String Phenomenology 2003
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On string gas cosmology at finite temperature

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based on:

"Aspects of string gas cosmology at finite temperature",
B. Bassett, M. Borunda, M. Serone and S. Tsujikawa
Phys. Rev. D **67** (2003) 123506, hep-th/0301180

String cosmology

- ✔ An alternative to answer the basic questions of cosmology
- ✔ At the same time it offers a possibility to confront string theory with physics

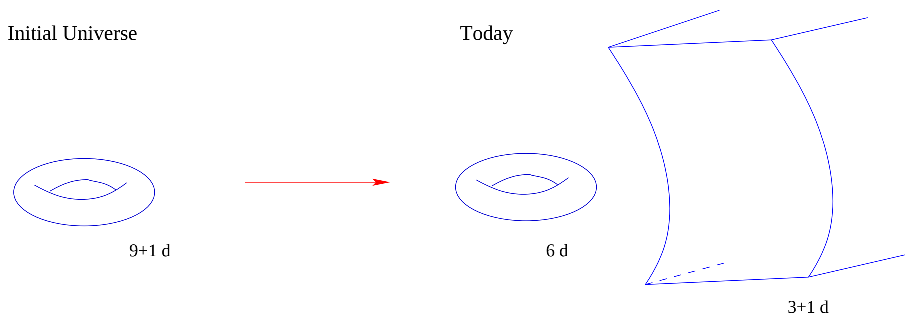
➠ Lots and lots of work

- ✧ Brandenberger & Vafa (1989)
- ✧ Pre-Big Bang (Veneziano & Gasperini, 1993)
- ✧ Ekpyrotic universe (Khoury, Ovrut, Steinhardt & Turok, 2001)
- ✧ Brane Cosmology
- •
- •
- •

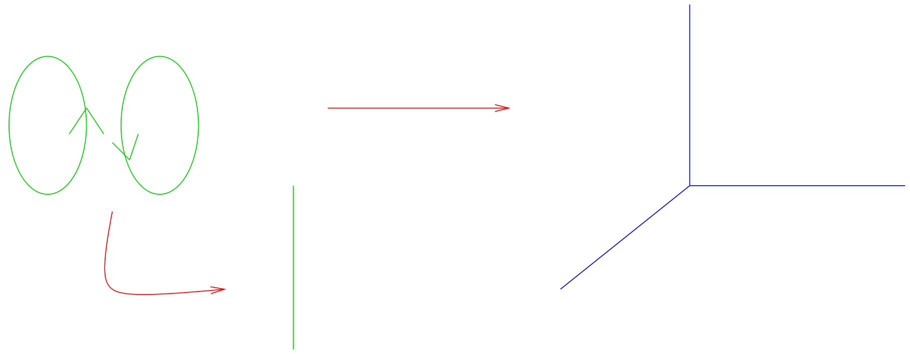
☞ Why today's observable universe is 3+1 d?

Perhaps the only proposal so far is

Brandenberger & Vafa model (1989)



• In non-thermal equilibrium annihilation of winding & antiwinding takes place in $3+1d$



whereas in $6d$

Motivations

✔ Inspired by B&V model

✔ Study, within a thermodynamical treatment in our toy model, the fate of the "small" and "large" dimensions.

Outline

➥ Model

Dilaton-gravity system

➥ A note on string thermodynamics

➥ Analysis

$$\xrightarrow{\hspace{6cm}} t$$

high T low T

- Hagedorn regime (high T)

  1 scale factor

  ⋆ Microcanonical string ensemble

  ⋆ Dynamics of the scale factor

- Almost radiation regime (low T)

  2 scale factors

  ⋆ Canonical string ensemble

  ⋆ Dynamics of the scale factors

➥ Conclusions

Model

Start

- Toy model: type IIA/B closed string theory on a T^9 torus

Assumptions

- Weak string coupling $\rightarrow$ ideal string gas.
- Thermodynamical equilibrium.
- Adiabatic evolution $\rightarrow$ constant entropy.

