk

Will not discuss several interesting effects:

- jet broadening (shower profiles)
- impact of underlying event on the jet energy

These are (well?) described by a tunable shower MC.

Will instead focus on the description of the **hard event**, and in particular on observables not well described by pure **NLO** DGLAP. Specifically **not** discussing a breakdown of DGLAP factorisation - only the fixed (NL-) order description.

*Which regions of phase space receive large corrections from **hard perturbative corrections** (= additional jet activity)*

Compare the description of hard jet activity from NLO, NLO+shower, High Energy Jets.
Dijets, W+Dijets, H+Dijets; Similarities in Jet Activity

Multiple (≥ 2) hard jets...

Smaller number of jets solved satisfactory (?) already... (POWHEG, MC@NLO, NNLO,...)
Special radiation pattern from **current-current** scattering
Look into **higher order corrections beyond** "inclusive K-factor"
Concentrate on the **hard, perturbative corrections** relevant for a description of the final state **in terms of jets**.

Goal

Build framework for **all-order summation** (virtual+real emissions). Exact in another limit than the usual soft&collinear. Better suited for describing **radiation relevant for multi-jet** production.

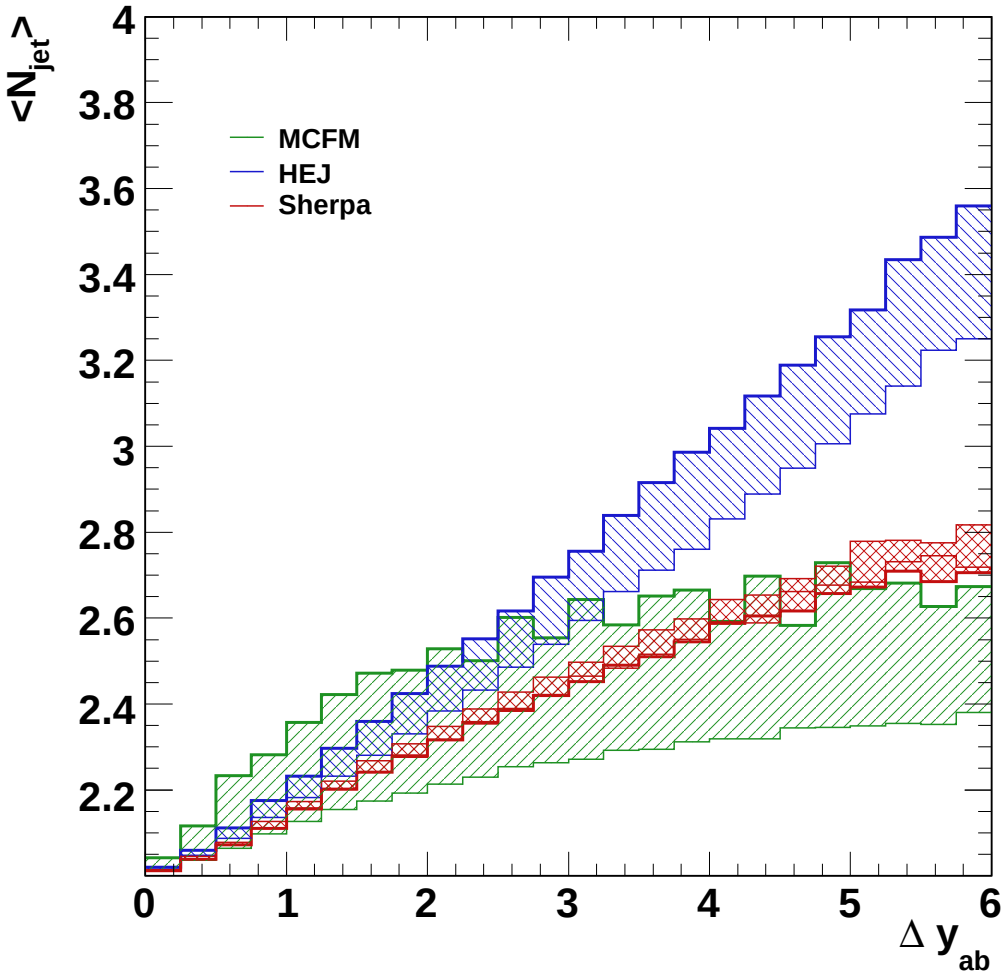
Insight

Can use the insight gained from studying the relevant limit to **guide and improve** analyses: CP-properties of the Higgs-boson couplings

Drivers of Emission

1. Collinear (jet profile)
2. Soft (p_t-hierarchies)
3. Opening of phase space (semi-hard emissions - not related to a divergence of $|M|^2$).
   Think (e.g.) multiple jets of fixed p_t, with increasing rapidity span (span=max difference in rapidity of two hard jets=Δy).
   All calculations will agree that number of additional jets increases - but the amount of radiation will differ (wildly) - e.g. due to **limitations** on the **number** (NLO) or **hardness** (shower) of additional radiation **imposed by theoretical assumptions**.

Increasing Rapidity Span $\rightarrow$ Increasing Number of Jets



J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

h+dijets (at least 40GeV). Δy_{ab}: Rapidity difference between most forward and backward hard jet

Compare NLO (green), CKKW matched shower (red), and High Energy Jets (blue).

All models show a clear increase in the number of hard jets as the rapidity span Δy_{ab} increases.

Please recall this plot when I discuss the results of the ATLAS study of $\langle N_{\text{jets}} \rangle$

HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but. . .

Sufficiently **accurate** that the description is relevant

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Collinear limit $\rightarrow$ enters many resummation formalisms, parton showers. . . .

Like all good limits, the collinear approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider. . .

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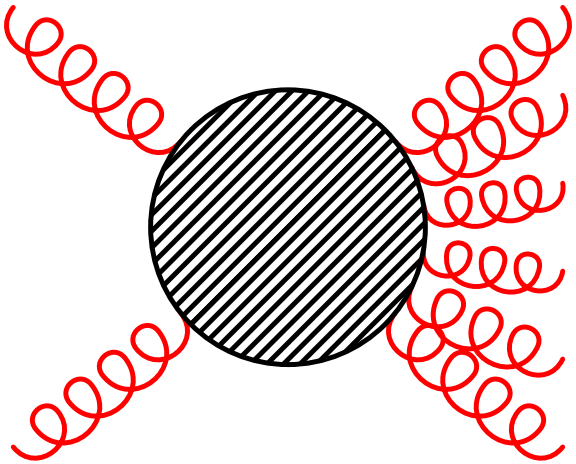
The Possibility for Predictions of n-jet Rates

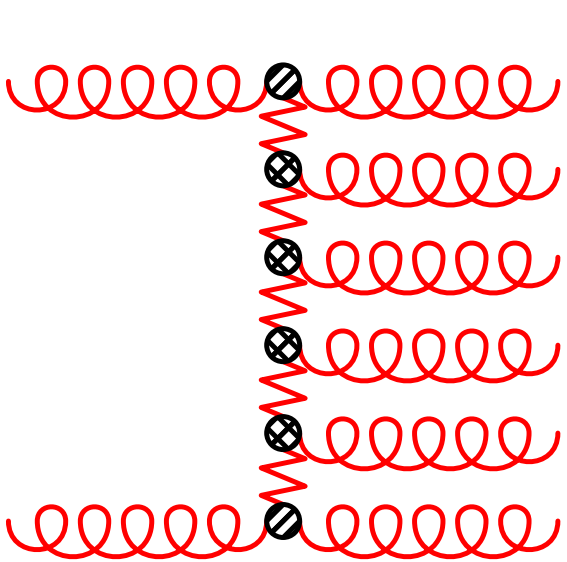
The Power of Reggeisation



High Energy Limit

$$\longrightarrow$$

$|\hat{t}|$ fixed, $\hat{s} \to \infty$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\,\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right) e^{\omega(q_{n+1})(y_n-y_{n+1})}\,\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet hard scattering matrix elements

Universal behaviour of the hard scattering matrix element in the High energy (MRK) limit:

$$\forall i \in \{2, \ldots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}_{gg\to g\cdots g}\right|^2 \longrightarrow \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2}.$$

$$\left|\overline{\mathcal{M}}_{qg\to qg\cdots g}\right|^2 \longrightarrow \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2},$$

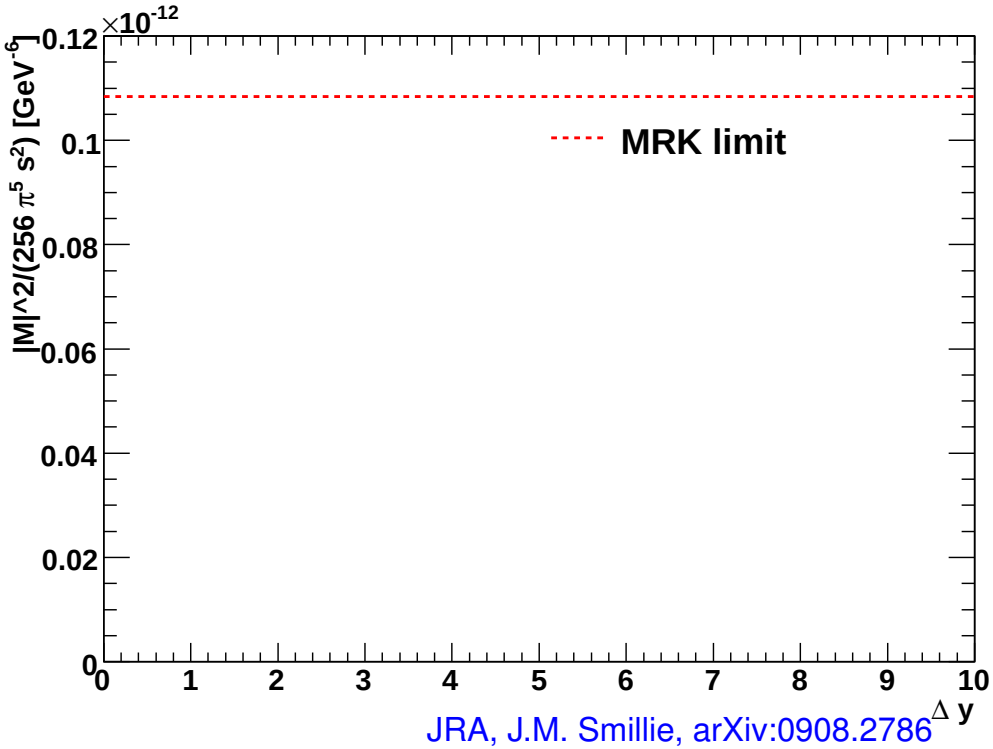
$$\left|\overline{\mathcal{M}}_{qQ\to qg\cdots Q}\right|^2 \longrightarrow \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Comparison of 3-jet hard scattering matrix elements

Study just a slice in phase space, and compare full tree-level with α_s^3-approximation from resummation:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

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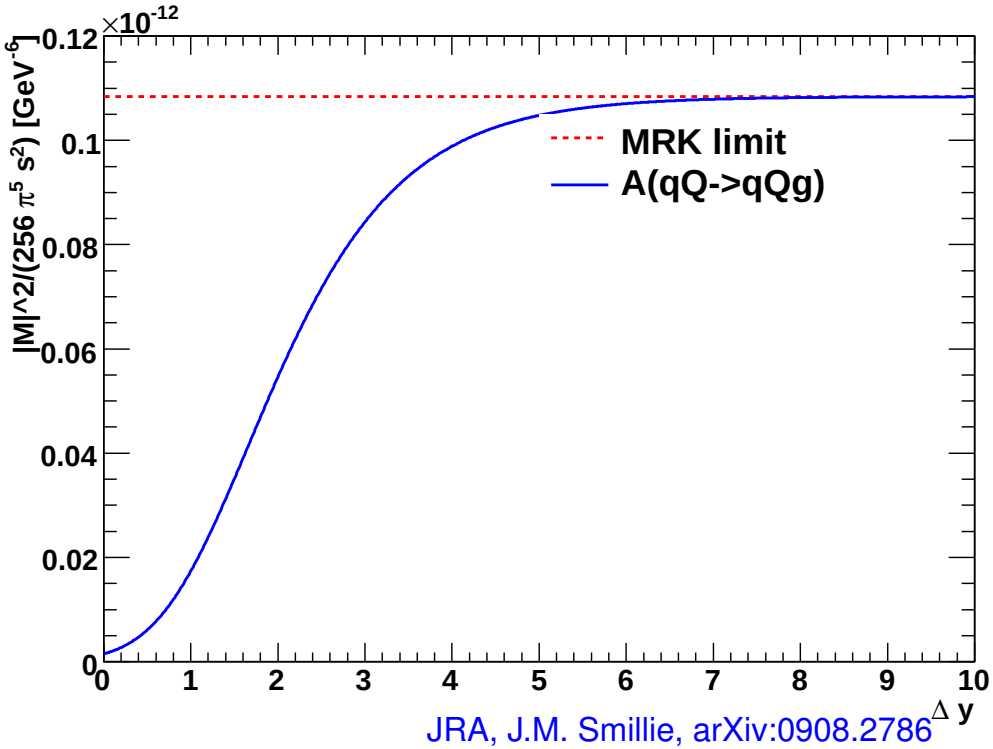

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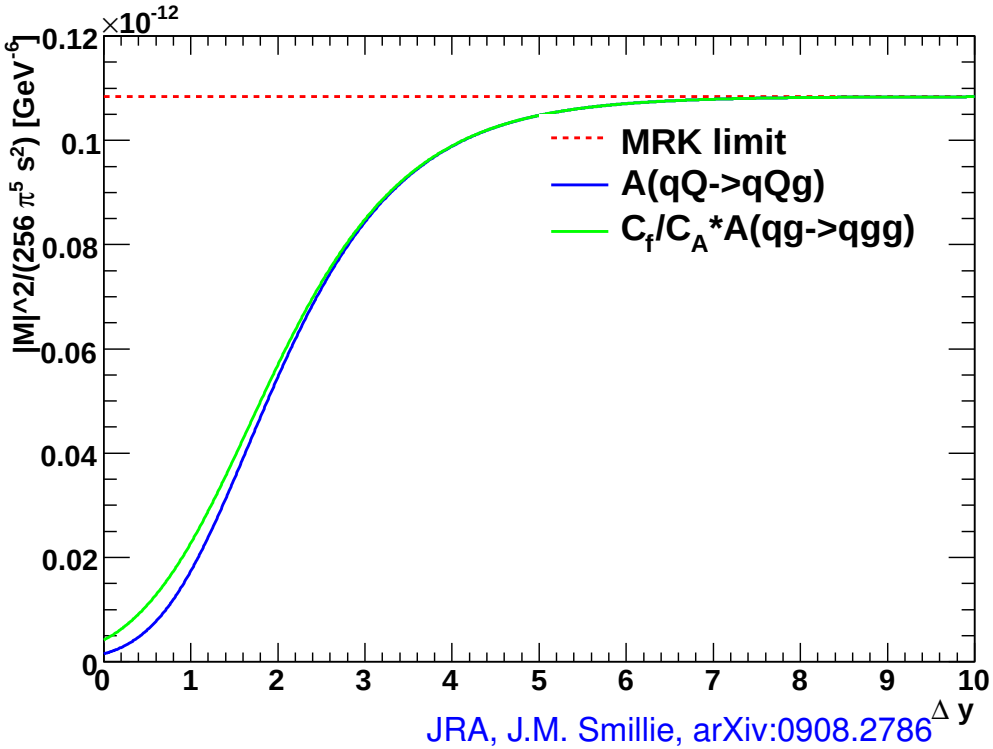

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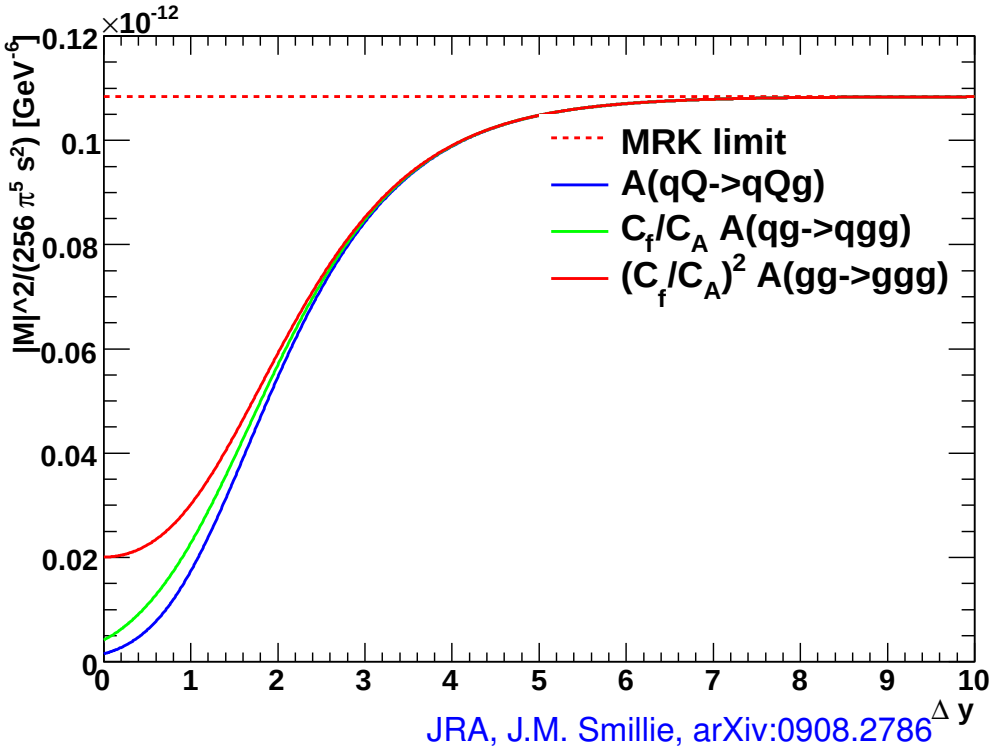

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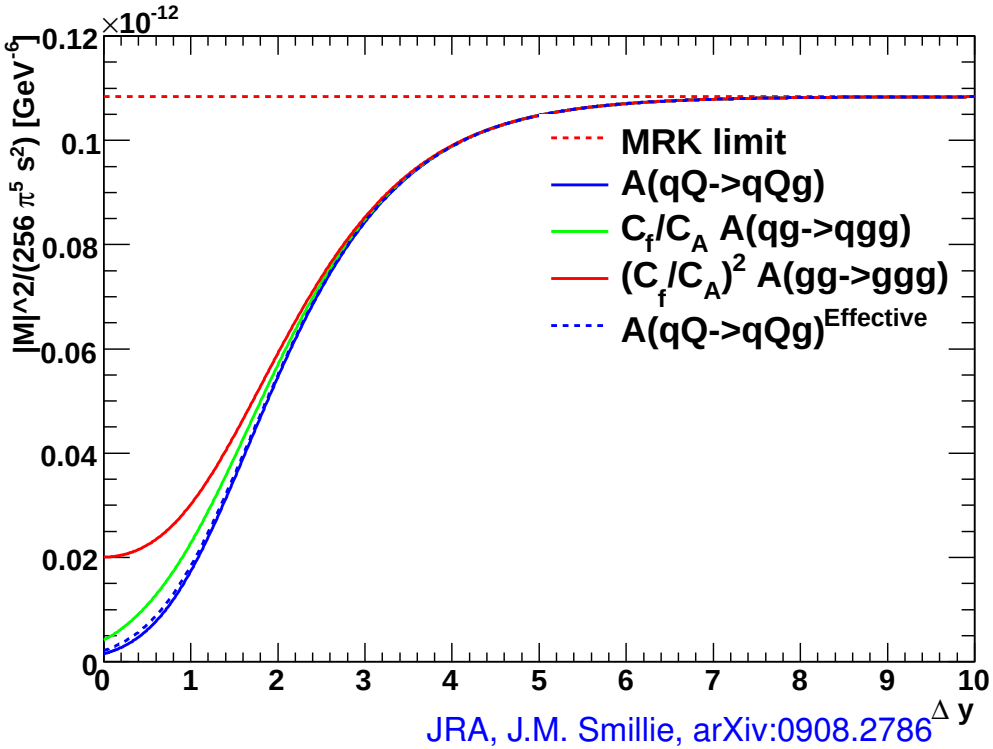

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High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: dominance by *t*-channel colour octet exchange
2) No kinematic approximations in invariants
3) Accurate definition of currents (coupling through *t*-channel exchange)
4) Gauge invariance. Not just asymptotically.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^-Q^-\to q^-Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

***t*-channel factorised**: Contraction of (local) currents across t-channel pole

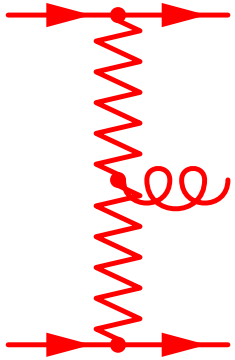
$$\left|\overline{\mathcal{M}}^{t}_{qQ\to qQ}\right|^2 = \frac{1}{4\,(N_C^2-1)} \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\, C_F\, \frac{1}{t_1}\right) \cdot \left(g^2\, C_F\, \frac{1}{t_2}\right).$$

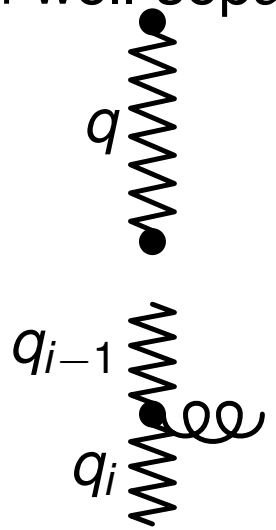
Extend to $2 \to n$ . . .

J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

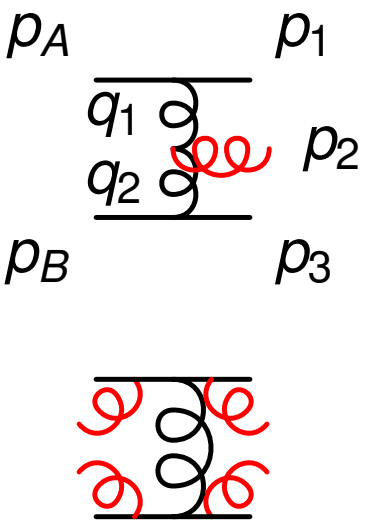




$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

$$\mu \quad V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



$$V^\rho(q_1, q_2) = -(q_1 + q_2)^\rho + \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A} + \frac{p_2\cdot p_B}{p_A\cdot p_B} + \frac{p_2\cdot p_n}{p_A\cdot p_n}\right) + p_A \leftrightarrow p_1 - \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B} + \frac{p_2\cdot p_A}{p_B\cdot p_A} + \frac{p_2\cdot p_1}{p_A\cdot p_1}\right) - p_B \leftrightarrow p_3.$$

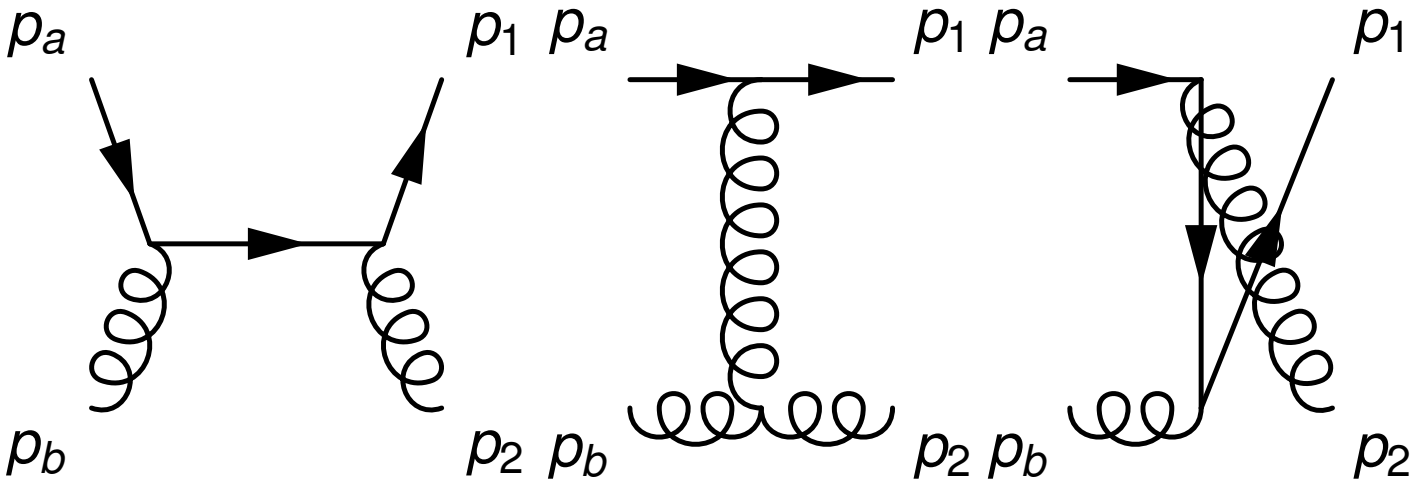
Building Blocks for an Amplitude

$p_g \cdot V = 0$ can easily be checked (**exact** gauge **invariance**)
The approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^{t}_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\,\left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\,C_F\,\frac{1}{t_1}\right)\cdot\left(g^2\,C_F\,\frac{1}{t_2}\right) \cdot \left(\frac{-g^2 C_A}{t_1\,t_2}\,V^{\mu}(q_1,q_2)\,V_{\mu}(q_1,q_2)\right).$$

Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p^*_{2\perp}}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p^-_b}{p^-_2}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p^-_2}{p^-_b}} \right) \langle b|\sigma|2\rangle \times \langle 1|\sigma|a\rangle .$$

Complete t-channel factorisation!

J.M.Smillie and JRA

Quark-Gluon Scattering

The t-channel current generated by a gluon in qg scattering is that genersated by a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

instead of C_F. Tends to C_A in the MRK limit.

Similar results for e.g. $g^+g^- \to g^+g^-$ (well-defined t-channel): **Exact, complete** t-channel **factorisation**.

By using the formalism of **current-current scattering**, we get a better description of the t-channel pole than by using just the MRK kinematic limit of BFKL.