Dilaton-gravity

$$S = \int d^{10}x\sqrt{-g}[e^{-2\phi}\{R + 4(\nabla\phi)^2\} + \mathcal{L}_M],$$

where $\mathcal{L}_M$ is the contribution from the string gas and

$$\phi = \phi(t).$$

$$ds^2 = -dt^2 + \sum_{i=1}^{9} R_i^2(t)dx_i^2,$$

with scale factors such that $R_i = e^{\lambda_i(t)}$

☞ Introduce the shifted dilaton: $\psi = 2\phi - \sum_{i=1}^{9} \lambda_i$

Equations of motion

$$- \sum_{i=1}^{9} \dot{\lambda}_i^2 + \dot{\psi}^2 = e^{\psi} E \,,$$

$$\ddot{\lambda}_i - \dot{\psi}\dot{\lambda}_i = \frac{1}{2} e^{\psi} P_i \,,$$

$$\ddot{\psi} - \sum_{i=1}^{9} \dot{\lambda}_i^2 = \frac{1}{2} e^{\psi} E \,,$$

Tseytlin & Vafa (1992)

where

$$E = -2 \frac{\delta S_m}{\delta g_{00}} F + \beta \frac{\partial F}{\partial \beta} \,,$$

$$P_i = - \frac{\delta S_m}{\delta \lambda_i} = - \frac{\partial F}{\partial \lambda_i}$$

and F is the free energy of the string gas.

Duality symmetry $\quad R \rightarrow 1/R$

$$\begin{array}{ccc} \lambda_i & \rightarrow & -\lambda_i \\ \phi & \rightarrow & \phi - \sum_i \lambda_i \end{array}$$

NON SINGULAR COSMOLOGY

✢ Adiabaticity condition

$$\frac{d}{dt} S = \frac{d}{dt} \left(\beta^2 \frac{\partial F}{\partial \beta} \right) = 0.$$

A note on string thermodynamics

In analogy to field theory the free energy of a thermal gas of strings

➲ world-sheet path integrals for string propagation on $R^{d-1} \times S$ $\Rightarrow$ euclidean time

➥ circunference $\beta = 1/T$

Atick & Witten (1988)

✧ periodic bosons ✧ antiperiodic fermions

➥ $x^0 \to x^0 + \beta$ & 2π spatial rotation

- $$X^0(\sigma,\tau) = X^0 + \frac{2\pi m\tau}{\beta} + \frac{n\beta\sigma}{\pi} + \cdots$$

$$\left\{ \begin{array}{rcl} \frac{1}{4}M^2 & = & N + \frac{1}{2}\left(\frac{m\pi}{\beta} + \frac{n\beta}{2\pi}\right)^2 + \tilde{N} + \frac{1}{2}\left(\frac{m\pi}{\beta} - \frac{n\beta}{2\pi}\right)^2 - 1 \\ N & = & \tilde{N} + mn \end{array} \right.$$

For $N = \tilde{N} = 0, m = 0, n = 1$

$$M^2 = -4 + \frac{\beta^2}{\pi^2} = -4 + \frac{1}{\pi^2 T^2}$$

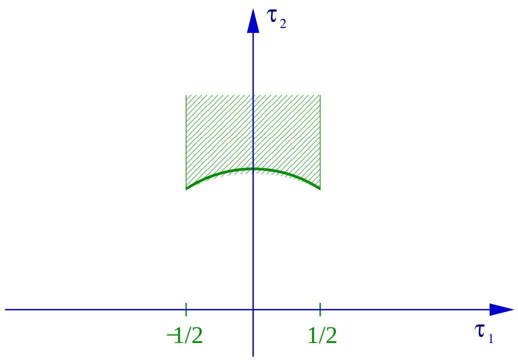
$$\boxed{T_H = \frac{1}{2\pi\sqrt{2}}}$$ Hagedorn Temperature

Free energy

$$F\beta = -\ln Z\,; \qquad\qquad Z = Tr\, e^{-\beta H}.$$

$$\frac{F}{VT} = \frac{\beta}{4}\left(\frac{1}{4\pi^2}\right)^5 \int \frac{d^2\tau}{\tau_2^6} \sum_{(n,m,\mu,\nu)} I_\mu(\tau)\bar{I}_\nu(\tau)\, e^{-\frac{\beta^2}{4\pi\tau_2}|m-n\tau|^2}$$

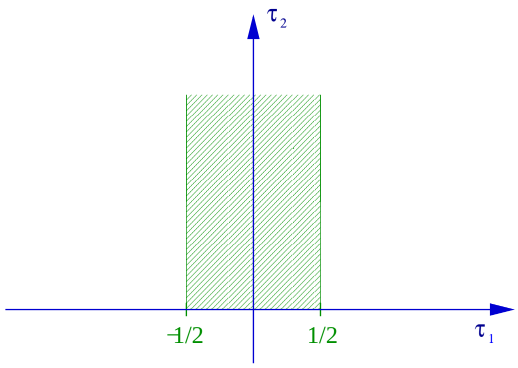
where $I_\mu(\tau) = \left(\frac{1}{\eta(\tau)}\right)^8 \left(\frac{\theta_\mu(0,\tau)}{\eta(\tau)}\right)^4$.