Performing the Explicit Resummation

Analytic subtraction of soft divergence from real radiation:

$$\left|\mathcal{M}_t^{p_a p_b \to p_0 p_1 p_2 p_3}\right|^2 \xrightarrow{\mathbf{p}_1^2 \to 0} \left(\frac{4 g_s^2 C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_t^{p_a p_b \to p_0 p_2 p_3}\right|^2$$

Integrate over the soft part $\mathbf{p}_1^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

$$\int_0^{\lambda} \frac{\mathrm{d}^{2+2\varepsilon}\mathbf{p}\, \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon}\, 4\pi} \left(\frac{4 g_s^2 C_A}{\mathbf{p}^2}\right) \mu^{-2\varepsilon}$$
$$= \frac{4 g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi}\, \Delta y_{02}\, \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^{\varepsilon}$$

Pole in ε cancels with that from the **virtual corrections**

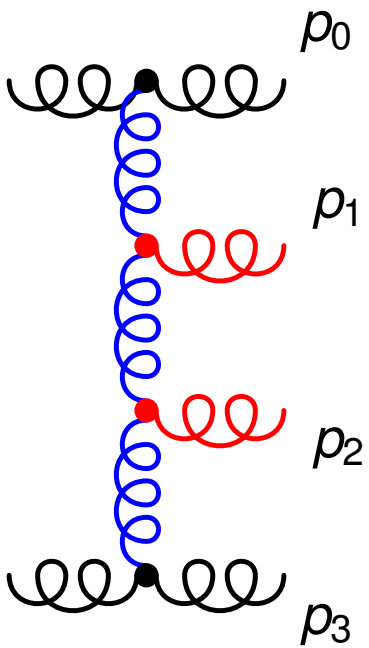
$$\frac{1}{t_1} \to \frac{1}{t_1} \exp\left(\hat{\alpha}(t) \Delta y_{02}\right) \qquad \hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^{\varepsilon}.$$

Expression for the Regularised Amplitude

$$\overline{\left|\mathcal{M}^{\text{reg}}_{\text{HEJ}}(\{p_i\})\right|^2} = \frac{1}{4\,(N_C^2-1)}\,\|S_{f_1 f_2\to f_1 f_2}\|^2 \cdot \left(g^2\,K_{f_1}\,\frac{1}{t_1}\right)\cdot\left(g^2\,K_{f_2}\,\frac{1}{t_{n-1}}\right)$$

$$\cdot\prod_{i=1}^{n-2}\left(g^2 C_A\left(\frac{-1}{t_i t_{i+1}}V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\,\theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right)$$

$$\cdot\prod_{j=1}^{n-1}\exp\left[\omega^0(q_j,\lambda)(y_{j-1}-y_j)\right], \qquad \omega^0(q_j,\lambda) = -\frac{\alpha_s N_C}{\pi}\log\frac{\mathbf{q}_j^2}{\lambda^2}.$$



All-Order Summed (and Matched) Cross Section

The cross section is calculated as the sum over the phase space integrals of the explicit n-body phase space

$$\sigma_{2j}^{\text{sum,match}} = \sum_{n=2}^{\infty} \sum_{f_1,f_2} \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2}$$

$$\times \quad \mathcal{O}_{2j}(\{p_i\}) \times \sum_m \mathcal{O}_{mj}^e(\{p_i\})\, w_{m-\text{jet}}$$

$$\times \quad x_a f_{A,f_1}(x_a, Q_a)\, x_2 f_{B,f_2}(x_b, Q_b)\, (2\pi)^4\, \delta^2\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp} \right).$$

Matching to fixed order (tree-level so far) is obtained by clustering the n-parton phase space point into m-jet momenta and multiply by the ratio of full to approximate matrix element:

$$w_{m-\text{jet}} \equiv \frac{\overline{\left|\mathcal{M}^{f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}{\overline{\left|\mathcal{M}^{t, f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}.$$

Summary: All-Orders, Regularisation, etc.

- Have prescription for $2 \to n$ matrix element, including virtual corrections: Lipatov Ansatz $1/t \to 1/t \exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can **calculate** the **sum over the n-particle phase space** explicitly ($n \sim 30$) to get the **all-order corrections** (just as if one had provided all the $N^{30}LO$ matrix elements and a regularisation procedure)
- **Merge** n-jet tree-level MEs (by merging m-parton momenta to n hard jet-momenta) where these can be evaluated in reasonable time
  Extension of merging mechanism to **NLO** ongoing
- HEJ recently merged with a **dipole shower** (Ariadne)

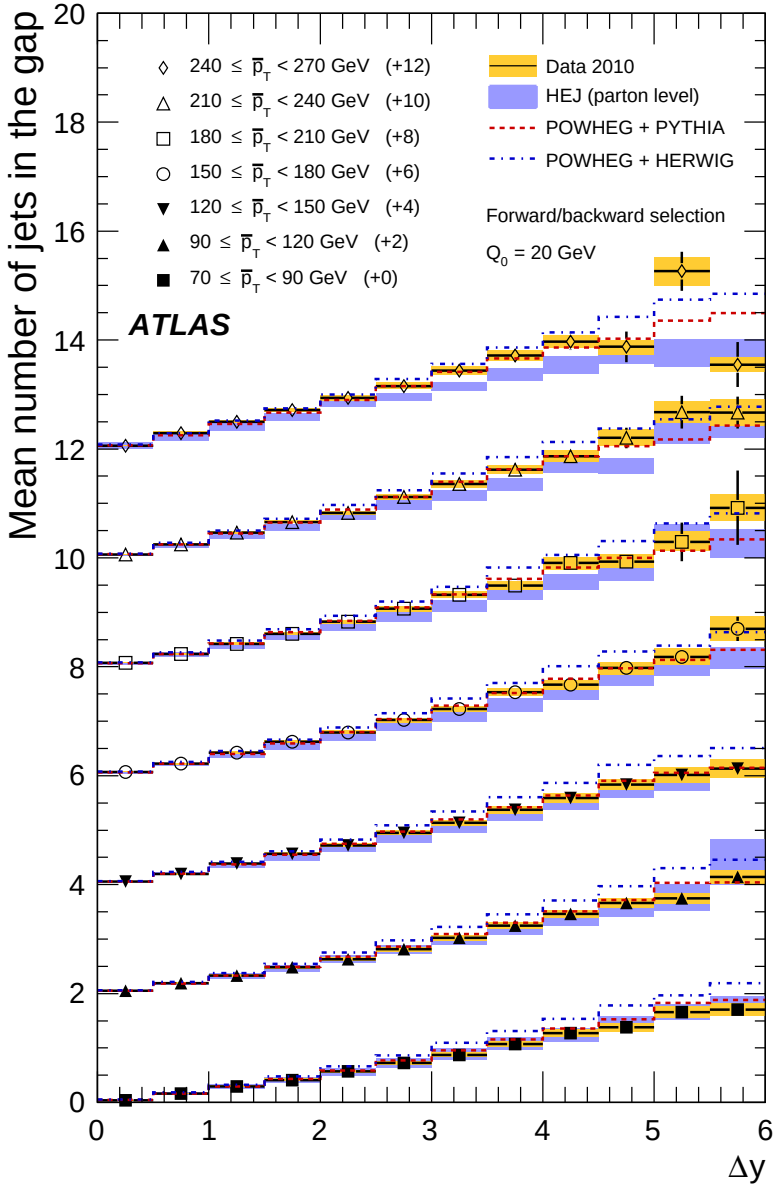
Comparison to data

Two drivers for multi-jet production:

- large ratio of transverse scales (shower resummation)
- Colour exchange over a range in rapidity

Both the Tevatron and the LHC has the energy to explore the second mechanism.
Several interesting studies already, and more to come!
New results from ATLAS for W+jets presented at ICHEP on Friday.

ATLAS: Study of Further Jet Activity in Dijet Events



CERN-PH-EP-2011-100

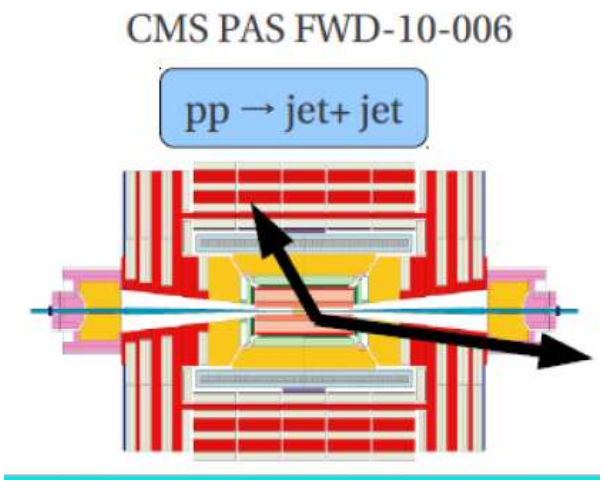
This Atlas analysis tests **both** of the two "drivers" of jet production. (cut on $\bar{p}_t$ induces large p_t-hierarchy on forward/backward jet, besides the hierarchy between large $\bar{p}_t$ and Q_0, the general jet scale)

HEJ slightly undershoots the jet activity when large ratios of transverse scales are imposed (shower region).

Very good agreement in the most important regions of phase space

Obviously **beyond** NLO (more than one extra jet **on average** at $\Delta y \geq 3$!)

CMS: Simultaneous prod. of central and forward jet

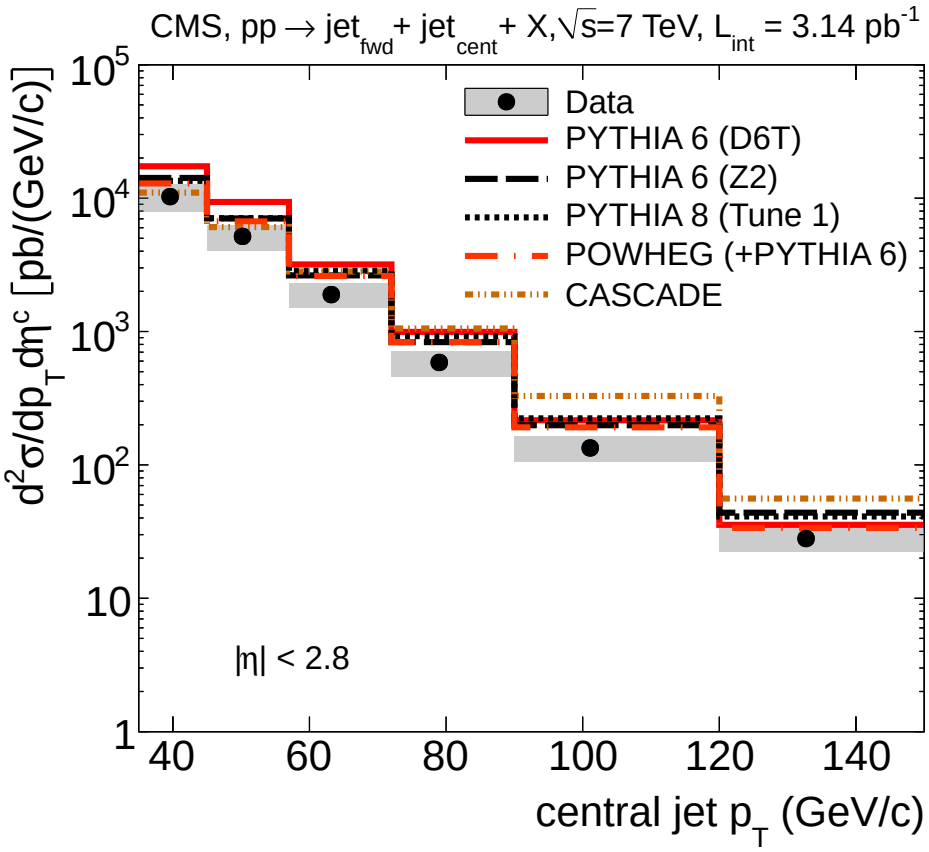


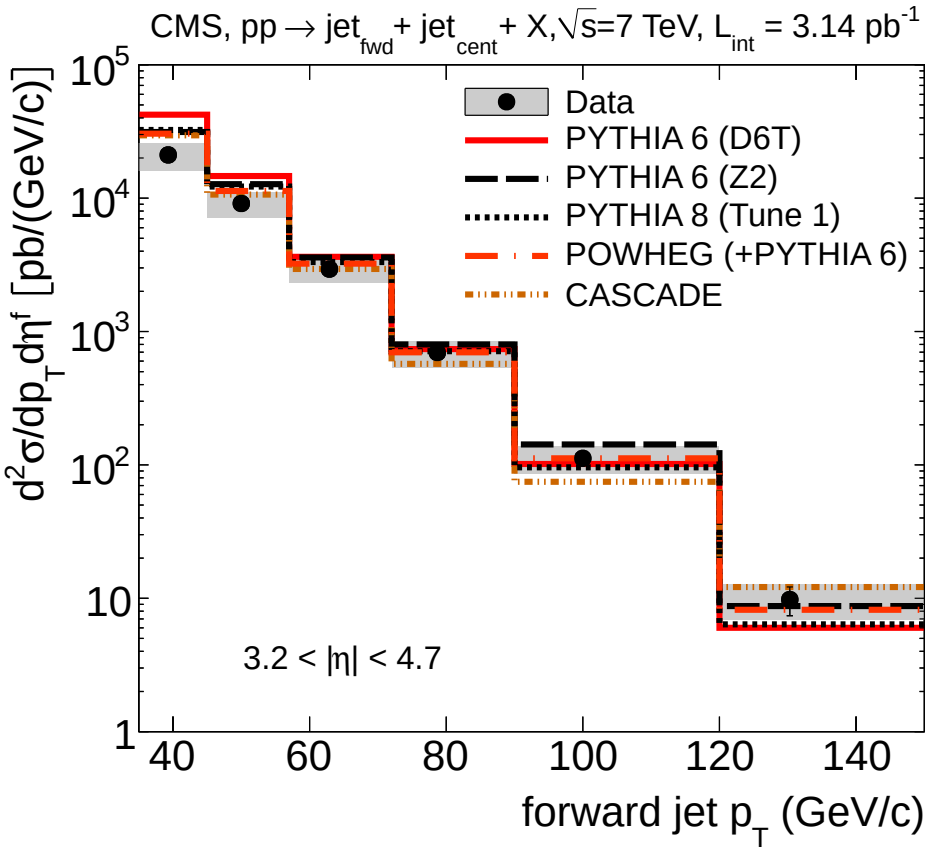
Jets: anti-kt, R=.5, $p_t > 35$GeV

$$\text{central} : |\eta| < 2.8$$
$$\text{forward} : 3.2 < |\eta| < 4.7$$

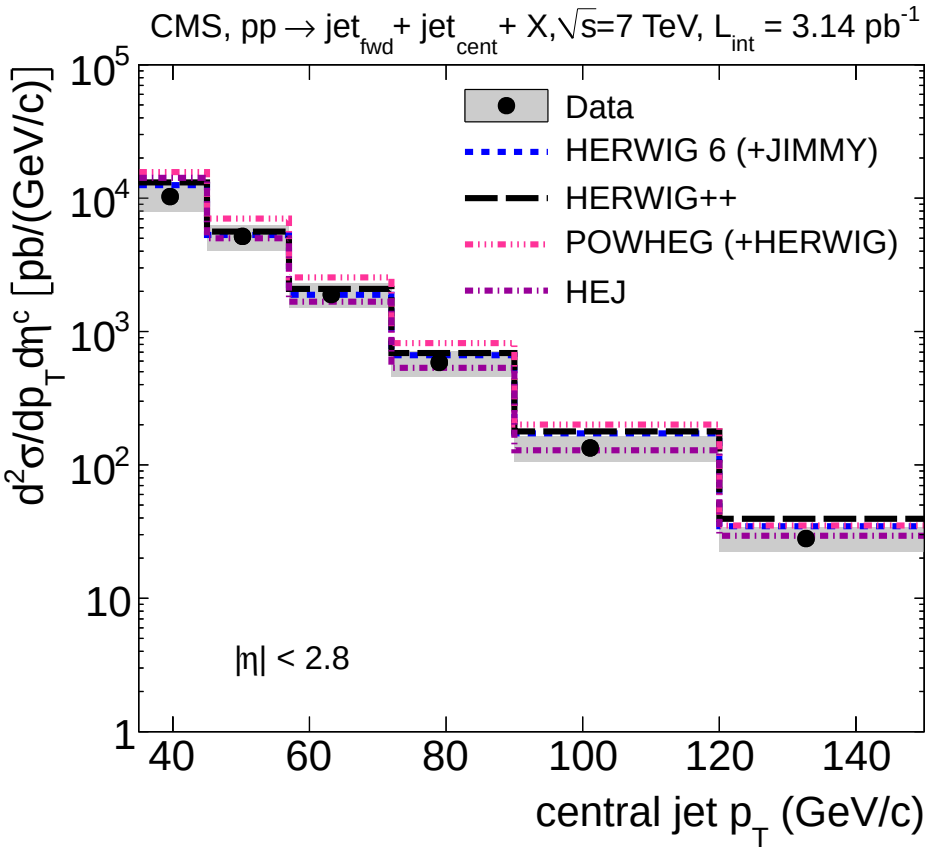
(not particularly large rapidity spans, typically 1 unit).
Measure the p_t-spectrum of the central and the forward jet. Any difference is obviously due to additional radiation.

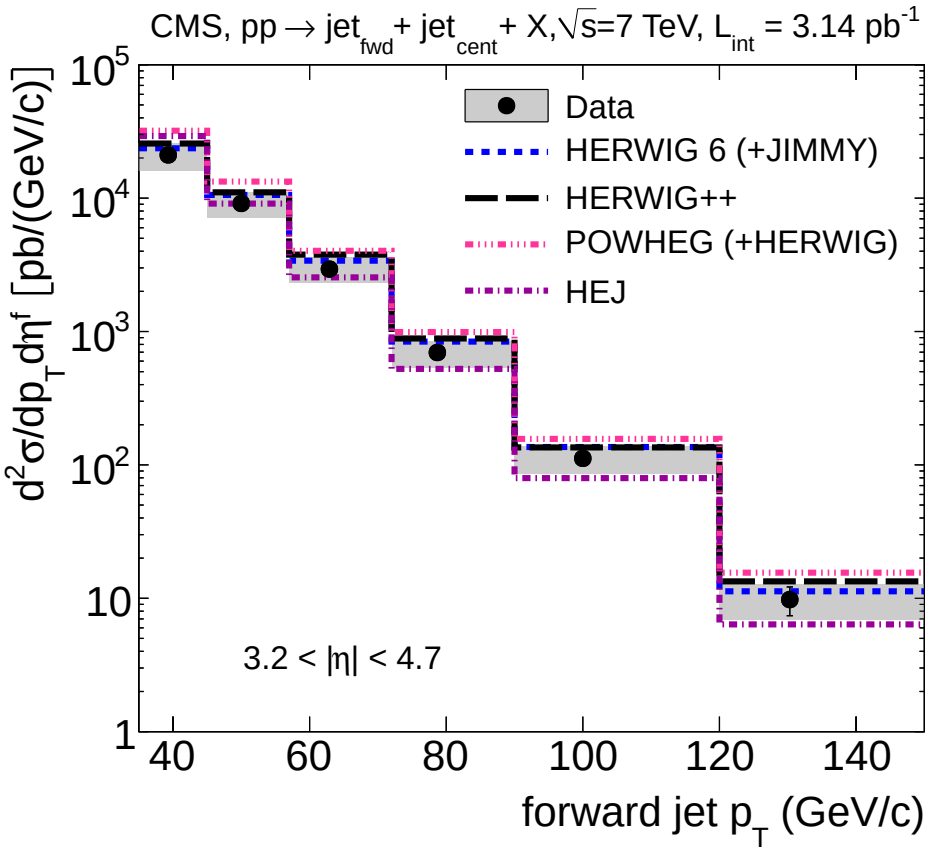
Comparison to Theory, I





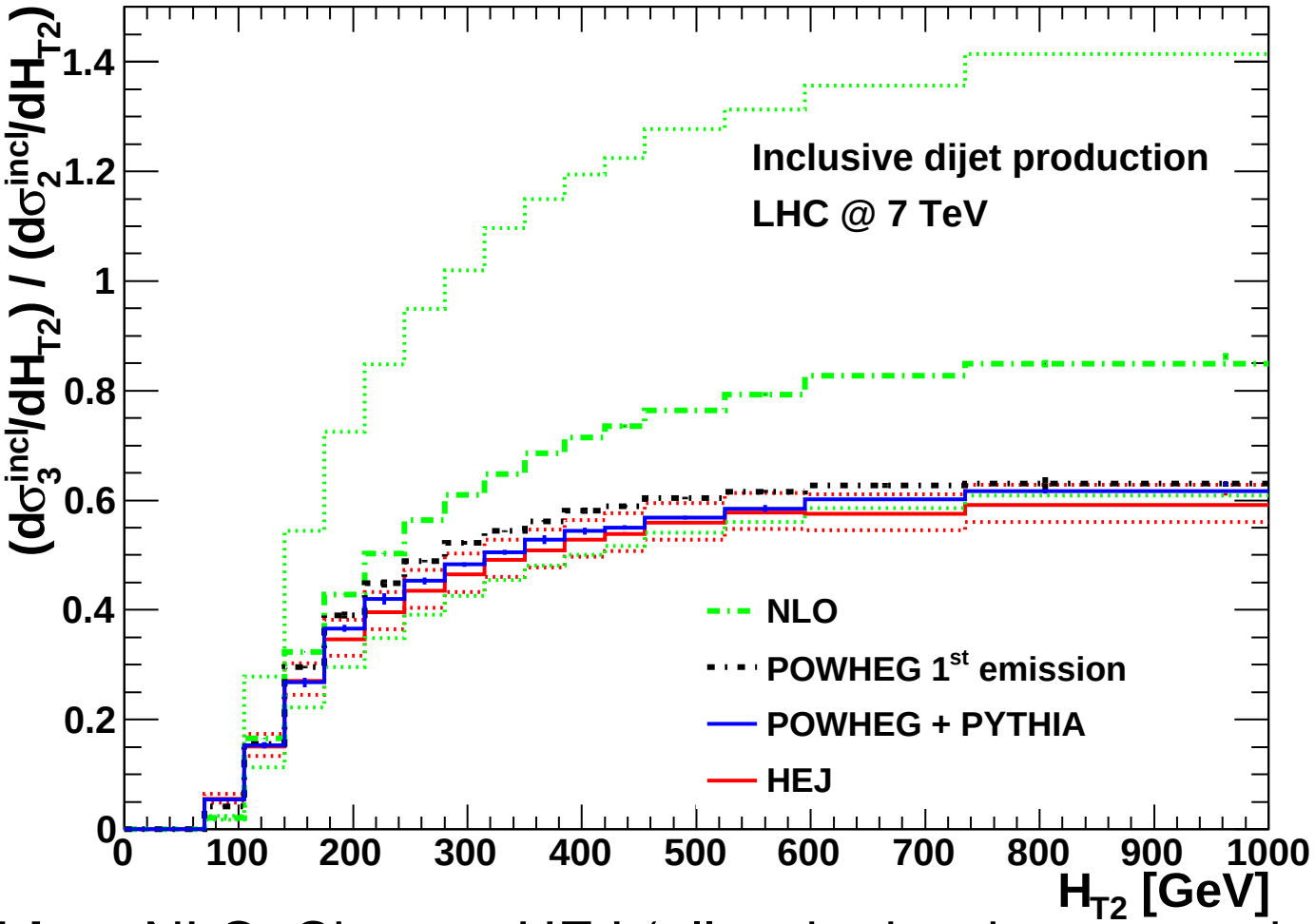
Comparison to Theory, II





Predictions for ratio of Inclusive Jet Rates vs. H_{T2}

S. Alioli, E. Re, J.M. Smillie, C. Oleari, JRA; arXiv:1202.1475

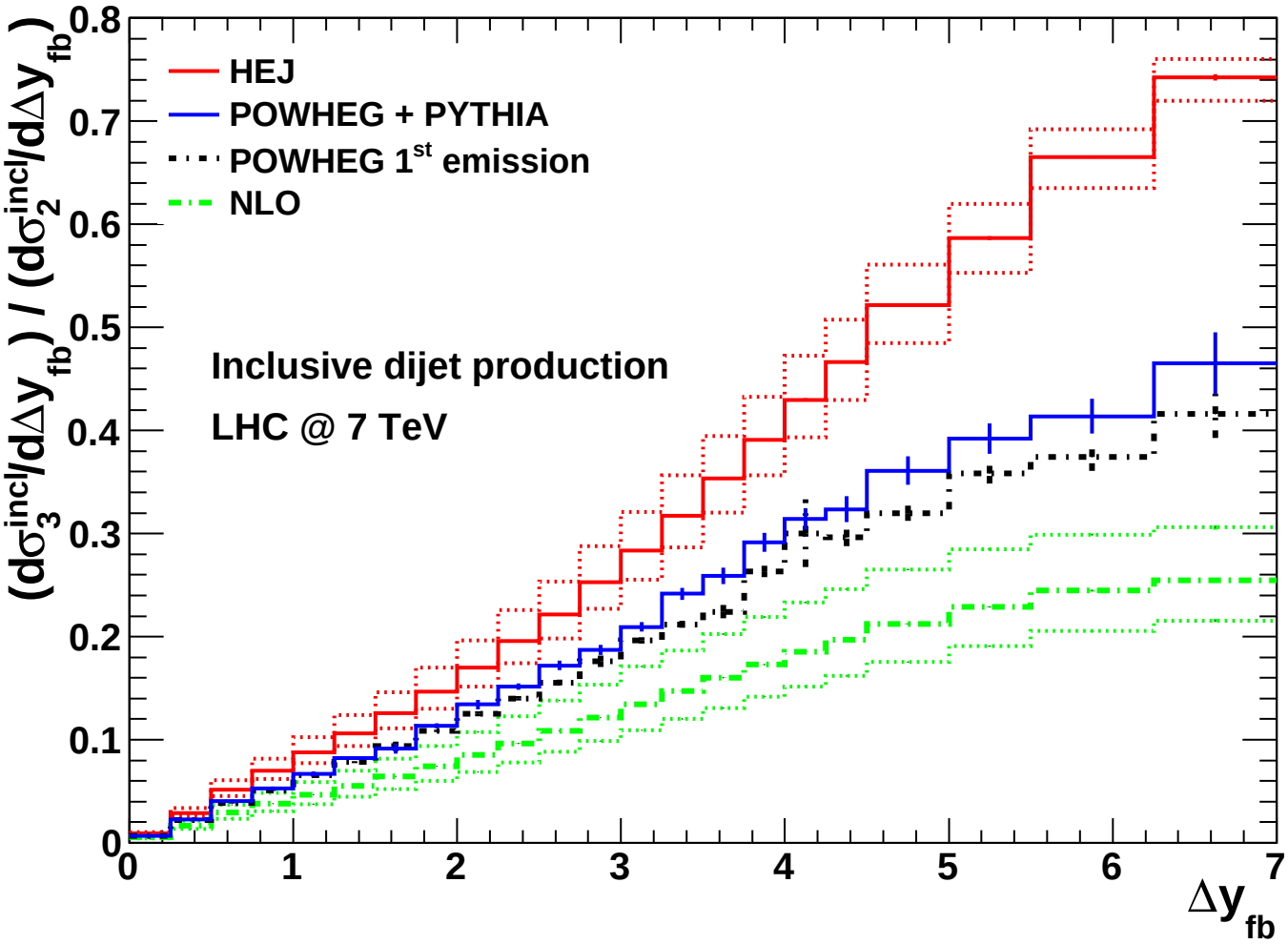


Similarities: NLO+Shower, HEJ (all-order hard resummation)