Unfolding technique

O'Brien & Tan (1987)

$$\frac{F}{VT} = \frac{\beta}{4}\left(\frac{1}{4\pi^2}\right)^5 \int_{-1/2}^{1/2} \tau_1 \int_0^\infty \frac{d\tau_2}{\tau_2^6} \sum_{p=1}^{\infty} \left(\frac{\theta_2^4(\tau)}{\eta^{12}(\tau)}\right)^2 e^{-\frac{\beta^2 p^2}{4\pi\tau_2}}$$



Consider $(9-d)$ compact spatial dimensions $\to r_i$
the free energy $F = -\beta^{-1} \ln Z$, is $\qquad (Z = \mathrm{Tr} e^{-\beta H})$

$$F(r_i, \beta) = \int_{1/2}^{1/2} d\tau_1 \int_0^\infty d\tau_2 \frac{1}{\tau_2^{(3+d)/2}} \prod_{i=d+1}^{9} \Lambda(r_i, \tau) \sum_{p=1}^{\infty} |M^2(\tau)| \, e^{-\frac{\beta^2 p^2}{4\pi\tau_2}} ,$$

where

$$\Lambda(r_i, \tau) = \sum_{m,n=-\infty}^{\infty} q^{\frac{1}{4}\left(\frac{m}{r_i}+nr_i\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{m}{r_i}-nr_i\right)^2} ,$$

$$|M^2(\tau)| = \left(\frac{\theta_2^4(\tau)}{\eta^{12}(\tau)}\right)^2 = \sum_{N,\bar{N}=0}^{\infty} D(N) D(\bar{N}) q^N \bar{q}^{\bar{N}} ;$$

$q = e^{2\pi i \tau}$ and $D(N)$ is the degeneracy factor at level N.

✦ Free energy diverges for $\beta < \beta_H$, $\quad \beta_H = 2\sqrt{2}\pi$

$\beta = \frac{1}{T}$

➥

- Canonical ensemble
- Microcanonical ensemble

☞ For weakly interacting strings the equivalence between *canonical ensemble* and *microcanonical ensemble* fails at high energy densities!

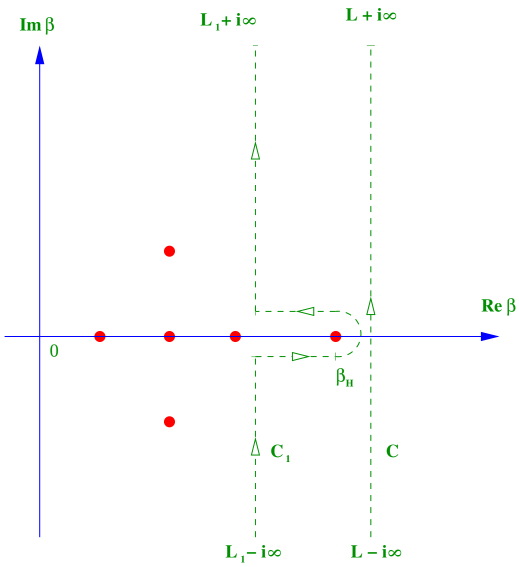
Hagedorn Regime

Microcanonical ensemble

The energy density of states $\Omega(E)$ is governed by the analytic structure of the canonical partition function, $Z = e^{-\beta F}$, in the complex β plane.