Difference: NLO

Ratio of Inclusive Jet Rates vs. Rapidity

S. Alioli, E. Re, J.M. Smillie, C. Oleari, JRA; arXiv:1202.1475



Clear differences: NLO, POWHEG, HEJ

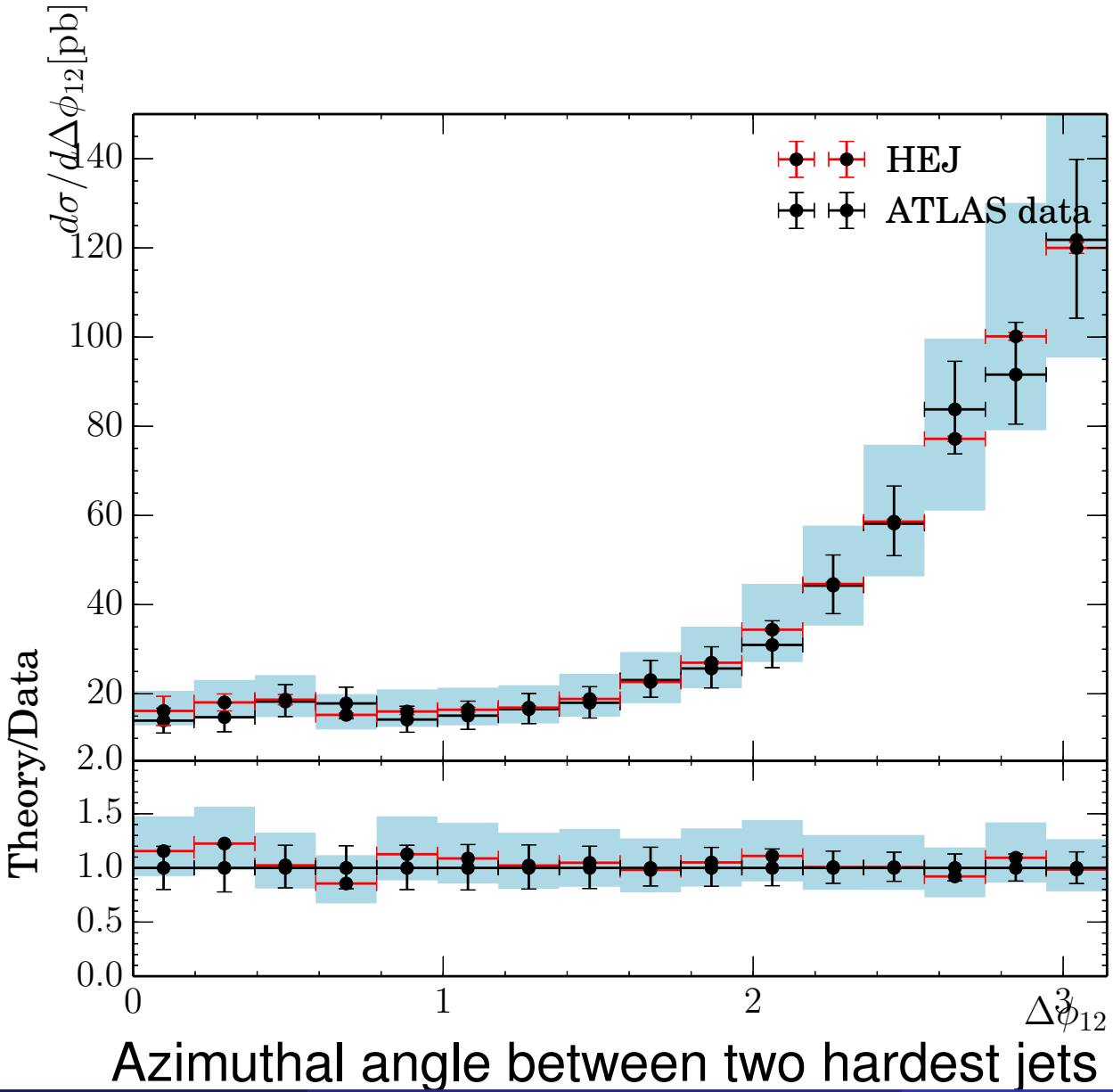
Summary of dijet study

Simple set of cuts, combined with a **exclusive dijet-analyses** can discriminate clearly between the **mechanisms of perturbative corrections** implemented in NLO, POWHEG (NLO+Shower) and High Energy Jets.