$$\Omega(E) = \sum_\alpha \delta(E - E_\alpha)$$

$$= \int_{L-i\infty}^{L+i\infty} \frac{d\beta}{2\pi i} e^{\beta E} Z(\beta)$$



Leading singularity at $\beta_H = 2\sqrt{2}\pi$,

next leading singularities at $\beta_W = 2\sqrt{2}\pi\sqrt{1 - \frac{\alpha'}{2r^2}}$,

$$\beta_K = 2\sqrt{2}\pi\sqrt{1 - \frac{r^2}{2\alpha'}}.$$

$$
\begin{aligned}
Z(\beta) &= \frac{e^{\Lambda(\beta,r)}}{\beta-\beta_H}\left(\frac{\eta_K}{\beta-\beta_K}\right)^{18}\left(\frac{\eta_W}{\beta-\beta_W}\right)^{18}, \\
S(E,V) &= \ln\Omega(E) \\
&\simeq \beta_H E + \log\left\{1 - \frac{1}{\Gamma(18)\eta_{KW}{}^{18}}\left[(\eta_K E)^{17}\eta_W{}^{18}e^{-\eta_K E}\right.\right. \\
&\quad \left.\left. + (\eta_W E)^{17}\eta_K{}^{18}e^{-\eta_W E}\right]\right\}
\end{aligned}
$$

where $\eta_K = \beta - \beta_K$, $\eta_W = \beta - \beta_W$ and $\eta_{KW} = \beta_K - \beta_W$.

Then, the thermodynamical quantities are

➥

$$
\begin{cases}
\frac{1}{T} = \frac{\partial S}{\partial E} \sim \frac{1}{T_H} + C_1 E^{17} e^{-\eta E} \sim \frac{1}{T_H} \\
P = T\frac{\partial S}{\partial V} \sim C_2 E^{17} e^{-\eta E} \sim 0
\end{cases}
$$

for $r \sim 1$ where $\eta \approx \sqrt{2}\pi[2 - \sqrt{4 - \frac{2}{r^2}}]$

➡ Pressureless dust

Conservation laws

To consider the total winding number n_i and the discrete momenta m_i in each compact spatial dimension as conserved quantities

➥ chemical potential μ for each conserved charge Q.

$$\mu Q \to \mu_i m_i + \nu_i n_i$$

such that

$$\begin{aligned} Z(\beta,\mu) &= \mathrm{Tr}\, \mathrm{e}^{-\beta \mathrm{H} + 2\pi \mathrm{i} \mu \mathrm{Q}} \\ \beta_H(\mu,\nu) &\sim \beta_0 - \sqrt{\tfrac{1}{2}}\pi \textstyle\sum_i \left(\frac{{\mu_i}^2}{r_i^2} + {\nu_i}^2 r_i^2 \right) \end{aligned}$$

Imposing $Q = c$,

$$Z(\beta, c) = \int_{-1/2}^{1/2} d\mu e^{-2\pi i \mu c}$$

at leading order

$$S(E) = 2\sqrt{2}\pi E - \frac{1}{2\sqrt{2}\pi E} \sum_{i=1}^{9} \left({n_i}^2 r^2 + \frac{{m_i}^2}{r^2} \right) - 9 \ln E$$

➥

$$\begin{cases} \frac{1}{T} = \frac{1}{T_0} - \frac{9}{E} + \frac{1}{2\sqrt{2}\pi} \frac{1}{E^2} \sum_{i=1}^{9} \left({n_i}^2 r^2 + \frac{{m_i}^2}{r^2} \right) \\ P = \frac{T}{9} \frac{1}{\sqrt{2}\pi} \frac{1}{E} \sum_{i=1}^{9} \left(-{n_i}^2 r^2 + \frac{{m_i}^2}{r^2} \right) \end{cases}$$

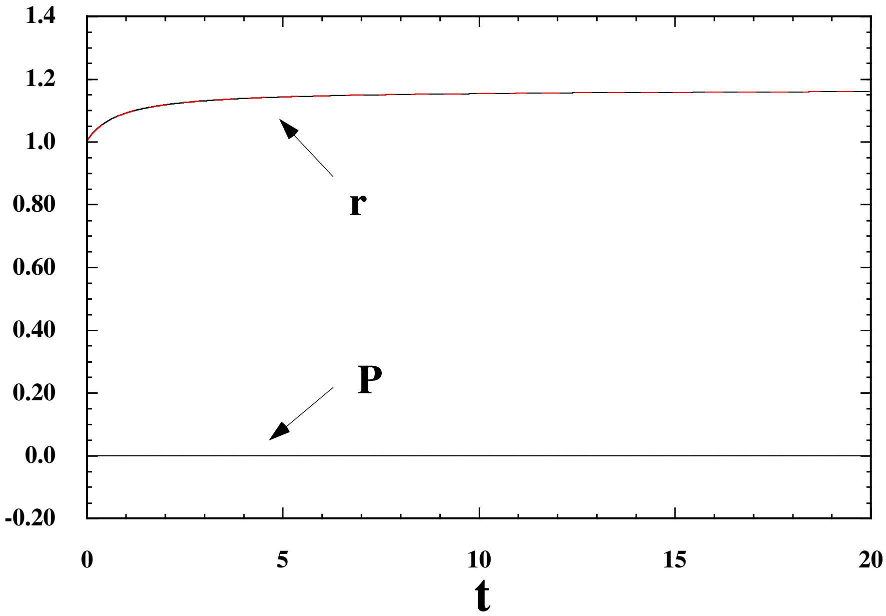
At high energies ➡ Pressureless dust

Dynamics in the Hagedorn Regime

Consider all dimensions start off equal radius r.

$$-9\dot{\lambda}^2 + \dot{\psi}^2 = e^{\psi} E \,,$$
$$\ddot{\lambda} - \dot{\psi}\dot{\lambda} = \tfrac{1}{2} e^{\psi} P \,,$$
$$\ddot{\psi} - 9\dot{\lambda}^2 = \tfrac{1}{2} e^{\psi} E$$

Pressureless dust



TIME EVOLUTION OF ALL 9 DIMENSIONS OF RADIUS r.
$\lambda_0 = 10^{-3}$, $\dot{\lambda}_0 = 0.2$, $\psi_0 = -5$, $E_0 = 10^3$

$$r_\infty = e^{\lambda_0} \left| \frac{B + \sqrt{d}A}{B - \sqrt{d}A} \right|$$

Almost Radiation Regime

Canonical ensemble $\rightarrow$ canonical partition function

$$
\begin{aligned}
Z(\beta) &= \sum_\alpha e^{-\beta E_\alpha}\,, \\
F &= F^{rad} + F^{mat}\,,
\end{aligned}
$$

with

$$
\begin{aligned}
F^{rad} &\propto \beta^{-d-1}\,, \qquad (N = \bar{N} = m_i = n_i = 0) \\
F^{mat} &\propto -\sum_{m_i,n_i,N,\bar{N}} \left(\frac{M}{\beta}\right)^{(d+1)/2} D(N)D(\bar{N})\,K_{\frac{d+1}{2}}(\beta M)\,,
\end{aligned}
$$

where $K_{\frac{d+1}{2}}$ are the modified Bessel functions, and

$$
M = M(m_i, n_i, r_i, N, \bar{N}) \sim \sqrt{\sum_{i=1}^{9-d}\left(\frac{m_i^2}{r_i^2} + n_i^2 r_i^2\right) + 2(N + \bar{N})}\,.
$$

The energy and pressure are then computed by

$$
\begin{aligned}
E &= F + \beta\frac{\partial F}{\partial \beta}\,, \\
P &= -\frac{\partial F}{\partial \ln r}\,.
\end{aligned}
$$

➥ pure radiation

$$\begin{aligned} E^{rad} &= \beta^{-d-1}\,, \\ P_d^{rad} &= \frac{E^{rad}}{d}\,, \\ P_{9-d}^{rad} &= 0\,. \end{aligned}$$

Radiation-like equation of state for the big dimensions whereas zero pressure for the small ones.

➥ massive $(N=\bar{N}=0, m_i=(1,0,\ldots,0), n_i=0, m_i\to n_i)$

$$\begin{aligned} E^{mat} &\sim \frac{f(\beta)}{r^{(d+1)/2}}\Big[K_{(d+1)/2}\left(\frac{\beta}{r}\right)+\frac{\beta}{r}K'_{(d+1)/2}\left(\frac{\beta}{r}\right) \\ &\quad +r^{d+1}\left\{K_{(d+1)/2}\left(\beta r\right)+\beta r K'_{(d+1)/2}\left(\beta r\right)\right\}\Big]\,, \\ P_d^{mat} &= \frac{f(\beta)}{r^{(d+1)/2}}\left[K_{(d+1)/2}\left(\frac{\beta}{r}\right)+r^{d+1}K_{(d+1)/2}\left(\beta r\right)\right], \\ P_{9-d}^{mat} &= \frac{g(\beta)}{r^{(d+3)/2}}\left[K_{(d-1)/2}\left(\frac{\beta}{r}\right)-r^{d+3}K_{(d-1)/2}\left(\beta r\right)\right]. \end{aligned}$$

Winding modes leads to a *positive/negative* contribution to *large/small* dimensions.