W+DiJets

ATLAS: Phys. Rev. D85 (2012) 092002

T. Hapola, J.M. Smillie, JRA (JHEP 1209 (2012) 047)

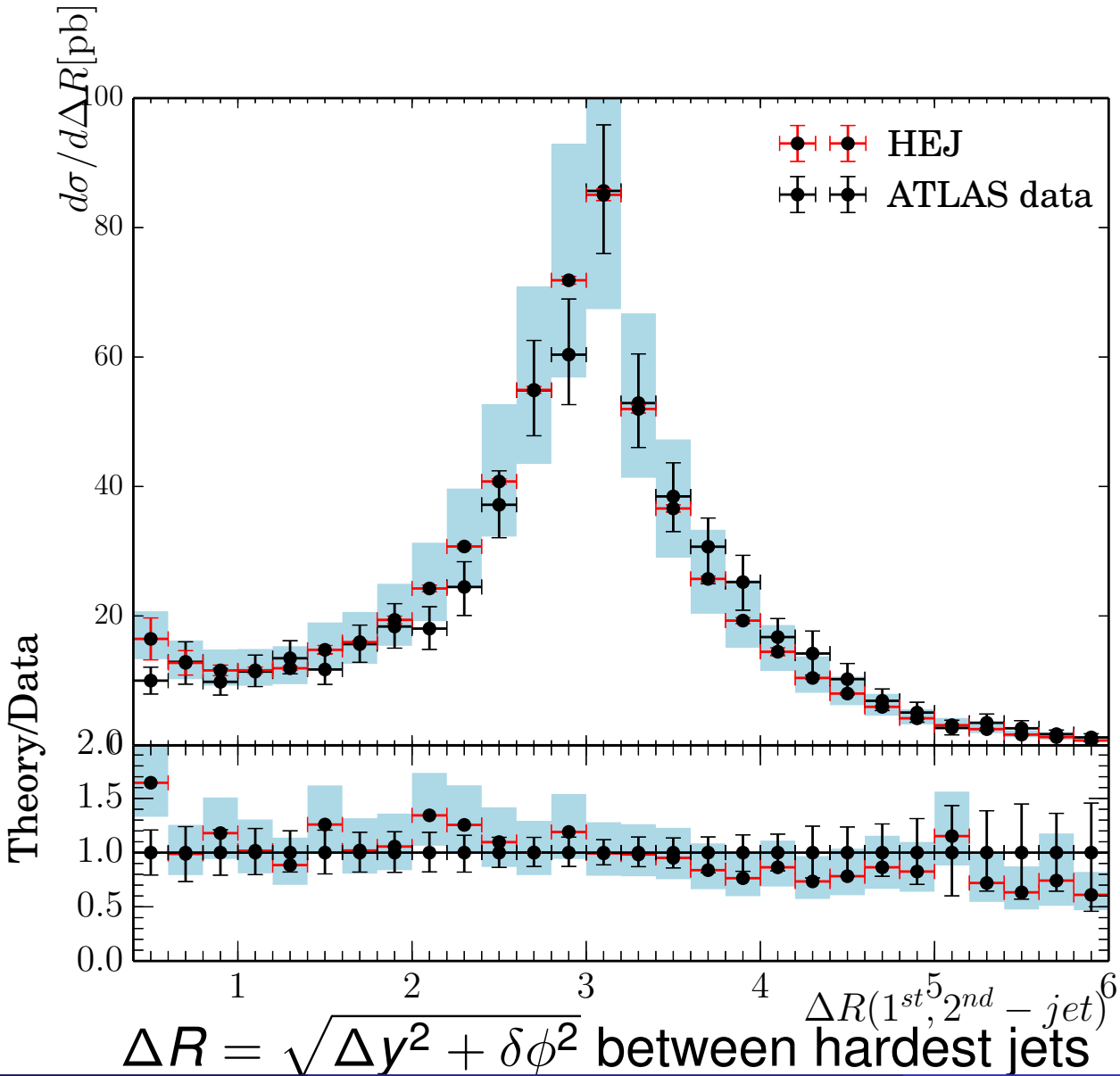


Azimuthal angle between two hardest jets

W+DiJets

ATLAS: Phys. Rev. D85 (2012) 092002

T. Hapola, J.M. Smillie, JRA (JHEP 1209 (2012) 047)

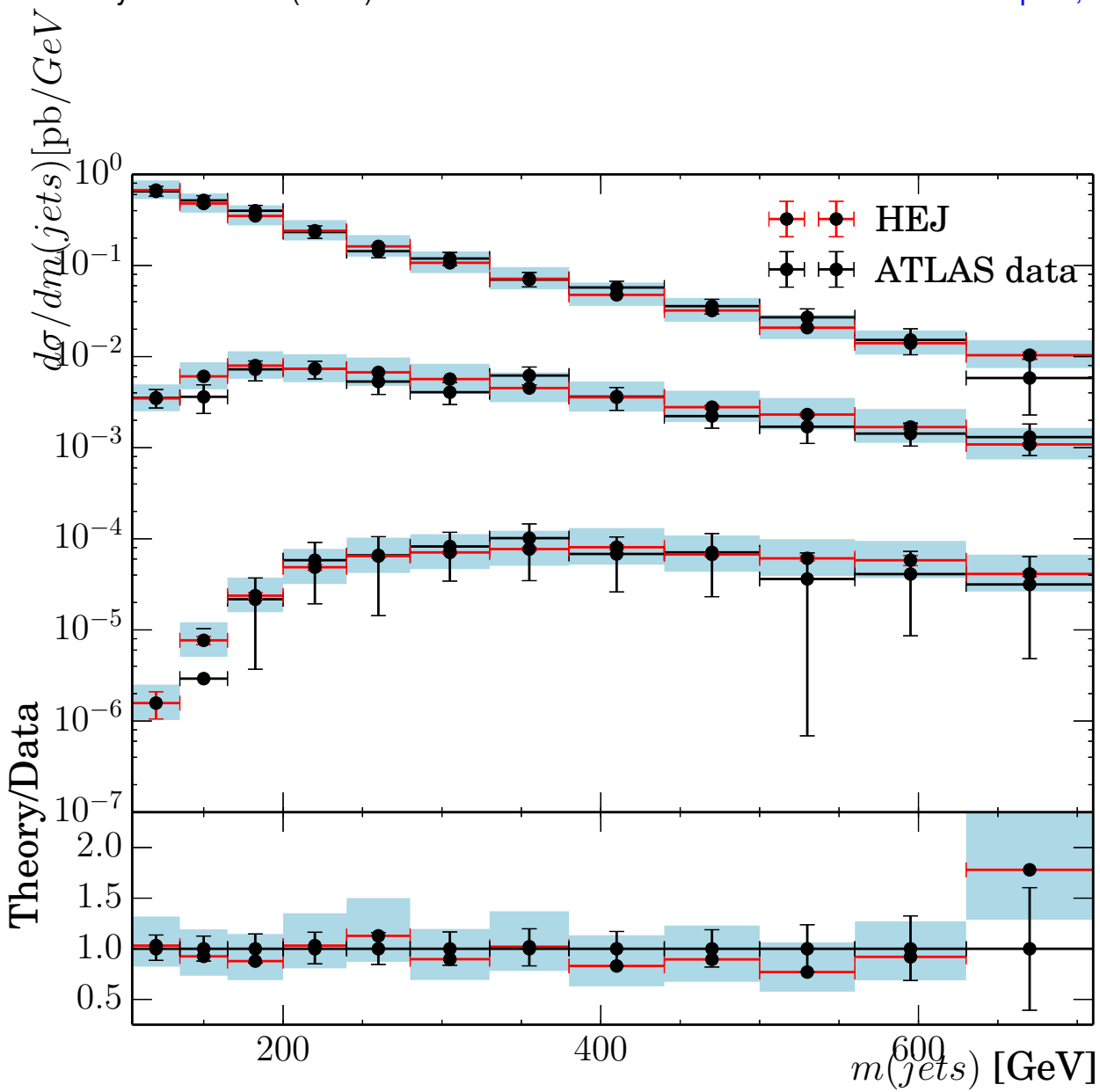


$\Delta R = \sqrt{\Delta y^2 + \delta\phi^2}$ between hardest jets

W+DiJets

ATLAS: Phys. Rev. D85 (2012) 092002

T. Hapola, J.M. Smillie, JRA (JHEP 1209 (2012) 047)

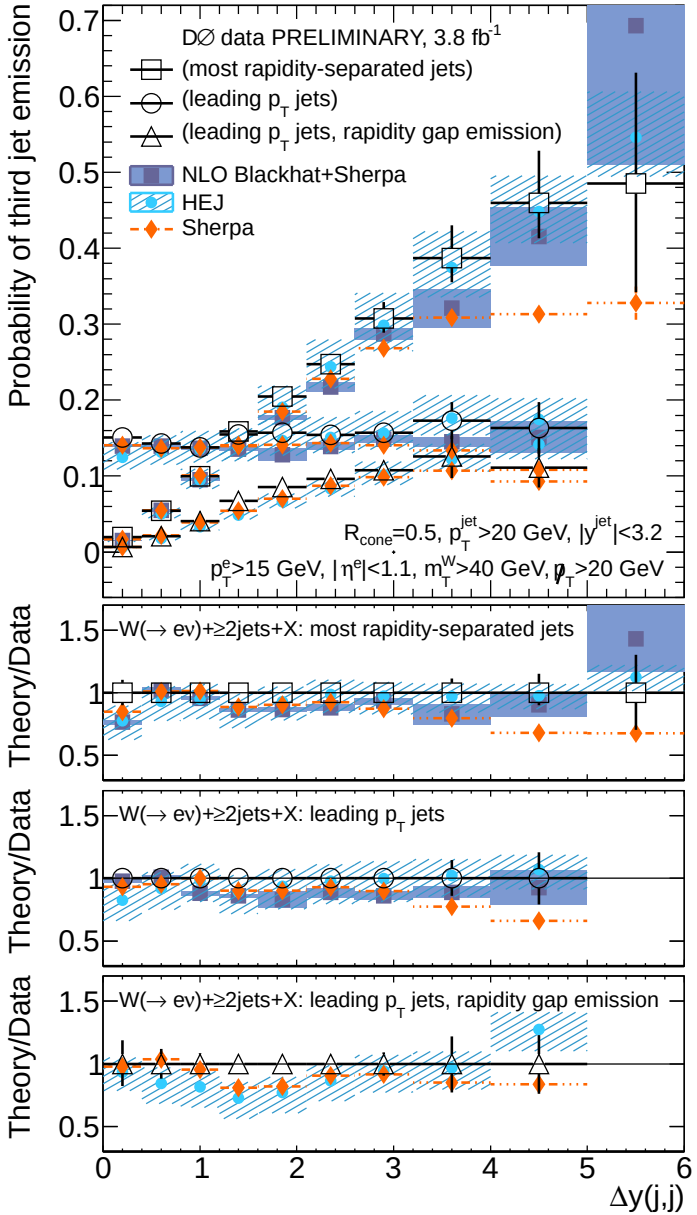


Good description everywhere, in particular also at large invariant mass between jets.
Important region for HJJ-analyses (see later).

D0: W+Jets Phys. Rev. D 88, 092001

D0 measurement of the probability of at least one additional jet when requiring just a *W* in association with two jets. Probability measured vs. rapidity separation of

1. the two most rapidity separated jets
2. the two hardest (in pt) jets
3. the two hardest (in pt) jets, counting additional jets only in the rapidity interval between the two hardest jets



CP Properties of Higgs-Boson Couplings from Hjj through Gluon Fusion

Stabilising the Extraction against Higher Order Corrections

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

. . . in a region of phase space where the **perturbative corrections are large**.
How do we deal with events with **three or more** jets?

The Solution

By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

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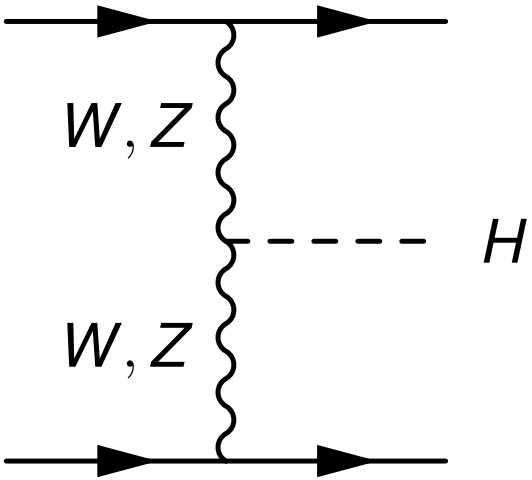
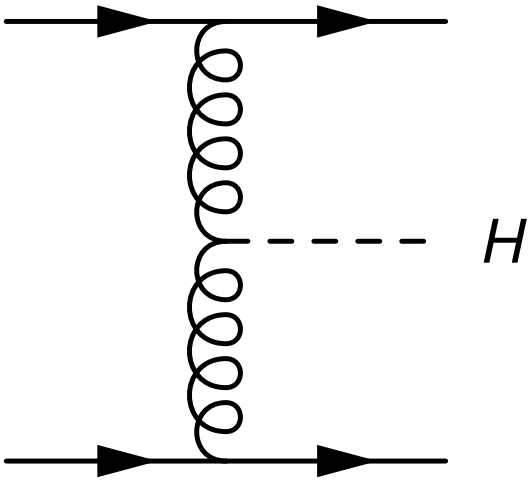
Higgs Couplings through Azimuthal Correllations



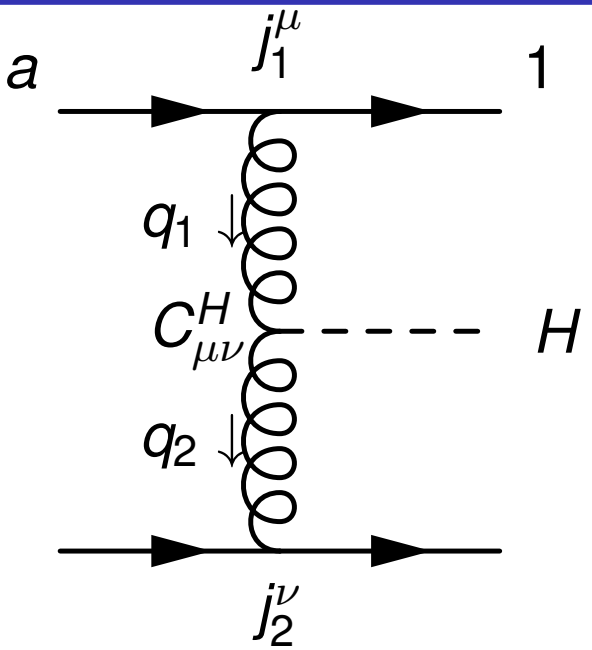
Considerations for Weak Boson Fusion

Higgs Couplings through Azimuthal Correllations



. . . and gluon fusion (Higgs coupling to gluons through top loop)

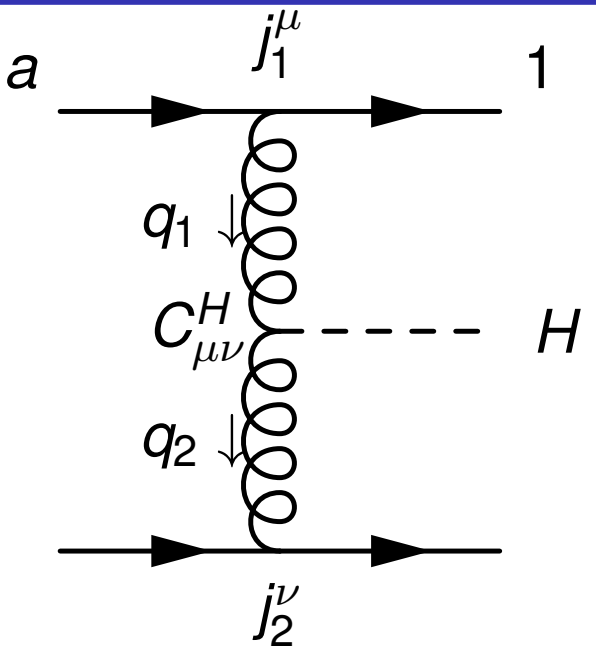
Higgs Couplings through Azimuthal Correllations



$$\mathcal{M} \propto \frac{j_1^\mu \; C^H_{\mu\nu} \; j_2^\nu}{t_1 \; t_2}, \qquad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a$$

$$C_H^{\mu\nu} = a_2 \left(q_1 q_2 g^{\mu\nu} - q_1^\nu q_2^\mu \right) + a_3 \; \varepsilon^{\mu\nu\rho\sigma} \; q_{1\rho} \; q_{2\sigma}.$$

Higgs Couplings through Azimuthal Correllations



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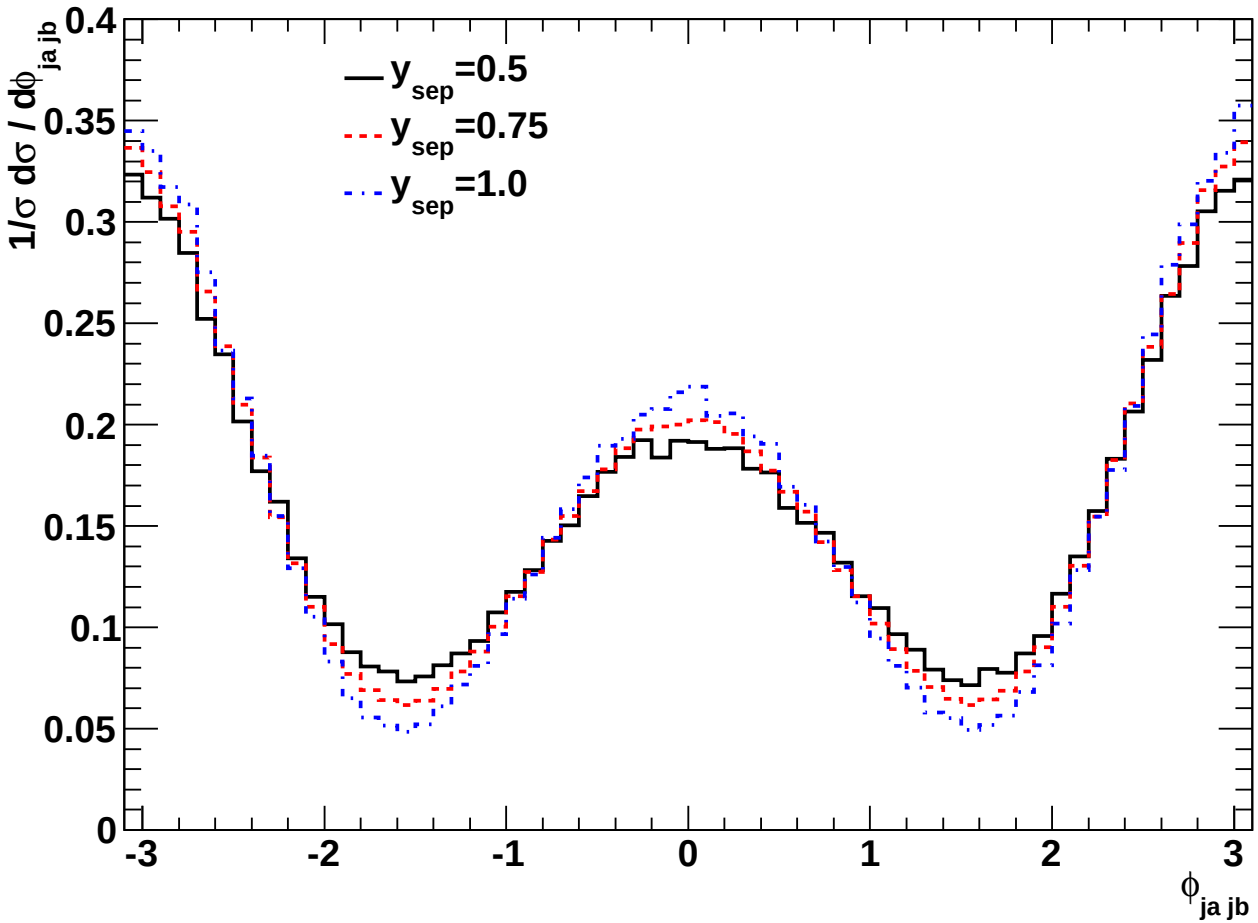
Take e.g. the term $\varepsilon^{\mu\nu\rho\sigma} \; q_{1\rho} \; q_{2\sigma}$: for $|p_{1,z}| \gg |p_{1,x,y}|$ and for small energy loss (i.e. $\overline{\psi}_1 \gamma^\mu \psi_a \to 2p_a, \overline{\psi}_2 \gamma^\mu \psi_b \to 2p_b, p_{a,e} \sim p_{1,e}$):

$$\left[j_1^0 \; j_2^3 - j_1^3 \; j_2^0\right] \left(\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp}\right).$$

In this limit, the azimuthal dependence of the propagators is also suppressed: $|\mathcal{M}|^2$:$\sin^2(\phi)$ **(CP-odd)**, $\cos^2(\phi)$ **(CP-even)**.

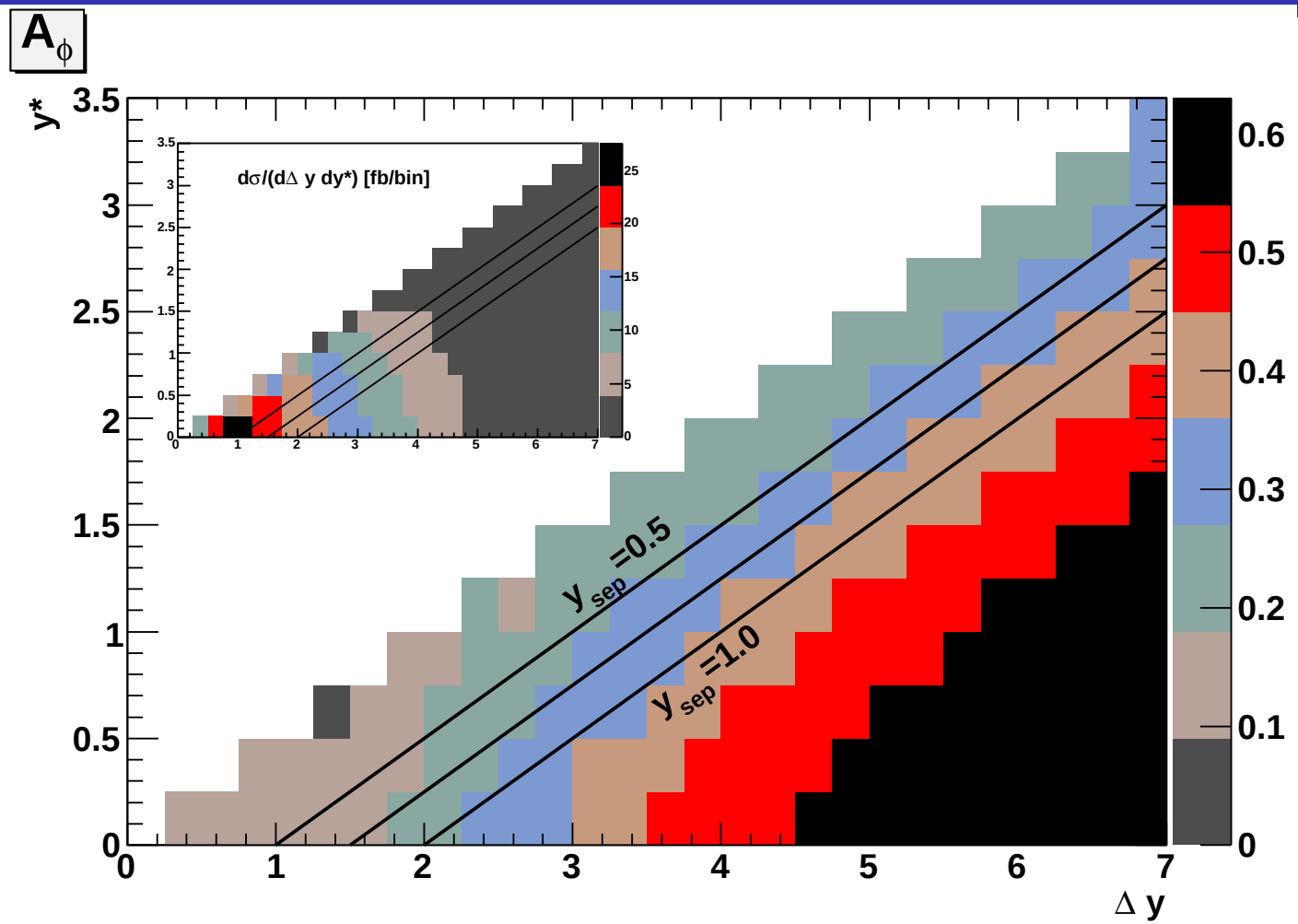
Azimuthal distribution



JRA, K. Arnold, D. Zeppenfeld (JHEP 1006 (2010) 091)

$$CP\text{-even},\ p_{j\perp} > 40\ \text{GeV}, \quad y_{ja} < y_h < y_{jb},$$
$$|y_{ja,j_b}| < 4.5, \min\left(|y_h - y_{ja}|, |y_h - y_{j_b}|\right) > y_{\text{sep}}.$$

Signature and Cross Section

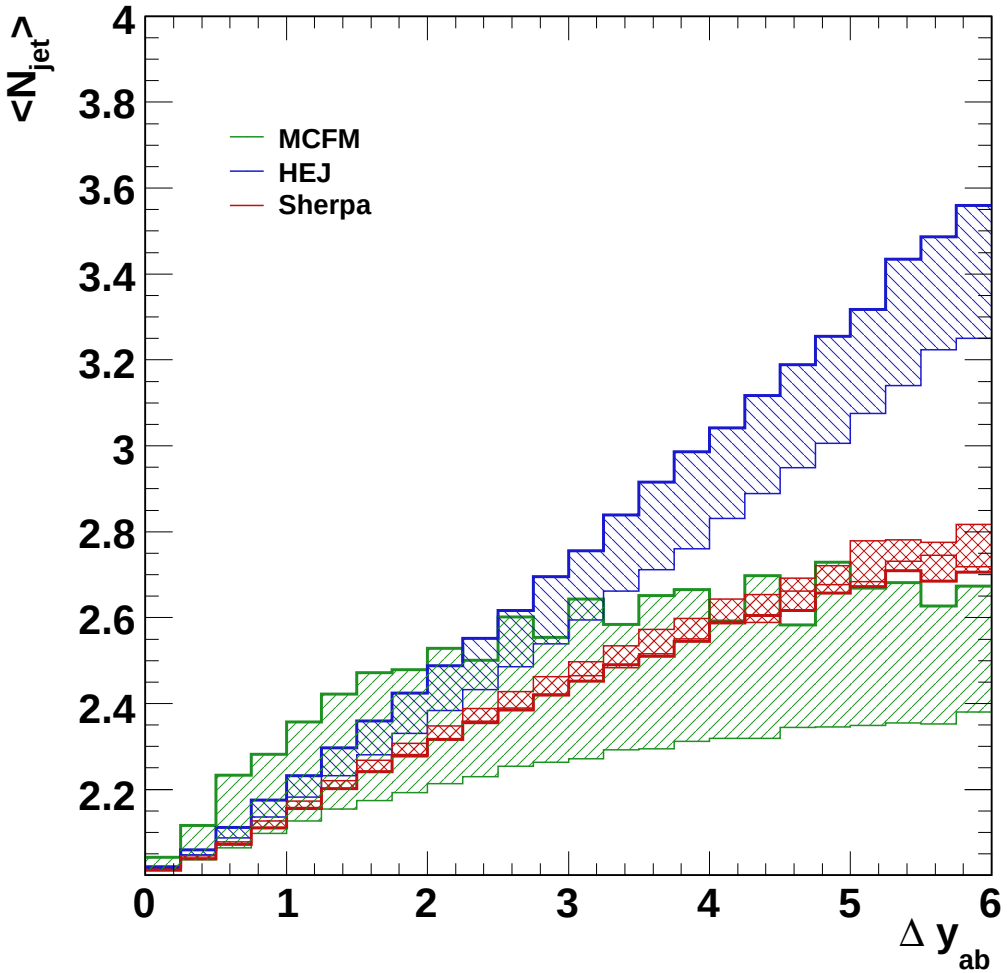


$$\Delta y = |y_{j_a} - y_{j_b}|, \quad y^* = y_h - \frac{y_{j_a}+y_{j_b}}{2}.$$

JRA, K. Arnold, D. Zeppenfeld

Rapidity separation between the jets and the Higgs Boson **enhance the azimuthal correlation**.

Increasing Rapidity Span $\rightarrow$ Increasing Number of Jets

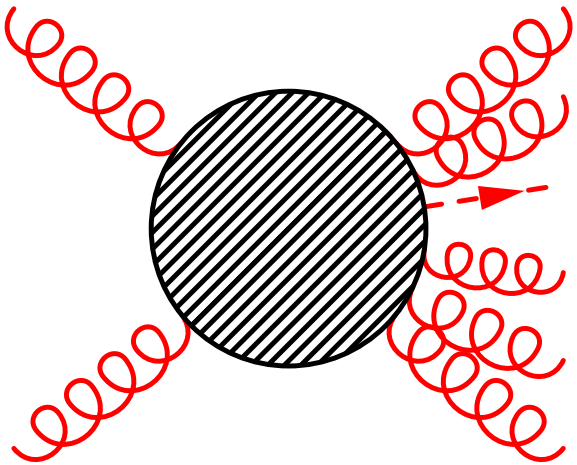


All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

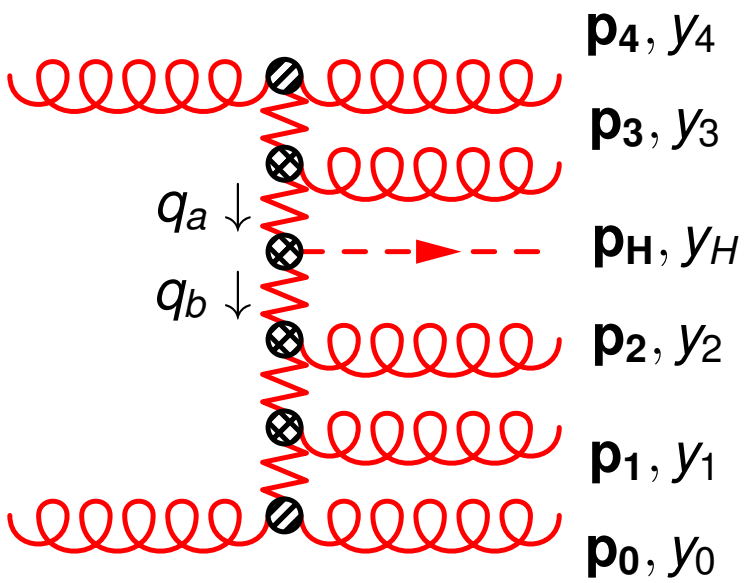
Develop Insight Into the Perturbative Corrections



High Energy Limit

$$\longrightarrow$$

$$|p_{\perp,i}| \text{ fixed}, \ \hat{s}_{ij} \to \infty$$

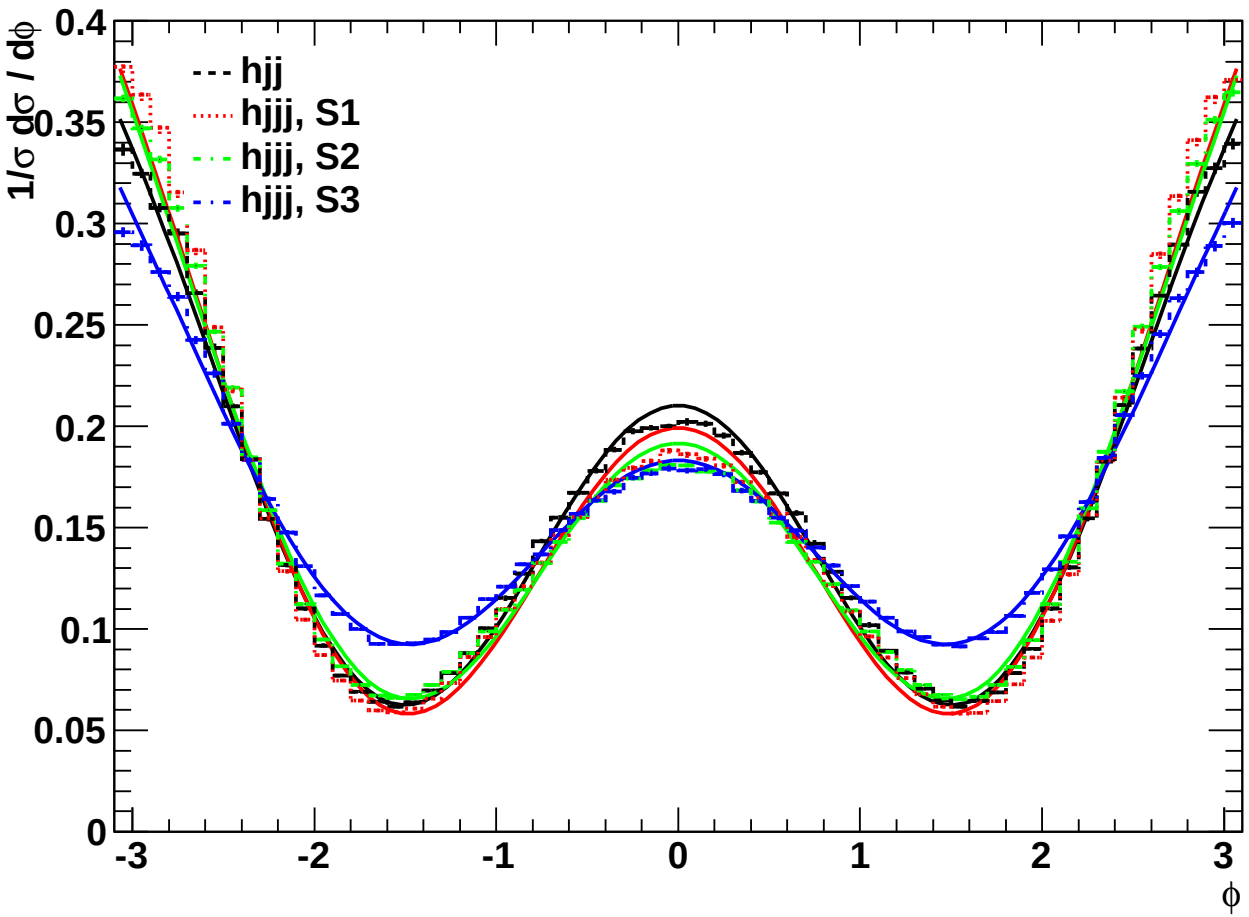


$$\left|\mathcal{M}_{gg\to g\cdots ghg\cdots g}\right|^2 \to \frac{4\hat{s}^2}{N_C^2-1}\left(\prod_{i=1}^{j}\frac{C_A\, g_s^2}{\mathbf{p}_{i\perp}^2}\right)\frac{|C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp})|^2}{\mathbf{q}_{a\perp}^2\,\mathbf{q}_{b\perp}^2}\left(\prod_{i=j+1}^{n}\frac{C_A\, g_s^2}{\mathbf{p}_{i\perp}^2}\right)$$

$$C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp}) \;=\; -i\,\frac{\alpha_s}{3\pi v}\,\mathbf{q}_{a\perp}\cdot\mathbf{q}_{b\perp}, \quad y_0 < \cdots < y_j < y_H < y_{j+1} < y_n$$

The **High Energy Limit** tells us to investigate the **azimuthal angle** between the **sum of the jet vectors** either side in rapidity of the Higgs Boson!