✧ Conservation laws winding & KK numbers → same behaviour.

Dynamics in the almost Radiation Regime

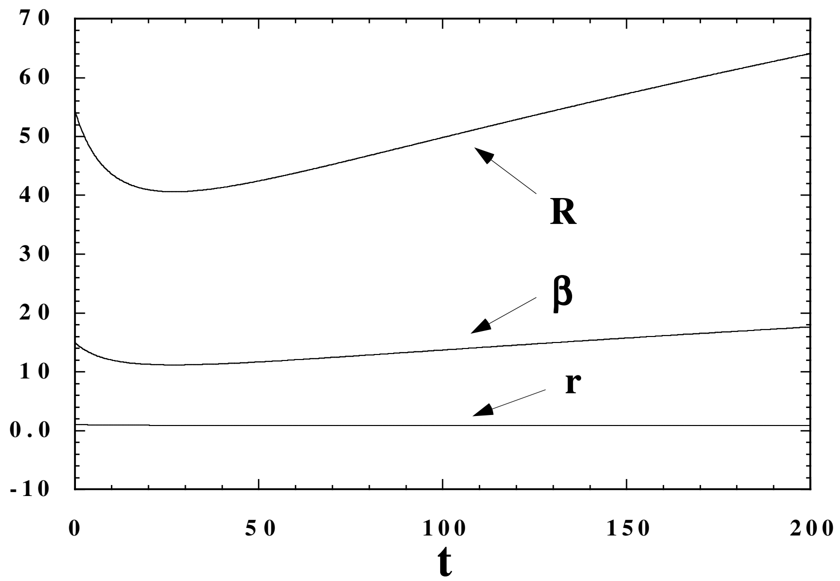
Assume d large dimensions $R = e^{\mu}$,

and $9-d$ small dimensions $r = e^{\nu}$

$$\begin{aligned}
-d\dot{\mu}^2 - (9-d)\dot{\nu}^2 + \dot{\psi}^2 &= e^{\psi} E\,, \\
\ddot{\mu} - \dot{\psi}\dot{\mu} &= \tfrac{1}{2} e^{\psi} P_d\,, \\
\ddot{\nu} - \dot{\psi}\dot{\nu} &= \tfrac{1}{2} e^{\psi} P_{9-d}\,, \\
\ddot{\psi} - d\dot{\mu}^2 - (9-d)\dot{\nu}^2 &= \tfrac{1}{2} e^{\psi} E\,.
\end{aligned}$$

Pure radiation $(N = \bar{N} = m_i = n_i = 0)$

$$E = E^{rad} \sim \beta^{-d-1}, \qquad P_d = P_d^{rad} = E^{rad}/d, \qquad P_{9-d} = P_{9-d}^{rad} = 0.$$