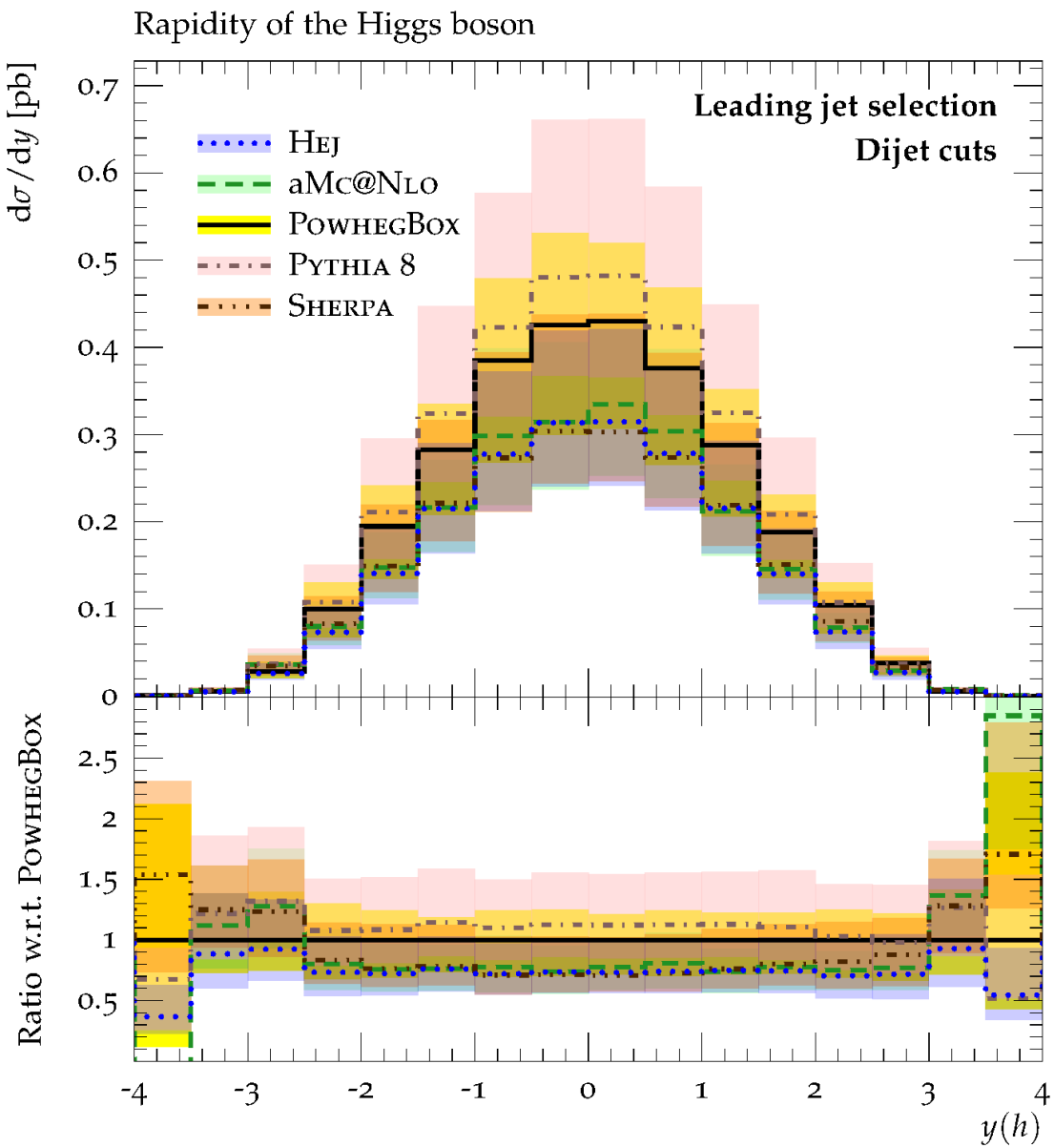
And It Even Works!



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

Three subsamples of tree-level three-jet events: two jets on same side of the Higgs boson parallel (S1), perpendicular (S2) or anti-parallel (S3). Azimuthal correlation almost unchanged from hjj.

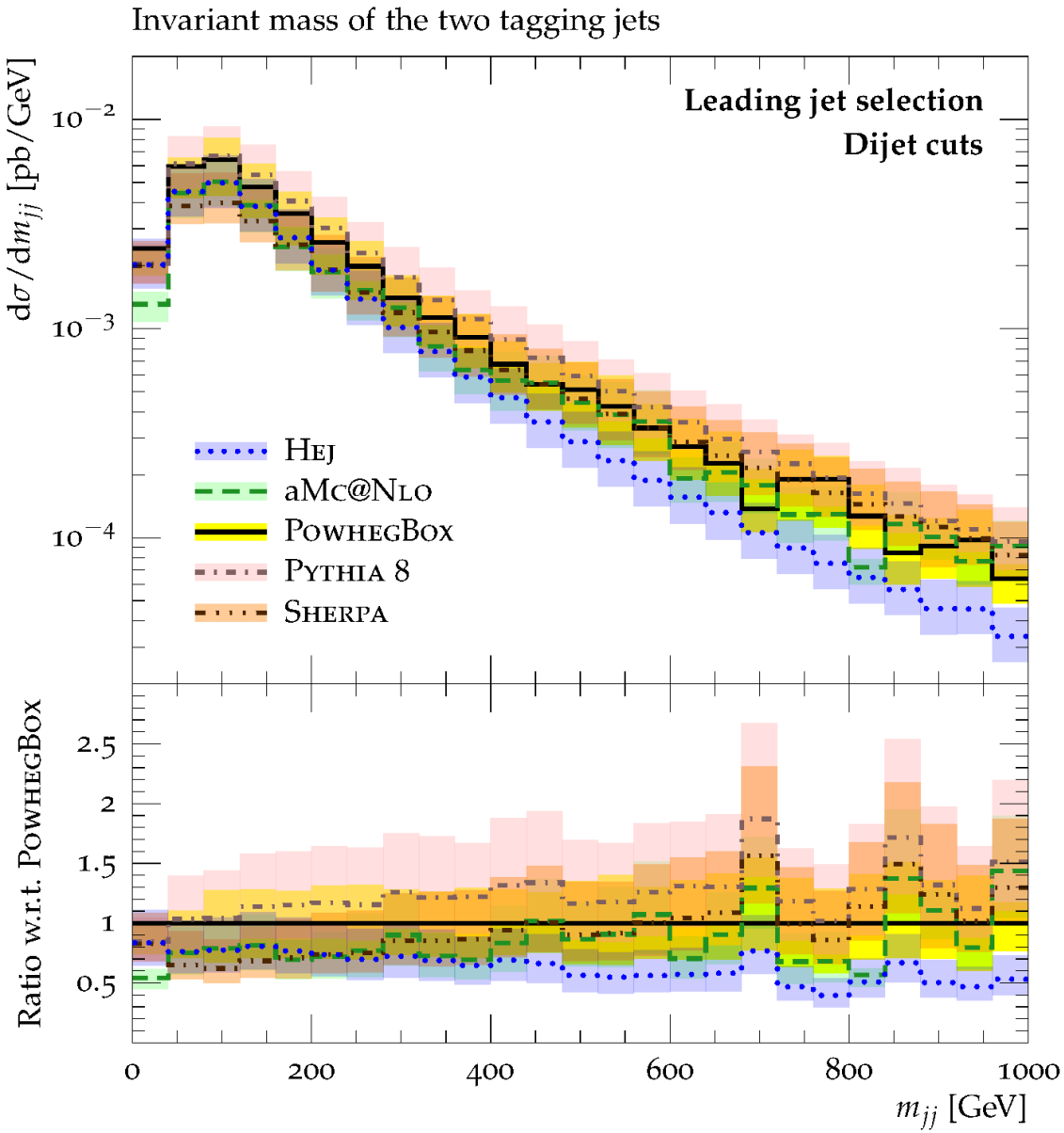
Les Houches Comparison of HJJ Predictions



Good agreement of inclusive *Hjj*-cross section and differential distributions (e.g. rapidity of the Higgs boson).

Variations within the uncertainty quoted for each calculation.

Les Houches Comparison of HJJ Predictions

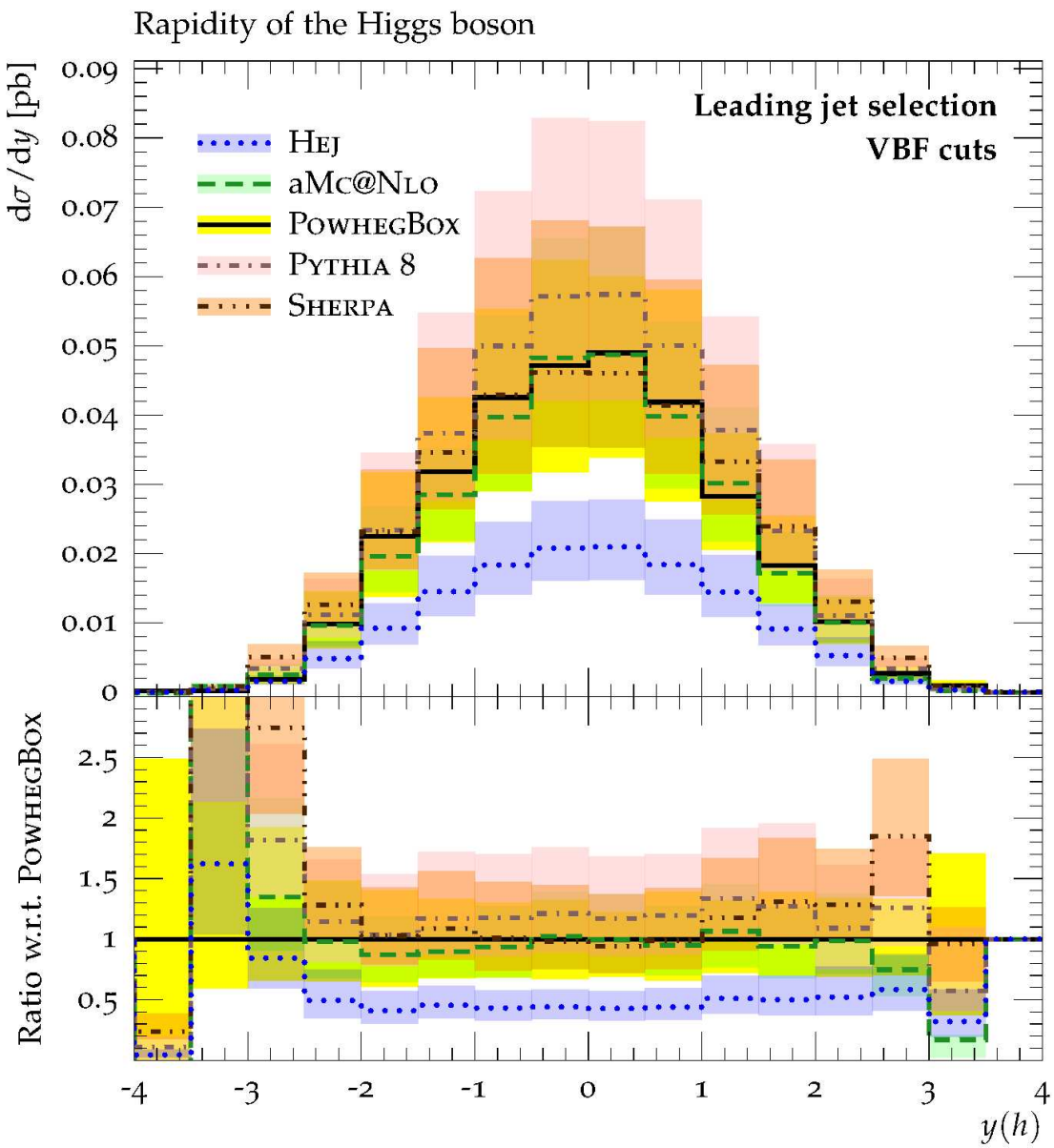


Differences arising at large invariant mass between the hard jets.
(as expected)

Vector-Boson-Fusion cuts select region of large m_{jj}.

(Please recall that HEJ gives a good description of WJJ at large invariant mass).

Les Houches Comparison of HJJ Predictions



The difference in the distribution of m_{jj} (and Δy_{12}) induce a difference in the cross section after VBF-cuts.

The difference in behaviour between shower-approaches and HEJ appear at large rapidities and large m_{jj} - where HEJ resums virtual corrections that are not treated systematically in any of the other approaches.

Conclusions

- Hadron colliders probes hard (=jets) perturbative corrections beyond pure NLO . . . already at 2, 7TeV!
- ***High Energy Jets**** provides a new approach to the perturbative description of proton collider physics
  . . . and compares favourably to data in several analyses
  . . . several ongoing improvements in the formal accuracy of the perturbative approximations

* http://cern.ch/hej