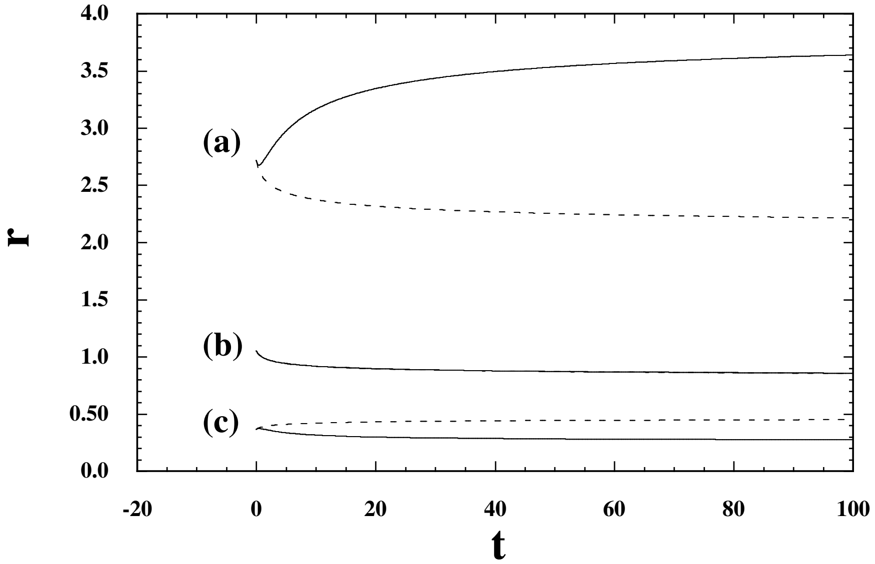


TIME EVOLUTION OF THE "LARGE", R, AND "SMALL", r, DIMENSIONS.
$\mu_0 = 4,\ \nu_0 = 0,\ \dot{\mu}_0 = -0.04,\ \dot{\nu}_0 = -0.01,\ \psi_0 = -16,\ \beta_0 = 15$

Asymptotic behaviour $R \propto t^{2/(d+1)}$

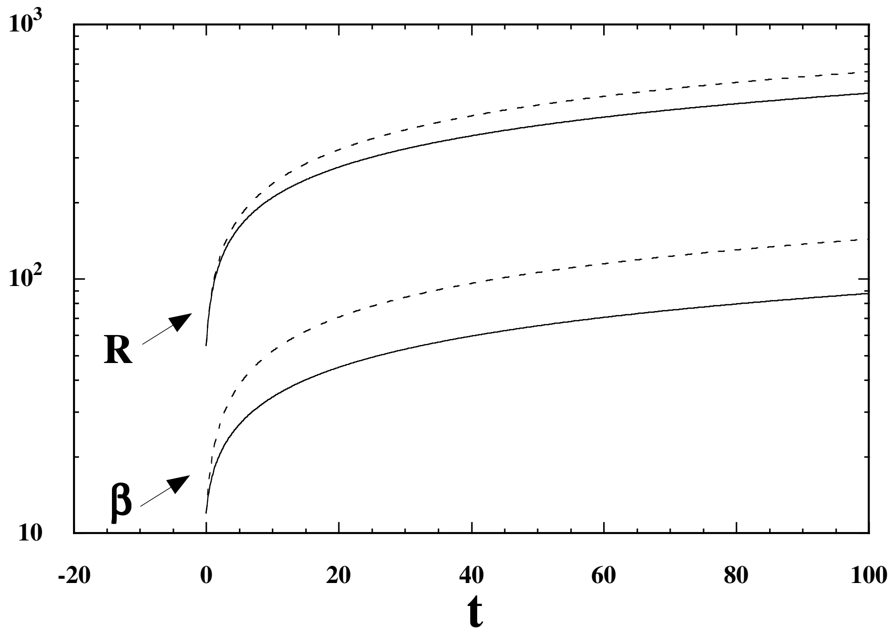
Inclusion of matter

$$E = E^{rad} + E^{mat}, \qquad P_d = P_d^{rad} + P_d^{mat}, \qquad P_{9-d} = P_{9-d}^{rad} + P_{9-d}^{mat}$$



Time evolution of the "small", r, dimensions. a) $\nu_0 = 1$, $\dot{\nu}_0 = -0.1$, b) $\nu_0 = 0.05$, $\dot{\nu}_0 = -0.1$, c) $\nu_0 = -1.0$, $\dot{\nu}_0 = 0.1$, $\psi_0 = -16$, $\beta_0 = 12$

Dashed line: massless. Solid line: massless + massive



Time evolution of the "large", R, dimensions. a) $\mu_0 = 4$, $\dot{\mu}_0 = 1.0$, $\psi_0 = -16$, $\beta_0 = 12$

Dashed line: massless. Solid line: massless + massive

Conclusions

☞ Thermodynamical treatment of dilaton-gravity equations of motion, satisfying $R \to 1/R$ duality under an adiabatic evolution.

The universe, initially homogeneous, isotropic and in thermal equilibrium is filled with an ideal gas of closed strings.

- In the Hagedorn regime all 9 radius equal, exhibit a very slow time evolution with nearly constant energy and negligible pressure $\Rightarrow$ out of thermal equilibrium to get interesting dynamics, in accordance with Brandenberger & Vafa model.

- In the Almost radiation regime there are solutions where the large dimensions expand whereas the small are kept small.

$\rightarrow$ Same behaviour independent of d dimensions.

Things to do

- Out of equilibrium.
- Inflation.
- Dilaton stabilization.
- Chiral theories $\rightarrow$ more general topologies.